

ESSAYS ON THE DESIGN OF PROCUREMENT AUCTIONS

by

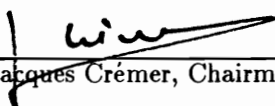
In-Gyu Kim

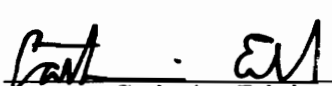
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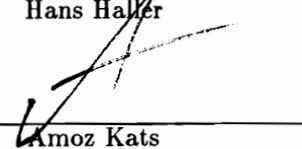
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Economics

(ABSTRACT)

This dissertation is a collection of articles on the design of procurement auctions. Chapter 1 provides a primer to the subsequent three essays. Rather than addressing all the issues involved, it illustrates some basic concepts about auctions, both institutionally and theoretically. It also highlights some problems that arise when auction theory is applied to procurement auctions.

The first essay (Chapter 2) deals with price discrimination against foreign firms in auctioning government procurement, known as price-preference policies. We demonstrate that when each firm has private cost information, its expected profits are the same under both tariff and price-preference policies. Thus, policy implications of tariffs in a Bertrand-like model apply directly to price-preferences in a procurement auction model. We also show that improvement of “the Agreement on Government Procurement” in the spirit of free trade is desirable, given the institutional practice of setting linear price-preferences and of procuring governments’ demands via a standard type of procurement auction. This is because eliminating price-preference policies enhances both domestic and world welfare.

The second essay (Chapter 3) presents a theory of selective tendering. Under contractual incompleteness, a bid-taker needs to depend on a self-enforcing contract where a winning bidder

puts his reputation at stake. In this case the winning bidder will renege on contractual obligations if a one-shot profit from opportunism exceeds his expected profit from maintaining his reputation. First we demonstrate that this profit from maintaining his reputation is a decreasing function of the number of competing bidders. We then derive an optimum number of bidders which ensures the self-enforcing contract at the lowest expected procurement cost. We also show how excessive bidding competition leads to the phenomenon of cost overruns.

The third essay (Chapter 4) investigates the role of competitive bribery in government procurement auctions. We present two models of two-dimensional (bid-prices and side-payments) auctions: the game of “bribes,” in which all the bidders pay competitive side-payments regardless of the event of winning or losing; and the game of “kickbacks,” in which only the winner makes the side-payment that he promised. We argue that if the preference of the corrupt procuring official is known to the risk-neutral bidders, then an isomorphism (in terms of bidders’s expected payoffs) between competitive bribery and competitive bidding, first proved by Beck and Maher (1986), continues to hold in the game of bribes as well as in the game of kickbacks. We further show that, roughly, other things equal, the risk-neutral corrupt procuring official is indifferent between the two games.

To  
My Parents  
for  
Their Unfailing Love and Support

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# Chapter 1

## Auctioning Procurement Contracts: A Prelude

### 1. INTRODUCTION

Consider a government facing the problem of procuring a single indivisible object (a highway construction project, say) from one of a set of potential suppliers. If there existed a competitive market for this object, then the government as a price-taker would simply purchase it at the equilibrium (or market-clearing) price. Even if such a market did not exist, the government (as a monopsonist) could simply tackle the problem by ordering from the lowest-cost supplier on a take-it-or-leave-it basis, if the government knew each supplier's expected cost to supply the object. In most cases, however, government procurement is characterized by informational asymmetry between the government and the potential suppliers. That is, the potential suppliers are better informed than the government is about the expected production costs.<sup>1</sup>

Then, in the presence of informational asymmetry, which mechanism would allow the government to minimize its expected procurement costs? An obvious answer, both theoretically and institutionally, would be the auction (or competitive bidding) mechanism.<sup>2</sup> McAfee and McMillan (1987, p. 701) define an auction as “a market institution with an explicit set of rules determining resource allocations and prices on the basis of bids from the market participants.” Riley and Samuelson (1981, p. 381) also argue that “the auction model is a useful description of

‘thin markets’ characterized by a fundamental asymmetry of market position (or by [imperfect] competition on only one side of the market).”

Cassady (1967) has been an excellent source for historical and institutional description of many auctions. The first rigorous analysis of competitive behavior of bidders (who are informed of their opponents’ parameter values only probabilistically) is due to Vickrey (1961). Since Vickrey’s pioneering study entitled, “Counterspeculation, auctions, and competitive sealed tenders,” the literature on auctions has grown extensively. Since it is not our purpose here to review this literature, we will briefly mention some excellent survey articles later in Section 5.

In auctions there are two polar cases: (i) low-bid auctions where the bidders are potential suppliers and the auctioneer (or bid-taker) is a single buyer (the government in our example), the lowest bidder being the designated winner; and (ii) high-bid auctions where the bidders are potential buyers and the auctioneer is a single seller, the highest bidder being the designated winner. As we are dealing with procurement auctions, we shall adopt, until indicated otherwise, low-bid auctions throughout the thesis. (High-bid auctions can be seen as the mirror image of low-bid auctions).

The aim of this chapter is to provide a primer to the subsequent chapters. More specifically, we aim for two goals in the current chapter. The first is to illustrate some basic concepts about auctions. To do this, we borrow liberally from the existing literature on auctions, particularly from Cassady (1967) for institutional descriptions and from Holt (1980) and Myerson (1981) for theoretical analysis.

The second goal is to highlight some problems that arise when auction theory is applied to procurement auctions. It seems fair to say that most research has placed more emphasis on the theoretical aspects of auctions (under somewhat ideal environments) than on the practical ones. A shift in emphasis leads to three relatively neglected but none the less important

questions. These questions are the following: (i) Can a government reduce its expected procurement cost through a discriminatory procurement policy against foreign bidders? (Chapter 2); (ii) Why does a bid-taker in procurement auctions usually limit entry in bidding competition to a certain group of selected bidders? (Chapter 3); and (iii) What would be the effects of competitive bribery in procurement auctions? (Chapter 4).

This chapter proceeds as follows. In Section 2 we begin by reviewing four major auction institutions. In Section 3 we set forth a conceptual scheme for the description and analysis of competitive bidding. Section 4 addresses some important issues of procurement auctions. Section 5 recommends some readings in the literature on auctions.

## 2. AUCTION INSTITUTIONS

Auctions (as economic institutions) have been widely used since ancient times. Examples abound. In 193 A.D., after having slain their emperor, Pertinax, the leaders of the Roman Empire guard agreed that they would give the throne to the Roman who offered the largest donation. Didius Julianus was chosen as the new emperor when he outbid all his rivals by promising each man 6,250 drachmas [Cassady (1967, p. 29)]. During the last several decades, the rapid growth of government purchases of goods and services has led to governments becoming the single largest purchasers in the economy.<sup>3</sup> A significant proportion of government purchases is implemented by procurement auctions. Billions of dollars of United States Treasury bills are sold through auctions each week. The U.S. federal government also uses auctions to sell federally owned timber and mineral rights. Sales of artwork and antiques at auction houses (such as Sotheby's, Christie's and Phillips) frequently receive extensive media coverage. Auctions are also common methods for sales of perishables such as fresh fish and cut flowers.

Auction methods vary between countries and across auctioned objects [Cassady (1967)]. In this chapter, however, we restrict our attention to the four most frequently observed institutions: the first-price sealed-bid, Dutch, Vickrey (or second-price sealed-bid), and English (or open) auctions. The Dutch and English auctions take place openly, while the other two auctions are sealed-bid procedures, as their names imply. In this section we describe the rules and uses of these auctions. For expositional simplicity, we assume that the government's reserve price (over which the trade cannot be made) does not bind, i.e., the reserve price is greater than or at least equal to the lowest bid-price (in low-bid auctions).

Let us begin with probably the most common institution in procurement auctions, the first-price sealed-bid auction. Again consider a government which wishes to purchase an indivisible object by the use of this auction. Each bidder independently and privately submits one bid without knowing his opponents' bid-prices. Then, the bidder who bids the lowest price wins and must provide the object at his bid-price.<sup>4</sup>

The U.S. government uses this first-price sealed-bid auction both to sell federally owned properties (high-bid auctions) and to purchase its necessities (low-bid auctions). To show the use of first-price sealed-bids in procurement, Hansen (1988) quotes data from the NAPA Handbook. According to it, 77 % of corporate buyers surveyed used the institution of first-price sealed-bid to procure their necessities. These observations justify restricting most parts of this thesis to the rules of the first-price sealed-bid auction.

In a low-price Dutch auction the auctioneer begins by calling some very low price and gradually increases the price until some bidder stops him to claim the object. Then the object is awarded to him at the stopped price. The name Dutch comes from the fact that this institution has been used in Holland to sell cut flowers in bulk (high-price Dutch auctions in this case).

The rules of a Vickrey auction are basically the same as those of the first-price sealed-bid

auction. But in the Vickrey auction, the lowest bidder as the winner is paid the second lowest bid-price. Thus, the Vickrey auction is also called the second-price sealed-bid auction.

Rothkopf, Teisberg and Kahn (1990) observe that, though the Vickrey auction is theoretically appealing, it is rare in real-life situations. They report that, under Secretary George Schultz (1973-1974), the U.S. Treasury experimented with nondiscriminatory sealed bidding, which is essentially a second-price sealed-bid auction.

When most people hear the word auction, the rules of an English (open) auction come across their mind. This is because they are quite familiar with the open bidding process at auction houses. In low-bid English auction, the auctioneer starts the price at the highest acceptable level (or the reserve price) and then gradually lowers the price until only one bidder remains active. The remaining bidder as the winner must provide the object at the last called price.<sup>5</sup>

### 3. GAME MODELS OF BIDDING COMPETITION

The central feature of the literature on auctions is its use of game theory as an analytic framework. Auctions are traditionally modeled as a class of games of incomplete and asymmetric information. To formulate bidding competition as a non-cooperative game, one has to specify it in terms of (i) the players, (ii) the information available to each player, (iii) the strategy space, and (iv) the payoff functions. The players in a typical auction are the bidders.<sup>6</sup> Most auction literature assumes that the number of bidders is commonly known.<sup>7</sup>

The assumption of the information structure classifies the auction literature generally into two categories: common and independent private values models. First, we look at the difference between the two models. Then we describe two different modeling techniques: "standard" and

“optimal” auctions.

### 3.1. *Common vs. Independent Private Values Models*

Consider the U.S. government auctions of oil and gas leases for the outer continental shelf. Each bidding firm obtains an estimate of the unknown value of the lease under contract, using its own seismic information. If all the bidding firms have the same opportunity cost (to extract oil from the lease), the *ex post* value (or economic rent) from the lease is *common* to all the firms. Since the common value is not known *ex ante* (until oil is extracted) and since the bidding firms have different estimates about the common value, bidding competition occurs on the basis of both each bidder’s estimate and his belief about his opponents’ estimates. This is a typical example of a common value auction.

Theoretical models of common value auctions can be best found in Wilson (1977) and Engelbrecht-Wiggans, Milgrom and Weber (1983), among others. Based on these models, Reece (1978) and Hendricks and Porter (1988) analyze oil firms’ bidding for offshore oil leases.

Now let us consider a sale of a Picasso at an auction house. Collectors (as bidders) have different *private* values for the Picasso, which reflect differences in their subjective tastes. If their tastes (or valuations) are independent of each other, an auction for this sale is modeled as an independent private-values model. The private values assumption implies that each bidder’s valuation (cost in procurement auctions) parameter will not be affected by learning about his opponents’ parameters, though the learning may affect his strategic behavior in bidding competition. The independence assumption means that learning about some bidder’s parameters does not help in guessing other bidders’ parameters, i.e., that there is no correlation between bidders’ parameters.

Common and independent private values are the two polar cases in auction models. In

most real-life situations, auctions may incorporate elements belonging to both common and private values [see Milgrom and Weber (1982) for a general model involving both components]. As always in modeling, however, we must forego the realism for simplicity. Common vs. private values could be two obvious candidates for an appropriate simplification.

Researchers frequently use intuitive reasons when they have to choose between common and independent private values for their model specifications. For instance, Holt (1980, pp. 434-5) adopts the independent private values for his model, because “the opportunity cost of undertaking the procurement project is the profit from the best alternative use of the firms’ resources. ..., this opportunity cost is assumed to vary firm to firm.” In his study of bidding competition in the highway construction industry, however, Thiel (1989) presumes that all bidders have the same unknown *a priori* value of the contract (or the same unknown *a priori* cost of performing the contract).

The information structure of a procurement auction may lie somewhere in the middle of the two extreme values, in that each bidder has a different private opportunity cost as well as different sample information about the common value. Thus, it is somewhat troublesome to decide between the two approaches.<sup>8</sup> For expositional simplicity, however, we shall use the independent private-values model throughout the thesis.

### 3.2. *Standard Auctions*

The logical place to start the standard analysis is with Vickrey (1961). In the context of independent private values, his auction model is based on the following two assumptions: (i) the bidders are risk-neutral; and (ii) the parameter values of the bidders are independent and identically distributed. The second assumption (known as the IID assumption) states that each bidder has private information about his parameter (production cost in our example) but his

opponents and the bid-taker view the value of the parameter as a random draw from a common-knowledge distribution  $F(c)$  with  $F(\underline{c}) = 0$ ,  $F(\bar{c}) = 1$ , and  $F(c)$  strictly increasing and differentiable over the interval  $[\underline{c}, \bar{c}]$ .

We consider the situation in which a government wishes to purchase an indivisible object from  $n$  bidders by the use of first-price sealed-bid auction. Uncertain about his opponents' cost parameters, bidder  $i$  (who observes cost  $c_i$ ) determines his bid  $\hat{b}$  by maximizing his expected profit:

$$(1) \quad \text{Max } \pi_i = \text{Pr}[i \text{ wins} \mid c_i](\hat{b} - c_i), \quad i = 1, 2, \dots, n.$$

Together with the above two assumptions, we can confine ourselves to a symmetric equilibrium in which all the bidders use the same strategy function. Assume that the symmetric equilibrium strategy  $b_i = B(c_i)$  is a strictly increasing and continuous function, and that all the bidders except bidder  $i$  use this equilibrium strategy. Bidder  $i$ 's probability of winning,  $\text{Pr}[i \text{ wins} \mid c_i]$ , is then  $[1 - F(B^{-1}(\hat{b}))]^{n-1}$  since bidder  $i$  can win only when all the other bidders bid higher than  $\hat{b}$ . Substituting this into (1), we then have the first-order condition for the problem of bidder  $i$ :

$$(2) \quad d\pi_i/d\hat{b} = [1 - F(B^{-1}(\hat{b}))]^{n-1} \cdot (n-1)(\hat{b} - c_i)[1 - F(B^{-1}(\hat{b}))]^{n-2} f(B^{-1}(\hat{b})) B^{-1'}(\hat{b}) = 0,$$

where  $f(\cdot) \equiv F'(\cdot)$ . By the symmetry (or by the assumption of identical bidders), at equilibrium, bidder  $i$ 's bid  $\hat{b}$  must be based on the equilibrium bidding function  $B(c_i)$ .<sup>9</sup> Substituting this into (2) and since  $B^{-1'}(\hat{b}) = 1/B'(B^{-1}(\hat{b}))$ , the equation (2) can be rewritten as

$$(3) \quad B'(c_i)[1 - F(c_i)]^{n-1} = (n-1)[B(c_i) - c_i][1 - F(c_i)]^{n-2} f(c_i).$$

Solving the differential equation (3), one obtains

$$(4) \quad B(c_i) = c_i + \frac{\int_{c_i}^{\bar{c}} [1 - F(c_i)]^{n-1} dc}{[1 - F(c_i)]^{n-1}}.$$

The second term of the right-hand side of (4) shows, at equilibrium, how much surplus (or economic rent) bidder  $i$  can enjoy with the event of winning. (One can refer to Holt (1980) for a formal derivation process of the symmetric equilibrium bidding strategy.)

Vickrey (1961) proves that, with the assumptions of IID and risk-neutrality, the first-price sealed-bid and English auctions yield the same equilibrium outcomes for both the bid-taker and bidders [Revenue Equivalence Theorem].<sup>10</sup>

### 3.3. *Optimal Auctions*

Beginning with Myerson (1981) and then continuing perforce with many researchers [see, e.g., Crémer and McLean (1985, 1988), Maskin and Riley (1984), and Riley and Samuelson (1981)] into the 1980s, research trends turned from the standard auction to the optimal auction paradigm. Based on the celebrated “direct revelation principle,” the optimal auctions mechanism ensures that a bid-taker maximizes his expected payoff, while the standard auction paradigm focuses on the bidders’ strategic behaviors within the context of the existing institutions.<sup>11</sup>

Let  $p_i(\cdot)$  denote the probability that bidder  $i$  wins, and let  $x_i(\cdot)$  denote the expected payment from the bid-taker to bidder  $i$ . With the two assumptions in Vickrey (1961), then the problem the risk-neutral bid-taker faces can be written:

$$(5) \quad \text{Min}_{p_i, x_i} \sum_{i=1}^n x_i(c_i, c_{-i})$$

subject to

$$(6) \quad x_i(c_i, c_{-i}) - p_i(c_i, c_{-i})c_i \geq 0,$$

$$(7) \quad x_i(c_i, c_{-i}) - p_i(c_i, c_{-i})c_i \geq x_i(\hat{c}_i, c_{-i}) - p_i(\hat{c}_i, c_{-i})c_i,$$

where  $c_{-i} \equiv (c_1, \dots, c_{i-1}, c_{i+1}, \dots, c_n)$ ,  $p_i(\cdot) \geq 0$ , and  $\sum_{i=1}^n p_i(\cdot) \leq 1$ . Inequality (6) is bidder  $i$ 's individual rationality. Bidder  $i$ 's incentive compatibility constraint (7) states that, in equilibrium, he will be induced to report his cost parameter  $c_i$  truthfully. (See Myerson (1981) for a formal description and analysis of optimal auctions.)

Riley and Samuelson (1981) show that if the two assumptions in Vickrey (1961) hold, and if the bid-taker sets a reserve price optimally (to maximize his expected payoff), then the four auctions mentioned in Section 2 result in the same optimal auctions mechanism. This result confirms the Revenue Equivalence Theorem in the context of optimal auctions.

One virtue of the optimal auctions approach is that the symmetry of bidders does not have to be assumed [Myerson (1981)]. Maskin and Riley (1984) prove that if the bidders are risk-averse, then optimal auctions with the first-price rule provide the bid-taker a better outcome than optimal auctions with the second-price rule (i.e., the Revenue Equivalence Theorem breaks down without the assumption of risk-neutrality). Relaxing the independence assumption, Crémer and McLean (1985, 1988) show that a bid-taker can extract all surplus from the bidders.<sup>12</sup> Many authors (e.g., Laffont and Tirole (1987), and Riordon and Sappington (1987), among others) apply the optimal auction technique to design procurement auctions.

Though the optimal auctions approach has established certain benchmark results, it has a

limitation: it is very sensitive to the specifications of auction environments.<sup>13</sup> Thus, it is natural to seek models which are robust enough for a broad class of underlying auction environments. Milgrom (1985, p. 287) relates that: “Too much recent research effort in auctions has been simply applying the latest techniques (principally ‘mechanism design’) to ever more complicated models; too little has been devoted to the very real and important economic questions that auctions raise.”

Following this line of criticism, our models in the subsequent analyses are based on the standard auction approach.

#### 4. SOME ISSUES IN PROCUREMENT AUCTIONS

This section illuminates a number of interesting situations in auctioning procurement contracts that have not been easily analyzed with existing models.

With the assumption of identical or symmetric (or homogeneous) bidders, most the existing literature treats bidders anonymously in the bidding process. In many cases, however, bidders may not be homogeneous. Even if we assume that bidders are homogeneous (in that their cost parameters are drawn from the same distribution function), price discrimination against foreign bidders makes a group of foreign bidders different from a group of domestic bidders, as if the two groups come from different distributions. Put differently, the assumption of symmetric bidders is stronger and possibly more problematic here than it is in a standard auction. In Chapter 2 we investigate asymmetric equilibrium bidding strategies associated with price discrimination against foreign bidders.

As Spulber (1990) points out, auctioning a procurement contract involves both a spot exchange (or a bid-price) and future responsibility (or contractual terms), while auctioning an

artwork concerns only a spot exchange. If contracts are complete, and if the legal system of liability is perfect, then the contractual terms are automatically executed so that the bidding model of a procurement contract is reduced to that of an artwork. In reality, however, contracts are substantially incomplete due to “transaction costs” of making state-contingent contracts [see Williamson (1985) for the exposition of transaction costs], and the legal system of liability is not that perfect. Thus, if the “premium” for performing the contractual terms were not high enough, the winning contractor might want to deviate from future responsibility in order to reduce performance cost. Chapter 3 investigates how the number of competing bidders affects incentives of performing contractual terms.

In less developed countries, bids for government procurement frequently are more than one-dimensional, involving side-payments as well as bid-prices. This is especially true when an object under contract is not well specified, and consequently a corrupt procuring official has discretionary powers over the definition of the object. Chapter 4 presents two models of two-dimensional (bid-prices and side-payments) auctions: the game of “bribes,” in which all the bidders pay competitive side-payments regardless of the event of winning or losing; and the game of “kickbacks,” in which only the winner pays the side-payment that he promised.

There are many other important issues in procurement auctions that we have omitted from our considerations. For instance, the problem of collusion among bidders may have great practical importance, as collusion is rampant and increases procurement costs substantially in real-life auctions.<sup>14</sup>

## 5. FURTHER READING

This section intends to familiarize readers with the auction literature. Milgrom (1989)

provides a very readable introductory. Myerson (1983) presents a good explanation of the logic behind optimal auctions. Bulow and Roberts (1989) simplify the exposition of optimal auctions by connecting optimal auctions and the monopolist's problem in third-degree price discrimination.

An excellent survey of the development up to 1980 can be found in Engelbrecht-Wiggans (1980), who stresses an auction as a non-cooperative game. McAfee and McMillan's (1987) survey makes a comprehensive but lucid exposition on recent development. Wilson's (1992) survey interprets auction research in terms of game theory. The surveys by Milgrom (1985, 1987) cover a narrower range of topics but are more thorough than other surveys.

## FOOTNOTES

1. This informational asymmetry is partly due to the absence of a market. If a market for the object existed, then communication (through an equilibrium price) somehow would transmit relevant information to each trader in the market and thus, most problems associated with informational asymmetry would disappear.
2. There are some other alternative mechanisms, such as posting a fixed-price by the government or bargaining over the price between the government and the designated supplier. Under informational asymmetry, however, an auction mechanism brings the government a better outcome than any other alternative [McAfee and McMillan (1987)].
3. For instance, in 1974 the U.S. federal and local governments' purchases were \$308 billion, or 22 percent of the GNP [Amick (1976, p. 75)].
4. If the auctioned objects are more than one unit and if each bidder is allowed to submit one or more bids, then the first-price sealed-bid auction is called a discriminatory auction, as each unit may be auctioned at a different price. The U.S. government currently uses this kind of auction to sell U.S. Treasury bills to the major dealers.
5. Another version is that, after the auctioneer's initial calling, each bidder is free to revise and call his bid downwards.
6. The bid-taker behaves as a Stackelberg leader in that he announces the rules of the game and commits himself to them prior to bidding competition. In this sense, the bid-taker can be

regarded as a *passive* player. Though, in some situations, a bid-taker hires an auctioneer to delegate auctions [see Laffont and Tirole (1991) for more details on delegation in auctions], in this thesis we assume that the auctioneer is also the bid-taker.

7. McAfee and McMillan (1987a) discuss an implication of a stochastic number of bidders, and Harstad (1990) compares auction institutions with an endogenous number of bidders.

8. In order to overcome such somewhat arbitrary intuitions, Paarsch (1992) proposes empirical specification tests to decide between the two approaches.

9. Holt (1980) proves that this strategy is a symmetric Nash equilibrium in that no bidder can increase his expected payoff by deviating from this strategy. Maskin and Riley (1986) show that (in the case of two bidders) the symmetric equilibrium is the unique equilibrium.

10. The Revenue Equivalence Theorem also holds for the Dutch and Vickrey auctions, as the game modeling of the first-price sealed-bid and Dutch auctions are identical, and as the English auction is conventionally modeled as a Vickrey auction [see Milgrom (1989) for more details].

11. The revelation principle states that the outcome of any mechanism that is not incentive-compatible can be mimicked by one that is incentive-compatible. This result justifies restricting attention to a truth-telling mechanism [see Myerson (1985) for more details].

12. Robert (1991) generalizes Crémer and McLean (1988). McAfee, McMillan and Reny (1989) also proves the full extraction of surplus in the context of common values when bidders' information is correlated. Note that the IID assumption in a common value model implies that,

conditional on the common (or true) value, information samples of all bidders are independent and identically distributed.

13. Rothkopf et. al. (1990) investigate a few other reasons why Vickrey auctions (which are truth-telling optimal auctions under some conditions) are rare in practice.

14. In their study of collusion, McAfee and McMillan (1992) quote a story from *New York Times* (April 4, 1988, p. 29): “According to the U.S. Justice Department’s antitrust chief, bid-rigging by highway constructors increased the cost of building roads by 10 percent or more.” Graham and Marshall (1987) also provide a theoretical investigation on collusion.

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## Chapter 2

# Procurement Auctions and Price Discrimination: An Economic Analysis of “the Agreement on Government Procurement”\*

### 1. INTRODUCTION

The Tokyo Round of multilateral trade negotiations, under the auspices of the General Agreement on Tariffs and Trade (GATT), was concluded in April 1979. The most widely acclaimed progresses have taken place in the area of non-tariff barriers to trade, among which “the Agreement on Government Procurement” [hereafter the Agreement] has a particular importance in terms of economic efficiency and resource allocation. The Agreement which became effective January 1, 1981, is designed to insure that governments do not discriminate against foreign firms in favor of domestic firms. To achieve this goal, the Agreement introduces the principle of non-discrimination, although it still leaves much room for improvement.<sup>1</sup> The primary objective of this chapter is to develop a theoretical foundation for understanding the Agreement. To do this, this chapter deals with the price discrimination against foreign firms in government procurement auctions, known as price-preference policies, by the use of the auction and game theory.

During the last several decades, government purchases of goods and services have experienced a rapid growth, which led to governments being the single largest consumer in the

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economy. At the time of the Tokyo round negotiations, the market potential affected by government procurement was estimated to be several hundred billion dollars each year [Graham (1983)].<sup>2</sup> A significant proportion of government purchases is governed by procurement auctions. For instance, when the U.S. Gross Domestic Product was 4081 billion dollars in 1985, federal and local government purchases were 355 and 466 billion dollars, respectively. In the same year, the U.S. federal government paid out 187 billion dollars to procurement contractors.

Given these figures, the importance of efficiency in the government procurement market cannot be overemphasized. Discriminatory government procurement policies that favor domestic firms have long been criticized. They are accused of weighting government decisions in favor of inefficient domestic firms to the detriment of the public interest (via a misuse of public funds). For example, consider a government requesting bids on a highway construction project. Suppose that the government announces a 50 percent price-preference. The project will be awarded to a domestic firm at the price of its bid tendered if it is no higher than 50 percent of the lowest foreign firm's bid. As a result of the protective price-preference policy, the government may have to spend more public funds when the domestic firm wins.

However, the governments' favoritism toward domestic firms may be justified by non-price considerations such as after-sales service, assurances of delivery date and product quality, and accessibility in case of legal difficulties. Governments also often use their purchases as a conscious policy tool either to correct the balance-of-payment or to promote economic development for less developed regions [de Mestral (1982)]. However, this chapter assumes that the government is only concerned with minimizing its expected procurement cost or maximizing domestic welfare, which is defined as the sum of the possible procurement cost savings and domestic firms' expected profits.

There are also political reasons why governments favor domestic firms. Most elected

governments try to minimize their political costs by stressing the domestic employment and welfare stemming from increased exports; thus, they are presumably vulnerable to the charges of “exporting jobs” and being “uncaring of domestic industry.” In order to overcome such a political risk of antagonizing interest groups (representing the import-competing sector), international cooperation and multilateral negotiations are strongly needed. Thus, governments must be prepared to persuade the public that reciprocal negotiations on the elimination of non-tariff barriers in the government procurement market are necessary in order to enhance economic efficiency and proper use of public funds. This chapter provides a theoretical reasoning why such international cooperation is required. Furthermore, from a normative perspective, this chapter suggests that the Agreement should be improved upon in the spirit of the GATT (or of free trade).

As the rules of the game in the economy are affected by the institutional environment, an economic model should incorporate the essence of the underlying structure of institutions’ social practices, which binds players’ decision making. To understand the institutional environment of the Agreement, it is helpful to look at its historical background. In the mid 1960s, the countries of the Organization for Economic Cooperation and Development (OECD) set out to investigate the possibility of forming an international agreement on government procurement contracts. This investigation originated from the OECD countries’ criticism to the Buy American Act where American firms are given a price-preference ranging from 6 to 12 percent. This price-preference is raised up to 50 percent for military and agricultural equipment procurements. The U.S. protested that the U.S. was the only country which maintained a fixed statutory price-preference for domestic firms and that all other governments had practiced discriminatory procurement policies by more covert ways. The U.S. refutation was right in that there were several ways of covert favoritism toward domestic firms (or non-tariff barriers against foreign firms) such as residence

requirements on bidding firms, stringent technical and financial requirements being applied to foreign firms, too short notices for foreign firms to apply, and selective tendering procedures [Lowinger (1976)].

The Agreement (as the outcome of OECD's such efforts) removes these covert favoritism and provides "transparency" (or the degree of publicity) of the procedures of government procurements. Under the transparency of the Agreement, governments must explain the procedures of procurement to the potential suppliers prior to bidding competition and must be prepared to explain why and how the winner was determined *ex post* via publications.<sup>3</sup> Since the Agreement replaces non-tariff barriers with tariffs via transparency [see GATT (1986) for details], the institution of the Agreement looks very similar to that of the Buy American Act. One of the main goals of this chapter is to show the equivalence between price-preferences and tariffs so that, ironically enough, in equilibrium, the Agreement is institutionally identical to the Buy American Act.

Another important institutional environment in government procurement is the economic institution of auctions: auctions are the most prominent way in which governments procure their necessities from the private sector. This is because a government as a prospective buyer can enjoy some monopsony power but potential suppliers (firms) are better informed than the government about the expected procurement cost. Most theoretical models of procurement auctions assume that the competing firms are symmetric: firms differ only with respect to a cost parameter, but the parameter is regarded as a random draw from the *same* distribution function. Once price-preferences are applied to procurement auctions, however, the firms in the favored class (domestic firms) are no longer symmetric to the firms in the non-favored class (foreign firms), since at least the support of the distribution function will be changed. We consider this type of asymmetry both in the first-price sealed-bid and the second-price sealed-bid auctions.<sup>4</sup>

Before proceeding, we note the relationship between this chapter and McAfee and McMillan's (1989) analysis, which is the only article we are aware of that is explicitly devoted to a theoretical investigation of price-preference policies.<sup>5</sup> McAfee and McMillan (1989), based on Myerson's (1981) revelation principle in auctions, examine the potential cost savings from the optimal discrimination in procurement auctions when there are two types of asymmetric bidders--domestic and foreign firms. They argue that the analogy between price-preferences and tariffs is inaccurate: "unlike the zero tariff, the zero preference is not the appropriate benchmark for evaluating the effects of price-preferences."<sup>6</sup> Favoring domestic firms may raise the probability that a domestic firm will win, but it also presses foreign firms (which are assumed to have more favorable cost distributions than domestic firm do) to bid lower. Because of this kind of trade-off, they believe that policies such as the 6 percent buy-American preference could allow the government to minimize its expected procurement cost.

Though we find their results intriguing, we believe that their normative analysis does not fully consider a few important institutional aspects. For example, since they analyze the functional relationship between optimal discrimination and cost structure, their optimal price-preferences in general are non-linear in final bid-prices, while linear price-preferences are widely adopted in the real world. Often the sophisticated type of optimal auctions may be of limited use to the government. As Milgrom (1989) correctly points out, optimal auction theory is too sensitive to the specifications of the auction environment, while the real-life auction institutions are all simple but stable in a variety of environments. This sensitivity may make governments not use optimal auctions in real-life situations.

Unlike McAfee and McMillan (1989), our auction model does not use the direct revelation mechanism. We accept the institutional restriction that a government announces a linear price-preference prior to bidding competition. Our model shows how McAfee and McMillan's (1989)

policy recommendation of a positive price-preference depends critically on details of their model: contrary to their policy recommendation of positive price-preferences, eliminating price-preferences minimizes the government's expected procurement cost in our model. We also consider the bilateral price-preferences with two governments, while McAfee and McMillan (1989) only consider the monopoly price-preferences with one government.

An outline of this chapter is as follows. In the next section, we begin by presenting the formal structure of the model. We shall then show that the two seemingly different regimes, price-preference and tariff, result in the same outcome in equilibrium. Section 3 presents an asymmetric equilibrium of a very simple discrete model both in the first-price sealed-bid and the second-price sealed-bid auctions. Despite the simplicity of the model, some suggestive results arise: a price-preference policy may transfer some of the government's expected payment from a foreign firm to a domestic firm so that it increases the domestic firm's expected profit. However, this type of transfer (or rent shifting) may be possible only when the government is willing to pay more public funds for the auctioned object. Since the simultaneous differential equations that determine a Nash equilibrium in an asymmetric first-price sealed-bid auction do not have a closed form solution in general, in Section 4 we formalize a rebate auction, (which is a modified version of the first-price sealed-bid auction) in order to derive an analytical approximation for the closed form solution. We show that, in a rebate auction, the expected procurement cost to the government is an increasing function of the discriminatory price-preferences. Thus, the bigger the price-preference, the greater the expected procurement cost. In Section 5, we analyze the price-preference policies in a general setting of second-price sealed-bid auctions. In Section 6, we provide a model of bilateral price-preference policies between two countries. With the introduction of a foreign government, the model becomes a two-stage game in which the domestic and the foreign governments must arrive at a Nash equilibrium in price-preference policies prior

to bidding competition of firms. We prove that zero price-preference will be the best policy for a government, if its objective is to minimize its expected procurement cost. We also show that each country has an incentive to engage in monopoly price-preferences if its objective is to maximize expected domestic welfare. Thus, it is shown that international coordination such as the Agreement is strongly needed in order to overcome such an incentive of monopoly price-preferences. Section 7 contains concluding remarks.

## 2. EQUIVALENCE BETWEEN PRICE-PREFERENCES AND TARIFFS

In this section we first describe the formal structure of a procurement auction model with price-preferences. For simplicity, we assume that there are only two potential suppliers, a domestic firm (firm 1) and a foreign firm (firm 2). (Generalization of more than two firms is not a trivial exercise, but similar principles apply). To show the equivalence between price-preferences and tariffs, we shall then characterize an asymmetric Bertrand duopoly model for a tariff regime, where two firms simultaneously and independently name prices, and where the government is not only the consumer but also the tariff setter.

### 2.1. *A price-preference regime*

We consider the situation in which a government must procure an indivisible object (a highway construction project, say) from one of the two firms by the use of a first-price sealed-bid auction. The government as the Stackelberg leader announces a  $d$  percent price-preference to the domestic firm's bid-price prior to bidding competition. To normalize price-preferences, we define  $k = 1/(1 + 0.01d)$ , and therefore,  $0 \leq k \leq 1$ . Higher values of  $k$  indicate lower degree of price discrimination against the foreign firms. Thus,  $k = 0$  (or  $d = \infty$ ) is the most extreme form of

discrimination and  $k = 1$  (or  $d = 0$ ) corresponds to the absence of discrimination.

The rules of the first-price sealed-bid auction with a price-preference  $k$  are as follows. The two firms submit their sealed-bids both simultaneously and noncooperatively. Let  $b_1$  and  $b_2$  be the domestic and the foreign firm's sealed bids, respectively. If  $kb_1 < b_2$ , then the domestic firm wins and must provide the auctioned object at  $b_1$ . It should be noted that  $kb_1 < b_2$  can be consistent with  $b_1 > b_2$ . In other words, the payment by the government is determined by the pair of bids tendered,  $(b_1, b_2)$ , while the choice of the winner (or designated supplier) is based on the comparison of  $(kb_1, b_2)$  or equivalently, of  $(b_1, k^{-1}b_2)$ . If  $kb_1 = b_2$ , a coin is flipped to determine the winner.

We assume that the government and two firms are risk-neutral and that the two firms produce equal-quality products. We also assume that the government and two firms are expected cost minimizer and expected profit maximizers, respectively. For analytical simplicity, we employ an independent private-value model for the auction.<sup>7</sup> As in Holt (1980), the (opportunity) cost of bidding for a procurement project is the profit from the best alternative investment of a firm's resources. This cost is private information to each firm and differs from firm to firm. Assumption 1 summarizes each firm's belief about its opponent's cost.

*Assumption 1.* Firm  $i$  knows its own cost parameter,  $c_i$ , which is considered by its opponent and the government to have been drawn from a common-knowledge distribution  $G_i$  with  $G_i(c_i) = 0$  and  $G_i(\bar{c}_i) = 1$ ,  $i = 1, 2$ . The distribution  $G_i(c_i)$  is strictly increasing and continuously differentiable over the interval  $[c_i, \bar{c}_i]$ . Each firm's draw is independently distributed from the other firm's draw.

Assumption 1 describes informational and cost asymmetries. Since the assumption of asymmetric

bidders usually causes computational complexity, most of the literature on auctions have assumed symmetric bidders. In the case of procurement auctions with a price-preference policy, however, the assumption of symmetric bidders does not extricate us from such a complexity. That is, even if we assume symmetric firms with the same cost distribution, the price-preference makes the domestic firm no longer symmetric to the foreign firm. In addition, the assumption of asymmetric bidders is not only weaker but also more suited to model possible international differences in input prices. For simplicity, we maintain the assumption of independence.

Since the two firms submit their bids both simultaneously and noncooperatively, each firm cannot observe the bid-price of its opponent when submitting its bid. Rather, each firm anticipates the bid-price of its opponent at the bidding stage. Uncertain about its rival's bid-price, firm  $i$  determines his bid-price  $b_i$  by maximizing its expected profit, given its cost observation  $c_i$  and the price-preference policy  $k$ :

$$(1) \quad \pi_i(b_i; c_i, k) = \Pr[i \text{ wins} \mid b_i](b_i - c_i), \quad i = 1, 2,$$

where  $\Pr[i \text{ wins} \mid b_i]$  stands for the probability that firm  $i$  wins with bid-price  $b_i$ .

In order to calculate  $\Pr[i \text{ wins} \mid b_i]$  explicitly, firm  $i$  must guess how its opponent would bid with any possible private cost observation. The existence of asymmetric equilibrium bidding strategy is now assumed:

*Assumption 2.* In an asymmetric first-price sealed-bid auction, there exists a pair of equilibrium bidding strategies,  $(B_1(c_1), B_2(c_2))$ , which are strictly increasing and continuously differentiable.<sup>8</sup> Firm  $i$  is correctly believed to follow  $B_i(c_i)$  by its opponent.

Assumption 2 implies that the two firms are Bayesian players in the bidding game: each firm anticipates the bidding behavior of its opponent when it has to make a bid with its cost observation.

Since Assumption 2 postulates the existence of a pair of strictly increasing equilibrium bidding strategies, there must exist a corresponding pair of inverse functions such that  $B_i^{-1}(B_i(c_i)) = c_i$ ,  $i = 1, 2$ . Thus, the probability of winning for each firm,  $\Pr[i \text{ wins} \mid b_i]$ , is

$$[1 - G_2(B_2^{-1}(kb_1))] \text{ for firm 1,}$$

(2)

$$[1 - G_1(B_1^{-1}(k^{-1}b_2))] \text{ for firm 2.}$$

The reasoning behind equation (2) can be seen quite easily in Figure 2.1. On the horizontal axis,  $c_i$  represents each firm's cost structure and on the vertical axis,  $b_i$  represents each firm's bidding structure. The curve  $B_i(c_i)$  is the locus of each firm's bidding schedule based on  $c_i$ . Thus, for example, with its bid of  $b_2^0$ , the foreign firm can win only when the deflated value of the domestic firm's bid is greater than  $b_2^0$ , i.e., when  $b_2^0 < kB_1(c_1)$  or  $c_1 > B_1^{-1}(k^{-1}b_2^0)$  as the domestic firm is believed to follow  $B_1(c_1)$  by Assumption 2. And the probability of this occurring is  $(1 - G_1(B_1^{-1}(k^{-1}b_2^0)))$ . Then, substituting (2) into (1) yields firm  $i$ 's expected profit as a function of its bid,  $b_i$ :

$$\pi_1 = [1 - G_2(B_2^{-1}(kb_1))](b_1 - c_1),$$

(3)

$$\pi_2 = [1 - G_1(B_1^{-1}(k^{-1}b_2))](b_2 - c_2).$$

A Nash equilibrium can now be defined.

*Definition 1.* A pair of bidding strategy  $(B_1(c_1), B_2(c_2))$  is a Nash equilibrium if and only if  $B_i(c_i) = \operatorname{argmax} \pi_i(b_i(c_i), B_j(c_j))$  for all  $c_i \in [\underline{c}_i, \bar{c}_i]$ ,  $i, j = 1, 2; i \neq j$ .

In order for  $b_i$  to also be firm  $i$ 's optimal strategy, it must be the case that  $b_i = B_i(c_i)$ . That is, in equilibrium, firm  $i$ 's best bid  $b_i$  which maximizes  $\pi_i$  in (3) must be equal to  $B_i(c_i)$  for all  $c_i \in [\underline{c}_i, \bar{c}_i]$ ,  $i = 1, 2$ .

## 2.2. A tariff regime and a comparison

Consider a Bertrand model where the two firms choose prices and compete for the right to sell. The firm with the lowest price wins and must provide the object demanded at that price. If the two firms must to name prices simultaneously and independently, the Bertrand model becomes nothing more than a model of the first-price sealed-bid auction. Now suppose that a proportional tariff,  $0 \leq t \leq 1$ , is levied upon the winning bid of the foreign firm. But, again, the government as the Stackelberg leader announces a tariff  $t$  prior to bidding competition. If firm 1 and firm 2 submit sealed bids,  $\beta_1$  and  $\beta_2$  non-cooperatively, given the announcement of  $t$ , then the right to sell is awarded to the firm which submits the lowest bid. However, the payment to each firm by the government is based on  $(\beta_1, (1 - t)\beta_2)$ , as the foreign firm is taxed  $t\beta_2$  upon its winnings. Note that the tariff regime protects the domestic firm by collecting tariff revenues from the foreign firm after it wins. On the other hand, the price-preference regime can discriminate against the foreign firm without such a tariff, and prior to bidding competition. Thus, tariffs distort the prices paid but not the choice of the supplier directly, while price-preferences distort the choice of the supplier but not the prices paid directly.

Suppose that Assumption 2 is applied to a pair of equilibrium bidding strategies,  $\tilde{B}_i(c_i)$ ,  $i = 1, 2$ , in the tariff regime. Then, there must exist a pair of inverse functions such that  $\tilde{B}_i^{-1}(\tilde{B}_i(c_i)) = c_i$ . Firm  $i$ 's problem is to maximize its expected profit (but, in this regime, the foreign firm tries to maximize its net expected profit after tariff deduction):

$$\begin{aligned} \tilde{\pi}_1(\beta_1; c_1, t) &= [1 - G_2(\tilde{B}_2^{-1}(\beta_1))](\beta_1 - c_1), \\ \tilde{\pi}_2(\beta_2; c_2, t) &= [1 - G_1(\tilde{B}_1^{-1}(\beta_2))][(1 - t)\beta_2 - c_2]. \end{aligned} \tag{4}$$

We can now compare the tariff regime with the price-preference regime via the comparison of (1) and (4). Proposition 1 describes the equivalence between tariffs and price-preferences.

*Proposition 1. In equilibrium,  $\pi_i(b_i; c_i, k) = \tilde{\pi}_i(\beta_i; c_i, t)$ , if  $t = 1 - k$ . That is, under both tariff and price-preference policies, each firm's expected profits are the same if  $t = 1 - k$ .*

*Proof.* Let  $\hat{\beta}_2$  denote net payment to the foreign firm after tariff deduction, i.e.,  $\hat{\beta}_2 \equiv (1 - t)\beta_2$ . Similarly, define  $\hat{B}_2(c_2) = (1 - t)\tilde{B}_2(c_2)$ . Making these substitutions and using  $t = 1 - k$ , (4) can be rewritten as

$$\begin{aligned} \tilde{\pi}_1 &= [1 - G_2(\hat{B}_2^{-1}(k\beta_1))](\beta_1 - c_1), \\ \tilde{\pi}_2 &= [1 - G_1(\tilde{B}_1^{-1}(k^{-1}\hat{\beta}_2))](\hat{\beta}_2 - c_2). \end{aligned} \tag{5}$$

Then, one can easily recognize that  $\tilde{B}_1(c_1) = B_1(c_1)$  and  $\hat{B}_2(c_2) = k\tilde{B}_2(c_2) = B_2(c_2)$  are a solution of (5) if and only if  $B_1(c_1)$  and  $B_2(c_2)$  are a solution to (3). Therefore,  $B_i(c_i)$  is a Nash equilibrium bidding strategy in both regimes, and the two regimes yield the same expected profit to each bidder. This means that the structure of (5) is identical to that of (3), although the comparison schedule for the choice of the supplier under each regime,  $(kB_1(c_1), B_2(c_2))$  and  $(\tilde{B}_1(c_1), \tilde{B}_2(c_2))$ , appears different from the other.  $\square$

Since  $B_i(c_i)$  is not only firm  $i$ 's Nash equilibrium bidding strategy but also the payment schedule by the government, Proposition 1 implies that there exists a one-to-one correspondence between the two policies in terms of the government's expected payment. Consequently, if the government can minimize its expected procurement cost with a zero tariff ( $t = 0$ ), it can also achieve the same goal with a zero price-preference ( $d = 0$  or  $k = 1$ ), and vice versa.<sup>9</sup> Later we will show that the zero price-preference policy minimizes the government's expected procurement cost.

The Agreement allows governments to impose tariffs on foreign contractors [see GATT (1986)]. Due to Proposition 1, we can continue our analysis on procurement policies in terms of price-preferences instead of in terms of tariffs. Unfortunately, a closed form characterization of equilibrium  $B_i(c_i)$  in (3) in general has not been obtained yet except for the very limited case discussed by Griesmer, Levitan and Shubik (1967). In Section 4, we derive a closed form solution for  $B_i(c_i; k)$  in a modified version of the model. In the next section, however, we consider the simplest case where each bidder's cost observation can assume only two values.

### 3. A DISCRETE CASE

In this section we first analyze the effects of price-preference policies at the second-price sealed-bid auction and then at the first-price sealed-bid auction. We use what is perhaps the simplest model capable of capturing the essential features of price-preference policies.

Again, consider the government which procures a single indivisible object from two potential suppliers, a domestic firm (firm 1) and a foreign firm (firm 2). The basic assumptions made in Section 2 are also applied here. Though we do not need Assumption 2 here, we need an equivalent of Assumption 1: The cost parameter,  $c_i$ , of firm  $i$ ,  $i = 1, 2$ , is independent and identically distributed, drawn from a common-knowledge two-point probability distribution on  $\{0, \bar{c}\}$ ,  $\bar{c} > 0$ . Let  $p = \Pr[c_i = \bar{c}]$ . Note that both firms are symmetric in terms of their cost distribution. Again, each firm's objective is to maximize its expected profit given its cost realization, as specified in (1).

### 3.1. *Second-price sealed-bid auction*

Consider a second-price sealed-bid auction: each firm submits a sealed-bid, all bid-prices are revealed simultaneously, and the lowest bidder wins and is rewarded the auctioned object at the second lowest bid-price. If both firms submit the same bid-price, a coin is flipped to determine the winner. It is well known that, in the second-price sealed-bid auction (or equivalently in the open auction), an equilibrium strategy for each firm is to bid its cost realization, i.e.,  $B(c) = c$ . This equilibrium bidding strategy is also a (weakly) dominant strategy since it is firm  $i$ 's best response to any bidding strategy employed by its opponent.<sup>10</sup>

The result of Proposition 1 continues to hold at the second-price sealed-bid auction by the Revenue Equivalence Theorem [see McAfee and McMillan (1987) for an exposition of this theorem], one implication of which tells us that the first-price and the second-price sealed-bid auctions yield the same expected profit to each firm. Let us now consider the effects of a

discriminatory price-preference policy  $k$  on the government's expected payment via the equilibrium bidding strategy. The government announces that the comparison schedule to determine the winner is  $(b_1, k^{-1}b_2)$  and that the payment to firm 1 is  $k^{-1}b_2$  upon its winning but the payment to firm 2 is  $b_1$  upon firm 2's winning. Then we can observe the following:

*Claim 1.* If each firm's cost observation is either zero or  $\bar{c}$  with positive probabilities  $1 - p$  and  $p$ , respectively, then the government's expected payment in the second-price sealed-bid auction is increasing in  $k^{-1}$ .

*Proof.* If there is no discriminatory policy, we can represent the set of the two bidders' observations on their costs by a state set  $S = \{(0, 0), (\bar{c}, 0), (0, \bar{c}), (\bar{c}, \bar{c})\}$ , where the first (respectively, the second) element of each state denotes firm 1's (respectively, firm 2's) cost observation. Since the government's expected payment is the sum of the expected payment to each state, the government payment will be  $p(2 - p)\bar{c} [= (1 - p)^2 0 + 2p(1 - p)\bar{c} + p^2\bar{c}]$  in equilibrium. Next impose the discriminatory rule  $k$  upon this auction. Then the payment schedule is affected by  $k$ , while the equilibrium strategy,  $B(c) = c$ , is unchanged. Thus, the government's payment on the first two possible states in  $S$  is not affected by  $k$ . However, the last two possible states in  $S$  is affected by  $k$ . Thus, the government's expected payment with  $k$  will be  $p\bar{c}(1 - p + k^{-1})$ , which is increasing in  $k^{-1}$  □

With the discriminatory policy  $k$ , the domestic firm's (or firm 1's) probability of winning increases from  $1/2$  to  $(1 + p^2)/2$ , while that of foreign firm (or firm 2) decreases from  $1/2$  to  $(1 - p^2)/2$ . In order to increase the domestic firm's probability of winning through  $k$ , however, the government must be willing to pay more expected procurement cost. This type of transfer of the

government's expected payment from the foreign firm to the domestic firm through  $k$  will be further analyzed in Section 5.

### 3.2. *First-price sealed-bid auction*

The structure of the game is basically the same as in Section 2, except that Section 2 deals with a continuous cost distribution. As the cost parameter  $c_i$  is regarded as a random draw from a common-knowledge two-point distribution on  $\{0, \bar{c}\}$ , the high-cost bidder will be used as shorthand for a bidder who observes cost of  $\bar{c}$ , while the low-cost bidder will refer to a bidder who observes cost of zero. It would be convenient to think that a bidder (a low- or a high-cost type) has two opponents: a high-cost opponent with probability  $p$  and a low-cost opponent with  $1 - p$ .

Our analysis with two bidders builds on Maskin and Riley (1986), though they deal with a high-bid auction. With the discrimination rule  $k$ , the government announces that the comparison schedule to determine the winner is  $(b_1, k^{-1}b_2)$  and that the payment to firm 1 is  $k^{-1}b_2$  upon its winning but the payment to firm 2 is  $b_1$  upon firm 2's winning. We first show that, in equilibrium, a high-cost domestic firm bids  $k^{-1}\bar{c}$  and a high-cost foreign firm cannot win. To see why, suppose that the high-cost foreign firm bids less than  $\bar{c}$ . Then it might win the auction with positive probability, but it must expect a negative expected profit. Now suppose that the high-cost domestic and foreign firms bid  $b_1 > k^{-1}\bar{c}$  and  $b_2 > \bar{c}$ , respectively, and that  $b_1 = k^{-1}b_2$ . Then firms 1 and 2's profits are  $(k^{-1}b_2 - \bar{c})/2$  and  $(b_1 - \bar{c})/2$ , respectively, as each firm's winning probability is  $1/2$ . If one firm bids infinitesimally smaller than its previous bid, then it can increase its chance of winning from  $1/2$  to one; thus, it can increase its expected profit. Foreseeing these results, the high-cost foreign firm cannot bid higher than  $\bar{c}$ . The domestic firm can win the auction with probability one if both firms observe high costs and if the domestic firm bids  $k^{-1}\bar{c} - \epsilon$ , where  $\epsilon$  is infinitesimally small. For mathematical convenience, we set this highest

feasible price of the high-cost domestic firm is  $k^{-1}\bar{c}$ .<sup>11</sup>

Next consider low-cost firms (or firms which observe cost of zero). Since the distribution of cost observation is discrete, there is no equilibrium in pure strategy for low-cost firms. To see this, assume that both firms observe low-cost and that firm 1 bids  $k^{-1}(\bar{c} - \epsilon)$ . Then firm 2's best response is to bid  $\bar{c} - 2\epsilon$ , in which case firm 1 would deviate to  $k^{-1}(\bar{c} - 3\epsilon)$ , and so on. However, if the decreasing bid-price hits some low price, a low-cost firm's best response would be to bid the original price, realizing it would be better to beat only a possible high-cost opponent. Then the cycle of price-cutting continues.

Suppose that both firms observe low-cost. Let  $F_i(b)$  be the cumulative distribution function of firm  $i$ 's bid. Let  $\underline{b}_i$  and  $\bar{b}_i$  be the infimum (or largest lower bound) and the supremum (or smallest upper bound), respectively, for firm  $i$ . Maskin and Riley (1986) prove that  $F_i(b)$  is continuous over  $[\underline{b}_i, \bar{b}_i]$  and that there can be no subinterval of  $[\underline{b}_i, \bar{b}_i]$  over which  $F_i(b)$  is constant.<sup>12</sup> Since a high-cost domestic firm bids  $k^{-1}\bar{c}$  in equilibrium, a low-cost foreign firm can bid up to  $\bar{c} - \epsilon$ , without losing the auction to the high-cost domestic firm. For mathematical convenience, we set  $\bar{b}_2 = \bar{c}$  [Refer to Footnote 11]. Lemmas 1 and 2 in Section 4 and the proof of Claim 2 will show that, in equilibrium,  $\underline{b}_1 = k^{-1}\underline{b}_2$  and  $\bar{b}_1 = k^{-1}\bar{b}_2$ . For notational simplicity, set  $\underline{b} = \underline{b}_1 = k^{-1}\underline{b}_2$  and  $\bar{b} = \bar{b}_1 = k^{-1}\bar{b}_2$ . Thus,  $b \in [\underline{b}, \bar{b}]$ . The equilibrium  $F_i(b)$  will be characterized by equation (7) in the proof of Claim 2.

*Claim 2.* Suppose that firm  $i$ 's cost observation is either zero with positive probability  $1 - p$  or  $\bar{c}$  with positive probability  $p$  and that the discriminatory rule  $k$  is imposed. Then, in equilibrium, the first-price and second-price sealed-bid auctions yield the same expected procurement cost.

*Proof.* We will first show that, in equilibrium, the high-cost domestic firm will not bid lower

than  $k^{-1}\bar{c}$ .<sup>13</sup> Suppose to the contrary that it bids less than that, and thus there is overlap with the support of  $F_i(b)$ . Choose any two distinct bids,  $b^0$  and  $b^{00}$  from the overlapped support. Then it must be true that (by the definition of a mixed-strategy equilibrium):

$$(b^0 - 0)\Pr(\text{firm 1 wins} \mid b^0) = (b^{00} - 0)\Pr(\text{firm 1 wins} \mid b^{00}),$$

(6)

$$(b^0 - \bar{c})\Pr(\text{firm 1 wins} \mid b^0) = (b^{00} - \bar{c})\Pr(\text{firm 1 wins} \mid b^{00}).$$

However, the two equations in (6) cannot be compatible with each other, since  $\Pr(\text{firm 1 wins} \mid b^0)$  of the first equation cannot be equal to that of the second equation, contradicting the assertion of overlap.

From the above proof, it is obvious that  $F_i(b)$  has a common supremum  $\bar{b} = k^{-1}\bar{c}$ . As Lemma 2 in Section 4 will show, it is quite easy to prove that  $F_i(b)$  has a common infimum  $\underline{b}$ . When both firms observe cost of zero, any bid made as part of a mixed strategy must generate the same expected payoff; thus, the following equations must be true:

$$(b - 0)[p + (1 - p)(1 - F_2(b))] = (\bar{b} - 0)[p + (1 - p)(1 - F_2(\bar{b}))],$$

(7)

$$(b - 0)[p + (1 - p)(1 - F_1(b))] = (\bar{b} - 0)[p + (1 - p)(1 - F_1(\bar{b}))].$$

By definition,  $F_i(\underline{b}) = 0$  and  $F_i(\bar{b}) = 1$ . Thus, from (7),  $\underline{b} = p\bar{b}$ .

When the domestic firm observes high cost, its expected receipt from the government is  $k^{-1}\bar{c}$ . Since the probability that both firms observe  $\bar{c}$  is  $p^2$ , the government's expected payment to the high-cost foreign firm is  $p^2 k^{-1}\bar{c}$  when both firms observe high cost. Since the right-hand

side of (7) is independent of  $b$ , each firm's expected profit (which is the integration of the left-hand side of (7) over the equilibrium support) can be easily calculated. Since the probability that each firm observes low cost is  $(1 - p)$ , from (7) the government's expected payment to the low-cost domestic firm is  $p(1 - p)k^{-1}\bar{c}$ , while that to the low-cost foreign firm is  $p(1 - p)\bar{c}$ . Thus, the government's expected payment in the first-price sealed-bid auction is the same as in the second-price sealed-bid auction, (i.e.,  $p^2k^{-1}\bar{c} + p(1 - p)(k^{-1} + 1)\bar{c} = p\bar{c}(1 - p + k^{-1})$ ).  $\square$

As in the second-price sealed-bid auction, with the discriminatory policy  $k$ , the domestic firm's probability of winning increases from  $1/2$  to  $(1 + p^2)/2$ , while that of foreign firm decreases from  $1/2$  to  $(1 - p^2)/2$ . Consequently, as seen in the second-price sealed-bid auction, the government must be prepared to pay more expected procurement cost in order to transfer its expected payment from the foreign firm to the domestic firm through  $k$ .

The results in Claims 2 and 1 will be reaffirmed in the case of continuous distribution in the next section and in Section 5, respectively.

#### 4. REBATE AUCTIONS

As mentioned at the end of Section 2, the first-order condition of (3) with respect to  $b_i$ , (which determines a Nash equilibrium in an asymmetric first-price sealed-bid auction) does not have a closed form solution in general. In order to understand the effects of a discriminatory price-preference policy, therefore, we may have to depend on a numerical method. However, usually a numerical method does not give us a more general understanding of how the price-preference policy affects the Nash equilibrium (which is necessary to calculate a possible minimum expected procurement cost to the government). Thus, we derive an analytical approximation for the closed form solution in an asymmetric, first-price sealed-bid auction. To do this, we make a

number of specific rules and assumptions.

Let us revert to the continuous distributions in Section 2. To study the equilibrium bidding strategy in asymmetric auctions, however, it will help to suppose that the government announces the following auction rules: (i) before both firms bid at a first-price sealed-bid auction, they are promised to receive a fixed amount  $z$  upon the event of winning and; (ii) the firm which submits the lower bid wins and receives  $z$ . The winning firm must provide the object demanded at its bid-price.

If a submitted bid price is negative, the auction is called a “rebate auction.” Usually procurement bids include not only bid-prices but also several non-price concessions such as a description of after-sales service and a performance schedule (including delivery date). We may think of a procurement auction where the government announces the fixed-price of an auctioned project in advance and seeks the most favorable non-price concessions at the auction. Assuming that non-price concessions can be converted into a price-equivalent, we may interpret  $z$  as the pre-fixed contract price for the project and the rebate as the price-equivalent of the non-price concessions. While the description seems to be restrictive, it is often a good approximation for the real world. In 1990, for example, South Korea chose the General Dynamics F-16 fighter over the McDonnell Douglas F/A-18 for the next-generation support fighter. Since price-tags of both fighters were quite well-known and South Korea had considered a \$3 billion contract, the two firms competed for the contract mainly with the means of two non-price concessions: degrees of technology-transfer and “offset.”<sup>14</sup> In this example, \$3 billion may correspond to the contract price  $z$ , and the two non-price concessions to the rebate. Bribes associated with international procurement contracts will be another example for rebate auctions. Usually corrupt procurement officials (implicitly) promise a fixed price  $z$  for a contract to potential suppliers. Then a supplier who offers the highest bribe (kickback or illegal rebate) wins.

Since the firm which bids the highest rebate (or the lowest negative bid-price) wins, a rebate auction can be interpreted as an isomorphic transformation of a high-price sealed-bid auction. To maintain the logical consistency of low-price auctions in this paper, however, we use the argument of the lowest negative bid-price instead of the highest rebate. To provide a closed form characterization of the equilibrium, we modify Assumption 1 so that both firms have a common upper bound  $\bar{c}$  for their cost distributions. This may be justified by the fact that each firm has made different technological progress, which is characterized by  $G_i(\cdot)$ , with a common old technology  $\bar{c}$ . Furthermore, we assume that  $G_i(\cdot)$  is uniform on  $[\underline{c}_i, \bar{c}]$ . Then, Lemma 1 shows that the equilibrium strategy of each firm at this rebate auction is the same as the one at the first-price sealed-bid auction without the pre-fixed contract price  $z$ , and thus there is no change in the government's expected payment.

*Lemma 1. With the above auction rule, a pre-announced, fixed amount  $z$  does not change each firm's winning probability and expected profit.*

*Proof.* If firm  $i$  observes its cost  $c_i$  and its opponent follows the equilibrium bidding strategy  $B_j(\cdot)$ , then its choice of a bid  $b_i$  nets an expected profit of

$$(8) \quad \pi_i = [1 - G_j(B_j^{-1}(b_i))][b_i - c_i], \quad i = 1, 2, j \neq i.$$

Let  $b_i^z$  and  $B_j^z$  denote firm  $i$ 's bid and its opponent's equilibrium bidding strategy, respectively, with the pre-fixed contract price  $z$ . Then, firm  $i$  determines  $b_i^z$  by maximizing its expected profit:

$$(9) \quad \pi_i(b_i^z; c_i, z) = [1 - G_j(B_j^{z^{-1}}(b_i^z))][b_i^z - c_i + z].$$

Consider first a symmetric case. From the first-order condition of (8) and of (9), we have

$$(10) \quad \begin{aligned} B(c) &= c + \frac{\int_c^{\bar{c}} (1 - G(\tilde{c})) d\tilde{c}}{1 - G(c)} \quad \text{for } c \in [\underline{c}, \bar{c}], \\ B^z(c - z) &= c + \frac{\int_{c-z}^{\bar{c}-z} (1 - G(\tilde{c} - z)) d\tilde{c}}{1 - G(c - z)} - z \quad \text{for } c \in [\underline{c} - z, \bar{c} - z]. \end{aligned}$$

As we have assumed a uniform distribution,  $G(c - z) = (c - \underline{c})/(\bar{c} - \underline{c}) = G(c)$  and  $\int_c^{\bar{c}} (1 - G(\tilde{c})) dc = \int_{c-z}^{\bar{c}-z} (1 - G(\tilde{c} - z)) dc$ . Thus,  $B^z(c - z) = B(c) - z$ .  $\square$

As the proof of Lemma 1 shows that the pre-fixed contract price  $z$  shifts only the support of  $G_i$  to the left by the scale of  $z$ , the introduction of  $z$  does not change the basic game structure of the first-price sealed-bid auction.

We now assume that the government promises the contract price  $z$  equal to  $\bar{c}$ ; thus, the cost interval can be regarded as shifted to  $[\underline{c}_i - \bar{c}, 0]$ . Then we shall derive the solution for the equilibrium along the lines of Griesmer et. al. Based on the drawn cost  $c_i \in [\underline{c}_i - \bar{c}, 0]$ , a bid of  $b_i$  (or  $i$ 's rebate) is made by each firm:

$$(11) \quad b_i = B_i(c_i) \leq 0, \quad B_i'(c_i) > 0, \quad i = 1, 2.$$

Now suppose that the government announces the discrimination rule  $k$ ; thus, firm  $i$ 's probability of winning with  $b_i$ ,  $i = 1, 2$ , is based on the comparison schedule  $(b_1, kb_2)$ . Note that the method of applying  $k$  here looks different from the one at (2), where the comparison schedule is  $(kb_1, b_2)$ . The reasoning is as follows. If firm 1 and firm 2 submit sealed bids,  $b_1$  and  $b_2$ ,

given the announcement of  $k$ , then the government chooses the lower bid between  $b_1$  and  $kb_2$ . Since  $b_i$  is negative (or since it is a concession), multiplying  $b_2$  by  $k$  means that firm 2 is nonfavored. Thus, the winning probability of firm 1 is

$$\begin{aligned}
 (12) \quad \Pr(\text{firm 1 wins} \mid b_1) &= \Pr(k|b_2| < |b_1|) \\
 &= \Pr(|c_2| < |B_2^{-1}(k^{-1}b_1)|) \\
 &= \Pr[c_2 > B_2^{-1}(k^{-1}b_1)] \\
 &= 1 - \Pr[c_2 < B_2^{-1}(k^{-1}b_1)] \\
 &= 1 - G_2(B_2^{-1}(k^{-1}b_1)).
 \end{aligned}$$

Similarly, firm 2's probability of winning is

$$(13) \quad \Pr(\text{firm 2 wins} \mid b_2) = 1 - G_1(B_1^{-1}(kb_2)).$$

Rather than working directly with the bidding functions, it proves more convenient to define the inverse functions:

$$\begin{aligned}
 \psi_1(b) &= B_1^{-1}(kb), \\
 (14) \quad \psi_2(b) &= B_2^{-1}(b),
 \end{aligned}$$

where  $\psi_i(b)$  is the cost of firm  $i$ ,  $b \in [\underline{b}, \bar{b}]$ . Lemma 2 shows that  $k^{-1}b_1$  and  $b_2$  must have the common lowest bid.

*Lemma 2.*  $k^{-1}b_1 = b_2 = \underline{b}$ .

*Proof.* Suppose  $k^{-1}\underline{b}_1 < \underline{b}_2$ . Then firm 1 always wins with a bid  $k^{-1}b_1 \in [k^{-1}\underline{b}_1, \underline{b}_2)$ . But such  $k^{-1}b_1$  is strictly dominated by a bid  $(k^{-1}\underline{b}_1 + \underline{b}_2)/2$ . Of course, a symmetric argument applies for firm 2 so that  $\underline{b}_2 < k^{-1}\underline{b}_1$  cannot be true.  $\square$

Lemma 3 shows that  $k^{-1}b_1$  and  $b_2$  must have the common highest bid of zero.

*Lemma 3.*  $k^{-1}\bar{b}_1 = \bar{b}_2 = \bar{b} = 0$ .

*Proof.* Note that the upper bound of the cost (with the introduction of the contract price  $z$ ) is 0 regardless of  $k$ . If both  $k^{-1}\bar{b}_1$  and  $\bar{b}_2$  are equal to or greater than 0, then  $k^{-1}\bar{b}_1 = \bar{b}_2 = 0$ , since, otherwise, either firm could improve its expected profit at essentially zero cost by lowering its bid slightly. If both  $k^{-1}\bar{b}_1$  and  $\bar{b}_2$  are less than 0, then they result in  $k^{-1}\bar{b}_1 = \bar{b}_2 = 0$ , since, otherwise, they receive negative profit.  $\square$

From Lemmas 2 and 3, the equilibrium inverse bid functions must satisfy  $\psi_i(0) = 0$ ,  $\psi_i(\underline{b}) = \underline{c}_i - \bar{c}$ ,  $G_i(\psi_i(0)) = 1$  and  $G_i(\psi_i(\underline{b})) = 0$ . Each firm's expected profit maximization with respect to  $b_i$  yields the following pair of differential equations.

$$(15) \quad \frac{1 - G_2(\psi_2(b))}{(b - k^{-1}\psi_1(b))g_2(\psi_2(b))} = \psi_2'(b),$$

$$\frac{1 - G_1(\psi_1(b))}{(b - \psi_2(b))g_1(\psi_1(b))} = \psi_1'(b).$$

It is convenient to redefine the cost interval  $[\underline{c}_i - \bar{c}, 0]$  as  $[-1/a_i, 0]$ ,  $a_i > 0$ ,  $i = 1, 2$ .

Thus, firm  $i$ 's cost  $c_i$  is regarded as a random draw from a uniform distribution over support  $[1/a_i, 0]$ . Assume that  $a_1 > a_2$ , i.e., firm 2 has a more favorable cost distribution. Then,

$$(16) \quad 1 - G_i(c) = -a_i c \quad \text{and} \quad G_i'(c) = g_i(c) = a_i.$$

Making the substitution of (16) into (15) yields

$$(17) \quad \begin{aligned} \psi_2 &= \psi_2' \cdot (k^{-1} \psi_1 - b), \\ \psi_1 &= \psi_1' \cdot (\psi_2 - b). \end{aligned}$$

Multiplying both sides of the second equation in (17) by  $k^{-1}$  and defining  $\phi_1 = k^{-1} \psi_1$ , we have

$$(18) \quad \begin{aligned} \phi_1 \psi_2' - (b \psi_2)' &= 0, \\ \phi_1' \psi_2 - (b \phi_1)' &= 0. \end{aligned}$$

Adding the two equations in (18) and integrating, we obtain

$$(19) \quad \phi_1 \psi_2 = b(\phi_1 + \psi_2) + L,$$

where  $L$  is an arbitrary constant of integration. But,  $L = 0$ , since  $\psi_i(0) = 0$ , and thus  $\phi_1(0) = 0$ .

Since  $G_i(\psi_i(b)) = 0$ ,

$$\phi_1(b) = k^{-1}\psi_1(b) = -\frac{1}{a_1 k},$$

(20)

$$\psi_2(b) = -\frac{1}{a_2}.$$

Substituting (20) into (19), we obtain

$$(21) \quad \underline{b} = -\frac{1}{a_1 k + a_2}.$$

It is easily checked that  $\partial \underline{b} / \partial k > 0$ . That is, the less the domestic firm is favored, the less the lower bound for the equilibrium bid is. Since firm 1's rebate schedule is  $kb$ , we have to consider both  $\partial \underline{b} / \partial k > 0$  and  $\partial k\underline{b} / \partial k < 0$  together in order to determine the effect of  $k$  on the government's expected payment. There is a trade-off. Favoring firm 1 through imposing  $k$  makes firm 2 bid more aggressively, and thus causes  $\underline{b}$  to go down. But it also allows firm 1 to enjoy favored status, and thus causes  $k\underline{b}$  to go up. In order to determine the effect of  $k$  on the government's expected payment, we also have to consider the effect of  $k$  both on each firm's winning probability and on the shape of equilibrium bid functions. Proposition 2 shows us the effect of  $k$  on a pair of asymmetric equilibrium bid functions.

*Proposition 2.* *If firm  $i$ 's cost  $c_i$  is uniformly distributed over the interval  $[-1/a_i, 0]$ ,  $a_i > 0$ ,  $i = 1, 2$ , then a pair of equilibrium inverse bid functions are*

$$\psi_1(b) = \frac{2k\underline{b}^2 b}{\underline{b}^2 - b^2(1 + 2a_1 k\underline{b})} \quad \text{and} \quad \psi_2(b) = \frac{2\underline{b}^2 b}{\underline{b}^2 - b^2(1 + 2a_2 \underline{b})},$$

where  $\underline{b} = -\frac{1}{a_1 k + a_2}$ .

*Proof.* From (18),  $\psi_2 = \psi_2' \cdot (\phi_1 - b)$ . But  $\phi_1 = b\psi_2/(\psi_2 - b)$  by (18). Thus,  $d\psi_2/db = \psi_2' \cdot (\psi_2 - b)/b^2 = (\psi_2'/b)[(\psi_2/b) - (\psi_2'/b)/(\psi_2/b)]$ . Since this is a homogeneous equation, we can use the following technique.

$$(22) \quad \psi_2(b) = b\mu(b), \quad \psi_2' = \mu + b\mu'.$$

Substituting (22) into (18), we have  $b\mu = (\mu + b\mu')[b^2\mu/(b\mu - b) - b]$  or  $2b\mu = [(b\mu)^2 + b^3\mu']/(b\mu - b)$  or  $\mu^2 - 2\mu = b\mu'$  or  $d\mu/(\mu - 2) - d\mu/\mu - 2db/b = 0$ . Integrating, we obtain  $\ln(\mu - 2) - \ln\mu - 2\ln b = \ln t_2$ , where  $t_2$  is an arbitrary constant of integration, or  $\mu = 2/(1 - t_2 b^2)$ . Thus,  $\psi_2 = b\mu = 2b/(1 - t_2 b^2)$ . But  $\psi_2(b) = 2\bar{b}/(1 - t_2 \bar{b}^2) = -1/a_2$ . Thus  $t_2 = (2a_2 \bar{b} + 1)/\bar{b}^2$  and  $\psi_2(b) = 2\bar{b}^2 b / [\bar{b}^2 - b^2(1 + 2a_2 \bar{b})]$ . Then substituting this  $\psi_2(b)$  into (18), we obtain  $\phi_1(b) = 2b/(1 + t_1 b^2)$ , where  $t_1$  is an arbitrary constant of integration. But  $\phi_1(b) = 2\bar{b}/(1 + t_1 \bar{b}^2) = -1/a_1 k$ . Thus,  $t_1 = -(1 + 2a_1 \bar{b} k)/\bar{b}^2$  and  $\phi_1(b) = -2\bar{b}^2 b / (b^2 + 2a_1 k \bar{b} b^2 - \bar{b}^2)$ . It is easily verified that  $t_1 = t_2$ .  $\square$

Figures 2.2 and 2.3 depict the equilibrium inverse bid functions,  $\psi_i(b)$ . Support of each firm's bid-price is indexed along the horizontal axis, and the support of the cost distribution along the vertical axis. Note that  $\psi_1(b) > \psi_2(b)$  for all  $b \in [\bar{b}, 0)$ , and thus  $B_2(c_2) > k^{-1}B_1(c_1)$  or  $kB_2(c_2) > B_1(c_1)$ . Therefore, firm 2, whose cost observation is perceived by firm 1 to be drawn from a more favorable distribution, bids more everywhere except the origin.

Using the characterization of equilibrium bid functions for two asymmetric bidders in Proposition 2, we can calculate the government's expected procurement cost from the rebate auction:

$$\begin{aligned}
(23) \quad H(k; a_1, a_2) &= m + \int_{\underline{b}}^0 b \, dF(b; k, a_1, a_2) \\
&\quad - \left[ \int_{\underline{b}}^0 b \{1 - G_2(\psi_2(b))\} \, dG_1(\psi_1(b)) - k \int_{k\underline{b}}^0 b \{1 - G_2(\psi_2(b))\} \, dG_1(\psi_1(b)) \right] \\
&= m + P_1 - P_2,
\end{aligned}$$

where  $F$  is the distribution function for the winning bid and thus  $P_1$  is the expected rebate from the winner. The last term  $P_2$  indicates that the expected rebate from firm 1 should be adjusted when it is the winner. This is because the comparison schedule,  $(b_1, kb_2)$ , is different from the rebate schedule,  $(b_1, b_2)$ .

For expositional simplicity, we set  $a_1 = 1$  and  $0 < a_2 \leq 1$ . Since we are dealing with the low bid auction, the distribution function for the winning bid is

$$\begin{aligned}
(24) \quad F(b; k, a_2) &= \Pr[\text{the low bid between } k^{-1}b_1 \text{ and } b_2 \text{ is less than } b] \\
&= 1 - \Pr[\text{both bids exceed } b] \\
&= 1 - \Pr[k^{-1}b_1 > b \text{ and } b_2 > b] \\
&= 1 - \Pr[c_1 > \psi_1(b)] \cdot \Pr[c_2 > \psi_2(b)] \\
&= 1 - [1 - G_1(\psi_1(b))] [1 - G_2(\psi_2(b))].
\end{aligned}$$

Applying integration by parts, and using (16) and (24), we can rewrite  $P_1$  as

$$(25) \quad P_1 = bF|_{\underline{b}}^0 - \int_{\underline{b}}^0 F \, db$$

$$\begin{aligned}
&= - \int_{\underline{b}}^0 F \, db \\
&= - \int_{\underline{b}}^0 \left(1 - \frac{l_2}{1 - l_1 b^2} + \frac{l_2}{1 + l_1 b^2}\right) db \\
&= \underline{b} - \frac{a_2 k}{\sqrt{l_1}^3} \left[ \ln \left| \frac{1 + \sqrt{l_1} \underline{b}}{1 - \sqrt{l_1} \underline{b}} \right| - 2 \arctan(\sqrt{l_1} \underline{b}) \right],
\end{aligned}$$

where  $l_1 = |a_2^2 - k^2|$  and  $l_2 = 2a_2 k / l_1$ . Substituting from (16), we can rewrite the first integral in the adjustment  $P_2$  as

$$\begin{aligned}
(26) \quad &-4a_2 k \int_{\underline{b}}^0 \frac{b^2}{(1 - l_1 b^2)^2} db \\
&= \begin{cases} \frac{2a_2 k}{l_1} \left[ \frac{\underline{b}}{1 - l_1 \underline{b}^2} - \frac{1}{2\sqrt{l_1}} \ln \left( \frac{\sqrt{l_1}^{-1} + \underline{b}}{\sqrt{l_1}^{-1} - \underline{b}} \right) \right], & \text{if } a_2 > k. \\ \frac{2a_2 k}{\sqrt{l_1}^3} \left[ \arctan(\sqrt{l_1} \underline{b}) - \frac{\underline{b}}{\sqrt{l_1}(b^2 + l_1^{-1})} \right], & \text{if } a_2 < k. \end{cases}
\end{aligned}$$

The second integral in the adjustment  $P_2$  is basically the same as the first except for the lower bound.

Although the government's expected payment  $H$  can be expressed as a function of  $k$ ,  $\partial H / \partial k$  is possible only numerically because of functional complexity. Table 2.1 reports the results of the expected procurement cost from the first-price sealed-bid auction with  $a_1 = 1$  and  $a_2 = 0.5$ . The effects of price-preference policies can be easily checked, as zero price-preference (or  $k = 1$ ) provides the minimum expected cost to the government. Claim 2 in Section 3 has already shown the same result. An interesting fact from Table 2.1 is, however, that there is a local minimum around  $k = 0.5$ , where the price-preference  $k$  counterbalances the distributional

asymmetry of costs.

The relevance of Lemma 1 is confirmed when  $a_1 = a_2 = 1$  and  $k = 1$ , as the first-price sealed-bid and the rebate auctions result in the same expected procurement cost of 0.667 to the government. (But the government's expected payment  $H$  at  $k = a_2$  are approximate values as  $l_1$  cannot be zero at (25) and (26).)

The reason we have analyzed rebate auctions is to introduce zero as the common upper bound for the two firms' cost distributions. (The basic idea of using zero is due to Maskin and Riley (1986)). Although it can explain expected cost differential between countries, zero as the common upperbound could be a strong assumption. Another weak point is that the analysis is mostly constrained to the case of uniform distribution. Unlike a first-price sealed-bid auction, a second-price sealed-bid auction always provides a dominant strategy for each firm, regardless of the assumption of symmetry. Thus, equilibrium analysis in a more general setting, such as with more than two firms and without any more restrictions on Assumption 1, would be possible.

## 5. SECOND-PRICE SEALED-BID AUCTIONS WITH $n \geq 2$ BIDDERS

It has been assumed up to now that there are two bidders, a domestic and a foreign firm. The model can now be generalized by allowing more than two bidders. There are  $n$  potential suppliers,  $n^d \geq 1$  of which are domestic firms and  $n^f \geq 1$  foreign,  $n = n^d + n^f \geq 2$ . The basic assumptions made in Section 2 are also applied here, including continuous cost distributions.

Consider the situation in which the government must procure the object from one of  $n$  potential suppliers by the use of a second-price sealed-bid auction. The government announces the discrimination rule  $k$  prior to bidding competition. Then the rules of the auction with  $k$  are as follows. Each firm submits its sealed-bid both simultaneously and non-cooperatively. Let  $b_{1i}$

and  $b_{2j}$  denote domestic firm  $i$ 's and foreign firm  $j$ 's sealed-bids, respectively,  $i = 1, 2, \dots, n^d$ ,  $j = 1, 2, \dots, n^f$ . According to the discrimination rule  $k$ , the comparison schedule to determine the winner is  $(b_{1i}, k^{-1}b_{2j})$ . Suppose that domestic firm 1's bid  $b_{11}$  and domestic firm 2's bid  $b_{12}$  are the lowest and the second lowest bids, respectively, among domestic firms' bids. Also suppose that foreign firm 1's bid  $b_{21}$  and foreign firm 2's bid  $b_{22}$  are the lowest and the second lowest bids, respectively, among foreign firms' bids. Then, if  $b_{11} < k^{-1}b_{21}$ , domestic firm 1 as the winner receives the second lowest bid-price,  $\min \{b_{12}, k^{-1}b_{21}\}$ . But, if  $b_{11} > k^{-1}b_{21}$ , foreign firm 1 as the winner receives the second lowest bid-price,  $\min \{b_{11}, b_{22}\}$ .

With the above description of the model, one can easily recognize that each firm's equilibrium (and dominant) strategy is to bid its cost observation, as shown in Footnote 10. To calculate the government's expected procurement cost, it would be convenient to arrange domestic firms' cost observations, and foreign firms cost observations multiplied by  $k^{-1}$ , in increasing rank order. Let  $x_1$  be the smallest of the cost observations,  $x_2$  the second smallest and thus,  $x_1 < x_2 < \dots < x_n$ , in the absence of ties. Let  $G_i(\cdot)$  be the distribution function associated with  $x_i$ . By the rules of the second-price sealed-bid auction, the firm which bids  $x_1$  as the winner will receive the bid price  $x_2$  and must provide the object demanded. Thus, the expected procurement cost for the government will be the expected value of  $x_2$ ,  $E_n(x_2)$ , where subscript  $n$  indicates that the expected value depends on the number of bidding firms.<sup>15</sup>

If the random draws are independent but not necessarily identically distributed (i.n.n.i.d.) as we have assumed here, the probability of an event concerning the corresponding order statistic  $x_i$  is very complicated. Due to Guilbaud's (1982) interesting theorem, however, we can make a connection between the distribution of order statistics of i.n.n.i.d. random variables and of independent and identically distributed (i.i.d.) random variables [see the Appendix for details].

Using Guilbaud's theorem, Proposition 3 shows that non-discrimination (or  $k = 1$ ) is the

best policy for the government, which tries to minimize its procurement costs by the use of a second-price sealed-bid auction.

*Proposition 3.* Suppose that  $n^d \geq 1$  and  $n^f \geq 1$ , and that  $G_i(\cdot)$  satisfies Assumption 1,  $i = 1, 2, \dots, n$ . Then  $E_n(x_2; k') \geq E_n(x_2; k)$  for every  $k, k' \in (0, 1)$ , if  $k > k'$ .

*Proof.* From (A1) in the Appendix,

$$(27) \quad E_n(x_2; k') - E_n(x_2; k) = \int x_2 dF(x_2; k') - \int x_2 dF(x_2; k).$$

Note that the right-hand side of (27) is directly related to the notion of stochastic dominance. Thus, (27) is greater than zero for all  $k \in (0, 1)$ , if and only if  $F(x_2; k) \geq F(x_2; k')$ . As Example A in the Appendix shows, the distribution function for the second order statistic is  $F(x_2; k) = \sum_{m=2}^n (-1)^{2n-m} (m-1) \sum_{|s|=m} \prod_{s \in i} G_i(x_2; k)$ ,  $i = 1, \dots, n$ . We prove Proposition 3 for the cases  $n = 2$  and  $n = 3$ . If  $n = 2$  and  $k > k'$ , then  $F(x_2) = G_1(x_2)G_2(x_2)$  and thus  $F(x_2; k) \geq F(x_2; k')$ . If  $n = 3$ ,  $F(x_2)$  is the one in Example A. In this case,  $\partial F / \partial G_1 = -2G_2G_3 + G_2 + G_3 = G_2(1 - G_3) + G_3(1 - G_2) \geq 0$ . Similarly, this is true for other partial derivatives with respect to  $G_2$  and  $G_3$ . Thus,  $F(x_2; k) \geq F(x_2; k')$ , if  $k > k'$ .  $\square$

Unlike in the case of a first-price sealed-bid auction with price discrimination policy  $k$ , the asymmetric second-price sealed-bid auction does not show any trade-off: price discrimination  $k$  against foreign firms raises the probability that domestic firms will win and thus tends to raise the domestic firms' expected procurement costs, but it does not change foreign firms' bidding behavior. Thus, as shown in Proposition 3, the government's expected procurement cost is a

decreasing function of  $k$ . This also means that the transfer of the government's expected payment from the foreign firms to the domestic firms is feasible only when the government is willing to spend more public funds for the auctioned object. Section 3 has already shown this result.

Up to now, we have considered the domestic government's monopoly price-preferences. In the next section, we introduce a foreign government's retaliatory price-preference policy to the analysis.

## 6. BILATERAL PRICE-PREFERENCES AT THE SECOND-PRICE SEALED-BID AUCTION

We now extend the analysis in Section 5 to the case where two governments--domestic and foreign--impose the price discrimination  $k$  against the other country's firms. As mentioned in the Introduction, governments are usually concerned with both cost minimization and domestic firms' profit maximization. That is, an alternative objective function for the government would be to maximize expected domestic welfare, which is defined as the sum of possible procurement cost savings and domestic firms' expected profits from both domestic and foreign governments' procurement contracts. This type of objective function could be more reasonable than that of cost minimization when the welfare effects of trade policies are evaluated.

In Section 3 we have shown that favoring domestic firms through the price discrimination  $k$  could shift some of foreign firms' profits to domestic firms. However, this profit shifting motive of the domestic government may provoke the competing foreign country's retaliation, i.e., the foreign government's price-preference policy  $k_f \in (0, 1)$ . This means that the amount of the revenue transfer shrinks with the retaliative  $k_f$ .

We basically follow the game structure of Section 5. That is, each government procures a single indivisible project by the use of the second-price sealed-bid auction. With the introduction

of the foreign government, however, the game structure becomes a two-stage game in which both governments first arrive at a Nash equilibrium in price-preference policies and then oligopolistic competition among firms takes place.

We now add the following notation in the model.

$W_i(\cdot)$ : country  $i$ 's social welfare,  $i = d, f$ .

$\gamma$  : relative size of foreign procurement market as compared to domestic procurement market.  $0 \leq \gamma < \infty$ .

$q_i^j$  : probability that country  $i$ 's firms win country  $j$ 's procurement auction,  $i, j = d, f$ . Thus,  $q_d^d + q_f^d = 1$  and  $q_d^f + q_f^f = 1$ .

Then a domestic country's social welfare  $W_d$  may be measured as the sum of the expected procurement cost saving and the domestic firms's expected profits. Now we define the domestic welfare:

$$(28) \quad W_d(k_d; k_f, \gamma, n^d, n^f) = \\ [E_n(x_2; k_d = 1) - E_n(x_2; k_d)] + q_d^d \cdot [E_n(x_2; k_d) - E_n(x_1; k_d)] + \gamma q_d^f \cdot [E_n(x_2; k_f) - E_n(x_1; k_f)] \\ = \pi_d + \pi_d^d + \gamma \pi_d^f,$$

where  $x_1$  and  $x_2$  are the smallest and the second smallest cost observations, respectively, given price-preferences. Note that  $E_n(x_2; k_d = 1)$  is used as a reference point to measure expected cost saving. From (28), the domestic country's social welfare  $W_d$  consists of three components; the domestic government's expected cost savings ( $\pi_d$ ), domestic firms' expected profits from domestic procurement contracts ( $\pi_d^d$ ), and domestic firms' expected profits from foreign procurement

contracts ( $\gamma\pi_d^f$ ). Given  $\gamma$ ,  $n^d$ ,  $n^f$ , and that the foreign government engages in seeking the best  $k_f$ , the domestic government tries to find the best  $k_d$  which maximizes  $W_d$ . Since the domestic firms' profits from foreign contracts,  $\gamma\pi_d^f$ , is not affected by domestic price discrimination  $k_d$ , the necessary condition for the best  $k_d$  is

$$(29) \quad -\frac{\partial E_n(x_2; k_d)}{\partial k_d} + \frac{\partial q_d^d(k_d)[E_n(x_2; k_d) - E_n(x_1; k_d)]}{\partial k_d} = 0.$$

Assuming that the distribution function  $G_i(\cdot)$  associated with  $x_i$  is uniform greatly simplifies the following analysis, although it does create some problems for establishing inner solutions, as shown in Example 1.

*Example 1.* Suppose that there are two countries, a domestic  $d$  and a foreign  $f$  country. Assume that there are two firms, a domestic firm (firm 1) and a foreign firm (firm 2). Both firms can serve their domestic and foreign markets. Each firm's cost realization is believed to be drawn independently from  $G$ , which is uniform over  $[1, 2]$ . Assumption 1 is applied here. Then, after a domestic price-preference  $k_d$  is imposed, firm 2's cost realization is regarded as a random draw from a new distribution  $G_2$ , which is uniform over  $[k_d^{-1}, 2k_d^{-1}]$ . The range of  $k_d$  is  $[0.5, 1]$  since firm 2 is totally excluded from the domestic procurement auction if  $k_d \leq 0.5$ . Then  $W_d$  in terms of  $k_d$  is

$$W_d(k_d) = \left[ \frac{5}{3} - \left( -2 + \frac{4k_d}{3} + \frac{5}{2k_d} - \frac{1}{6k_d^2} \right) \right] \\ + \left[ \left( 3 - 2k_d - \frac{1}{2k_d} \right) \left( -5.5 + \frac{8k_d}{3} + \frac{7}{2k_d} - \frac{1}{3k_d^2} \right) \right]$$

$$+ \gamma q_d^f [E_n(x_2; k_f) - E_n(x_1; k_f)].$$

Figure 2.4 reports the graphical result of domestic welfare  $W_d(k_d)$  in this example. Domestic welfare is maximized when both domestic price discrimination  $k_d = 0.5$  (i.e., when the foreign firm is prevented from attending the domestic procurement auction) and  $k_d = 1$  (i.e., when there is no price-preference). Compare these two extreme cases. As the domestic firm's expected profit with domestic price discrimination  $k = 0.5$  is  $\pi_d^d(k = 0.5) = 1.5$ , and that with  $k = 1$  is  $\pi_d^d(k = 1) = 0.167$ , the domestic firm has a strong incentive to seek a monopoly position in the domestic procurement contract. However, in order for the domestic firm to receive  $\pi_d^d = 1.5$ , the domestic government must spend 1.334 of more public funds. This is the way in which public resources are misused with a protective procurement policy. This type of protection makes the foreign government close its procurement market against the domestic firm in retaliation.

Suppose, alternatively, that both governments agree to remove price-preferences and thus  $k_d = k_f = 1$ . Also assume a symmetry of both procurement markets so that  $\gamma = 1$ . Then, each country's domestic welfare will double from 0.167 to 0.333 as  $\gamma \pi_d^f = 0.167$ . World welfare,  $W_d + W_f$ , will also double from 0.333 to 0.667. In addition, production efficiency is obtained as the winner's expected cost  $E_n(x_1)$  is minimized at  $k_d = k_f = 1$ . Thus, from this example, we may draw a normative conclusion: the Agreement should be improved upon in the spirit of free trade (i.e.,  $k_d = k_f = 1$ ) in order to maximize world welfare and to discourage the misuse of public funds.

Table 2.2 summarizes the results from Example 1, using a normal form. The game structure is a simple two-party model. Once country  $i$ 's welfare  $W_i$  is constructed, each country's decision of price-preference policies determines the payoffs according to  $W_i$ . Here we assume that each country is aware of its competitor's choice of price-preference policies. The first component

of each cell in Table 2.2 refers to the domestic country's welfare  $W_d$ , and the second to the foreign country's welfare  $W_f$ . From Example 1,  $W_i(k_d = 0.5, k_f = 0.5) < W_i(k_d = 0.5, k_f = 1) = W_i(k_d = 1, k_f = 1)$ ,  $i = d, f$ . Interestingly enough, if there is no political pressure from interest groups (domestic firms), and thus each government's objective is to maximize its domestic welfare, then zero price-preference ( $k_d = k_f = 1$ ) can be supported as a Nash equilibrium, i.e., as an equilibrium in a non-cooperative game. However, if both countries are afraid of antagonizing domestic firms, total exclusion of foreign firms ( $k_d = k_f = 0.5$ ) will be a Nash equilibrium.

## 7. CONCLUDING COMMENTS

The analysis presented above takes the institutional practice of setting linear price-preferences and of procuring governments' demand via a standard type of procurement auction as given. Its main thrust is to provide an equilibrium analysis of the Agreement. We have provided an explanation of why international coordination on procurement policies such as the Agreement is needed in order to prevent governments from the misuse of public funds. We show that, in empirical studies, the use of a zero price-preference as a bench mark to evaluate welfare loss is appropriate, given the institutional environment. Improvement of the Agreement in the spirit of free trade is desirable, since zero price-preference enhances both domestic and world welfare.

A major drawback of this chapter is its implicit assumption that the government can make complete or state-contingent contracts with the potential supplier. In other words, we have neglected the problems of possible disputes between parties (governments and firms) related to long term contract or to the enforcement of contracts. Thus, the design of government procurement auction is reduced to the problem of purchasing a single indivisible object. However,

this may ignore the important fact that bidding competition for a procurement contract involves both a bid-price and a promise to perform the contract. Because of the transaction costs of making complete contracts, the government procurement auction could be much different from an auction for an artwork.<sup>16</sup> To deal with this contractual problem, therefore, we have to consider not only the bid-price but also the ability of both parties' (i.e., between the government and the designated contractor) commitment to their ongoing relationship. Some of the government's favoritism toward domestic firms might be justified on the basis of this type of commitment between the government and the reputable domestic firms. In that sense, it may be reasonable that the Agreement allows governments to select reputable suppliers *ex ante*, provided the governments adhere to the "transparency."

Figures on the use of first-price sealed-bid auction in government procurement are difficult to find. However, most government procurement contracts are believed to be made by the use of a first-price sealed-bid auction. In Section 4 we could obtain only analytical approximations on asymmetric first-price sealed-bid auctions. General results seem to be unavailable at the moment, as even the existence of asymmetric equilibrium bids in general has not been proven.

## FOOTNOTES

1. Although economic studies of international procurement market have not shown much development, law studies have produced substantial progress for the Agreement [see, for example, 'International symposium on government procurement law' edited by Cibinic, part I&II (1987)].
2. As the Uruguay Round of GATT talks attempted to draw up rules for service contracts, the potential of international procurement markets was expected to be further increased. The Canada-U.S. Defense Production Sharing Agreement (1961) shows the possibility for mutual access and international competition even in defense contracts [see de Mestral (1982)].
3. The Agreement includes two more basic issues: coverage and dispute settlement [see de Mestral (1982) for more details].
4. There are two more commonly observed institutions--the Dutch and English (or open) auctions. The game model of the open auction is identical to that of the second-price sealed-bid auction. For a more detailed description of auctions both institutionally and theoretically, see McAfee and McMillan (1987).
5. We do not consider the interesting issue arising from the interaction between the government and the private sector's consumption. Miyagawa (1991) examines the effects of discriminatory government procurement policy on domestic output when demands originate from both the private and government sectors.
6. Before McAfee and McMillan (1989), researchers tried to measure the potential costs of price-

preference policies, using a zero price-preference as a benchmark to evaluate welfare loss [see, e.g., Joson (1985) among others].

7. There are two polar cases in auction models: independent private and common values. An auction is called an independent private-value model if bidders have different opportunity costs for the procurement project which will not be affected by knowledge of competitors' opportunity costs. The independent private values do not imply, however, that a bidder's knowledge about his opponents' costs does not affect his strategic behavior. A common-value model deals with the case in which a true cost of providing the project is common but unknown.

8. Lebrun (1991) proves the existence of a Nash equilibrium for the first-price sealed-bid auction in the independent private-value model with two asymmetric bidders whose reservation value probability distributions are continuous and have the same support. Kimmel (1989) derives the necessary condition for the existence of a pair of asymmetric equilibrium bidding strategies. But existence in general has not been shown yet. Thus, we confine our analysis to the case that satisfies Kimmel's condition.

9. Using a Cournot model, Brander and Spencer (1984) argue that when a market is imperfectly competitive, the world welfare maximum may involve positive tariffs even if a cooperative equilibrium between countries is considered. With a Bertrand competition model, however, Eaton and Grossman (1986) show that free trade could be optimal policy under oligopoly.

10. To show this, suppose that firm  $i$ 's cost observation is equal to or less than firm  $j$ 's bid-price, i.e.,  $c_i \leq b_j$ . Then firm  $i$ 's submitting  $b_i > b_j$  (rather than  $c_i$ ) reduces its profit by  $b_j - c_i$ , while its submitting  $b_i \leq b_j$  (including  $b_i = c_i$ ) does not change its profit. Similarly, if  $c_i > b_j$ , firm

$i$ 's submitting  $b_i < b_j$  reduces its profit by  $c_i - b_j$ , while its submitting  $b_i > b_j$  (including at  $b_i = c_i$ ) does not change its profit. Thus, firm  $i$ 's (weakly) dominant strategy is  $B_i(c_i) = c_i$ . Note that the rewarded price to the winner is independent of his bid-price.

11. Let  $\hat{b}$  denote the highest feasible price of the high-cost domestic firm. Suppose that  $\hat{b} < k^{-1}\bar{c}$ . Then any  $b$  between  $\hat{b}$  and  $k^{-1}\bar{c}$  has the same chance of winning as  $\hat{b}$ , and so is preferable. Thus,  $\hat{b}$  must be the closest number to  $k^{-1}\bar{c}$ . Because of the continuum property of the real numbers, it is troublesome to fix the closest number to  $\hat{b}$ . To avoid this problem, it will be convenient to suppose that a low-cost bidder wins with  $\hat{b}$  if both a low-cost and a high-cost bidders submit  $\hat{b}$ .

12. The existence of a symmetric mixed-strategy equilibrium in general is proved by Theorem 6 in Dasgupta and Maskin (1986). The reasoning of no atom over the interval is roughly as follows. Suppose to the contrary that a low-cost-bidder  $i$  submits  $b_i \in [\underline{b}_i, \bar{b}_i]$  with positive probability. Then there must be some interval  $[b_i, b_i + \epsilon]$  over which his low-cost opponent  $j$  will not bid since the probability of  $j$ 's winning increases discontinuously if a bid in this interval is replaced with one that is infinitesimally smaller than  $b_i$ . Then bidder  $i$  can increase  $b_i$  to  $b_i + \epsilon$  without reducing his probability of winning, which contradicts the assertion that the original  $b_i$  with positive probability was an equilibrium.

13. The discussion which follows owes much to comments by Bernard Lebrun.

14. Under a system known as "offset," the U.S. has to buy back a certain portion of the contract's total value in the good of the country to which the U.S. has sold arms [Wall Street Journal (May 10, 1990)].

15. If any two foreign firms' inflated bid-prices, say  $k^{-1}b_{21}$  and  $k^{-1}b_{22}$ , are the lowest and the second lowest, respectively, then foreign firm 1 as the winner receives the price of  $b_{22}$  instead of  $k^{-1}b_{22}$ . However, we can ignore this type of detail, since this situation is the same as a situation without  $k$ .

16. Spulber (1990) analyzes how the effectiveness of the auction mechanism hinges on *ex post* incentives for performance, without proper legal enforcement. Chapter 3 argues that *ex post* incentives for performance limit the auctioneer's ability of to introduce competition in procurement auctions.

## APPENDIX

*Guilbaud Theorem.* Let  $X_1 \leq \dots \leq X_n$  be the order statistics of independent but not necessarily identically distributed (i.n.n.i.d.) random variables  $C_1, \dots, C_n$ . Let  $G_i$  be the continuous distribution function for  $C_i$ . Then, for every Borel set  $\mathfrak{B}$ ,

$$\Pr[(X_1, \dots, X_n) \in \mathfrak{B}] = \sum_{m=1}^n (-1)^{n-m} \frac{m^n}{n!} \sum_{|s|=m} \Pr[(X_1^s, \dots, X_n^s) \in \mathfrak{B}],$$

where the summation runs over all subsets  $s$  of  $\{1, \dots, n\}$  with  $m$  elements, and where  $X_1^s, \dots, X_n^s$  are the order statistics of  $n$  of independent and identically distributed (i.i.d.) random variables with common distribution function  $F_s = \sum_{i \in s} G_i/|s|$ .

*Proof.* See Guilbaud (1982).

Guilbaud's theorem enables us to express the probability of any event in the i.n.n.i.d. case in terms of the simpler corresponding probability in the i.i.d. case. The following example shows the way the i.n.n.i.d. case corresponds to the i.i.d. case.

*Example A.* Suppose that there are three firms. Let  $c_i$  be firm  $i$ 's cost observation from the distribution function  $G_i$ ,  $i = 1, 2, 3$ . Let  $x_i$  be the corresponding order statistics of  $c_i$ . Then the distribution of the second order statistic is

$$\begin{aligned} F(x_2) &= \Pr [\text{at least two of } c_i \text{ are less than or equal to } x_2] \\ &= G_1 G_2 [1 - G_3] + G_1 G_3 [1 - G_2] + G_2 G_3 [1 - G_1] + G_1 G_2 G_3 \end{aligned}$$

$$= -2G_1G_2G_3 + G_1G_2 + G_2G_3 + G_1G_3.$$

To ease the notational burden, we have dropped  $x_2$  from  $G_i(x_2)$ . Using the above expression for  $F(x_2)$ , we can confirm the relevance of the Guilbaud Theorem:

$$\begin{aligned} F(x_2) &= \frac{1}{6}[-2(G_1^3 + G_2^3 + G_3^3) + 3(G_1^2 + G_2^2 + G_3^2)] \\ &\quad - \frac{8}{6}[-2\{(\frac{G_1 + G_2}{2})^3 + (\frac{G_2 + G_3}{2})^3 + (\frac{G_1 + G_3}{2})^3\} \\ &\quad + 3\{(\frac{G_1 + G_2}{2})^2 + (\frac{G_2 + G_3}{2})^2 + (\frac{G_1 + G_3}{2})^2\}] \\ &\quad + \frac{27}{6}[-2(\frac{G_1 + G_2 + G_3}{3})^3 + 3(\frac{G_1 + G_2 + G_3}{3})^2] \\ &= -2G_1G_2G_3 + G_1G_2 + G_2G_3 + G_1G_3. \end{aligned}$$

Applying the Guilbaud Theorem to order statistics, we derive a density function for the  $r^{\text{th}}$  order statistic.

*Corollary 1.* Let the probability density function (p.d.f.) for  $F_s$  be  $f_s (= \sum_{s \in i} g_i/|s|)$ . Then, the p.d.f. for an order statistic  $r$  is

$$f(x_r) = \sum_{m=1}^n (-1)^{n-m} \frac{m^n}{n!} \sum_{|s|=m} \frac{n!}{(r-1)!(n-r)!} f_s(x_r) F_s(x_r)^{r-1} [1 - F_s(x_r)]^{n-r}.$$

Applying Corollary 1 to our model, we can express the expected procurement cost for the government as

$$(A1) \quad E_n(x_2; k) = \int x_2 dF(x_2; k),$$

where  $dF(x_2; k) = n(n-1) \sum_{m=1}^n (-1)^{n-m} \frac{m^n}{n!} \sum_{|s|=m} F_s(x_2; k) [1 - F_s(x_2; k)]^{n-2} f_s(x_2; k) dx_2$ . To illustrate the way in which Corollary 1 is applied, consider the following example.

*Example B.* Suppose that there are just two firms, firm 1 and firm 2, as we have assumed previously. Assume that  $G_1(c_1)$  is uniform over  $[0, 2]$  and that  $G_2(c_2)$  is uniform over  $[1, 2]$  after a discrimination rule  $k$  was imposed. Since the distribution function for the second order statistic is  $F(x_2) = G_1(x_2)G_2(x_2)$ , the expected procurement cost for the government from the second-price sealed-bid auction is

$$\begin{aligned} E_2(x_2) = & -\frac{1}{2} \left[ \int_0^2 x_2 \{G_1(x_2)g_1(x_2) + G_1(x_2)g_1(x_2)\} dx_2 \right] \\ & + \int_1^2 x_2 \{G_2(x_2)g_2(x_2) + G_2(x_2)g_2(x_2)\} dx_2 \\ & + 2 \left[ \frac{1}{2} \left\{ \int_0^2 x_2 \{G_1(x_2)g_1(x_2) + G_2(x_2)g_2(x_2)\} dx_2 \right\} \right]. \end{aligned}$$

This is because  $F_s$  are

$$F_{\{1\}} = G_1, \quad F_{\{2\}} = G_2 \text{ and}$$

$$F_{\{1,2\}} = \begin{cases} \frac{0 + G_1}{2} & \text{if } 0 \leq c_1, c_2 \leq 1, \\ \frac{G_1 + G_2}{2} & \text{if } 1 < c_1, c_2 \leq 2. \end{cases}$$

$$\begin{aligned} \text{Thus, } E_2(x_2) &= -\frac{1}{4} \int_0^2 x_2^2 dx_2 - \int_1^2 x_2(x_2 - 1) dx_2 + \frac{1}{4} \int_0^1 x_2^2 dx_2 + \frac{3}{4} \int_1^2 x_2(3x_2 - 2) dx_2 \\ &= \frac{19}{12}. \end{aligned}$$

$E_n(x_2)$  can also be obtained using the technique of “the sample maximum,” as there are only two random variables,  $c_1$  and  $c_2$  in this example. Let  $E_m(x_2)$  be the sample maximum. Then,

$$E_m(x_2) = \int_1^2 x_2 dF_m(x_2),$$

where  $F_m$  is the distribution of  $\max\{c_1, c_2\}$ . Since  $c_1$  and  $c_2$  are assumed to be independent from each other,

$$\begin{aligned} F_m(x_2) &= \Pr [\text{both } c_1 \text{ and } c_2 \text{ are less than } x_2] \\ &= \Pr [c_1 < x_2] \cdot \Pr [c_2 < x_2] \\ &= G_1(x_2)G_2(x_2). \end{aligned}$$

But note that the lower bound for the integration is 1 as  $\max\{c_1, c_2\}$  cannot be less than 1. Substituting numerical values, we can easily confirm that  $E_2(x_2) = E_m(x_2)$ .

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## Chapter 3

# On the Use of Selective Tendering in the Procurement Market\*

Having a few highly qualified bidders creates a more effective competition than having a very large number of bidders--in contrast with a normal free-market environment, where, as the number of bidders increases, the competition becomes more intense.

[Gansler (1989), p. 181]

### 1. INTRODUCTION

Most studies of procurement auctions have focused on the ways that stimulating competition can benefit bid-takers' interests. Maximizing the number of competing bidders is one way for a bid-taker to secure the most competitive price of an auctioned project. This is because the expected procurement cost is a decreasing function of the number of bidders (see, e.g., Holt [1980]). Based on the fundamental nature of bidding competition, the U.S. Congress enacted the Competition in Contracting Act (CICA) of 1984, providing procurement contractors with better access to the government procurement market. Specifically, under the CICA, excluded contractors and losing bidders can protest the actions of U.S. government procuring agencies.<sup>1</sup> Marshall, Meurer and Richard [1990] offers an economic analysis of the CICA, supporting the enactment.

On the other hand, in most real-life situations, bid-takers usually limit entry into bidding competition to a certain group of selected contractors. In the United Kingdom, for instance, 50 percent of government procurement funds in 1962-63 were allocated by this type of selective

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tendering, and 49 percent by single tendering where a single, selected contractor is invited to tender (Baldwin [1970, p. 60]). During the first half of fiscal year 1985, contracts through public tendering, which means free entry of potential contractors, accounted for only 4.6 percent of procurement funds of the U.S. Department of Defense (source: U.S. Department of Defense [1986]). In the case of government procurement, we may have to consider a political motive for selective tendering.<sup>2</sup> In her study of the construction industry, however, Hillebrandt [1974, p. 79] observes that the commercial sector more frequently uses selective tendering than the government sector and that there is a steady upward trend in the use of selective tendering.

Why then is a bid-taker reluctant to exploit his monopsony position through public tendering which most researchers believe reduces the expected procurement cost? To study this puzzling phenomenon, we develop a model based on the following idea: *ex ante* bidding competition *could* cause a winning bidder's opportunism *ex post*. (Hereafter we shall call the winning bidder the contractor.) Though it may not be possible to write a procurement contract that ensures contractual obligations, the contractor is willing to abide by contractual obligations because otherwise he would face exclusion from future procurement auctions. However, such a fear of exclusion is credible only when a present value of the expected profits from attending the future auctions exceeds a one-shot profit from renegeing on the contractual obligations. While undoubtedly the main purpose of selective tendering is to choose "qualified" bidders *ex ante*, the analysis in this paper will show that the use of selective tendering may have another justification: to implement a self-enforcing contract *ex post* via the credible threat of exclusion.

To be more precise our model runs as follows. A bid-taker, who wishes to procure an infinite sequence of projects, uses a sealed-bid auction to award each procurement project to one of many potential contractors. We assume that all the potential contractors are well qualified to produce high-quality products. The bid-taker lexicographically prefers high quality over cost

minimization.<sup>3</sup> However, transaction costs of making and enforcing a state-contingent contract make it difficult to ensure that the project procured is of high quality.<sup>4</sup> More precisely, we focus on a situation in which the product quality is observable *ex post* but non-verifiable; thus, the product quality is non-contractible and hard to enforce by legal means.

This situation involves two types of potential opportunism: (i) the bid-taker may threaten to cancel the procurement *ex post* in order to renegotiate the contract price, and (ii) the contractor may provide low quality to cut down on production cost. To rule out the bid-taker's opportunism, we will assume that the bid-taker precommits himself to a fixed price.<sup>5</sup> We will further assume that any contractor who provides low quality will face exclusion from future auctions. Then reputable bidders can put their reputations at stake in order to show their commitment to high quality.<sup>6</sup> On the other hand, to the extent the bid-taker perceives the potential contractors' opportunism *ex ante*, he will organize procurement auctions "so as to economize on bounded rationality while simultaneously safeguarding them against the hazard of opportunism (Williamson [1985, p. 32])". We define a "quality premium" to be a minimum safeguard (in terms of economic rent) which makes the contractor willing to provide high quality.

As in Holt [1980], a bidder faces an opportunity and production costs for the auctioned project. The opportunity cost, which is private information and varies across bidders, is the profit from the best foregone alternative opportunity of his resources. All the bidders have a common technology characterized by high production cost for high quality and low production cost for low quality. Then the question is whether bidding competition will affect the bidders' quality decision *ex ante*, i.e., whether bidding strategy will depend on *ex post* opportunism as well as bidders' opportunity costs. The answer is positive if the bid-taker fails to provide the quality premium. As we will show later in Section 4, other things equal, a reputable bidder's expected profit from maintaining his reputation is a decreasing function of the number of bidders. If this

profit becomes less than the quality premium, a contractor will provide low quality to save on production costs. To avoid this hazard of opportunism, therefore, the bid-taker must control the number of bidders prior to bidding competition.

This argument will also throw light on the problem of cost overruns, as the phenomenon of cost overruns can be seen as the mirror image of the underbidding phenomenon (Tirole [1986]). If the contractor provides low quality because of his underbidding at the bidding stage, then the government with the lexicographic preference will pay additional costs to get high quality (for instance, by the use of second sourcing). Several explanations may exist for cost overruns: inefficient monitoring of procurement contracts with agency problems and unanticipated adoption of superior design (Tirole [1986]); and a bid-taker's and bidders' lack of commitment to a contract (Lewis [1986]). Tirole [1986] and Lewis [1986] agree, however, that cost overruns occur because renegotiation is possible after signing a contract. On the contrary, this paper shows that, even when renegotiation is impossible (as with a fixed-price contract), cost overruns (corresponding to quality reduction in our context) can occur in equilibrium if the bid-taker fails to provide the quality premium. This is because excessive bidding competition through public tendering may force bidders to submit bid-prices for low quality instead of prices for high quality, though they are well qualified *ex ante* to provide high quality and there are no unforeseen cost overruns on average.

Before proceeding, we note the relationship between this paper and Spulber [1990], which is the only article we know that explicitly analyzes how the effectiveness of auction mechanism hinges on *ex post* incentives for performance, though his analysis differs from ours in scope and focus.<sup>7</sup> He considers an auction model where bidders have private information about their own abilities to perform, characterized by the size of potential cost overruns in the performance stage. He shows that, in the absence of proper legal enforcement, adverse selection problems cause the

bidding process to fail.<sup>8</sup> He examines a number of alternative legal compensation schemes to regain the effectiveness of auctions when the contractor reneges on the contractual obligations. To do this, Spulber [1990] assumes that the legal system of liability is perfect and that the bid-taker is better informed about the details (more specifically, a nonrecoverable project-specific investment) of the project than the bidders. These assumptions fundamentally differentiate his model from ours.

In the next section we begin by describing the timing and assumptions of the model. Section 3 examines the benefits for the bid-taker of bidding competition when contracts are complete and the liability system is perfect. This provides a benchmark for the subsequent analysis. In Section 4 we analyze a static version of dynamic auctions in which the bid-taker has discretionary power to set the number of competing bidders prior to bidding competition. We show that when the product quality is not contractible, different numbers of competing bidders result in three different equilibria. The phenomenon of cost overruns is explained with these equilibria. We also derive an optimum number of competing bidders and explore its comparative statics. Section 5 discusses several issues associated with our model, especially the role of the bid-taker's discretionary power and incumbent bidders' advantages over entrants.

## 2. DESCRIPTION OF THE MODEL

Consider a problem facing a prospective buyer who wishes to procure an infinite sequence of projects.<sup>9</sup> The buyer whom we shall call the bid-taker uses a first-price sealed-bid auction to award each procurement project to one of many potential contractors.<sup>10</sup> The risk-neutral bid-taker lexicographically prefers high quality over expected cost minimization and precommits himself to a fixed-price. Though, in general, quality of a procurement project consists of a

number of characteristics such as product quality, delivery date and after-sale service, here we assume that these have been converted into a quality equivalent, which takes only two values, high ( $H$ ) and low ( $L$ ).

We formulate each procurement as a three-stage process. Figure 3.1 describes the timeline summarizing the sequence.

*Stage 1 (Pre-auction stage).* Prior to bidding competition, the bid-taker either selects an appropriate number of bidders from the pool of potential contractors or allows free entry of bidders.<sup>11</sup> We assume that all the potential contractors are equally well qualified to produce high-quality products. At the end of stage 1, the number of competing bidders  $n$  is commonly known.

*Stage 2 (Bidding stage).* At the beginning of stage 2, each bidder observes his opportunity cost, which can be interpreted as his own alternative profit opportunity. The bidders are risk-neutral and differ only with respect to their opportunity costs. The opportunity cost parameter,  $c_i$ , of bidder  $i$ ,  $i = 1, 2, \dots, n$ , is independent and identically distributed, drawn from a common-knowledge two-point probability distribution on  $\{0, \bar{c}\}$ ,  $\bar{c} > 0$ . Let  $p = \Pr(c_i = \bar{c}) > 0$ . Since the winning bidder whom we shall call the contractor must forego his best alternative profit opportunity regardless of his potential opportunism in stage 3, the opportunity cost is a sunk cost equivalent to the contractor. Note that, however, it is a variable cost equivalent to losing bidders.

All the bidders have a common technology characterized by high production cost ( $c_H$ ) for high quality  $H$  and low production cost ( $c_L$ ) for low quality  $L$ . (Note that a bidder's production cost depends on his decision of quality.) A bidder's cost for the project is the sum of his

opportunity cost in stage 2 and a production cost in stage 3. As we have assumed a fixed-price contract, a bidder's decision of bid-price must consider his production cost as well as his opportunity cost.

By the rules of the first-price sealed-bid auction, then each bidder submits his bid both simultaneously and noncooperatively, and the lowest bidder wins. The bid-taker then makes a contract for high quality with the contractor at the lowest bid-price. If two bidders quote the same lowest price, the flip of a coin determines the winner.

*Stage 3 (Production Stage).* At the beginning of stage 3, nature resolves technological uncertainty about the production cost. We represent this by the state set  $S = \{S_B, S_G\}$ , where the subscript  $B$  denotes "bad" state and  $G$  "good" state. We may interpret a state of the world as an economic environment in stage 3, such as the uncertain input price for the production. It is commonly known that  $p_B = \Pr(S = S_B) > 0$ . We assume that the realization of a state in stage 3 is independent of the bidders' observation of their opportunity costs in stage 2.

Given this realization of state, the contractor produces the project using one of the two alternative strategies: a "faithful" and a "hit-and-run." With the faithful strategy, the contractor provides high quality  $H$  at production cost  $c_H$  if the state is good, and at  $\gamma c_H$  if bad, where  $\gamma > 1$  represents the effects of the technological uncertainty on opportunism. Similarly, with the hit-and-run strategy, the contractor provides low quality  $L$  at production cost  $c_L$  if  $S = S_G$ , and at  $\gamma c_L$  if  $S = S_B$ , where  $c_H > c_L$ . Thus, higher values of  $\gamma$  indicate higher degree of lucrativeness of opportunism due to the technological uncertainty.

At the end of stage 3, the bid-taker accurately assesses the product quality, though it may not be observable to third parties. If the contractor provides low quality, the bid-taker will

permanently exclude him from future auctions. Otherwise, the bid-taker allows him to attend the future auctions. Then the play proceeds to the subsequent procurement auction. Table 3.1 summarizes the structure of information and costs in stages 2 and 3.

We assume that bidders draw their parameter of opportunity costs anew at the beginning of stage 2 in each procurement auction, so that it is the same three-stage-game as described above that is infinitely repeated. With this game structure, a bidder may bid below his cost estimation in attempting to discourage his opponents from attending future auctions. In order to rule out this type of predatory pricing, we shall assume that the event of winning or losing does not change a bidder's reputation. That is, reputation is only related to a contractor's possible opportunism of quality reduction. To make our model tractable, we also assume that there is no sequential learning about production costs between projects. Then, for a bidder at a specific auction, receiving zero profit with either losing or refraining from that auction is better than receiving negative expected profit through predatory pricing. This assumption of reputation makes the exposition of the model much easier, because it allows the dynamic structure of the game to be reduced to a static model.<sup>12</sup>

### 3. COMPLETE CONTRACT AND FULL ENFORCEMENT: THE BENCHMARK

In this section we analyze the three-stage-game described in the previous section under the assumptions that contracts are complete and that the liability system is perfect. We use this analysis, in the following section, as the benchmark to study the role of selective tendering.

We employ the standard backward argument to tackle the three-stage-game. However, the problem of quality provision in stage 3 is automatically resolved, since the assumptions of

complete contract and full enforcement implies that a contractor is bound to provide high quality only at his bid-price. Therefore we can now move into the problems in stage 2.

For expositional simplicity, in this paper, we use an independent private-value model for the first-price sealed-bid auction.<sup>13</sup> We employ a Nash equilibrium as the solution concept for the auction. We first derive equilibrium bid-functions with two bidders. Our analysis of a low-bid auction with two bidders builds on that of Maskin and Riley (1985), although they deal with a high-bid auction.

The structure of information and costs in Table I tells us that, at the bidding stage, each bidder has informational advantage only about his own opportunity cost. Hereafter, the high-cost bidder will refer to as shorthand for a bidder who observes cost of  $\bar{c}$  in stage 2, and the low-cost bidder a bidder with of zero in stage 2. It would be helpful to think that a bidder (a low- or a high-cost type) has two opponents: a high-cost opponent with probability  $p$  and a low-cost opponent with  $1 - p$ . As a contractor is bound to provide high quality, at the bidding stage, a bidder's estimation of expected cost of providing high quality is  $\hat{c}_H = p_B \gamma c_H + (1 - p_B) c_H$ .

We first show that high-cost bidders can only bid  $\bar{c} + \hat{c}_H$  at equilibrium. To see why, suppose that they bid less than that. Then they might win the auction with positive probability, but they have to expect a negative expected profit. Now suppose that they bid more than  $\bar{c} + \hat{c}_H$ , i.e., their bid-price is  $b > \bar{c} + \hat{c}_H$ . Then a high-cost bidder's profit is  $(b - \bar{c} - \hat{c}_H)/2$ . If the other bidder reduces his bid-price slightly to  $b - \epsilon$ , his profit becomes  $(b - \bar{c} - \hat{c}_H - \epsilon)$  which is greater for small  $\epsilon$ . In other words, he can increase his chance of winning discontinuously if he bids infinitesimally smaller than his opponent's bid-price; thus, he can increase his expected profits. Foreseeing these results, in equilibrium, high-cost bidders bid just  $\bar{c} + \hat{c}_H$ . That is, in equilibrium, competition among bidders forces high-cost bidders to earn expected profit of zero.

As we will use this type of logic repeatedly, we call it the “Bertrand-like competition” for high-cost bidders. One can easily recognize that this result is independent of the number of bidders  $n \geq 2$ .

Now let us consider low-cost bidders. Since the distribution of cost observations is discrete, there is no equilibrium in pure strategy for low-cost bidders. To see this, assume that a low-cost bidder, say bidder 1, submits  $\bar{c} + \hat{c}_H - \epsilon$ . Then the other low-cost bidder’s best response is to bid  $\bar{c} + \hat{c}_H - 2\epsilon$ , in which case bidder 1 again would deviate to  $\bar{c} + \hat{c}_H - 3\epsilon$ , and so on. However, if the decreasing bid-price hits some low price, a low-cost bidder’s best response is to bid  $\bar{c} + \hat{c}_H - \epsilon$  again, realizing it would be better to beat only a *possible* high-cost opponent. Then the cycle of price cutting continues.

Let  $G(b)$  be the cumulative distribution function of a low-cost bidder’s bid  $b$ . Let  $\underline{b}$  and  $\bar{b}$  be the lowest and the highest bids, respectively. Since the two bidders are symmetric *ex ante*, they should have a common support in equilibrium. Maskin and Riley prove that  $G(b)$  is continuous over a common support  $[\underline{b}, \bar{b}]$  and that there can be no subinterval of  $[\underline{b}, \bar{b}]$  over which  $G(b)$  is constant.<sup>14</sup> Since high-cost bidders always bid  $\bar{c} + \hat{c}_H$  in equilibrium, low-cost bidders can bid up to  $\bar{c} + \hat{c}_H - \epsilon$ , without losing the auction to high-cost bidders. For mathematical convenience, we set  $\bar{b} = \bar{c} + \hat{c}_H$ .<sup>15</sup> Since, in equilibrium, any bid  $b$  as part of a mixed strategy must generate the same expected payoff, the following equation must be true:

$$(1) \quad \Pr(\text{win}|b)(b - \hat{c}_H) = \Pr(\text{win}|\bar{b})(\bar{b} - \hat{c}_H).$$

A low-cost bidder’s probability of winning,  $\Pr(\text{win}|b)$ , is  $p + (1 - p)(1 - G(b))$ , since he can beat a high-cost type opponent with probability one (which results in the first term  $p$  of the probability) and beat a low-cost opponent when the opponent bids higher than  $b$  (which results in the second

term  $(1 - p)(1 - G(b))$  of the probability). And since  $\bar{b} = \bar{c} + \hat{c}_H$ , we can rewrite equation (1) as

$$(2) \quad [p + (1 - p)(1 - G(b))](b - \hat{c}_H) = p\bar{c}.$$

Since (2) must hold for any  $b$  in an equilibrium support and since  $G(b) = 0$  by definition, the infimum (or largest lower bound) of  $b$ ,  $\underline{b} = p\bar{c} + \hat{c}_H$ . Thus, the equilibrium support of  $b$  is the interval  $[p\bar{c} + \hat{c}_H, \bar{c} + \hat{c}_H]$ .

Let  $\underline{b}_n$  be the infimum and  $G_n(b)$  the cumulative distribution function of a low-cost bidder's bid  $b$  with  $n$  bidders. Using the same logic with two bidders, we then characterize a symmetric Nash-equilibrium bidding strategy with  $n$  bidders:

*Lemma 1. Suppose that contracts are complete and that the liability system is perfect. Then, in equilibrium, (i) every high-cost bidder always bids  $\bar{c} + \hat{c}_H$ , and (ii) every low-cost bidder bids according to the strategy*

$$(3) \quad [p^{n-1} + (n - 1)p^{n-2}(1 - p)(1 - G_n(b)) + \dots + (1 - p)^{n-1}(1 - G_n(b))^{n-1}] (b - \hat{c}_H) = p^{n-1}\bar{c}$$

for all  $b \in [\underline{b}_n, \bar{b}]$ .

When  $b = \underline{b}_n$ , the right-hand side of (3) is  $p^{n-1} + (n - 1)p^{n-2}(1 - p) + \dots + (1 - p)^{n-1} = [p + (1 - p)]^{n-1} = 1$ . By definition,  $G_n(\underline{b}_n) = 0$ . Thus,  $\underline{b}_n = p^{n-1}\bar{c} + \hat{c}_H$ , which converges to  $\hat{c}_H$  if  $n \rightarrow \infty$ .

Let  $K_n$  denote the bid-taker's expected procurement cost with  $n$  bidders. Then Lemma 2 proves the folk wisdom of bidding competition: As the number of competing bidders increases,

the bid-taker's expected procurement cost  $K_n$  decreases.

*Lemma 2.* If  $0 < p < 1$ ,  $K_n$  converges to  $\hat{c}_H$  as  $n \rightarrow \infty$ .

*Proof.* To prove this lemma, we use the Revenue Equivalence Theorem (see, e.g., McAfee and McMillan [1987]), one implication of which tells us that both the first-price sealed-bid and the second-price sealed-bid auctions result in the same expected procurement cost to the bid-taker. We regard bidder  $i$ 's cost observation  $c_i$  as an independent draw from the given distribution. Arrange the sample of  $c_i$  in increasing rank order, and let  $x_1$  be the smallest of  $c_i$ ,  $x_2$  the same or the next  $c_i$ , and thus,  $x_1 \leq x_2 \leq \dots \leq x_n$ . Then, in the second-price sealed-bid auction, the bidder who submits  $x_1$  is the winner and is awarded the contract at the bid-price  $x_2$ . By the Revenue Equivalence Theorem, therefore,  $K_n = Ex_2$ , where the notation E is for expected values. To emphasize the sample size, we denote the second order statistic of a sample size  $n$  as  $x_{2:n}$ . Then,  $Ex_{2:n+1} \leq Ex_{2:n}$  since  $x_{2:n+1} \leq x_{2:n}$ . Since  $Ex_{2:n}$  is monotonically decreasing and bounded, it converges to the infimum of  $\underline{b}_n$ ,  $\hat{c}_H$ , as  $n \rightarrow \infty$  (see Theorem 4.17 in Binmore [1977] for this property of convergence).  $\square$

From the proof of Lemma 2, the following lemma is immediate:

*Lemma 3.* A contractor's expected profit with  $n$  bidders,  $Ex_{2:n} - Ex_{1:n}$ , converges to zero as  $n \rightarrow \infty$ .

Now let us consider the bid-taker's problem in stage 1. As shown in Lemma 2, if contracts are complete, and if the liability system is perfect, then the bid-taker can minimize his

expected procurement cost for the project by the use of public tendering.

#### 4. REPUTATION AND SELF-ENFORCING CONTRACTS

In the previous section, we have maintained the assumptions of complete contract and full enforcement so that a contractor's decision about which quality to produce is exogenously given rather than endogenous. In this section we relax these assumptions. Specifically, the product quality is non-contractible and non-verifiable, though it is observable *ex post* by the bid-taker. Also remember that the bid-taker precommits himself to a fixed price. With these assumptions, a contractor is now free to choose a product quality from the quality set  $Q = \{H, L\}$ . However, the bid-taker permanently prohibits a contractor from attending future auctions if he provides low quality  $L$ .

As in Section 3, we employ the standard backward argument by starting with the production stage (or stage 3). Let  $V$  be the present value of a reputable contractor's expected profits from all possible future contracts, which is feasible only when he has provided high quality  $H$  and thus is allowed to attend the infinite stream of future auctions. We call  $V$  the present value of incumbency. In deciding whether to provide  $H$  or  $L$ , then the contractor chooses between obtaining some one-shot profit from a "hit-and-run" strategy of providing  $L$  or receiving  $V$  from a "faithful" strategy of providing  $H$ .

After nature resolves technological uncertainty by choosing a state from the state set  $S = \{S_B, S_G\}$  at the beginning of stage 3, the contractor chooses a product quality from the quality set  $Q = \{H, L\}$ . Suppose that nature selects bad state  $S_B$ . If the contractor decides to provide  $H$ , then he will incur cost  $\gamma c_H$ . However, if he decides to provide  $L$ , he must incur  $\gamma c_L + V$ , as

he will face a permanent exclusion from the future auctions. Thus, the contractor chooses a product quality based on the following rule when nature selects bad state  $S_B$  (assuming that he chooses  $H$  in the border case):

$$(4) \quad \begin{aligned} Q = H & \text{ if } \gamma(c_H - c_L) \leq V \\ & L \text{ if } \gamma(c_H - c_L) > V. \end{aligned}$$

Similarly, if nature selects good state  $S_G$ , the contractor's choice of quality is based on:

$$(5) \quad \begin{aligned} Q = H & \text{ if } c_H - c_L \leq V \\ & L \text{ if } c_H - c_L > V. \end{aligned}$$

We now examine the problem in the bidding stage. Let us revert to the Nash equilibrium in Lemma 1, but with two bidders. It would be convenient to divide a bidder's bid-price into two components: bid-price based on the opportunity cost and bid-price based on the production cost. Let  $b_3$  denote the bid-price based on the production cost. Then one can easily recognize that  $b_3$  in Lemma 1 is the expected production cost for high quality,  $\hat{c}_H$ , as the contractor is bound to provide high quality  $H$  only in the previous section. As bidders are now free to choose between  $H$  and  $L$ , it is natural to expect that equilibrium bid-price is influenced by this new environment. However, this influence is limited only to the production cost, since the opportunity cost in stage 2 is a sunk cost equivalent to the contractor. We thus focus on the derivation of an equilibrium  $b_3$ .

Let  $\hat{c}_L = p_B \gamma c_L + (1 - p_B)c_L$  and  $\hat{c} = p_B \gamma c_L + (1 - p_B)c_H$ . Note that  $\hat{c}_L$  denotes the expected production cost for low quality, while  $\hat{c}$  is the expected production cost for low

quality with probability  $p_B$  and for high quality with  $1 - p_B$ . Then the following lemma summarizes an equilibrium  $b_3$ .

*Lemma 4. Suppose that the product quality is not contractible. Then, in equilibrium, a high-cost bidder's bid-price for a product quality is:*

$$(6.1) \quad \hat{c}_H \quad \text{if } V \geq \gamma(c_H - c_L)$$

$$(6.2) \quad b_3(V) = \begin{cases} \hat{c} + p_B V & \text{if } c_H - c_L \leq V < \gamma(c_H - c_L) \\ \hat{c}_L + V & \text{if } V < c_H - c_L. \end{cases}$$

$$(6.3) \quad \hat{c}_L + V \quad \text{if } V < c_H - c_L.$$

*Proof.* The basic idea of the proof is again the “Bertrand-like competition” for high-cost bidders. We will prove only (6.2), since the proofs of (6.1) and (6.3) are essentially the same as that of (6.2). Suppose that the two bidders observe high cost in stage 2 and that a bidder wins with  $\bar{c} + b_3$ . Then the first inequality of the condition in (6.2),  $V < \gamma(\hat{c}_H - \hat{c}_L)$ , tells us that the contractor will provide low quality  $L$  if the state of nature is bad, while the second inequality  $c_H - c_L \leq V$  implies  $Q = H$  if  $S = S_G$ . It is then obvious that the production cost for this strategy is  $\hat{c}$  and that the contractor will not lose  $V$  with probability  $(1 - p_B)$ . Thus, the following equation must be true in the bidding stage:

$$(7) \quad (1 - p_B)V + b_3 - \hat{c} \geq V.$$

Note that the left-hand side of (7) is the sum of a high-cost contractor's expected profits from the present and future auctions by bidding  $\bar{c} + b_3$ , while the right-hand side of (7) is the present value of a losing but reputable bidder's incumbency. By the “Bertrand-like competition” for

high-cost bidders, in equilibrium, (7) becomes the equality in (6.2).  $\square$

Figure 3.2 represents the graph of  $b_3(V)$ . After determining the value of  $b_3(V)$ , the next step to find an equilibrium strategy for low-cost bidders is essentially the same as in (1) and (2).

See Figure 3.2. First consider the case where  $V \geq \gamma(c_H - c_L)$ . From (4) and (5), the contract for high quality is self-enforcing; consequently, bidding competition yields the same equilibrium as in Lemma 1.

When the value of reputation is relatively small, i.e., when  $V < c_H - c_L$ , the description of the Nash equilibria in Lemma 4 pertains to the recognition of Tirole's [1986] conjecture about the problem of underbidding (or cost overruns). Remember that cost overruns corresponds to quality reduction in this paper, as explained in the Introduction. To explain cost overruns in the presence of rational expectations, Tirole [1986] proposes a hypothesis that, if renegotiation is possible after signing a contract, the production-cost estimate at the bidding stage represents only a lower bound on the transfer in case of implementation. But Lemma 4 suggests that, even when renegotiation is impossible (as with a fixed-price), cost overruns (quality reduction in our context) can occur in equilibrium if the bid-taker fails to provide a necessary value of incumbency  $V$  for high quality. (As we will show it later in Proposition 2, the bid-taker can decide the size of  $V$  by controlling the number of competing bidders.) The following proposition then summarizes the main implication of Lemma 4.

*Proposition 1. Suppose that the product quality is not contractible. If the present value of expected profits from maintaining reputation is relatively small (more precisely, if  $V < c_H - c_L$ ), then, in equilibrium, bidders cannot bid their bid price for a quality ( $b_3$ ) high enough to cover the*

expected cost of high quality ( $\hat{c}_H$ ), though there is no unforeseen cost overrun for high quality  $H$ . Rather, bidding competition forces the bidders to bid so low that a designated contractor can only provide low quality  $L$  (via the hit-and-run strategy).

Next consider the intermediate case where  $c_H - c_L \leq V < \gamma(c_H - c_L)$ . By the conditions (4) and (5), the contractor will provide low quality  $L$  with probability  $p_B$  and high quality  $H$  with probability  $1 - p_B$  at stage 3. This type of partial failure of providing high quality  $H$  explains how uncertainty can affect cost overruns: the phenomenon of cost overruns can occur as a result of unfavorable economic environment,  $S_B$ , in the production stage.

We now move into the bid-taker's problem in stage 1. As the bid-taker lexicographically prefers high-quality performance to expected cost minimization, he tries to minimize the expected procurement cost under the condition that no contractor provides low quality at any circumstances. Lemma 3 shows that a contractor's expected profit from a specific contract is a decreasing function of the number of bidders  $n$ . Since  $V$  is a reputable contractor's present value of expected profits from all feasible future contracts, we then have  $V = V(n)$  and  $V'(n) < 0$ . We assume that the size of  $V$  is large enough to allow at least two bidders in the auction. Using (6.1) and Lemma 1, we define a high-quality equilibrium:

*Definition 1.* A high-quality equilibrium is a pair of a number of bidders and a bidding strategy  $(n, B(n))$  such that

$$(9) \quad V(n) - \gamma \cdot (c_H - c_L) \geq 0,$$

and bidding strategy  $B(n)$  satisfies the Nash equilibrium in Lemma 1.

Inequality (9) is a necessary condition (or incentive compatibility) for the high-quality equilibrium, since it implies that the expected profit from the hit-and-run strategy is less profitable than that from the faithful strategy. To do some comparative statics, it is convenient to ignore the restriction that  $n$  must be an integer.

Proposition 2 describes the existence of the high-quality equilibrium. An asterisk will indicate the optimal value of  $n$ .

*Proposition 2.* Fix the values of  $p, p_B, r, \gamma, \bar{c}$  and  $c_H - c_L$ . Then there exists a unique high-quality equilibrium,  $(n^*(p, p_B, r, \gamma, \bar{c}, c_H - c_L), B(n^*(\cdot)))$ , which ensures a self-enforcing contract at the lowest expected procurement cost.

*Proof.* Suppose that  $V(n) - \gamma \cdot (c_H - c_L) \geq 0$ . Then, in equilibrium, no contractor will provide low quality, which results in the Nash equilibrium in Lemma 1. Again, recall the “Bertrand-like competition” for high-cost bidders, which implies that only low-cost contractors receive positive expected profits. But the probability that a bidder observes low-cost is  $1 - p$ . From (3), then a low-cost bidder’s expected profit with  $n$  bidders is  $(1 - p)p^{n-1}\bar{c}$  (which is the integration of the left-hand side of (3) over the equilibrium support). Thus, when  $n$  is the number of bidders for the present and future auctions, the value of reputation is:

$$(10) \quad V(n) = \frac{(1-p)p^{n-1}\bar{c}}{1+r} + \frac{(1-p)p^{n-1}\bar{c}}{(1+r)^2} + \dots = \frac{(1-p)p^{n-1}\bar{c}}{r}.$$

Since  $V(n)$  is monotonically decreasing in  $n$  while  $\gamma \cdot (c_H - c_L)$  is constant, there must be the

unique  $n^*$  such that  $V(n^*) - \gamma \cdot (c_H - c_L) = 0$ . □

Lemma 2 shows that the expected procurement cost is a decreasing function of the number of bidders  $n$ . If the hit-and-run strategy of quality reduction is available to a contractor, however, the cost minimization implication of Lemma 2 is substantially limited, since now a conflicting force is at work. Namely, a contractor becomes tempted to follow the hit-and-run strategy, as the the bid-taker tries to reduce the expected procurement cost by admitting more bidders to an auction. This is because an increase in  $n$  makes a contractor's expected profits from maintaining reputation less important than his feasible profits from the hit-and-run strategy. This trade-off between the assurance of a quality premium  $V$  and the maximization of competition determines the optimum number of competing bidders,  $n^*$ , when the product quality is not contractible.

From (9) and (10), we have

$$(11) \quad \frac{(1-p)p^{n^*-1}\bar{c}}{r} - \gamma \cdot (c_H - c_L) = 0.$$

One can then easily check that Proposition 2 has the following comparative statics.

*Proposition 3.* (i)  $\partial n^*/\partial r < 0$ ; (ii)  $\partial n^*/\partial \bar{c} > 0$ ; (iii)  $\partial n^*/\partial (c_H - c_L) < 0$ ; (iv)  $\partial n^*/\partial \gamma < 0$ .

*Proof.* Using logarithmic transformation, we can rewrite (10) as

$$(12) \quad n^* = \frac{1}{\ln p} [\ln r - \ln(1-p) - \ln \bar{c} + \ln \gamma + \ln(c_H - c_L)] + 1 \geq 2.$$

Thus,

$$(13) \quad \frac{\partial n^*}{\partial r} = \frac{1}{\ln p} \cdot \frac{1}{r} < 0.$$

$$(14) \quad \frac{\partial n^*}{\partial \bar{c}} = - \frac{1}{\ln p} \cdot \frac{1}{\bar{c}} > 0.$$

$$(15) \quad \frac{\partial n^*}{\partial (c_H - c_L)} = \frac{1}{\ln p} \cdot \frac{1}{(c_H - c_L)} < 0.$$

$$(16) \quad \frac{\partial n^*}{\partial \gamma} = \frac{1}{\ln p} \cdot \frac{1}{\gamma} < 0.$$

□

With a given technology and self-enforcing contracts, several parameters affect the optimum number of competing bidders  $n^*$ . From (13), the discount rate  $r$  is inversely proportional to  $n^*$ . This result is quite well-known in the literature of reputation. A smaller  $r$  can result from a short time period of a project, as for a given  $r$  per unit of time,  $r$  per period grows with the information lag. The shorter the time period, the easier it is for the bid-taker to introduce competition, because he is able to observe the product quality earlier. Thus, the feasible profits from quality reduction becomes smaller.

The sign of (14) implies that the optimum number  $n^*$  is an increasing function of  $\bar{c}$ , that is, as the opportunity-cost differential increases, so does  $n^*$ . This is because the increase in the opportunity cost differential makes the value of the incumbency become more valuable and thus, the bid-taker can exploit some of the value of the incumbency by the introduction of more competition.

The inequality (15) says that as the difference  $c_H - c_L$  becomes larger, the optimum number  $n^*$  becomes smaller. Note that  $c_H - c_L$  reflects the degree of the lucriveness of the hit-and-run strategy in stage 3. If the process of the production is well established, i.e., if the difference  $c_H - c_L$  is relatively small, then a contractor is less able to exploit his *ex post* monopoly position for the project. In other words, the bid-taker's ability to introduce more competition decreases with the relative lucriveness of the hit-and-run strategy.

Since  $\gamma$  represents the relative size of the technological uncertainty affecting the relative lucriveness of the hit-and-run strategy, the same argument as in the difference  $c_H - c_L$  applies to  $\gamma$  in (16). If a procurement contract involves both higher technological uncertainty  $\gamma$  and greater lucriveness  $c_H - c_L$ , the benefit of bidding competition is easily limited and incumbent bidders can enjoy a higher premium from their *ex post* monopoly positions.

If the optimum number of bidders is  $n^* < 2$ , then the bid-taker with the lexicographic preference prefers bilateral negotiation (or single tendering) to an auction mechanism.

## 5. CONCLUDING REMARKS

Our relatively simple model has provided a straightforward explanation of why selective tendering is so widely used in the procurement market. Some extensions of our model seem important: (i) considering asymmetry between incumbent and entrant bidders; (ii) adjusting a bid-taker's discretionary powers to legal environments; and (iii) comparing selective tendering with some alternative institutions. We shall comment briefly on each of these.

For expositional simplicity, we have assumed that the number of potential contractors is large enough to allow selective tendering and that the potential contractors are equally well

qualified to provide high quality. Suppose that there are two types of contractors--qualified (incumbent contractors) and less qualified (entrants). Also assume that the number of the incumbents is less than an optimum number of bidders. Then the introduction of the entrants into bidding competition would be beneficial to the bid-taker. The basic argument in support of the introduction of the entrants is that concern over losing the contract to an entrant will cause the incumbent bidders to bid more aggressively. However, two important factors favor the incumbents. First, it is natural that, on average, the incumbents may have lower production costs (or superior technology) than the entrants, since the incumbents have accumulated production experience in the industry. Second, the entrants may also have larger financing requirements than the incumbents. Furthermore, the incumbents can finance at lower financial costs because of their established reputation in the product market. If an entrant, realizing high cost at the bidding stage, believes that his wealth including the financing ability is insufficient to invest in reputation-building, then he may try to mimic a low-cost entrant's strategy. This mimicry results from the high-cost entrant's anticipation that he can employ the hit-and-run strategy of quality reduction when he cannot provide high quality in the production stage. This opportunistic behavior of the high-cost entrant influences the incumbents' strategies as well as other low-cost entrants' strategies, which then have feedback effects on the high-cost entrant's opportunism. If the bid-taker cannot screen this type of high-cost entrant, the bid-taker's belief about an entrant must be independent of the entrant's realization of opportunity cost at the beginning of the bidding stage. This "lemon" effect may cause the bid-taker to block potentially beneficial entry.

The second issue we have not developed in this paper is the bid-taker's discretionary powers. As in our model, bid-takers in the commercial world can force contractors who provide low quality to exit from the industry by excluding them from future bidding. As long as a bid-

taker has such discretionary powers (corresponding to the consumer sovereignty in the commercial world), the reputation mechanism works very well regardless of contractual incompleteness. In the case of government procurement, however, some legal environments may limit the bid-taker's discretionary powers. In order to discourage possible misuse of the procuring agencies' discretion, the U.S. Congress (with the enactment of the Competition in Contracting Act (CICA) of 1984) has encouraged excluded contractors and losing bidders in certain federal procurement auctions to protest the procuring agencies' decisions. Under the CICA, it may not be easy for a government procuring agency (as a bid-taker) to ensure that the previous performance of a contractor should play a major role in evaluating his qualifications for present and future procurement auctions. This is because the procuring agencies cannot punish the poor-performance contractors unless poor performance is proved by third parties (e.g., by a court). This may encourage contractors' opportunistic behaviors. In his study of buying military equipment, Gansler [1989] argues that quality deteriorated as the CICA tended to overemphasize competition.<sup>16</sup>

Finally, though the development of our model is based on the bid-taker's direct control of the number of bidders, we have a few alternatives such as a minimum financial or technical requirement for bidders, and price discrimination against foreign bidders. Comparing selective tendering with these alternatives could produce some policy implications in the design of procurement auctions.

## FOOTNOTES

1. Bidders refer to potential contractors invited to a procurement auction.
2. Governments often use their purchases as a conscious policy tool either to correct balance-of-payments inequities in international trade or to promote economic development in less developed regions (de Mestral [1982]). Price discrimination against foreign bidders and selective tendering through residence requirements on bidders are typical examples of such conscious procurement policies (see Baldwin [1970] for more details).
3. Although simplified, this assumption of lexicographic preference is not far from reality. In the real world, governments have paid cost overruns in government contracts *ex post* in order to avoid quality reduction, if the associated costs are not too high.
4. It might be feasible to write and enforce complete (or state-contingent) contracts, but simply too costly to do so. Contracts cannot be complete because of transaction costs that result from: (i) *ex ante*, bounded rationality associated with uncertainty, and (ii) *ex post*, opportunism and imperfect legal enforcement. The new institutional economics in terms of transaction costs is now popular in economics. See Williamson [1975, Ch. 2; 1985, Ch. 1] for more detailed description of transaction costs.
5. Here the assumption of the fixed-price contract helps us concentrate only on the effect of the bidders' opportunism. However, the fixed-price contract is indeed the most common form of contract in the real world. With the absence of the bid-taker's potential opportunism, in the

remainder of the paper, we will use the terminology opportunism to denote a contractor's quality reduction *ex post*.

6. Klein and Leffler [1981] first recognize and Shapiro [1983] develops further the idea of reputation in a competitive market where product quality is unobservable prior to purchase: Sellers have an incentive to provide high quality because of reputation related to repeated purchases. A frequent and long-lived relationship between the bid-taker and bidders makes this mechanism more effective. Even if the bid-taker is a one-shot player, the reputation mechanism still works if he can communicate with other bid-takers by using word-of-mouth so that all bid-takers in the industry know a specific contractor's reputation.

7. We do not consider the interesting issues arising from the bidders' *ex ante* need to gather information about the true value of the auctioned object. French and McCormick [1984] argue that the bid-taker may deter free entry of bidders if he must bear the costs associated with the bidders' efforts to gather information. In a similar situation, Harstad [1989] compares the bid-taker's revenues from different auction institutions, with endogenous entry of bidders.

8. We may roughly summarize his argument as follows. If contract law cannot force bidders to abide by the contractual obligations, high-cost-overrun bidders will submit bid-prices too low to cover their costs of performance, foreseeing that they can renege on the contractual obligations when a cost overrun occurs. This changes the bidding behavior of low-cost-overrun bidders to the point at which both types' bid-prices as well as their possible opportunism become identical. Thus, the auction mechanism fails to distinguish between bidders on the basis of their private information about anticipated cost overruns.

9. The assumption of an infinite sequence is justifiable in the present context since the bid-taker is a government or a firm and not an individual.

10. Though there are other auction institutions such as open and Vickrey auctions, Hansen [1988] notes that the first-price sealed-bid auction is the most popular institution in the procurement market. He also provides an excellent theoretical explanation why this is so.

11. In the case of selective tendering, it is somewhat troublesome to determine which potential contractors should be selected and entitled to be incumbent bidders. (As we will explain at the description of stage 3, the bid-taker allows reputable bidders to be incumbents.) It may be plausible to assume that selection and incumbency result from both the bid-taker's random selection and the history of the industry.

12. The dynamic approach may capture long-run behavior more accurately but it is too complicated to consider problems such as Nash equilibrium bidding strategies and the optimum number of competing bidders in each procurement auction, because of the possibility of predatory pricing and of the sequential learning about production costs.

13. Independent private and common values are the two polar cases in auction models. An auction is called an independent private-value model if a bidder's opportunity cost is statistically independent from other bidders' opportunity costs and if his knowledge of other bidder bidders' opportunity costs does not change his opportunity cost. The independent private values do not imply, however, that a bidder's knowledge about his opponents' costs does not affect his strategic

behavior. A common-value model deals with the case in which a true cost of providing the project is common but unknown (see, for e.g., French and McCormick [1984] and Harstad [1989], among others).

14. Theorem 6 in Dasgupta and Maskin [1986] is a general proof of the existence of a symmetric mixed-strategy equilibrium. The reasoning of no atom over the interval is roughly as follows. Suppose to the contrary that a low-cost-bidder  $i$  submits  $b_i \in [\underline{b}, \bar{b}]$  with positive probability. Then there must be some interval  $[b_i, b_i + \epsilon]$  over which his low-cost opponent  $j$  will not bid since bidder  $j$ 's probability of winning increases discontinuously if he replaces his bid in this interval with one that is infinitesimally smaller than  $b_i$ . Then bidder  $i$  can increase  $b_i$  to  $b_i + \epsilon$  without reducing his probability of winning, which contradicts the assertion that the original  $b_i$  with positive probability was an equilibrium.

15. Let us call  $\bar{b}$  the supremum (or smallest upper bound) of  $b$ . Suppose that  $\bar{b} < \bar{c} + \hat{c}_H$ . Then any  $b$  between  $\bar{b}$  and  $\bar{c} + \hat{c}_H$  has the same chance of winning as  $\bar{b}$ , and so is preferable. Thus,  $\bar{b}$  must be the closest number to  $\bar{c} + \hat{c}_H$ . Because of the continuum property of the real numbers, however, it is troublesome to fix the closest number to  $\bar{b}$ . To avoid this problem, it will be convenient to suppose that a low-cost bidder wins with  $\bar{b}$  if both a low-cost and a high-cost bidders submit  $\bar{b}$ .

18. The CICA also increases the delays and bottlenecks in the procurement system. Gansler [1989, p. 191] quotes an article from the Washington Post as an example of such problems. According to it, the number of protests went from a few hundred a year to over 3,000 a year within four years of the law's passage. One company, with only 10 employees, submitted 50

protests in the three years following the passage of the law.

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## Chapter 4

# Government Procurement and Competitive Bribery in Less Developed Countries

DISCRETION. Government cannot work without it. But it can be unmercifully abused.

[Amick 1976, p. 76]

### 1. INTRODUCTION

The auction mechanism is generally regarded as one of the most efficient ways to deter corruption in procuring governments' necessities. This is because bidding competition makes it difficult for a corrupt procuring official to be bribed by a specific supplier.<sup>1</sup> But in many cases regarding government procurement, especially in less developed countries (LDCs), the auction mechanism also can be misused to serve the corrupt procuring official's goal. For instance, *The Economist* (January 4, 1992) reports a typical example of the adverse effects of bribery in procurement auctions in Thailand:

The government led by Chatichai Choonhavan, which was overthrown in February, had become a byword for corruption. Its ministers' tactic of squeezing bribes out of rival bidders for Bangkok's infrastructure projects has ensured that none of those projects is yet under way. As a result, the capital is almost at a standstill.

If a commodity for procurement is well specified, and thus there is no room for a corrupt procuring official to exercise his discretionary powers over the definition of the commodity, then the auction mechanism will avert bribery. Even with a great effort to deter corruption, however, it seems to be almost impossible for a government to eliminate such discretionary powers of the

official completely, since the definition of the commodity involves various non-price attributes such as quality, design features and performance. Consequently the official can evaluate multi-dimensional bids subjectively.<sup>2</sup> Our major concern in this chapter is the nature of feasible equilibria when the official has subjective criteria in choosing the winner in procurement auctions.

There are two polar cases of bribery (or side-payments) in auctioning government procurement: “bribes” and “kickbacks.” Bribes (more precisely, illegal entry-fees) refer to bidders’ *ex ante* side-payments to the corrupt procuring official to influence the outcome of bidding competition in their favor. Bribes are a type of sunk-costs to the bidders as they lose the amounts whether or not they are ultimately successful. Kickbacks (more precisely, illegal rebates) refer to *ex post* side-payments from the winner to the official.<sup>3</sup> Since the losing bidders do not pay them, kickbacks can be viewed as a variable-cost equivalent to the bidders. In most real-life situations, bribery probably incorporates elements belonging to both bribes and kickbacks, so it would be desirable to have a unified framework. As always in modeling, however, we must forego a bit of realism for simplicity. Bribes vs. kickbacks could be a simple contrast worth exploring.

In his study of corruption in government, Amick (1976) observes that kickbacks are the preferred form of bribery in procurement auctions in the U.S. This is no wonder, given that the U.S. is one of the few countries where “whistleblowing” (or public disclosure of bribery) is widely accepted [Glazer and Glazer (1989)]. Roughly, other things equal, losing bidders have stronger *ex post* incentives to expose corrupt practices with bribes than with kickbacks. The procurement auction scandal in Thailand and some case studies in Rose-Ackerman (1978), however, show that the game of bribes is dominant over that of kickbacks in LDCs, where corruption is likely to be widespread.<sup>4</sup>

The problem of bribery in government procurement was first formally considered in the

context of auction theory by Beck and Maher (1986), where bidders have private cost information. Using a first-price sealed-bid auction, they concentrate on a model of kickbacks in which a corrupt procuring official sets a final contract price and then induces competition between the bidders for the highest kickback.<sup>5</sup> With this description, the model of competitive bribery can be transformed onto that of bidding competition. Since competitive bribery allows the official to extract all the surplus associated with a higher contract price and since the largest kickback can be paid by the lowest cost bidder, competitive bidding and bribery achieve an isomorphism (both in terms of bidders' expected payoffs and of the selection of the lowest bidder as the winner).

It is true that a corrupt procuring official may inflate a contract price in order to increase his revenue from competitive bribery. At the same time, however, he must not raise the contract price high enough to arouse public suspicions.<sup>6</sup> We present a simpler, but more general, model of bribery both in the games of bribes and kickbacks. There are two main differences between our approach and that of the existing literature represented by Beck and Maher. First, rather than focusing on one-dimensional side-payments, we explicitly model a two-dimensional auction, i.e., a simultaneous decision of bid-prices and side-payments. The second difference is that the official sets a reserve (or ceiling) price above which a contract cannot be made regardless of the amounts of side-payments. This reserve price, limited by environments for corruption (such as a budget constraint for the project and riskiness of bribery due to anti-corruption policies), is not necessarily equal to the final contract price, while the existing literature presumed an equivalence between the reserve price and the final contract price.

Our model specification has two immediate implications. First, the isomorphism in Beck and Maher holds in the game of bribes as well as that of kickbacks. This implies that the risk-neutral bidders are indifferent between the two games, and that from the bidders' viewpoints the

two-dimensional auction can be mapped into the standard one-dimensional auction if the official's preference is known to the bidders. Secondly, the risk-neutral corrupt procuring official may also be indifferent between the two games. Furthermore, if a reserve price is a decreasing function of the probability and severity of punishment for bribery, the official must prefer the less-risky game of kickbacks to that of bribes. This result raises a puzzling and unsolved issue: Why does the corrupt procuring official in LDCs (where the official is likely endowed with all the power to choose the game structure) prefer the seemingly riskier game of bribes to the game of kickbacks?

This chapter is organized as follows. In the next section we begin by modeling a standard low-price auction, which provides a benchmark for our subsequent analysis of competitive bribery. In Section 3 we present the formal structure of two alternative bribery models: bribes and kickbacks. Section 4 characterizes a Nash equilibrium in each model and compares them. Section 5 contains concluding remarks.

## 2. A ONE-DIMENSIONAL AUCTION: THE BENCHMARK

We consider the situation in which a government (or an *honest* procuring official) must procure an indivisible object from two potential suppliers (bidders) by the use of first-price sealed-bid auction.<sup>7</sup> We assume that the government and the two bidders are risk-neutral in its monetary payment and in their monetary income, respectively, and that the government and the two bidders are expected cost minimizer and expected profit maximizer, respectively. We also assume that the two bidders produce the same quality products. The rules of the first-price sealed-bid auction are as follows. The two bidders submit their bids both simultaneously and noncooperatively, and the bidder with the lowest bid-price wins and must provide the object demanded at that price. If the two bidders quote the same lowest price, a coin is flipped to

determine the winner.

For simplicity, we employ an independent private-value model for the auction. Each bidder has private information about his production cost. The cost parameter,  $c_i$ , of bidder  $i$ ,  $i = 1, 2$ , is independent and identically distributed, drawn from a common-knowledge two-point probability distribution on  $\{0, \bar{c}\}$ ,  $\bar{c} > 0$ . Let  $p = \Pr(c_i = \bar{c}) > 0$ . Our analysis with two bidders builds on that of Maskin and Riley (1985), but they deal with a high-bid auction.

With the above description of the information structure, it would be helpful to think that a bidder (a low- or a high-cost type) has two opponents: a high-cost opponent with probability  $p$  and a low-cost opponent with  $1 - p$ . We employ a Nash equilibrium as the solution concept for the auction. We first show that the high-cost bidders (or the bidders who observe cost  $\bar{c}$ ) can only bid  $\bar{c}$  at equilibrium. To see why, suppose that they bid less than that. Then they might win the auction with positive probability, but they have to expect a negative expected profit. Now suppose that they bid more than  $\bar{c}$ , i.e., their bid-price is  $b > \bar{c}$ . Then a high-cost bidder's profit is  $(b - \bar{c})/2$ . If the other bidder reduces his bid-price slightly to  $b - \epsilon$ , his profit becomes  $(b - \epsilon - \bar{c})$  which is greater for small  $\epsilon$ . In other words, he can increase his chance of winning discontinuously if he bids infinitesimally smaller than his opponent's bid-price; thus, he can increase his expected profits. Foreseeing these results, high-cost bidders bid just  $\bar{c}$ . That is, in equilibrium, competition among bidders forces high-cost bidders to earn expected profit of zero. As we will use this type of logic repeatedly, we call it the "Bertrand-like competition" for high-cost bidders.

Now let us consider low-cost bidders (or bidders who observe zero cost). Since the distribution of cost observations is discrete, there is no equilibrium in pure strategies for low-cost bidders. To see this, assume that a low-cost bidder, say bidder 1, submits  $\bar{c} - \epsilon$ . Then the other

low-cost bidder's best response is to bid  $\bar{c} - 2\epsilon$ , in which case bidder 1 again would deviate to  $\bar{c} - 3\epsilon$ , and so on. However, if the decreasing bid-price hits some low price, a low-cost bidder's best response is to bid  $\bar{c} - \epsilon$  again, realizing it would be better to beat only a *possible* high-cost opponent. Then the cycle of price-cutting continues.

Let  $F(b)$  be the cumulative distribution function of a low-cost bidder's bid  $b$ . Let  $\underline{b}$  and  $\bar{b}$  be the lowest and the highest bids, respectively. Since the two bidders are symmetric *ex ante*, they should have a common support in equilibrium. Maskin and Riley prove that  $F(b)$  is continuous over a common support  $[\underline{b}, \bar{b}]$  and that there can be no subinterval of  $[\underline{b}, \bar{b}]$  over which  $F(b)$  is constant.<sup>8</sup> Since high-cost bidders always bid  $\bar{c}$ , in equilibrium, low-cost bidders can bid up to  $\bar{c} - \epsilon$ , where  $\epsilon$  is infinitesimally small but positive, without losing the auction to high-cost bidders. For mathematical convenience, we set  $\bar{b} = \bar{c}$ .<sup>9</sup> Since, in equilibrium, any bid  $b$  as part of a mixed strategy must generate the same expected payoff, the following equation must be true:

$$(1) \quad \Pr(\text{win}|b)(b - 0) = \Pr(\text{win}|\bar{b})(\bar{b} - 0).$$

A low-cost bidder's probability of winning,  $\Pr(\text{win}|b)$ , is  $p + (1 - p)(1 - F(b))$ , since he can beat a high-cost type opponent for sure (the first term  $p$ ) and beat a low-cost opponent when the opponent bids higher than  $b$ . And since  $\bar{b} = \bar{c}$ , equation (1) can be rewritten as

$$(2) \quad [p + (1 - p)(1 - F(b))]b = p\bar{c}.$$

Since (2) must hold for any  $b$  in an equilibrium support and since  $F(\underline{b}) = 0$  by definition, the infimum (or largest lower bound) of  $b$ ,  $\underline{b} = p\bar{c}$ . Thus, the equilibrium support of  $b$  is the interval  $[p\bar{c}, \bar{c}]$ .

Note that, in equilibrium, a high-cost bidder's expected profit is zero because of the Bertrand-like competition. A low-cost bidder's expected profit is obtained as follows:

*Observation 1.* Since the right-hand side of (2) is independent of  $b$ , a low-cost bidder's expected profit (which is the integration of the left-hand side of (2) over the equilibrium support) is  $p\bar{c}$ .

### 3. DESCRIPTION OF THE MODEL

Now suppose that a corrupt procuring official as a delegate for the government holds the auction. It should be noted that while we recognize that heterogeneity of bidders' products increases the room for the official's discretionary powers over the definition of the commodity and consequently, the room for side-payments, we assume for the sake of tractability that the products are homogeneous. We assume, however, that the complexity of the definition of the commodity allows the official, through rigging specifications, to exploit side-payments from the bidders.<sup>10</sup> We also assume that the probability distribution on cost parameters is known only to the official and the two bidders. That is, the government has no idea what the cost distribution is and consequently, the official can set a reserve price (or a ceiling price over which transaction cannot be made) higher than  $\bar{c}$ . But his ability to set the reserve price is limited by corruption environments (e.g., budget constraints and anti-corruption policies).

We may describe the procurement problem with two stages. In the first stage, in order to maximize his expected utility, the risk-neutral official acts as a Stackelberg leader both in setting a reserve price and informing bidders about his preference over bid-prices and side-payments. In the second stage, the two bidders then compete with each other on the two-dimensional space of bid-prices ( $b$ ) and side-payments ( $s$ ). Since each bidder does not know his rival's  $b$  and  $s$

submitted prior to his own bid, the sequential order of  $b$  and  $s$  does not matter strategically. The winner and expected payments are determined by the rules of the first-price sealed-bid auction, together with (3), (4) and (5).

Let superscripts  $f$  and  $k$  denote the games of bribes and kickbacks, respectively. The official ranks the bids  $(b, s)$  according to a utility function (or a scoring function):

$$(3) \quad U(b^j, s^j) = \gamma^j s^j - b^j, \quad \gamma^j \geq 0, b^j \geq 0, s^j \geq 0,$$

where  $\gamma^j$ ,  $j = f, k$ , represents the official's preference over  $(b^j, s^j)$ .<sup>11</sup> Higher values of  $\gamma^j$  indicate higher degrees of corruption. Thus,  $\gamma^j = 0$  means that the procuring official is not corrupt or cannot be corrupt. We assume that the value of  $\gamma^j$ , which is influenced by corruption environments, is exogeneously given. Note that  $\gamma^j$  is the marginal rate of substitution between  $b^j$  and  $s^j$  (or the slope of the corresponding indifference curve). The bidder who gives the official the highest utility  $U(b^j, s^j)$  wins the auction.

We now formulate two different games: In the first, the game of bribes (or of illegal entry-fees), both bidders bid and pay bribes  $s^f$  regardless of the event of winning or losing. Thus, bidder  $i$ 's cost,  $i = 1, 2$ , is defined by

$$(4) \quad C(s^f; c_i) = \begin{cases} \tau^f s^f + c_i & \text{if he wins} \\ \tau^f s^f & \text{if he loses,} \end{cases}$$

where  $\tau^f \geq 1$ . Higher values of  $\tau^f$  indicate higher degrees of penalties to the bidders upon being caught for bribery. We assume that  $\tau^f$  is given exogeneously.

In the second version, the game of kickbacks (or of illegal rebates), both bidders bid side-payments  $s^k$  competitively but only the winner pays the promised  $s^k$ . Thus, the cost of each bidder is defined by

$$(5) \quad C(s^k; c_i) = \tau^k s^k + c_i \quad \text{if he wins}$$

$$0 \quad \text{if he loses.}$$

*Remark 1.* As mentioned in the Introduction, the games of bribes and kickbacks are two polar cases of bribery. More general description of bribery would be a combination of the two polar cases:

$$(6) \quad C(s^f, s^k; c_i) = \tau(s^f + s^k) + c_i \quad \text{if he wins}$$

$$\tau s^f \quad \text{if he loses.}$$

Note that (6) reduces to (4) if  $s^k = 0$  and that (6) reduces to (5) if  $s^f = 0$ .

#### 4. PROPERTIES OF EQUILIBRIUM BIDS

This section provides a simple method for finding the set of Nash equilibrium bids (bid-prices and side-payments) and associated outcomes. We first analyze mixed-strategy equilibria in the game of bribes and then in the game of kickbacks. We then compare outcomes of the two games both from the bidders' and the corrupt procuring official's viewpoints.

##### 4.1. *The game of bribes*

By the rules of the bribes game both bidders have to pay bribes to the official whether or not they win, while the bidder who gives the official the highest utility of (3) wins and must provide the object demanded at his bid-price. Consider a bidder who pays a bribe  $s^f$  and submits a bid-price  $b^f$ . Uncertain about his rival's cost parameter, bidder  $i$  determines  $s^f$  and then  $b^f$  by maximizing his expected profit:

$$(7) \quad \text{Max } \pi_i = \text{Pr}[i \text{ wins} | (b_i^f, s_i^f)](b_i^f - c_i) - \tau s_i^f$$

for all  $c_i \in \{0, \bar{c}\}$ , and  $i = 1, 2$ . Since bidder  $i$  does not know his rival's decision of  $s^f$  prior to his own bid  $b_i^f$ , the decision of  $s^f$  and  $b^f$  are viewed as being taken simultaneously rather than sequentially.

We begin by analyzing high-cost bidders' bidding strategy. Suppose that the official sets a reserve price  $\bar{b}^f > \bar{c}$ . Note that within the constraints of corruption environments, the official tries to set  $\bar{b}^f$  as high as possible if it increases his expected utility. Given the official's utility function (3), equilibrium bids for high-cost bidders are then summarized as follows:

*Observation 2.* Suppose that  $\gamma^f > \tau^f$ . Then, high-cost bidders's equilibrium bidding strategy is to submit a bid-price  $\bar{b}^f$  (the reserve price) and a bribe  $ps^f$ , where  $s^f$  is a random variable from a uniform distribution over the interval  $[0, (\bar{b}^f - \bar{c})/\tau^f]$ . In equilibrium, the high-cost bidders receive expected profit of zero.

*Proof.* Refer to Figure 4.1. Recall that  $\gamma^f$  and  $\tau^f$  are the slope of the official's indifference curve and the slope of bidders' isoprofit curve, respectively. Consider a strategy  $(b^f, s^f)$  of point A in Figure 4.1. This strategy dominates any point on the curve AD since a bidder can increase his

profit without changing his probability of winning as he moves toward point A. For the same reason, any point below the curve  $\bar{b}^f C$  is dominated by the corresponding point on the curve  $\bar{b}^f C$ . This establishes that the high-cost bidders' two-dimensional problem can be mapped into the one dimensional-problem of bribes. That is, competition occurs only on the curve  $\bar{b}^f C$ . Since the reserve price is  $\bar{b}^f$ , it is obvious that a high-cost bidder would not pay a bribe greater than  $(\bar{b}^f - \bar{c})/\tau$ .

Clearly, there is no pure-strategy equilibrium of this game, as high-cost bidders engage in changing bribes. Let  $G^f(s^f)$  over the interval  $[0, (\bar{b}^f - \bar{c})/\tau]$  be the cumulative distribution function of a high-cost bidder's bribe  $s^f$ . Then, when his high-cost type opponent submits  $(\bar{b}^f, \hat{s}^f)$ , a high-cost bidder expected profit with submitting  $(\bar{b}^f, s^f)$  is

$$\begin{aligned}
 (8) \quad \Pr[s^f > \hat{s}^f](\bar{b}^f - \bar{c} - \tau s^f) - \Pr[s^f < \hat{s}^f]\tau s^f \\
 &= G^f(s^f)(\bar{b}^f - \bar{c} - \tau s^f) - [1 - G^f(s^f)]\tau s^f \\
 &= G^f(s^f)(\bar{b}^f - \bar{c}) - \tau s^f.
 \end{aligned}$$

Since, in equilibrium, any  $s^f \in [0, (\bar{b}^f - \bar{c})/\tau]$  as part of a mixed strategy must generate the same expected payoff, (8) is independent of  $s^f$ . Thus, a partial derivative of (8) with respect to  $s^f$  must be zero:

$$(9) \quad g^f(s^f)(\bar{b}^f - \bar{c}) - \tau = 0,$$

where  $g^f(s^f)$  is a density function associated with  $G^f(s^f)$ . From (9), one can easily recognize that  $g^f(s^f)$  is a uniform density function. Integrating (8) over the equilibrium support  $[0, (\bar{b}^f - \bar{c})/\tau]$  shows that high-cost bidders receive expected profit of zero.

Next consider a high-cost bidder's belief about his opponent's cost type. Since a low-cost bidder can beat a high-cost bidder for sure (as will be shown in the Proof of Observation 3), a high-cost bidder adjusts his strategy in (8) according to this belief:  $p[G^f(s^f)(\bar{b}^f - \bar{c}) - \tau^f s^f]$ . That is, since a high-cost bidder's probability of winning against either a low- or high-cost type of his opponent is  $pG^f(s^f)$ , his random bribe is also multiplied by  $p$ .  $\square$

Using the same logic in the beginning part of the above proof, one can easily check that if  $\gamma^f < \tau^f$ , submitting  $(\bar{c}, 0)$  is a pure-strategy equilibrium of this game, i.e., both bidders compete only on the dimension of bid-prices. Hereafter, we shall assume until indicated otherwise that  $\gamma^f > \tau^f$ .

Now let us convert to low-cost bidders (or bidders who observe zero cost). Let  $F^f(b^f)$  be the cumulative distribution function of a low-cost bidder's bid-price  $b^f$ . Then equilibrium bids for low-cost bidders are summarized as follows:

*Observation 3.* Suppose that  $\gamma^f > \tau^f$ . Then, in the game of bribes, low-cost bidders's equilibrium bidding strategy is to submit a bribe  $p(\bar{b}^f - \bar{c})/\tau^f$  (the maximum bribe payable by a high-cost bidder) and to randomize a bid-price  $b^f \in [p\bar{b}^f, \bar{b}^f]$  to satisfy  $[p + (1 - p)(1 - F^f(b^f))]b^f = p\bar{b}^f$ . In equilibrium, a low-cost bidder's expected profit is  $p\bar{c}$ .

*Proof.* Refer to Figure 4.1. Using a similar argument as in the proof of Observation 2 establishes that competition occurs only in the dimension of bid-prices. Thus, in equilibrium, low-cost bidders always pay the maximum bribe  $p(\bar{b}^f - \bar{c})/\tau^f$ , which must be recouped from their expected revenue.

Since any bid  $b^f$  as part of a mixed-equilibrium strategy must generate the same expected payoff, from (4) the following equation must be true:

$$(10) \quad \Pr[\text{win}|(b^f, (\bar{b}^f - \bar{c})/\tau^f)](b^f - 0) - p\tau^f(\bar{b}^f - \bar{c})/\tau^f \\ = \Pr[\text{win}|(\bar{b}^f, (\bar{b}^f - \bar{c})/\tau^f)](\bar{b}^f - 0) - p\tau^f(\bar{b}^f - \bar{c})/\tau^f.$$

Since a low-cost bidder's probability of winning,  $\Pr[\text{win}|(b^f, (\bar{b}^f - \bar{c})/\tau^f)]$ , is  $p + (1 - p)(1 - F^f(b^f))$ , and since  $\Pr[\text{win}|(\bar{b}^f, (\bar{b}^f - \bar{c})/\tau^f)] = p$ , equation (10) can be rewritten as

$$(11) \quad [p + (1 - p)(1 - F^f(b^f))]b^f = p\bar{b}^f.$$

Since (11) must hold for any  $b^f$  in an equilibrium support, the infimum of  $b^f$  is  $p\bar{b}^f$ . Thus, the equilibrium support of  $b^f$  is the interval  $[p\bar{b}^f, \bar{b}^f]$ .

Since the right-hand side of (11) is independent of  $b$ , a low-cost bidder's expected revenue (which is the integration of the left-hand side of (11) over the equilibrium support) is  $p\bar{b}^f$ . Since a low-cost bidder pays the maximum bribe  $p(\bar{b}^f - \bar{c})/\tau^f$ , his expected profit is  $p\bar{c}$ .  $\square$

#### 4.2. The game of kickbacks

We now consider a bidder's problem in the game of kickbacks. The two games have a similar structure, except that in the game of kickbacks only the winner pays the side-payment that he promised in bidding competition. Thus, we can apply the logic used in Section 4.1 to this section without much modification. Let  $b^k$  and  $s^k$  denote bid-prices and kickbacks, respectively. As in the game of bribes, we begin by maximizing bidder  $i$ 's expected profit.

$$(12) \quad \text{Max } \tilde{\pi}_i = \Pr[i \text{ wins} | (b_i^k, s_i^k)] (b_i^k - c_i - \tau^k s_i^k)$$

for all  $c_i \in \{0, \bar{c}\}$ , and  $i = 1, 2$ .

Let us first analyze high-cost bidders' bidding strategy. Suppose that the official sets a reserve  $\bar{b}^k > \bar{c}$ . Then, like in Observation 1, competition occurs on the curve  $\bar{b}^k C$ . By the Bertrand-like competition, however, high-cost bidders in this game have pure-strategy equilibrium bids  $(\bar{b}^k, (\bar{b}^k - \bar{c})/\tau^k)$ , which allow them expected profit of zero.

Let us now convert to low-cost bidders. Let  $F^k(b^k)$  be the cumulative distribution function of a low-cost bidder's bid-price  $b^k$ . Then equilibrium bids for low-cost bidders are summarized as follows:

*Observation 4.* Suppose that  $\gamma^k > \tau^k$ . Then, in the game of kickbacks, low-cost bidders' equilibrium bidding strategy is to submit the maximum kickback  $(\bar{b}^k - \bar{c})/\tau^k$  and to randomize  $b^k \in [p\bar{b}^k + (1-p)(\bar{b}^k - \bar{c}), \bar{b}^k]$  to satisfy  $[p + (1-p)(1 - F^k(b^k))](b^k - \tau^k s^k) = p\bar{c}$ . In equilibrium, a low-cost bidder's expected profit is  $p\bar{c}$ .

*Proof.* Using a similar argument as in the proof of Observation 2, from (5) we have (13) as an equivalent of (10):

$$(13) \quad \Pr[\text{win} | (\bar{b}^k, (\bar{b}^k - \bar{c})/\tau^k)] [b^k - 0 - \tau^k (\bar{b}^k - \bar{c})/\tau^k] \\ = \Pr[\text{win} | (\bar{b}^k, (\bar{b}^k - \bar{c})/\tau^k)] [b^k - 0 - \tau^k (\bar{b}^k - \bar{c})/\tau^k].$$

Since a low-cost bidder's probability of winning,  $\Pr[\text{win} | (b^k, (\bar{b}^k - \bar{c})/\tau^k)]$ , is  $p + (1-p)(1 -$

$F^k(b^k)$ , equation (13) can be rewritten as

$$(14) \quad [p + (1 - p)(1 - F^k(b^k))](b^k - \bar{b}^k + \bar{c}) = p\bar{c}.$$

Since, in equilibrium, (14) must hold for any  $b^k$  in an equilibrium support, the infimum of  $b^k$  is  $\bar{b}^k + \bar{c}(p - 1)$  or  $p\bar{b}^k + (1 - p)(\bar{b}^k - \bar{c})$ .

Since the right-hand side of (14) is independent of  $b^k$ , a low-cost bidder's expected profit is  $p\bar{c}$ .  $\square$

#### 4.3. A comparison of the games

A comparison of Observations 1 and 4 shows that the standard auction and the game of kickbacks bring the same outcome (both in terms of bidders' expected profits and the selection of the lowest-cost bidder as the winner), like the isomorphism in Beck and Maher. A comparison of Observation 1 and 3 also establishes an isomorphism between the standard auction and the game of bribes. Proposition 1 then results from these two isomorphisms.

*Proposition 1. Suppose that  $\gamma^j > \tau^j$ ,  $j = f, k$ , and that the corrupt procuring official sets a reserve price  $\bar{b}^j > \bar{c}$ . Then risk-neutral bidders are indifferent to which game--that of bribes or kickbacks--is chosen.*

Remember that if  $\gamma^j < \tau^j$ , the two games are reduced to a one-dimensional game of bidding competition, regardless of the size of  $\bar{b}^j \geq \bar{c}$ . This means that the bidders' expected profits are independent of both the game structure and the parameter values. One implication of

Proposition 1 is that if the official's preference over bid-prices and side-payments is known to the bidders, the two-dimensional auction can be mapped into the standard one-dimensional auction (from the bidders' perspective).

Next consider the corrupt procuring official's choice between the two bribery games. A comparison of Observations 2,3 and 4 leads to the following proposition.

*Proposition 2. Suppose that  $\gamma^j > \tau^j$ ,  $j = f, k$ , and that the corrupt procuring official sets a reserve price  $\bar{b}^j > \bar{c}$ . Suppose that the two games have the same parameter values, i.e.,  $j = f = k$ . Then the risk-neutral, corrupt procuring official is indifferent to the choice between the two games.*

*Proof.* We begin with high-cost bidders in the game of bribes. Recall that the probability that a bidder observes high cost is  $p$  and that the official receives bribes from both bidders. Then, from (3) the official's expected utility from high-cost bidders is  $p[2\gamma E(s^f) - p\bar{b}^f]$  or  $p^2[\gamma^f(\bar{b}^f - \bar{c})/\tau^f - \bar{b}^f]$ .

We now consider high-cost bidders in the game of kickbacks. From symmetry of bidders, in equilibrium a high-cost bidder's expected probability of winning (averaged over all bids) is  $p/2$ . Recall that high-cost bidders have a pure-strategy equilibrium  $(\bar{b}^k, (\bar{b}^k - \bar{c})/\tau^k)$ . Then, from (3) the official's expected utility from high-cost bidders is  $p^2[\gamma^k(\bar{b}^k - \bar{c})/\tau^k - \bar{b}^k]$ . Thus, the high-cost bidders in both games yield the same utility to the official if  $f = k$ .

Now let us convert to low-cost bidders. Proposition 1 implies that the low-cost bidders in both games obtain the same expected profits. Note that the low-cost bidders' only expected cost is side-payments to the official. This means that if the low-cost bidders in both games have the

same expected revenue and if the two games have the same parameter values, then the official obtains the same utility from the two games. From (11), a low-cost bidder's expected revenue in the bribes game is  $p\bar{b}^f$ . The same is true in the kickbacks game, from (13) and (14).  $\square$

Propositions 1 and 2 imply that the official's expected utility depends only on the magnitudes of  $\gamma^j$  and  $\bar{b}^j$  as long as  $\gamma^j > \tau^j$ ,  $j = f, k$ . Though there is no *ex ante* difference in incentives of "whistleblowing" between the two games, losing bidders have stronger *ex post* incentives to expose bribery with the game of bribes than with the game of kickbacks, under a *ceteris paribus* condition. This means that the official must prefer the game of kickbacks to the game of bribes since it is reasonable to believe that the official's ability to raise a reserve price  $\bar{b}^j$  and his preference parameter  $\gamma^j$  are a decreasing function of the probability and severity of punishment for bribery, i.e.,  $\bar{b}^k \geq \bar{b}^f$  and  $\gamma^k \geq \gamma^f$ . This result runs counter to the usual instance of government procurement in LDCs, including the Thailand procurement scandal described in the Introduction.

## 5. CONCLUDING REMARKS

The auction mechanism itself may not deter bribery in government procurement as long as the corrupt procuring official uses discretion. But it is also equally true that no other alternative is superior to the auction mechanism in reducing corruption. This paper has investigated bidding behaviors under corruption environments. But the result of the corrupt procuring official's indifference between the two bribery games (Proposition 2) is inconsistent with most case studies of government corruption in LDCs. Thus, we may need a different model to address this issue, which will be a necessary extension to this analysis.

Squeezing bribes out of competing bidders is an open secret in LDCs. This means that the role of the LDC procuring official is very close to the role of the auctioneer in “optimal auctions.” If we relax the independence assumption of bidders’ cost information, the model of Crémer and McLean (1985) would be useful to explain how the official can extract the full surplus from the bidders. We may be able to show that it is easier to implement Crémer and McLean with the game of bribes than with the game of kickbacks. If so, the official has the reason to prefer the game of bribes to that of kickbacks.

## FOOTNOTES

1. Bribe-takers usually consist of more than one individual, such as administrative hierarchies. For simplicity, we define the corrupt procuring official to represent collective bodies of corruptive hierarchies.
2. The official's discretionary powers may be limited by the introduction of a scoring function which maps the multi-dimensional components of the bids into a single number. However, the adoption of a particular scoring function by which bidders are ranked is presumably influenced by side-payments prior to bidding competition. Even with two dimensions of quality and price, a quality test to determine potential suppliers can be easily manipulated to exclude unwanted suppliers or to include wanted ones.
3. Though we assume here that an exchange of favors through side-payments is enforceable, a legal enforcement seems to be impossible. There may be a few alternative enforcement mechanisms such as reputation, "word of honor," and fear of retaliation [Tirole (1990)]. Since the model we develop analyzes strategic behaviors in a single auction, however, an enforcement mechanism is implicitly assumed. In this chapter, we leave this enforcement problem unexplained as our interest is not in the enforcement of side-contracts.
4. Rose-Ackerman (1978, p. 144) quotes a story from Andreski (1968, p. 355): "In Africa competition for jobs sometimes takes the form of a kind of auction in which the prize goes to the highest bidder while the rest forfeit their bids. Private African firms sometimes openly charge a fee for considering an application, even if it is turned down." It is also interesting to note that

the game of bribes (prevalent in today's LDCs) did exist in the past in the U.S. According to Smith (1958, p. 316) [quoted in Rose-Ackerman (1978, p. 21)], "In New York State in the 1860s, the assembly was asked to approve legislation which would legalize stock issues by the Erie Railroad designed to frustrate a takeover bid by Cornelius Vanderbilt. One important legislator accepted \$75,000 from Vanderbilt and \$100,000 from the Erie, voted for the Erie, but kept Vanderbilt's money."

5. Though Lien (1990) considers a model of bribes, he basically follows Beck and Maher in using a predetermined contract price.

6. Rose-Ackerman (1976, p. 113) argues that the corrupt procuring official's net gain from receiving a bribe is the value of the bribe minus the costs associated with being caught and punished.

7. Though there are other auction institutions such as open and Vickrey auctions, Hansen (1988) notes that the first-price sealed-bid auction is the most popular institution in procurement auctions.

8. The reasoning of no atom over the interval is roughly as follows. Suppose to the contrary that a low-cost-bidder  $i$  submits  $b_i \in [b, \bar{b}]$  with positive probability. Then there must be some interval  $[b_i, b_i + \epsilon]$  over which his low-cost opponent  $j$  will not bid since the probability of  $j$ 's winning increases discontinuously if a bid in this interval is replaced with one that is infinitesimally smaller than  $b_i$ . Then bidder  $i$  can increase  $b_i$  to  $b_i + \epsilon$  without reducing his probability of winning, which contradicts the assertion that the original  $b_i$  with positive

probability was an equilibrium.

9. Let us call this supremum (or smallest upper bound)  $\bar{b} \leq \bar{c}$ . Suppose that  $\bar{b} < \bar{c}$ . Then any  $b$  between  $\bar{b}$  and  $\bar{c}$  has the same chance of winning as  $\bar{b}$ , and so is preferable. Thus,  $\bar{b}$  must be the closest number to  $\bar{c}$ . Because of the continuum property of the real numbers, it is troublesome to fix the closest number to  $\bar{b}$ . To avoid this problem, it will be convenient to suppose that a low-cost bidder wins with  $\bar{b}$  if both a low-cost and a high-cost bidder submit  $\bar{b}$ .

10. See Amick for examples of rigging specifications.

11. More general models of non-linear utility involve much more complicated manipulations, though they raise few additional conceptual problems.

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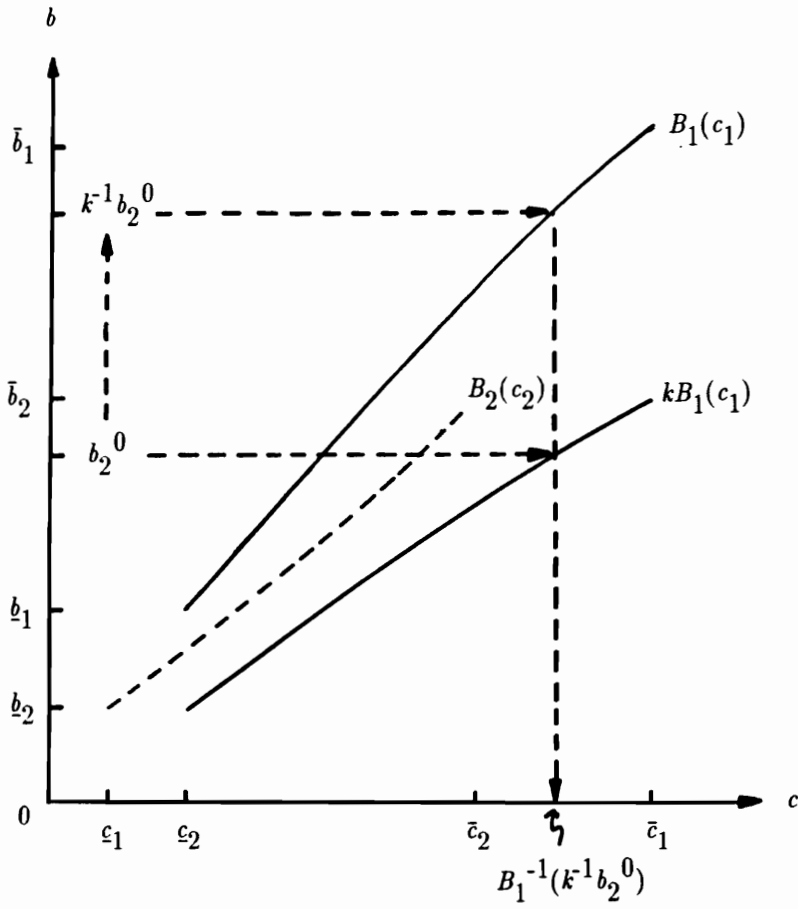


Figure 2.1

Each firm's bidding schedule based on its cost observation, given  $k$

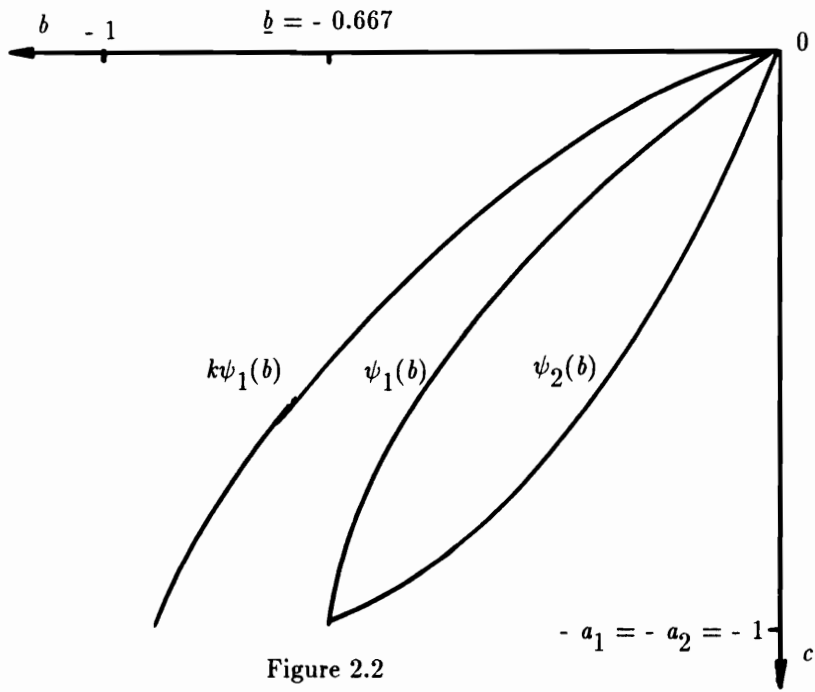


Figure 2.2

Equilibrium inverse bid functions when  $a_1 = a_2 = 1, k = 0.5$

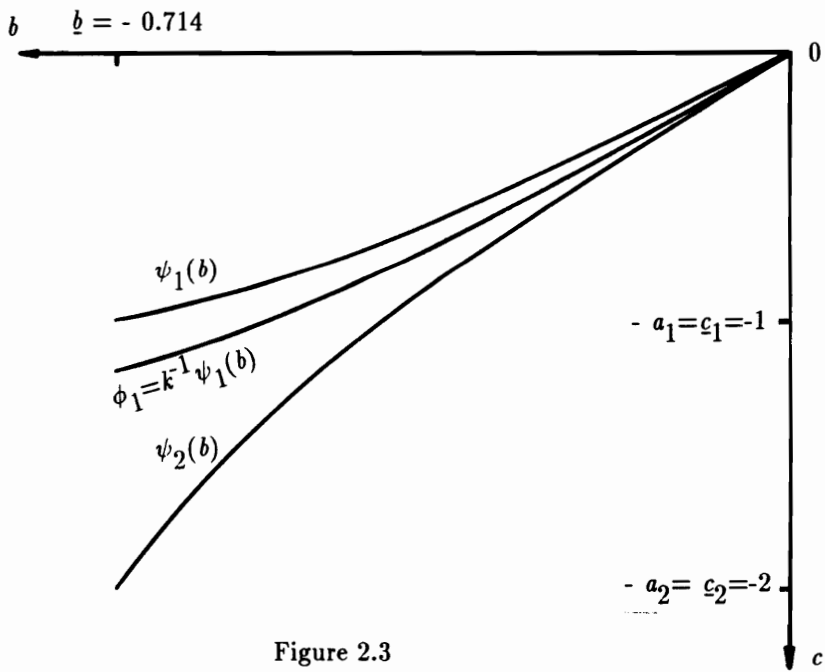


Figure 2.3

Equilibrium inverse bid functions when  $a_1 = 1, a_2 = 0.5, k = 0.9$

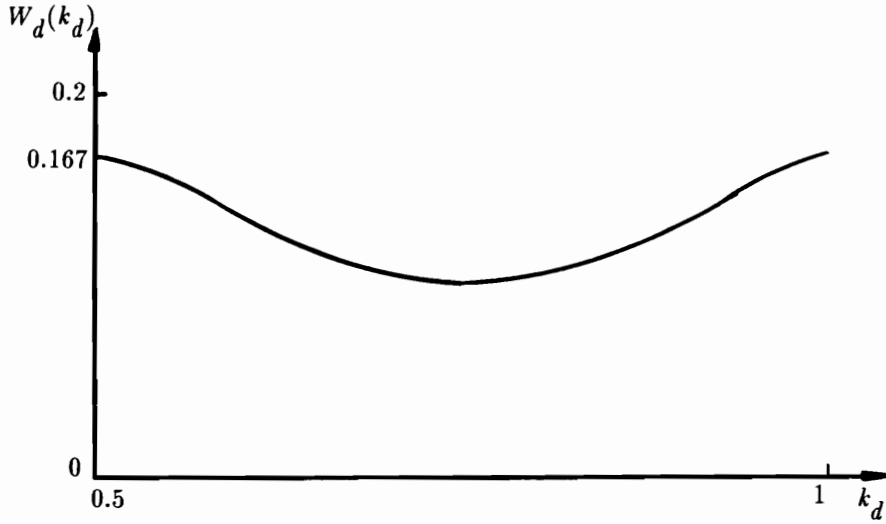


Figure 2.4

Domestic country's welfare  $W_d(k_d)$  in the two-country, two-firm world

Table 2.1

The expected procurement costs  $H$  from the rebate auction

$k$	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
$H$	0.541	0.642	0.691	0.704	0.689	0.646	0.668	0.699	0.747	0.835

Table 2.2

Domestic and foreign welfare in the two-country, two-firm world

		Foreign country $f$	
		monopoly price-preference ( $k_f^*$ )	bilateral price-preference ( $k_f = 1$ )
Domestic country $d$	monopoly $k_d^*$	$W_d(k_d^*, k_f^*)$ $W_f(k_d^*, k_f^*)$	$W_d(k_d^*, k_f=1)$ $W_f(k_d^*, k_f=1)$
	bilateral $k_d = 1$	$W_d(k_d=1, k_f^*)$ $W_f(k_d=1, k_f^*)$	$W_d(k_d=k_f=1)$ $W_f(k_d=k_f=1)$



Table 3.1

The structure of information and costs

quality & probability	stage 1 (asymmetric information)		stage 2 (uncertainty)	
	low cost ( $0$ )	high cost ( $\bar{c}$ )	good state	bad state
$H$	$0$	$\bar{c}$	$c_H$	$\gamma c_H$
$L$	$0$	$\bar{c}$	$c_L$	$\gamma c_L$
probability	$1 - p$	$p$	$1 - p_B$	$p_B$

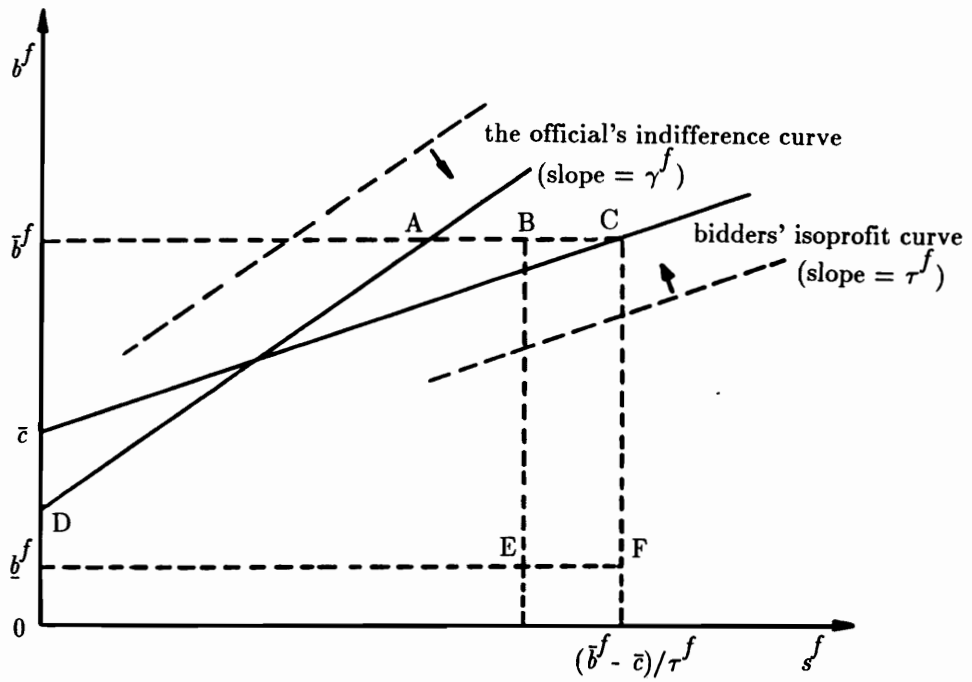


Figure 4.1

Bidders' isoprofit curve and the official's indifference curve

## Vita

In-Gyu Kim was born in Kyungbuk, Korea, on February 5, 1956. He received his B.A. in Economics and Statistics in 1981, and M.A. in Economics in 1984, from Yonsei University, Seoul, Korea. He entered the VPI & SU in 1989 and received his Ph.D. in Economics in 1992.

A handwritten signature in black ink that reads "In-Gyu Kim". The signature is written in a cursive style with a large initial 'I' and 'K'.