Re-envisioning Accessibility for Math-Intensive OER

Anita Walz, Assistant Director of Open Education & Scholarly Communication Librarian
Open Education Conference (Online) https://sched.co/fCpG
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University Libraries, Virginia Tech

With thanks

Talea Anderson, Washington State University

Apurva Ashok, Rebus Foundation

Ed Beck, SUNY-Oneonta

Robert Browder, Virginia Tech

Corinne Guimont, Virginia Tech

Christa Miller, Virginia Tech

Volker Sorge, University of Birmingham

Format for today

Getting to know you (5 minutes)

- Level of expertise with math and accessibility
- What do you hope to learn in this session?

Philosophy of accessibility (2 minutes)

Case studies (10 minutes)

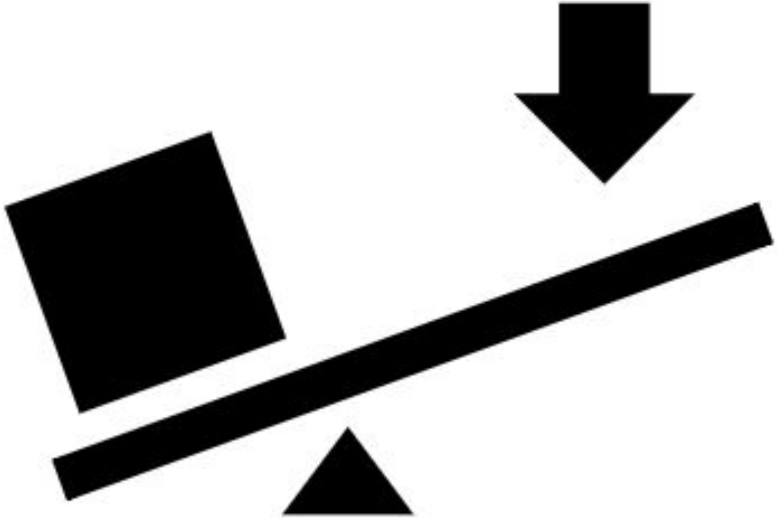
Practices/solutions from your own experience (7 minutes)

Hallway conversations

Getting to know you

https://bit.ly/mathoer

Accessibility Philosophy



Levers for readers to do more

Public domain. By Jakob Ukrop. https://thenounproject.com/term/lever/4927

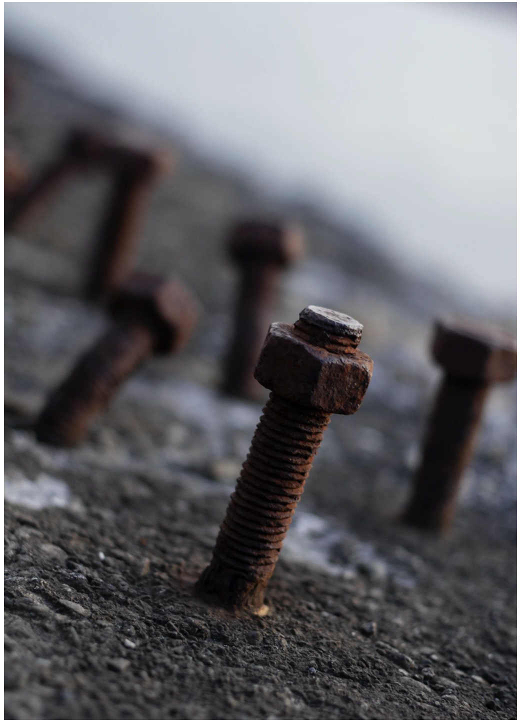
Accessibility Philosophy

Jesper Aggergaard https://unsplash.com/photos/j6cMbjtCqvE (Unsplash license)

Level of effort

- Ours
- For adaptation

Accessibility Philosophy



Luke Hodde Unsplash license https://unsplash.com/photos/VC-VBlI6UQM

Bolted on?

Accessibility Philosophy

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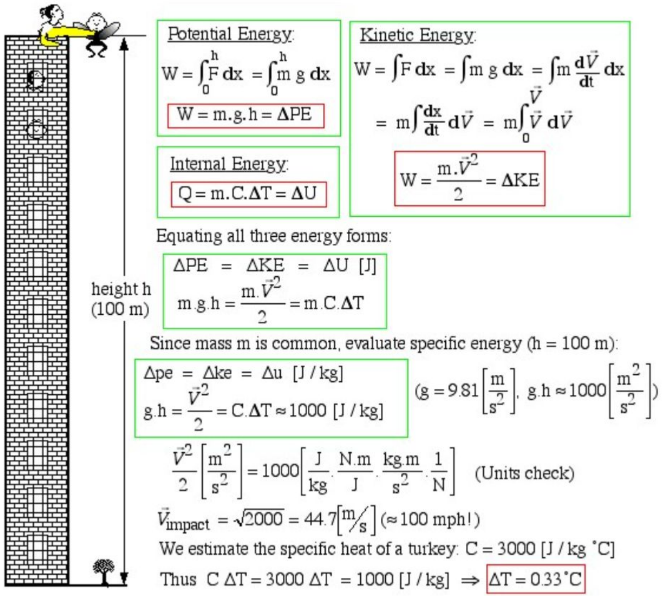
Baked in!

Cooking with Potential Energy

In order to gain an intuitive appreciation for the relative magnitudes of the different forms of energy we consider the (tongue-in-cheek) example of an attempt to cook a turkey by potential energy. The turkey is brought to the top of a 100 m building (about 30 stories) and then dropped from the ledge. The potential energy is thus converted into kinetic energy, and finally on impact the kinetic energy is converted into internal energy. The increase in internal energy is represented by an increase in temperature, and hopefully, if this experiment is repeated enough times the temperature increase will allow the turkey to cook. This remarkable experiment was first reported by R.C.Gimmi and Gloria J Browne – "**Cooking with Potential Energy**", published in the **Journal of Irreproducible Results** (Vol. 33, 1987, pp 21-22).



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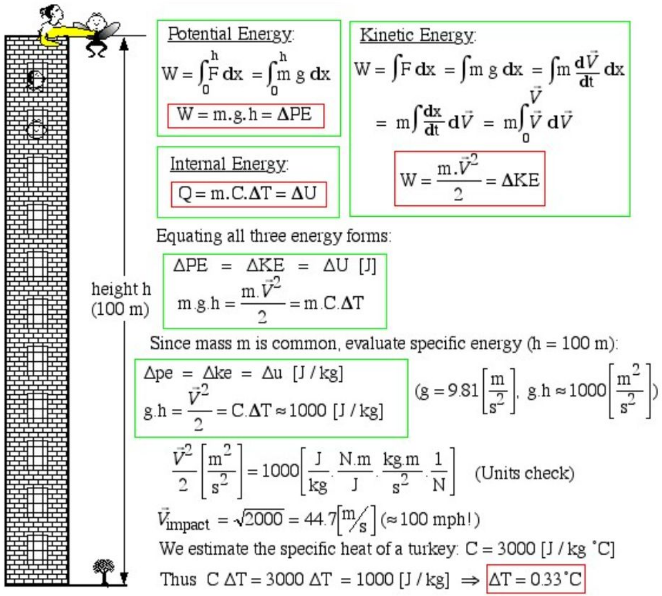


Potential Energy

$$W = \int_0^h F \, dx = \int_0^h mg \, dx$$

$$\boxed{W = m \cdot g \cdot h = \Delta PE}$$

Internal Energy

$$\boxed{Q = m \cdot C \cdot \Delta T = \Delta U}$$

Kinetic Energy

$$W = \int F \, dx = \int mg \, dx = \int m \frac{d\vec{V}}{dt} \, dx$$

$$= m \int \frac{dx}{dt} \, d\vec{V} = m \int_0^{\vec{V}} \vec{V} \, d\vec{V}$$

$$\boxed{W = \frac{m \cdot \vec{V}^2}{2} = \Delta KE}$$

Equating all three energy forms:

$\Delta PE = \Delta KE = \Delta U$ [J]

$$m \cdot g \cdot h = \frac{m \cdot \vec{V}^2}{2} = m \cdot C \cdot \Delta T$$

Since mass m is common, evaluate specific energy (h=100 m):

$\Delta pe = \Delta ke = \Delta u$ [J/kg]

$$g \cdot h = \frac{\vec{V}^2}{2} = C \cdot \Delta T \approx 1000 \text{ [J/kg]}$$

$$\frac{\vec{V}^2}{2}\left[\frac{m^2}{s^2}\right] = 1000\left[\frac{J}{kg} \cdot \frac{N\,m}{J} \cdot \frac{kg\,m}{s^2} \cdot \frac{1}{N}\right]$$

$$\vec{V}_{impact} = \sqrt{2000} = 44.7 \,[m/s]\ (\approx 100 \text{ mph!})$$

We estimate the specific heat of a turkey.

$C = 3000$ [J/kg °C]

Thus

$$C\Delta T = 3000\ \Delta T = 1000 \text{ [J/kg]} \implies \boxed{\Delta T = 0.33°C}$$

What a disappointment! At 0.33°C per fall it will require repeating the experiment 600 times just to reach the cooking temperature of 200°C.

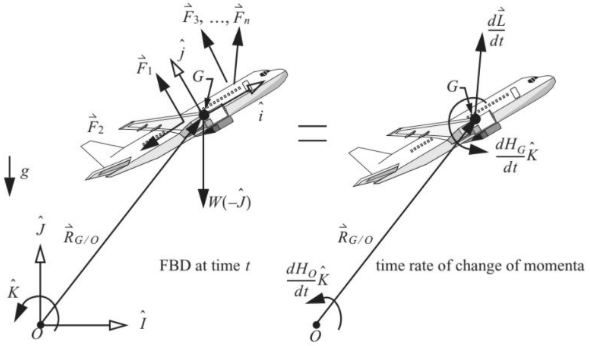


Fig. 2.1 Free body and rate of momenta diagrams for symmetrical motion of an aircraft at time *t*

The local acceleration due to gravity, g, is in the negative *Y*-direction, and the weight of the total airplane is $W = mg$ with m denoting the total mass of the airplane. Let $\vec{R}_{G/O}$ denote the position vector of the mass center G with respect to fixed point O, $\vec{L}$ the linear momentum of the body, $H_G\hat{K}$ the angular momentum of the body about the mass center, and let $H_O\hat{K}$ denote the angular momentum of the body about the fixed point. Newton second laws at time t are

$$\sum \vec{F} = \frac{d\vec{L}}{dt} \qquad \sum \vec{M}_G = \frac{dH_G}{dt}\hat{K} \qquad \sum \vec{M}_O = \frac{dH_O}{dt}\hat{K} \tag{2.1}$$

where the time rate of change of the linear momentum is

$$\frac{d\vec{L}}{dt} = m\vec{a}_G \tag{2.2}$$

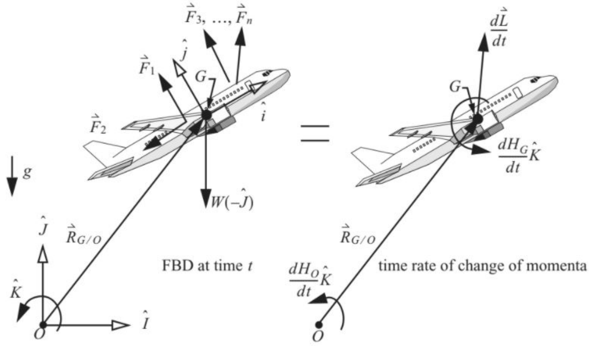
In the body fixed system the acceleration of the mass center is written as

$$\vec{a}_G = a_{Gx}\hat{i} + a_{Gy}\hat{j} \tag{2.3}$$

The time rate of angular momenta about the mass center and fixed point, respectively, are given by

$$\frac{dH_G}{dt}\hat{K} = I_G\frac{d^2\theta}{dt^2}\hat{K} \qquad \frac{dH_O}{dt}\hat{K} = \vec{R}_{G/O} \times \frac{d\vec{L}}{dt} + \frac{dH_G}{dt}\hat{K} \tag{2.4}$$

where the mass moment of inertia of the airplane about the *z*-axis is denoted by I_G, and $\frac{d^2\theta}{dt^2}\hat{K}$ denotes the angular acceleration in pitch.



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I^ J^ K^ i^j^G G F1 F2 WJ-()^ dt dHG dt ----------K^ FBD at time t dHO dt ----------K^ g time rate of change of momenta RGO⁄RGO⁄
<Caption>
<P>Fig. 2.1 Free body and rate of momenta diagrams for symmetrical motion of an aircraft at time t </P>
</Caption>
</Figure>

<P>The local acceleration due to gravity, g , is in the negative Y-direction, and the weight of the total airplane is </P>
<Figure>

<ImageData src="images/AeroStructures-16.book_img_13.jpg"/>
</Figure>

<P>W = mg with m denoting the total mass of the airplane. Let RGO denote the position vector of the mass center </P>

<P>⁄ </P>

<P><Figure>

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</Figure>
^</P>

<P>G with respect to fixed point O, L the linear momentum of the body, HGK the angular momentum of the body ^</P>

<P>about the mass center, and let HOK denote the angular momentum of the body about the fixed point. Newton second laws at time t are </P>

<P>dHO </P>

<P>^</P>

<P>= ------= MO = ----------K (2.1)</P>

<P>dtdt dt </P>

<P>where the time rate of change of the linear momentum is </P>

<P>------= maG (2.2)</P>

<P>dt </P>

<P>In the body fixed system the acceleration of the mass center is wr
<Figure>

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<P>aG =+ (2.3)</P>

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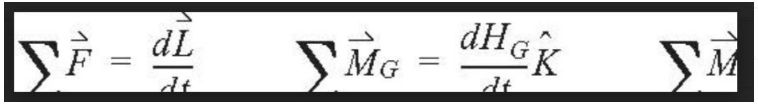

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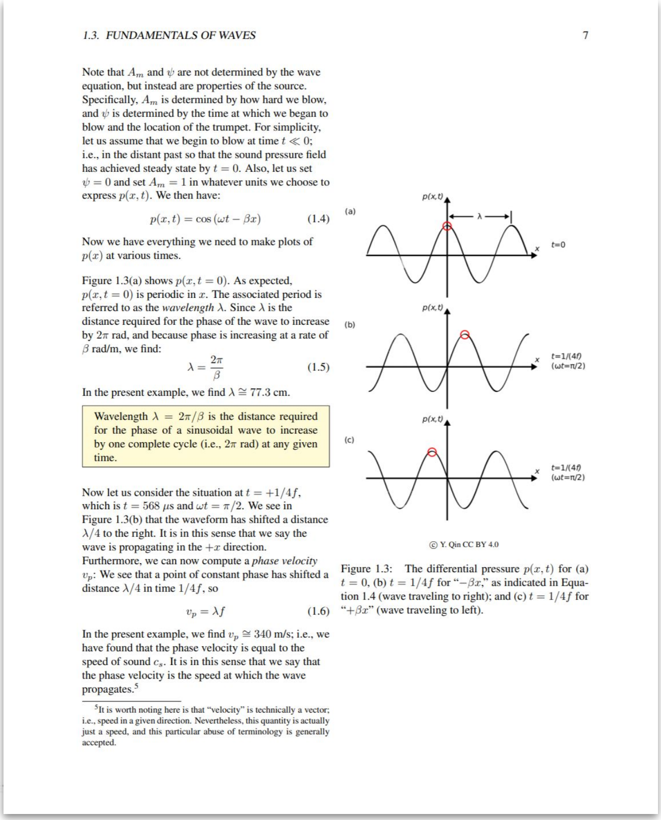


1.3. *FUNDAMENTALS OF WAVES* 7

Note that A_m and ψ are not determined by the wave equation, but instead are properties of the source. Specifically, A_m is determined by how hard we blow, and ψ is determined by the time at which we began to blow and the location of the trumpet. For simplicity, let us assume that we begin to blow at time $t \ll 0$; i.e., in the distant past so that the sound pressure field has achieved steady state by $t = 0$. Also, let us set $\psi = 0$ and set $A_m = 1$ in whatever units we choose to express $p(x, t)$. We then have:

$$p(x, t) = \cos(\omega t - \beta x) \quad (1.4)$$

Now we have everything we need to make plots of $p(x)$ at various times.

Figure 1.3(a) shows $p(x, t = 0)$. As expected, $p(x, t = 0)$ is periodic in x. The associated period is referred to as the *wavelength* λ. Since λ is the distance required for the phase of the wave to increase by 2π rad, and because phase is increasing at a rate of β rad/m, we find:

$$\lambda = \frac{2\pi}{\beta} \quad (1.5)$$

In the present example, we find $\lambda \cong 77.3$ cm.

Wavelength $\lambda = 2\pi/\beta$ is the distance required for the phase of a sinusoidal wave to increase by one complete cycle (i.e., 2π rad) at any given time.

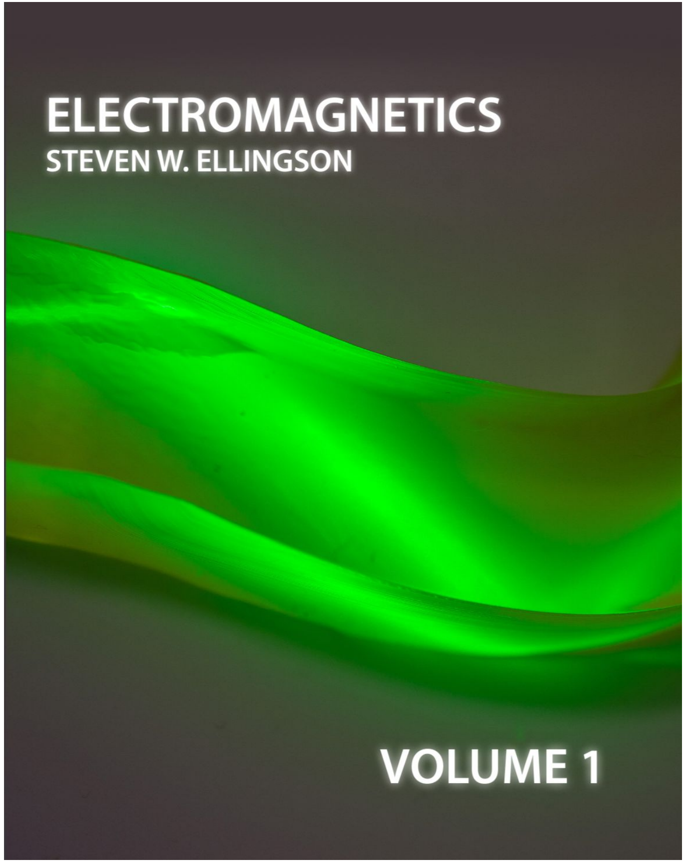
Now let us consider the situation at $t = +1/4f$, which is $t = 568$ μs and $\omega t = \pi/2$. We see in Figure 1.3(b) that the waveform has shifted a distance $\lambda/4$ to the right. It is in this sense that we say the wave is propagating in the $+x$ direction. Furthermore, we can now compute a *phase velocity* v_p: We see that a point of constant phase has shifted a distance $\lambda/4$ in time $1/4f$, so

$$v_p = \lambda f \quad (1.6)$$

In the present example, we find $v_p \cong 340$ m/s; i.e., we have found that the phase velocity is equal to the speed of sound c_s. It is in this sense that we say that the phase velocity is the speed at which the wave propagates.[5]

[5]It is worth noting here is that "velocity" is technically a vector; i.e., speed in a given direction. Nevertheless, this quantity is actually just a speed, and this particular abuse of terminology is generally accepted.

Figure 1.3: The differential pressure $p(x, t)$ for (a) $t = 0$, (b) $t = 1/4f$ for "$-\beta x$," as indicated in Equation 1.4 (wave traveling to right); and (c) $t = 1/4f$ for "$+\beta x$" (wave traveling to left).



http://hdl.handle.net/10919/84164

density, change with altitude in the Troposphere. We start with the differential form of the hydrostatic equation and combine it with the Perfect Gas equation to eliminate the density term.

$$dP = -\rho g\,dh, \quad P = \rho RT$$

or,

$$dP/dh = -\rho g = -(P/RT)g$$

which is rearranged to give

$$dP/P = -(g/RT)dh.$$

Now we substitute in the lapse rate relationship for the temperature to get

$$dP/P = \{g/[R(T_{SL}-Lh)]\}dh.$$

This is now a relationship with only one variable (P) on the left and only one (h) on the right. It can be integrated to give

$$\mathbf{P_{alt}/P_{SL} = [T_{alt}/T_{SL}]^{g/LR}}$$

In a similar manner we can get a relationship to find the density at any altitude in the troposphere

$$\boldsymbol{\rho_{alt}/\rho_{SL} = [T_{alt}/T_{SL}]^{(g-LR)/LR}}$$

So now we have equations to find pressure, density, and temperature at any altitude in the troposphere. Care has to be taken with units when using these equations. All temperatures must be in absolute values (Kelvin or Rankine instead of Celsius or Fahrenheit). ***The exponents in the pressure and density ratio equations must be unitless. Exponents cannot have units!***

We can use these equations up to the top of the Troposphere, that is, up to 11,000 meters or 36,100 feet in altitude. Above that altitude is the Stratosphere where temperature is modeled as being constant up to roughly 100,000 feet.

The Stratosphere

We can use the temperature lapse rate equation result at 11,000 meters altitude to find the temperature in this part of the Stratosphere.

$$\mathbf{T_{stratosphere} = 216.5^{o}K = 389.99^{o}R = constant}$$

The equations for determining the pressure and density in the constant temperature part of the stratosphere are different from those in the troposphere since temperature is constant. And, since temperature is constant both pressure and density vary in the same manner.

$$\mathbf{P_2/P_1 = \rho_2/\rho_1 = e^{g(h1-h2)/RT}}$$

Problems

- The math **renders as a picture** in ALL of the examples.
- Math was created in a **variety of authoring tools** -- an image editor, an obsolete software page, LaTex, MSWord
- Math was **displayed in different file types** (PNGs in HTML, PDF, MSWord)
- In three cases, none of the math is ~~**editable**~~. In one case, the math is only editable if you have the LaTeX sourcefiles.

Solutions

1) Reader-side solution

2) Content-developer side solution

Solutions

- HTML Output
  - LaTeX, AsciiMath or MathML + MathJax = HTML/CSS
  - LaTeX, Word, Markdown + Pandoc = HTML
- (opensource) Pandoc LaTeX, Word, Markdown

1.5 The Standard Atmosphere

We said we were starting with the Ideal Gas Equation of State, P=RT. We will also make use of the ***Hydrostatic Equation***, another relationship you have seen before in chemistry and physics:

$$\Delta \mathbf{P} = -\rho g \Delta \mathbf{h}$$

This tells us how pressure changes with height in a column of fluid. This tells us how pressure changes as we move up or down through the atmosphere.

These two equations, the Perfect Gas Equation of State and the Hydrostatic Equation, have three variables in them; pressure, density, and temperature. To solve for these properties at any point in the atmosphere requires us to have one more equation, one involving temperature. This is going to require our ***first assumption***. We must have some relationship that can tell us how temperature should vary with altitude in the atmosphere.

Many years of measurement and observation have shown that, in general, the lower portion of the atmosphere, where most airplanes fly, can be modeled in two segments, the ***Troposphere*** and the ***Stratosphere***. The temperature in the troposphere is found to drop fairly linearly as altitude increases. This linear decrease in temperature continues up to about 36,000 feet (about 11,000 meters). Above this altitude the temperature is found to hold constant up to altitudes over 100,000 ft. This constant temperature region is the lower part of the ***Stratosphere***. The troposphere and stratosphere are where airplanes operate, so we need to look at these in detail.

EDIT

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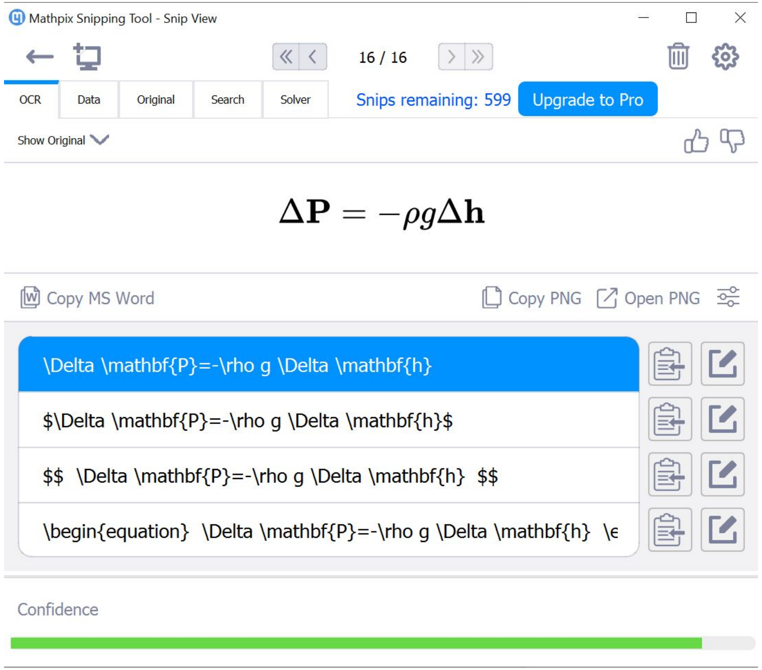

https://mathpix.com

Solutions for Creators

- Understanding what formats are already accessible by screen readers
- Hand coding the math OR finding visual recognition software
- Defining your workflow (what document type(s) are you starting with, and where do you wish to end up?)

https://bit.ly/mathoer

Thank you!

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