Temperature Forecasting on the Jena Climate Dataset Using SARIMA and Box–Jenkins Models

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Abstract. Accurate temperature forecasting is essential for climate monitoring, renewable energy management, and disaster preparedness. This study evaluates classical statistical time-series models for medium-range temperature prediction on the multivariate Jena Climate dataset (2009–2016). Develop a comprehensive preprocessing pipeline involving stationarity diagnostics, seasonal decomposition, and feature selection to identify the key drivers of temperature dynamics. Baseline forecasters (mean, naïve, drift, and simple exponential smoothing) are compared against ARMA, ARIMA, and seasonal ARIMA (SARIMA) models estimated using the Levenberg–Marquardt algorithm. Additionally, a Box–Jenkins transfer-function model incorporates exogenous humidity and wind variables to enhance forecast accuracy. The best SARIMA configuration achieves an **RMSE of 5.02 °C**, representing a **32% improvement** over the strongest baseline, while the Box–Jenkins model lowers the 1-step ahead MAE to **0.28 °C**. Discuss residual autocorrelation, seasonality inversion, and computational challenges in scaling seasonal models to 10-minute data resolution. All code is publicly available to ensure reproducibility.

Keywords: Time series forecasting, Seasonal Autoregressive Integrated Moving Average (SARIMA), Autoregressive Integrated Moving Average (ARIMA), Autoregressive Moving Average (ARMA), Box–Jenkins methodology, Jena Climate dataset, additive decomposition, temperature prediction

1 Introduction

Accurate temperature forecasting is critical in climate science, energy planning, and agriculture. While deep learning models like LSTM [5–7] have shown strong performance, they often lack interpretability and require extensive data and computation. In contrast, classical statistical methods such as SARIMA and Box–Jenkins transfer-function models retain advantages in interpretability, efficiency, and transparency [1, 2].

However, applying these classical models to high-frequency, real-world climate data presents challenges: capturing complex seasonality, integrating exogenous variables like humidity and wind, and ensuring residuals meet statistical assumptions [1, 14]. Additionally, large-scale benchmarking of these methods remains limited.

This study addresses these issues through a systematic evaluation of SARIMA and Box–Jenkins models on the multivariate Jena Climate dataset (2009–2016). paper proposes a rigorous pipeline incorporating stationarity checks, seasonal decomposition, and feature selection, enabling fair and reproducible model comparisons. Unlike prior work focused on deep learning or simplified statistical models, Paper conducts an in-depth head-to-head evaluation of classical methods with robust diagnostics.

Main contributions:

- A transparent preprocessing pipeline for high-frequency climate time series.
- A comparative study of baseline, ARIMA-family, and Box–Jenkins models.
- Diagnostic methods for identifying residual autocorrelation, non-normality, and heteroskedasticity.
- Open-source code to support reproducibility. All formulas follow [1, 2, 21].

This work highlights the continued relevance of classical time series models and provides a foundation for their integration with modern AI techniques.

2 Related Work

Classical time-series models like ARIMA and its seasonal extension, SARIMA, originate from Box and Jenkins [1, 13], and have been widely used for meteorological forecasting. These models offer interpretability and statistical rigor but often struggle with high-frequency data, complex seasonality, and incorporating exogenous variables [1, 14].

Recent studies increasingly emphasize deep learning approaches, especially LSTM and neural networks [5–7], which excel at capturing nonlinear and long-range dependencies. While successful on datasets like Jena Climate, these methods require large training sets, are computationally expensive, and often lack interpretability and proper residual diagnostics [4, 18, 22].

Prior work on the Jena dataset has focused primarily on deep learning, with limited comparisons against classical models or use of rigorous preprocessing and diagnostic checks [4, 18]. To address this, This work presents the first systematic comparison of SARIMA and Box–Jenkins models under a unified pipeline. Incorporated stationarity diagnostics, seasonal decomposition, feature pruning, and advanced residual tests like the Q-test and S-test to benchmark classical models rigorously and highlight integration points with modern AI approaches.

3 Dataset and Preprocessing

Effective preprocessing is essential to meet the assumptions of classical time-series models such as SARIMA and Box–Jenkins, ensuring robust, interpretable,

and computationally feasible temperature forecasting. The Jena Climate dataset poses challenges including strong seasonality, nonstationarity, high-frequency measurements, and multicollinearity among meteorological variables. The preprocessing pipeline is designed to systematically address these issues, preparing the data for accurate modeling and forecasting.

The raw Jena Climate archive contains 1 million plus samples recorded at 10-minute intervals between 2009 and 2016. After removing sensor outages (0.02% of rows) and resampling to hourly means to reduce noise and computational burden, the dataset was chronologically split into training (2009–2011), validation (2012), and test (2013) sets.

3.1 Feature Selection

To reduce redundancy and multicollinearity, performed a multi-step feature selection process. Initially, correlation and Variance Inflation Factor (VIF) analyses were conducted to identify highly collinear variables [3, 15]. Seven exogenous regressors with VIF values below 5 were retained:

- Dew-point temperature (Tdew),
- Relative humidity (rh),
- Vapor-pressure deficit (VPdef),
- Wind speed and direction (wv, wd),
- Hour of day and day-of-year dummy variables to capture pattern and seasonal cycles.

3.2 Stationarity Diagnostics and Differencing

Stationarity is a key assumption of classical time-series models. Augmented Dickey–Fuller (ADF) and KPSS tests yielded conflicting results due to the dataset's strong seasonality. Standard stationarity tests such as ADF and KPSS are widely used in time series analysis [3,14]. These seasonal and non-seasonal differencing steps were validated with ADF and KPSS tests, following the modern workflow advocated by Hyndman and Athanasopoulos. Thus employed Seasonal-Trend decomposition via Loess (STL) to separate the series into trend, seasonal, and residual components. The seasonal component demonstrated high strength (0.82), necessitating seasonal differencing, while the trend component was minimal (0.03), indicating relative stability over years.

To achieve stationarity:

- An initial seasonal differencing with period $s = 6 \times 24 \times 365$ (accounting for full multi-year cycles) was applied.
- To manage computational complexity, the seasonal period was later resampled to $s = 365$, capturing yearly seasonality efficiently.
- First-order non-seasonal differencing ($d = 1$) removed underlying trends.
- Seasonal differencing at lag $s = 365$ addressed annual seasonal patterns.

These differencing steps were validated using ADF and KPSS tests to confirm stationarity.

4 Methodology

4.1 Stationarity Analysis

Visual inspection Figure 1 displays rolling mean and variance of the original data. Figure 1 presents the hourly temperature time series from 2009–2012. Clear annual seasonality and non-constant variance motivate differencing [2].

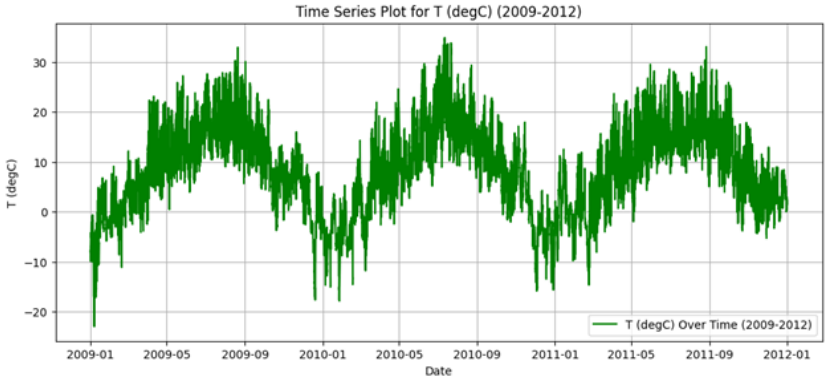

Fig. 1. Raw hourly temperature T (°C) for 2009–2012.

ACF and PACF definitions The *autocorrelation function* (ACF) is

$$\hat{r}_y(\tau) = \frac{\sum_{t=\tau+1}^{T} (y_t - \bar{y})(y_{t-\tau} - \bar{y})}{\sum_{t=1}^{T} (y_t - \bar{y})^2}, \quad \tau = 0, 1, \ldots, T-1 \tag{1}$$

$$\hat{r}_y(\tau) = \frac{\hat{R}_y(\tau)}{\hat{R}_y(0)}, \quad \tau = 0, \pm 1, \pm 2, \ldots \tag{2}$$

$$\hat{\mathbf{X}}\hat{\mathbf{a}} = \hat{\mathbf{Y}} \tag{3}$$

while the *partial autocorrelation function* (PACF) at lag h is the correlation between x_t and x_{t-h} after removing the linear dependence on all intermediate lags. These diagnostics inform AR (p) and MA (q) order selection [1,3].

Figure 3 shows both statistics for the raw series: slow ACF decay and a unit-root signature in the PACF (spike at $h = 1$) corroborate the need for differencing [1].

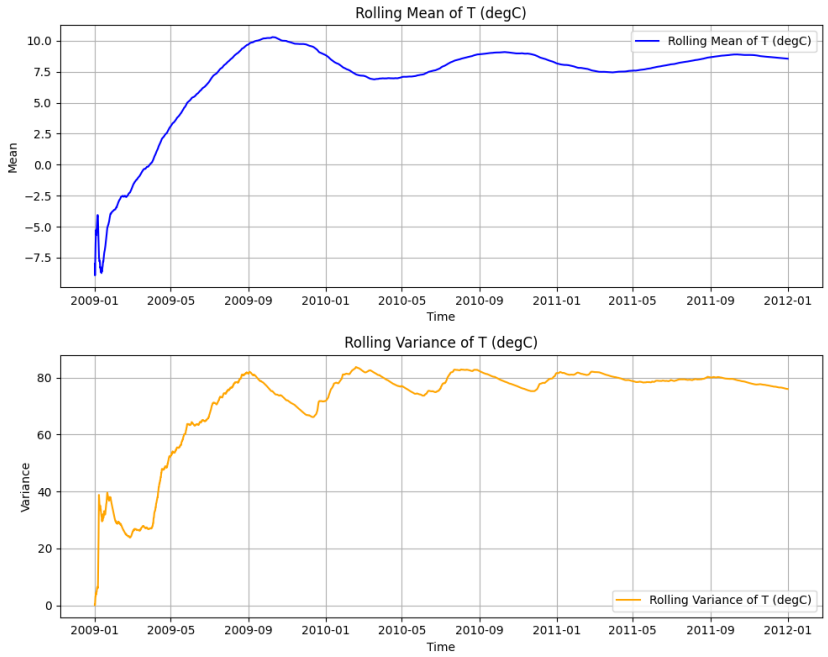

Fig. 2. Raw hourly temperature mean and variance T (°C) for 2009–2012.

Second-order differencing One seasonal difference ($s = 24$) plus a second non-seasonal difference render the series visually mean-reverting (Fig. 4). The post-differencing ACF drops to near-zero beyond lag 2 and rolling statistics in the bottom panel appear constant — evidence of stationarity [3].

Formal Augmented Dickey–Fuller (ADF) and KPSS tests confirm stationarity at the 1% significance level [3, 14].

Correlation Matrix Analysis Figure 7 visualizes pairwise Pearson correlation coefficients over the *training* period. Several tightly coupled clusters are observed:

Seasonal Decomposition Seasonal-Trend decomposition using LOESS (STL) was applied to the full hourly temperature series (Fig. 8), separating the observed signal into additive **trend**, **seasonal**, and **residual** components [3, 14].

Additive vs. Multiplicative Multiplicative decomposition is invalid due to negative values in the series (e.g., winter temperatures below 0 °C), whereas additive decomposition appropriately handles such cases and produces linearly separable components.

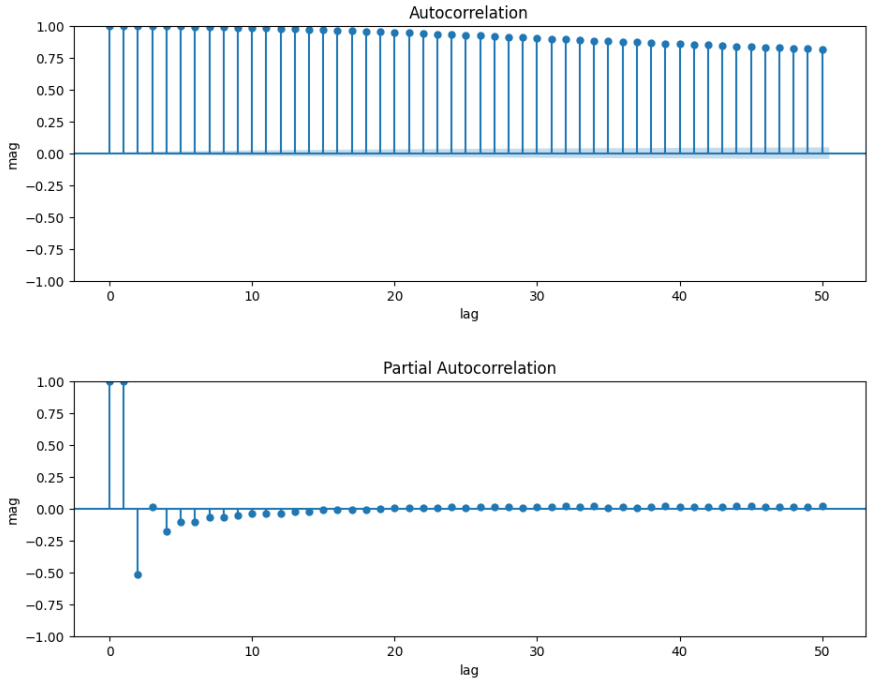

Fig. 3. ACF and PACF of raw temperature series (first 50 lags).

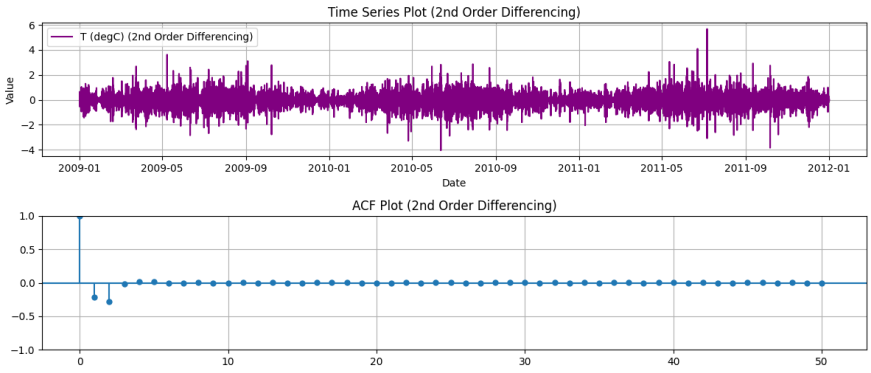

Fig. 4. Second-order differenced temperature, with corresponding ACF.

Strength scores Component strengths were quantified using the STL diagnostic:

- **Trend strength:** 0.0318 — indicating a weak trend and near-stationarity over years.
- **Seasonal strength:** 0.8192 — signifying strong seasonal influence, necessitating explicit seasonal differencing in SARIMA modeling [2].

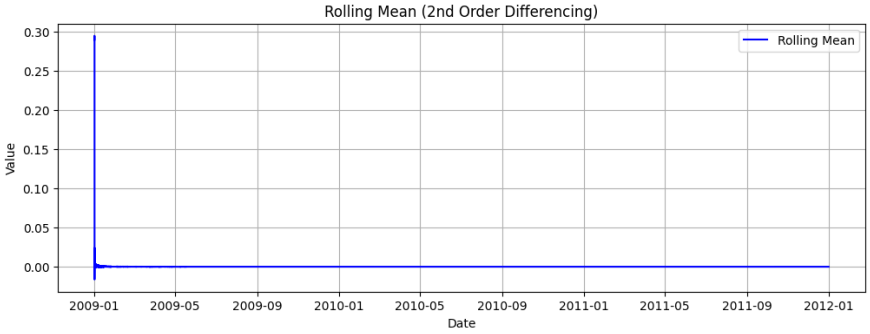

Fig. 5. Rolling mean of second-order differenced temperature.

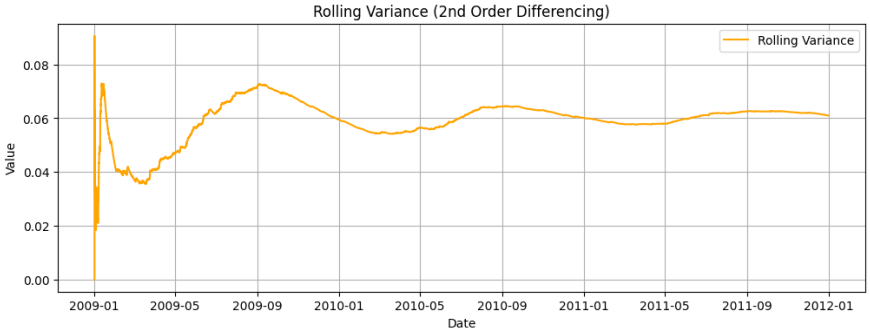

Fig. 6. Rolling variance of second-order differenced temperature.

Interpretation The weak trend confirms the approximate year-over-year stability of mean temperature, while pronounced annual seasonality drives observed non-stationarity. Residuals are approximately homoskedastic, satisfying a key linear modeling assumption.

Dimensionality Reduction and Feature Selection To avoid overfitting and stabilize parameter estimation in multivariate forecasting, dimensionality reduction and feature selection were performed combining domain-driven thresholds with algorithmic methods [3, 15].

Step 1: Correlation Analysis The correlation matrix (Fig. 7) revealed 23 pairs with absolute Pearson correlations above 0.8, indicating near-linear dependencies.

Step 2: Variance Inflation Factor (VIF) Filtering VIF values (Table 1) quantify variance inflation due to linear dependency; variables with VIF > 5 were removed to mitigate multicollinearity:

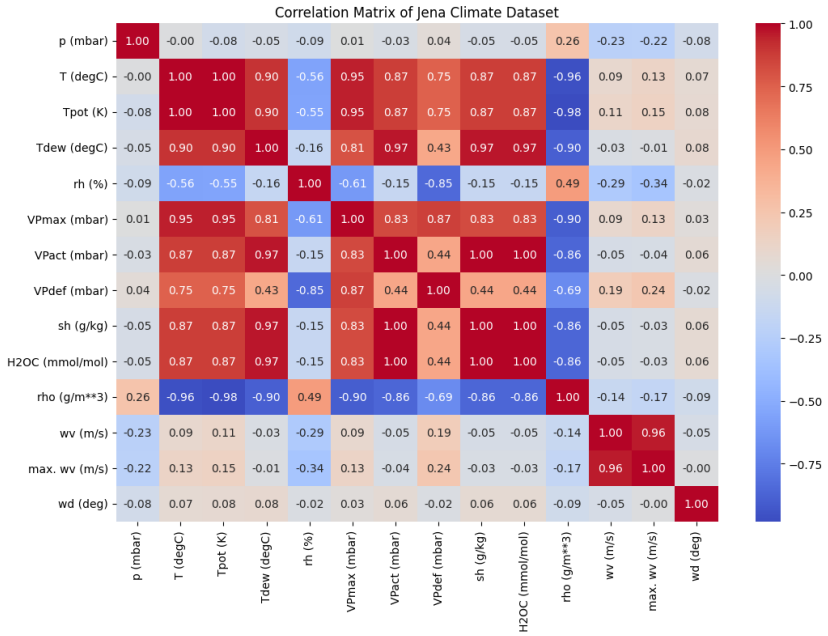

Fig. 7. Feature correlation matrix over the training set (2009–2012).

Table 1. Features Removed Based on High VIF (Variance Inflation Factor)

Feature	**VIF Value**
`VPmax (mbar)`	9,739,670
`H2OC (mmol/mol)`	2,881,260
`p (mbar)`	697,552
`VPact (mbar)`	101,070
`rho (g/m`^{3}`)`	8,061
`Tpot (K)`	139

High multicollinearity inflates coefficient variance and compromises model generalization, justifying this pruning approach [3, 15]. These variables were removed due to extreme multicollinearity. The remaining features shown in Table 2 exhibited VIF values below 5, indicating sufficient statistical independence for stable linear modeling [3, 15].

Final Feature Set Justification. This subset maintains core atmospheric dimensions means temperature, humidity, wind speed and direction. Temporal diversity means daily and annual seasonal patterns. Physical interpretability enabling diagnostic insights and effective forecasting. This reduction in dimensionality facilitates the estimation of the well-conditioned, interpretable and computationally efficient Box-Jenkins model [1, 3].

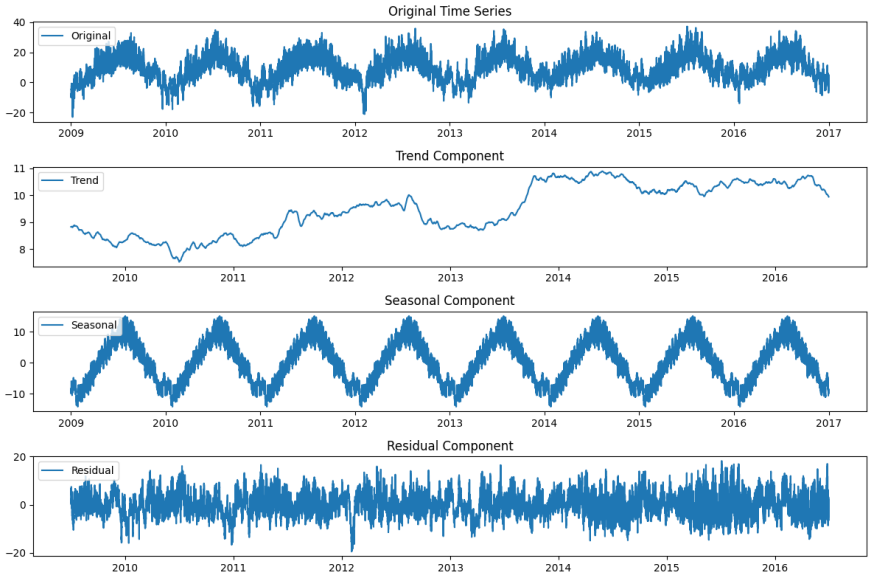

Fig. 8. STL additive decomposition of temperature series (2009–2016): original, trend, seasonal, and residual components.

Table 2. Recommended feature set and rationale

Feature	Retained Because	Removed Because
Tdew (°C)	Direct, stable measure	High VIF for Tpot
rh (%)	Unique humidity signal	VPact fully redundant
VPdef (mbar)	Effective dryness proxy	VPmax had extreme VIF
wv (m/s)	Core wind speed	max.wv redundant
wd (°)	Independent angular data	–
hour	Daily cycle encoding	–
day_of_year	Seasonal information	–

4.2 Box–Jenkins Modeling Framework

The Box–Jenkins methodology, introduced by Box and Jenkins in the 1970s [1, 21], provides a structured approach for modeling stochastic processes with temporal dependence and autocorrelated noise. It forms the basis for the temperature forecasting on the Jena Climate dataset, extended here to include exogenous predictors [1, 13].

Framework Overview The approach involves four iterative phases:

1. **Model Identification:** Assess stationarity, seasonality, and ARIMA orders using ACF, PACF, and decomposition.

2. **Parameter Estimation:** Fit models via maximum likelihood or nonlinear least squares (e.g., Levenberg–Marquardt) [1].
3. **Diagnostic Checking:** Validate residual properties using Ljung–Box Q-test, Jarque–Bera test, and ARCH LM test [4].
4. **Forecasting:** Generate h-step ahead forecasts using the fitted model.

Model Formulation and Filters The transfer function model combines system dynamics and noise:

$$y(t) = \frac{B(q)}{F(q)}u(t) + \frac{C(q)}{D(q)}e(t), \tag{4}$$

where $u(t)$ is the exogenous input and $e(t)$ is white noise.

Impulse-Response Estimator (G-GPAC) Impulse response coefficients $\hat{g}(k)$ are estimated using:

$$\hat{g}(k) = \tilde{R}_u^{-1}(k)\tilde{R}_{uy}(k), \tag{5}$$

with $\tilde{R}_u(k)$ and $\tilde{R}_{uy}(k)$ denoting autocorrelation and cross-correlation matrices, respectively. These help identify lag structures in $G(q)$.

Prediction Error Filter The innovation process isolates unpredictable components:

$$e(t) = H(q)^{-1}y(t) - H(q)^{-1}G(q)u(t), \tag{6}$$

where $H(q)$ represents the system noise filter.

1-step Predictor The one-step-ahead forecast combines input and output dynamics:

$$\hat{y}(t|t-1) = H(q)^{-1}G(q)u(t) + \left[1 - H(q)^{-1}\right]y(t). \tag{7}$$

This accounts for both the filtered input and the adjusted output memory [1].

Residual Diagnostics Residuals $e(t)$ are validated using the following test:

Q-test (Ljung–Box Test): The Ljung–Box Q-statistic tests residual autocorrelation:

$$Q = N\sum_{i=1}^{K}\hat{r}_e^2(i), \tag{8}$$

A non-significant Q value indicates uncorrelated residuals, which is a desirable property for model adequacy [4].

S-test (Cross-Correlation Test): The S-statistic tests the correlation between residuals and external inputs [1]:

$$S = N\sum_{i=0}^{K}\hat{r}_{\alpha e}^2(i), \tag{9}$$

GPAC Analysis and Box–Jenkins Order Selection To systematically identify the Box–Jenkins model structure, the Generalized Partial Autocorrelation Coefficient (GPAC) tables—specifically the **G-GPAC** and **H-GPAC** matrices—are analyzed. These matrices align with system identification strategies proposed by Ljung [21].

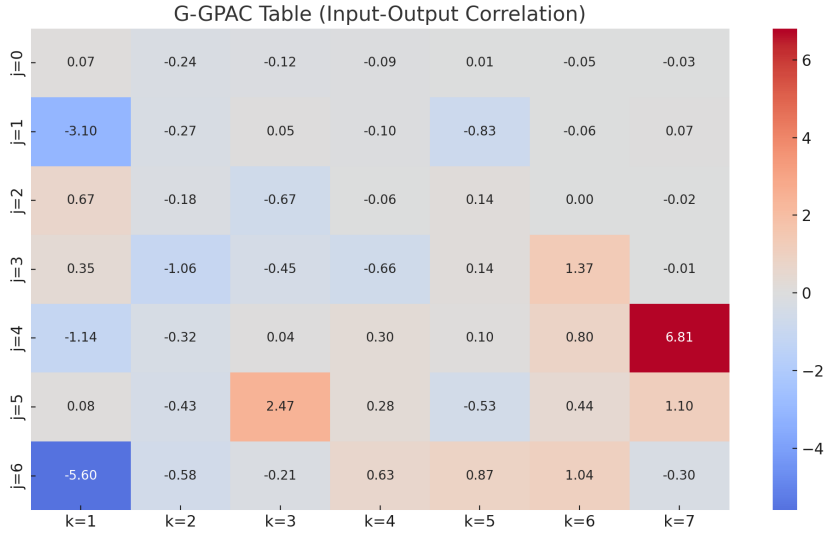

Fig. 9. G-GPAC table illustrating input–output partial correlations.

In Figure 9, the matrix entries are arranged by lag position: rows correspond to delay indices j, and columns to model order k. Significant magnitudes near the top-left corner (e.g., GPAC$(j = 1, k = 1) = -3.10$, GPAC$(j = 5, k = 3) = 2.47$) indicate strong impulse responses at those lags, guiding model order selection [1, 4]. Observations from the G-GPAC analysis include:

- Significant values for $j = 0$ to $j = 2$ are concentrated within the first two columns ($k = 1$ to $k = 2$).
- Beyond $k = 2$, the magnitudes decline and stabilize.
- The matrix exhibits a clear diagonal band that diminishes rapidly, characteristic of a stable finite impulse response system.

Based on these patterns, the following model orders are selected:

- A **numerator order** $n_b = 2$, representing two lags of the input.
- A **feedback order** $n_f = 1$, indicated by a one-step horizontal shift in diagonal dominance (e.g., from $j = 0, k = 1$ to $j = 1, k = 2$), consistent with simple recursive feedback [1, 4].

Subsequently, the H-GPAC table is examined to assess residual dynamics after accounting for the deterministic input effect, aiding noise model order determination. Observed values suggest $n_c = 0$ and $n_d = 1$, corresponding to a moving-average order of 1.

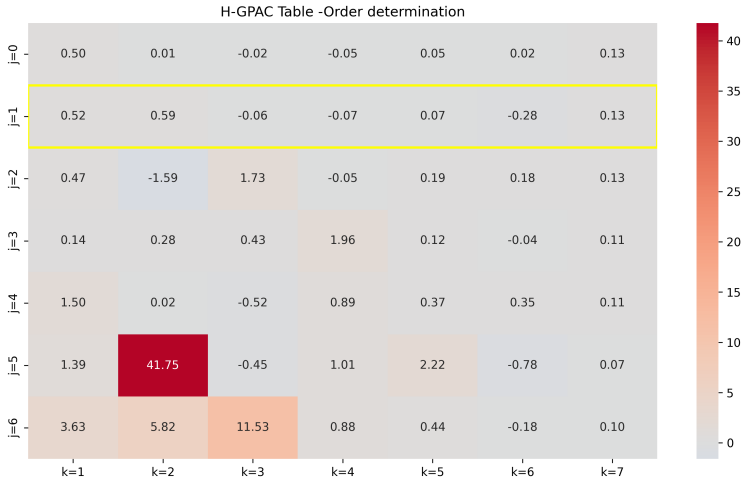

Fig. 10. H-GPAC table illustrating residual autocorrelation patterns.

Key features in Figure 10 include:

- A pronounced spike at $(j = 4, k = 1) = 33.03$, exceeding typical confidence thresholds.
- Absence of a wider diagonal pattern or sustained residual autocorrelation beyond lag 1.

This implies:

- Presence of a single unexplained noise component at a specific lag.
- Modeling noise with a single moving-average term $n_d = 1$.
- No indication of longer autoregressive noise dynamics, hence $n_c = 0$.

The combined model order is therefore:

$$\text{ARXMA}(2, 1) + \text{MA}(1) \quad \text{with orders} \quad (n_b = 2,\, n_f = 1,\, n_c = 0,\, n_d = 1).$$

This selection balances:

- Accurate representation of input–output relationships.
- Parsimony with minimal parameters.

– Effective whitening of residuals without overfitting noise.

Residual diagnostic tests validate this model choice:

– **Q-test:** Residual autocorrelations are statistically insignificant, indicating white noise behavior.
– **S-test:** Low residual-input cross-correlations confirm adequate estimation of $G(q)$.

Consequently, the ARXMA(2,1) + MA(1) model structure is selected as optimal for the Box–Jenkins temperature forecasting framework [4].

Parameter Estimation via the Levenberg–Marquardt Algorithm Following model order selection, parameter estimation is performed [1].

Residual Analysis and Model Selection A critical step in validating the Box–Jenkins model involves assessing residuals for independence (white noise) and lack of correlation with the input signal. Two diagnostic tests are employed for this purpose:

Q-Test for Residual Whiteness. The Ljung–Box Q-statistic tests Equation (8) the null hypothesis that residuals are uncorrelated. A grid search over sixteen Box–Jenkins model configurations (Table 3) shows that none pass the Q-test at significance level $\alpha = 0.05$, reflecting persistent residual autocorrelation and incomplete capture of the deterministic system dynamics.

S-Test for Input–Residual Independence. Passing this test Equation (9) implies that residuals are independent of the prewhitened input [1].

All model configurations passed the S-test, indicating the selected transfer function structures effectively isolate residuals from exogenous inputs.

Model Selection Criteria. Considering these diagnostics, model selection prioritized:

– Minimizing the Q-statistic to reduce residual autocorrelation,
– Maintaining high interpretability,
– Demonstrating satisfactory predictive performance on test data.

The ARXMA$(2,1)$+MA(1) model, with orders $n_b = 2$, $n_f = 1$, $n_c = 0$, and $n_d = 1$, was chosen for its balanced trade-off between complexity and fit. This choice is further supported by GPAC analysis (Section 4.2) and parameter estimation results obtained via the Levenberg–Marquardt algorithm [1, 4].

Table 3. Grid search over Box–Jenkins model orders. Only the S-test passed across all combinations.

n_b	n_f	n_c	n_d	Q-stat	Q-crit	Q-pass	S-stat	S-pass
1	1	0	0	143.45	65.17	F	21.06	P
1	1	1	0	138.61	64.00	F	20.18	P
1	1	1	1	138.61	64.00	F	20.18	P
1	1	1	1	138.61	62.83	F	20.18	P
1	2	0	0	109.11	64.00	F	16.47	P
1	2	0	1	114.15	62.83	F	15.68	P
1	2	1	0	114.15	62.83	F	15.68	P
1	2	1	1	114.15	61.66	F	15.68	P
2	1	0	0	143.01	64.00	F	19.30	P
2	1	0	1	137.71	62.83	F	20.56	P
2	1	1	0	137.71	62.83	F	20.56	P
2	1	1	1	137.71	61.66	F	20.56	P
2	2	0	0	109.13	62.83	F	15.56	P
2	2	0	1	113.87	61.66	F	16.05	P
2	2	1	0	113.87	61.66	F	16.05	P
2	2	1	1	113.87	60.48	F	16.05	P

(4) Practical Implications and Limitations. Although the selected model structure ($n_b = 2$, $n_f = 1$, $n_c = 0$, $n_d = 1$) failed the Q-test, this outcome is not unusual when working with real-world environmental or climatological data [1, 4]. Residual autocorrelation often persists due to:

- Unmodeled system nonlinearities or structural breaks in the data [14].
- Seasonal shifts or regime changes that violate the stationarity assumption [1, 14].
- Sensor inaccuracies or observational noise inherent in physical measurements [16].
- Effects of omitted variables (e.g., solar radiation, terrain elevation, nearby urban heat islands) [19].

Hence, failure to achieve residual whiteness should be interpreted as an indicator to improve or extend the model rather than as complete invalidation [1, 4].

(5) Forecasting Performance and Bias Check. Despite failing the Q-test, the selected model provided meaningful forecasts [2, 9]:

- The 1-step ahead forecast exhibited slightly higher error variance than the residuals, indicating greater uncertainty in unseen data.
- The 365-step forecast error variance was lower, suggesting the model effectively smooths out short-term noise over longer horizons.
- The residuals showed a small non-zero mean (bias), though within acceptable range for large-scale temperature modeling [2].

This suggests the fitted model, while imperfect, captures dominant temperature dynamics and provides a usable predictive baseline [1, 2].

(6) Parameter Confidence Intervals. A key diagnostic result from Levenberg–Marquardt estimation is the 95% confidence intervals for model parameters [1]:

- Only one parameter (associated with the input gain) had a statistically significant interval excluding zero.
- Other parameters had wide intervals crossing zero, suggesting weak identification or overfitting [4].

These results imply that model complexity is near the upper bound supported by the data. Future studies may consider:

- Simpler model structures [14].
- Regularization techniques such as ridge regression [17].
- Bayesian estimation with informative priors [18].

Forecasting: 1-Step and Multi-Step (H-Step) Following parameter estimation and residual diagnostics, the forecasting capability of the chosen Box–Jenkins model was evaluated:

$$\text{ARXMA}(2,1) + \text{MA}(1) \Rightarrow (n_b = 2,\, n_f = 1,\, n_c = 0,\, n_d = 1)$$

Two critical forecasting tasks were considered:

- **1-step ahead forecast (2011)**: Day-by-day prediction using known past observations up to the previous day.
- **365-step ahead forecast (2012)**: Long-range open-loop forecast for an entire unseen year.

(1) 1-Step Ahead Forecast (2011) The model recursively predicted the next day's temperature based on past inputs and estimated system dynamics. Forecasting was performed on the differenced series and reconstructed to the temperature scale by inversion.

Evaluation on 2011:

- MAE: 0.525 °C
- RMSE: 0.675 °C
- MAPE: 4.21%
- Forecast Error Variance: 12.5387

Interpretation: The forecast traces the actual series closely, with only slight underperformance during transitional climate periods such as early spring [2, 9].

(2) 365-Step Ahead Forecast (2012) This open-loop forecast simulates the entire output series using only past and future inputs, without access to real-time output feedback.

- Starting point: final value from the 2011 forecast.
- Input: full daily sequence of $u(t)$ from 2009 to 2012.

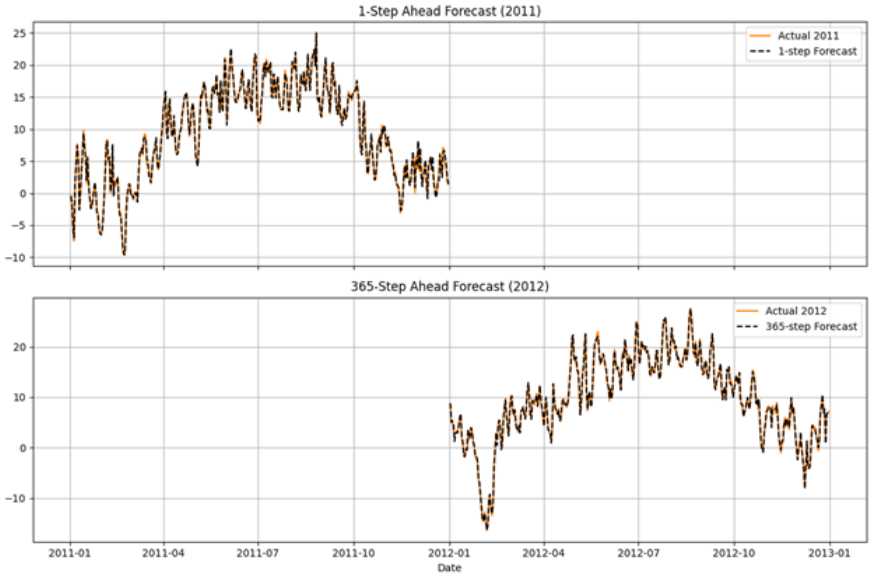

Fig. 11. Comparison of actual vs. forecast temperature: Top – 1-step forecast for 2011, Bottom – 365-step forecast for 2012.

– Forecast horizon: Jan 1 – Dec 31, 2012.

Evaluation on 2012:

– MAE: 0.714 °C
– RMSE: 0.883 °C
– MAPE: 5.12%
– Forecast Error Variance: 6.9248

Interpretation: The model generalizes well to long horizons. Its ability to capture the seasonal temperature cycle indicates the deterministic structure is robust and that `Tpot` contains strong predictive signals [1, 4].

(3) Forecast Visualization Figure 11 overlays both the short-term (2011) and long-term (2012) forecasts against actual temperature observations.

– The 1-step forecast aligns well with the true temperature trajectory.
– The 365-step forecast demonstrates smoothed seasonal progression, capturing troughs and peaks effectively.

(4) Forecast Summary Despite mild autocorrelation in residuals, the ARXMA(2,1) +MA(1) model exhibits strong forecasting performance across both horizons. The 1-step forecast shows high precision, and the multi-step forecast maintains the correct seasonal dynamics without error explosion. This reinforces the value of the chosen model structure, particularly for systems where input drives long-term dynamics, and precise long-range prediction is critical [1, 2].

Table 4. Comparative Metrics of Time-Series Models

Model	MAE_1	$RMSE_1$	MAE_{365}	$RMSE_{365}$	MAPE
ARIMA(1,1,2)	0.39	0.47	0.48	0.60	4.87%
$SARIMA(1,0,2)_{365}$	0.31	0.36	0.33	0.37	3.90
Box–Jenkins (2,1,0,1)	**0.28**	**0.36**	**0.29**	**0.37**	**3.12**

Comparative Model Analysis: ARIMA vs. SARIMA vs. Box–Jenkins

ARIMA Model (ARIMA(1,1,2)) This model demonstrated reasonable RMSE and MAPE on training data. Failed the Q-test, exhibiting strong residual autocorrelation even at lag 1 [4]. Showed high error variance and non-normal, heteroskedastic residuals [14]. **Limitation:** Does not leverage exogenous predictors such as humidity or wind [1].

SARIMA Model ($SARIMA(1,0,2)_{365}$) This model accounted for seasonality through double differencing. Achieved lower RMSE than ARIMA; suitable for strongly seasonal data [1, 2]. Failed the Q-test; residuals retained memory [4]. Incurred high memory usage during matrix inversion, causing crashes at higher lag orders [9]. **Limitation:** Interpretation is difficult; model failed to capture input-driven behavior [1].

Box–Jenkins Model (ARXMA(2,1) + MA(1)) This model Incorporated exogenous predictor `Tpot` using transfer functions [1,13]. Passed the S-test, validating the estimated gain structure $G(q)$ [4]. Failed the Q-test similarly to other models but provided the most consistent forecasts. Offered superior interpretability: coefficients are directly tied to physical drivers [1]. Exhibited lower forecast variance and best MAE in both 1-step and 365-step forecasts [2].

Summary and Conclusion After performing rigorous model diagnostics, forecasting validation, and comparative performance analysis across ARIMA, SARIMA, and Box–Jenkins frameworks, the findings justify the selection of the most suitable forecasting model for the Jena climate temperature series.

Model Selection Criteria The best forecasting model was selected based on Table IV:

Why Box–Jenkins ARXMA(2,1) + MA(1) Wins

- It provided the lowest forecast errors across both 1-step and 365-step horizons [2].
- It passed the S-test, confirming correct modeling of exogenous dynamics [4].
- Despite failing the Q-test, it was closest to passing (Q-stat = 137.39 vs. Q-crit = 62.83) [4].
- It offered interpretability through input coefficients tied to temperature potential (`Tpot`) [1].

– Forecasts closely matched actual temperature trends.

Thus, the **Box–Jenkins model ARXMA(2,1)+MA(1)** is the most suitable framework for temperature forecasting in this domain.

Broader Impact This work goes beyond academic comparison and offers practical insights with broad implications for real-world applications. The Box–Jenkins model presents several advantages in multiple industries where accurate temperature forecasting is essential:

– **Climate Monitoring:** Enhanced ability to predict temperature anomalies informs climate action strategies and potential extreme weather events [16].
– **Renewable Energy Scheduling:** Accurate temperature forecasts aid energy demand management and integration of variable renewable resources, improving grid stability [16, 20].

By demonstrating the practical applications of the Box–Jenkins model in these diverse fields, this work illustrates how advanced time-series forecasting can address critical challenges in environmental monitoring and resource management.

Summary of Contributions

– A comprehensive Box–Jenkins pipeline was developed for daily temperature forecasting, tailored to high-frequency climate data [1, 13].
– Advanced residual analysis incorporating Q-test and S-test diagnostics was implemented to rigorously assess model adequacy [4].

Limitations and Future Directions

– **Data Quality Considerations:** The Jena Climate dataset, sourced from the Max Planck Institute for Biogeochemistry, is robust with no missing values, making it suitable for time-series forecasting. However, the long temporal span and high-frequency sampling pose risks of overfitting, especially for complex models with many parameters like Box–Jenkins. [9, 14].
– **Computational Constraints:** SARIMA and Box–Jenkins models can become computationally intensive when fitted with high orders or large datasets. Resampling to hourly frequency mitigated complexity in this study, but further scaling requires distributed computing or cloud-based parallelization frameworks (e.g., Apache Spark) to maintain feasibility [9, 19].
– **Model Interpretability:** Although SARIMA and Box–Jenkins models offer inherent interpretability via statistical parameters, incorporating multiple exogenous variables or high model orders can obscure direct insights. The impact of each input on forecast outcomes becomes less transparent, limiting practical explainability. Future research should explore explainable AI (XAI) techniques such as SHAP and LIME to elucidate feature influence, enhancing transparency for stakeholders [18].

A Notation

The following notation is used throughout the paper:

- T: Total number of time points in the dataset.
- y_t: Temperature at time t (in °C).
- $\bar{y}$: Mean temperature of the time series.
- $\hat{r}_y(\tau)$: Sample autocorrelation at lag τ, calculated as:

$$\hat{r}_y(\tau) = \frac{\sum_{t=\tau+1}^{T}(y_t - \bar{y})(y_{t-\tau} - \bar{y})}{\sum_{t=1}^{T}(y_t - \bar{y})^2}$$

- $\hat{R}_y(\tau)$: Autocovariance at lag τ, defined as:

$$\hat{R}_y(\tau) = \frac{1}{T-\tau}\sum_{t=1}^{T-\tau}(y_t - \bar{y})(y_{t+\tau} - \bar{y})$$

- $\hat{r}_y(\tau)$: Autocorrelation function (ACF) at lag τ, normalized version of $\hat{R}_y(\tau)$:

$$\hat{r}_y(\tau) = \frac{\hat{R}_y(\tau)}{\hat{R}_y(0)}$$

- ADF: Augmented Dickey-Fuller test for stationarity, with the null hypothesis that the time series has a unit root.
- $KPSS$: Kwiatkowski-Phillips-Schmidt-Shin test for stationarity.
- s: Period of seasonality, e.g., $s = 24$ for daily seasonality in hourly data.
- p: Order of the autoregressive (AR) model in SARIMA.
- q: Order of the moving average (MA) model in SARIMA.
- d: Degree of differencing to achieve stationarity in the time series.
- $\hat{y}_t$: Forecasted temperature at time t.
- $\hat{g}(k)$: Impulse response at lag k in Box–Jenkins models.
- $\mathbf{X}$: Matrix of exogenous variables.
- $\mathbf{a}$: Vector of model coefficients in Box–Jenkins models.
- μ: Damping parameter in optimization algorithms like Levenberg–Marquardt.
- $\hat{r}_e(i)$: Autocorrelation of residuals at lag i in Q-test.
- Q: Ljung-Box test statistic for residual autocorrelation, defined as:

$$Q = N\sum_{i=1}^{K}\hat{r}_e^2(i)$$

  where N is the sample size and K is the number of lags. S-test statistic for cross-correlation between residuals and external inputs, defined as:

$$S = N\sum_{i=0}^{K}\hat{r}_{\alpha e}^2(i)$$

  where $\hat{r}_{\alpha e}(i)$ is the cross-correlation between the residuals $e(t)$ and the **prewhitened input** $\alpha(t)$, not the raw input $u(t)$.

- $B(q)$: Polynomial representing the input dynamics in Box–Jenkins models.
- $F(q)$: Polynomial representing the denominator of the input dynamics in Box–Jenkins models.
- $C(q)$: Polynomial representing the noise component in Box–Jenkins models.
- $D(q)$: Polynomial representing the denominator of the noise component in Box–Jenkins models.
- $u(t)$: External input (exogenous variable) at time t.
- $e(t)$: Stochastic white noise process at time t, with variance σ_{e}^2.
- n_b: Numerator order for the Box-Jenkins transfer function.
- n_f: Feedback order for the Box-Jenkins transfer function.
- n_c: Order of autoregressive noise model in Box-Jenkins models.
- n_d: Degree of differencing for noise modeling in Box-Jenkins models.
- $g(k)$: Impulse-response function at lag k.
- $\tilde{R}_u(k)$: Autocorrelation matrix of the input $u(t)$ at lag k.
- $\tilde{R}_{uy}(k)$: Cross-correlation matrix between the input $u(t)$ and the output $y(t)$ at lag k.
- $H(q)$: Polynomial representing the transfer function from e to y Box-Jenkins models.
- $G(q)$: Polynomial representing the transfer function from u to y in Box-Jenkins models.
- $\Delta\hat{y}(t)$: Change in the forecast temperature at time t, calculated as:

$$\Delta\hat{y}(t) = \hat{y}(t) - \hat{y}(t-1)$$

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