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The Impact of Computational Labs on Conceptual and Contextual Understanding in a Business Calculus Course

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Abstract

This study investigates the impact of computational labs on students' conceptual understanding of calculus in a one-semester Business Calculus course. The computational labs integrated Jupyter Notebook as the modeling tool. Using the Calculus Concept Inventory, quantitative analysis was performed to measure differences in conceptual knowledge between a control and experimental group based on whether the student engaged in computational labs compared to traditional classwork. Qualitative analysis was conducted to understand student perspectives about the value of participating in computational labs during the course. The qualitative data involved student reflections at the end of each lab experience. Although the quantitative analysis did not produce statistically significant results, the qualitative analysis revealed the students perceived the computational labs as beneficial regarding their understanding of the content and practical applications of the material. Notably, the students reported the labs offered a unique way to solve problems, allowed for connections to real-life mathematical situations, and helped to visualize calculus concepts. This paper describes the research project and offers practical applications of computational labs in Business Calculus courses, as well as suggestions for future research.

Keywords Business calculus · Computational labs · Conceptual understanding · Calculus concept inventory · Jupyter notebook

Introduction

Calculus has long served as a stumbling block and a "critical filter" that is "blocking access to professional careers for the vast majority of those who enroll" (The Mathematical Association of America, 1988, p. xi). This dilemma applies to both underrepresented and overrepresented students (Alexander et al., 2009) and appears

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to remain as a filter today, even for some strong students (Bressoud et al., 2013) in Science, Technology, Engineering, and Mathematics (STEM) and business majors. Although there have been major changes in who is expected to complete calculus among college students, what is taught in a calculus course and how it is taught has been relatively stagnant, leading to an increasingly poor fit for the over 300,000 students in the United States that enroll in a calculus course each semester (Bressoud, 2015; Kaput, 1997). Since calculus is a gateway to other courses in many students' plans of study, having a foundational understanding of the content is critical. Due to a weak understanding of prior knowledge, resulting from a lack of quality accumulation of high school mathematics content (Douglas, 1985), many first-semester college students enter their first calculus courses with less basic conceptual knowledge than expected by the instructor (Epstein, 2013). This leads to a failure to gain the understanding needed to scaffold their knowledge through subsequent courses, especially when considering "problem-solving skills, mathematical modeling, and communication of ideas" (Axtell et al., 2016, pp. 33–34). To improve the relevance of the calculus content within the curricula, there has been sustained advocacy among educators, policymakers, and instructors for improved pedagogy as a means of transforming the calculus experience from a filter for numerous disciplines to an avenue of expanded professional opportunities through the emphasis of conceptual understanding (Axtell et al., 2016). Conceptual understanding is referred to as the "implicit or explicit understanding of the principles that govern a domain and of the interrelations between units of knowledge in a domain" (Rittle-Johnson et al., 2001, p. 346). This research study uses conceptual understanding in the context of calculus content.

Technology integration can support students' conceptual understanding of calculus content (Zahner et al., 2012). Given the importance of technology in instruction, especially its potential to increase students' engagement, improve student-student and student-teacher collaboration, and accommodate individual differences (Cox, 2024), course activities that intentionally integrate technology to solve calculus-based mathematical problems, referred to in this study as computational labs, may offer rich opportunities for turning the tide in students' favor. This comes in tandem with Lockwood et al. (2019) suggestion that "With mathematics seeing an increasing focus on computation, mathematics education should not be far behind in its pursuit to understand the teaching and learning of computing within mathematics" (p. 1). Heeding this call, researchers have sought to enhance students' positive experiences with calculus through technology-based computational approaches. For example, Krause et al.'s (2021) adoption of computational labs offered students "a rich learning experience' amidst vivid institutional constraints" (p. 434). Vestal et al. (2015) found that computational labs might offer an alternative pedagogical approach to existing calculus instruction due to their potential for improving student outcomes. Although these studies offer preliminary support for the instructional benefits of computational labs in calculus, their inferences were drawn from a restricted population of students. To address this, we sought to measure the impact of incorporating computational labs using Jupyter Notebook on conceptual understanding in a Business Calculus course. Our view of conceptual understanding draws on Hiebert and Lefevre's (2013) notion of conceptual knowledge as a "connected web of knowledge" (p. 3) characterized by a grasp of procedures as well as the rationales behind them. Although this innovation is intended for calculus, findings from empirical research on the potential for students' successful experience with calculus may inspire positive attitudinal change in their mathematics (Duran et al., 2024) and an attendant increase in retention across STEM fields (Johnson et al., 2024).

Literature Review

Students worldwide need help with conceptual understanding of calculus, often finishing the course with little to no conceptual knowledge intended by the curriculum (Epstein, 2013). A nationwide study in the United States found that in any introductory calculus class, about one-fourth of students either withdraw or earn a grade of D or F[1] (Bressoud, 2015). This same study found that about two-thirds of calculus students in the United States self-reported difficulty in understanding derivatives and integrals. Only 40% of these students felt confident using derivatives and integrals in problem-solving scenarios (Bressoud, 2015), raising concerns about the extent of learning occurring in these courses (Breidenbach et al., 1992; Tallman et al., 2016; Thompson, 1994). Consequently, students remain marginally prepared for other mathematics courses due to the lack of understanding of the ideas foundational to calculus (Carlson, 1995; Selden et al., 2000; Thompson, 1994), resulting in a further lack of interest in mathematics in general (Bressoud et al., 2015; Seymour, 2006).

Students' struggles with such core aspects of cognitive (e.g., conceptual understanding) and affective (e.g., interest) functioning suggest the need for a radical shift in the form and substance of calculus instruction to serve a diverse student population better. Incorporated in this view is the imperative for calculus instruction to tap into the broader call for student-centered pedagogical practices. Research suggests that embedding calculus instruction within formative practices enhances conceptual understanding and students' mathematical skill development (Fuchs & Sahmbi, 2024). Lodged within this is an argument that such a course that serves students from several fields should employ diverse instructional strategies, suggestive of the potential for calculus to serve both academic and equity needs. To this end, the teaching and learning of calculus should incorporate active strategies that allow students to monitor, assess, self-regulate, and interact with one another, their instructor, and mathematics content. (Bowers et al., 2019; Fuchs & Sahmbi, 2024).

While calculus has a reputation for being responsible for the attrition of students in STEM fields due to what many students describe as instructors' uninspiring delivery (President's Council of Advisors on Science and Technology, 2012), it can also serve as a screening course for business and social science students (Brito & Goldberg, 1988). Given the complementary roles between what are considered STEM and non-STEM fields, improved methods of learning are needed, especially given the increasing demand for business professionals who are capable of integrating STEM into business operations and management (Ledley, 2012; Ledley & Holt, 2014; Ledley & Oches, 2013; McCann, 2006). Leaders in mathematics education have long called upon mathematics departments to collaborate with other disciplines, such as busi-

[1] U.S. grading system: A: 90–100; B: 80–89; C: 70–79; D: 60–69; F < 60.

ness, to revise course content based on mutual interests (Egerer & Cannon, 1988). As Bouldin et al. (2015) argue, "business is inherently interdisciplinary…the practices, policies, and norms that govern business are grounded in social science, and the goods and services that businesses produce are themselves the fruits of science, engineering, arts, and humanities" (p. 17). Collaboration among business and mathematics experts would produce such an interdisciplinary curriculum that it would depart radically from the existing ones that have been devoid of context and relevance to business majors (Bouldin et al., 2015). Leaders in mathematics education call for technology in business mathematics to allow students to use realistic data, use technology as an analytic tool, and encourage alternative approaches to solve problems (Lamoureux et al., 2000). With such innovation, students are likely to complete Business Calculus with a feeling far removed from merely fulfilling a requirement to take a mathematics course but would allow them to apply the content in a variety of contexts and increase the student's appreciation of the relevance of mathematics (Lamoureux et al., 2000).

The literature is replete with evidence-based research on improving Business Calculus courses. Some recommendations for using technology to foster interest include enhancing the efficiency of the learning process, providing rich opportunities through introductory data analysis, and creating models and applications such as optimization and simulations (Lamoureux et al., 2000). One recommendation by the Mathematical Association of America's (MAA) Curriculum Renewal Across the First Two Years (CRAFTY) Project suggests the incorporation of lab activities to support student learning (Lamoureux et al., 2000). Labs in calculus courses do not refer to a physical space, such as a science lab, but to activities that take various forms. This may include integrating various technologies into the curriculum and can cover a wide range of content (Leinbach, 1991). Labs incorporating real data provide opportunities to connect mathematical content to other content areas, courses, and disciplines (Basson et al., 2006; Kowalczyk & Hausknecht, 1994).

Research from the 1970s and 1980s recommended introducing students to calculus concepts using computers. This was particularly useful for non-STEM majors (Gordon, 1979; Hoffman, 1989; Ralston, 1984). Gordon (1979) claims that students gain an appreciation of the relationship between math and the computer, which "provides an ideal context in which to develop several simple, yet useful, numerical algorithms for approximating functions and for actually finding where all those 'given' functions come from" (p. 23). Using computational tools through labs can relieve the students of the need to do some of the work through symbolic manipulation with paper and pencil, thus allowing for introducing more advanced topics and scenarios than many students could do by hand (Hoffman, 1989). Teachers often reduce problems to simplified versions so that they are at a level where students can complete them with the tools at their disposal, but using computational tools could allow for more advanced examples (Hoffman, 1989). Many university mathematics professors reported that technology significantly influenced mathematics teaching and recommended incorporating software such as MATLAB, Maple, and Python (Quinlan, 2016). It is often difficult for students to engage in mathematical modeling or translating a real-world situation into a mathematical representation because they have learned math decontextualized and have a hard time switching between mathematics theory and real-world application because of the lack of practice they get with this in school (Couch & Haines, 2004). These essential skills cannot be ignored as “when mathematics is applied in the modern world for a practical purpose, we almost always require a computer to deal with a realistic level of complexity or to manage the data involved” (Cline et al., 2020, p. 735).

Research indicates the positive impacts of using computational tools in the mathematics classroom. Common themes across these studies have, on one part, been the beneficial effects of technology on students’ learning as manifested in their acquisition of requisite skills, conceptual understanding, and peer collaboration and, on the other, the enormous support it offers teachers in the design of instructional environments and meeting of set goals (Bray & Tangney, 2017; Drijvers & Sinclair, 2023). They also enable students to understand “real-world quantitative situations” (Heid et al., 2002, p. 588). Research from the 1990s on using computers and various programming languages in calculus courses has yet to discover a significant improvement in student learning. However, many found positive results based on students’ reflections and did not see adverse effects on performance (e.g., Höft & James, 1990; Schwingendorf & Dubinsky, 1990). Fenton and Dubinsky (1996) developed ISETL, a programming language, to help students more effectively learn mathematics, beginning with the argument that “communicating with a computer requires a level of precision that will help illuminate important mathematical ideas for students” (Lockwood et al., 2019, p. 17). Cetin and Dubinsky (2017) found that students learned concepts such as functions more effectively using ISETL. Visualization is at the root of many powerful mathematical ideas, such as the development of the concepts of functions, limits, continuity, and the fundamental theorem of calculus, among others (Perkins, 2012; Tall, 1991). By writing and running their code, they had to think about what the computer was doing with the code, which could help internalize concepts. They explained that students had to define the function correctly in the program, reflect when entering a particular code, and think about how the computer got the result (Cetin & Dubinsky, 2017). Benakli et al. (2017) also report similar results when using hands-on computer programming in *R* to solve problems of calculus, probability, statistics, and data analysis. Using computational tools can improve conceptual understanding of difficult concepts, including complex and abstract problems, and can improve problem-solving skills (Benakli et al., 2017). Rich et al. (2014), in studying the impacts that learning computer programming can have on the way students approach mathematics, claim that learning to code “provided participants with context, application, structure, and motivation for mathematics” (p. 103) that was long-lasting. In calculus courses where students explored calculus topics through programming in BASIC, Crowell and Prosser (1991) did not find overall improvement in understanding as measured by final exam scores but did see an increase in overall student engagement. Further developing tools that utilize computing power at most students’ fingertips could provide valuable learning opportunities.

Research Question

This research study builds on previous research concepts, suggesting that using computational labs to integrate computational tools in a calculus course increases students' conceptual understanding of course content. This study was designed to determine the impact of computational labs on students' conceptual understanding of calculus in a one-semester undergraduate Business Calculus course. The research question for this study is: What effect do computational labs have on students' conceptual understanding of calculus content and on students' perceptions regarding the learning of concepts in a Business Calculus course?

Methodology

This project utilized a quasi-experimental, multi-method design, using both quantitative and qualitative data collected and analysis. The Calculus Concept Inventory (CCI) was used in a pre/post-test format with a control and experimental group to collect quantitative data on students' conceptual calculus knowledge, and the data was examined to determine the impact of the labs on this variable. The control group participated in traditional classroom activities and the experimental group participated in the computational labs. A description and analysis of the CCI is provided by Epstein (2013). The qualitative data was collected from students' reflections at the end of each lab for the experimental group.

Description of Computational Labs

The labs in this project refer to the activities in which Jupyter Notebook was incorporated into the course assignments, which involved coding in Python. Jupyter Notebook is considered an easy-to-use program, even for teachers with little to no background in coding and computing, and students can quickly discover how technology can help them solve and communicate solutions to challenging and real-world problems (Koehler & Kim, 2018). "The Jupyter Notebook is an open-source web application that allows you to create and share documents that contain live code, equations, visualizations, and narrative text. Uses include data cleaning and transformation, numerical simulation, statistical modeling, data visualization, machine learning, and much more" (Project Jupyter, 2024). Jupyter Notebook supports multiple programming languages, can be easily shared, can produce interactive output with images, videos, LaTex, and more, and can support big data tools (Project Jupyter, 2024). The Notebook used in this project ran Python, which was chosen since Python is a popular, commonly used programming language that focuses on ease of use and readability (Meurer et al., 2017). Technology is playing a growing role in the financial and business industries, and many companies, such as big banks, are requiring Python courses for analysts and other employees (Hilpisch, 2016; Rayome, 2019). Exposing business majors to this computing language could introduce them to a valuable skill they may be expected to know in the industry.

Students in the experimental group completed six computational labs throughout the semester using Jupyter Notebook with Python. The labs in this project are used to introduce students to calculus topics and are situated in the context of business or financial problems. One defining element of the computational labs in this project was to begin instruction of calculus topics within the lab and then extend the topic in the lecture. This was done for several reasons. One reason was that students would not view the labs as an extension activity or something else to do, but rather an integrated learning tool. This also followed the recommendation of the MAA CRAFTY projects (Lamoureux et al., 2000). Another important reason was introducing students to calculus concepts beginning with finite elements and discrete data points. This was done in the introduction of derivatives, applications of derivatives, summation and integration, and area between curves labs. The goal was to introduce students to these concepts concretely and then extend them to more abstraction in the lecture. During the first part of the labs, students worked together as a class with the instructor. Students used their personal computers as the instructor projected Jupyter Notebook on the board and worked through it with them. Then, each lab had a problem set for the students to complete on their own. Students were typically given one and a half weeks to complete the problem set in the lab. In addition to other course activities, students were given some time at each class meeting to work on the lab problems while the instructor circulated. Some labs included applying real-life data to demonstrate the practical application of analyzing information using computational modeling. At the end of each lab, students wrote a paragraph reflecting on their learning, which became part of the qualitative data. Table 1 outlines the topics and concepts covered in each lab and any practical applications of problems.

Table 1 Course outline for computational labs

Lab	Topics	Concepts	Applications
1	• Introduction to Jupyter Notebook	• Using code cells • Lists and arrays • Simple loops • Define and plot functions	• Plotting functions for a specific domain and format
2	• Functions	• Linear, quadratic, and exponential • Sequences • Fitting curves to data	• Using functions to model situations
3	• Introduction to derivatives	• Limits, slopes, and finite differences • Slope of tangent line	• Develop rules for computing derivatives
4	• Applications of derivatives	• Shapes of curves • Finding relative and absolute extrema • Optimization • Python code to graph functions and derivatives	• Analyze sales of smokeless tobacco products • Apply law of diminishing returns • Optimization for maximizing revenue or profit
5	• Integration	• Area under a curve • Definite integral	• Numerical integration to determine net change and average value
6	• Area between curves	• Graphing and analyzing curves	• Income inequality • Consumer and producer surplus • Future value of income streams

Course Delivery

The research project occurred over a fifteen-week semester in a Business Calculus course comprising four sections taught by the same instructor. All four sections of the course met twice a week for 90 min each class period. Students self-selected the sections with no prior knowledge of how the course would be taught. The only course variation the students saw when registering for classes was the different times the course was offered. None of the sections were held at the extremes of the day. To randomly select which course section received the intervention, the course sections were assigned numbers one through four and then selected with a random number generator. Two of the course sections were taught with traditional, or lecture-based, course pedagogy and considered part of the control group. Two sections of the course incorporated computational labs and were considered part of the experimental group. Course content between the control and experimental groups largely overlapped throughout the semester. Some content was modified to allow time for the experimental group to work on labs, but the order in which topics were covered mostly remained the same to maintain consistency between the sections. The time spent on each topic was around the same number of class meetings, but the experimental group spent part of the class meetings on the labs, while the control spent part of their class time combining additional lectures and collaborative work time on traditional problems.

In addition to slight content differences between the course sections, there were some pedagogical differences. The control group was taught predominantly by lecture, which is still the dominant style of teaching Calculus I nationwide (Larsen et al., 2015). Outside of lecture, students in the control group spent part of the class meetings working collaboratively on problems. To complete course assignments, the control group used graphing calculators. They also completed four teacher-generated projects, three of which included solving a calculus problem related to business and writing a letter about their findings, while the fourth involved researching and writing a paper. Project one covered an analysis of cost, revenue, and profit functions. Project two was a marginal analysis of cost, revenue, and profit functions. Project three was an optimization of how to construct a pipeline. Project four was a research paper on how mathematics and calculus could potentially be used in their intended future careers.

Participants

Participants were undergraduate students at a medium-sized private university in the United States, with an undergraduate enrollment of around 4,500 students during the spring of 2019. At this university, the liberal arts general education curriculum currently includes a mathematics requirement, which approximately 39% of students meet by taking Calculus I or Business Calculus during their first year. Although students in all four sections of the course range from freshmen to seniors, the majority of students are freshmen.

Data were collected from four sections of Business Calculus taught by the same instructor during one semester. Thirty students were enrolled in each section at the

Table 2 Student demographic characteristics

	Gender		Major			Previous calculus	
	M	F	Business	Undecided	Other	Yes	No
Control	25	14	35	3	1	17	22
Experimental	21	19	36	4	0	12	28
Total	46	33	71	7	1	29	50

Table 3 Student Major and Concentration

College Major and Concentration	Number of Respondents		
	Control ($n=39$)	Experimental ($n=40$)	Total ($n=79$)
Business	35	36	71
Business Admin.	13	12	26
Marketing	9	6	15
Accounting	2	5	7
Entrepreneurship	5	4	9
Finance	2	4	6
International Bus.	3	4	7
Sales	1	1	2
Non-Business Major	1	0	1
Undecided	3	4	7

beginning of the semester. Therefore, 120 students began the semester with two sections of the course in each of the experimental and control groups. Of the 120 students enrolled at the beginning of the semester, 113 elected to participate in the initial completion of the CCI. By the end of the semester, enrollment across the four sections was down to 106 students, of whom 94 participated in the end-of-semester CCI. Despite 94 responses on the post-test, only 79 responses gave consent and were successfully matched to their corresponding pre-test code, as shown in Table 2.

In addition to demographic information, students were asked to provide their intended major and the concentration of their major, if applicable. As shown in Table 3 below, 39 and 40 students were in the control and experimental groups, respectively, with the majority spread across seven business majors.

Data Collection

Students in both groups were given consent forms and took the pre-CCI on the first day of class. The pre and post-tests were completed online via Qualtrics, a secure survey collection software system approved by the institution's Institutional Review Board (IRB) protocol. After the pre-tests were completed, the semester progressed with the control group receiving lecture and the experimental group receiving labs to introduce calculus topics that were extended in lecture. The two groups covered material in the same order and followed the same timeline throughout the semester. Each group took identical tests and final exams at the same time. The two groups also had similar grading scales, with the only difference being that the control group completed projects while the experimental groups completed computational labs. However, both types of activities carried the same weight in the grade book. On the final exam day, students in both groups took the post-CCI on the day when all students were present. This strategy was used to avoid student apathy and missing data (Anderson & Brennan, 2015).

Calculus Concept Inventory Instrument

The CCI was used to collect data on conceptual calculus knowledge administered in a pre and post-test format. The CCI was the first instrument developed to measure basic understanding from a first-semester differential calculus course (Epstein, 2013). This instrument has been used to determine the impact of different teaching methodologies and interactive engagement (e.g., flipped and inverted classroom, team-based learning) on students' conceptual calculus understanding (Anderson & Brennan, 2015; Bagley, 2014; Epstein, 2013; Peters et al., 2020).

The CCI has twenty-two items and is said to have two dimensions that "correlated well internally but not as well with each other" (Epstein, 2007, p. 168). The two dimensions of the CCI are functions and derivatives and limits, ratios, and the continuum. The CCI is an appropriate measurement tool for this project since it is typically used to study different teaching methods on undergraduate students in first-semester calculus courses, which is the same population on which this study focused. The CCI was also found to have only one factor, which can be explained as "overall knowledge of calculus content" (Bagley et al., 2017). While this could be seen as a drawback, this study did not look at the effects of the intervention on specific sub-categories of calculus knowledge but rather on overall conceptual calculus knowledge; therefore, the one factor aspect of the CCI was a proper fit.

Student Reflections

As students in the experimental group progressed through the labs, they were asked to reflect on what they learned in the lab as part of the qualitative data collection. For this study, since the research question related to students' perceptions regarding the implementation of the computational labs, qualitative data was only collected from the experimental group. Comparing qualitative data between the control and experimental groups was beyond the scope of this study.

Data Analysis

Demographic information and CCI scores were collected and inputted into SPSS to compile and analyze the data. The quantitative data were analyzed using independent samples t-tests, chi-square tests, and ANCOVA tests. To better understand the students' perception of how the labs affected their experience in the Business Calculus course, the qualitative data from the reflections students wrote at the end of the labs were analyzed using NVivo to implement pattern coding and determine emerging themes (Saldaña, 2021) about what students learned from participating in the labs, including self-reported conceptual understanding, success and challenges of technology integration, and recommendations for improvement of the labs. The reflection paragraphs were read for content only, and students received credit for completion and did not lose points for what was written. The analytic passes made through the data informed initial codes with participants' own language (Saldaña, 2021). A second cycle of coding was done using pattern coding to group the data segments from the first cycle into smaller categories (Saldaña, 2021).

Results

The following sections describe the quantitative and qualitative results of the data analysis.

Equivalence of Research Groups

To begin the data analysis, statistical tests were performed to determine the two groups' equivalence before the intervention. On the pre-CCI, students answered whether they had previously taken calculus. Using a chi-square test for independence to determine if there was a statistical difference for the number of students that had previously taken calculus, the results revealed no statistically significant difference between the groups, $\chi^2(1, N=79)=1.570$, $p=0.219$. There was also no statistically significant difference between the groups on gender, $\chi^2(1, N=79)=1.093$, $p=0.296$, or college major, $\chi^2(1, N=79)=1.144$, $p=0.564$. Data was also analyzed to determine if there was a statistically significant difference in the pre-CCI scores of the two groups. This was done to measure previous calculus knowledge as, even though students may have had previous exposure to calculus, their conceptual knowledge may be varied (Schroeder et al., 2015). An independent samples t-test was run for this comparison. The pre-CCI percent scores for the control group ($M=29.02$, $SD=9.48$) and the experimental group ($M=28.29$, $SD=10.67$) were not significantly different, $t(77)= -0.319$, $p=0.75$.

Analysis of CCI Scores

The following sections describe the results of the quantitative data analysis of the pre and post-CCI scores. An initial comparison between the two groups for the homogeneity of variances was conducted with the Levene's F Test for Equality of Variance. The results show an equality of variance with $p>0.05$.

Pre and Post-CCI

An independent samples t-test was run to examine the change in pre-CCI to post-CCI scores to statistically assess the overall change in calculus conceptual understanding between the two groups. The results show no statistically significant difference in CCI scores between the group without labs and those with labs; $t(77)= -0.309$, $p=0.758$. To further analyze the test results, the difference between the groups on just the post-CCI were examined, revealing no significant differences between the experimental and control groups, $t(77)= -0.580$, $p=0.563$.

Controlling for Prior Calculus Experience

Additional analysis of the CCI data was performed to control for (i) pre-CCI score and (ii) prior calculus courses taken. A one-way ANCOVA was conducted to determine if these factors determined a difference in post-CCI scores between the control and experimental groups. The results show there was not a statistically significant difference in post-CCI scores between the two groups when controlling for pre-CCI scores, $F(1, 78)=0.232, p=0.632$ and when controlling for previously taken calculus, $F(1, 78)=0.112, p=0.739$.

Controlling for Gender, Major, and Concentration

Other variables such as gender, college major, and major concentration were of interest as covariates. There were no statistically significant differences in post-CCI scores between the control and experimental groups when controlling for gender: $F(1,76)=0.063$, $p=0.802$; college major: $F(1,76)=0.226$, $p=0.636$; and major concentration: $F(1,76)=0.188, p=0.667$.

Qualitative Results

While there were no statistically significant differences between the two groups in conceptual knowledge as measured by the CCI, students in the experimental group were able to articulate how the computational labs impacted their experience in the course through the reflection paragraphs. The qualitative data analysis resulted in two major themes: real-world connections and visualization.

Theme 1: Real-World Connections

The most prominent theme throughout the students' reflections was connecting course content to real-world applications, as indicated by 64% of the student responses mentioning something about "real." Specific words suggestive of that included *real-world*, *real-world applications*, *outside of the classroom*, *real life stuff*, and *business problems*. Two ideas emerged from the Real-World Connection theme: (1) students reflected on being able to connect the mathematical concepts to real-world scenarios, and (2) students felt problem-solving was more realistic, authentic, and closer to how they might approach problems in the working world. These ideas were supported by the comments from the reflections students made throughout Lab 1 through 6. A student remarked:

> Through this Notebook, I have been able to actually see how mathematical models and exponential equations are actually applicable to real-world situations since usually random scenario word problems in a textbook don't do a good job of illustrating these things for me.

Approximately 50% of the lab reflections mentioned the students liked that some of what they were doing connected directly to the content they were learning in some of their business classes. Students expressed that making connections between the material of this class and their business classes helped them form important connections and aided their understanding.

> This lab clearly was designed to challenge students into looking at concepts from class from a different, real-life perspective. I really like doing these labs and trying to figure out how functions, graphs, exponents and other 'class material' will be used after college in the 'business world.' I look forward to some of the future labs. The truth of the matter is: I am not really fond of math, but these labs help me want to learn more. I have always been a believer that a lot of these concepts will never be used after graduation. I am starting to change my mind!

Making real-world connections can be valuable for students and may have aided their understanding. In general, students who participated in these labs commented that the labs did help them make connections between the calculus content and practical applications. Some students offered very specific explanations of how they were able to use a calculus concept and apply it to a real-world situation. A comment that exemplifies this is:

> I was able to take a company's profit data, fit a curve to it, find the derivative, and use the derivative and critical points to determine when the profits were increasing, decreasing, and where it was at a maximum. From that I was able to think about under their current business model how many units they should aim to sell to maximize profits.

Similarly, another student remarked:

> I liked that we learned about derivatives early on with the real data. Like the smokeless tobacco sales problem where I had to fit a curve to the year and sales, then find the derivative of that curve, and then tell you about the rate of change of sales in certain years and when that was the greatest; that really helped remind me throughout the chapter about the derivative being a rate of change. I just thought back to this example when the derivative was positive the sales were increasing. It also made me think about what else might be going on in those years that would have made sales higher.

These two quotes provide support for students seeing the relevance in the connection between calculus and business practices, thus establishing connections that may not have otherwise been obvious in existing calculus instruction. For example, comments from the reflections suggested the possibility of a business model to induce change in the mathematics of a situation as occasioned by outside factors.

Another student commented on how they believed that doing some lab problems about the derivative helped them learn the material:

> Using Jupyter Notebook, I learned how to apply derivatives to math problems without the question flat out saying that you need to find the derivative. I learned how to work with derivatives when dealing with functions and slope and rate of change. Learning how to apply the equations to real life problems is something that I will be able to use in the future.

This excerpt demonstrates that some students learned calculus concepts and could state how they used these concepts in applications. Other students, however, did not find value in this and may have thought that it was a meaningless addition to the course. In four responses, students stated that they could not understand why coding had anything to do with business and questioned why a business major needed to be exposed to this.

Theme 2: Visualization

Throughout students' reflections across many of the labs, words such as *visualize*, *see*, *showed*, *picture*, *image*, and *graphs* provided evidence of the visualization theme, indicative of the extent to which the labs fostered conceptual understanding. One way this theme emerged was when students talked about connecting real data points to models of that data. Each lab involved at least one scenario where students had to fit a curve given data, and about 45% of comments from the reflections on each of these labs made mention that fitting a function to these data points and graphing allowed them to "see" where the "given functions come from." A student remarked:

> I guess I never really thought about where the functions my teacher gave me came from. But doing this I can literally see that I was able to fit a model that goes through or near a lot of the data points, so it did actually come from somewhere.

Another student stated, "I also enjoy how I can literally SEE what is happening in equations, functions, and graphs and what inputs create specific outputs when I change them." A student stated, "being able to easily see the derivatives helped me with the increasing, decreasing, max/min, and concavity parts. It was very good for the visualization of that because I was struggling to get those with the lecture in class." A similar sentiment was expressed regarding integration. Students expressed value in quickly seeing the rectangles under the curve and being able to expediently increase the number of rectangles and observe how that changed the approximation.

Other Considerations

The reflection on content and the themes discussed above represent important impacts these labs had on students' understanding of calculus concepts and the development of certain habits of mind. In addition, students' reflections highlighted several ways they evaluated the labs and recommendations for future use. Comments of this nature were analyzed through evaluation coding, which assigns judgments about the merit, worth, or significance of programs (Rallis & Rossman, 2003). According to Rallis and Rossman (2003), evaluation data can describe, compare, and predict. Describing "focuses on the patterned observations or participant responses of attributes and details that assess quality. Comparison explores how the program measures up to a standard or ideal. Prediction provides recommendation for change, if needed, and how those changes might be implemented" (Saldaña, 2016, p. 141). These comments from the reflections provide valuable insight into how the students believed the labs contributed to their understanding and how the intervention could be improved. The following sections describe how the students felt the labs helped them with their conceptual knowledge (problem solving and course sequencing) and their perception of the integration of Jupyter Notebook (technology integration).

Problem-Solving

Some students come into these Business Calculus classes having previously taken calculus. It is always a goal to challenge these students to think of different problem-solving strategies, or to get them to walk away from the class learning something different than what they had seen in their high school calculus courses. While the business applications are usually new for students, previous discussions with students have revealed that they had seen most of the content before. The labs alleviated this issue to an extent because most students had not had practice using such tools, and the few that had did not use them to solve problems in a math class. These students were also better able to step up to the challenge of learning through engaging in the labs and the content because they had previously learned the material and were not as challenged by learning both. Looking at reflections from only students who had taken calculus before revealed they learned a new way of problem-solving and were indeed challenged in the course.

The students in the experimental group mentioned the new experiences of seeing the calculus concepts in concert with business applications and how the labs helped clear up confusion they still had and how they were able to solve problems differently. A quote demonstrative of this point is:

> Overall, I feel like the mathematical content from this lab was information I have seen in my previous calc course, but the way the questions in the lab presented the same information has helped me to see what the point of learning that mathematical content was. In high school if we did word problems to reinforce the topics we were learning, the situations seemed too made up for me to find them relevant. But some of these they just seem more real.

Another student who had taken calculus before commented that doing these labs gave them a different problem-solving method. This student stated that in doing their homework problems, they no longer pulled out paper and pencil; instead, they opened a Jupyter Notebook and worked through the steps. This student remarked that it seemed easier and quicker to do it that way and that it was a "totally different way of doing things."

For some students, this "totally different way of doing things" seemed to help them understand some of the content and how it was related to calculations they had previously seen. The labs also showed how the content connected to outside of the classroom, which helped to give the problems more meaning and context and could then help them transfer the knowledge to other settings. This was certainly not all students and seemed most impactful for students that had previously taken calculus because it provided them with a new way of doing mathematics they had already learned, reinforced content they had seen before, provided them with opportunities to explore mathematical situations, and they were most able to focus on the coding and not as burdened by the new mathematics.

Course Sequencing

Beginning instruction with the labs, extending the concepts during lectures, and allowing students time to pull this knowledge together to complete the problems was intended to provide concrete examples of the topics. Student reflections supported this and expressed ways they believed helped them learn. Many students reflected on the placement of the lab at the beginning of new content and voiced their support for the sequence because it helped them understand the material better when it was extended in the lecture.

Here is a student's remark on Lab 3, which is a typical comment from of all six labs:

> I like that we started this lab before learning the material because it made it easier to understand the content when we went to take notes in class. I liked that I got a little experience with these topics, then took 'paper' notes on them, and then was able to go back and practice with the questions at the end of the lab.

Technology Integration

When starting this project, it was known that there would be challenges and a steep learning curve to get students to use the computer in a way many had probably never done before. A concern was getting students to learn calculus concepts while balancing that with learning new coding skills. This proved true throughout the semester and was supported in the students' reflections. Referring to Lab 3, a student mentioned:

> It was really tricky having to go back and forth to find exactly what to type in due to how one mistake can mess up a whole strain of coding. It was also tough trying to figure out which exact lines of code were needed because of the lack of labeling on the problems, which was intentional. It is really like trying to learn a whole new language with how intricate and different it is for someone who has never had experience with it before this class… This isn't a coding class, and it is just making kids struggle that much more. Yes, I do admit that I can see the helpfulness of learning this program, I just think it should be offered in a different class.

Other students certainly felt this way but did not express their thoughts quite so explicitly in their reflections. Words such as *frustrating*, *difficult*, *stressful*, *intimidating*, *hard*, and *time-consuming* were especially present throughout Labs 1–3 when students were still getting used to working in Jupyter Notebook. These types of comments from the reflections decreased greatly in Labs 4–6 and were, in some instances, replaced with sentiments such as "this is still hard but I'm starting to get the hang of it a little" and "I'm beginning to be a little more confident in using this program." By the end of the semester, some students began to use Jupyter Notebook to complete assignments outside of lab questions. One student defaulted to Jupyter Notebook over paper and pencil. In observing the student work through some problems, the instructor noted the student had become comfortable using the technology and the techniques they had learned. In watching the student work through an optimization problem, the instructor asked the student to discuss what they were doing. The student said:

> I just think it is easier on here and to me it reinforces the steps. See like here I defined the function, and then I know I need to take the derivative, so I type the command for that. Defined the derivative as a function so I can use it later. Then I need to find the critical points, well how do you do that you see where it doesn't exist or equals zero so I then used the solve command. Now I can easily plug points in around those to the derivative using the function I defined and make determinations about max or min. I think typing in these commands helps me remember what I am doing.

While the labs provided some with a new and effective way to learn the content, it was not very easy for others, which they felt may have hindered them from learning the concepts. Descriptors characterizing such included *frustrating*, *difficult*, *challenging*, *hard*, *intimidating*, and *stressful*. Unsurprisingly, many of the negative statements were made at the beginning of the semester, which became more positive as the semester progressed and students became more comfortable using the technology. A comment that typifies those states: "Although this lab was difficult, I think I'm getting better at understanding how to use coding to do math." Other students, however, remained frustrated and extremely challenged all semester. For example, a participant noted, "After doing several of these labs, I figured out that I do not like this and am not good at it at all." This shows that some students were challenged by the technology in addition to the mathematics and were burdened by it all semester. These students may not have shown gains in understanding because they were hampered by trying to understand the technology and did not feel they could devote enough time to both understanding the technology and the calculus concepts.

Discussion

The quantitative data analysis of the CCI scores between the two groups revealed no statistically significant differences in conceptual understanding. Several factors and limitations of this study may have contributed to this outcome. The questions on the

CCI are not worded to situate the problems in real-world business situations. Results from the qualitative data analysis revealed that students were able to discuss the calculus concepts in the context of business situations such as profits, sales, and supply and demand and make valuable connections regarding these situations. But when asked about concepts on the CCI, the questions did not relate to the field of business. Since this course was taught with authentic application as the foundation, perhaps the disconnect between how students had seen calculus concepts used in class and the scenarios in which they were asked on the CCI contributed to the lack of gains in the quantitative analysis. Perhaps a deeper analysis of the CCI components may inform the design of a future study.

The likelihood of students randomly guessing on the pre-CCI and demonstrating apathy on the post-CCI is an issue that needs to be addressed. As seen in the description of the participants, most students in this study had not taken calculus at any level before: 56.4% in the control group and 70.7% in the experimental group. With most students unfamiliar with the notations and terminologies of calculus on the pre-CCI, random guessing likely affected the scores. Miller et al. (2010) reported that losses in gains do not necessarily come from actual conceptual losses but rather "correct guesses on the pre-test that, by chance, became incorrect on the post-test" (p. 232). Random guessing is a factor of great relevance when interpreting data from the CCI since most students in both groups had not taken calculus before, and the test contained numerous questions with calculus terminologies and notations. Some of the stronger students even remarked on how hard they thought the CCI was and multiple students commented on how they thought the wording of the questions was confusing. Even students who had taken calculus before and knew the specific notations and terminologies on the pre-test did not score well. The wording of many of these questions was different than problems seen by students throughout the semester, so they may have had challenges translating their knowledge of calculus concepts and how they had been asked them during the semester to the types of questions on the CCI. The authors chose the CCI because it is the primary instrument used to measure conceptual understanding of calculus concepts. However, the authors acknowledge that the format of the course did not align well with the wording on the CCI in a way that was meaningful for the students. This suggests a new instrument is needed that aligns more with pedagogy that incorporates a more authentic approach to teaching calculus content. This idea is further echoed by Bagley (2014) and Gleason et al. (2015), calling for a better instrument for use in research to assess conceptual calculus knowledge. The post-CCI was also given on final exam day, which, although ensured all students were present, numerous students appeared to rush through it, closing their computers after less than ten minutes since it did not count toward their grade.

Although the quantitative analysis did not have statistically significant results, the results from the qualitative data from student reflections demonstrate that students perceived the labs as beneficial to their learning of calculus. The content of the computational labs allowed students to see calculus concepts used in what they deemed "a real-world setting" or application. Students demonstrated their understanding by articulating how different calculus concepts could be used in settings outside of the classroom. Being able to make real-world connections can be valuable for students and can aid in the understanding of mathematics concepts. Getting students to experience meaningful real-world connections makes them better equipped to use their mathematics knowledge and adapt to new situations (Boaler, 1998). Students reflected on their ability to articulate and discuss calculus concepts in connection with other situations and thought the real-world connections helped them better understand the course content. Some ways students discussed calculus concepts in connection to real-world scenarios demonstrated that they understood and could apply them. Using real-world information, such as importing live stock market data, analyzing rates of change, and the analysis of larger data sets, makes the connections to calculus concepts more authentic and relevant. The labs allowed for the use of more real data and "real-world problems" beyond the use of a typical word problem.

The integration of Jupyter Notebook had some challenges that may have adversely affected students' understanding of the material. Since new technology was being incorporated into the classroom, the burden of learning a new program may have distracted from learning the calculus concepts. The qualitative data highlights that balancing learning new technology with new mathematics concepts was a major challenge for numerous students. This challenge, combined with some students feeling the course was too much work, led some students to believe that the labs hindered their ability to grasp some calculus concepts effectively. The ability of the students to use technology was not measured as part of this study. Therefore, the technology integration could have impacted the ability to learn some of the concepts for students in the experimental group, while this particular challenge did not exist for the control group. This could have impacted the results on the CCI when comparing the two groups. Listening to what students said throughout the semester and adjusting as necessary led to changes throughout the semester, such as allowing more time for work in the labs in class. Most students articulated that combining the labs with content was challenging but not impossible. This finding aligns with previous research as Crowe and Zand (2000) noted that while computer programming helps students explore mathematics constructively, technology integration could be challenging and burdensome in rigorous mathematics courses.

Conclusions

Increasing students' conceptual understanding of calculus is certainly a goal of undergraduate calculus courses and was an objective of using computational labs in this project to introduce students to calculus concepts. There was no statistically significant difference in calculus conceptual understanding in this study when students participated in these computational labs. However, there was evidence from students' reflections that the labs did help many students learn, most notably making connections with business. Using computational labs in a Business Calculus course allows students to expand their understanding of calculus concepts to include more authentic connections to real-world situations. The computational labs used in this project also showed students new ways of solving problems, an essential skill for their future careers. These labs allowed students to see the applicability of calculus to the "real world" and left them not asking the question of its usefulness. Students could also see other methods of solving mathematics problems besides paper and pencil and algebraic manipulation. While students were not proficient in coding or programming when they left this class, they were at least exposed to some introductory computing skills and were able to discover some of the power and utility of these tools. Because of the mixed feelings and experiences, this course's use of computational tools should continue to undergo revisions to enhance students' experiences and learning. Further research is needed to understand how computational tools can be employed in undergraduate general education mathematics classes to help students better understand mathematical concepts.

Declarations

Conflict of interest On behalf of all authors, the corresponding author states that there is no conflict of interest.

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