LOAD AND RESISTANCE FACTOR DESIGN OF SHALLOW FOUNDATIONS FOR BRIDGES

by

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(ABSTRACT)

Load Factor Design (LFD), adopted by AASHTO in the mid-1970, is currently used for bridge superstructure design. However, the AASHTO specifications do not have any LFD provisions for foundations. In this study, a LFD format for the design of shallow foundations for bridges is developed.

Design equations for reliability analysis are formulated. Uncertainties in design parameters for ultimate and serviceability limit states are evaluated. A random field model is employed to investigate the combined inherent spatial variability and systematic error for serviceability limit state. Advanced first order second moment method is then used to compute reliability indices inherent in the current AASHTO specifications. Reliability indices for ultimate and serviceability limit states with different safety factors and dead to live load ratios are investigated. Reliability indices for ultimate limit state are found to be in the range of 2.3 to 3.4, for safety factors between 2 and 3. This is shown to be in good agreement with Meyerhof's conclusion (1970). Reliability indices for serviceability limit state are found to be in the range of 0.43 to 1.40, for ratios of allowable to actual settlement between 1.0 to 2.0. This ap-

pears to be in good agreement with what may be expected. Performance factors are then determined using target reliability indices selected on the basis of existing risk levels.

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Chapter 1

INTRODUCTION

1.1 GENERAL

Load factor design (LFD), adopted by AASHTO in the mid-1970, is currently used for bridge superstructure design. The AASHTO specifications do not have any LFD provisions for foundations, and engineers use Allowable Stress Design (ASD). The use of two different design methods - LFD for superstructures and ASD for foundations - has resulted in inconsistences and duplication of effort, since different sets of loads are required for the two methods, and has hindered the widespread adoption of Load Factor Design. Another limitation of the current AASHTO specifications and other deterministic design procedures is that uncertainties in load and resistance are not explicitly considered in the design process, resulting in nonuniform risk levels. Thus, a study to develop a rational load factor design method for bridge foundations was necessary in order to develop a consistent approach to the design of bridge superstructures and foundations.

1.2 OBJECTIVE AND SCOPE

The main objective of this study is to develop performance factors for shallow bridge foundations for both ultimate and serviceability limit states. A reliability-based approach is used to calibrate the performance factors with existing allowable stress method. This study begins with a review of current design practice and basic reliability-based design concepts. Next, the mathematical formulation for computing reliability indices for different limit states is presented. This is followed by a discussion of the statistical analyses performed to obtain uncertainties in loads, soil properties and design equations. The next step is to compute reliability indices for existing designs for different load combinations and dead to live load ratios. The last task is code calibration and determination of performance factors.

1.3 ORGANIZATION

In Chapter 1, the motivation, approach and scope of the study is described. Chapter 2 discusses current design practice for shallow foundations, and limit states and reliability-based design concepts. Chapter 3 presents the reliability model for determining ultimate bearing capacity; the uncertainties in modeling and design parameters are also evaluated. Chapter 4 presents the reliability model for serviceability limit state and the evaluation of model uncertainty, inherent variability, and systematic error resulting from soil exploration. Statistics of dead and live loads on bridges are presented in Chapter 5. Chapter 6 presents the results of the reliability analysis, code calibration and the determination of performance factors consistent with selected target reliability indices. Chapter 7 contains a summary of the work, a discussion of the results of reliability analysis, and suggestions for future study.

Chapter 2

LITERATURE REVIEW

2.1 GENERAL

Many design codes based on the load and resistance factor design (LRFD) format have been developed in the past few decades. They include the Danish Code of Practice for Foundation Engineering (1985), the Ontario Highway Bridge Design (OHBD) code (1983), and AISC Steel Specifications (1986). These design codes have recognized the existence of uncertainties in design practice and were developed using reliability theory and the concept of limit states.

In this chapter, a review of current practice for shallow foundation design is presented. The limit states and the sources of uncertainty in current shallow foundation design are discussed. Finally, the concept of probability-based design is presented.

2.2 CURRENT PRACTICE FOR SHALLOW FOUNDATION DESIGN

Allowable stress design, also called working stress design, is the usual method used for the design of bridge foundations. In the allowable stress design method, the safety of a structure is determined by restricting the stress due to applied loads (calculated from an elastic analysis) to a value less than or equal to the allowable stress. The allowable stress is usually determined by dividing the yield or ultimate strength of the material by a suitable safety factor. The statistical nature of the loads, and the fact that different types of loads (for example, dead load and live load) have different levels of uncertainty are not incorporated in the design philosophy. Design loads are determined and their effects on the structure are then analyzed in a deterministic way.

Current design equations for determining the ultimate strength of shallow foundation, that is, the ultimate bearing capacity, are based on Terzaghi's (1943) superposition method. The prevailing equations in shallow foundation design include Terzaghi's equation

$$q_{ult} = 0.5\gamma B N_{\gamma} + \gamma D N_q + C N_c \tag{2.1}$$

Meyerhof's equation (1951, 1963)

$$q_{ult} = 0.5\gamma B N_{\gamma} S_{\gamma} d_{\gamma} + \gamma D N_q S_q d_q + C N_c S_c d_c \tag{2.2}$$

and Hansen's equation (1957, 1970)

$$q_{ult} = 0.5\gamma B N_\gamma S_\gamma d_\gamma I_\gamma + \gamma D N_q S_q d_q I_q + C N_c S_c d_c I_c \tag{2.3}$$

In the above equations, q_{ult} = ultimate bearing capacity of foundation, γ = unit weight of soil, B = footing width, D = footing depth, N_γ, N_q, N_c = bearing capacity factors, which are functions of frictional angle, C = cohesion, S_γ, S_q, S_c = shape factors, d_γ, d_q, d_c = depth factors, and I_γ, I_q, I_c = inclination factors. The expressions for the bearing capacity factors are different in each of the above equations and are shown in Table 1.

The equations for predicting the ultimate bearing capacity are partly theoretical and partly empirical, so that there is modeling uncertainty in these equations. There is also uncertainty resulting from the basic variability in soil parameters such as friction angle and cohesion. The difference between the predicted and the measured ultimate bearing capacity resulting from the uncertainties mentioned above can not be ignored. In a study conducted by Abdul-Baki et al. (1970), the difference between actual and predicted ultimate bearing capacity of shallow foundations on sand was investigated. Ten footings were used to compare the measured ultimate bearing capacity with results obtained by different prediction methods. The footings varied in width from 1-ft to 6-ft and had depth to width ratios in the range of 0.5 to 0.8. The angle of internal friction, ϕ_t, varied from 25° to 45° and the unit weight of soil varied from 100 pcf to 130 pcf. The foundation dimensions, soil properties, and results of predicted as well as measured ultimate bearing capacity are presented in Table 2. The comparison between the measured and predicted ultimate bearing capacity results is shown in Table 3. N in Table 3 is the ratio of measured to predicted ultimate bearing capacity. The coefficients of variation in Table 3 include the uncertainty associated with inherent randomness and random sampling error.

Table 1

Bearing Capacity Factors for the Different Prediction Equations

FACTOR	TERZAGHI'S METHOD	MEYERHOF'S METHOD	HANSEN'S METHOD
N_γ	$\tan^6(45 + \frac{\phi_f}{2}) - \tan(45 + \frac{\phi_f}{2})$	$(N_q - 1)\tan(1.4\phi_f)$	$1.5(N_q - 1)\tan\phi_f$
N_q	$\tan^4(45 + \frac{\phi_f}{2})$	$\exp(\pi \tan\phi_f)\tan^2(45 + \frac{\phi_f}{2})$	same as Meyerhof
N_c	$2(\tan^3(45 + \frac{\phi_f}{2}) + \tan(45 + \frac{\phi_f}{2}))$	$(N_q - 1)\cot\phi_f$	same as Meyerhof

Table 2
Ultimate Bearing Capacity of Shallow Foundation
on Sand as Obtained by Different Equations

Test	Soil and footing properties	Ultimate Bearing Capacity, q, in Kips per Square Foot			
		Terzaghi	Meyerhof	Hansen	Measured
1	$\phi = 25°, B = 6ft, \frac{D}{B} = 1, \gamma = 100pcf$	10.20	9.80	10.92	10.95
2	$\phi = 30°, B = 4ft, \frac{D}{B} = 3, \gamma = 100pcf$	30.94	36.5	38.9	34.8
3	$\phi = 30°, B = 4ft, \frac{D}{B} = 5, \gamma = 100pcf$	48.94	66.5	64.0	59.8
4	$\phi = 35°, B = 8ft, \frac{D}{B} = 0.5, \gamma = 100pcf$	34.4	35.8	35.3	40.2
5	$\phi = 35°, B = 2ft, \frac{D}{B} = 2, \gamma = 100pcf$	20.8	21.0	24.5	23.1
6	$\phi = 35°, B = 2ft, \frac{D}{B} = 6, \gamma = 110pcf$	59.3	87.4	94.2	77.9
7	$\phi = 40°, B = 1ft, \frac{D}{B} = 6, \gamma = 110pcf$	59.2	79.3	91.3	77.1
8	$\phi = 40°, B = 5ft, \frac{D}{B} = 8, \gamma = 120pcf$	420.84	660	730	598
9	$\phi = 45°, B = 1ft, \frac{D}{B} = 2, \gamma = 130pcf$	64.4	58.7	79.7	71.2
10	$\phi = 45°, B = 2ft, \frac{D}{B} = 5, \gamma = 130pcf$	263.97	291	390	316

Table 3
Comparison of Ratio of Measured to Predicted Ultimate Bearing
Capacity on Sand for Different Prediction Methods

Prediction Equation	Test No										N		
	1	2	3	4	5	6	7	8	9	10	Mean	Standard Deviation	c.o.v.
Terzaghi	1.07	0.8	1.22	1.17	1.11	1.31	1.3	1.42	1.11	1.2	1.17	0.18	0.15
Meyerhof	1.12	0.95	0.9	1.12	1.1	0.89	0.97	0.91	1.21	1.09	1.03	0.12	0.12
Hansen	1	0.89	0.93	1.14	0.94	0.82	0.84	0.82	0.89	0.81	0.91	0.10	0.11

From Table 3, it can be seen that Terzaghi's and Meyerhof's equations underestimate the ultimate bearing capacity while Hansen's equation overestimates the ultimate bearing capacity of shallow foundations on sand. Comparing the statistics of measured to predicted ultimate bearing capacity one may conclude that Hansen's equation is the best method among the three methods for predicting ultimate bearing capacity of shallow foundations on sand.

Besides ultimate bearing capacity, bridge designers also need to make sure that foundation settlements can be assessed as accurately as possible. It is well known that the design of shallow foundations on sand is controlled by settlement criteria in most cases, except for small footings, say $B < 4$ feet, at or near the surface. Currently, there are many methods for predicting settlements. These methods are based on different soil exploration methods. According to Kovacs and Salomone (1982) the Standard Penetration Test (SPT) is the most widely used. They estimated that up to 80-90 percent of routine foundation designs in the United States are carried out using the N_{SPT} value. The method has been standardized by ASTM D1586 as "Standard Method for Penetration Test and Split-Barrel Sampling of Soil" (Bowles, 1982). The typical setup for the SPT test is shown in Figure 1.

The standard penetration test consists of (Bowles, 1982) using a 140-lb (63.5 kg) driving hammer falling free from a height of 30 inches (762 mm) to hit an anvil, and driving the standard split-barrel sampler a distance of 18 inches (460 mm) into the soil at the bottom of the boring. The N_{SPT} value is the number of blows required to drive the tube the last 12 inches (305 mm). The sampler is pushed a distance of 6 inches to rest it on undisturbed soil with the blow count recorded. The blow count for each of the next two 6-in increments is used as the penetration count unless the last in-

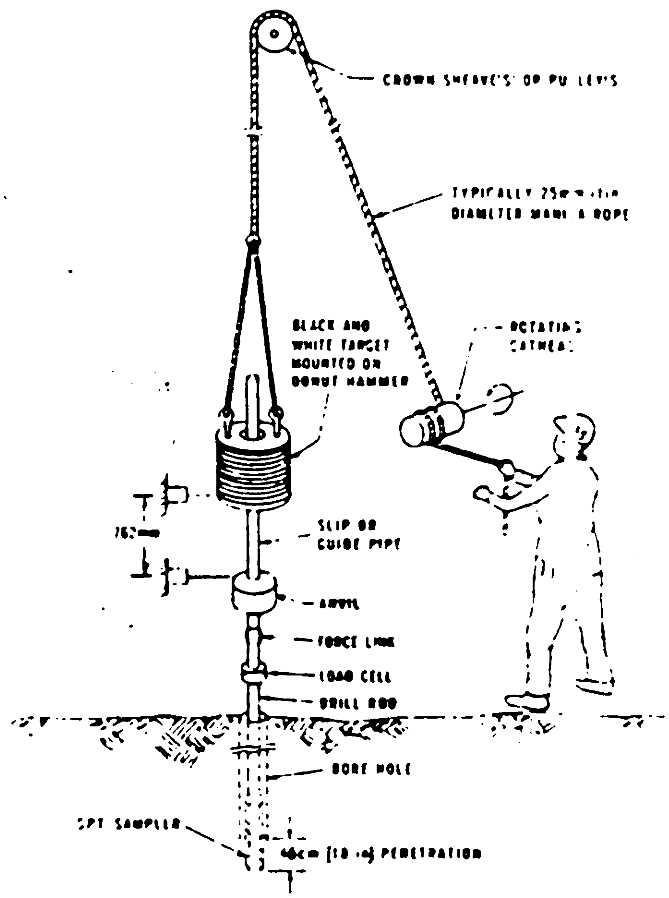


Figure 1. Typical Instrument Set Up for the Standard Penetration Test: (After Kovacs & Salomone, 1982)

crement can not be completed (either from encountering rock or because the blow count exceeds 100). In this case the blow count for the last 12 inches is computed and used as the N_{SPT} value.

Current standard penetration test procedures have some inherent variations between tests that, according to Ireland et al. (1970), makes this test a "non-standard" standard.

Terzaghi and Peck (1948) were among the first to propose a settlement prediction method using the N_{SPT} value. The chart they developed is shown in Figure 2.

Meyerhof (1965) modified Terzaghi and Peck's equation and proposed the following expressions for the settlement of spread footings on sand:

$$S_a = \frac{8P_a}{N_{SPT}} \qquad \text{for } B \leq 4\,ft \qquad (2.4a)$$

or

$$S_a = \frac{12P_a}{N_{SPT}}\left(\frac{B}{B+1}\right)^2 \qquad \text{for } B > 4\,ft \qquad (2.4b)$$

in which S_a = settlement, in inches; P_a = allowable bearing pressure, in tons per square foot; B = footing width, in foot; N_{SPT} = SPT blow count, in blows per foot.

D'Appolonia et al. (1968) conducted a study and found that there were large differences between the measured settlement and the settlement predicted by the above two methods. Standard penetration tests were carried out to investigate the settlement of 50 footings. These footings varied in width from 10-ft to 22-ft, with depth to

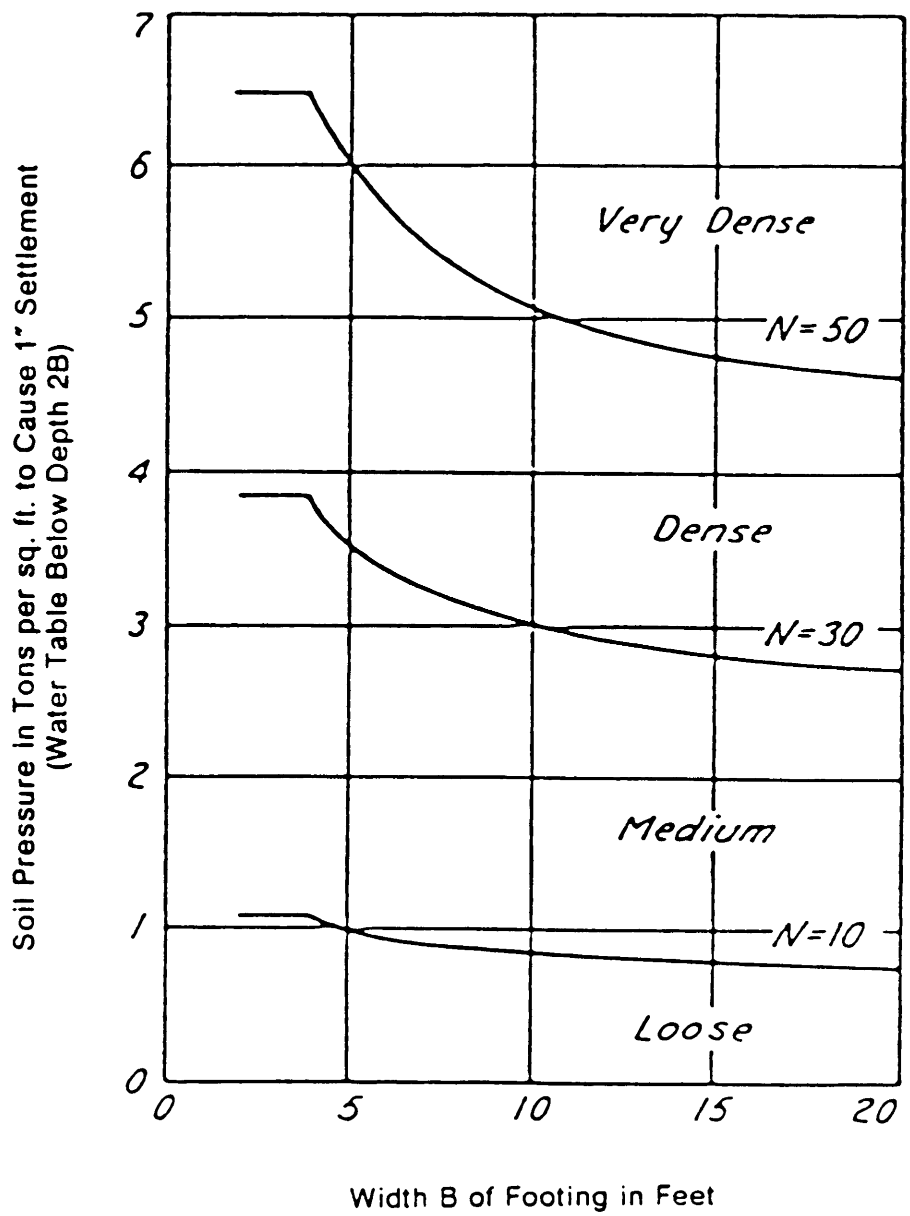


Figure 2. Chart for Estimating Allowable Soil Pressure for Footing on Sand on the Basis of Results of Standard Penetration Test.: (After Terzaghi & Peck, 1948)

width ratios in the range of 0.4 to 1.0. The depth of the ground water level below the base of the footing varied from the footing base to 1.0 B. The loads applied to the footings ranged from 1.5 tsf to 2.5 tsf. The comparison of measured to predicted settlement from this study is shown in Table 4.

Duncan and Buchighnani (1976) proposed a formula based on Terzaghi's theory and Meyerhof's modifications to Terzaghi's equation as

$$S_a = \frac{5P_a}{(N_{SPT} - 1.5)CB} \tag{2.5}$$

in which

$$CB = width\ correction\ factor$$
$$= 1.0 \quad for\ B \leq 4\ ft$$
$$= 1.0 - 0.025(B - 4) \quad for\ 4 < B \leq 12\ ft$$
$$= 0.8 \quad for\ B > 12\ ft$$

As previously mentioned, uncertainties in SPT are unavoidable and can influence the accuracy of Eq. (2.5). The model uncertainty also may contribute to the error in predicting settlement of footings. In this study, Eq. (2.5) will form the basis for predicting the relationship between allowable bearing pressure and settlement, and will be used in the development of performance factors for serviceability limit states.

Besides uncertainties in design equations, soil properties and soil exploration methods, uncertainties may also come from applied loads. Many authors have claimed that different loads have different degrees of uncertainty associated with them (e.g. Allen, 1975, 1981; Galambos et al., 1982; Meyerhof, 1970, 1984, etc). In conventional

Table 4. Comparison of the Ratio of Measured to Predicted Settlement (After D'Appolonia et al., 1968)

Ratio of Measured to Predicted Settlement, N	Prediction Method	
	Terzaghi and Peck	Meyerhof
Mean	0.26	0.52
Standard Deviation	0.11	0.24
Coefficient of Variation	0.42	0.46

geotechnical design, the loads that are applied to the foundation are the unfactored service or working loads. In developing a LRFD format for shallow foundation design, the nature of the uncertainty in different types of loads should be included to account for the different probability of occurrence of these loads. There have been a number of studies on the statistics of bridge loads. A notable example is the work done by Grouni and Nowak (1984) which in part forms the basis for the load factors given in the 1983 edition of Ontario Highway Bridge Design (OHBD) code.

From the above discussion, it is evident that working stress and other deterministic procedures have a number of limitations. Some of the disadvantages of deterministic design procedures as pointed out by MacGregor (1979) are as follows:

1. Failure to justify the variability of loads and resistance,
2. Failure to consider the variations in loadings that increase at different rates, and
3. Failure to take into account the type of failure or the consequences of failure.

Load and Resistance Factor Design (LRFD) was introduced to overcome some of the limitations of working stress design methods. LRFD is based on the concept of limit states, with factors of safety being obtained from a reliability analysis. The LRFD format is expressed as

$$\phi R \geq \sum_{i=1}^{n} \gamma_i Q_i \tag{2.6}$$

where ϕ is the resistance factor, R is the nominal resistance, Q_i is the load effect due to the ith load component, γ_i is the corresponding load factor, and n is the number of load components involved in the limit state under consideration. The resistance and load factors are determined from a reliability analysis. The resistance factor ϕ, which is usually less than unity, together with R reflects the uncertainties associated with R. The γ factors, which are usually greater than unity, reflect the effect of potential overloads and uncertainties inherent in the calculation of load effects. Many countries such as Canada, Denmark and the United States have employed LRFD in some of their design codes. Other countries such as Austria and Japan are developing LRFD codes. Some of the advantages of LRFD (Ellingwood et al., 1980) include:

1. More consistent reliability is achieved for different design cases because the different variabilities of the various design variables are evaluated explicitly and independently.
2. The consequences of failure can be reflected from selected reliability.

3. The designer can gain a better insight of the fundamental structural requirements and of the behavior of the structure in meeting those requirements.
4. It is a tool for incorporating judgment in non-routine situations.
5. It provides a basis for updating standards in a rational manner.

2.3 LIMIT STATES AND UNCERTAINTIES IN SHALLOW FOUNDATION DESIGN

All structures have in common two basic functional requirements, namely, safety and serviceability. Limit states define the various types of collapse and unserviceability that are to be avoided (Allen, 1975). According to Meyerhof (1984), collapse or failure of the structure and failure of the soil are ultimate limit states (ULS). Examples of ultimate limit states include instability by sliding, overturning, bearing capacity failure, uplift, seepage and piping. The occurrence of excessive deformation and deterioration are called serviceability limit states (SLS) and include excessive total or differential settlement, cracking, excessive vibration and corrosion. Although limit state concepts have only recently been introduced, many researchers have claimed that they are not new (Simpson et al., 1981; Ellingwood, 1982). For example, in conventional foundation design, the ultimate limit state is considered in the estimation of bearing capacity, and the serviceability limit state is included in settlement analysis. The occurrence of a limit state is an event that has a probability of failure associated with it. This probability of failure can be evaluated by a systematic analysis of the uncertainties inherent in design models, equations, and soil parameters.

As we know, the actual outcomes in geotechnical engineering problems are to some degree unpredictable due to the random nature of soil. Foundation performance is usually dominated by the spatial average value of soil property over an appropriate domain. According to Tang (1984) there are three types of uncertainties associated with the spatial average soil properties. These are:

1. Inherent variability- This is the actual variability of soil property in a given domain although the variability is reduced somewhat from the point variability through the averaging effect.
2. Statistical uncertainty- This is due to:(i) inherent variability, and (ii) random sampling and measurement errors in tests on a limited number of soil samples. The statistical uncertainty can be reduced through increasing the number of test samples.
3. Systematic uncertainty- This is the uncertainty resulting from the incapability of a test to reproduce the in situ property due to factors such as sample disturbance, size of specimen, and different stress conditions, etc. This error may not be diminished as the number of test samples increases because the same kind of test condition is likely to continue to yield consistently high or low property values. This also applies to the estimation of soil properties using an empirical calibration formula.

In estimating the ultimate bearing capacity of shallow foundations on sand, uncertainties may be due to the model error, footing shape effect, footing depth effect, load inclination effect and soil friction angle ϕ_t. The sources of uncertainty that may result in differences between actual and predicted ultimate bearing capacity are listed in Figure 3.

Figure 3. Sources of Uncertainty in the Prediction of Ultimate Bearing Capacity

SOURCES OF UNCERTAINTY IN THE PREDICTION OF THE ULTIMATE BEARING CAPACITY OF SHALLOW FOUNDATIONS

1. MATHEMATICAL MODELING
   - MODEL SIMPLIFICATION
   - FOOTING SHAPE EFFECT
   - FOOTING DEPTH EFFECT
   - LOAD INCLINATION EFFECT
2. SOIL PROPERTIES
   - FRICTION ANGLE ϕ_f

In settlement analysis, uncertainties may be due to the model error, footing width effect, and inherent soil variability, as well as systematic uncertainties in the SPT method. Simplifying assumptions made in the empirical models for analysis of settlement may result in significant uncertainties. Also, soil properties may be quite different from point to point even in a supposedly homogeneous soil stratum (Vanmarcke, 1977). Systematic uncertainties in the SPT method have been well studied. These uncertainties are from both equipment and test procedures adopted (Orchant et al., 1988.) The sources of uncertainty in settlement analysis are listed in Figure 4.

Due to these unavoidable uncertainties, it is not surprising to observe differences between actual and predicted results for shallow foundations as shown in Tables 3

Figure 4. Sources of Uncertainty in Settlement Analysis

SOURCES OF UNCERTAINTY IN SETTLEMENT

ANALYSIS OF SHALLOW FOUNDATIONS

1. MATHEMATICAL MODELING
   - MODEL SIMPLIFICATION
   - FOOTING WIDTH EFFECT
2. SOIL
   - INHERENT VARIABILITY
   - SYSTEMATIC UNCERTAINTY, SPT

and 4. Probabilistic methods are a convenient and useful tool for dealing with uncertainties in foundation design.

2.4 RELIABILITY-BASED DESIGN

It has been recognized by many authors that probability theory is a valuable tool for dealing with uncertainties in engineering problems and many probability-based design formats have been developed (e.g. Cornell, 1969; Ellingwood et al., 1980; Ravindra et al., 1978; Tang et al., 1976, etc.) In a probabilistic approach, the design parameters are treated as random variables rather than deterministic constants. The safety of a structure may be assured in terms of the probability of failure which is defined as the probability that the system fails to perform its designed function. The probability of failure can be computed from the distributions of the load and resistance.

The probability of failure is determined from a systematic analysis of uncertainties in all the design parameters. It differs from the deterministic safety factor whose derivation depends mainly upon experience and judgment. Although the probabilistic approach is more complicated than the deterministic approach, it is more economical and provides consistent risk levels between different structures and different materials.

A typical reliabilty-based design procedure may consist of the following steps:

1. Develop mathematical expressions for the limit states of interest.
2. Perform uncertainty analyses for all design variables.
3. Compute the reliability index β for the limit states.
4. Define the load and resistance factors for the selected target reliability index β_T.

Typically, the limit state equation for a given problem is expressed as

$$G(x) = R - Q \tag{2.7a}$$

in normal format, or as

$$G(x) = lnR - lnQ \tag{2.7b}$$

in lognormal format; in which R is the resistance and Q is the load effect. In the case of foundation design, R is the ultimate bearing load for the ultimate limit state. For the serviceability limit state, R is the bearing load for specified tolerable settlement.

Several methods have been proposed for computing the reliability index β. The mean value first-order second-moment (MVFOSM) methods were widely employed during early versions of reliability-based design codes for structural design, for example, LRFD for steel (Ravindra and Galambos, 1978). In the MVFOSM, the reliability index is based on the mean and standard deviation (the first and second moment) of the design variables. For the normal format

$$\beta = \frac{\overline{R} - \overline{Q}}{\sqrt{\sigma_R^2 + \sigma_Q^2}} \tag{2.8}$$

For the lognormal format

$$\beta = \frac{\ln \frac{\overline{R}}{\overline{Q}}}{\sqrt{V_R^2 + V_Q^2}} \tag{2.9}$$

in which $\overline{R}$, σ_R, V_R and $\overline{Q}$, σ_Q, V_Q are the mean, standard deviation and coefficient of variation (cov) of resistance and load, respectively. The coefficient of variation is the ratio of the standard deviation to the mean. MVFOSM methods have two basic shortcomings (Hasofer and Lind, 1974) :

1. The limit state function G( ) is linearized at the mean values of the design variables instead of at a point on the failure surface. This can result in considerable errors when G( ) is nonlinear because the higher order terms are neglected during the linearization.
2. Given the same problem, the MVFOSM methods are not invariant for mechanically equivalent formulations.

Advanced FOSM methods have been developed to overcome the limitations of MVFOSM methods. The basic concepts and analysis procedure for the advanced FOSM methods have been presented in Ditlevsen (1974), Ellingwood et al. (1980), Hasofer and Lind (1974), and Rackwitz and Fiessler (1978). For a general case where the limit state function is a function of several variables, the limit state function can be expressed as

$$G(X) = G(X_1, X_2, X_3, \dots, X_n) = 0 \tag{2.10}$$

where $X = (X_1, X_2, X_3, \dots, X_n)$ is the vector of basic state or design variables of the system. The function G(X) determines the performance of the system. Geometrically, the limit state function is a n-dimensional surface called the "failure surface". One side of the failure surface is the safe state which corresponds to $G(X) > 0$, and the other side is the failure state, which corresponds to $G(X) < 0$.

The reliability analysis starts with the transformation of the X_i variables to a space of reduced variables using

$$x_i = \frac{(X_i - \overline{X}_i)}{\sigma_{X_i}} \tag{2.11}$$

where x_i are reduced variables with zero mean and unit variance. The limit state function in reduced variables is

$$G_1(x_1, x_2, x_3, \dots, x_n) = 0 \tag{2.12}$$

Failure occurs when $G_1() = 0$. A two-variable problem is shown in Figure 5. The reliability index is defined as the shortest distance between the origin and the failure surface. The point $(x_1^*, x_2^*, x_3^*, \dots, x_n^*)$ on $G_1() = 0$ corresponding to this shortest distance is called the design or checking point. The design point must be determined by simultaneously solving the following equations

$$\alpha_i = \frac{\frac{\partial G_1}{\partial x_i}}{[\sum (\frac{\partial G_1}{\partial x_i})^2]^{\frac{1}{2}}} \tag{2.13}$$

$$x_i^* = -\alpha_i \beta \tag{2.14}$$

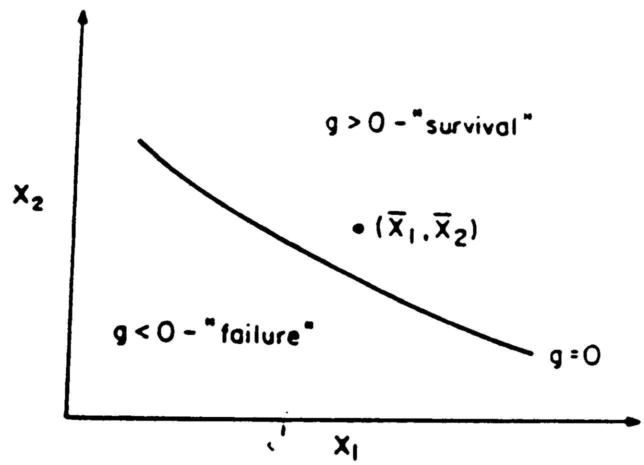
$$G_1(x_1^*, x_2^*, x_3^*, \ldots, x_n^*) = 0 \tag{2.15}$$

and searching for the direction cosines α_i, that minimize β. The partial derivatives are evaluated at the design point.

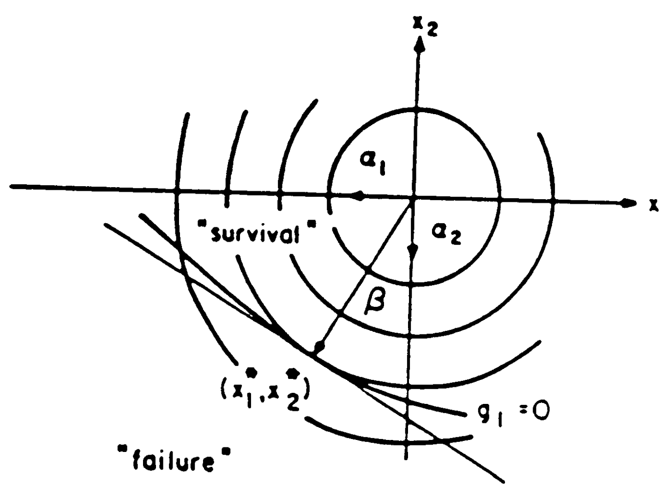
Since many design random variables do not have a normal probability distribution, methods for including information on probability distributions have been developed (Rackwitz and Fiessler, 1978). The basic idea is to transform all non-normal variables into equivalent normal variables before solving the system of equations (2.13) - (2.15).

The algorithm for computing the reliability index using the advanced FOSM methods can be summarized as follows (Ellingwood et al., 1980) :

1. Define the appropriate limit state; $G(X_1, X_2, X_3, \ldots, X_n) = 0$ where the X_i's are the design variables.
2. Make an initial guess of the safety index β.
3. Let the initial checking point $(X_1^*, X_2^*, X_3^*, \ldots, X_n^*)$ be equal to the point at the mean values $(\overline{X}_1, \overline{X}_2, \overline{X}_3, \ldots, \overline{X}_n)$.



(a)



(b)

Figure 5. Safety Analysis of Two-variable Problem Using Advanced Method in (a) Original Coordinates and (b) Reduced Coordinates.

4. Calculate the mean and standard deviation (X_i^N and σ_i^N, respectively) of the equivalent normal distribution for all non-normal variables.
5. Compute the partial derivatives $\frac{\partial G}{\partial X_i}$, evaluated at the checking point X_i^*.
6. Compute the direction cosines, α_i, using Eq. (2.13).
7. Determine new checking point values from : $X_i^* = X_i^N - \alpha_i \beta \sigma_i^N$ and repeat steps 4 to 7 until the estimates of the direction cosines α_i stabilize.
8. Calculate the value of β necessary for $G(x_1^*, x_2^*, x_3^*, \ldots, x_n^*) = 0$, and repeat steps 4 to 8 until the values of β on successive iterations differ by some small tolerance.

A computer program was developed to compute the reliability index using the above procedure.

In order to maintain continuity, it was decided to use the same load factors as those used in the current AASHTO load factor design specifications. The performance factors are determined by using the reliability indices inherent in current working stress design methods for shallow foundations and the same load factors as in the current AASHTO specifications.

Chapter 3

UNCERTAINTY ANALYSIS FOR ULTIMATE LIMIT STATE

3.1 GENERAL

Deterministic methods have traditionally been adopted to predict the ultimate bearing capacity of shallow foundations. The accuracy of the results predicted by these methods depends on a number of factors such as the model used, the choice of soil parameters and the type of shear failure assumed, etc. As indicated in Chapter 2 there are a number of sources of uncertainty. In this chapter, the uncertainties associated with the prediction of the ultimate bearing capacity are investigated.

3.2 MODEL FOR ULTIMATE BEARING CAPACITY PREDICTION

This study is limited to shallow foundations on sand since most bridge shallow foundations are on sand. When other types of soil such as clay is encountered, deep foundations or pile foundations are used instead of shallow foundations. From the comparison presented in Chapter 2, it can be observed that Hansen's equation exhibits less scatter than Terzaghi's and Meyerhof's equation in predicting ultimate bearing capacity of shallow foundations on sand. Using Hansen's equation as the basis for the analysis, the ultimate bearing capacity for a shallow footing on sand can be expressed as:

$$R = \{[0.5\gamma B S_\gamma d_\gamma I_\gamma N_\gamma(\phi_f)] + [\gamma D S_q d_q I_q N_q(\phi_f)]\}A \qquad (3.1)$$

In Eq. 3.1, N_γ and N_q are bearing capacity factors and are functions of the angle of internal friction ϕ_f ; S_γ and S_q are shape factors; d_γ and d_q are depth factors; I_γ and I_q are load inclination factors; B and D are the width and depth of the footing and A is the footing area, L is the footing length and i = inclination angle of load. These factors are defined as

$$N_\gamma = \exp(-2.445 + 0.172\phi) \qquad (3.2a)$$

$$N_q = \exp(-1.078 + 0.133\phi) \qquad (3.2b)$$

$$S_\gamma = 1 - 0.4 \times \frac{B}{L} \qquad (3.2c)$$

$$S_q = 1 + \sin \phi_f \times \frac{B}{L} \tag{3.2d}$$

$$I_y = I_q = (1 - 0.5 \tan i)^5 \tag{3.2e}$$

$$d_q = 1 + 2 \tan \phi_f \times (1 - \sin \phi_f)^2 \times (\frac{D}{B}) \qquad \frac{D}{B} \leq 1 \tag{3.2f}$$

$$d_y = 1 \tag{3.2g}$$

Introducing model correction factors N_1 and N_2 for bearing capacity due to weight and surcharge gives

$$R = \{N_1 \times [0.5\gamma B S_y d_y I_y N_y(\phi_f)] + N_2 \times [\gamma D S_q d_q I_q N_q(\phi_f)]\}A \tag{3.3}$$

The model correction factors N_1 and N_2 can further be expressed as products of component correction factors N_j 's accounting for modeling errors in N_γ, S_γ, I_γ and N_k's accounting for modeling errors in N_q, S_q, d_q and I_q, respectively. Thus,

$$N_1 = \prod_{j=1}^{m} N_j \tag{3.4a}$$

$$N_2 = \prod_{k=1}^{n} N_k \tag{3.4b}$$

where $\prod$ denotes the product of N_j's and N_k 's. Each of these factors has a mean value equal to $\overline{N}_j$ or $\overline{N}_k$ and a coefficient of variation Ω_{N_j} or Ω_{N_k}. Thus, the expression for the bearing capacity can be written as

$$R = \{[\prod_{j=1}^{m} N_j][0.5\gamma B S_\gamma d_\gamma I_\gamma N_\gamma(\overline{\phi}_f)] + [\prod_{k=1}^{n} N_k][\gamma D S_q d_q I_q N_q(\overline{\phi}_f)]\}A \quad (3.5)$$

The uncertainty in the estimation of the soil parameter ϕ_f can also have a considerable influence on the accuracy of the predicted ultimate bearing capacity. The unit weight of soil, γ, has a small coefficient of variation and therefore can be treated as a deterministic variable.

The mean value of the ultimate bearing capacity R can be expressed as

$$\overline{R} = \{[\prod_{j=1}^{m} \overline{N}_j][0.5\gamma B S_\gamma d_\gamma I_\gamma N_\gamma(\overline{\phi}_f)] + [\prod_{k=1}^{n} \overline{N}_k][\gamma D S_q d_q I_q N_q(\overline{\phi}_f)]\}A \quad (3.6)$$

and the variance of R is given by

$$\begin{aligned}
\sigma_R^2 &= \{[\sum_{j=1}^{m}(\Omega_{N_j}^2 \times \overline{N}_j^2)][0.5\gamma B S_\gamma d_\gamma I_\gamma N_\gamma(\overline{\phi}_f)]^2\}A^2 \\
&+ \{[\sum_{k=1}^{n}(\Omega_{N_k}^2 \times \overline{N}_k^2)][\gamma D S_q d_q I_q N_q(\overline{\phi}_f)]^2\}A^2 \\
&+ \{[\prod_{j=1}^{m} \overline{N}_j][0.5\gamma B S_\gamma d_\gamma I_\gamma \frac{\partial N_\gamma}{\partial \phi_f}] + [\prod_{k=1}^{n} \overline{N}_k][\gamma D S_q d_q I_q \frac{\partial N_q}{\partial \phi_f}]\}^2(\sigma_{\phi_f} \times A)^2
\end{aligned} \quad (3.7)$$

in which $\overline{\phi}_f$ and σ_{ϕ_f} are the mean and standard deviation of the friction angle ϕ_f.

3.3 UNCERTAINTY ANALYSIS

In this section the analysis of uncertainties in the various parameters that affect the ultimate bearing capacity of shallow foundations on sand is presented. The model error due to weight effect in the equation for predicting the ultimate bearing capacity is incorporated by introducing a random correction factor N_1 as in

$$R_r = N_1[0.5\gamma B I_\gamma N_\gamma(\phi_f)] \qquad (3.8)$$

where R_r = the contribution to the bearing capacity from the first term on the right hand side of Eq. 3.1. Introducing component correction factors N_3, N_4 and N_5 to account for the model errors in N_γ , S_γ, I_γ, we get

$$R_r = (0.5\gamma B d_\gamma)(N_3 N_\gamma)(N_4 S_\gamma)(N_5 I_\gamma) \qquad (3.9)$$

The model error due to surcharge effect in the equation for the ultimate bearing capacity is incorporated by introducing a random correction factor N_2 as in

$$R_q = N_2(\gamma D S_q d_q I_q N_q(\phi_f)) \qquad (3.10)$$

in which R_q = the contribution to the bearing capacity from the second term on the right hand side of Eq. 3.1. Since S_q and d_q have less influence than N_q on the overall bearing capacity, they can be treated as deterministic variables. Introducing component correction factors N_5 and N_6 to account for model errors in N_q and I_q gives

$$R_q = (\gamma D)(N_6 N_q S_q d_q)(N_5 I_q) \qquad (3.11)$$

Ingra and Baecher (1983) collected data on bearing capacity tests to study the model error for surface footings on sand. De Beer (1970) also employed some experiments to determine the shape factors and the bearing capacity factors of sand. A total of 662 tests were performed with dry Mol sand, 350 at the surface, and 312 with an overburden pressure. The model correction factors for N_γ, S_γ, I_γ and $N_q S_q d_q$ as given by Eq 3.2a to Eg.3.2g can be determined using the data from these studies. The original data and computed values for N_γ, S_γ, I_γ and $N_q S_q d_q$ are listed in Appendix A through D.

Twelve data for ϕ_f ranging from 36° to 47° were used to study the model error of N_γ . If the ratios of measured N_γ to predicted N_γ are arranged in ascending order as shown in Table 5 and plotted on normal probability paper as shown in Figure 6, the mean and standard deviation of the underlying population can be determined graphically from the line of the best fit. The ratio of the measured over the predicted value, which is denoted as N, on the line corresponding to $\Phi_s(s) = 0.50$ is the estimate of the mean value $\bar{N}$, whereas the slope of the line is the estimate of the standard deviation s_N; thus, $s_N \simeq N_{0.84} - \bar{N}$. The mean ratio of measured N_γ to predicted N_γ, $\bar{N}_3$, of sample data is 1.05 and sample standard deviation, s_{N_3}, is 0.11. The coefficient of variation, δ_{N_3}, therefore is equal to $\frac{0.11}{1.05} = 0.10$. The random error of $\bar{N}_3$ is the standard error of $\bar{N}_3$, which is $\sigma_{\bar{N}_3} = \frac{0.11}{\sqrt{12}} = 0.03$ and the uncertainty associated with random sampling error is $\Delta_{N_3} = \frac{0.03}{1.05} = 0.03$. Hence, the total uncertainty in the prediction of N_3 becomes $\Omega_{N_3} = \sqrt{0.1^2 + 0.03^2} = 0.1$.

Table 5. Ratio of Measured over Predicted N_Y

m	$\frac{(N_\gamma)_m}{(N_\gamma)_p}$	$\frac{m}{N+1}$
1	0.96	0.077
2	0.96	0.154
3	0.97	0.231
4	0.99	0.308
5	0.99	0.385
6	1.0	0.462
7	1.03	0.538
8	1.05	0.615
9	1.10	0.692
10	1.16	0.769
11	1.18	0.846
12	1.28	0.923

Thirty-eight values with $\frac{B}{L}$ ratios ranging from 0.12 to 1 were used to study the model error of S_γ. If the ratios of measured S_γ over predicted S_γ are arranged in ascending order as shown in Table 6 and plotted on normal probability paper as shown in Figure 7, the sample mean, $\overline{N}_4$, is 1.2 and the sample standard deviation, s_{N_4}, is 0.2. The coefficient of variation, δ_{N_4}, is equal to $\frac{0.2}{1.2} = 0.167$. The random error of $\overline{N}_4$, $\sigma_{\overline{N}_4}$, is $\frac{0.2}{\sqrt{38}} = 0.03$, and the uncertainty associated with the random sampling error is $\Delta_{N_4} = \frac{0.03}{1.2} = 0.025$. The total uncertainty in the prediction of N_4 becomes $\Omega_{N_4} = \sqrt{0.167^2 + 0.03^2} = 0.17$.

Thirty-three data for tan i ranging from 0.11 to 0.58 were used to study the model error of I_γ. If the ratios of measured I_γ over predicted I_γ are arranged in ascending order

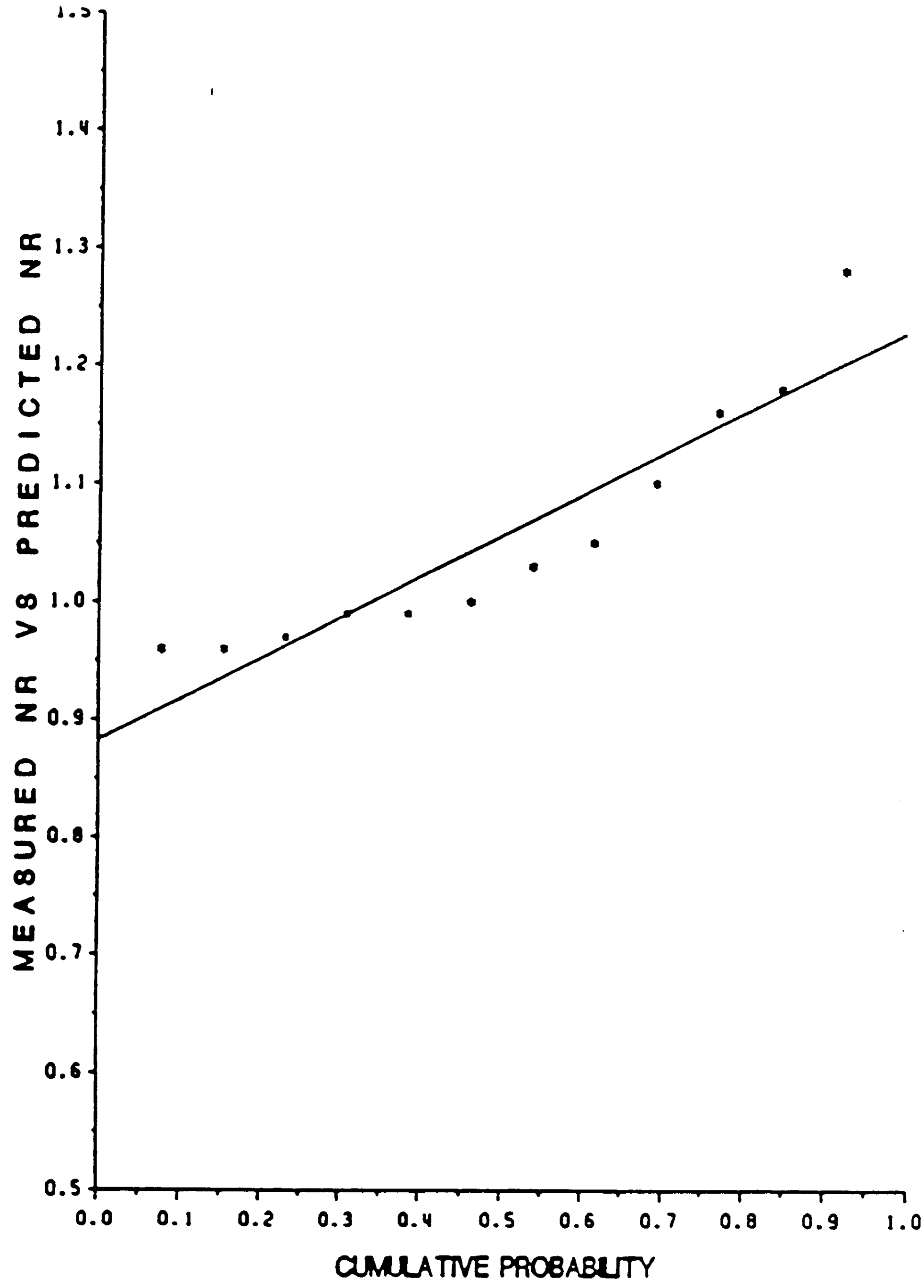


Figure 6. Measured over Predicted N_γ Plotted on normal Probability Paper

Table 6. Ratio of Measured over Predicted S_Y

m	$\frac{(S_Y)_m}{(S_Y)_p}$	$\frac{m}{N+1}$
1	0.94	0.026
2	0.97	0.051
3	1.0	0.077
4	1.03	0.103
5	1.03	0.128
6	1.04	0.154
7	1.05	0.179
8	1.05	0.205
9	1.05	0.231
10	1.07	0.256
11	1.08	0.282
12	1.09	0.308
13	1.09	0.333
14	1.10	0.359
15	1.11	0.385
16	1.11	0.410
17	1.12	0.436
18	1.15	0.462
19	1.17	0.487
20	1.17	0.513
21	1.18	0.538
22	1.20	0.564
23	1.20	0.590
24	1.20	0.615
25	1.22	0.641
26	1.23	0.667
27	1.23	0.692
28	1.25	0.718
29	1.33	0.744
30	1.34	0.769
31	1.37	0.795
32	1.38	0.821
33	1.38	0.846
34	1.40	0.872
35	1.42	0.897
36	1.50	0.923
37	1.67	0.949
38	1.70	0.974

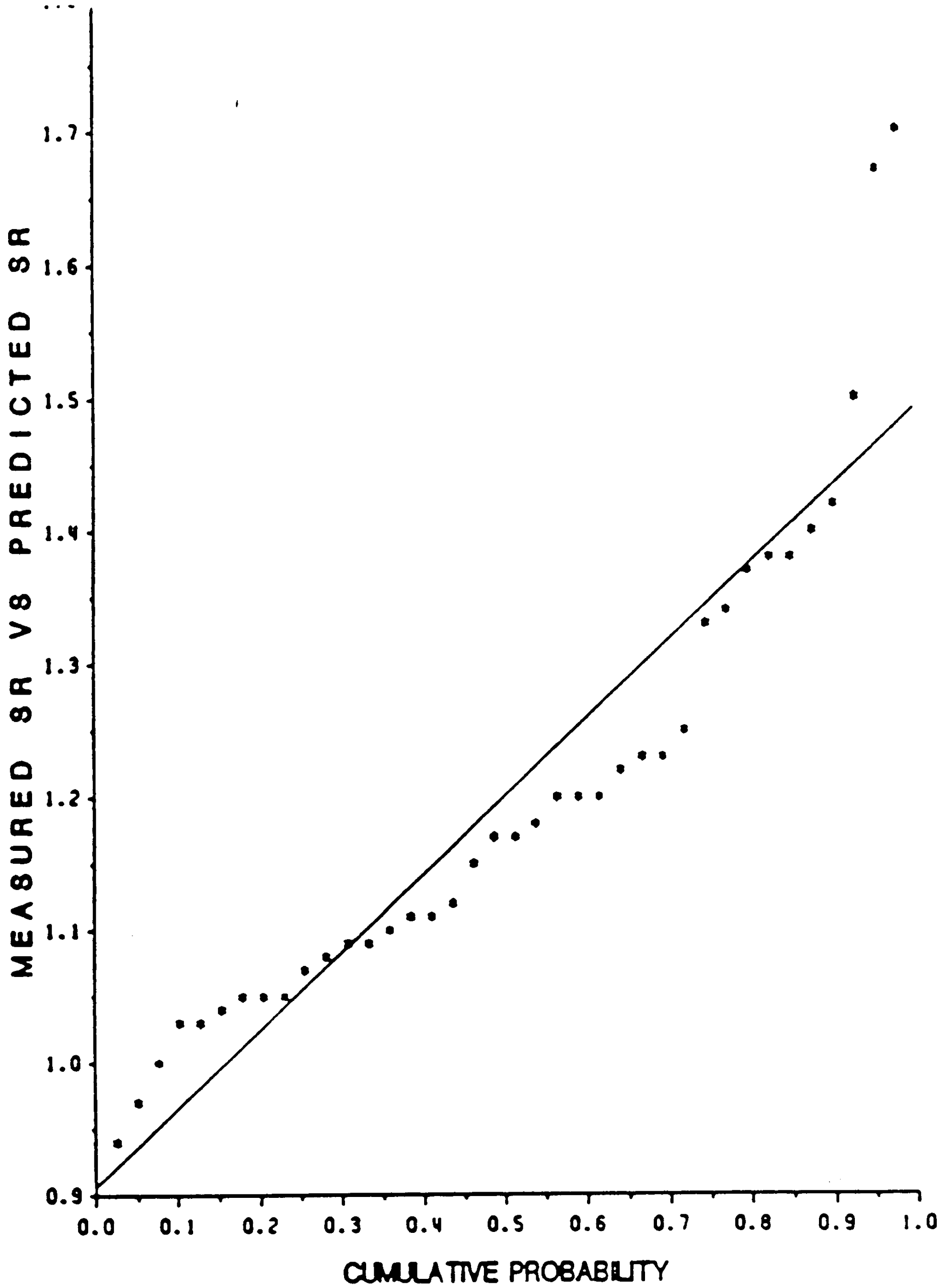


Figure 7. Measured over predicted S_Y Plotted on Normal Probability Paper

as shown in Table 7 and plotted on the normal probability paper as in Figure 8, the sample mean, $\overline{N}_5$, is 0.95 and the sample standard deviation, s_{N_5}, is 0.19. The coefficient of variation, δ_{N_5}, is equal to $\frac{0.19}{0.95} = 0.2$. The random error of $\overline{N}_5$, $\sigma_{\overline{N}_5}$, is $\frac{0.19}{\sqrt{33}} = 0.03$ and the uncertainty associated with random sampling is $\Delta_{N_5} = \frac{0.03}{0.95} = 0.03$ The total uncertainty in the prediction of N_5 is then $\Omega_{N_5} = \sqrt{0.2^2 + 0.03^2} = 0.2$.

Data with $\frac{B}{L}$ ratios ranging from 1 to $\frac{1}{6}$, and ϕ_f ranging from 33° to 41° were used to study the model error of $N_qS_qd_q$. If the ratios of measured $N_qS_qd_q$ over predicted $N_qS_qd_q$ are arranged in ascending order as shown in Table 8 and plotted on normal probability paper as in Figure 9, the sample mean, $\overline{N}_6$, is 0.93 and the sample standard deviation, s_{N_6} , is 0.15. The coefficient of variation is equal to $\frac{0.15}{0.93} = 0.16$. The random error of $\overline{N}_6$, $\sigma_{\overline{N}_6}$, is $\frac{0.15}{\sqrt{16}} = 0.0375$ and the uncertainty associated with the random sampling is $\Delta_{N_6} = \frac{0.0375}{0.93} = 0.04$. The total uncertainty in the prediction of N_6 is $\Omega_{N_6} = \sqrt{0.16^2 + 0.04^2} = 0.16$.

The mean values and coefficients of variation for the correction factors are summarized in Table 9.

Table 7. Ratio of Measured over Predicted I_Y

m	$\frac{(I_y)_m}{(I_y)_p}$	$\frac{m}{N+1}$
1	0.60	0.03
2	0.60	0.06
3	0.61	0.09
4	0.66	0.12
5	0.67	0.15
6	0.67	0.18
7	0.76	0.21
8	0.81	0.24
9	0.82	0.26
10	0.85	0.29
11	0.86	0.32
12	0.88	0.35
13	0.89	0.38
14	0.89	0.41
15	0.89	0.44
16	0.90	0.47
17	0.90	0.5
18	0.92	0.53
19	0.94	0.56
20	0.95	0.59
21	1.0	0.62
22	1.04	0.65
23	1.1	0.68
24	1.11	0.71
25	1.13	0.74
26	1.15	0.76
27	1.15	0.79
28	1.16	0.82
29	1.17	0.85
30	1.19	0.88
31	1.19	0.91
32	1.21	0.94
33	1.27	0.97

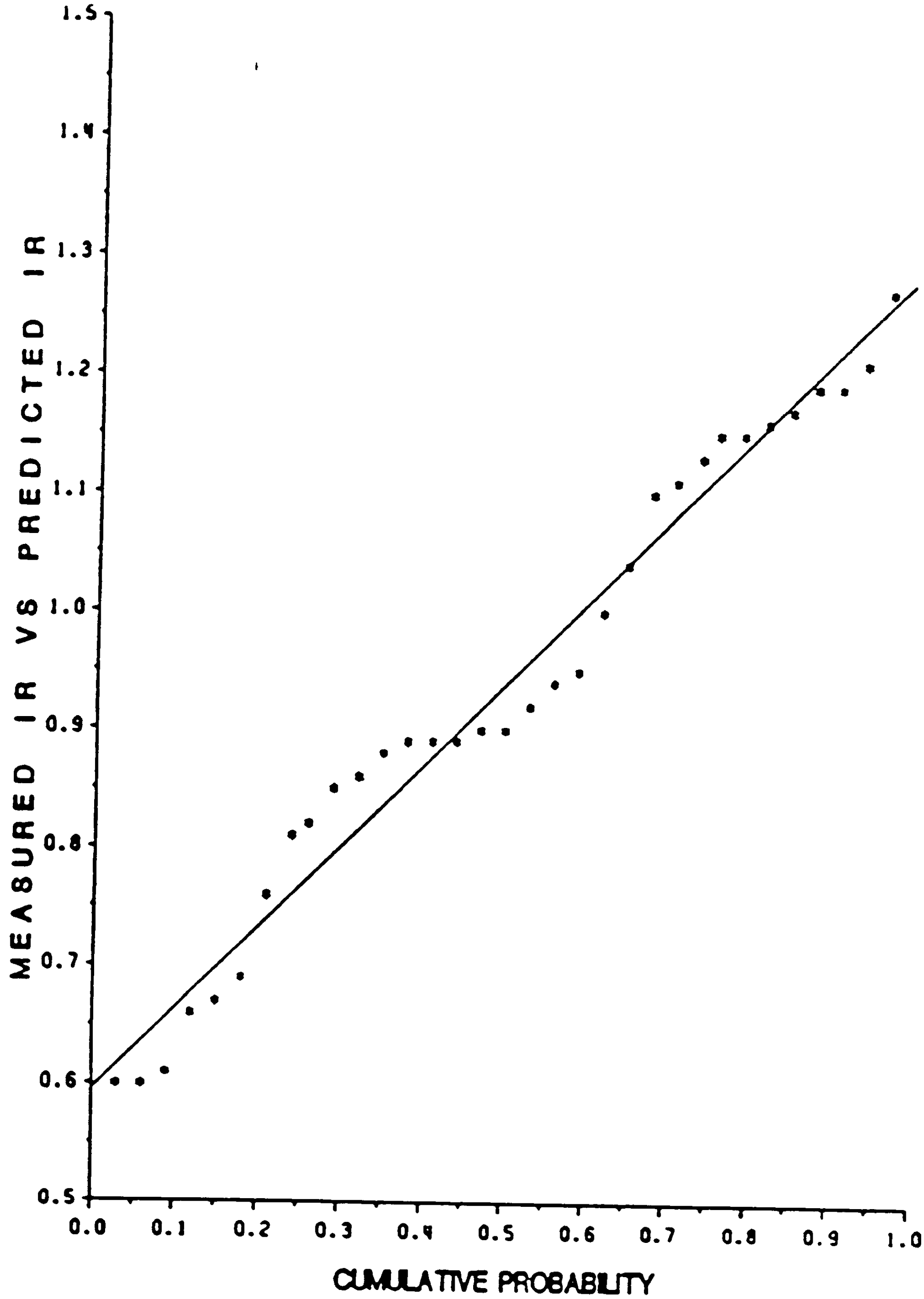


Figure 8. Measured over Predicted I_Y Plotted on Normal Probability Paper

Table 8. Ratio of Measured over Predicted $N_q S_q d_q$

m	$\dfrac{(N_q S_q d_q)_m}{(N_q S_q d_q)_p}$	$\dfrac{m}{N+1}$
1	0.77	0.06
2	0.78	0.12
3	0.78	0.18
4	0.81	0.24
5	0.81	0.29
6	0.82	0.35
7	0.82	0.41
8	0.87	0.47
9	0.87	0.53
10	0.92	0.59
11	0.95	0.65
12	1.01	0.71
13	1.04	0.76
14	1.12	0.82
15	1.18	0.88
16	1.26	0.94

3.4 UNCERTAINTY FROM SOIL PARAMETER

Friction angle of sand is usually correlated with the standard penetration resistance, N_{SPT}, value. Thus, the uncertainty in ϕ could be estimated from a statistical analysis of the N_{SPT} value. However, this procedure is tedious and the inherent spatial variability of soil and the systematic error of SPT are not taken into account. Thus, an alternate way is to directly investigate the uncertainty in the friction angle of sand.

Singh and Lee (1970) studied the variability in soil properties and investigated the effect of such a variation on the design of earth structures. Twenty-eight values of ϕ_f for both loose and dense Ottawa sand as determined by direct shear tests were analyzed. Two extremely unreasonable values were discarded. The unit weights

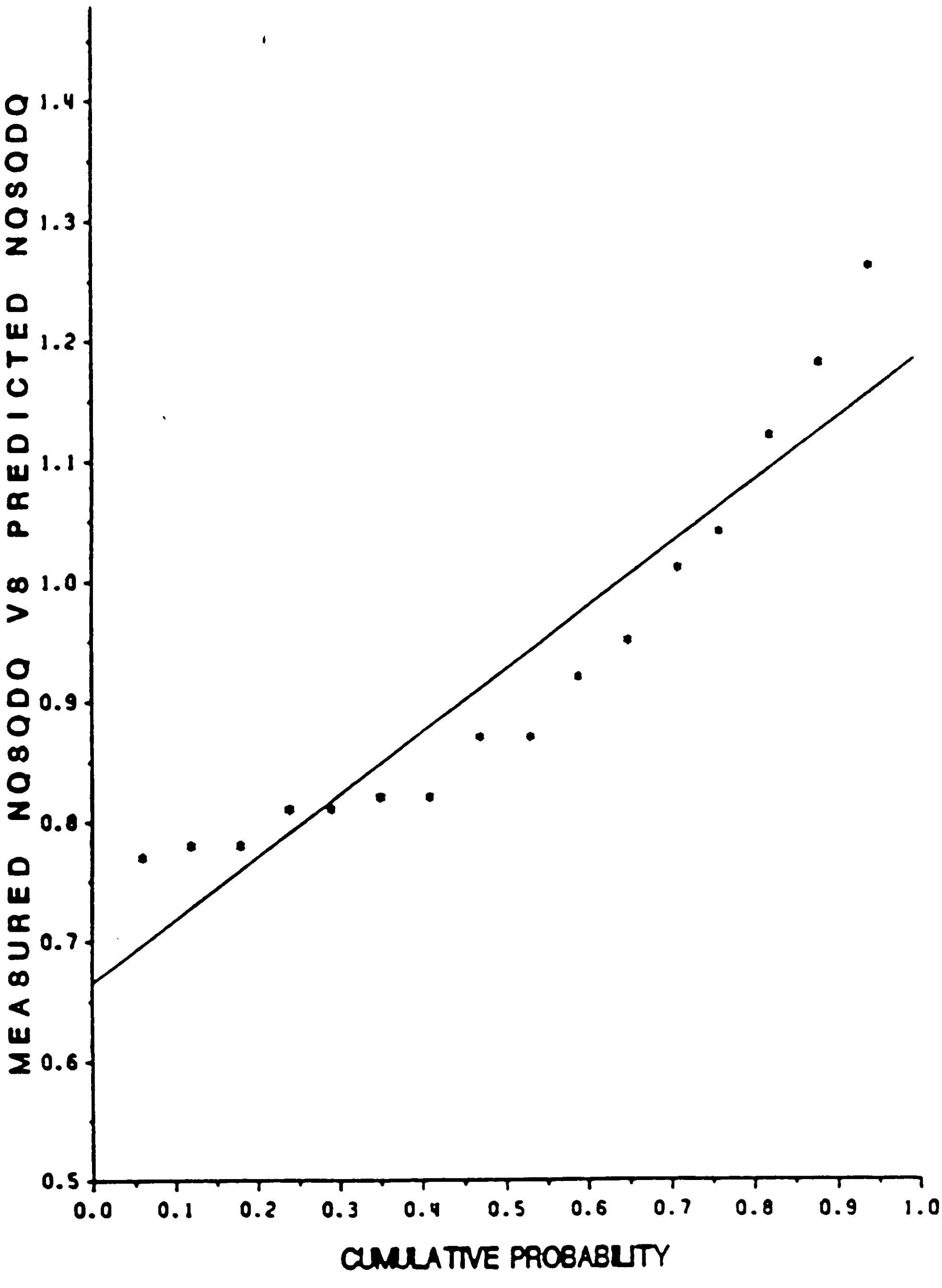


Figure 9. Measured vs. Predicted $N_qS_qd_q$ Plotted on Normal Probability Paper

Table 9. Summary of Model Errors for Ultimate Bearing Capacity

Variable	Correction Factor	Mean	C.O.V.
N_γ	N_3	1.05	0.1
S_γ	N_4	1.2	0.17
I_γ	N_5	0.95	0.2
$N_\gamma S_\gamma I_\gamma$	N_1	1.2	0.28
I_q	N_5	0.95	0.2
$N_q S_q d_q$	N_6	0.93	0.16
$N_q S_q d_q I_q$	N_2	0.88	0.26

ranged from 88 pcf to 103 pcf for loose sand and from 93 pcf to 122 pcf for dense sand. Six triaxial tests were also performed to compare the friction angle obtained from triaxial tests with the values obtained from direct shear tests. Results from these two different tests were found out to be in good agreement. The tests on dense sand are shown in Table 10, while the results for loose sand are shown in Table 11. From the analysis of the above data, the mean friction angle of loose sand can be estimated to be 30° with a c.o.v. of 0.17. The mean friction angle of dense sand is 36° with a c.o.v. of 0.18. The statistics of friction angle of sand are summarized in Table 12. The results can be assumed to be the spatial statistics of sand.

Because of the plain strain effect, a large value of ϕ_f should be used for long footings (Meyerhof, 1965). The relationship can be expressed as follows:

$$\phi_{BC} = \phi_t(1.1 - 0.1\frac{B}{L}) \tag{3.12}$$

where ϕ_t is the value of ϕ obtained from triaxial tests and ϕ_{BC} is the value of ϕ used for computing bearing capacity.

Table 10. Results of Direct Shear Test on Dense Sand (After Singh and Lee, 1970)

Interval	Number of Observations	Fraction of total Observation
25°~27°	1	0.038
27°~29°	1	0.038
29°~31°	0	0
31°~33°	3	0.116
33°~35°	4	0.154
35°~37°	2	0.077
37°~39°	6	0.231
39°~41°	6	0.231
41°~43°	2	0.077
43°~45°	0	0
45°~47°	0	0
47°~49°	0	0
49°~51°	1	0.038
	Total = 26	1.00

Table 11. Results of Direct Shear Test on Loose Sand (After Singh and Lee, 1970)

Interval	Number of Observations	Fraction of total Observation
17°~19°	1	0.038
19°~21°	0	0
21°~23°	0	0
23°~25°	2	0.077
25°~27°	2	0.077
27°~29°	2	0.077
29°~31°	3	0.115
31°~33°	10	0.385
33°~35°	3	0.115
35°~37°	2	0.077
37°~39°	0	0.0
39°~41°	1	0.038
	Total = 26	1.00

3.5 SUMMARY OF UNCERTAINTIES IN BEARING CAPACITY PREDICTION

In the previous sections, uncertainty analyses of the major factors affecting ultimate bearing capacity for shallow foundations on sand were presented. The statistics of the model error and friction angle, ϕ_f , for the prediction of the ultimate bearing capacity are summarized in Table 13. The overall uncertainty of ultimate bearing capacity of shallow foundations on sand then can be evaluated using equations 3.6 and 3.7.

Table 12. Statistics of Friction Angle of Sand

Soil Type	$\bar{\phi}_f$	σ_{ϕ_f}	Ω_{ϕ_f}
Loose Sand	30°	5°	0.17
Dense Sand	36°	6.5°	0.18

Table 13. Summary of Uncertainties in Prediction of Ultimate Bearing Capacity

Factor	Mean	C.O.V.
$N_1(N_\gamma S_\gamma I_\gamma)$	1.2	0.28
$N_2(N_q S_q I_q d_q)$	0.88	0.26
Friction Angle, ϕ_f		
Loose Sand	30	0.17
Dense Sand	36	0.18

Chapter 4

UNCERTAINTY ANALYSIS FOR SERVICEABILITY LIMIT STATE

4.1 GENERAL

Settlement is an important consideration in the design of bridge foundations. In fact, for most shallow foundations, settlement controls the design rather than bearing capacity. In this chapter the model for the serviceability limit state is presented. Then the results of the investigation of model uncertainty, inherent variability and systematic error is presented. To evaluate the individual contribution of inherent spatial variability, N_i , and systematic error, N_s, to the overall uncertainties, an equivalent random field model for combined correction factors N_iN_s is developed and the relative contribution of N_i and N_s is studied. The final section presents a summary of the results of the uncertainty analysis.

4.2 MODEL FOR SERVICEABILITY LIMIT STATE

As mentioned in Chapter 2, Eq 2.5 is used as the basis for the uncertainty analysis for serviceability limit state. The bearing load for a shallow foundation on sand for given tolerable settlement can be expressed as

$$R = [\frac{(N_{SPT} - 1.5)CB \times S}{5}] \times A \times 2 \qquad in\ Kips \tag{4.1}$$

where S = given tolerable settlement, in inches, A = footing area, in square feet, N_{SPT} = SPT blow count, in blows per foot, and CB = width correction factor.

Introducing a lumped model correction factor N for the settlement model, width correction factor, inherent spatial variability and systematic error gives

$$R = N[\frac{(N_{SPT} - 1.5)CB \times S}{5}]A \times 2 \qquad in\ Kips \tag{4.2}$$

The model correction factor N can be expressed as the product of component correction factors N_p's accounting for model error, error in CB, systematic error of standard penetration resistance, and inherent spatial variability. Thus,

$$N = \prod_{p=1}^{t} N_p \tag{4.3}$$

Each of these factors has a mean value equal to $\overline{N}_p$ and coefficient of variation Ω_{N_p}. Thus, the expression of bearing load for settlement analysis can be written as

$$R = [\prod_{p=1}^{t} N_p][\frac{(N_{SPT} - 1.5)CB \times S}{5}] \times A \times 2 \qquad in\ Kips \tag{4.4}$$

To evaluate the variance of R, it is easier to rewrite as

$$R = [\prod_{p=1}^{t} N_p]R_1 \tag{4.5}$$

in which $R_1 = \frac{(N_{SPT} - 1.5) \times CB \times S \times 2}{5} A$. The effect of random systematic error and inherent spatial variability of soil parameter on bearing load is incorporated in the evaluation of the statistics of R_1. However, it is desirable that each of these effects be represented by one random variable similar to N_p so that the influence of each factor on the bearing load can be conveniently investigated and measured. Therefore, the bearing load for given tolerable settlement can be written as

$$R_N = \frac{(\mu_{SPT} - 1.5)\overline{CB} \times S \times A \times 2}{5} \tag{4.6}$$

where μ_{SPT} is the mean value of N_{SPT}. Observe that R_N is simply the bearing load determined by Eq. 4.4 with the penetration resistance, N_{SPT}, and CB set equal to their mean values, μ_{SPT} and $\overline{CB}$, and all model correction factors set equal to one. In fact, R_N is the deterministic bearing load. Thus, the bearing load in Eq. 4.4 can be expressed as

$$R = [\prod_{p=1}^{t} N_p]\{\frac{R_1}{R_N}\}R_N$$
$$= [\prod_{p=1}^{t} N_p]N_i N_s R_N \qquad (4.7)$$
$$= [\prod_{p=1}^{w=t+2} N_p]R_N$$

in which the product of $N_i N_s = \frac{R_1}{R_N}$; and N_i and N_s represent the individual effect of inherent spatial variability and systematic error of soil properties on the bearing load. Thus, the mean value of bearing capacity in Eq. 4.7 can be expressed as

$$\bar{R} = [\prod_{p=1}^{w} \bar{N}_p]R_N \qquad (4.8)$$

and the variance of R is given by

$$\sigma_R^2 = \sum_{p=1}^{w}(\Omega_{N_p}^2 \times \bar{N}_p^2 \times (\frac{\partial R}{\partial N_p})^2) \qquad (4.9)$$

where w = t + 2, based on the assumption that all the correction factors are statistically independent random variables.

4.3 MODEL UNCERTAINTY

In a probabilistic approach, the model error is assumed to be dominated by the penetration resistance N_{SPT}. Thus, the model error N_m in the bearing capacity equation may be evaluated from measured pressure (q_o) obtained from test data

$$N_m = \frac{q_o}{\frac{(N_{SPT} - 1.5)CB \times S}{5}} \tag{4.10}$$

provided that N_{SPT}, S and foundation dimensions are available.

For this study, a data base of 200 records of settlement of foundation collected by Burland and Burbridge (1985) was used to estimate the model error. The data set included observed settlements of shallow foundations estimated using different types of testing methods, including SPT, CPT (cone penetration test), oedometer and plate loading test. Only the data from tests performed using the SPT were analyzed. Data which gave extremely large or extremely small measured over predicted ratios of bearing pressure were excluded from this analysis. A ratio of measured over predicted bearing pressure greater than 2.5 is considered extremely large while ratio smaller than 0.5 is considered extremely small. A total of 27 values was included in this study. The ratio of measured to predicted bearing pressure, N_m, ranged from 0.55 to 2.47 as shown in Table 14. The mean, $\overline{N}_m$, of sample data is 1.4 and sample standard deviation s_{N_m} is 0.55. The coefficient of variation δ_{N_m}, therefore, is equal to

$0.55/1.4 = 0.393$. The random error of $\overline{N}_m$, $\sigma_{\overline{N}_m}$, is $\frac{0.55}{\sqrt{27}} = 0.106$ and the uncertainty associated with the random sampling error is $\Delta_{N_m} = 0.106/1.4 = 0.076$. Hence, the total uncertainty in the prediction of N_m becomes $\Omega_{N_m} = \sqrt{0.393^2 + 0.076^2} = 0.4$.

The expression enclosed in the second square bracket in Eq. 4.4 was obtained by interpolation from the curves shown in Figure 10 originally developed by Terzaghi and Peck (1948). Thus, errors may exist when Eq. 4.4 is used. It is reasonable to assume that the curves represent the real values of bearing pressure because the original data base was based on many observations. The variability can be evaluated by drawing three straight lines as shown in Figure 10 according to the values of CB in Eq. 2.5 and comparing the values of curves with those of straight lines. The mean $\overline{N}_{CB}$ is observed to be slightly greater than unity. Therefore, it is reasonable to assume $\overline{N}_{CB}$ equal to 1 with a coefficient of variation $\Omega_{N_{CB}}$ equal to 0.03. Because Eq. 4.4 is obtained from interpolation, the analysis above represents the best estimate of the actual mean value of CB and its corresponding uncertainty.

4.4 INHERENT SPATIAL VARIABILITY AND SYSTEMATIC ERROR

Sidi (1986) studied the inherent spatial variability and systematic error for pile foundations in clay. The technique he used, "random field model" (see e.g. Vanmarcke, 1977), is employed in this study to obtain statistics of the spatial average soil property of shallow foundations on sand.

Table 14. Ratio of Measured to Predicted Bearing Pressure

m	N_m	$\frac{m}{m+1}$
1	0.55	0.0357
2	0.63	0.0714
3	0.68	0.1071
4	0.72	0.1429
5	0.77	0.1786
6	0.78	0.2143
7	0.88	0.25
8	0.93	0.2857
9	0.96	0.3214
10	1.03	0.3571
11	1.13	0.3929
12	1.32	0.4286
13	1.42	0.4643
14	1.5	0.5
15	1.53	0.5357
16	1.56	0.5714
17	1.66	0.6071
18	1.73	0.6429
19	1.79	0.6786
20	1.82	0.7143
21	1.85	0.75
22	1.89	0.7857
23	1.89	0.8214
24	1.93	0.8571
25	2.0	0.8929
26	2.26	.9286
27	2.47	0.9643

Soil parameters are partly correlated between any two points within the scale of fluctuation in both horizontal and vertical directions (Vanmarcke, 1977). Besides the spatial variation, uncertainties in soil properties also include systematic errors. Therefore, a model that is able to incorporate both inherent spatial variability and systematic error is needed for a realistic evaluation of these uncertainties. In the next section, an attempt is made to express these new components of uncertainties in a form so that their statistics could be easily combined with those obtained earlier for the other modeling errors.

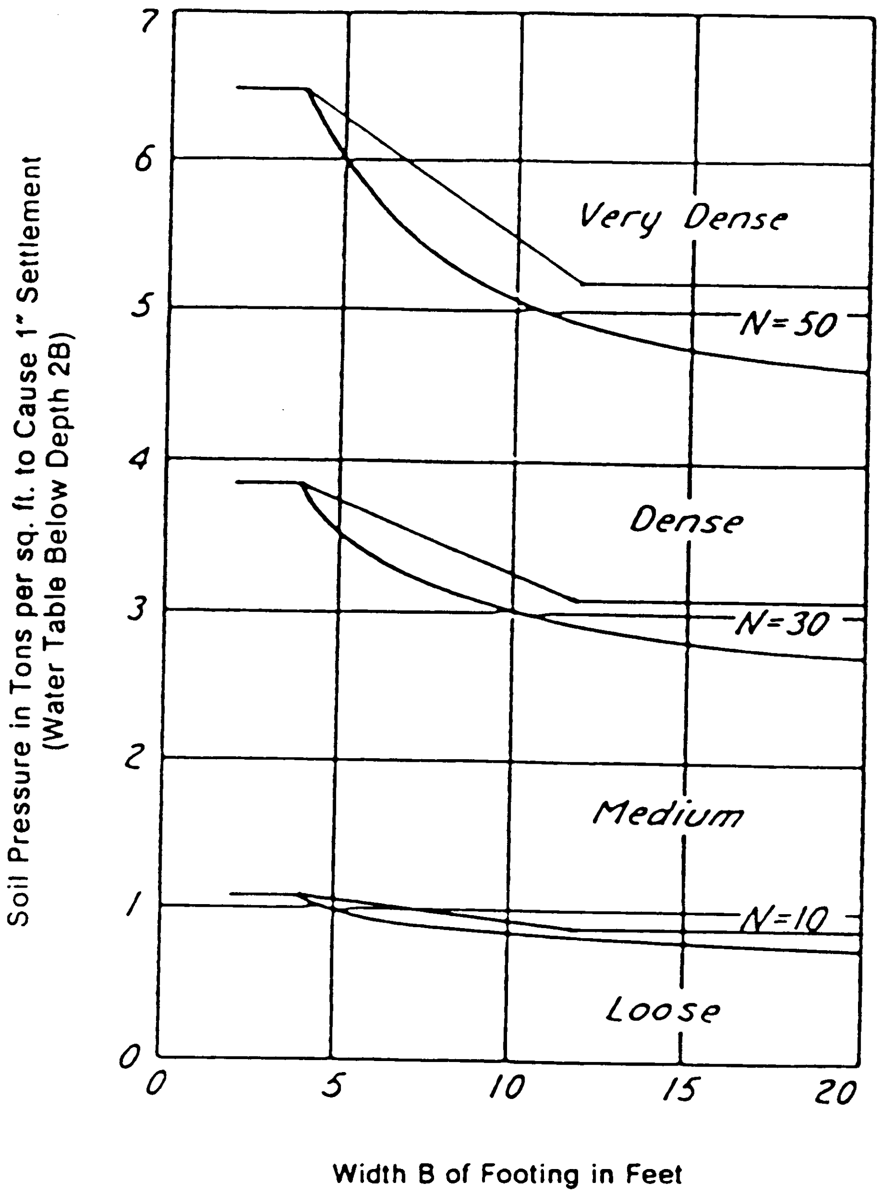


Figure 10. Chart for Estimating Allowable Soil Pressure for Footing on Sand on the Basis of Results of Standard Penetration Test.: (After Terzaghi and Peck, 1948)

4.4.1 Equivalent Random Variable Model

The effect of inherent spatial variability (N_i) and random systematic error (N_s) of soil properties associated with settlement analysis of shallow foundation on sand is given by

$$N_i N_s = \frac{R_1}{R_N} \tag{4.11}$$

where R_N and R_1 are factors previously defined in section 4.2. From basic theory of probability, the mean and variance of $N_i N_s$ are given by

$$E[N_i N_s] = \frac{1}{R_N} \{\bar{R}_1\} \tag{4.12}$$

and

$$VAR[N_i N_s] = \frac{1}{R_N^2} \{\sigma_{R_1}^2\} \tag{4.13}$$

Vanmarcke (1977) has claimed that there is always a reduction in the coefficient of variation of the spatial average as compared to that of the average point. The amount of reduction is dependent on the variance function which is a function of the scale of fluctuation. The scale of fluctuation, θ, is a measure of the distance within which the soil property demonstrates a strong correlation from point to point in a soil stratum. For a shallow foundation on sand, a 3-dimensional random field model can be used to express the relationship between inherent spatial variability and point variability as (Vanmarcke, 1983)

$$\Omega_{\text{spatial}} = \Gamma(x)\Gamma(y|x)\Gamma(z|x, y)\Omega_{\text{point}} \tag{4.14}$$

in which $\Omega_{spatial}$ = spatial c.o.v.; $\Gamma(x)$ = variance function in x-direction; $\Gamma(y|x)$ = conditional variance in y-direction given the variance function in x-direction; $\Gamma(z|x, y)$ = conditional variance function in z-direction given the variance functions in x and y directions; Ω_{point} = point c.o.v.

It is difficult to obtain the conditional variance functions in geotechnical problems. However, to study the problem it is convenient and reasonable to assume that the variance functions are independent. Also, it can be assumed that soil properties are the same in horizontal directions; that is, the scale of fluctuation (θ_x and θ_y) are the same. Therefore, Eq. 4.14 can be rewritten as

$$\Omega_{\text{spatial}} = \Gamma_x\Gamma_y\Gamma_z\Omega_{\text{point}} = \Gamma_x^2\Gamma_z\Omega_{\text{point}} \tag{4.15}$$

In order to obtain the variance functions in x and z directions, consider a 1-dimensional random field model for the z direction first. It is known that penetration resistance varies with depth even though both soil density and components are the same. Thus, the uncertainties of the penetration resistance with depth may be modeled by a one dimensional random field $\hat{N}_{SPT}(z)$ as shown in Figure 11. According to the 1-dimensional model proposed by Vanmarcke (1977), the mean value of N_rN_s can be expressed as

$$E[N_iN_s] = \frac{1}{R_N} E[R_1]$$

$$= \frac{1}{R_N} E[\frac{(\frac{1}{L}\int_0^L N_{SPT}(z)\,dz - 1.5)\overline{CB} \times A \times S \times 2}{5}]$$

$$= \frac{1}{R_N} E[\frac{(\frac{1}{L}\int_0^L N_N\hat{N}_{SPT}(z)\,dz - 1.5)\overline{CB} \times A \times S \times 2}{5}] \qquad (4.16)$$

$$= \frac{1}{R_N} [\frac{(\overline{N}_N\overline{N}_{SPT} - 1.5)\overline{CB} \times A \times S \times 2}{5}]$$

The variance is

$$VAR[N_iN_s] = \frac{1}{R_N^2}\{VAR[R_1]\}$$

$$= \frac{1}{R_N^2} VAR[\frac{(\frac{1}{L}\int_0^L N_N\hat{N}_{SPT}(z)dz - 1.5)\overline{CB} \times S \times A \times 2}{5}]$$

$$= \frac{\overline{CB}^2S^2A^2 \times 4}{25 \times R_N^2}\{E[N_N^2]\int_0^L\int_0^L \frac{E[\hat{N}_{SPT}(z_1), \hat{N}_{SPT}(z_2)]dz_1dz_2}{L^2} - \overline{N}_N^2\overline{N}_{SPT}^2\}$$

$$= \frac{\overline{CB}^2S^2A^2 \times 4}{25 \times R_N^2}\{(\Delta_{N_{SPT}}^2 + 1)\overline{N}_{SPT}^2[\int_0^L\int_0^L \frac{\rho_{N_{SPT}}(z_1, z_2)\sigma_{N_{SPT}}^2 dz_1dz_2}{L^2} + \overline{N}_{SPT}^2] - \overline{N}_N^2\overline{N}_{SPT}^2\} \qquad (4.17)$$

$$= \frac{\overline{CB}^2S^2A^2 \times 4}{25 \times R_N^2}\{(\Delta_{N_{SPT}}^2 + 1)[\int_0^L\int_0^L \frac{\rho_{N_{SPT}}(z_1, z_2)\sigma_{N_{SPT}}^2 dz_1dz_2}{L^2} \times \delta_{N_{SPT}}^2 + 1] - 1\}\overline{N}_N^2\overline{N}_{SPT}^2$$

$$= \frac{\overline{CB}^2S^2A^2 \times 4}{25 \times R_N^2}\{(\Delta_{N_{SPT}}^2 + 1)(\Gamma_A^2(L)\delta_{N_{SPT}}^2 + 1) - 1\}\overline{N}_N^2\overline{N}_{SPT}^2$$

in which L = averaging distance; $\rho_{N_{SPT}}$ = correlation function of $\hat{N}_{SPT}(z)$ between z_1 and z_2; $\sigma_{N_{SPT}}$ = point standard deviation of $\hat{N}_{SPT}(z)$ accounting for random spatial variability at a point; $\Delta_{N_{SPT}}$ = coefficient of variation of systematic model error of $\hat{N}_{SPT}(z)$ and $\Gamma_A^2(L)$ is the variance function of $\hat{N}_{SPT}(z)$ defined as

$$\Gamma_A^2(L) = \frac{\int_0^L \int_0^L \rho_{N_{SPT}}(z_1, z_2) dz_1 dz_2}{L^2} \tag{4.18}$$

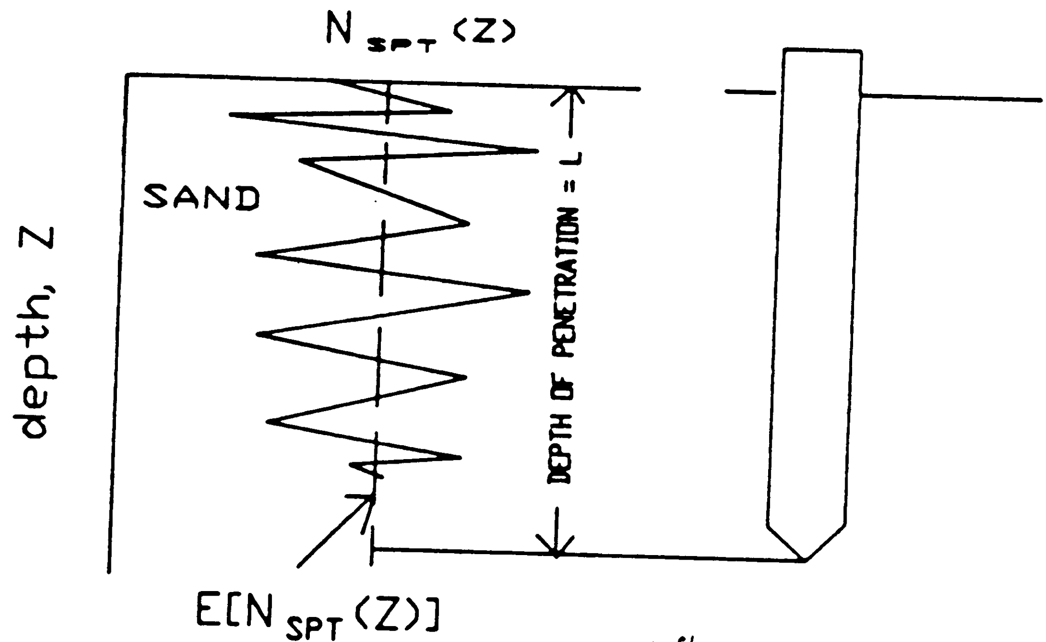
From Eq. 4.16, it can be seen that both the inherent spatial variability and random systematic error in soil parameter associated with bearing capacity of shallow foundations on sand are included in the model. The influence of inherent spatial variability can thus be analyzed by using an appropriate variance function. Vanmarcke (1979) pointed out that the scale of fluctuation can be a significant factor in random field problems. Mathematically, it can be expressed as

$$\theta = \lim_{L \to \infty} L\Gamma_A^2(L) = \int_{-\infty}^{\infty} \rho_\mu(\mu) dr \tag{4.19}$$

where $\rho_\mu(\mu)$ = correlation function. If L goes to infinity, then $\Gamma_A^2(L)$ will approach $\frac{\theta}{L}$. From observations of the relationship between the scale of fluctuation θ and averaging distance L, Vanmarcke (1979) proposed an expression for $\Gamma_A(L)$ as shown in Figure 12, namely,

$$\Gamma_A(L) = 1 \qquad \theta \geq L \tag{4.20a}$$

$$\Gamma_A(L) = [\frac{\theta}{L}]^{\frac{1}{2}} \qquad \theta \leq L \tag{4.20b}$$



$$R = \frac{\left(\frac{1}{L}\int_0^L N_{SPT}(z)\,dz - 1.5\right)\overline{CB} \times A \times S \times 2}{5}$$

Figure 11. Stochastic Process $N_{SPT}(z)$ and Bearing Capacity

This approximation gives satisfactory results in most cases.

4.4.2 Evaluation of N_i and N_s

From Eq. 4.16 and 4.17, the combined effect of the inherent spatial variability and random systematic error can be assessed. However, it is desirable to evaluate the individual contribution of N_i and N_s to the overall statistics of R. Assuming that there is no systematic error in determining the soil parameter, i.e., $\overline{N}_N = 1$ and $\Delta_{N_{SPT}} = 0$, Eqs. 4.16 and 4.17 reduce to the mean and variance of N_i. Similarly, statistics of N_s may be obtained by assuming that there is no inherent spatial variability of soil property included, namely by letting the variance function $\Gamma_A(L)$ equal to unity.

In order to evaluate N_i, the c.o.v. of N_{SPT} was assumed to be 0.42 which represents a reasonable but conservative estimate (Briaud and Tucker, 1984). Then, from previous derivations, the overall variance function can be expressed as

$$\Gamma(D) = \Gamma^2(x)\Gamma(z) \tag{4.21}$$

where $\Gamma(D)$ = variance function of the interested 3-D domain. According to Vanmarcke (1977), the value of the scale of fluctuation θ of SPT is about 8 feet (2.4 meter). For shallow foundations, it is reasonable to assume that the averaging distance L is equal to two times the footing width (2B). Currently, there is no available data for the scale of fluctuation of SPT in the horizontal direction. However, data from other tests such as CPT can be employed to estimate the scale of fluctuation for SPT in horizontal direction. A value of 20 meters to 35 meters of correlation parameter, b, of CPT on sand in horizontal direction has been proposed in Tang's work (1979)

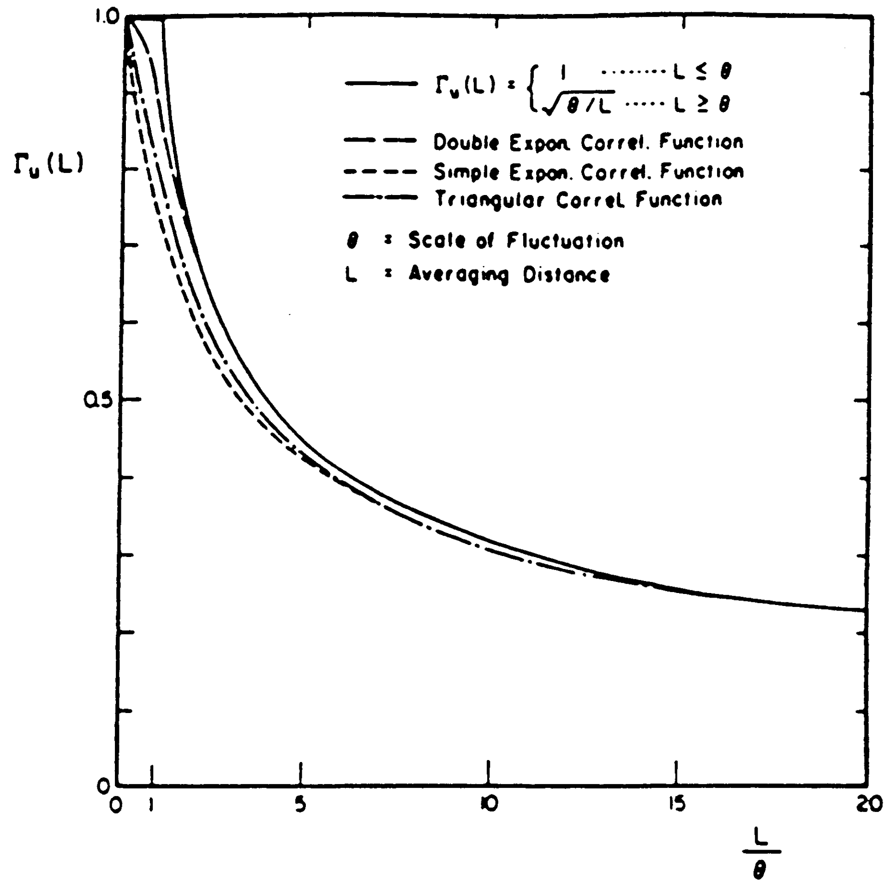


Figure 12. C.O.V. Reduction Factor, Type-A Variance Function: (After Vanmarcke, 1979)

where b is equal to $\frac{\theta}{\sqrt{\pi}}$. A value of 30 meters for b was used in the study of offshore site exploration in North Sea (Wu et al., 1986). Therefore, it is reasonable but conservative to assume that the correlation parameter b is equal to 20 meters for SPT on sands in the horizontal direction. The value of θ for SPT in the horizontal direction then is equal to $20 \times \sqrt{\pi} \times 3.28 = 116$ feet. For a shallow foundation of a bridge, the width of the foundation seldom exceeds 30 feet. Thus, the averaging distance L is smaller than 60 feet in most cases. Then, according to Eq. 4.20a, the variance function in the horizontal direction, $\Gamma(x)$ is equal to 1. For a homogeneous soil media, the mean of the soil parameter at a point is identical to the mean value of the spatial average (Vanmarcke, 1979), and so the evaluation of $\overline{N_i}$ for sands from Eq. 4.16 gives a value of one (that is $\overline{N_i} = 1$). The coefficient of variation of inherent spatial variability N_i can be approximately expressed as

$$\begin{aligned} \Omega_{N_i} &= \Gamma^2(x) \times \Gamma(z)\Omega_{N_{SPT}} \\ &= 1 \times \left(\frac{\theta}{L}\right)^{\frac{1}{2}} \times 0.42 \\ &= \left(\frac{8}{2B}\right)^{\frac{1}{2}} \times 0.42 \qquad (B \text{ in feet}) \qquad (4.22) \\ &= \frac{1.2}{\sqrt{2B}} \end{aligned}$$

In addition to inherent spatial variability, the variation in the way standard penetration tests are performed also contribute to the uncertainty in the prediction of bearing capacity. There are several factors influencing the SPT and they are called systematic errors. Orchant et al.(1988) studied the test variability in SPT by reviewing the previous work (e.g. Bieganousky et al., 1976, 1977; Schmertmann and Palacious, 1979; Kovacs and Salomone, 1982, etc.) They classified the sources of uncertainties in SPT into two groups: those due to equipment and those due to test procedures. The factors that influence the SPT results are listed in Table 15 and 16.

Three sets of SPT data were collected to study systematic errors. The first data set was collected to study the relationship between relative density of cohesionless soils and N_{SPT} values. A total of 32 test results (Bieganousky and Marcuson, 1976; 1977) with different types of sand, overconsolidation ratio, dry density, relative density, vertical stress, and depth were obtained. The second data set was collected to investigate energy transfer of different types of hammer during the standard penetration test. A total of 109 measurements (Palacios, 1977) with the combination of safety hammer, donut hammer and A-weight rod, AW-weight rod and N-weight rod were used. The third data set was collected to study the energy delivered by different drilling systems. Thirty five series (Kovacs et al., 1981) of tests with different combinations of drill rig model, number of turns of rope around cathead, hammer fall height, cathead speed and rotation direction, rope size, rope age and type of hammer were obtained.

Statistical analyses of these three data sets showed that the coefficient of variation for equipment variability was 5-75 percent and for procedural variability 5-75 percent. The low value of c.o.v. represents the use of identical equipment and procedures in each test. The high value of c.o.v. results from no control on equipment and test procedures. A 12-15 percent correction factor added was included to take into account random test error. According to Orchant et al. (1988), the c.o.v. due to systematic error is on the order of 14 to 45 percent. Thus, it is conservative but reasonable to assume that the mean correction factor for systematic error is equal to 1.0 with c.o.v. of 0.30.

Table 15. SPT Equipment Variables (After Orchant et al., 1988)

Variable	Relative Effect on SPT Results
Non-standard sampler	moderate
Deformed or damaged sampler	moderate
Rod diameter/weight	minor
Rod length	minor
Deformed drill rods	minor
Type of hammer	moderate to significant
Hammer drop systems	significant
Hammer weight	minor
Size of anvil	minor
Type of drill rig	minor

4.5 SUMMARY OF UNCERTAINTIES FOR SETTLEMENT ANALYSIS

In the previous sections, uncertainty analyses of major factors affecting bearing capacity for given settlement of shallow foundations was presented. Statistics of each of these correction factors were obtained in terms of the mean and coefficient of variation as shown in Table 17. By using the relevant components of the correction factors, the total uncertainty in the allowable bearing load of shallow foundations for given tolerable settlement can be evaluated using equations 4.8 and 4.9. For exam-

Table 16. SPT Procedural/Operator Variables (After Orchant et al., 1988)

Variable	Relative Effect on SPT Results
Size of borehole	moderate
Method of maintaining hole	minor to significant
Cleaning of borehole	moderate to significant
Failure to maintain sufficient Hydrostatic head	moderate to significant
Seating of sampler	moderate to significant
Hammer drop method	moderate to significant
Error in counting blows	minor

ple, for a 12 feet by 12 feet square footing put on loose sand (N_{SPT} = 10) with embedment depth equal to 3 feet, the mean value of resistance is 548.4 kips and the standard deviation is 333 kips. Therefore, the overall coefficient of variation of resistance is equal to $\frac{333}{548.4} = 0.607$

Table 17. Summary of Uncertainties in Bearing Pressure in Settlement Analysis

CORRECTION FACTOR	MEAN	C.O.V.
Model Error, N		
Duncan & Buchighnani method	1.4	0.4
Width effect, N_{CB}	1.0	0.03
Inherent Variability, N_i	1.0	$\frac{1.2}{\sqrt{2B}}$
Systematic Error, N_s (Standard Penetration Test)	1.0	0.30

Chapter 5

STATISTICS OF LOADS

GENERAL

Loads on bridges exhibit more variability than loads on other structures. In this study loads are assumed to be transferred from the superstructure directly to the foundation without any lateral transfer through the abutments. Therefore, the available statistical data for loads on the superstructure can be directly applied to the foundation.

The different bridge design codes and specifications use different load classifications. The load classification employed by Grouni and Nowak (1984) for the Ontario Highway Bridge Design (OHBD) code will be adopted in this study. Grouni and Nowak classified load components into five categories : (1) dead load, (2) live load and other loads related to moving trucks, (3) environmental effects, (4) strain-related effects, and (5) prestressing force. This classification is somewhat different from that in current

AASHTO specifications. In this study, only dead and live loads are considered. Also, not both dead and live loads are normally distributed. However, by central limit theorem (Ang and Tang, 1975), their combinations can be considered to be normally distributed.

5.2 DEAD LOAD

The dead load, D, is from the self weight of the superstructure, sidewalks, appurtenances, substructure, and other non-structural elements. Dead loads may impose more influence on larger span bridges than short to medium span bridges due to the higher dead-to-live load ratio of larger span bridges as compared to that of short to medium span bridges. Variability in dead load results from uncertainties in the unit weight of different materials. Grouni and Nowak (1984) subdivided the dead load, D, into three components (i) D_1, weight of factory-made members, (ii) D_2, weight of structural components produced in the field and weight of mass concrete, and (iii) D_3, weight of wearing surface. Dead load components are essentially time-invariant and can be treated as normally distributed random variables.

The distribution parameters, ratios of the mean to nominal (λ) and coefficients of variation (V) of dead load were established by an OHBD working committee and evaluated on the basis of available data. Values of the parameters are λ = 1.03, 1.05 and 1.05 and V = 0.04, 0.08 and 0.25 for D_1, D_2, D_3, respectively. The highest degree of uncertainty in dead load, D, comes from the weight of wearing surface, D_3. However, D_3 is a small portion of the overall dead load and can be assumed to have less influence than D_1 and D_2 on the overall dead load D. Therefore, it is conservative to assume λ = 1.05 and V = 0.10 for the combined dead load effects transferred from the superstructure to the foundation.

5.3 LIVE LOAD

Live load, L, consists of weight of vehicles on a bridge as well as other loads such as impact or dynamic effect of a moving truck, I, braking force, centrifugal force, etc. In the current AASHTO specifications, impact, I, is not considered in foundation design. Other live loads such as braking forces and centrifugal force have almost no influence on foundations when load effects are transferred from the superstructure to the foundation. Therefore, only live loads from vehicles need to be considered. Live load is a time-varying load with a short time span depending on the volume and composition of traffic using the bridge, which in turn depends on the geographical location of site and the probability of potential overloads of vehicles. There is some uncertainty associated with conversion from loads to load effect since the actual bridge response can not be accurately predicted (Verma and Moses, 1988).

Two types of live load are considered in this study. One is the maximum live load for a design life of 50 years for ultimate limit state. The other is the maximum live load for a return period of one month for serviceability limit state. In short to medium span bridges, the maximum live load effect is usually due to the simultaneous occurrence of two or more heavy trucks on the bridge. Moses and Ghosn (1985) developed a model to predict the maximum live load effects, $L_{max}(T)$, over a specified period of time, T as follows:

$$L_{\max}(T) = a \times g \times m \times W_{0.95} \times H \times G_r \times (1 + I) \tag{5.1}$$

where a is the load to load effect conversion factor, g is component load distribution, m is the influence of dominant vehicle type and configuration at a site, $W_{0.95}$ is a 95th percentile characteristic value of gross truck weight at a site, H is the influence of multiple presence of trucks, G_r is the potential growth factor, and (1 + I) is the dynamic effect. The coefficient of variation of these parameters are also presented in their report. However, the bias factor for maximum live load is not available.

To determine the bias factor for maximum live load, data presented by Nowak (1989) are employed in this study. These are shown in Table 18. The 50-year maximum live load for span length of 40 feet, 80 feet, 120 feet, and 160 feet are used in reliability analysis to investigate the variation of reliability index with different span lengths. The bias factors for span length of 40 feet, 80 feet, 120 feet, and 160 feet are 1.22, 1.19, 1.10 and 0.97, respectively. It is known that for one-month maximum live load the bias factor should be smaller than and the coefficient of variation should be larger than for the 50-year maximum live load. These data can also be obtained from Nowak's study (1989). The bias factor is assumed to be 1 and the coefficient of variation is assumed to be 0.2 for one-month maximum live load.

Table 18. Bias and Coefficient of Variation for 50-Year Maximum Live Load (After Nowak, 1989)

Span (ft)	Span (m)	Bias	C.O.V.
10	3	1.22	0.14
20	6	1.22	0.13
30	9	1.22	0.11
40	12	1.22	0.11
60	18	1.21	0.11
80	24	1.19	0.11
100	30	1.15	0.11
120	36	1.10	0.11
130	40	1.05	0.11
160	50	0.97	0.11
200	60	0.93	0.11
300	100	0.85	0.11
500	150	0.72	0.11
600	200	0.60	0.11

5.4 SUMMARY OF STATISTICS OF LOADS

The mean to nominal ratio and coefficients of variation for dead and live loads are summarized in Table 19. The total load, Q, can be considered as the sum of dead load, D, and live load, L.

$$Q = D + L \qquad (5.2)$$

According to Turkstra's rule (1970), the distribution of Q can be shown as

$$Q = D_{apt} + L_{\max} \qquad (5.3)$$

where D_{apt} is the arbitrary-point-in-time value of D in the design period, and L_{max} is the extreme value of L in the design period. As previously mentioned, according to the central limit theorem, the total load Q can be assumed to be normally distributed.

Table 19. Summary of Statistics of Load Component

Load Component	$\frac{\bar{X}}{X_n}$	V_x
D	1.05	0.1
L_{max} (40 feet) (50 years)	1.22	0.11
L_{max} (80 feet) (50 years)	1.19	0.11
L_{max} (120 feet) (50 years)	1.10	0.11
L_{max} (160 feet) (50 years)	0.97	0.11
L_{max} (1 month)	1.0	0.2

Chapter 6

RELIABILITY ANALYSIS AND CODE CALIBRATION

6.1 GENERAL

The process of assigning values to the code parameters is called calibration. Codes may be calibrated by judgment, fitting, optimization or a combination of these approaches (Madsen, Krenk and Lind, 1986). Calibration by judgment or fitting has some basic disadvantages. A more formal procedure of explicit optimization to calibrate a code has been presented by Ravindra and Lind (1983). The procedure can be summarized as follows (Ellingwood et al., 1980) :

1. Define the class of structures to which the code is meant to employ, and specify the relevant failure modes.
2. Specify the code objective.
3. Investigate the frequency of occurrence of the particular safety check.

4. Choose a measure of closeness between a code realization and its objective.
5. Choose a sequence of trial code formats arranged in order of decreasing simplicity. Based on the criterion of closeness the simplest format that meets the criterion is then selected.

Several design codes such as the Ontario Highway Bridge Design Code (Grouni and Nowak, 1984) and the American National Standard Institute Loading Standard, ANSI A58 (Ellingwood et al., 1980) have been calibrated with this basic philosophy.

Calibrating with existing codes ensures a proper design evolution and avoids drastic change in designs by the new procedures from designs by current procedures. Instead of designating a new level of reliability, the new design procedures are based on risk levels inherent in current codes. After evaluating the levels of reliability inherent in current designs, a target reliability index is selected as a basis for the determination of the probability-based load and resistance factors. The code parameters are evaluated so that the resulting reliability indices are close to the target reliability indices. To estimate load and resistance factors, a basic procedure employing the above philosophy consists of the following steps (Ellingwood et al., 1980) :

1. Estimate the levels of reliability inherent in current design practices.
2. Observe the reliability levels over ranges of material, limit states, nominal load ratios and load combination from reliability analyses.
3. Based on the observed reliability levels, select target reliability indices.
4. Determine load and resistance factors that are consistent with the selected target reliability indices.

In this chapter, the results of the reliability analysis are presented. Then target reliability indices are determined. Finally, performance factors consistent with selected target reliability indices are obtained.

6.2 RELIABILITY ANALYSIS

In working stress design, the safety of a structure is assumed by restricting the applied loads so that they are less than or equal to the allowable stress. This approach can be applied to both settlement and ultimate bearing capacity. Currently, the recommended safety factor for shear failure of spread footings is between 2 and 3. There is no explicit safety factor for settlement analysis. However, it is reasonable to assume that the tolerable settlement should be at least equal to or greater, say 1.0 to 2.0 times, than the actual settlement. Therefore, the ratio of tolerable settlement to actual settlement can be treated as the safety factor for serviceability limit state.

From the discussion above, the relationship between resistance and load in the working stress design method can be expressed as follows:

$$R_{all} = \frac{R}{SF} \geq Q_{applied} = (D + L) \tag{6.1}$$

where R is the ultimate bearing capacity for ultimate limit state; for serviceability limit state, R is the bearing load for specified tolerable settlement; R_{all} is the allowable bearing load, $Q_{applied}$ is the applied load, D, L are the dead and loads, and SF is the safety factor.

Then, the nominal live load for a given ratio of dead to live load can be expressed as follows:

$$L = \frac{R}{SF(1 + RI)} \tag{6.2}$$

where RI = ratio of nominal dead load to live load. The mean value of dead and live loads, therefore, can be determined using the bias factors for dead and live loads, respectively. Thus, for given footing dimensions and soil parameters, the reliability indices for both the ultimate and serviceability limit states can be determined. This has been done using a computer program that employs the advanced FOSM method.

The reliability index β for different footing dimensions of equal footing area for both the ultimate and serviceability limit states was evaluated using the resistance and load statistics presented earlier. The width to length ratio of 1 represents circular and square footings while the ratio of $\frac{1}{6}$ represents long footings. Safety indices corresponding to safety factors of 2, 2.5 and 3 for the ultimate limit state, and 1.0, 1.5, and 2.0 for serviceability limit state were computed. The variation of β with dead to live load ratio for the ultimate limit state is shown in Figures 13, 14, 15, and 16 for span length of 40 feet, 80 feet, 120 feet, and 160 feet, respectively. The results for ultimate limit state are summarized in Tables 20, 21, 22, and 23, respectively. The results for serviceability limit state are shown in Figures 17, 18, and 19. A summary of the results for serviceability limit state is given in Table 24.

From the analysis above, it is observed that reliability indices for ultimate limit state are in good agreement with results presented by other investigators. The probability of failure for safety factors between 2 and 3 is in the range of 10^{-2} to 10^{-4} which agrees with Meyerhof's conclusions (1970). It is also observed that the reliability indices for

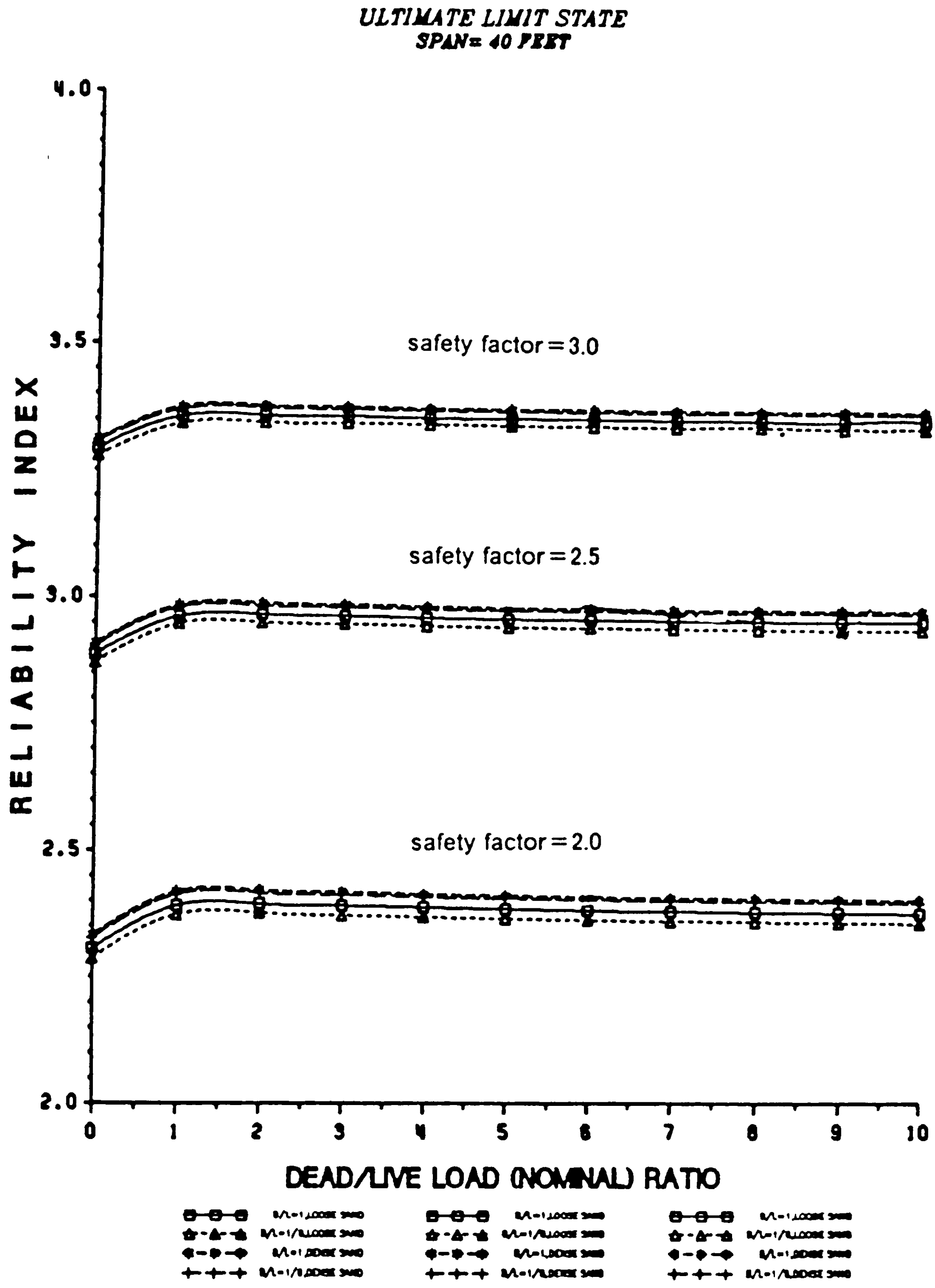


Figure 13. Reliability Index β versus Different Dead to Live Load Ratios for Ultimate Limit State: Span Length = 40 feet

Table 20
Summary of Reliability Analysis for
Ultimate Limit State for Span Length of 40 feet

Safety	Loose Sand				Dense Sand			
	$\frac{B}{L} = 1$		$\frac{B}{L} = \frac{1}{6}$		$\frac{B}{L} = 1$		$\frac{B}{L} = \frac{1}{6}$	
Factor	β	p_f	β	p_f	β	p_f	β	p_f
2 0	2 31~2.37	.010~.0088	2.29~2.36	.011~ 0092	2.33~2 40	.0098~.0082	2.33~2.40	.0098~.0083
2.5	2 89~2 95	.0019~.0016	2.87~2.93	.0020~.0017	2.91~2.97	.0018~.0015	2.90~2.97	.0018~.0015
3.0	3 29~3.34	.00050~.00041	3.28~3.33	.00052~.00043	3.31~3.36	.00046~.00039	3.31~3.36	.00047~.00039

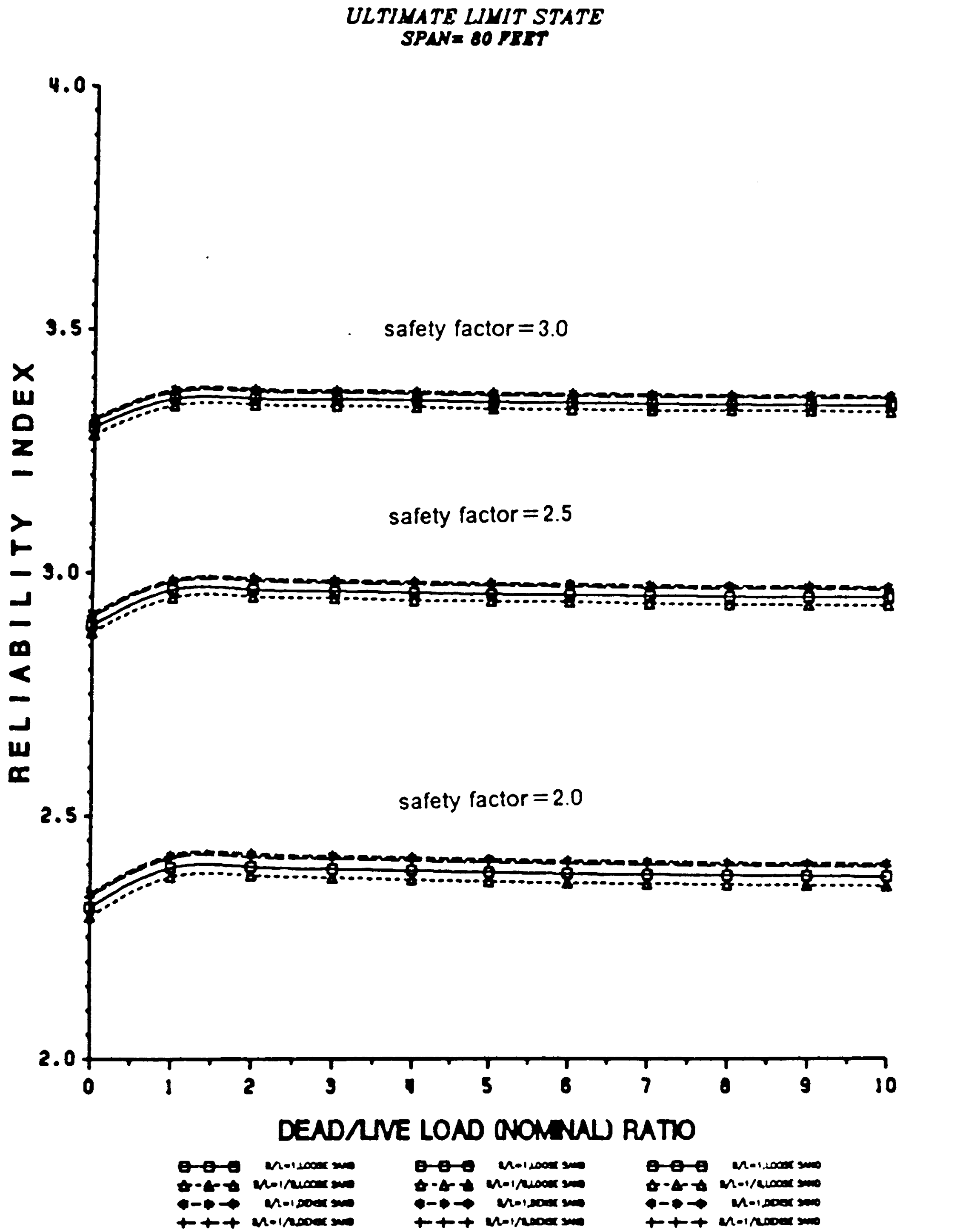


Figure 14. Reliability Index β versus Different Dead to Live Load Ratios for Ultimate Limit State: Span Length = 80 feet

Table 21
Summary of Reliability Analysis for
Ultimate Limit State for Span Length of 80 feet

Safety Factor	Loose Sand				Dense Sand			
	$\frac{B}{L} = 1$		$\frac{B}{L} = \frac{1}{6}$		$\frac{B}{L} = 1$		$\frac{B}{L} = \frac{1}{6}$	
	β	p_f	β	p_f	β	p_f	β	p_f
2.0	2.31~2.37	.010~.0088	2.29~2.36	.011~.0092	2.34~2.40	.0096~.0082	2.33~2.40	.0098~.0083
2.5	2.89~2.95	.0019~.0016	2.88~2.93	.0020~.0017	2.92~2.97	.0018~.0015	2.91~2.97	.0018~.0015
3.0	3.29~3.34	.00048~.00041	3.28~3.33	.00050~.00043	3.32~3.36	.00045~.00039	3.31~3.36	.00046~.00039

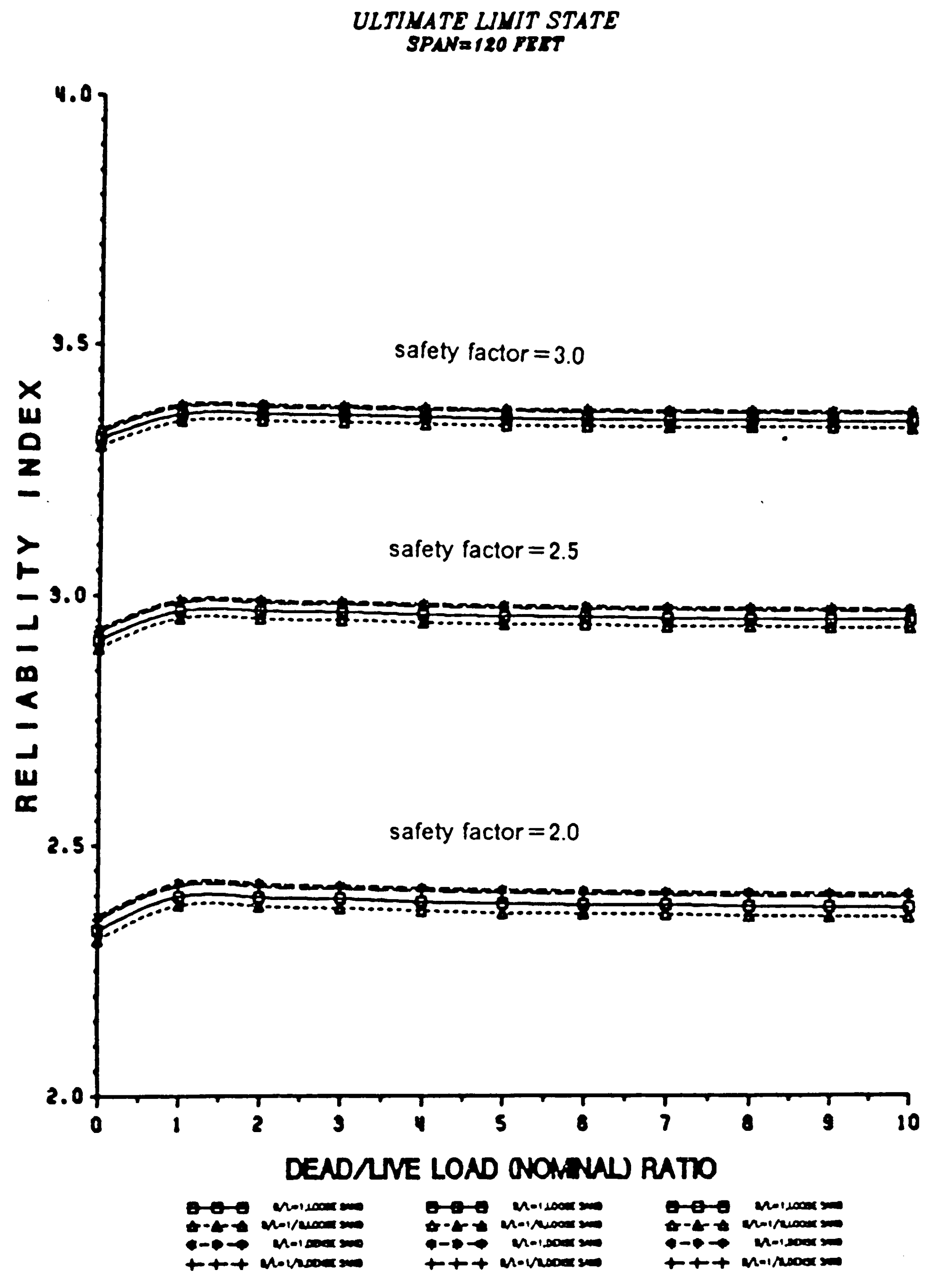


Figure 15. Reliability Index β versus Different Dead to Live Load Ratios for Ultimate Limit State: Span Length = 120 feet

Table 22
Summary of Reliability Analysis for
Ultimate Limit State for Span Length of 120 feet

Safety Factor	Loose Sand				Dense Sand			
	$\frac{B}{L}=1$		$\frac{B}{L}=\frac{1}{6}$		$\frac{B}{L}=1$		$\frac{B}{L}=\frac{1}{6}$	
	β	p_f	β	p_f	β	p_f	β	p_f
2.0	2.33~2.37	.0096~.0088	2.31~2.36	.010~.0092	2.36~2.40	.0092~.0082	2.35~2.40	.0093~.0083
2.5	2.91~2.95	.0018~.0016	2.90~2.93	.0019~.0017	2.93~2.97	.0017~.0015	2.93~2.97	.0017~.0015
3.0	3.31~3.34	.00046~.00041	3.30~3.33	.00048~.00043	3.33~3.36	.00043 ~ .00039	3.32~3.36	.00043~.00039

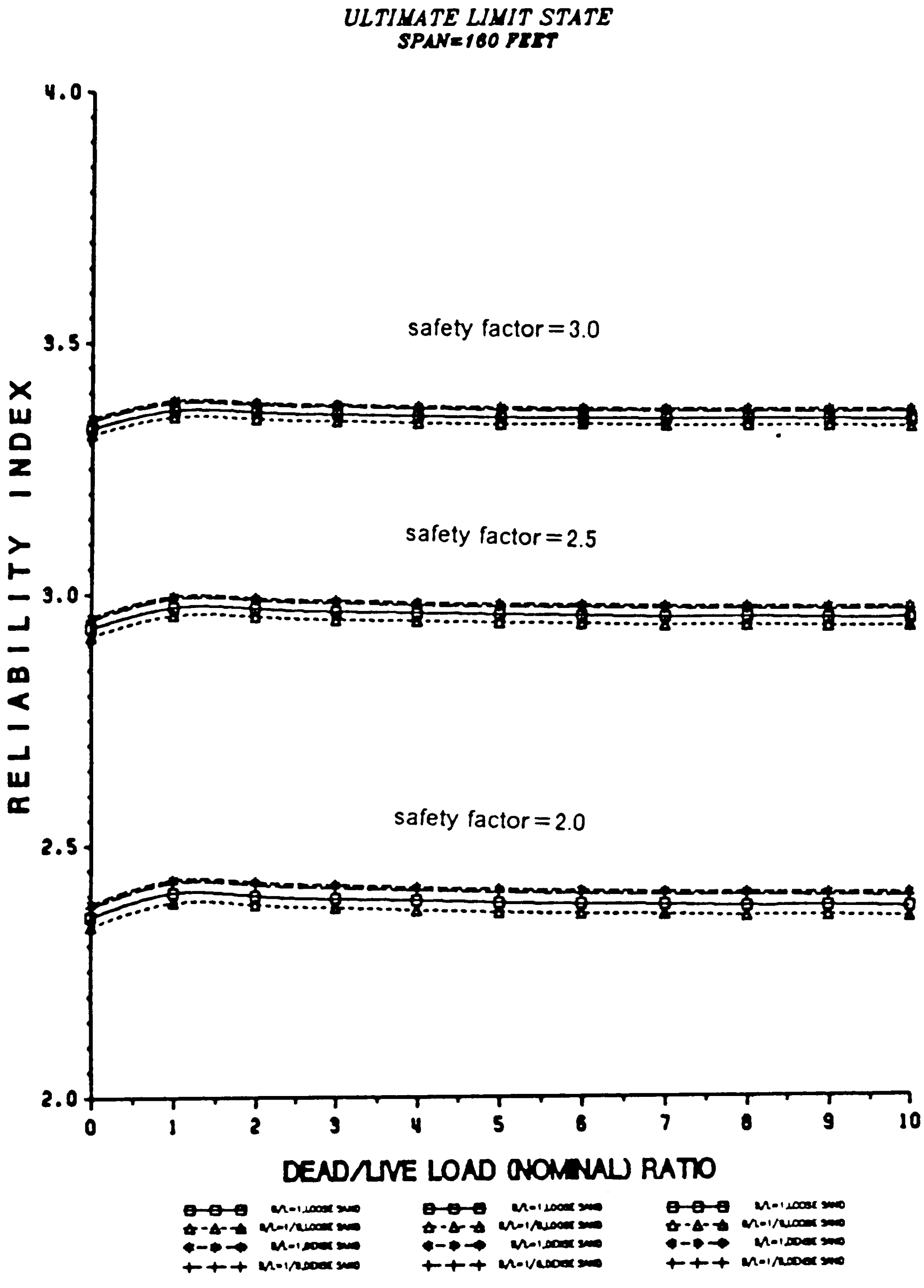


Figure 16. Reliability Index β versus Different Dead to Live Load Ratios for Ultimate Limit State: Span Length = 160 feet

Table 23
Summary of Reliability Analysis for
Ultimate Limit State for Span Length of 160 feet

Safety	Loose Sand				Dense Sand			
	$\frac{B}{L} = 1$		$\frac{B}{L} = \frac{1}{6}$		$\frac{B}{L} = 1$		$\frac{B}{L} = \frac{1}{6}$	
Factor	β	p_f	β	p_f	β	p_f	β	p_f
2.0	2.36~2.37	.0092~.0089	2.33~2.36	.0096~.0092	2.38~2.40	.0086~.0082	2.38~2.40	.0087~.0083
2.5	2.93~2.95	.0017~.0016	2.92~2.93	.0018~.0017	2.95~2.97	.0016~.0015	2.95~2.97	.0016~.0015
3.0	3.33~3.34	.00043~.00041	3.32~3.33	.00045~.00043	3.35~3.36	.00040 ~ .00039	3.35~3.36	.00041~.00039

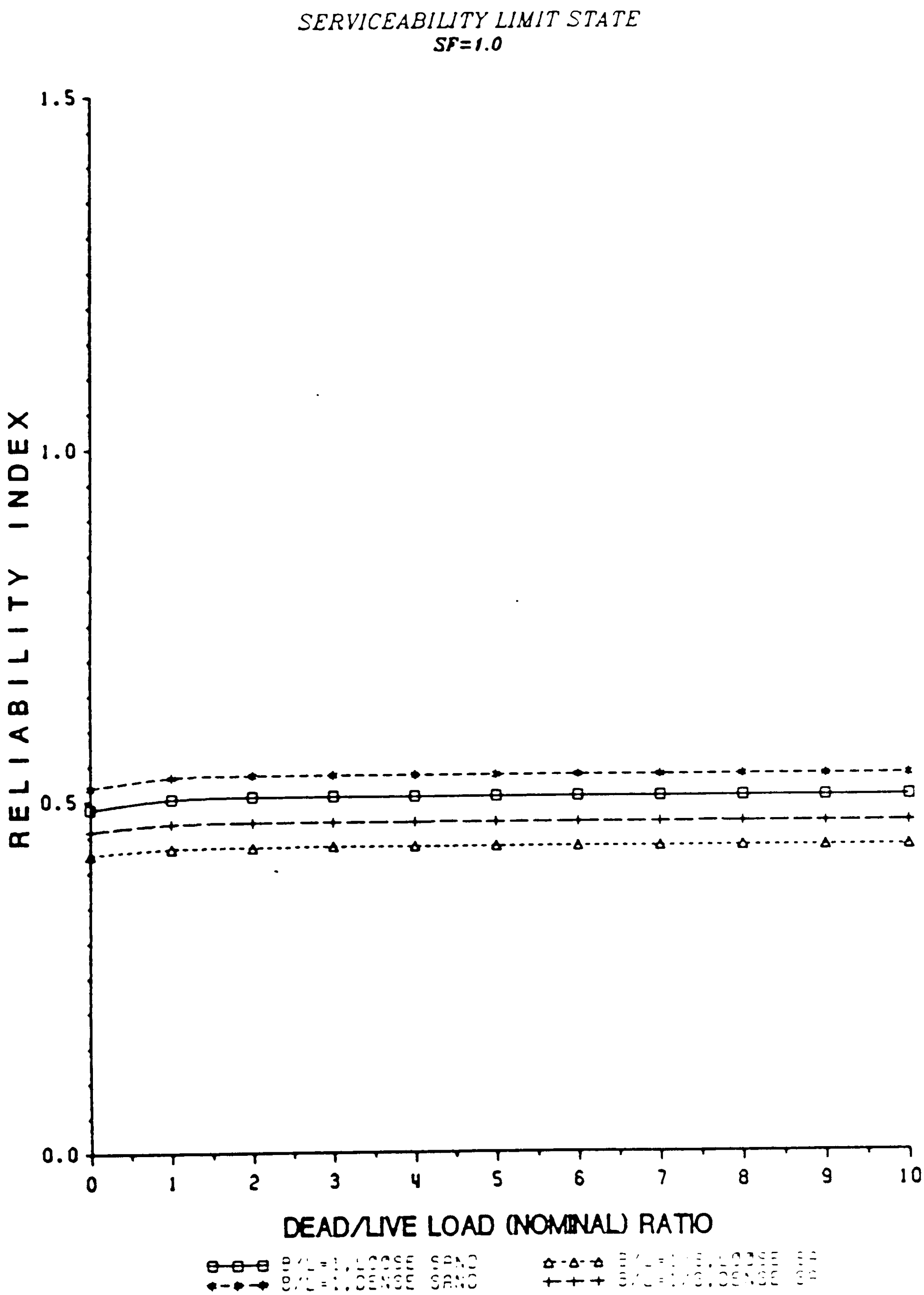


Figure 17. Reliability Index β versus Different Dead to Live Load Ratios for Serviceability Limit State: Ratio of allowable to actual settlement = 1.0

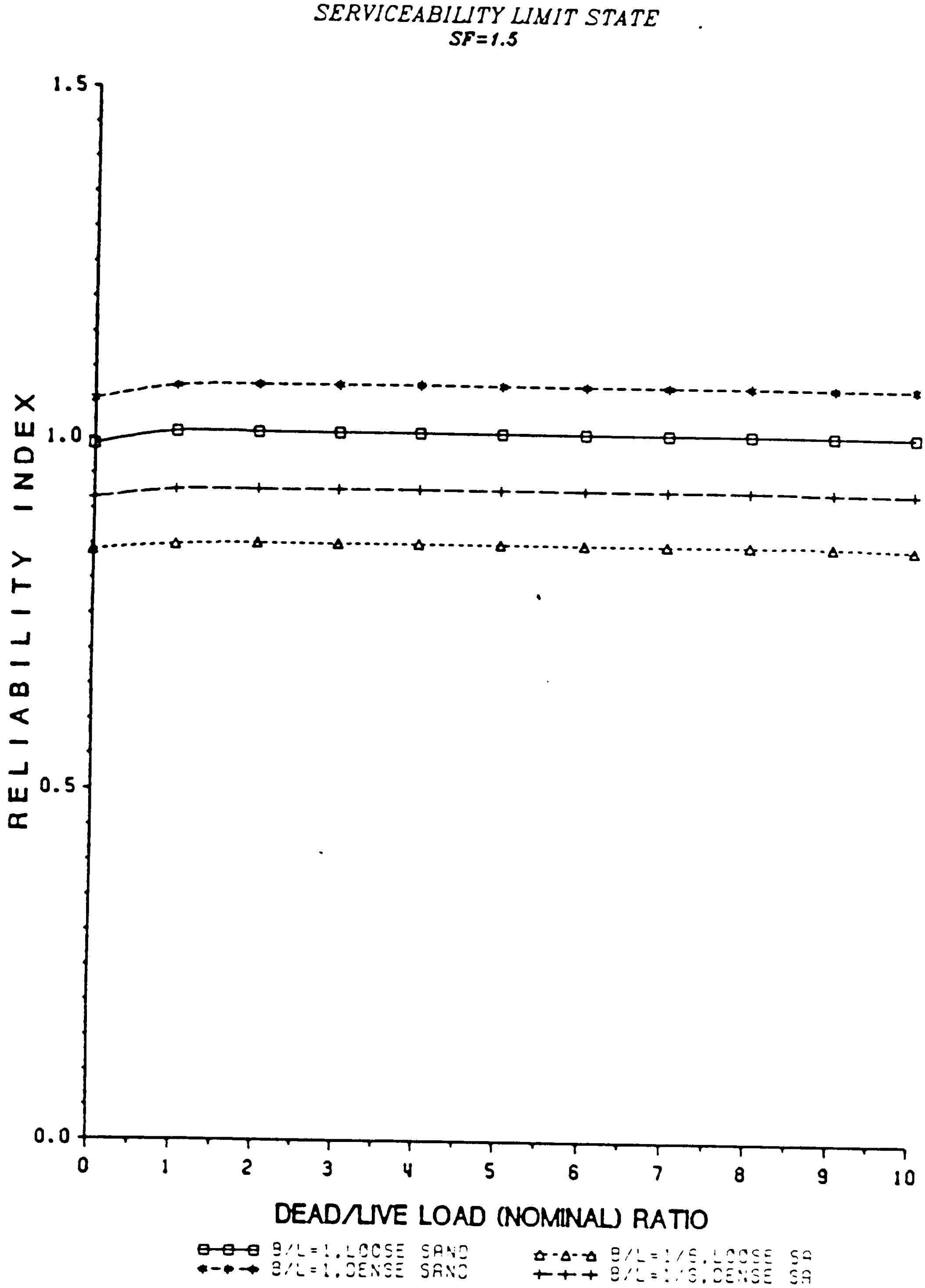


Figure 18. Reliability Index β versus Different Dead to Live Load Ratios for Serviceability Limit State: Ratio of allowable to actual settlement = 1.5

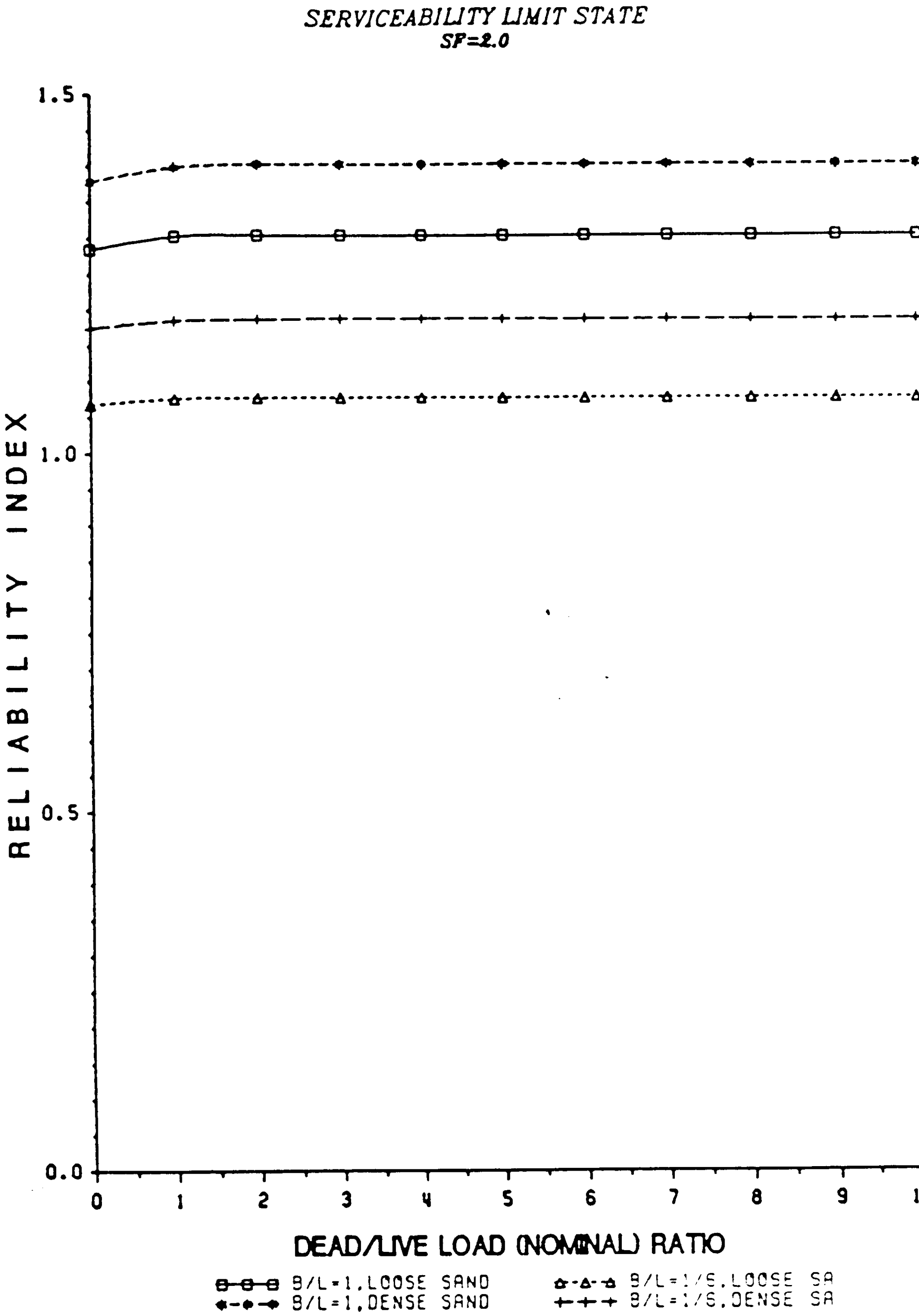


Figure 19. Reliability Index β versus Different Dead to Live Load Ratios for Serviceability Limit State: Ratio of allowable to actual settlement = 2.0

Table 24
Summary of Reliability Analysis
for Serviceability Limit State

$\frac{P_{act}}{P_{prd}}$	Loose Sand				Dense Sand			
	$\frac{B}{L} = 1$		$\frac{B}{L} = \frac{1}{6}$		$\frac{B}{L} = 1$		$\frac{B}{L} = \frac{1}{6}$	
	β	p_f	β	p_f	β	p_f	β	p_f
1.0	0.49~0.51	.312~.306	0.43~0.44	.335~.331	0.52~0.54	.301~.293	0.46~0.47	.323~.319
1.5	0.99~1.01	.161~.158	0.84~0.85	.20~.197	1.06~1.08	.146~.141	0.92~0.93	.180~.176
2.0	1.29~1.30	.10~.096	1.07~1.08	.143~.141	1.38~1.40	.084~.081	1.18~1.19	.120~.118

P_{act} = actual bearing pressure

P_{prd} = predicted bearing pressure

footings on dense sand are larger than those of footing on loose sand. This is reasonable because the probability of failure for footings on dense sand should be less than that for footings on loose sand for the same factor of safety. Reliability indices for square footing were slightly higher than those for long footings. This is more evident for loose sand than for dense sand and might be explained as follows: a square footing has larger width than a long footing for the same area. Thus, a square footing should have a larger bearing capacity than a long footing. The difference is decreased due to a consideration of the plain strain effect for a long footing. A larger value of friction angle is used for long footings (see Eq. 3.12). The difference in the friction angle between a long footing and a square footing is larger for dense sand than for loose sand. Thus, the difference in the reliability index for a square footing and a long footing is less for dense sand than for loose sand.

Reliability indices vary slightly with dead to live load ratio when the dead to live load ratio is less than 1. This is more evident for shorter spans bridges than for longer spans bridges. It is due to the larger bias factors shorter span bridges have than do longer span bridges. There is less variation in the reliability index for larger dead to live load ratios. The largest value of reliability index occurs at dead to live load ratio equal to 1 for all four different span bridges. The variation of reliability index at low dead to live load ratios is attributable to the larger value of overall coefficient of variation for applied load. The overall coefficient of variation for applied load reaches its smallest value at a dead to live load ratio equal to 1. When dead to live load ratio increases, the dead load becomes dominant and the overall c.o.v. for applied load approaches a constant value for all four different span bridges. The reliability index varies more for short span bridges than for medium to long span bridges; however, the difference is not significant.

Reliability indices for serviceability limit state appear to be reasonable. The probability that the actual settlement will exceed the predicted settlement is in the range of 0.08 to 0.33 which is in good agreement with what may be expected. It is observed that the reliability index for footings on dense sand is higher than that for footings on loose sand. This is in good agreement with what we know. For a footing on the same type of soil, it is observed that a square footing has a larger reliability index than a long footing for the same footing area. This is because a square footing has larger footing width. It is also found that reliability indices for serviceability limit state are not sensitive to the dead to live load ratio.

SELECTION OF TARGET RELIABILITY INDEX

The selection of target reliability index, β_T, for each limit state with different safety factors should be based on a measure of closeness between the code realization and its objective. For a level II objective, a criterion for closeness is that the expected value of $(\beta_T - \beta)^2$ should not exceed a given value, say 0.05, for ultimate and serviceability limit states. Based on this procedure, the target reliability indices are selected as β_T = 2.3, 2.9 and 3.3 for safety factors equal to 2.0, 2.5 and 3.0 for ultimate limit state for all span lengths. The target reliability indices for serviceability limit state are 0.45, 0.85, and 1.1 for safety factors equal to 1.0, 1.5, and 2.0, respectively.

PERFORMANCE FACTORS

As previously mentioned, the load factors will be the same as those in the current AASHTO specifications in order to maintain continuity. Thus, the performance factors are obtained using the current load factors and selected target reliability indices. The resistance factors are called performance factors because they are applied to the overall resistance side of the equation for each limit state instead of to the individual design variable in the equation.

For given load factors, $\gamma_1, \gamma_2, \ldots, \gamma_n$, the required resistance has to satisfy the following equation:

$$\phi R_n = \sum_{i=1}^{n} \gamma_i Q_i \tag{6.3}$$

in which ϕ is the performance factor for a given limit state, R_n is the nominal resistance, and Q_i is the load effect due to load i. From equation (2.8), the following expression can be obtained:

$$\overline{R} = \overline{Q} + \beta_T \sqrt{\sigma_R^2 + \sigma_Q^2} \tag{6.4}$$

In design it is typical to use nominal values rather than mean values. The ratio of the mean value to nominal value of R, also called the bias factor, can be defined as

$$\lambda_R = \frac{\overline{R}}{R_n} \tag{6.5}$$

From equations (6.3), (6.4) and (6.5), after transformation, the performance factor can be expressed as:

$$\phi = \frac{\lambda_R \sum_{i=1}^{n} \gamma_i Q_i}{\overline{Q} + \beta_T \sqrt{\sigma_R^2 + \sigma_Q^2}} \tag{6.6}$$

The dead load factor, γ_D, is 1.3 and the live load factor, γ_L, is 2.17 in current AASHTO specifications for one ultimate limit state. For the serviceability limit state the dead load and live load factors are both equal to 1. Figures 19 and 20 show the performance factors corresponding to the different safety factors for the ultimate and serviceability limit states.

It is observed that performance factors for the ultimate limit state almost decrease exponentially when dead to live load ratio increases. In other words, performance factors decrease exponentially as bridge span increases. For a given value of the dead to live load ratio the performance factor can be obtained from Figure 20. The performance factors for serviceability the limit state do not vary very much with dead to live load ratio.

Thus, the ULS design criterion for dead and live load can be written as

$$\phi R \geq 1.3 \times D + 2.17 \times L \tag{6.7}$$

The relationship of performance factor ϕ, safety factor, and target reliability indices for ultimate limit state is shown in Table 25.

The SLS design criterion for dead and live load with calibrated load and resistance factors are

$$\phi R \geq D + L \tag{6.8}$$

The relationship between the performance factor ϕ, the traditional safety factor, and target reliability indices for serviceability limit state is shown in Table 26.

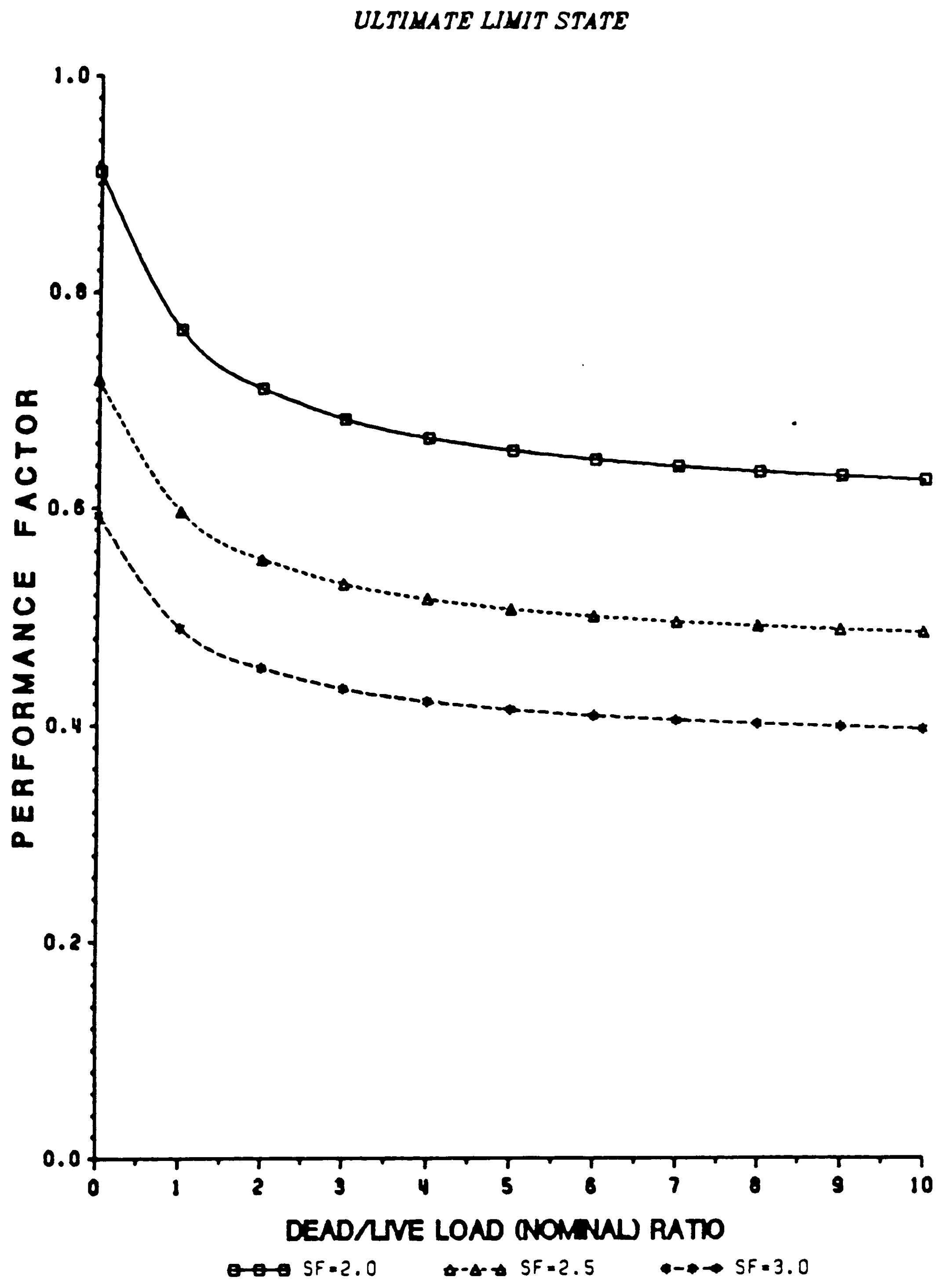


Figure 20. Performance Factors for Ultimate Limit State

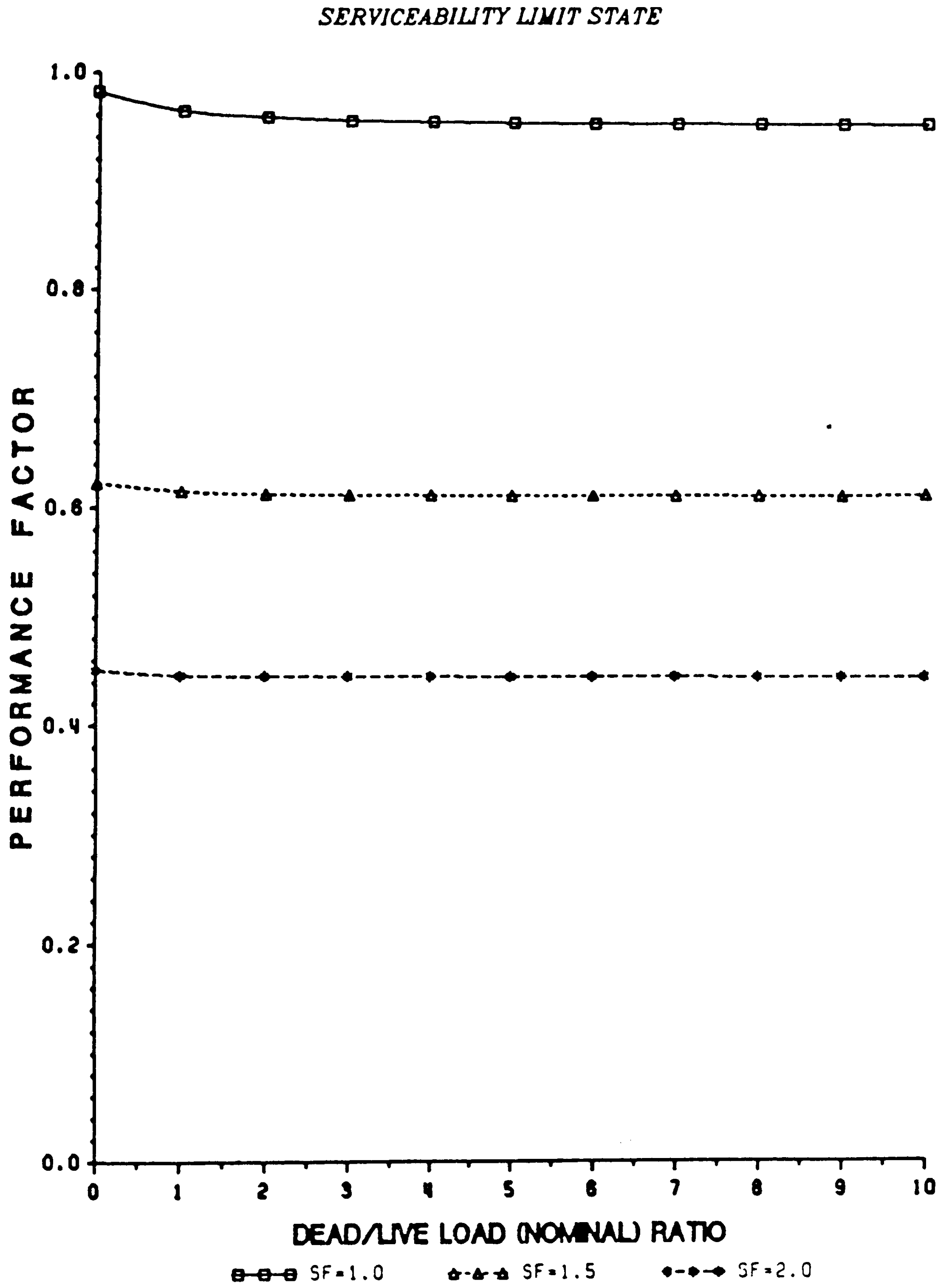


Figure 21. Performance Factors for Serviceability Limit State

Table 25. Relationship of Safety Factor, Target Reliability Index and Performance Factor for Ultimate Limit State

Safety Factor	Target Reliability Indices	Performance Factor
2.0	2.3	0.6~0.9
2.5	2.9	0.5~0.7
3.0	3.3	0.4~0.6

Table 26. Relationship of Ratio of Actual to Predicted Bearing Pressure, Target Reliability Index and Performance Factor for SLS

Ratio of Actual to Predicted Bearing Pressure	Target Reliability Indices	Performance Factor
1.0	0.45	1.0
1.5	0.85	0.6
2.0	1.1	0.5

Chapter 7

CONCLUSIONS

A LFD format for the design of shallow foundations for bridges is developed in order to provide a consistent approach to the design of the bridge superstructure and foundations. The code calibration was performed using reliability theory. Uncertainties in the design equations and design parameters for ultimate and serviceability limit states were investigated and their statistics were obtained. Reliability indices inherent in the current allowable stress design method for shallow foundations of bridges were determined. Reliability indices for ultimate limit state are in the range of 2.3 to 3.4, for safety factors between 2 and 3. The corresponding probability of failure is in the range of 10^{-2} to 10^{-4}. This is in good agreement with Meyerhof's conclusions (1970). The corresponding performance factors were found to be in the range of 0.4 to 0.9.

Reliability indices for serviceability limit state are in the range of 0.43 to 1.40, for the ratios of allowable to actual settlement between 1.0 and 2.0. The corresponding probability of failure is in the range of 0.34 to 0.08. This appears to be in good agreement with what may be expected Generally speaking, the results of the reliability analyses are in good agreement with results presented by other investigators; that is, the design of shallow foundations on sand is controlled by settlement criteria in almost all cases.

Reliability-based design procedures appear to be a useful tool for dealing with geotechnical engineering problems. Although this study concentrated only on dead and live load combinations, future extensions that include different load combinations are possible as long as the statistics of these load combinations and suitable load ratios are available. This study illustrates the application of reliability-based design to the design of shallow foundations of bridges.

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Appendix A

Experimental and Theoretical Values of N_γ

Table 27. Experimental and Theoretical Values of N_γ (After De Beer, 1970)

ϕ_f	Experimental N_γ	Theoretical N_γ
36°	47.2	40.1
37°	52	47.4
38°	57.6	56.17
39°	66	66.76
40°	78.4	79.5
41°	91.	95
42°	110.	114
43°	133.4.	137.1
44°	166.6.	165.6
45°	211.6.	200.8
46°	283.4.	244.6
47°	383.4.	299.5

Appendix B

Experimental and Theoretical Values of S_Y

Table 28. Experimental and Theoretical Values of S_γ (After Ingra and Baecher, 1983)

No.	$\frac{B}{L}$	Experimental S_γ	Theoretical S_γ
1	0.12	1.3	0.95
2	0.14	1.1	0.94
3	0.15	0.98	0.94
4	0.16	0.94	0.94
5	0.16	1.13	0.94
6	0.19	1	0.92
7	0.19	1.06	0.92
8	0.2	0.97	0.92
9	0.22	0.96	0.91
10	0.24	1	0.9
11	0.25	1.06	0.9
12	0.25	1.08	0.9
13	0.26	0.97	0.9
14	0.27	1	0.89
15	0.29	0.94	0.88
16	0.31	0.98	0.88
17	0.32	0.95	0.87
18	0.32	1.04	0.87
19	0.33	0.84	0.87
20	0.33	1.17	0.87
21	0.48	1	0.81
22	0.49	0.75	0.8
23	0.49	0.88	0.8
24	0.49	1	0.8
25	0.49	1.06	0.8
26	0.49	1.1	0.8
27	0.49	1.12	0.8
28	0.68	0.98	0.73
29	1	0.62	0.6
30	1	0.62	0.6
31	1	0.63	0.6
32	1	0.7	0.6
33	1	0.73	0.6
34	1	0.83	0.6
35	1	0.85	0.6
36	1	0.9	0.6
37	1	1	0.6
38	1	1.02	0.6

Appendix C

Experimental and Theoretical Values of I_Y

Table 29. Experimental and Theoretical Values of I_Y (After Ingra and Baecher, 1983)

No.	tan *i*	Experimental I_Y	Theoretical I_Y
1	0.11	0.67	0.75
2	0.12	0.66	0.73
3	0.13	0.58	0.71
4	0.14	0.63	0.7
5	0.14	0.77	0.7
6	0.14	0.83	0.7
7	0.14	0.85	0.7
8	0.14	0.89	0.7
9	0.15	0.6	0.68
10	0.16	0.57	0.66
11	0.17	0.72	0.64
12	0.18	0.55	0.62
13	0.18	0.57	0.62
14	0.18	0.71	0.62
15	0.2	0.44	0.59
16	0.21	0.51	0.57
17	0.27	0.50	0.48
18	0.27	0.55	0.48
19	0.27	0.56	0.48
20	0.27	0.57	0.48
21	0.28	0.38	0.47
22	0.29	0.39	0.46
23	0.31	0.26	0.43
24	0.36	0.43	0.37
25	0.37	0.25	0.36
26	0.38	0.21	0.35
27	0.41	0.21	0.32
28	0.53	0.14	0.21
29	0.56	0.18	0.19
30	0.57	0.19	0.19
31	0.57	0.21	0.19
32	0.58	0.11	0.18
33	0.58	0.17	0.18

Appendix D

Experimental and Theoretical Values of $N_q S_q d_q$

Table 30. Experimental and Theoretical Values of $N_q S_q d_q$ (After De Beer, 1970)

$\frac{B}{L}$	ϕ_f	Experimental $N_q s_q d_q$	Theoretical $N_q S_q d_q$
1	30°	22.6	29.0
1	31°	25.7	33.0
1	32°	29.0	37.7
1	33°	35.2	43.0
1	34°	43.0	49.3
1	35°	54.0	56.6
1	36°	68.1	65.2
1	37°	89.0	75.3
$\frac{1}{6}$	30°	16.3	20.2
$\frac{1}{6}$	31°	18.4	22.7
$\frac{1}{6}$	32°	21.0	25.6
$\frac{1}{6}$	33°	25.0	28.9
$\frac{1}{6}$	34°	30.2	32.7
$\frac{1}{6}$	35°	37.7	42.3
$\frac{1}{6}$	36°	47.4	42.3
$\frac{1}{6}$	47°	60.7	48.3

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