

An Activity- Based Costing and Theory of Constraints Model for Product- Mix Decisions

by

AYSE PINAR GURSES

Thesis submitted to the Faculty of the
Virginia Polytechnic Institute and State University
in partial fulfillment of the requirements for the degree of

Master of Science

in

Industrial and Systems Engineering

Dr. William G. Sullivan
Dr. Larry N. Killough
Dr. Eileen M. Van Aken

June 29, 1999
Blacksburg, Virginia

Keywords: Activity-Based Costing, Theory of Constraints, Product- Mix, Integer
Programming

An Activity- Based Costing and Theory of Constraints Model for Product-Mix Decisions

by

AYSE PINAR GURSES

(ABSTRACT)

The objective of this thesis is to demonstrate the use of the Activity-Based Costing (ABC) approach together with the Theory of Constraints (TOC) philosophy in determining the optimal product-mix and restrictive bottlenecks of a company. The contribution of this thesis is a new product-mix decision model that uses activity-based cost information. This new model is proposed to be used with the TOC philosophy in order to improve the financial performance of a company.

Four case studies, all of which are based on hypothetical data, are prepared in this research to show the applicability of the proposed model in different manufacturing environments. Specifically, the first case study shows that the conventional product-mix decision model and the model developed in this thesis can give significantly different results regarding the best product-mix and associated bottlenecks of a company. The second case study demonstrates the use of the proposed product-mix decision model in a complex manufacturing environment. Specifically, this case study shows how companies should consider alternatives such as activity flexibility and outsourcing to improve their profitability figures. The third case study is an extension of the second case study, and it is prepared to illustrate that the proposed model can be extended to include more than one time period. The final case study demonstrates the applicability of the proposed model in a lean manufacturing environment.

Using the proposed model developed in this research will give managers more accurate information regarding the optimum product-mix and critical bottlenecks of their companies. By applying the TOC philosophy based on this information, managers will be able to take the right actions that will improve the profitability of their companies. Specifically, they will be able to observe the effects of several alternatives, such as activity flexibility and outsourcing, on the throughput of the whole system. In addition, the proposed model should help managers to prevent making decisions that sub-optimize the system. This may occur, for example, when using only the most efficient methods to produce each product even though the capacities of these methods are limited and some other less efficient methods are currently available in the company. By extending the

model to include more than one time period, managers will be able to estimate the potential bottlenecks and the amount of idle capacities of each non-bottleneck activity performed in the company ahead of time. This information is powerful and can give companies a substantial advantage over their competitors because the users of the new model will have enough time to improve the performance of their potential bottlenecks and to search for more profitable usage of excess capacities before the actual production takes place.

Dedicated to

My parents, Nuran and Naci Gurses

My sister, Bengi Gurses

My grandmother, Done Erdogan

TABLE OF CONTENTS

Abstract	ii
Dedication	iv
Table of Contents	v
List of Figures	ix
List of Tables	x

Chapter I

Introduction.....	1
1.1 Introduction.....	1
1.2 The Need for Research.....	2
1.3 The Elements of the Research.....	4
1.3.1 Research Question.....	4
1.3.2 Research Purpose.....	4
1.3.3 Research Objective.....	4
1.4 Scope of the Research.....	4
1.5 Major Assumptions.....	5
1.6 Plan of Presentation.....	6

Chapter II

Literature Review	8
2.1 A General Look at ABC.....	8
2.1.1 What is Activity-Based Costing?.....	8

2.1.2	Fundamentals of ABC.....	9
2.1.3	Why is ABC Needed?.....	10
2.1.4	When is ABC Needed?.....	12
2.2	Implementation of ABC.....	14
2.2.1	A Model for Implementation.....	14
2.2.2	Vital Factors in ABC Implementation.....	17
2.2.2.1	Top Management Support.....	17
2.2.2.2	Other Major Initiatives.....	18
2.2.2.3	Linkage to Performance Evaluation and Compensation.....	18
2.2.2.4	Training.....	18
2.2.2.5	Non-accounting ownership.....	19
2.2.2.6	Resources.....	19
2.2.2.7	Information Technology Sophistication.....	20
2.3	Activity-Based Management.....	20
2.4	Case Studies from the Literature.....	22
2.4.1	ABC at Dayton Technologies.....	22
2.4.2	Colter Industries- Choosing the Right Customer.....	25
2.5	What is Theory of Constraints?.....	27
2.6	Integrating ABC and TOC.....	30
2.6.1	Limitations of ABC.....	30
2.6.2	Limitations of TOC.....	30

2.6.3 How do ABC and TOC complement each other?.....	30
2.6.4 Past Research Performed on Integrating ABC and TOC.....	31
2.7 What is Capacity?.....	34
2.8 Activity Mapping to Identify Bottlenecks.....	38
2.9 An ABC and TOC Model To Measure the Costs of Resource Usage	43
Chapter III	
Model Formulation.....	46
3.1 The Conventional Product-Mix Problem	46
3.2 ABC Hierarchical Model.....	48
3.3 The Proposed Approach.....	49
3.3.1 Mutually Exclusive Products.....	54
3.3.2 Activity Flexibility.....	56
3.3.3 Outsourcing Decision.....	61
3.3.4 Fixed Costs.....	67
3.3.5 The Multi-Period Product-Mix Decision Model.....	72
Chapter IV	
Case Studies.....	80
4.1 Case Study I.....	80
4.2 Case Study II.....	87
4.3 Case Study III.....	102

4.4 Case Study IV.....	113
Chapter V	
Conclusions and Recommendations.....	121
5.1 Conclusions.....	121
5.2 Recommendations for Future Research.....	125
References.....	127
Appendixes.....	134
Vita.....	183

List of Figures

Figure 2.1	The proportion of material, direct labor and overhead costs in today's world	11
Figure 2.2	Where the traditional systems focus their attention	11
Figure 2.3.	Definitions of capacity	35
Figure 2.4	The summary capacity model developed by CAM-I	36
Figure 2.5	The Responsibilities of the Teams in Capacity Management	37
Figure 2.6	Gantt Chart	41
Figure 2.7	Dependency Grid	41
Figure 4.1	The Fixed Costs Associated with P_I	99

List of Tables

Table 2.1	The Changes in Manufacturing Environment	10
Table 2.2	ABC Adoption Factors	13
Table 2.3	Value- Added Decision Rankings	24
Table 2.4	Five- Step Focusing Process of TOC	29
Table 3.1	The Two Routing Alternatives for P_1	56
Table 4.1	The Three Main Products of the XYZ Company	81
Table 4.2	The Calculation of the Charge Rates for each Overhead Activity	82
Table 4.3	The Overhead Activity Consumption of Product 1, Product 2, and Product 3	82
Table 4.4	The General Information regarding the Five Main Products of the AYBEN Company	87
Table 4.5	The Calculation of the Charge Rates for each Overhead Activity of the AYBEN Company	89
Table 4.6	The Overhead Activity Consumption of the Five Products of the AYBEN Company	90
Table 4.7	The Two Routing Alternatives for P_1	92
Table 4.8	The Activity Consumption amounts of per unit P_5 with and without the outsourcing option	98
Table 4.9	The information regarding the batch sizes, and the material and direct labor costs of the five main products of the AYBEN Company	102
Table 4.10	The information regarding the quarterly selling prices and demands of five main products of the AYBEN Company	103

Table 4.11	The calculation of charge rates for each overhead activity of the AYBEN Company	104
Table 4.12	The overhead activity consumption of the five products of the AYBEN Company	105
Table 4.13	The activity consumption amounts of per unit P_5 with and without the outsourcing option	109
Table 4.14	The two routing alternatives for P_1	110
Table 4.15	The information regarding the quarterly selling prices and demands of the five main products of the GUROTO Company	114
Table 4.16	The information regarding the batch sizes, and the material and direct labor costs of the five main products of the GUROTO Company	114
Table 4.17	The calculation of charge rates for each overhead activity of the GUROTO Company	116
Table 4.18	The overhead activity consumption of the five products of the GUROTO Company	117

Chapter I

INTRODUCTION

1.1 Introduction

One of the most important decisions that a manufacturing company should make is to determine the product mix that will maximize profits. Given that a company has capacity constraints, it may not be able to produce every unit of product demanded by the market. The best action to take in this case is to focus on the most profitable products for the company and to use all the existing resources of the company to produce these products. In this way the company can increase its profitability because it will use its existing resources to produce the most profitable products.

The product mix problem is one of the most well-known applications of linear programming [58]. The problem includes determining both the quantity and the identification of each product to produce. The main structure of the problem is to maximize profit from the mix of manufactured products subject to constraints on the available capacity of resources [37]. This is the type of problem that is investigated in this thesis.

Determining the best product mix of a company correctly is an important requirement to increase the profitability of a company. To make right decisions, management needs more accurate information about the optimal product mix and the restrictive bottlenecks of a company. Activity-based costing (ABC), together with the theory of constraints philosophy (TOC) and mathematical programming, can provide management with more accurate information about the optimal product mix of a company and can help to identify the right bottlenecks that should be focused on to improve the system.

Activity-based costing is a method that is increasingly used to improve the accuracy of product cost information. Traditional costing systems allocate overhead costs arbitrarily, primarily based on direct labor hours. However, direct labor hours often do not adequately represent the percentage of indirect resources consumed by a certain cost object in a certain period. As a result, product cost distortion occurs. Activity-based costing provides a solution to this problem by viewing the manufacturing system as being composed of activities. It assigns the costs of these activities to cost objects by using cost drivers that represent the consumption of indirect resources by cost objects more accurately than arbitrary allocation bases.

The real power of activity-based costing arises from its ability to support managerial decisions. Specifically, ABC generates the data necessary to support the theory of constraints [53]. TOC is a management philosophy developed by Goldratt [41]. The basic concept of TOC is that the goal of a company is to make money, and there are various constraints that prevent each company from reaching this goal. Goldratt emphasizes the importance of constraints and argues that management should focus on these constraints in order to improve the performance of the company.

Various studies have been conducted to demonstrate how linear programming (LP) and TOC philosophy can be used together to determine the best product mix of a company [33, 34, 35]. However, none of these studies indicate that accurate overhead cost allocation may be crucial in getting the right information about the optimal product mix and bottlenecks of a company. This research illustrates how using traditional overhead cost allocation approach or ABC may give completely different results about the best product mix and the bottlenecks.

1.2 The Need for Research

In today's competitive world of business, having accurate information may be the key factor in distinguishing between the loser and the winner. Using more accurate cost information while determining the optimal product mix of a company may lead

management to make better decisions, and as a result, may have a great effect on the success of a company.

Mabin and Gibson argue that TOC and spreadsheet LP approaches can complement each other and provide effective decision aids. Particularly they argue that the linear programming technique can be used together with the TOC philosophy to model product mix problems [35]. They consider LP as a good starting point for the production plan and suggest that the five step improvement process of TOC be used to improve the system based on the results obtained from linear programming.

Although combining TOC and LP should improve product mix decisions, the information obtained from this combination may be incorrect, especially for companies with high overhead costs and a large variety of products, if a traditional overhead cost allocation method is used in calculating the profitability of a product. The result of an LP solution may even suggest that management should produce unprofitable products. What is needed is a model that will help managers in determining the optimal product mix by using the more accurate product cost information. This new model must be able to give management the right information about the capacity-constrained product mix of a company. It must also be able to determine right processes the management should focus on to improve the performance of the system. Finally, it must be able to demonstrate the effects of various alternatives, such as outsourcing one or more of the activities performed in the company or increasing the capacity of the bottleneck operation by using an inefficient method to produce products, on the performance of the whole system.

This research demonstrates that inaccurate product cost information may lead managers to choose a non-optimal product mix and to focus improvement efforts on the wrong processes. As a result, they may decrease the profitability of a company substantially. A mathematical model that uses the activity-based cost information is developed in this thesis to determine the optimal product mix. This model is proposed to be used in combination with the TOC approach to focus on the right activities to improve the system.

1.3 The Elements of the Research

1.3.1 Research Question

Can ABC, together with mathematical modeling and the TOC approach, help to improve product mix decisions and performance of a system?

1.3.2 Research Purpose

The purpose of this research is to provide relevant information to managers that will enable them to make better decisions regarding the product mix and to focus their efforts on the most restrictive bottlenecks of a system.

1.3.3 Research Objective

The objective of this research is to develop and demonstrate a mathematical model that uses activity-based cost information and the TOC approach to determine the optimal product-mix and the bottlenecks of a company.

1.4 Scope of the Research

The study entails a literature survey regarding the activity-based costing and the theory of constraints approaches. Past research conducted on combining these two approaches is also reported herein. Moreover, the two different frameworks, the traditional framework and the new framework of the Consortium for Advanced Manufacturing International (CAM-I), developed to categorize the capacity of a plant are explained. The past research performed on capacity improvement by using the principles of activity-based costing and the theory of constraints is also described in detail.

In addition, the thesis includes the conceptual development of an activity-based cost model that may help companies to improve their product mix decisions and explains how this model can be used together with the TOC approach to improve a company's profitability. Four case studies are included in this research to demonstrate the use of the proposed product-mix decision model in different manufacturing environments. All of the four case studies utilize hypothetical data that are based on the existing literature in activity-based costing and the theory of constraints. There has been no effort to implement the concepts developed in this research in companies with real costs, activities, and products. The major difficulty that is expected in real-life implementation of this research is to collect the data necessary for the proposed product-mix decision model.

All of the four case studies included in this thesis demonstrate the new model in hypothetical manufacturing environments. However, the concepts developed in this research should be both useful and applicable to the service industry as well. Furthermore, activity-based costing and the theory of constraints approaches have been implemented successfully in service type organizations in the past [13, 23]. The thesis also includes conclusions derived from the literature survey and the four case studies. Future research recommendations are the final topic of this study.

1.5 Major Assumptions

The following eight basic assumptions have been made while developing the new product-mix decision model:

1- To eliminate facility-sustaining expenses, a plant would have to be permanently closed. In this study, it is assumed that closing the plant is not an option. Therefore, facility-sustaining activities are not included in the model developed in this research.

2- No finished goods inventory is carried from one period to the next.

- 3- A batch of product i has to be finished completely once that batch is started to be processed.
- 4- Whenever one unit of product i is started to be manufactured by using a specific route, all the other parts in that batch must follow the same route.
- 5- Products with an activity flexibility option do not have an outsourcing option and vice versa.
- 6- An activity of product i must be either outsourced completely, or performed in-house for each unit of product i .
- 7- The elements of the sets PS , $PS1$, $PS2$, $PS3$, $PF1$, $PF2$, $PO1$, $PO2$, $FC1$, $FC2$, and $FC3$ (See Chapter 3) do not vary by time. For example, if product i has an activity flexibility option in period I , it will continue to have this flexibility option for the remaining $T-I$ periods.
- 8- Whenever a batch of product i is started to be manufactured during period t , that batch has to be completed by the end of that period.

1.6 Plan of Presentation

Chapter 1 introduces the problem and the need for this research. It identifies the research question, the purpose, the objective, and the scope of this study. A plan of presentation is also included in this chapter.

Chapter 2 gives a review of the relevant literature. Specifically, it gives information about the principles of activity-based costing and the theory of constraints approaches. Chapter 2 also includes the past research conducted on how ABC and TOC can be used together to improve a system. Furthermore, it explains the significance of effective capacity management in the success of a company in today's competitive

business environment and describes the different frameworks used to define capacity. Chapter 2 also covers the past research performed on capacity improvement that used the principles of activity-based costing and the theory of constraints.

Chapter 3 develops the proposed model that can be used to support the decisions regarding the product mix of a company. Specifically, this chapter explains how the model is developed based on the current literature and how the model can be expanded to include several alternatives such as activity flexibility and outsourcing.

Chapter 4 presents four case studies to show how to implement the product-mix decision model developed in Chapter 3 in different situations. All of the four case studies are based on fictitious company data. The first case study illustrates that using traditional costing method and ABC can give two completely different results for the optimal product mix of a company. The second case study shows how the proposed model can be used in a complex manufacturing environment where several alternatives such as outsourcing and activity flexibility exist. The third case study illustrates that the new product-mix decision model can be extended to include more than one time period. Finally, the fourth case study demonstrates how to use the proposed model in a lean manufacturing environment.

Chapter 5 includes the conclusions derived from this research and the contribution of this research to the existing body of knowledge. It also presents a summary of the study and identifies future research opportunities.

Chapter II

LITERATURE REVIEW

2.1 A General Look at ABC

2.1.1 What is Activity-Based Costing?

Traditionally, the costs of manufacturing a product have been categorized as direct material, direct labor, and overhead. Traditional cost systems, also called volume based cost systems (VBC systems) trace overhead costs to the product based on the assumption that products cause the costs. Very few allocation bases have historically been used. The most common allocation base used in VBC is direct labor hours. The amount of overhead allocated to a batch of products increases linearly with the volume produced. So, it is assumed that as volume of output increases, direct labor hours increase in a linear fashion [57].

ABC, on the other hand, focuses on activities performed in manufacturing the product. ABC is defined by Computer Aided Manufacturing-International (CAM-I) as “the collection of financial and operating performance information tracing the significant activities of the firm to product costs” [47, p.2]. ABC is a concept in which overhead is assigned to products based on the number of activities consumed by the products [24]. Liggett et al. states the underlying philosophy of ABC as follows:

“ Certain activities are carried out in the manufacture of products. Those activities consume a firm’s resources, thereby creating costs. The products, in turn, consume activities. By determining the amount of resource (and the resulting cost) consumed by an activity and the amount of activity consumed in manufacturing a product, it is possible to directly trace manufacturing costs to products ” [32, p.4].

2.1.2 Fundamentals of ABC

ABC is a two-stage procedure (See Appendix A). In the first stage it assigns all costs of resources to the activities in activity centers based on the resource drivers [43]. The amount paid for a resource and assigned to an activity is called a cost element [11]. CAM-I Glossary of Activity Based Management defines a cost pool as a “grouping of all cost elements associated with one activity” [47]. However, Novin argues that a cost pool does not have to contain only one activity. It can be formed by classifying a large number of activities into a few groups. Regression analysis can be used in forming cost pools. A basic assumption of ABC is that cost pools are homogenous, which means that the costs of activities in each cost pool should have the same cause-and-effect relationship with the chosen cost driver [42].

In the second stage, costs assigned to the cost pools are then assigned to the products based on the product’s consumption of each activity and the level of the activity in the ABC hierarchy, which will be discussed in the following section. The final costs assigned to a product are called a cost object. Cost drivers are used to assign the costs of activities to products. A cost driver is any factor that causes costs to be incurred, such as number of machine setups, number of engineering change notices, and number of purchase orders. At least one cost driver is required for each activity. More can be used to increase the accuracy of the information [42, 43, 46].

It is the second stage described above that separates ABC from traditional systems. In traditional systems, costs are allocated to products based on just one cost driver, direct labor hours, instead of using more realistic cost drivers [24].

Another unique feature of ABC is that the focus of this approach is on activities and the cost of those activities, rather than on products as in the traditional costing systems. It is this feature of ABC that gives management the necessary information to identify opportunities for process improvements and cost reductions. By using ABC information, managers can see the cost of each major overhead activity performed in a

plant separately, and therefore can give right decisions about where to focus efforts to reduce costs [6].

2.1.3. Why is ABC Needed?

Sullivan lists the characteristics of the new manufacturing environment (see Table 2.1), and states that in today’s world, manufacturing companies are changing and becoming more information intensive, highly flexible, and immediately responsive to the customer expectations [56]. Due to the changing manufacturing environment, traditional cost accounting is rapidly disappearing. Traditional accounting systems were developed at a time when direct labor was a large percentage of the total product costs. Changes in manufacturing technologies, such as the just-in-time philosophy, robotics, and flexible manufacturing systems decreased the direct labor component of production and increased overhead costs. In today’s manufacturing environment, direct labor accounts for only 10% of the costs, whereas material accounts for 55% and overhead 35% (See Figure 2.1 and 2.2). As a result, product cost distortion occurs due to allocating overhead costs to the products arbitrarily on the basis of direct labor hours used by each product [24, 32]. Cooper reports several situations that can cause distortions to occur, such as production volume diversity, complexity diversity, material diversity, and setup diversity [11, 12].

Table 2.1: The Changes in Manufacturing Environment [56, p.6]

Yesterday	New Paradigm
High volume, long production runs, long product life cycles	Low volume, short product runs, short product life cycles
Small number of product variations in a domestic market	Large number of product variations in an international market
Large direct labor component; high cost of processing information	Relatively high technology costs; relatively low information processing costs
Small indirect/overhead costs in relation to direct labor	Large indirect/overhead costs in relation to direct labor

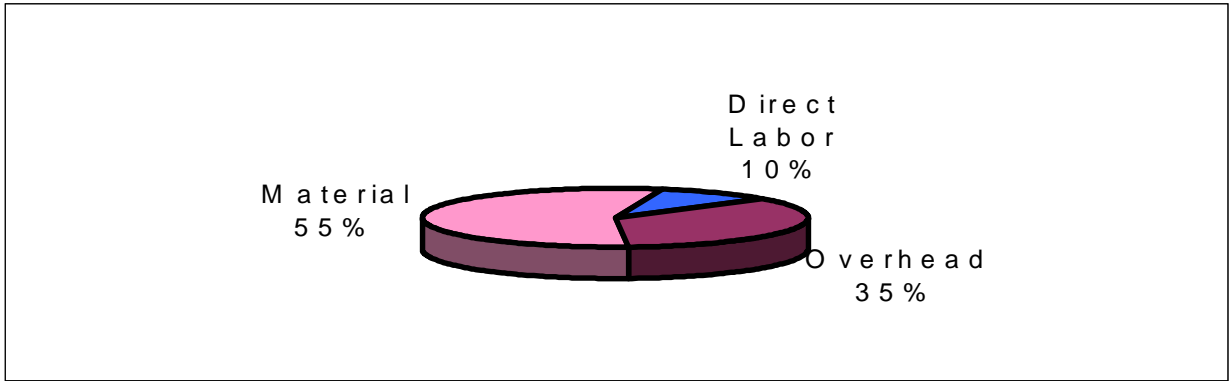


Figure 2.1 The proportion of material, direct labor and overhead costs in today's world [45].

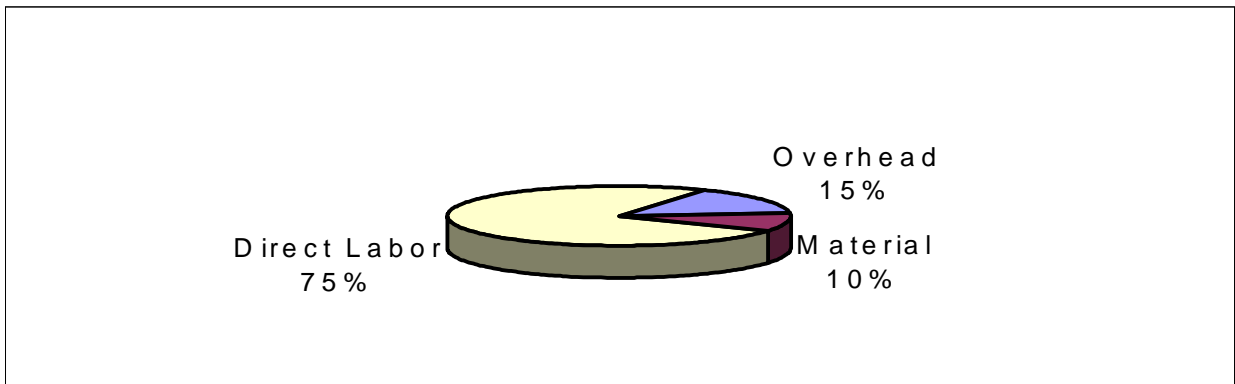


Figure 2.2 Where the traditional cost systems focus their attention [45].

The solution to the product cost distortion problem is ABC. ABC provides the information to identify the components of overhead more precisely such that product design, development, production, and distribution decisions are better grounded. ABC assigns resource costs to products more accurately, and as a result it acts as a decision support tool for companies. Decisions are not arbitrary, which is the case in traditional accounting systems, but based on facts [24, 27].

Ioannou and Sullivan state that the information obtained by using ABC can improve decision-making in manufacturing companies. Specifically, they use a two-stage method for the justification of investment in automated material handling systems. The first stage collects information regarding the necessary costs and benefits that will result from the implementation of a new material handling system in a manufacturing company. In this stage, an ABC process management software, DaCapo, is used for modeling the material handling activities in the company. In the second stage, the information obtained from the first stage is used to justify the investment for the proposed material handling system by using an investment decision model that performs an Economic Value Analysis (EVA) while comparing different material handling alternatives. Based on the results of this study, the authors conclude that using activity-based costing information together with EVA can, most probably, improve decisions regarding the investments in new technologies such as automated material-handling equipment [26].

2.1.4 When is ABC Needed?

As with any course of action, the implementation of ABC is justified if the costs of installing and operating the system are more than offset by the long-term benefits. The cost drivers used in an ABC system require the measurement of some unique attributes of each product. For example, using raw material invoices as a cost driver requires measuring the number of raw material invoices consumed by each product. Measuring these attributes can be expensive. On the other hand, not using an ABC system may increase the cost of erroneous decisions made with inaccurate product costs. As the diversity of products increases, both the measurement costs associated with ABC and the cost of erroneous decisions associated with traditional costing system increase. Cost of measurement, cost of errors, and product diversity are continuously changing over time. The decision of when to implement an ABC system should be made by analyzing and balancing the costs of using ABC against the costs of not using it [10].

In fact, quantifying these costs is very difficult. Fortunately, management can identify the need for ABC by observing the symptoms listed below [10]:

- Products that are very difficult to produce are reported to be very profitable, although they are not premium priced.
- Profit margins are difficult to explain.
- The results of bids are difficult to explain
- The competitor’s high volume products are priced at unrealistically low levels.
- Customers do not react to price increases, although there is no corresponding increase in costs.

A study conducted by the Cost Management Group of the Institute of Management Accountants (IMA) revealed that there are four basic factors that separate the companies that adopt ABC from those that do not: potential for cost distortions, decision usefulness of cost information, lack of system initiatives, and size of the organization (See Table 2.2).

Table 2.2 ABC Adoption Factors [31, p.34]

Potential for cost distortions	39% had above-average potential	71% had above-average potential
Decision usefulness of cost information	54% had above-average usefulness	65% had above-average usefulness
Lack of system initiatives	15% report major system or software initiative occurring	7% report major system software initiative occurring
Size of the organization	\$51M-\$100M (average)	\$101M-\$500M (average)

Potential for cost distortions: The IMA study found that as the potential for cost distortions increases, the organization gets more motivated to adopt ABC. In the study, the potential for cost distortions was a composite measure based on questions relating to the diversity of products, support departments, processes, and volumes. The companies that adopted ABC had generally a higher distortion potential than those that did not. The study also showed that cost distortions affect the decisions of many companies, including

customer profitability analyses. Most of the companies implementing ABC found that some of their customers who were thought to be very profitable and were given special efforts were, in fact, unprofitable [31].

Decision usefulness of cost information: The companies participating in the IMA survey were asked questions about why they have implemented ABC, and 65% of them indicated decision usefulness of ABC information as one of the main reasons. Decision usefulness is measured based on questions relating to how ABC helped the companies in cost-reduction efforts and pricing decisions, and how it helped to gain an advantage over their competitors [31].

Lack of system initiatives: In the IMA survey, 15% of all nonadopters reported that their company was in the midst of installing a new or upgraded information system. On the other hand, only 7% of adopters reported an ongoing system initiative. This result supports the claim that the probability of success of ABC implementation will increase if the system upgrade is complete. There are two reasons why system initiatives hinder ABC adoption. First, ABC implementations take considerable time and effort. Second, since ABC systems require more detailed information than traditional cost systems, it is better to wait until the infrastructure for ABC is ready in the company [31].

Size of the organization: Among the companies that participated into the survey, ABC adopters had larger company sizes than nonadopters. Availability of resources and economies of scale in implementing ABC at multiple sites are shown as the possible reasons for the effect of size of the company on ABC implementations [31].

2.2 Implementation of ABC

2.2.1 A Model for Implementation

There are six steps in the implementation of ABC. The model presented below is adapted from Compton, 1996.

1- Forming the team: A multidisciplinary team must be assembled to design the ABC system to reflect correctly the workings of the company. This team should be selected and assigned to the project full time. It should be composed of three to five people, and should include representatives from manufacturing, marketing, management information systems (MIS) and accounting. All team members must be open-minded, knowledgeable about the company's operations, and well respected. The benefits of the team approach include smoother implementation and greater design effectiveness. Members from across the organization also ensure a broader acceptance due to better communication, transfer of knowledge, and awareness of the benefits [8, 16, 43].

2- Deciding on Design Choices: At least six major decisions should be made before an ABC system can be implemented. These are as follows [8, 14]:

- Should the system be integrated with the existing system or should it be a stand-alone system?
- Should a formal design be approved before implementation?
- Who should take the "ownership" of the final system?
- How precise should the system be?
- Should the system report historical or future costs?
- Should the initial design be complex or simple?

3- Training: Training is essential for effective implementation, execution, use, and acceptance of an ABC system. Training is an ongoing process throughout the life of the project. Three groups should be addressed in designing a training program [8].

- Management: Management must have enough knowledge about ABC and its potential benefits.
- Implementers: The implementation team must have enough knowledge about technical design needs, software-modeling capabilities, and project organization.
- Users: Users should understand the benefits of the ABC information system and how this information can be used in decision-making.

4- Gathering Information: A vital component of the ABC implementation is identifying the information requirements of its users. Each phase of information gathering should include a member of the implementation team and the user. The requirements of the system and the information needed should be explored. Questions each user should attempt to answer include the following: What key decisions must I make, and how often do they arise? What other cost information would be useful in carrying out my responsibilities? Documentation and note taking of reviews of records, observations, interviews, questionnaires, and interfaces with the existing information systems are some of the techniques that can be used in this step [8].

5- Creating an ABC Systems Model: This is the most important step in the design of an ABC information system. Each model has a set of resources, resource drivers, activity centers, activities, cost drivers, cost elements, and cost objectives. Flowcharting is recommended to gain a better understanding of the complexities of cost allocation. As a starting point, the organization chart can be used in the model. The process of converting the organization chart to an ABC flowchart by breaking down tasks into divisions, resource centers, and activity centers is called functional decomposition [8].

Activity centers should be established in this step. To do this, all activities related to accomplishing a particular attribute should be grouped. These clusters of activities form the activity centers. Clustering activities will reduce the level of detail substantially, but it will also decrease the amount of effort required. A good rule of thumb is not to have more than 20-25 activity centers for an ABC project [8].

Analyzing and identifying the resource and cost drivers is a crucial step in building a model for an ABC project. A general rule to follow in selecting resource and cost drivers is to pick drivers that will show a cause-and-effect relationship. After resource and cost drivers are determined, resources are allocated to the activity centers. Finally activities in activity centers are allocated to the products depending on the types of activities [8, 14].

Computer technology should be utilized especially in this step. A computer model will provide an easy mechanism for manipulating the model and performing a sensitivity analysis [8].

6- Post-implementation Review: Like any other system initiatives, the ABC system must undergo a post-implementation period. Hidden problems must be uncovered and solved, system components must be fine-tuned, and users must learn how to operate the system. To assure that the desired results are achieved, members of the ABC implementation team should be assigned as troubleshooters. They can observe operations and assist in making any adjustments. Any necessary changes should be carefully controlled. The post-implementation evaluation enables the ABC project implementation team an opportunity to assess the degree to which the ABC project objectives have been met, to determine the necessary modifications, to evaluate the implementation team's performance, and to make recommendations about the improvements of the system in the future [8].

2.2.2 Vital Factors In ABC Implementation

There are many factors that affect the success of ABC implementations. Based on an extensive search of the literature, the most important eight factors are as follows: top management support, other major initiatives, linkage to performance evaluation and compensation, training, non-accounting ownership, resources, and information technology sophistication (See Appendix B) [3, 5, 31, 52]. These factors are explained in detail in the following paragraphs.

2.2.2.1 Top Management Support

Research shows that top management support is the most crucial factor in the success of ABC implementation [3, 52, 57]. This finding is, in fact, consistent with the more general finding that almost all successful innovations require the support of top management. Top management should focus resources, goals, and strategies on the

implementation of ABC. They must demonstrate a commitment to ABC by using it as the basis for decision making. To encourage the use of ABC information, top management must use ABC information in communications and agreements with other employees [52].

2.2.2.2 Other Major Initiatives

According to a recent survey conducted by the Cost Management Group of the Institute of Management Accountants (IMA), 62% of the firms trying to implement ABC but not having reached the usage stage report other major initiatives being implemented currently. Several companies expressed that they cannot commit enough resources to the implementation of ABC because of their need to implement other initiatives such as Total Quality Management (TQM), lean manufacturing, Just-in-Time (JIT), and balanced scorecard [31].

2.2.2.3 Linkage to performance evaluation and compensation

Shields and McEwen articulate that the importance of the linkage between performance evaluation and compensation, and ABC implementation is natural because employees pay attention to those things that affect their welfare. The welfare of most employees is affected by the system used to evaluate and compensate them. Therefore, when ABC is linked to performance measurement and compensation, and when employees believe that the resulting system fairly represents their performance, they will be motivated to help ABC succeed [52].

2.2.2.4 Training

Training is important to help people understand how ABC differs from traditional cost accounting and why ABC provides a superior economic measurement and information system. It also increases non-accounting ownership. ABC requires training

from the senior management to the shop floor. Training can include readings, lectures, hand-on projects, and on-the-job training [3].

2.2.2.5 Non-accounting ownership

When ABC is owned only by accountants, there is a danger that it might be used only to satisfy their needs. An important reason why some companies have not had good implementation experiences is that the accountants have retained ownership and have not succeeded in sharing ownership with non-accountants [43, 52]. For this reason, not only accountants but also non-accountants should be seen as the owners of the new system. Non-accountants should be involved in the initial decisions to invest in ABC, and in the design and implementation of ABC. In this way the chances that non-accountants will support and promote ABC, and be committed to its use and success will increase [52].

2.2.2.6 Resources

The process of designing and implementing an ABC system requires companies to have adequate resources. The necessary resources primarily include the time and commitment of accountants, top management, operating employees, software, and external consultants [48]. The implementation of ABC often takes more time than expected. The companies who have participated in the survey of IMA report an average of 3 years for implementation before they can start to use ABC. The amount of time necessary to reach the usage stage varies with the size of the company [31].

In a survey of 143 companies, Shields and McEwen found that having adequate employee resources is one of the most important factors for ABC success. Interestingly, however, the other types of resources, such as commercial or custom-made software and also external consultants, did not prove to be important to the success of ABC implementation. Most companies surveyed use commercial software to help structure their ABC design and to process ABC information. The availability and use of commercial software versus custom software did not prove important to the success of

ABC in the long run. Shields and McEwen argued in their study that the choice of software as a technical information system is important for accountants and Management Information System (MIS) specialists, but this choice is relatively unimportant to non-accountants or for the ultimate success of an ABC project [52].

2.2.2.7 Information Technology Sophistication

In the survey conducted by IMA, a high level of information technology (IT) sophistication appears to be an important factor in getting to the usage stage for the majority of the companies. Sixty-one percent of the usage-stage companies received an above average IT score, compared to only forty-six percent of the non-usage stage firms. ABC implementation will be much easier if the IT of the company has the following characteristics: good subsystem integration; user-friendly query capability; available sales, cost, and performance data going back 12 months; and updates of all these types of data [31].

2.3. Activity-Based Management (ABM)

Activity-based management is defined by CAM-I as “a discipline that focuses on the management of activities as the route to improving the value received by the customer and the profit achieved by providing this value” [47]. The terms “ABC” and “ABM” should not be used interchangeably. ABC is only a tool for determining the costs of activities and the outputs that those activities produce. ABC, by itself, is not enough for continuous improvement of the company. On the other hand, ABM is a management philosophy that focuses on the planning, execution and measurement of activities and helps companies to survive in the competitive world of business. ABM uses the information obtained through ABC to reduce or eliminate non-value added activities, and as a result, improve the overall process [48]. Cooper et al. explain this important difference as follows:

ABC information, by itself, does not invoke actions and decisions leading to improved profits and operating performance. Management must institute a conscious process of organizational change and implementation if the organization is to receive benefits from the improved insights resulting from an ABC analysis [16, p.308].

ABC information indicates the activities having the highest opportunity for cost reduction (See Appendix C). After identifying the target activity, management should determine whether the high cost of this activity is a problem of efficiency or of effectiveness, or whether there is a problem at all. In order to reduce cost, effectiveness should always be emphasized over efficiency. If a job is a non-value added activity, it should not be performed at all rather than being performed more efficiently. The management should first consider the necessity of performing an activity. Only after the management decides that the activity cannot be eliminated, should improving its efficiency be taken into account [43].

In order to determine whether a company or a customer requirement is causing the activity's frequency, O'Guin suggests investigating the trigger of the activity. If the high frequency is caused by the company, the management should either try to eliminate it or to reduce its frequency by increasing its efficiency. If the high frequency is caused by the customer, the management should try to come to an agreement with the customer limiting the activity. Flowcharting can be used in identifying the non-value added activities in a process [43].

Management should always use the information obtained by ABC with caution because using this information inappropriately can lead to sub-optimization of the system. The management should not try to optimize each component of the system, since optimizing each component probably means sub-optimizing the system. Reducing costs in one process can increase costs in another process. Also, management should not reduce cost at the expense of flexibility and customer satisfaction. This is an area where ABM is criticized because managers using the information obtained by ABC tend to improve processes without considering the negative effects of these improvements on flexibility and customer satisfaction [49].

A new method in the process improvement aspect of ABM has been introduced by Cooper and Kaplan. In their article “Activity-Based Systems: Measuring the Costs of Resource Usage”, they suggest not allocating the unused capacity costs to the products. The unused capacity costs should be isolated and not included in the product costs. In this approach, the unit cost of a product does not change as the number of units produced increases, however this change decreases the unused capacity cost associated with each activity used in the production of the specific product. ABM and TOC together can help to model the company so that the bottlenecks and the cost of unused capacity will be revealed. Management can use this model in deciding which activities to focus on for improvement. This new approach to capacity management will be explained later in detail [15].

2.4 Case Studies from the Literature

2.4.1 ABC at Dayton Technologies [44]

Dayton Technologies is a business unit within ALCOA, producing components for vinyl window manufacturers by the process of polyvinyl chloride extrusion. Like many other manufacturers, Dayton Technologies had relied on a single-driver, traditional costing system using direct material dollars. Since the company had high levels of product and volume diversity, the executives thought that ABC would be a good fit for the company. The company began the ABC model in 1989. Interviews with employees were conducted to collect data. At first, the ABC model that was developed was very complex and cumbersome. It contained 40 cost drivers, so it was impossible to communicate effectively to managers and staff. After careful reconsideration, the number of drivers was reduced to 11. The company had a number of major obstacles in the implementation of ABC. The following are a sample of the most challenging issues.

* *Defining and obtaining cost driver information:* Frequently the necessary data was not being collected, or the records were incomplete.

* *Obtaining valid and unbiased responses from employees:* Employees need to feel comfortable with the purpose of the data collection process. Otherwise, their responses can be defensive and biased. In the effort to capture a value-added ranking for activities, the company established a scale of 1 to 5. The questions were designed to help the employees provide honest feedback without fear. As is the case at most other companies, Dayton Technologies' employees are considered very customer-oriented. In determining whether an activity is value-added, the question generally asked is whether or not the customer receives any benefit. This approach to defining and quantifying value is detailed in the "Benefits Gained" section below.

* *Obtaining timely responses:* Another difficulty was the delay in obtaining follow-up responses after the initial activity data were provided. If information is not updated to reflect changes, the quality of the results will jeopardize the credibility of an ABC system.

* *Interruptions from other business priorities:* Even though the company assigned two employees to address the ABC program as their primary function, competing requests for them to deal with other urgent matters frequently disrupted the progress.

Benefits Gained:

As a result of interviews with employees, Dayton identified its value-added and non-value added activities by using the framework presented in Table 2.3. Dayton considers an activity as value-added (VA1, VA2) if it is either important for the external customer or necessary to meet corporate requirements. Otherwise, the activity is considered as non-value added. Non-value added activities can be divided into two categories. The first group (VA3, VA4) consists of the necessary activities for continuing the business, although they do not add any value to the product. The second group (VA5) includes activities that are not required for the business and just waste resources. These are the activities that should be totally eliminated in order to improve the business.

Table 2.3: Value-Added Decision Rankings [44, p.23].

	Value-added Activities
VA1:	Is the activity of value to the external customer?
VA2:	Is the activity required to meet corporate requirements?
	Nonvalue-added Activities
VA3	Is the activity required for sound business practices?
VA4	Is the activity of value to the internal customer?
VA5	Is the activity a waste?

In this way, the company achieved a greater awareness of value-added versus non-value added activities. An early output of the model was a list of costed activities (those that had a cost associated with them). The cost levels were surprisingly higher than expected. This awareness led to efforts to reduce redundancies, eliminate tasks, and combine steps to simplify and reduce troubleshooting by improving training. Some companies refer to this procedure as an example of process reengineering. Dayton Technologies calls this process activity-based cost reduction, and the decision model for this process can be seen in Appendix D. The process can be summarized as follows: If the activity is a value-added activity (VA1 or VA2), the company tries to improve it. If it is a first group non-value added activity (VA3 or VA4), the company tries to eliminate it or at least reduce its frequency. If it is a second group non-value added activity (VA5), it should absolutely be eliminated.

The ABC data also provided valuable support for a change in the product offering strategy. The company realized the weaknesses of some of its specialty products and took actions to eliminate or limit their availability. Also, for many years the company resisted the concept of outsourcing. But as the ABC data began to reveal core strengths and expose the greatest weaknesses, outsourcing became a new opportunity. Significant gains in product margin performance have been obtained from this outsourcing strategy.

As a result, managers have seen evidence of how a better understanding of costs and activities can help improve performance, and they then become committed to continuing the ABC journey. They are confident that ABC will provide essential information for business management that is not available from traditional accounting. The benefits gained can be a source of competitive advantage in the marketplace.

2.4.2 Colter Industries- Choosing the Right Customer [43]

Colter is a manufacturer of vibration isolation tables, mounts, and systems. In 1988, when a new president took office, Colter was a successful, profitable company, with a return on equity exceeding 18%. However, the new president was dissatisfied with the current performance, and laid out an aggressive plan for stronger performance. As a part of the plan, he wanted additional growth without additional distribution, sales and administrative resources. A strategy for achieving higher growth was to promote high margin products to desirable customers. The president intuitively knew that there was a great variation in the real cost of each customer order. To meet his stated goals, the president felt that he needed a new system to determine the real cost of each customer order. As a result, Colter implemented an ABC system as a part of its new strategy. To meet its goals, Colter had to understand which customers were the most profitable to serve, and which were not. A team was formed to implement ABC. After completing the study, the results surprised everyone. Only 32% of their customers were found to be profitable to serve (See Appendix E). The study identified order size as one of the key factors in determining the customer's profit contribution. The other was ordering non-stock items, which were roughly 80% of the product offering.

The sales force was very resistant to the new cost system. Most of the sales force just did not realize how much work was entailed in processing a small order or short production run. However, by educating and working with the salespeople, the team convinced the sales force of these true costs.

After developing the ABC system, Colter devised a plan for increasing customer profit contributions. Salespeople raised prices on small, custom orders with surcharges and handling fees. Product managers reduced the number of product offerings in various production lines. The company taught the salespeople to emphasize standard products, and not to accept small build-to-order requests. The minimum buying quantities were doubled. Some customers cut the number of different end items purchased and the frequency of small orders. Colter also gave a number of its largest customers direct on-line order entry terminals to reduce both firms' order entry costs.

The results were impressive: the average customer order size increased by 87 %. While the number of different end items sold was decreased by 40%, sales increased in 1988 and 1989 by 11% and 6%, respectively. In addition, Colter increased the number of profit-contributing customers by 15%.

The case of Colter shows the effects of ABC on customer selection and management. It shows the cost diversity of serving large numbers of customers. Customers with many small orders and a high percentage of non-stock items are not profitable to serve. Colter used its study results to maneuver its customers to greater margin contributions. Because Colter was the first company in its industry that used ABC, it was able to pick the market's most profitable customers. The company also dropped many of the transaction-intensive customers that were unprofitable to serve. This case also shows the success of having ABC supported by the company's president. The president also used the ABC information to take decisive and strategic actions.

Although dropping unprofitable customers and not accepting small build-to-order requests based on ABC information worked well in Colter's case, it may not work in other companies. Not focusing on customers can bring some unexpected results in the long term. Since customer focus is the principle of all Continuous Improvement (CI) programs such as Total Quality Management (TQM), Business Process Reengineering (BPR), Just in Time (JIT), it seems that ABC and CI programs have some conflicting points. But making decisions based solely on ABC can lead to wrong actions. I believe

that ABC and CI programs can work together, but further research should be performed to understand how to integrate them.

2.5 What is the Theory of Constraints?

The Theory of Constraints (TOC) is a system's management philosophy developed by Eliyahu M. Goldratt. In his book, called The Goal: A Process of Ongoing Improvement, Goldratt states that a firm's goal is to make money now and in the future. A company will not exist if it is not making money. Any activity that does not help make money is a waste of time and resources [29]. TOC is implemented through three measures: throughput, operating expenses, and inventory [21].

Throughput is the rate at which the system generates money through sales. Goldratt defines throughput as revenue less direct materials, since direct labor is often considered as a fixed cost in the short term [50]. In order to increase the throughput, all bottlenecks of the system should be identified. After this, management should focus on improving the efficiency of the bottleneck because this means improving the efficiency of the whole system.

A common approach in capacity management is to focus on the efficiency of each machine individually and try to maximize the number of hours the machines are working. This approach causes the sub-optimization of the system. It does not increase the overall efficiency of the system because it does not guarantee that the product will be completed on time. The solution to this problem is to understand that the capacity of a system is determined by the bottleneck of the system. The system cannot produce more than the bottleneck can. The bottleneck machine determines the pace of the system. So, no matter how much the management improves the efficiency of a non-bottleneck machine, this improvement will only decrease the cost of running that specific machine, but it will not have a great effect on the whole system. Just improving the efficiency of a non-bottleneck should not be the aim of the management. Management must focus on the

overall plant efficiency, rather than on individual machines. And this can be accomplished by focusing on the bottlenecks [50].

Weston studied the effect of the location of a constraint on the performance of a manufacturing cell. Specifically, he placed a single constraint first at the beginning, then in the middle, and finally at the end of a work cell. He observed the performance of these three scenarios by using simulation and concluded that placing the bottleneck resources at the beginning of a manufacturing cell generally improves the performance [59].

The second measure Goldratt uses to describe the costs of manufacturing products is operating expenses. Operating expenses can be defined as all the money the system spends in order to turn the inventory into throughput [21]. The cost of direct labor, raw materials, managers, machines, and material-handling activities are all included in this category [50]. A common false belief related to the operating expenses is that all components of the system must constantly be working all the time, otherwise the company will lose some profit. What this approach accomplishes is producing unnecessary parts continuously and increasing inventory. What it is not doing is increasing the throughput. So, this approach does not help the company to make more profit. On the contrary, it increases the costs of the company by rewarding managers for increasing the inventory level. TOC argues that only the bottleneck must be kept busy all the time. All other operations should be either idle at some point during the production or be used to decrease the workload of the bottleneck of the system.

The third concept in TOC is inventory. Goldratt defines inventory as “all the money the system invests in things it intends to sell.” Since most of the managers believe that everything in the system should be kept busy at all time, there will exist an excess amount of inventory most of the time. This will decrease the performance of the system substantially because in this case the money of a company will be tied up in the inventory although it could be invested in a profitable project [21]. In addition, excess inventory will mask the underlying problems of a company. So, to improve a system, the inventory

level should be reduced gradually, and the problems revealed with the decreasing inventory level should be solved one by one [40].

Goldratt states that there is always at least one constraint that restricts the company’s ability to achieve its goal [20]. When the system is improved such that it can produce as much as the demand, the active constraint will be an external constraint such as a lack of customer orders, logistical limitations, or availability of materials. TOC categorizes the resources into scarce bottleneck resources, non-bottleneck resources, and capacity constraint resources (CCR). A CCR is a resource that is not a bottleneck currently, but, if not managed properly, it could become a constraint [25].

Goldratt developed a five step process for managing constraints and improving the system continuously (See Table 2.4). This process can be used to increase throughput, while decreasing inventory and operating expenses. In this way the company will reach to its goal, namely making money [1].

Table 2.4. Five-Step Focusing Process of TOC [1, p.26]

1- IDENTIFY system constraints, whether physical or political constraints.
2- Decide how to EXPLOIT the system constraints. That is, get the most possible within the limit of the current constraints.
3- SUBORDINATE everything else to the above decision.
4- ELEVATE the system constraints. That is, reduce the effects of the current constraints; off-load some demand or expand capability; and make everyone aware of the constraints and its effects on the per performance of processes.
5- If in the previous steps a constraint has been broken, go back to step 1, but DON’T allow inertia to cause a system constraint.

2.6 Integrating ABC and TOC

2.6.1 Limitations of ABC

ABC is long-term oriented. It traces the cost of resources used in production to products [25, 29]. Under ABC, it is assumed that almost all of the costs are variable, so they change according to the output level. However, in the short run, there are many fixed costs such as the cost of labor, rent, equipment, etc. The company will incur these kinds of costs whether the product is produced or not. As a result, ABC may give wrong information about short-run decisions because of not reflecting the actual costs the company will incur in the short-run [28].

Another weakness of ABC is that it does not involve the constraints of a system into the analysis. In the short-run, the capacities of all the activities are fixed. But ABC omits this fact, and as a result, does not take into account the opportunity cost of using the bottleneck [29].

2.6.2 Limitations of TOC

TOC has a short-run time horizon. In the short-run, the capacity of a plant is fixed, and this fixed capacity will create the bottlenecks. However, in the long run, management can have an effect on capacity. Labor and overhead costs will not necessarily be fixed all the time. The weakness of TOC is that it does not include these costs and may give wrong information in the profitability analysis. The managers may decide to produce unprofitable products if they make decisions solely based on TOC [25, 29].

2.6.3 How do ABC and TOC complement each other?

ABC and TOC are based on different time horizons. They have different assumptions about labor and overhead costs and production capacity. These assumptions are valid depending on the time horizon. In the short-run, labor and overhead costs and

capacity can be considered as fixed. Under these circumstances, TOC can give the right information. However, in the long-run, all costs tend to be variable and the capacity of a plant can be either increased or decreased depending on the level of demand. Since the assumptions of ABC are long-term oriented, it can reflect the expected costs of a company correctly in this time frame. Since ABC and TOC are valid in different time horizons, they can complement each other. The weaknesses of one approach can be overcome by the strengths of the other approach, depending on the time horizon [29]. Spoede et al. states that the real power of ABC is its ability to generate the necessary information to support TOC [53]. Specifically, TOC can be used in the resource consumption ABM model that will be explained later in detail [1].

2.6.4. Past Research Performed on Integrating ABC and TOC

Although ABC and TOC may seem in contradiction with each other, various studies have been conducted on how these two approaches can be integrated to provide more accurate information to management. These studies are summarized below:

Holmen, in his article “ABC vs. TOC: It’s a Matter of Time” states that ABC and TOC are based on different time horizons. They have different assumptions about labor and overhead costs and production capacity. These assumptions are valid depending on the time horizon. In the short-run, labor and overhead costs and capacity can be considered fixed. Under these circumstances, TOC can give the right information. However, in the long-run, all costs tend to be variable and the capacity of a plant can be either increased or decreased depending on the level of demand. Since the assumptions of ABC are long-term oriented, it can reflect the expected costs of a company correctly in this time frame. Since ABC and TOC are valid in different time horizons, they can complement each other. The weaknesses of one approach can be overcome by the strengths of the other approach, depending on the time horizon [25].

Kee, in his article “Integrating ABC and the Theory of Constraints to Evaluate Outsourcing Decisions” identifies the problems associated with outsourcing and argues

that information developed from TOC and ABC may be integrated to evaluate the economic consequences of outsourcing commodity and strategic components. According to Kee, the information obtained from ABC and TOC may be used to evaluate the economic feasibility of outsourcing, the time frame over which it is feasible, and the reasons why it is economically feasible [29].

Although TOC and ABC philosophies differed in their approaches to improve the profitability of the companies in the early phase of their evolution, Gupta et al. believe that both philosophies have now evolved to a stage where consolidation can begin. Their study describes a successful application of ABC and TOC in a health care company and demonstrates that these two philosophies can work together in non-manufacturing companies [23].

MacArthur, in his article “Theory of Constraints and Activity-Based Costing: Friends or Foes?” states that ABC and TOC can be considered complementary rather than conflicting or contradictory. According to him, both of these approaches only provide information. Informed management action using that information is necessary to increase the profitability of a company [36].

Campbell et al. argue that ABC and TOC can play complementary roles in the design of a manufacturing information system. In their article, they discuss the development of a customer profitability model based on both ABC and TOC philosophies in a manufacturing company. ABC concepts are applied in those departments of the plant where employees perform supporting and customer service activities. TOC principles are applied to the factory floor, where machine-paced activities are performed. The authors also argue that understanding the principles of both ABC and TOC and applying these principles appropriately can help companies to close the communication gaps between the departments and support cross-functional decision making [5].

Demmy and Talbott criticize TOC because it does not segregate costs into variable and fixed components. They propose to provide a more accurate estimate of truly

variable costs than the materials-only assumption of standard TOC by combining ABC and TOC approaches and to address the distinction between direct and indirect fixed costs. According to them, all significant indirect fixed costs should be allocated to the product or product-line levels by using cost drivers. According to their proposal, the pool of indirect costs to be allocated would be much smaller than under standard ABC. The integration would require less effort than a traditional ABC implementation and would provide more information than the standard TOC approach. The authors believe that combining ABC and TOC approaches would permit companies to answer traditional cost-volume-profit questions, facilitate the evaluation of profitable product lines, and estimate the bottom-line figures more accurately [17].

Spoeede et al. argue that the real potential of ABC is its ability to generate data necessary to support the TOC approach. According to them, the activity-based method does not recognize the importance of an internal constraint in a system, and as a result may give wrong information about the optimal product mix of a company if it is not implemented together with a throughput-oriented management philosophy [53].

In his article, Campbell describes how a steel service center reduces its cycle time by using ABC and TOC in combination with each other. A pricing model that is developed by the company using the combined elements of both ABC and TOC is also included in the article [4].

Noreen et al. in their book The Theory of Constraints and Its Implications for Management Accounting state that product diversity is encouraged in TOC, in contrast to ABC. According to them, ABC generally discourages product diversity by shifting overhead costs to low-volume products. However, in TOC it is assumed that overhead functions, like other non-constraint work-centers, can handle additional diversity without new resources. If they cannot, the overhead resources themselves become the constraint and can be dealt with using the usual TOC approaches. As a result, they argue that TOC is able to cope with any situation without any need for information obtained from ABC [41].

Cokins in his paper “TOC vs. ABC: Friends or Foes?” articulates that ABC and TOC principles can be combined and can be applied collectively to people-intensive and machine-intensive activities for purposes of product costing and customer costing, customer and channel profitability analysis, process improvements, product mix and volume decisions, scheduling and level loading of work-center capacities, and focusing on scarce resources for throughput improvement initiatives. According to Cokins using only ABC or TOC approach provides partial solutions, and combining them creates the synergism that leads to a more complete solution. Linking these two approaches can help lessen the communication and credibility gaps among the marketing, sales, operations and accounting functions [7].

TOC can also be used in the resource consumption ABC model to improve capacity management. Past research regarding the integration of ABC and TOC to improve capacity management will be explained later in detail.

2.7 What is Capacity?

The largest asset any manufacturer has which allows it to make product for its customers is its capacity. Capacity of a plant includes all the facilities, equipment and people used to make product and the ways those facilities, equipment and people are used. It is a measure of a manufacturing enterprise's ability to provide products to its customers when needed, or a manufacturer's ability to meet demand. Effectively managing capacity can make the difference between loss and profit. For this reason, it is extremely important for managers to understand what capacity really means and how it is categorized.

Although there is no unique way to define capacity, there are two important frameworks that can be used by managers to categorize the capacity used in their plant. The traditional approach categorizes capacity into five groups: theoretical capacity, practical capacity, normal capacity, budgeted capacity, and actual capacity (See Figure

2.3) [38]. A new way of defining and categorizing the capacity of a plant is developed by CAM-I. Both approaches will be discussed in the following sections.

Theoretical capacity is the maximum output a plant can produce in a specific period. It assumes that all personnel and equipment will operate at peak efficiency. It does not allow for any downtime, waste, or idle time. Once the plant reaches this capacity level, the only way to increase capacity is by enlarging the plant [18, 38].

Practical capacity is the theoretical capacity adjusted for lost time due to nonworking days, plant breakdowns, repairs, and maintenance. Companies generally use “the practical capacity” as a measure of capacity. “Running at full capacity” usually means running at the practical capacity [18, 38].

Normal capacity is the average output of a plant over an extended period. It involves human and equipment inefficiencies and idle time [38]. While determining the normal capacity, enough years should be included to even out the cyclical patterns in sales [18].

Budgeted capacity refers to the estimate of capacity that will be utilized in a specific time period, whereas *actual capacity* refers to the work actually done in that time period. Budgeted capacity is determined by the planning process in a plant [38].

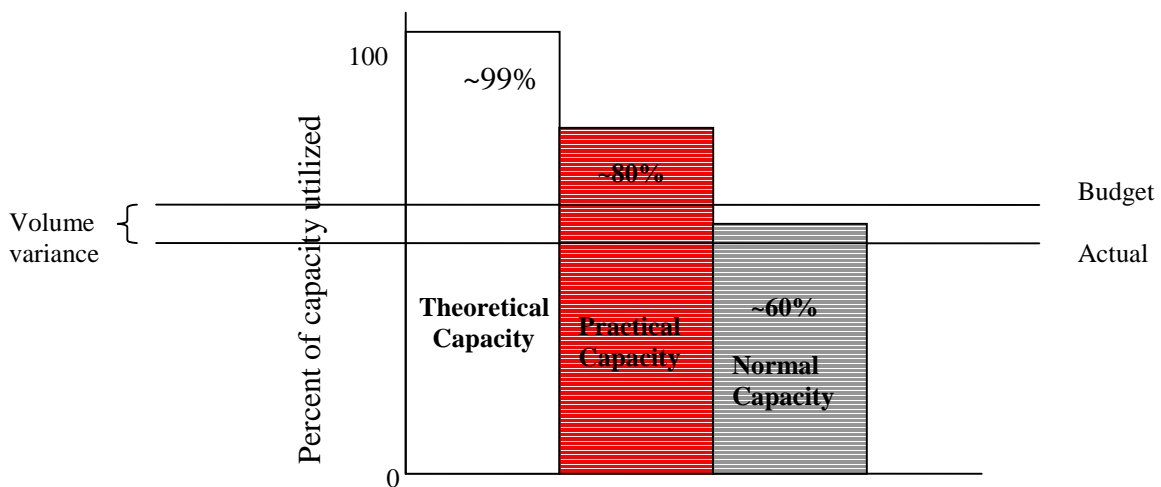


Figure 2.3 Definitions of capacity [38, p.13]

Another approach for defining capacity is developed by CAM-I (see Figure 2.4). CAM-I categorizes the capacity into three (productive, nonproductive, and idle) and uses the term “rated capacity” instead of the term “theoretical capacity” in its model. In other words, *rated capacity* is equal to the sum of the idle, nonproductive, and productive capacity in the CAM-I model [30].

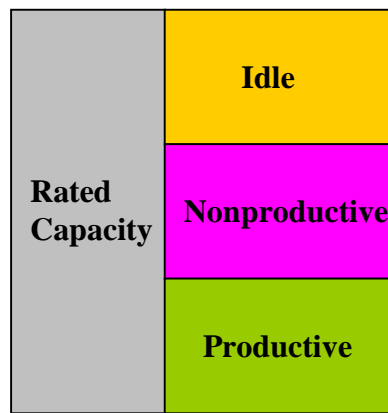


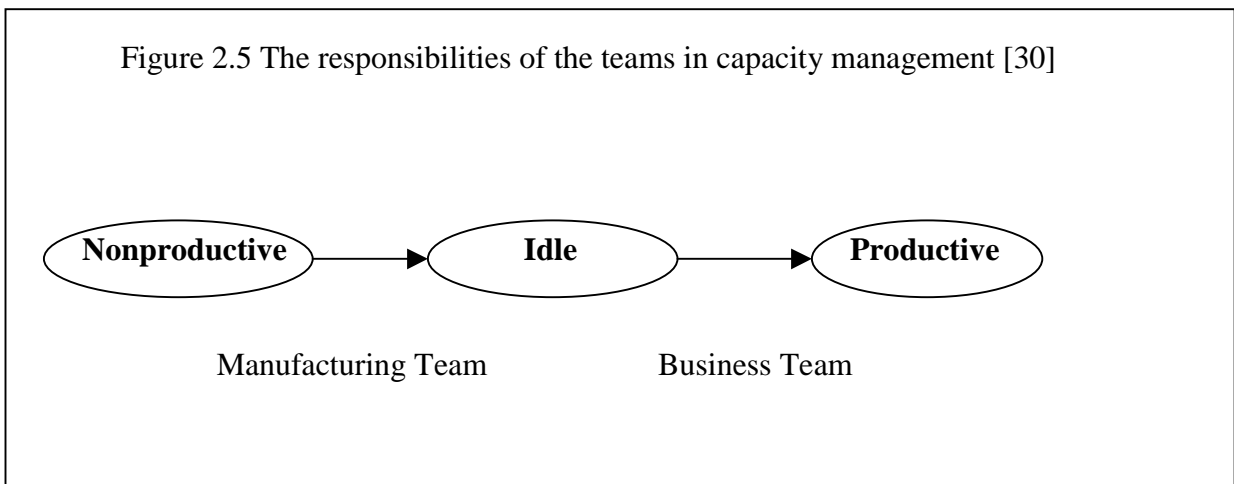
Figure 2.4. The summary capacity model developed by CAM-I [30, p.15]

Productive capacity is the capacity used to produce a product or provide a service. Tangible changes in the product or service that are of value to the customer are accomplished by using this type of capacity. The type of capacity used to develop and improve a process or product is also considered as productive [30, 55].

Nonproductive capacity includes the uses of capacity that do not result in the production of good products, or that are not included in one of the defined idle states of capacity. The setup or maintenance activities and the activities that result in scrap or rework are all considered as nonproductive [30]. These activities are also called “non-value added” activities.

Idle capacity includes idle marketable, idle not marketable, and idle off-limits capacity. *Idle marketable capacity* means that there is currently a market for this kind of capacity, but our capacity is still idle because of competitor market share, or price/cost constraints. *Idle not marketable* capacity refers to the capacity that is not demanded in the market currently or is not marketable because of the decision of management. *Idle off-limits* capacity is the capacity that is not available because of holidays, policies, contracts, etc. [30, 54].

CAM-I reports that there are two main groups that are primarily responsible for the use of capacity in a company: the business team and the manufacturing team. The manufacturing team focuses on operations and tries to increase the throughput of the system. It is the responsibility of the manufacturing team to reduce the nonproductive capacity by transforming it to the idle capacity. The business team gives strategic decisions about the capacity of the plant. The responsibility of this team is to reduce the idle capacity and increase the productive capacity (See Figure 2.5) [30].



Galloway and Waldron argue that, since theoretical capacity does not allow for any downtime, waste, or idle time, it is not possible for a plant to run at this capacity level because of the constant and challenging fluctuations and disruptions in the business. As a result, setting a goal such as “to utilize the capacity 100%” is not realistic in their

views [19]. However, McNair does not agree with this idea and argues that the goal should be to reach to the theoretical capacity since the lower the goal is, the less the improvements will be [38].

According to the TOC, the goal should be to balance the flow of production with demand, rather than maximizing the total available capacity at a plant, because increasing the capacity of a plant by increasing the utilization of a non-bottleneck resource will not change the company's throughput. In fact, this will have a negative effect on the plant's performance since it will result in an increased WIP. The goal, therefore, should be to maximize the flow of product through the system rather than to maximize the capacity of each independent resource [21]. Whether the theoretical capacity or the practical capacity should be used as the baseline for the management of bottlenecks is a debatable issue in the literature [30, 38, 55].

As mentioned before, TOC emphasizes the bottleneck because it is the bottleneck that determines the pace of the system. A system cannot produce more than the bottleneck can. In other words, the only way to balance the flow of production with demand is to reduce the bottlenecks in a system [50, 51]. Management needs information to identify the bottlenecks. Activity mapping can give this necessary information by providing us a view of the whole plant as composed of activities.

2.8 Activity Mapping To Identify Bottlenecks

There are several ways to identify the bottlenecks in a system. One way is to locate the resource where the WIP piles up. Salafatinos suggests using activity maps as a better way. An activity map is, in fact, a flowchart of activities that portrays the vertical relationships between activities in a department, horizontal connections between activities and cycle times necessary to perform each activity [51]. An activity map example showing the extra activities that are performed when an unexpected consignment is delivered to a plant is provided in Appendix F.

The cost object under ABC can be products, services, customers, etc. By considering the business process as the cost object, we can prepare “activity maps.” A business process is composed of many activities that work together to add value to a product or service. By using the “activity mapping” technique, the activities in a business process can be mapped in order to determine the flow of a product through the plant [39, 51].

Morrow and Hazell state that activity maps are adaptations of the process flowchart technique. They identify two main differences between an activity map and a process flowchart [39]:

- 1-.An activity map looks at activities rather than at detailed tasks.
- 2-.A process flowchart looks at the type of task undertaken at each stage, whereas an activity map focuses on the linkages and usage of resources.

The amount of resources used by each activity can be shown as the activity cost in the activity maps. Time used for each activity can also be shown on the chart. Both of these can be especially helpful when trying to decrease the cost and the overall cycle time of a business process. Different symbols and colors can be used in preparing the activity maps to differentiate between the activities associated with the process and the activities that support the process [39].

Activity maps can help managers to identify the bottlenecks correctly. Salafatinos defines a bottleneck as “the condition that exists when demand on a set of activities exceeds the capacity of that set of activities to support the demand” [51, p.63]. Therefore, a bottleneck does not necessarily have to be a single resource. It can occur as a result of a complex web of activities that cross the boundaries between departments. Thus, it can be said that locating a pile of WIP is not enough for identifying a bottleneck correctly. An activity map can help to find the bottleneck because it is a graphical representation of the interconnecting activities of a process. After identifying the location of a bottleneck

correctly, managers can focus on the activities that cause the problem and try to increase the throughput of the system [51].

In his article “Integrating the Theory of Constraints and Activity-Based Costing,” Salafatinos modified the way activity maps are prepared in order to make them applicable with the TOC. According to him, activity maps must have two important attributes so that they can be used with the TOC: They must illustrate gaps between activities for both overhead and production areas of the company, and they must be able to locate the set of activities that cause these gaps. In his article, Salafatinos suggests using a combination of Gantt charts and dependency grids as a way of activity mapping to ensure that these two attributes are satisfied, and he describes the three steps in preparing and using the activity maps to locate the bottlenecks. These three steps are summarized below [51]:

Step1- Prepare an activity list: First, all the activities should be identified. This can be accomplished by conducting interviews. The activities that form a business process are then grouped and numbered to make activity mapping easier. The difference between the activities identified for mapping purposes and the activities identified to implement ABC is that, in activity mapping, activities are identified in more detail compared to ABC. This is because not all activities are important to consider for cost purposes.

Step2- Prepare a Gantt chart: The next step is to prepare a Gantt chart. As can be seen in Figure 2.6, a Gantt chart lists the activities along the vertical axis and associated cycle times along the horizontal axis. Blank spaces are provided if no activity is performed at a specific time.

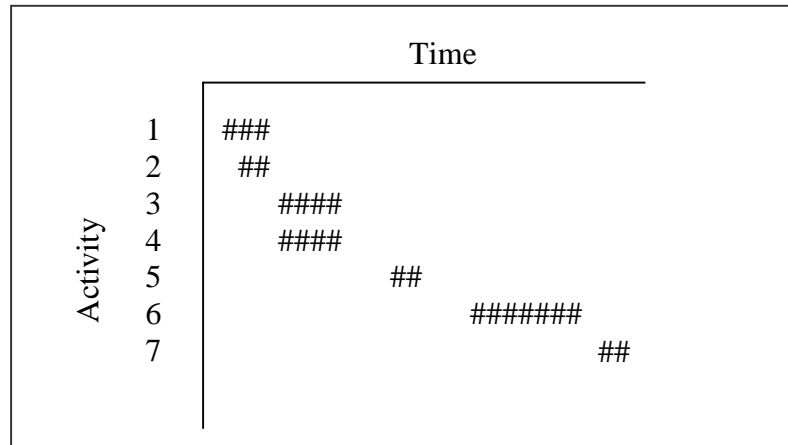


Fig. 2.6. Gantt Chart [51]

Step3- Prepare a dependency grid: The last step is to prepare a dependency diagram. A dependency diagram is used to understand the relationships between activities. An activity that supplies output to the other activities is called a "customer activity" and plotted on the horizontal axis of the dependency grid. An activity that uses the outputs of the other activities is called a "supplier activity" and is plotted on the vertical axis. Since a dependency grid reveals all the relations between activities, whenever a bottleneck activity is discovered by using the Gantt chart, that activity can be located on the dependency grid to find the other related activities that may be involved in the bottleneck (see Figure 2.7).

		Customer Activities		
		A	B	C
Supplier Activities	A	1		7
	B	2	3, 6	
	C		5	4

Figure 2.7 Dependency Grid [51]

After completing the activity maps, the processes with the longest cycle times should be identified by using the Gantt charts. Management should focus on the activities that are involved in these kinds of processes and that have the largest time gaps between their adjacent activities, since the throughput can be most increased by shortening the gap between these activities. Dependency grids should be used in order to find the other related activities that may cause this long waiting period. These activities should be located in the Gantt chart to investigate the reasons of long cycle times [51].

The last step in using activity maps is to find new alternatives and redesign the processes with the longest cycle times. Since Gantt charts and dependency grids portray the linkages between the activities more clearly, managers can identify the bottlenecks more easily and correctly, and increase the throughput of the system by focusing on the right resources [51].

In their article, "Activity-Based Cost Management: A Process Design Framework," Greenwood and Reeve use the term "activity network" instead of the term "activity mapping" and suggest using activity networks in developing the process-based activity structure of the company. Greenwood and Reeve use circles to represent the activities and one- or two-headed arrows to show the dependencies between activities, and they argue that activity networks can help management in identifying redundancies and waste in the processes more easily. The unique feature of the activity networks developed by Greenwood and Reeve is that they make a distinction between the two types of activities: the activities that have customary patterns, occur under normal operating conditions and on a repetitive basis; and the activities that occur routinely but not under normal conditions. Machine breakdowns, stockouts and customer returns can all be considered in this last category. Greenwood and Reeve use the term "event-dependent activity" in referring to the activities in this category and suggest determining and using a cost per event in order to identify the total cost of unexpected events. In fact, these costs show the cost of variation in a company, and therefore constitute crucial information for all continuous improvement programs [22].

2.9. An ABC and TOC Model to Measure the Costs of Resource Usage

In their article “Activity-Based Systems: Measuring the Costs of Resource Usage,” Kaplan and Cooper argue that a company should have two different reporting systems, the periodic financial statement and the ABC-based cost system, since they provide different information to management. The periodic financial statement gives information about the cost of activities supplied each period, and the ABC-based cost system provides information about both the estimated cost and the quantity of activities that are actually used in a certain period. The difference between the amount of activity supplied and the amount of activity used gives the amount of capacity that is not used during the period. This relation can be explained more clearly by the following formula:

$$\text{Activity availability} = \text{Activity Usage} + \text{Unused Capacity} \quad (1)$$

Cooper and Kaplan consider the term “activity availability” as the practical capacity of an activity in this formula [15].

To find the cost of activity consumed or the cost of unused capacity of the activity, an activity rate should be determined. The activity rate can be calculated by using the following formula:

$$\text{Activity rate} = (\text{Annual cost of an activity}) / (\text{Annual capacity of an activity}) \quad (2)$$

The most common approach in calculating this formula is to use the budgeted or actual capacity in the denominator. However, this approach may lead to highly variable and distorted product costs, and as a result, to incorrect decisions. As the actual or budgeted capacity decreases, the activity rate will increase, which leads to an increase in the cost of a product. In this situation, management will most probably raise the selling price of the product in order to keep the profitability constant. But, in fact this action is logically incorrect since the company will have a slack capacity at that time. Raising the selling price will not improve the profitability. On the contrary, it will lead to a decrease

in the current demand. If the company continues to act in this way (gradually increasing the selling price every time actual capacity used is dropped), the demand will continue to drop gradually [13, 15].

Cooper and Kaplan suggest using the practical capacity instead of the actual or budgeted capacity in the denominator of Equation (2) and determining the cost of products by using the activity rates calculated in this way [1]. One of the benefits of using this approach is that it does not cause a high variation and distortion in the product costs since the activity rate does not change with the demand level. Another benefit is that the results of continuous improvement efforts can be easily observed by using this approach. In the traditional approach, when demand increases, the activity rate will decrease, which may lead to the wrong conclusion that the process has been improved although there was no change in the efficiency of the process. By using an activity rate that is not dependent on the number of units produced or planned to be produced, the effects of continuous improvement efforts can be assessed more accurately [13].

Cooper and Kaplan suggest not assigning the cost of capacity that is not used in a certain period to the products produced in that period. Instead, this cost should be treated as a period cost. As the number of units produced decreases, the cost of unused (idle) capacity will increase [1, 13]. Since this approach does not assign the cost of unused capacity to the products, the expense of having idle capacity will become more apparent and stimulate the business team to find a way of utilizing this excess capacity [13].

This activity-based cost system, together with the TOC approach, can help to identify the bottlenecks, and therefore, increase the throughput. The activities that have the smallest unused capacity amounts will, most probably, form the bottleneck activities. Once these activities are identified, the five-step approach of TOC that is described in Table 2.4 should be followed to increase the throughput. As a result, it can be said that ABC-based cost system provides the necessary information for TOC. The system can also be used to perform sensitivity analysis, which will show management the effects of the small changes in the activity levels, selling prices, etc. on the bottom line figures [1].

One major weakness of Equation (1) is that it is not differentiating between the nonproductive and productive capacity of a plant. Equation (1) gives information about the total amount of the productive and nonproductive capacity of a company in the “Activity usage” component, but it is not specifying the percentage of the capacity that is used to perform the nonproductive tasks. This weakness can be overcome by using the activity maps together with the ABC and TOC model. Each process in the company can be mapped by using the activity charts. During the mapping process, different colors and symbols can be used to specify the productive and nonproductive activities. The cost and time associated with each activity can be shown on the map. By examining these activity maps, the manufacturing team can determine the processes with the highest improvement opportunities (i.e. the processes that contain the highest amount of nonproductive activities) and focus on them in order to convert the nonproductive activities to idle capacity.

Brausch and Taylor, in their article “ Who is Accounting for the Cost of Capacity? ” summarize their findings and conclusions of a study about the different approaches used in twelve companies to manage capacity. Their findings suggest that today, companies are not determining the cost of their unused or excess capacity through any special analysis. The twelve companies studied in this research have no policy statement or guideline regarding the capacity management. According to Brausch and Taylor, capacity-related decisions have a long-term effect on a company’s profitability, and therefore should be taken into consideration very carefully. They recommend that the costs of unused capacity be segregated into each manufacturing department and product group to highlight the cost and use of more specific cost centers. Segregating the costs of unused capacity will assist greatly in identifying the bottlenecks of a system. Finally, the authors suggest that any additions to the current capacity of a company should be assessed very carefully and should be tied to the evaluation of management performance [2].

Chapter III

MODEL FORMULATION

3.1 The Conventional Product-Mix Problem

Determining the best product mix that maximizes profits is one of the most fundamental decisions that a company should make. If a company does not have sufficient capacity to satisfy the demand for its products, the best action would be that it should use all of its existing resources and/or expand capacity through capital investment to produce products with the highest profit. The conventional approach to the product-mix problem is given below:

$$\text{Maximize } \sum_{i=1}^N R_i * X_i$$

subject to

$$\sum_{i=1}^N m_i * X_i \leq MT$$

$$\sum_{i=1}^N dl_i * X_i \leq DL$$

$$l_i \leq X_i \leq u_i$$

X_i is a nonnegative integer variable.

X_i : the number of units of product i (P_i) that is produced in a specific period
 N : the total number of different kinds of products that can be produced in the company

m_i : material cost per unit of product i

dl_i : direct labor cost per unit of product i

MT : the capital available for material purchase

DL : the direct labor dollars available

l_i : the lower limit of the number of units of product i that the company must produce during a specific period

u_i : the upper limit of the number of units of product i that can be produced during a specific period

R_i : the return (profit) per unit of product i

In the traditional costing approach, the overhead cost is allocated to products based on just one cost driver, generally direct labor dollars. Under this approach, the objective function of the above problem can be calculated as follows:

$$R_i * X_i = (s_i - m_i - dl_i - ovr * dl_i) * X_i$$

s_i : the selling price of one unit of product i

ovr : the overhead rate is calculated by dividing the total overhead activity capacity by the total direct labor dollars available

However, as described in Chapter 2, technological improvements and automation has increased the percentage of overhead costs greatly in the new manufacturing environment. Today, direct labor constitutes a very small portion of the total costs incurred in a plant. Consequently, using direct labor dollars as the only basis to allocate overhead costs may give incorrect or misleading information about the profitability of products and may result in poor decisions about the best product mix for a company.

As with any other model, using accurate information in the formulation is the first requirement for a successful product-mix problem. Activity-based costing can give this

required information by allocating overhead costs to products based on cost drivers that best represent the consumption of resources by products.

3.2 ABC Hierarchical Model

The costs of some activities are not always related to a unit of a product, but are attached to a higher level, such as to a batch of products or to a certain product directly [43]. In such cases, assigning the cost to each unit of product will give erroneous information. Cooper and Kaplan categorize activities into four levels according to the types of costs assigned to the products: unit-level activities, batch-level activities, product-sustaining activities, and facility-sustaining activities (See Appendix G) [13].

The costs of unit-level activities are assigned in proportion to the number of products produced. Therefore, as the volume of production increases, the costs of this kind of activities increase [43].

Batch-related activities are performed every time a batch of products is processed. In this kind of activity, resource consumption is proportional to the number of batches processed. The cost drivers used in batch-related activities are called “batch drivers.” A batch driver assigns the cost of an activity to a batch [13].

Product-sustaining activities are performed in order to continue to produce and sell individual products. The costs of these activities can be traced to each product but should not be allocated based on the number of units or batches produced, because they are not affected by the level of production volume. The only way to eliminate the cost of product-sustaining activities is to discontinue the product [13].

Facility-sustaining activities are the activities necessary for a plant to continue production. The costs of these activities are not related to the production volume or product mix. Since these activities are common to each product produced in the plant, their costs

must also be considered as common to all products. [13]. The only way to eliminate facility-sustaining expenses is to shut down (permanently close) the plant.

3.3 The Proposed Approach

In the proposed approach, three activity levels, the unit-level, the batch-level, and the product-sustaining level, are included in the product-mix decision model by using three different kinds of decision variables described below:

X_i : the number of units of product i produced in a given time period, $i= 1, 2, 3, \dots, N$.

Y_i : the number of batches of product i produced in a given time period, $i= 1, 2, 3, \dots, N$.

$$Z_i = \begin{cases} 1, & \text{if product } i \text{ is produced in a given time period, } i= 1, 2, 3, \dots, N. \\ 0, & \text{if product } i \text{ is not produced in a given time period, } i= 1, 2, 3, \dots, N. \end{cases}$$

X_i and Y_i are integer variables, Z_i is a binary variable.

Facility-sustaining expenses are not included in the analysis because closing the plant is not an option. These expenses do not change with the volume and mix of individual products [13]. The general structures of the three different types of capacity constraints including the three different activity levels are given below. Note that the subscript “ p ” is used for unit-level activities, “ q ” for batch-level activities, and “ s ” for product-sustaining level activities:

$$\sum_{i=1}^N ac_{ip} * cr_p * X_i \leq PC_p \quad p= 1, 2, \dots, UL$$

$$\sum_{i=1}^N ac_{iq} * cr_q * Y_i \leq PC_q \quad q = UL+1, UL+2, \dots, BL$$

$$\sum_{i=1}^N ac_{is} * cr_s * Z_i \leq PC_s \quad s = BL+1, BL+2, \dots, PL$$

ac_{ip}: per unit usage of activity *p* by product *i*

ac_{iq}: per batch usage of activity *q* by product *i*

ac_{is}: per product usage of activity *s* by product *i*

cr_p: the charge rate of unit-level activity *p* performed in a system

cr_q: the charge rate of batch-level activity *q* performed in a system

cr_s: the charge rate of product-sustaining level activity *s* performed in a system

PC_p: the periodic practical capacity of unit-level activity *p* performed in a system

(in \$'s)

PC_q: the periodic practical capacity of batch-level activity *q* performed in a

system (in \$'s)

PC_s: the periodic practical capacity of product-sustaining level activity *s*

performed in a system (in \$'s)

UL: the total number of unit-level activities performed in a system

BL: the total number of unit-level and batch-level activities performed in a system

PL: the total number of activities performed in a system

The charge rate of each activity (*cr_p*, *cr_q*, *cr_s*) is calculated by dividing the budgeted rate of each activity by its practical capacity for a given time horizon. Practical capacity is the maximum capacity of a system adjusted for lost time due to non-working days, plant breakdowns, repairs, and maintenance [18, 38].

In addition to the capacity-related constraints of the overhead activities described above, the following set of constraints must also be included in the new product-mix decision model:

$$(1) \quad \sum_{i=1}^N m_i * X_i \leq MT$$

$$(2) \quad \sum_{i=1}^N dl_i * X_i \leq DL$$

$$(3) \quad l_i \leq X_i \leq u_i \quad i= 1, 2, 3, \dots, N.$$

The first two constraints ensure that the amount of raw material and direct labor hours consumed during a specific period do not exceed the available capacities of these two resources in that period. Since the consumption amounts of raw material and direct labor are directly proportional to the number of units of product i produced during a specific period, X_i is used as the decision variable in these two constraints rather than the decision variables Y_i or Z_i . The third constraint guarantees two things:

- 1- The number of units of product i produced during a specific period will not exceed the demand for that period.
- 2- The number of units of product i produced during a specific period will be at least equal to the lower limit of the number of units of product i that must be produced during that period.

To define the batch sizes, the following constraint must be included in the model for each product manufactured in the system.

$$bs_i * Y_i \geq X_i \quad i= 1, 2, 3, \dots, N.$$

bs_i : the batch size of product i .

This constraint guarantees that whenever one unit of a product i is manufactured, the relevant batch-level activity costs are incurred in the objective function. If we can assume that a batch of product i has to be finished completely once that batch is started to

be processed, the inequality sign can be replaced with an equality sign in the above constraint.

When building the model, we also want to include a constraint that ensures that whenever a batch of product i is produced, the relevant product-sustaining activity costs are incurred in the objective function. The general structure of this type of constraint is given below:

$$Y_i \leq M * Z_i \quad i = 1, 2, 3, \dots, N.$$

M is a very big number.

This constraint forces Z_i to be equal to one whenever a batch of product i is produced in the system.

The objective function of the proposed product-mix decision model is to maximize profit. In the proposed model, profit can be calculated by subtracting the costs of direct labor, material, and overhead activities consumed by each product during a specific time period from the total revenue earned during the same period:

$$\begin{aligned} & \sum_{i=1}^N (s_i - m_i - dl_i) * X_i - \sum_{p=1}^{UL} \sum_{i=1}^N ac_{ip} * cr_p * X_i - \sum_{q=UL+1}^{BL} \sum_{i=1}^N ac_{iq} * cr_q * Y_i \\ & - \sum_{s=BL+1}^{PL} \sum_{i=1}^N ac_{is} * cr_s * Z_i \end{aligned}$$

The general structure of the proposed product-mix decision model is given below

[Formulation I]:

$$\text{Max} \sum_{i=1}^N (s_i - m_i - dl_i) * X_i - \sum_{p=1}^{UL} \sum_{i=1}^N ac_{ip} * cr_p * X_i - \sum_{q=UL+1}^{BL} \sum_{i=1}^N ac_{iq} * cr_q * Y_i$$

$$- \sum_{s=BL+1}^{PL} \sum_{i=1}^N ac_{is} * cr_s * Z_i$$

subject to

$$(1) \quad \sum_{i=1}^N ac_{ip} * cr_p * X_i \leq PC_p \quad p = 1, 2, \dots, UL$$

$$(2) \quad \sum_{i=1}^N ac_{iq} * cr_q * Y_i \leq PC_q \quad q = UL+1, UL+2, \dots, BL$$

$$(3) \quad \sum_{i=1}^N ac_{is} * cr_s * Z_i \leq PC_s \quad s = BL+1, BL+2, \dots, PL$$

$$(4) \quad \sum_{i=1}^N m_i * X_i \leq MT$$

$$(5) \quad \sum_{i=1}^N dl_i * X_i \leq DL$$

$$(6) \quad bs_i * Y_i = X_i \quad i = 1, 2, 3, \dots, N$$

$$(7) \quad Y_i \leq M * Z_i \quad i = 1, 2, 3, \dots, N$$

$$(8) \quad l_i \leq X_i \leq u_i \quad i = 1, 2, 3, \dots, N$$

X_i and Y_i are integer variables; Z_i is a binary variable.

M is a very big number.

All variables are greater than or equal to zero.

3.3.1 Mutually Exclusive Products

Let us denote the set of products a company can produce by PS . The number of elements in PS is equal to N . Assume that the product set, PS , can be partitioned into three subsets such that $PS = PS1 \cup PS2 \cup PS3$ where

$PS1 =$ the set of independent products

$PS2 =$ the first set of mutually exclusive products

$PS3 =$ the second set of mutually exclusive products

In other words, the company can produce as many different products from set $PS1$ as demanded during a specific time period (given that it has enough capacity). However, the company is allowed to produce only one type of product from set $PS2$ and only one type of product from set $PS3$. This assumption can be incorporated into Formulation I with the following modifications [Formulation II]:

$$\begin{aligned} \text{Max } & \sum_{i=1}^N (s_i - m_i - dl_i) * X_i - \sum_{p=1}^{UL} \sum_{i=1}^N ac_{ip} * cr_p * X_i - \sum_{q=UL+1}^{BL} \sum_{i=1}^N ac_{iq} * cr_q * Y_i \\ & - \sum_{s=BL+1}^{PL} \sum_{i=1}^N ac_{is} * cr_s * Z_i \end{aligned}$$

subject to

$$(1) \quad \sum_{i=1}^N ac_{ip} * cr_p * X_i \leq PC_p \quad p = 1, 2, \dots, UL$$

$$(2) \quad \sum_{i=1}^N ac_{iq} * cr_q * Y_i \leq PC_q \quad q = UL+1, UL+2, \dots, BL$$

$$(3) \quad \sum_{i=1}^N ac_{is} * cr_s * Z_i \leq PC_s \quad s = BL+1, BL+2, \dots, PL$$

$$(4) \quad \sum_{i=1}^N m_i * X_i \leq MT$$

$$(5) \quad \sum_{i=1}^N dl_i * X_i \leq DL$$

$$(6) \quad bs_i * Y_i = X_i \quad i = 1, 2, 3, \dots, N$$

$$(7) \quad Y_i \leq M * Z_i \quad i \in PS1$$

$$(8) \quad l_i \leq X_i \leq u_i \quad i \in PS1$$

$$(9) \quad Z_i * l_i \leq X_i \leq u_i * Z_i \quad i \in PS2, l_i \neq 0$$

$$(10) \quad \sum_{i=1}^N Z_i = 1$$

$i \in PS2$

$$(11) \quad Z_i * l_i \leq X_i \leq u_i * Z_i \quad i \in PS3, l_i \neq 0$$

$$(12) \quad \sum_{i=1}^N Z_i = 1$$

$i \in PS3$

X_i and Y_i are integer variables; Z_i is a binary variable.

M is a very big number.

All variables are greater than or equal to zero.

Constraints 9 and 11 are incorporated into the model for mutually exclusive products to guarantee that whenever the value of variable Z_i is equal to zero, the company

will not produce any product i during that period. Constraint 10 ensures that the company can produce at most one type of product from the set $PS2$, whereas constraint 12 ensures that the company can produce at most one type of product from the set $PS3$ in a given time period. When building the model, it is assumed that the value of “ l_i ” for the products in sets $PS2$ and $PS3$ are greater than zero. In other words, the company must produce a specific amount from one of the products in set $PS2$ and from one of the products in set $PS3$ during each time period.

3.3.2 Activity Flexibility

Let us assume that one of the products produced in the company, product 1 (P_1), has an activity flexibility option. In other words, the company can produce P_1 by using either route 1 or route 2, which are described in Table 3.1.

Table 3.1 The two routing alternatives for P_1

	Route 1	Route 2
Activity 1	ac_{111} hrs/ unit	ac_{112} hrs/ unit
Activity 2	ac_{121} hrs/ unit	ac_{122} hrs/ unit

ac_{ipr} = the activity p consumption amount of product i following route r
 r denotes routing alternatives. In this case, $r = 1, 2$.

The activity flexibility option of P_1 can be incorporated into the proposed product-mix decision model by partitioning the decision variable Y_1 into two new integer variables, F_{11} and F_{12} . These two new integer variables are defined below:

F_{11} : the number of batches of P_1 that follow route 1

F_{12} : the number of batches of P_1 that follow route 2

The new formulation can be written as follows:

$$\begin{aligned}
 \text{Max} \quad & \sum_{i=1}^N (s_i - m_i - dl_i) * X_i - \sum_{p=1}^2 \sum_{r=1}^2 ac_{1pr} * cr_p * bs_1 * F_{1r} - \sum_{p=1}^2 \sum_{i=2}^N ac_{ip} * cr_p * X_i \\
 & - \sum_{p=3}^{UL} \sum_{i=1}^N ac_{ip} * cr_p * X_i - \sum_{q=UL+1}^{BL} \sum_{i=1}^N ac_{iq} * cr_q * Y_i - \sum_{s=BL+1}^{PL} \sum_{i=1}^N ac_{is} * cr_s * Z_i
 \end{aligned}$$

subject to

$$(1) \quad \sum_{r=1}^2 ac_{1pr} * cr_p * bs_1 * F_{1r} + \sum_{i=2}^N ac_{ip} * cr_p * X_i \leq PC_p \quad p=1, 2$$

$$(2) \quad \sum_{i=1}^N ac_{ip} * cr_p * X_i \leq PC_p \quad p=3, 4, \dots, UL$$

$$(3) \quad \sum_{i=1}^N ac_{iq} * cr_q * Y_i \leq PC_q \quad q = UL+1, UL+2, \dots, BL$$

$$(4) \quad \sum_{i=1}^N ac_{is} * cr_s * Z_i \leq PC_s \quad s = BL+1, BL+2, \dots, PL$$

$$(5) \quad \sum_{i=1}^N m_i * X_i \leq MT$$

$$(6) \quad \sum_{i=1}^N dl_i * X_i \leq DL$$

$$(7) \quad bs_i * Y_i = X_i \quad i=1, 2, 3, \dots, N$$

$$(8) \quad Y_i \leq M * Z_i \quad i \in PSI$$

$$(9) \quad l_i \leq X_i \leq u_i \quad i \in PSI$$

$$(10) \quad Z_i * l_i \leq X_i \leq u_i * Z_i \quad i \in PS2, l_i \neq 0$$

$$(11) \quad \sum_{\substack{i=1 \\ i \in PS2}}^N Z_i = 1$$

$$(12) \quad Z_i * l_i \leq X_i \leq u_i * Z_i \quad i \in PS3, l_i \neq 0$$

$$(13) \quad \sum_{i=1}^N Z_i = 1$$

$i \in PS3$

$$(14) \quad Y_1 = \sum_{r=1}^2 F_{1r}$$

X_i , Y_1 , and F_{1r} are integer variables; Z_i is a binary variable.

M is a very big number.

All variables are greater than or equal to zero.

When developing the above model, it is assumed that a batch of P_1 always follows the same route. In other words, whenever one unit of P_1 is started to be manufactured by using a specific route, all the other parts in that batch must follow the same route. This assumption is included in the model by defining F_{11} and F_{12} as batch-level variables instead of unit-level variables. Note that the variable “ X_i ” is replaced by the terms “ $b_{s_1} * F_{11}$ ” and “ $b_{s_1} * F_{12}$ ” in the capacity constraints of activities with the flexibility option (constraint 1). Constraint 14 ensures that the total number of batches of product 1 produced during a specific time horizon is equal to the summation of the total number of batches of product 1 produced by following route 1 and the total number of batches of the same product produced by following route 2.

In the above formulation, we assumed that only product 1 has an activity flexibility option. If more than one product has this flexibility option, this feature can be incorporated into the model as follows:

We need to partition the decision variable Y_i into two new integer variables, F_{i1} and F_{i2} , such that:

F_{ir} : the number of batches of product i produced by using route r during a specific time period $i= 1, 2, 3, \dots, N$ $r=1, 2$.

Let us also define two sets of products, $PF1$ and $PF2$, such that $PS= PF1 \cup PF2$.

Remember that PS denotes the set of all products that can be produced in the company.

The sets $PF1$ and $PF2$ are defined below:

$PF1$: the set of products with the activity flexibility option

$PF2$: the set of products that do not have an activity flexibility option

The new formulation can be written as follows [Formulation III]:

$$\text{Max} \sum_{i=1}^N (s_i - m_i - dl_i) * X_i - \sum_{p=1}^2 \sum_{i=1}^N \sum_{r=1}^2 ac_{ipr} * cr_p * bs_i * F_{ir} - \sum_{p=1}^2 \sum_{i=1}^N ac_{ip} * cr_p * X_i$$

$i \in PF1$ $i \in PF2$

$$- \sum_{p=3}^{UL} \sum_{i=1}^N ac_{ip} * cr_p * X_i - \sum_{q=UL+1}^{BL} \sum_{i=1}^N ac_{iq} * cr_q * Y_i - \sum_{s=BL+1}^{PL} \sum_{i=1}^N ac_{is} * cr_s * Z_i$$

subject to

$$(1) \quad \sum_{i=1}^N \sum_{r=1}^2 ac_{ipr} * cr_p * bs_i * F_{ir} + \sum_{i=1}^N ac_{ip} * cr_p * X_i \leq PC_p \quad p= 1, 2$$

$i \in PF1$ $i \in PF2$

$$(2) \quad \sum_{i=1}^N ac_{ip} * cr_p * X_i \leq PC_p \quad p = 3, 4, \dots, UL$$

$$(3) \quad \sum_{i=1}^N ac_{iq} * cr_q * Y_i \leq PC_q \quad q = UL+1, UL+2, \dots, BL$$

$$(4) \quad \sum_{i=1}^N ac_{is} * cr_s * Z_i \leq PC_s \quad s = BL+1, BL+2, \dots, PL$$

$$(5) \quad \sum_{i=1}^N m_i * X_i \leq MT$$

$$(6) \quad \sum_{i=1}^N dl_i * X_i \leq DL$$

$$(7) \quad bs_i * Y_i = X_i \quad i = 1, 2, 3, \dots, N$$

$$(8) \quad Y_i \leq M * Z_i \quad i \in PS1$$

$$(9) \quad l_i \leq X_i \leq u_i \quad i \in PS1$$

$$(10) \quad Z_i * l_i \leq X_i \leq u_i * Z_i \quad i \in PS2, l_i \neq 0$$

$$(11) \quad \sum_{i=1}^N Z_i = 1$$

$i \in PS2$

$$(12) \quad Z_i * l_i \leq X_i \leq u_i * Z_i \quad i \in PS3, l_i \neq 0$$

$$(13) \quad \sum_{i=1}^N Z_i = 1$$

$i \in PS3$

$$(14) \quad Y_i = \sum_{r=1}^2 F_{ir} \quad , i \in PF1$$

X_i, F_{ir}, Y_i are integer variables; Z_i is a binary variable.

M is a very big number.

All variables are greater than or equal to zero.

3.3.3 Outsourcing Decision

Let us assume that one of the activities (activity #3) of product i can be outsourced. This outsourcing decision can be included in the model as follows:

Let us define two sets of products, $PO1$ and $PO2$, such that $PS = PO1 \cup PO2$. The sets $PO1$ and $PO2$ are defined below:

$PO1$: the set of products without the outsourcing option

$PO2$: the set of products with the outsourcing option

To simplify the model, it is assumed that the set $PO2$ and the set $PF1$ do not have any common elements, that is products with an activity flexibility option do not have an outsourcing option and vice versa. For $P_i \in PO2$, we can partition the variable X_i into two new integer variables, J_{i1} and J_{i2} , such that the summation of J_{i1} and J_{i2} is equal to X_i .

$$J_{i1} = \begin{cases} X_i, & \text{if activity \#3 of product } i \text{ is performed in the company.} \\ 0, & \text{if activity \#3 of product } i \text{ outsourced.} \end{cases}$$

$$J_{i2} = \begin{cases} X_i, & \text{if activity \#3 of product } i \text{ outsourced.} \\ 0, & \text{if activity \#3 of product } i \text{ is performed in the company.} \end{cases}$$

It is further assumed that activity #3 must either be outsourced completely, or performed in-house for each unit of product i . In other words, J_{i1} and J_{i2} must be mutually exclusive, that is one must be equal to zero when the other is equal to X_i . This assumption can be incorporated into the model by defining a new binary variable K_i such that:

$$K_i = \begin{cases} 0, & \text{if activity \#3 of product } i \text{ is performed in the company} \\ 1, & \text{if activity \#3 of product } i \text{ outsourced} \end{cases}$$

The new constraints that guarantee that J_{i1} and J_{i2} are mutually exclusive can be written as follows:

$$l_i * (1 - K_i) \leq J_{i1} \leq u_i * (1 - K_i) \quad , i \in PO2$$

$$l_i * K_i \leq J_{i2} \leq u_i * K_i \quad , i \in PO2$$

These two constraints also ensure that the amount of P_i produced during a specific period will be between the lower and upper limits of P_i demanded for that period.

Activities performed in a company are not independent. In other words, outsourcing one of the activities for P_i would also affect the consumption amount of other activities by this product. This feature can be incorporated into the proposed model as follows:

Let us define six new coefficients, ac_{ip1} , ac_{ip2} , ac_{iq1} , ac_{iq2} , ac_{is1} , ac_{is2} , for all P_i 's that are in the set of PO2 such that:

ac_{ip1} : per unit usage of activity p by product i if activity #3 of this product is performed in-house

ac_{ip2} : per unit usage of activity p by product i if activity #3 of this product is outsourced

ac_{iq1} : per batch usage of activity q by product i if activity #3 of this product is performed in-house

ac_{iq2} : per batch usage of activity q by product i if activity #3 of this product is outsourced

ac_{is1} : per product usage of activity s by product i if activity #3 of this product is performed in-house

ac_{is2} : per product usage of activity s by product i if activity #3 of this product is outsourced

In the new product-mix decision model, the coefficient of J_{i1} will be equal to the multiplication of ac_{ip1} by cr_p (the activity charge rate) in the capacity-related constraints of unit-level activities, and the multiplication of ac_{iq1} by cr_q / bs_i in the capacity-related constraints of batch-level activities. The coefficient of J_{i2} will be equal to the multiplication of ac_{ip2} by cr_p in the capacity-related constraints of unit-level activities, and the multiplication of ac_{iq2} by cr_q / bs_i in the capacity-related constraints of batch-level activities. The effect of the outsourcing option on the product-sustaining level activities can be included in the model as described below:

$$\sum_{\substack{i=1 \\ i \in PO1}}^N ac_{is} * cr_s * Z_i + \sum_{\substack{i=1 \\ i \in PO2}}^N \{ac_{is1} * cr_s * Z_i + (ac_{is2} - ac_{is1}) * cr_s * K_i\} \leq PC_s$$

$$s = BL+1, BL+2, \dots, PL$$

For products with the outsourcing option ($P_i \in PO2$), the coefficient of Z_i will be equal to the multiplication of ac_{is1} by cr_s . A new term ($ac_{is2} - ac_{is1}$) that will add the increase in the activity consumption amount of product i if activity #3 of this product is outsourced ($K_i = 1$) should be included in the capacity-related constraints of product-sustaining level activities.

Note that we do not need to include the decision variable J_{i2} in the capacity constraint of the activity that can be outsourced (activity #3). This is because, if activity #3 of P_i , which is an element of the set $PO2$, is outsourced, this product will not consume any amount of activity #3 available in the company. However, the purchase price of this activity (pp_i) from a vendor for each unit of P_i produced during a specific period must be included in the objective function of the new model. The new formulation including the outsourcing option for all the P_i 's in the set $PO2$ can be written as follows [Formulation IV]:

$$\begin{aligned}
& \text{Max } \sum_{i=1}^N (s_i - m_i - dl_i) * X_i - \sum_{p=1}^2 \sum_{i=1}^N \sum_{r=1}^2 ac_{ipr} * cr_p * bs_i * F_{ir} - \sum_{p=1}^2 \sum_{i=1}^N ac_{ip} * cr_p * X_i \\
& \hspace{15em} i \in PF1 \hspace{15em} i \in PF2 \cap PO1 \\
& - \sum_{p=1}^2 \sum_{i=1}^N (ac_{ip1} * cr_p * J_{i1} + ac_{ip2} * cr_p * J_{i2}) - \sum_{i=1}^N ac_{i3} * cr_3 * X_i - \sum_{i=1}^N ac_{i3} * cr_3 * J_{i1} \\
& \hspace{15em} i \in PO2 \hspace{15em} i \in PO1 \hspace{15em} i \in PO2 \\
& - \sum_{p=4}^{UL} \sum_{i=1}^N ac_{ip} * cr_p * X_i - \sum_{p=4}^{UL} \sum_{i=1}^N (ac_{ip1} * cr_p * J_{i1} + ac_{ip2} * cr_p * J_{i2}) \\
& \hspace{15em} i \in PO1 \hspace{15em} i \in PO2
\end{aligned}$$

$$- \sum_{q=UL+1}^{BL} \sum_{i=1}^N ac_{iq} * cr_q * Y_i - \sum_{q=UL+1}^{BL} \sum_{i=1}^N (ac_{iq1} * cr_q * J_{i1} / bs_i + ac_{iq2} * cr_q * J_{i2} / bs_i)$$

$i \in PO1 \qquad i \in PO2$

$$- \sum_{s=BL+1}^{PL} \sum_{i=1}^N ac_{is} * cr_s * Z_i - \sum_{s=BL+1}^{PL} \sum_{i=1}^N \{ac_{is1} * cr_s * Z_i + (ac_{is2} - ac_{is1}) * cr_s * K_i\}$$

$i \in PO1 \qquad i \in PO2$

$$- \sum_{i=1}^N pp_i * J_{i2}$$

$i \in PO2$

subject to

$$(1) \quad \sum_{i=1}^N \sum_{r=1}^2 ac_{ipr} * cr_p * bs_i * F_{ir} + \sum_{i=1}^N ac_{ip} * cr_p * X_i$$

$i \in PF1 \qquad i \in PF2 \cap PO1$

$$+ \sum_{i=1}^N (ac_{ip1} * cr_p * J_{i1} + ac_{ip2} * cr_p * J_{i2}) \leq PC_p \quad p=1,2$$

$i \in PO2$

$$(2) \quad \sum_{i=1}^N ac_{ip} * cr_p * X_i + \sum_{i=1}^N ac_{ip} * cr_p * J_{i1} \leq PC_p \quad p=3$$

$i \in PO1 \qquad i \in PO2$

$$(3) \quad \sum_{i=1}^N ac_{ip} * cr_p * X_i + \sum_{i=1}^N (ac_{ip1} * cr_p * J_{i1} + ac_{ip2} * cr_p * J_{i2}) \leq PC_p \quad p=4, 5, \dots, UL$$

$i \in PO1 \qquad i \in PO2$

$$(4) \quad \sum_{i \in PO1}^N ac_{iq} * cr_q * Y_i + \sum_{i \in PO2}^N (ac_{iq1} * cr_q * J_{i1}/bs_i + ac_{iq2} * cr_q * J_{i2}/bs_i) \leq PC_q$$

$$q = UL+1, UL+2, \dots, BL$$

$$(5) \quad \sum_{i \in PO1}^N ac_{is} * cr_s * Z_i + \sum_{i \in PO2}^N \{ac_{is1} * cr_s * Z_i + (ac_{is2} - ac_{is1}) * cr_s * K_i\} \leq PC_s$$

$$s = BL+1, BL+2, \dots, PL$$

$$(6) \quad \sum_{i=1}^N m_i * X_i \leq MT$$

$$(7) \quad \sum_{i=1}^N dl_i * X_i \leq DL$$

$$(8) \quad bs_i * Y_i = X_i \quad i = 1, 2, 3, \dots, N$$

$$(9) \quad Y_i \leq M * Z_i \quad i \in PS1$$

$$(10) \quad l_i \leq X_i \leq u_i \quad i \in PS1$$

$$(11) \quad Z_i * l_i \leq X_i \leq u_i * Z_i \quad i \in PS2, l_i \neq 0$$

$$(12) \quad \sum_{i=1}^N Z_i = 1$$

$i \in PS2$

$$(13) \quad Z_i * l_i \leq X_i \leq u_i * Z_i \quad i \in PS3, l_i \neq 0$$

$$(14) \quad \sum_{i=1}^N Z_i = 1$$

$i \in PS3$

$$(15) \quad Y_i = \sum_{r=1}^2 F_{ir} \quad , i \in PFI$$

$$(16) \quad l_i * (1 - K_1) \leq J_{i1} \leq u_i * (1 - K_1) \quad , i \in PO2$$

$$(17) \quad l_i * K_i \leq J_{i2} \leq u_i * K_i \quad , i \in PO2$$

$$(18) \quad X_i = J_{i1} + J_{i2}$$

X_i, Y_i, J_{i1}, J_{i2} , and F_{ir} are integer variables; Z_i, K_i are binary variables.

M is a very big number.

All variables are greater than or equal to zero.

3.3.4 Fixed Costs

Now assume that for some of the products, if P_i exceeds a certain production level, there is a one-time fixed cost associated with this product. Also, assume that for some of the products, if P_i exceeds a second level of production, there is an additional fixed cost that the company must pay. Formulation IV can be modified to include these two assumptions as follows:

Let us partition the set of all products that can be produced in the company, PS , into three new subsets, $FC1$, $FC2$, and $FC3$, such that $PS = FC1 \cup FC2 \cup FC3$. The three new subsets can be defined as follows:

FC1: the set of P_i 's with no fixed costs

FC2: the set of P_i 's with one-time fixed costs

FC3: the set of P_i 's with two-time fixed costs

For products in the set *FC2* or *FC3*, we will denote the amount of production level of P_i that the company can reach without paying a one-time fixed cost by fa_{i1} . For products in the set *FC3*, the amount of production level of P_i that the company can reach without paying a second fixed cost is denoted by fa_{i2} . Furthermore, we need to define a new binary variable, C_{i1} , for the products in the set *FC2* or *FC3*, and an additional binary variable, C_{i2} , for the products in the set *FC3* such that:

$$C_{i1} = \begin{cases} 0, & \text{if the number of units of } P_i \text{ produced by the company is less than } fa_{i1}. \\ 1, & \text{if the number of units of } P_i \text{ produced by the company exceeds } fa_{i1}. \end{cases}$$

$$C_{i2} = \begin{cases} 0, & \text{if the number of units of } P_i \text{ produced by the company is less than } fa_{i2}. \\ 1, & \text{if the number of units of } P_i \text{ produced by the company exceeds } fa_{i2}. \end{cases}$$

The three new constraints that must be included in the new formulation can be written as follows:

$$\begin{aligned} X_i &\leq fa_{i1} + (u_i - fa_{i1}) * C_{i1} && , i \in FC2 \cup FC3 \\ X_i &\leq fa_{i2} + (u_i - fa_{i2}) * C_{i2} && , i \in FC3 \\ C_{i1} &\geq C_{i2} && , i \in FC3 \end{aligned}$$

Let fc_{i1} denote the first-time fixed cost for product i that is an element of $FC2 \cup FC3$, and let fc_{i2} denote the second-time fixed cost for product i that is an element of $FC3$. In order to incorporate the fixed costs associated with products that are elements of the sets *FC2*

or $FC3$ into the proposed model, the following expression needs to be included in the objective function:

$$\sum_{\substack{i=1 \\ i \in FC2 \cup FC3}}^N fc_{i1} * C_{i1} - \sum_{\substack{i=1 \\ i \in FC3}}^N fc_{i2} * C_{i2}$$

The new model can be written as follows [Formulation V]:

$$\begin{aligned} & Max \sum_{i=1}^N (s_i - m_i - dl_i) * X_i - \sum_{\substack{p=1 \\ i \in PF1}}^2 \sum_{i=1}^N \sum_{r=1}^2 ac_{ipr} * cr_p * bs_i * F_{ir} - \sum_{\substack{p=1 \\ i \in PF2 \cap PO1}}^2 \sum_{i=1}^N ac_{ip} * cr_p * X_i \\ & - \sum_{\substack{p=1 \\ i \in PO2}}^2 \sum_{i=1}^N (ac_{ip1} * cr_p * J_{i1} + ac_{ip2} * cr_p * J_{i2}) - \sum_{\substack{i=1 \\ i \in PO1}}^N ac_{i3} * cr_3 * X_i - \sum_{\substack{i=1 \\ i \in PO2}}^N ac_{i3} * cr_3 * J_{i1} \\ & - \sum_{\substack{p=4 \\ i \in PO1}}^{UL} \sum_{i=1}^N ac_{ip} * cr_p * X_i - \sum_{\substack{p=4 \\ i \in PO2}}^{UL} \sum_{i=1}^N (ac_{ip1} * cr_p * J_{i1} + ac_{ip2} * cr_p * J_{i2}) \\ & - \sum_{\substack{q=UL+1 \\ i \in PO1}}^{BL} \sum_{i=1}^N ac_{iq} * cr_q * Y_i - \sum_{\substack{q=UL+1 \\ i \in PO2}}^{BL} \sum_{i=1}^N (ac_{iq1} * cr_q * J_{i1} / bs_i + ac_{iq2} * cr_q * J_{i2} / bs_i) \\ & - \sum_{\substack{s=BL+1 \\ i \in PO1}}^{PL} \sum_{i=1}^N ac_{is} * cr_s * Z_i - \sum_{\substack{s=BL+1 \\ i \in PO2}}^{PL} \sum_{i=1}^N \{ ac_{is1} * cr_s * Z_i + (ac_{is2} - ac_{is1}) * cr_s * K_i \} \\ & - \sum_{\substack{i=1 \\ i \in PO2}}^N pp_i * J_{i2} - \sum_{\substack{i=1 \\ i \in FC2 \cup FC3}}^N fc_{i1} * C_{i1} - \sum_{\substack{i=1 \\ i \in FC3}}^N fc_{i2} * C_{i2} \end{aligned}$$

subject to

$$(1) \quad \sum_{i \in PF1}^N \sum_{r=1}^2 ac_{ipr} * cr_p * bs_i * F_{ir} + \sum_{i \in PF2 \cap PO1}^N ac_{ip} * cr_p * X_i$$

$$+ \sum_{i \in PO2}^N (ac_{ip1} * cr_p * J_{i1} + ac_{ip2} * cr_p * J_{i2}) \leq PC_p \quad p=1,2$$

$$(2) \quad \sum_{i \in PO1}^N ac_{ip} * cr_p * X_i + \sum_{i \in PO2}^N ac_{ip} * cr_p * J_{i1} \leq PC_p \quad p=3$$

$$(3) \quad \sum_{i \in PO1}^N ac_{ip} * cr_p * X_i + \sum_{i \in PO2}^N (ac_{ip1} * cr_p * J_{i1} + ac_{ip2} * cr_p * J_{i2}) \leq PC_p \quad p=4, 5, \dots, UL$$

$$(4) \quad \sum_{i \in PO1}^N ac_{iq} * cr_q * Y_i + \sum_{i \in PO2}^N (ac_{iq1} * cr_q * J_{i1} / bs_i + ac_{iq2} * cr_q * J_{i2} / bs_i) \leq PC_q$$

$$q = UL+1, UL+2, \dots, BL$$

$$(5) \quad \sum_{i \in PO1}^N ac_{is} * cr_s * Z_i + \sum_{i \in PO2}^N \{ac_{is1} * cr_s * Z_i + (ac_{is2} - ac_{is1}) * cr_s * K_i\} \leq PC_s$$

$$s = BL+1, BL+2, \dots, PL$$

$$(6) \quad \sum_{i=1}^N m_i * X_i \leq MT$$

$$(7) \quad \sum_{i=1}^N dl_i * X_i \leq DL$$

$$(8) \quad bs_i * Y_i = X_i \quad i = 1, 2, 3, \dots, N$$

$$(9) \quad Y_i \leq M * Z_i \quad i \in PS1$$

$$(10) \quad l_i \leq X_i \leq u_i \quad i \in PS1$$

$$(11) \quad Z_i * l_i \leq X_i \leq u_i * Z_i \quad i \in PS2, l_i \neq 0$$

$$(12) \quad \sum_{i=1}^N Z_i = 1$$

$i \in PS2$

$$(13) \quad Z_i * l_i \leq X_i \leq u_i * Z_i \quad i \in PS3, l_i \neq 0$$

$$(14) \quad \sum_{i=1}^N Z_i = 1$$

$i \in PS3$

$$(15) \quad Y_i = \sum_{r=1}^2 F_{ir} \quad i \in PF1$$

$$(16) \quad l_i * (1 - K_i) \leq J_{i1} \leq u_i * (1 - K_i) \quad i \in PO2$$

$$(17) \quad l_i * K_i \leq J_{i2} \leq u_i * K_i \quad i \in PO2$$

$$(18) \quad X_i = J_{i1} + J_{i2}$$

$$(19) \quad X_i \leq fa_{i1} + (u_i - fa_{i1}) * C_{i1} \quad i \in FC2 \cup FC3$$

$$(20) \quad X_i \leq fa_{i2} + (u_i - fa_{i2}) * C_{i2} \quad i \in FC3$$

$$(21) \quad C_{i1} \geq C_{i2} \quad i \in FC3$$

$X_i, Y_i, J_i, J_{i2},$ and F_{ir} are integer variables; $Z_i, K_i, C_{i1},$ and C_{i2} are binary variables.

M is a very big number.

All variables are greater than or equal to zero.

3.3.5 The Multi-Period Product-Mix Decision Model

The product-mix decision model developed in this chapter can be extended to include more than one time period. Let t denote a specific time period and T denote the total number of time periods. The decision variables of the new model are described below:

X_{it} : the number of units of product i produced during period $t, \quad i = 1, 2, 3, \dots, N$
 $t = 1, 2, 3, \dots, T$

Y_{it} : the number of batches of product i produced during period $t, \quad i = 1, 2, 3, \dots, N$
 $t = 1, 2, 3, \dots, T$

$$Z_{it} = \begin{cases} 1, & \text{if product } i \text{ is produced during period } t \\ 0, & \text{if product } i \text{ is not produced during period } t \end{cases} \quad \begin{matrix} i = 1, 2, 3, \dots, N \\ t = 1, 2, 3, \dots, T \end{matrix}$$

F_{irt} : the number of batches of product i produced by using route r during period t

$$i = 1, 2, 3, \dots, N$$

$$r = 1, 2$$

$$t = 1, 2, 3, \dots, T$$

$$J_{i1t} = \begin{cases} X_{it}, & \text{if activity \#3 of product } i \text{ is performed in the company during} \\ & \text{period } t. \\ 0, & \text{if activity \#3 of product } i \text{ outsourced during period } t. \end{cases}$$

$$J_{i2t} = \begin{cases} X_{it}, & \text{if activity \#3 of product } i \text{ outsourced during period } t. \\ 0, & \text{if activity \#3 of product } i \text{ is performed in the company during period } t. \end{cases}$$

$$K_i = \begin{cases} 0, & \text{if activity \#3 of product } i \text{ is performed in the company} \\ 1, & \text{if activity \#3 of product } i \text{ outsourced} \end{cases}$$

$$C_{i1t} = \begin{cases} 0, & \text{if the number of units of } P_i \text{ produced by the company during period } t \text{ is} \\ & \text{less than } fa_{i1t} \\ 1, & \text{if the number of units of } P_i \text{ produced by the company during period } t \\ & \text{exceeds } fa_{i1t} \end{cases}$$

$$C_{i2t} = \begin{cases} 0, & \text{if the number of units of } P_i \text{ produced by the company during period } t \text{ is} \\ & \text{less than } fa_{i2t} \\ 1, & \text{if the number of units of } P_i \text{ produced by the company during period } t \\ & \text{exceeds } fa_{i2t} \end{cases}$$

The coefficients used in the proposed multi-period product-mix decision model are explained below:

s_{it} : the selling price of product i during period t

m_{it} : material cost per unit of product i during period t

dl_{it} : direct labor cost per unit of product i during period t

bs_i : the batch size of product i (It is assumed that the batch size of product i does not vary with time.)

MT_t : the capital available for material purchase during period t

DL_t : the direct labor dollars available during period t

l_{it} : the lower limit of the number of units of product i demanded during period t

u_{it} : the upper limit of the number of units of product i demanded during period t

ac_{ipt} : per unit usage of activity p by product i during period t , $i \in PO1$

ac_{iprt} : per unit usage of activity p by product i following route r during period t , $i \in PF1$

ac_{ip1t} : per unit usage of activity p by product i if activity #3 of this product is performed in-house during period t , $i \in PO2$

ac_{ip2t} : per unit usage of activity p by product i if activity #3 of this product is outsourced during period t , $i \in PO2$

ac_{iqt} : per batch usage of activity q by product i during period t , $i \in PO1$

ac_{iq1t} : per batch usage of activity q by product i if activity #3 of this product is performed in-house during period t , $i \in PO2$

ac_{iq2t} : per batch usage of activity q by product i if activity #3 of this product is outsourced during period t , $i \in PO2$
 ac_{ist} : per product usage of activity s by product i during period t , $i \in PO1$
 ac_{is1t} : per product usage of activity s by product i if activity #3 of this product is performed in-house during period t , $i \in PO2$
 ac_{is2t} : per product usage of activity s by product i if activity #3 of this product is outsourced during period t , $i \in PO2$
 cr_{pt} : the charge rate of unit-level activity p performed in a system during period t
 cr_{qt} : the charge rate of batch-level activity q performed in a system during period t
 cr_{st} : the charge rate of product-sustaining level activity s performed in a system during period t
 PC_{pt} : the practical capacity of unit-level activity p performed in a system during period t (in \$'s)
 PC_{qt} : the practical capacity of batch-level activity q performed in a system during period t (in \$'s)
 PC_{st} : the practical capacity of product-sustaining level activity s performed in a system during period t (in \$'s)
 pp_i : the purchase price of activity #3 per unit of product i during period t
 fc_{i1t} : the first-time fixed cost for product i during period t , $i \in FC2 \cup FC3$
 fc_{i2t} : the second-time fixed cost for product i during period t , $i \in FC3$

While developing the new model, it is assumed that the elements of the sets PS , $PS1$, $PS2$, $PS3$, $PF1$, $PF2$, $PO1$, $PO2$, $FC1$, $FC2$, and $FC3$ do not vary by time. For example, if product i has an activity flexibility option in period I , it will continue to have this flexibility option for the remaining $T-I$ periods. The proposed multi-period product-mix decision model is given below [Formulation VI]:

$$\begin{aligned}
& \text{Max} \sum_{t=1}^T \sum_{i=1}^N (s_{it} - m_{it} - dl_{it}) * X_{it} - \sum_{t=1}^T \sum_{p=1}^2 \sum_{i=1}^N \sum_{r=1}^2 ac_{iprt} * cr_{pt} * bs_i * F_{irt} \\
& \qquad \qquad \qquad i \in PF1 \\
& - \sum_{t=1}^T \sum_{p=1}^2 \sum_{i=1}^N ac_{ipt} * cr_{pt} * X_{it} - \sum_{t=1}^T \sum_{p=1}^2 \sum_{i=1}^N (ac_{ip1t} * cr_{pt} * J_{i1t} + ac_{ip2t} * cr_{pt} * J_{i2t}) \\
& \qquad \qquad \qquad i \in PF2 \cap PO1 \qquad \qquad \qquad i \in PO2 \\
& - \sum_{t=1}^T \sum_{i=1}^N ac_{i3t} * cr_{3t} * X_{it} - \sum_{t=1}^T \sum_{i=1}^N ac_{i3t} * cr_{3t} * J_{i1t} - \sum_{t=1}^T \sum_{p=4}^{UL} \sum_{i=1}^N ac_{ipt} * cr_{pt} * X_{it} \\
& \qquad \qquad \qquad i \in PO1 \qquad \qquad \qquad i \in PO2 \qquad \qquad \qquad i \in PO1 \\
& - \sum_{t=1}^T \sum_{p=4}^{UL} \sum_{i=1}^N (ac_{ip1t} * cr_{pt} * J_{i1t} + ac_{ip2t} * cr_{pt} * J_{i2t}) - \sum_{t=1}^T \sum_{q=UL+1}^{BL} \sum_{i=1}^N ac_{iqt} * cr_{qt} * Y_{it} \\
& \qquad \qquad \qquad i \in PO2 \qquad \qquad \qquad i \in PO1 \\
& - \sum_{t=1}^T \sum_{q=UL+1}^{BL} \sum_{i=1}^N (ac_{iq1t} * cr_{qt} * J_{i1t} / bs_i + ac_{iq2t} * cr_{qt} * J_{i2t} / bs_i) \\
& \qquad \qquad \qquad i \in PO2 \\
& - \sum_{t=1}^T \sum_{s=BL+1}^{PL} \sum_{i=1}^N ac_{ist} * cr_{st} * Z_{it} + \sum_{t=1}^T \sum_{s=BL+1}^{PL} \sum_{i=1}^N \{ac_{is1t} * cr_{st} * Z_{it} + (ac_{is2t} - ac_{is1t}) * cr_{st} * K_i\} \\
& \qquad \qquad \qquad i \in PO1 \qquad \qquad \qquad i \in PO2 \\
& - \sum_{t=1}^T \sum_{i=1}^N pp_{it} * J_{i2t} - \sum_{t=1}^T \sum_{i=1}^N fc_{i1t} * C_{i1t} - \sum_{t=1}^T \sum_{i=1}^N fc_{i2t} * C_{i2t} \\
& \qquad \qquad \qquad i \in PO2 \qquad \qquad \qquad i \in FC2 \cup FC3 \qquad \qquad \qquad i \in FC3
\end{aligned}$$

subject to

$$(1) \quad \sum_{i \in PF1}^N \sum_{r=1}^2 ac_{iprt} * cr_{pt} * bs_i * F_{irt} + \sum_{i \in PF2 \cap PO1}^N ac_{ipt} * cr_{pt} * X_{it}$$

$$+ \sum_{i \in PO2}^N (ac_{ip1t} * cr_{pt} * J_{i1t} + ac_{ip2t} * cr_{pt} * J_{i2t}) \leq PC_{pt}, p=1, 2 \quad t=1, 2, 3, \dots, T$$

$$(2) \quad \sum_{i \in PO1}^N ac_{ipt} * cr_{pt} * X_{it} + \sum_{i \in PO2}^N ac_{ip1t} * cr_{pt} * J_{i1t} \leq PC_{pt}, p=3 \quad t=1, 2, 3, \dots, T$$

$$(3) \quad \sum_{i \in PO1}^N ac_{ipt} * cr_{pt} * X_{it} + \sum_{i \in PO2}^N (ac_{ip1t} * cr_{pt} * J_{i1t} + ac_{ip2t} * cr_{pt} * J_{i2t}) \leq PC_{pt}$$

$$, p=4, 5, \dots, UL \quad t=1, 2, 3, \dots, T$$

$$(4) \quad \sum_{i \in PO1}^N ac_{iqt} * cr_{qt} * Y_{it} + \sum_{i \in PO2}^N (ac_{iq1t} * cr_{qt} * J_{i1t} / bs_i + ac_{iq2t} * cr_{qt} * J_{i2t} / bs_i) \leq PC_{qt}$$

$$q=UL+1, UL+2, \dots, BL$$

$$t=1, 2, 3, \dots, T$$

$$(5) \quad \sum_{i \in PO1}^N ac_{ist} * cr_{st} * Z_{it} + \sum_{i \in PO2}^N \{ac_{is1t} * cr_{st} * Z_{it} + (ac_{is2t} - ac_{is1t}) * cr_{st} * K_i\} \leq PC_{st}$$

$$s=BL+1, BL+2, \dots, PL$$

$$t=1, 2, 3, \dots, T$$

$$(6) \quad \sum_{i=1}^N m_{it} * X_{it} \leq MT_t, t=1, 2, 3, \dots, T$$

$$(7) \quad \sum_{i=1}^N dl_{it} * X_{it} \leq DL_t \quad , t= 1, 2, 3, \dots, T$$

$$(8) \quad bs_i * Y_{it} = X_{it} \quad , i= 1, 2, 3, \dots, N \quad t= 1, 2, 3, \dots, T$$

$$(9) \quad Y_{it} \leq M * Z_{it} \quad , i \in PS1 \quad t= 1, 2, 3, \dots, T$$

$$(10) \quad l_{it} \leq X_{it} \leq u_{it} \quad , i \in PS1 \quad t= 1, 2, 3, \dots, T$$

$$(11) \quad Z_{it} * l_{it} \leq X_{it} \leq u_{it} * Z_{it} \quad , i \in PS2 \quad t= 1, 2, 3, \dots, T, l_{it} \neq 0$$

$$(12) \quad \sum_{i=1}^N Z_{it} = 1 \quad , t= 1, 2, 3, \dots, T$$

$i \in PS2$

$$(13) \quad Z_{it} * l_{it} \leq X_{it} \leq u_{it} * Z_{it} \quad , i \in PS3 \quad t= 1, 2, 3, \dots, T, l_{it} \neq 0$$

$$(14) \quad \sum_{i=1}^N Z_{it} = 1 \quad , t= 1, 2, 3, \dots, T$$

$i \in PS3$

$$(15) \quad Y_{it} = \sum_{r=1}^2 F_{irt} \quad , i \in PF1 \quad t= 1, 2, 3, \dots, T$$

$$(16) \quad \sum_{t=1}^T l_{it} * (1 - K_i) \leq \sum_{t=1}^T J_{i1t} \leq \sum_{t=1}^T u_{it} * (1 - K_i) \quad , i \in PO2$$

$$(17) \quad \sum_{t=1}^T l_{it} * K_i \leq \sum_{t=1}^T J_{i2t} \leq \sum_{t=1}^T u_{it} * K_i \quad , i \in PO2$$

$$(18) \quad X_{it} = J_{i1t} + J_{i2t} \quad , t= 1, 2, 3, \dots, T$$

$$(19) \quad X_{it} \leq fa_{i1t} + (u_{it} - fa_{i1t}) * C_{it} \quad , t= 1, 2, 3, \dots, T \quad i \in FC2 \cup FC3$$

$$(20) \quad X_{it} \leq fa_{i2t} + (u_{it} - fa_{i2t}) * C_{i2t}, t = 1, 2, 3, \dots, T \quad i \in FC3$$

$$(21) \quad C_{it} \geq C_{i2t}, t = 1, 2, 3, \dots, T \quad i \in FC3$$

X_{it} , Y_{it} , J_{it} , J_{i2t} , and F_{it} are integer variables; Z_{it} , K_i , C_{it} , and C_{i2t} are binary variables.

M is a very big number.

All variables are greater than or equal to zero.

When developing the multi-period product-mix decision model, it is assumed that a company must either outsource activity #3 of P_i (given that $P_i \in PO_2$) for the next T periods or perform this activity of P_i completely in the company. This feature is incorporated into [Formulation VI] by using the constraints 16 and 17. Another important assumption that has been made when building the above model is that the company does not carry any finished goods inventory. Thus, the model does not allow the company to produce more than the amount demanded during a specific time period (constraints 10, 11, 13, 16, and 17). Finally, it is also assumed that whenever a batch of product i is started to be manufactured during period t , that batch has to be completed by the end of that period. In other words, the same batch cannot be processed during two different time periods.

Chapter IV

CASE STUDIES

Four case studies are included in this chapter to demonstrate how to use the proposed approach of the product-mix problem developed in Chapter 3 in different manufacturing environments. All of the four case studies are based on hypothetical data. The first case study has been prepared to demonstrate that the results of a product mix problem might differ greatly depending on the product-costing approach used in developing the model. The second case study illustrates the use of the proposed product-mix decision model in a more complex manufacturing environment. The third case study shows that the product-mix decision model used in Case II can be extended to include more than one time period. Finally, the fourth case study demonstrates the use of the model developed in this research in a lean manufacturing environment.

4.1 Case Study I

The purpose of this case study is to show how the traditional costing and activity-based costing approaches can give greatly differing results in terms of the best product mix and capacity constraints of a company. To demonstrate this difference between the two approaches, two mathematical models can be used to determine the optimal product mix of a company by using first the traditional costing method and then the activity-based costing based on an evaluation of the data derived from a hypothetical company. The XYZ Company has three main products, P_1 , P_2 , and P_3 . The current selling price, the monthly demands, the batch sizes, the material cost, and the direct labor cost information for each product are given in Table 4.1.

Table 4.1 The three main products of the XYZ Company

	Product 1 (P_1)	Product 2 (P_2)	Product 3 (P_3)
Selling price	\$27	\$32	\$75
Monthly Demands	150,000	100,000	60,000
Batch size	7500	2500	400
Material cost (per unit)	\$16	\$20	\$22
Direct labor cost (per unit)	\$1	\$2	\$8

Seven major overhead activities are performed in the plant to produce three products. The information regarding the budgeted rate, the practical monthly capacity, the cost driver, and the charge rate of each activity are given in Table 4.2. The charge rate is calculated by dividing the budgeted rate by the monthly practical capacities. As defined in Chapter 2, practical capacity is the maximum capacity of a plant adjusted for lost time due to non-working days, plant breakdowns, repairs, and maintenance. The monthly capacities of direct labor and material consumption activities are \$250000 and \$6000000, respectively. It should be noted that the setup, receiving, material handling, quality assurance, and packing and shipping activities are batch-related activities and are performed each time a batch of goods is processed. The demands for these kinds of activities are dependent on the number of batches produced, not on the number of units produced. However, direct labor, material, depreciation, and maintenance are unit-level activities and are dependent on the volume of units produced. The activity usage per unit or per batch of each product is given in Table 4.3. The information regarding the seven major overhead activities performed in the XYZ Company could be obtained mainly from the general ledger and through the interviews conducted in the company.

The first model uses a traditional costing approach (See Appendix H). In other words, it allocates the total overhead cost to products based on direct labor dollars. The overhead rate of the company, which is based on direct labor dollars, can be calculated as follows:

Table 4.2. The calculation of the charge rates for each overhead activity

Activity Number	Overhead Activities	Cost Drivers	Budgeted Rate	Monthly Practical Capacities	Charge Rates
1	Setup	No. of setup labor hours	\$8,000	500 setup labor hrs.	\$16/setup labor hr.
2	Receiving	No. of components	\$260,000	450 components	\$577.78/component
3	Material Handling	No. of components	\$312,000	250 components	\$1,248/component
4	Quality Assurance	No. of inspections	\$75,000	3750 inspections	\$20/inspection
5	Depreciation	No. of machine hours	\$700,000	16000 machine hrs.	\$50/machine hour
6	Maintenance	No. of machine hours	\$50,000	16000 machine hrs.	\$3.125/machine hour
7	Packing & Shipping	No. of shipments	\$600,000	1750 shipments	\$342. 86/shipment

Table 4.3. The overhead activity consumption of product 1, product 2, and product 3.

Activity Number	Overhead Activities	Product1 (P_1)	Product 2 (P_2)	Product 3 (P_3)
1	Setup	16 setup labor hrs. /batch	8 setup labor hrs. /batch	4 setup labor hrs. /batch
2	Receiving	10 components/batch	6 components/batch	5 components/batch
3	Material Handling	10 components/batch	6 components/batch	5 components/batch
4	Quality Assurance	250 inspection/batch	50 inspection/batch	10 inspection/batch
5	Depreciation	0.1 machine hrs./unit	0.04 machine hrs./unit	0.02 machine hrs./unit
6	Maintenance	0.1 machine hrs./unit	0.04 machine hrs./unit	0.02 machine hrs./unit
7	Packing & Shipping	50 shipments/batch	10 shipments/batch	16 shipments/batch

Overhead rate = Total overhead activity capacity/ Total direct labor dollars

Overhead rate = 2005000/ 250000 = 802%

The second model is a modified version of the first one (See Appendix J). It uses the activity-based costing approach and allocates overhead costs by using cost drivers that estimate the consumption of overhead activities by products more accurately than the direct labor dollars. The decision variables of the two models are explained below:

X_i : the number of units of product i produced in one period, i=1, 2, 3

Y_i : the number of batches of product i produced in one period, i=1, 2, 3

Both X_i and Y_i are integer variables.

The objective functions of both models are to maximize profit. Profit is calculated by subtracting the direct labor, material, and manufacturing overhead costs of products from the total revenue in both models. However, there is an important difference in the allocation of total manufacturing overhead costs to the products between the two models. The first model calculates the total overhead cost allocated to a product by the following formula:

*Overhead cost = Unit direct labor cost * Overhead rate * Number of units of product i
of product i of product i produced in one period.*

In the objective function of the second model (Appendix J), the cost of each overhead activity consumed by each product is subtracted from the sales figure, which is also equal to the total overhead capacity available minus the amount of slack capacity of each activity.

The first seven constraints of both models are capacity constraints and ensure that the amount of activities consumed during a period do not exceed the capacity of each

activity. The left-hand side coefficients of these constraints are calculated by multiplying the activity usage per unit or per batch of each product by the charge rate of the same activity. For example, the coefficient of Y_1 in the first constraint in Appendix H and Appendix J is calculated by multiplying the charge rate of setup labor activity, which is equal to \$16 per setup labor hours (see Table 4.2), by the activity consumption per batch of P_1 , which is 16 setup labor hours per batch (See Table 4.3). The right-hand side values of these constraints are the practical capacity figures of each activity. The eighth constraint ensures that the total number of direct labor hours do not exceed the total number of direct labor hours available. The ninth constraint is included in the model to limit the investment amount in material. Both models also include demand constraints to guarantee that the amount of products produced during a period do not exceed the amount demanded during that period (constraints 10-12). Constraints 13-15 in both models are batch size constraints and are used to define the batch sizes of the three products. It is assumed that whenever a batch of one product is started to be manufactured, it has to be finished. The equality signs in constraints 13 to 15 ensure this assumption.

The two variables, UCT_1 and UCA_1 , are included in the first model (Appendix H) to show the difference between the per unit cost figures of P_1 calculated by using the traditional costing approach and the activity-based costing approach. UCT_1 represents the unit cost of P_1 calculated by using the traditional approach and UCA_1 represents the unit cost of P_1 under the ABC approach. These two variables do not have objective function coefficients, hence do not affect the optimum solution. The variables “ TT_1 and TA_1 ” represent the total costs assigned to P_1 under the traditional and activity-based approaches, respectively. These variables are divided by the value of the X_1 at the optimal solution in order to obtain UCT_1 and UCA_1 . It should be noted that UCT_1 and UCA_1 are included in the model after the optimal solution is determined by the computer. Two other variables, UCA_2 and UCA_3 , are used in the second model (Appendix J) to determine the unit cost of P_2 and P_3 under the ABC approach. Again, these two amounts are calculated by the model after the optimal solution is found. Note that the variables the TA_2 and TA_3 represent the total costs assigned to P_2 and P_3 under the ABC approach.

Both models are solved by using CPLEX. The output of the model using the direct labor dollars as the only cost driver is given in Appendix I. According to this model, the most profitable product is P_1 and the best action is to spend all of the available resources of the plant to produce 112500 units of P_1 . The bottleneck in this case is the quality assurance activity because the amount of slack capacity of this activity is equal to zero (See Appendix I). Under the traditional costing method, the cost of producing one unit of P_1 is \$25.02 (the value of UCT_1 in Appendix I). Since the selling price of P_1 is \$27, the profit per unit of P_1 is \$1.98 under the traditional approach. When the per unit cost of P_1 is calculated by the ABC approach, it is found to be \$27.73 (the value of UCA_1 in Appendix I). Under ABC, the company appears to be losing \$0.73 for each unit of P_1 that it produces. This suggests that allocating overhead costs arbitrarily to the products (based on only one cost driver) gives the company incorrect information about the profitability of a product and could lead management to make wrong decisions such as using all the available resources of the company to produce an unprofitable product.

The management might even use the theory of constraints approach (TOC) based on the information obtained from the traditional costing method and focus on the quality assurance activity in order to improve the whole system. However, increasing the capacity of this activity will only help the company to lose more money since the company will use the extra capacity added to the quality assurance activity to produce more of P_1 , which appears to be an unprofitable product under the ABC approach. As a result, this example shows how the TOC approach can even decrease the profitability of a company if it is implemented by using the wrong cost data. Using mathematical modeling with the TOC approach may not always be sufficient to provide the right information about the system to management. Accurate cost data can be the key factor in the success of a TOC implementation.

The output of the model developed by using the activity-based costing approach suggests that management should act in a completely different way (See Appendix K). Under the ABC approach, the most profitable product is P_2 . The company should produce P_2 as much as the market demands, and then should use its existing resources to

produce two batches of P_3 . There are two binding constraints in the model shown in Appendix J, the constraint on the available material handling activity and the demand constraint on P_2 , which is called the market constraint under the TOC approach. Market constraint is defined by Stein as “a condition that exists when market demand is less than the capability of the company to produce” [54, p.296]. To improve the performance of the system, the company should focus on these two constraints and should try to increase the capacity of the material handling activity and the demand for P_2 .

Overall, it can be said that using mathematical modeling with the TOC approach may cause management to take actions that will deteriorate the performance of a system if inaccurate product cost information is used in developing the model. So, it appears that activity-based costing information may be crucial in determining the best product mix and bottlenecks of a company. Using ABC together with the TOC approach and mathematical modeling are likely to help companies to take the right actions, and as a result to survive in the competitive world of business.

4.2 Case Study II

The purpose of this section is to show how the activity-based costing and the theory of constraints approaches can be used together with the integer programming technique to determine the best product mix and, therefore, to improve the performance of a complex manufacturing system. In order to accomplish this objective, a hypothetical case study of AYBEN Company is examined herein.

The AYBEN Company has five main products, P_1 , P_2 , P_3 , P_4 , and P_5 . The selling price, the quarterly demands, the batch sizes, the direct labor cost, and the material cost information for each product are given in Table 4.4. Assume that the AYBEN Company has a special agreement with the government, requiring the company to produce at least 30000 units of either P_3 or P_4 this quarter. After fulfilling this requirement, the company can use its current capacity to produce the other products.

Table 4.4 The general information regarding the five main products of the AYBEN Company

	Product 1 (P_1)	Product 2 (P_2)	Product 3 (P_3)	Product 4 (P_4)	Product 5 (P_5)
Selling Prices	\$50	\$58	\$62	\$60	\$60
Quarterly Demands	200,000	150,000	100,000	100,000	80,000
Batch Sizes	2000	1500	1000	1000	1250
Direct Labor Costs (per unit)	\$1.5	\$5.0	\$2.0	\$2.0	\$4.0
Material Costs (per unit)	\$8.0	\$5.0	\$10.0	\$11.0	\$6.0

The company performs twelve major overhead activities to produce five products. The information regarding the budgeted rate, the quarterly practical capacity, the cost driver, and the charge rate of each activity are given in Table 4.5. The charge rate is calculated by dividing the budgeted rate by the quarterly practical capacity. The twelve

major activities of the AYBEN Company can be grouped into three: the unit-level activities, the batch-level activities, and the product-sustaining activities. Maintenance, automatic machining, general machining, and assembly are unit-level activities. In other words, the consumption of these activities increases with the number of units produced. Set-up, material handling, receiving, quality assurance, packing and shipping, and production and inventory control are batch-level activities. The demands for these activities are dependent on the number of batches produced, not on the number of units produced. Finally, the engineering and vendor relations are product-sustaining level activities and are performed to enable individual products to be produced and sold. The activity usage of each product is given in Table 4.6. Facility-sustaining activities are necessary to provide a company that can produce products, but the resources consumed by these activities are not dependent on the volume and mix of individual products. To eliminate facility-sustaining expenses, a plant would have to be permanently closed. In this case study, it is assumed that closing the plant is not an option. Therefore, facility-sustaining activities are not included in the analysis.

The mathematical model that is used to determine the best product mix of the AYBEN Company based on the ABC information is given in Appendix L. The decision variables of the model are explained below:

X_i : the number of units of product i produced in one quarter, $i= 1, 2, 3, 4, 5$.

Y_i : the number of batches of product i produced in one quarter, $i= 1, 2, 3, 4, 5$.

$$Z_i = \begin{cases} 1, & \text{if product } i \text{ is produced, } i= 1, 2, 3, 4, 5. \\ 0, & \text{if product } i \text{ is not produced, } i= 1, 2, 3, 4, 5. \end{cases}$$

X_i and Y_i are integer variables, Z_i is a binary variable.

Table 4.5 The calculation of charge rates for each overhead activity of the AYBEN Company

Activity Number	Activities	Cost Drivers	Budgeted Rate	Quarterly Practical Capacities	Charge Rates
Unit-Level Activities					
1	Maintenance	No. of machine hours	\$1,500,000	214,286 machine hours	\$7.0/machine hour
2	Automatic Machining	Number of machine hours	\$1,697,500	24,250 machine hours	\$70.0/machine hour
3	General Machining	Number of machine hours	\$2,000,000	250,000 machine hours	\$8.0/machine hour
4	Assembly	Number of assembly labor hours	\$1,050,000	52,500 assembly labor-hours	\$20/assembly labor hour
Batch- Level Activities					
5	Set-up	Number of set-up labor hours	\$150,000	10,000 set-up labor hours	\$15.0/setup labor hour
6	Material Handling	Number of material moves	\$750,000	37,500 material moves	\$20.0/material movement
7	Receiving	Number of invoices	\$170,000	4,250 invoices	\$40.0/invoice
8	Quality Assurance	Number of inspections	\$500,000	50,000 inspection hours	\$10.0/inspection
9	Packing and Shipping	Number of shipments	\$550,000	5,500 shipments	\$100/shipment
10	Production and Inventory Control	Number of scheduling labor hours	\$350,000	8,750 scheduling labor hours	\$40/scheduling labor hour
Product-Sustaining Activities					
11	Engineering	Number of engineering change notices	\$80,000	400 engineering change notices	\$200/engineering change notice
12	Vendor Relations	Number of vendors	\$1,100,000	110 vendors	\$10,000/vendor

Table 4.6 The overhead activity consumption of the five products of the AYBEN Company

Activity Number	Activities	Product 1 (P_1)	Product 2 (P_2)	Product 3 (P_3)	Product 4 (P_4)	Product 5 (P_5)
Unit- Level Activities						
1	Maintenance	0.6 machine hours/unit	0.85 machine hours/unit	0.85 machine hours/ unit	0.75 machine hours/ unit	0.85 machine hours/ unit
2	Automatic Machining	0.2 machine hours/ unit	0.25 machine hours/ unit	0.35 machine hours/ unit	0.25 machine hours/ unit	0.15 machine hours/ unit
3	General Machining	0.4 machine hours/ unit	0.6 machine hours/ unit	0.5 machine hours/ unit	0.5 machine hours/ unit	0.7 machine hours/ unit
4	Assembly	0.2 assembly labor hrs/ unit	0.15 assembly labor hrs/ unit	0.075 assembly labor hrs/ unit	0.1 assembly labor hrs/ unit	0.125 assembly labor hrs/ unit
Batch- Level Activities						
5	Set-up	32 setup labor hours/ batch	20 setup labor hours/ batch	22 setup labor hours/ batch	25 setup labor hours/ batch	18 setup labor hours/ batch
6	Material Handling	250 material moves/ batch	150 material moves/ batch	90 material moves/ batch	100 material moves/ batch	130 material moves/ batch
7	Receiving	10 invoices/ batch	12 invoices/ batch	8 invoices/ batch	8 invoices/ batch	9 invoices/ batch
8	Quality Assurance	300 inspections/ batch	200 inspections/ batch	150 inspections/ batch	160 inspections/ batch	180 inspections/ batch
9	Packing and Shipping	25 shipments/ batch	30 shipments/ batch	20 shipments/ batch	20 shipments/ batch	27 shipments/ batch
10	Production and Inventory Control	40 scheduling labor hrs/ batch	30 scheduling labor hrs/ batch	20 scheduling labor hrs/ batch	23 scheduling labor hrs/ batch	26 scheduling labor hrs/ batch
Product-Sustaining Activities						
11	Engineering	10 ECN/ product	8 ECN/ product	15 ECN/ product	17 ECN/ product	20 ECN/ product
12	Vendor Relations	20 vendors/ product	25 vendors/ product	15 vendors/ product	18 vendors/ product	17 vendors/ product

The objective function of the model is to maximize profit. Profit is calculated by subtracting the direct labor, material, and activity costs of products from the total revenue. In other words, the left-hand sides of constraints 1- 14 are subtracted from the revenue to obtain the objective function.

The first fourteen constraints are capacity-related constraints and are included in the model to guarantee that the amount of activities consumed during a quarter do not exceed the capacity of each activity for that quarter. Constraints 15- 21 are demand constraints, which ensure that the amount of products produced by AYBEN during a period do not exceed the amount demanded during that period. For products 3 and 4, the demand constraints (constraints 17-20) are a little more complex since these two products are mutually exclusive and the company must produce at least 30000 units from either one. This feature can be incorporated into the model by using the following constraints:

$$X_3 \leq 100000 * Z_3$$

$$X_3 \geq 30000 * Z_3$$

$$X_4 \leq 100000 * (1 - Z_3)$$

$$X_4 \geq 30000 * (1 - Z_3)$$

Constraints 22- 26 are batch size constraints, which are used to define the batch sizes of the five products. Since it is assumed that a batch of product has to be completed whenever it is started to be manufactured, equality signs are used in these five constraints. Constraints 27- 31 are included in the model to ensure that whenever a batch of one product is produced, the relevant product-sustaining activity costs are incurred in the objective function. The general structure of this constraint is as follows:

$$Y_i \leq M * Z_i, \text{ where } M \text{ is a very big number.}$$

Whenever Y_i is greater than zero, this constraint forces Z_i to be equal to 1, and as a result ensures that the relevant product-sustaining activity costs will be subtracted from the revenue if product i is produced by the AYBEN Company during a certain quarter.

The model is solved by using CPLEX. The output of the model is given in Appendix M. As can be seen in Appendix M, the best action for the AYBEN Company is to produce 49000 units of product 4 and 80000 units of product 5. By taking this action, AYBEN will make \$1658035 of profit. The bottleneck of the whole system is the automatic machining activity. The company has a lot of surplus capacity in its other activities. So, if AYBEN eliminates, or at least improves the capacity of its bottleneck, there may be a substantial increase in its profitability figure.

Now assume that the management of the AYBEN Company has found an activity flexibility option for P_1 . As can be seen in Table 4.7, one unit of P_1 is required to be processed for 0.2 hours at the automatic machining department, and for 0.4 hours at the general machining department. This is the data that is used in the model given in Appendix L. The automatic machining activity is a much more efficient activity compared to the general machining activity. A part can be processed in a shorter time and at a lower cost by using this activity instead of using the general machining activity. However, after the automatic machining activity has been identified as the bottleneck, assume that management found an alternative to reduce the workload of this activity. This alternative was processing P_1 for 0.1 hours at the automatic machining department and for 1.5 hours at the general machining department, instead of processing it for 0.2 hours at the automatic machining department and for 0.4 hours at the general machining department (See Table 4.7). There was no other activity flexibility alternatives for other products because of the potential quality-related problems.

Table 4.7. The two routing alternatives for P_1

	Route 1 (old alternative)	Route 2 (new alternative)
Automatic Machining	0.2 hours/ unit	0.1 hours/ unit
General Machining	0.4 hours/ unit	1.5 hours/ unit

The cost of processing one unit of P_1 by using route 1 and route 2 can be calculated as follows (Remember that the charge rates of the automatic machining and the general machining activities are \$70.0 and \$8.0, respectively):

For route 1:

$$\$70.0 / \text{mc.hr.} * 0.2 \text{ mc.hrs./unit} + \$8.0 / \text{mc.hr.} * 0.4 \text{ mc.hrs./unit} = \$17.2 / \text{unit}$$

For route 2:

$$\$70.0 / \text{mc.hr.} * 0.1 \text{ mc.hrs./unit} + \$8.0 / \text{mc.hr.} * 1.5 \text{ mc.hrs./unit} = \$19.0 / \text{unit}$$

Although the new routing alternative was less effective compared to the first routing alternative (long processing times, high costs), the management decided to consider this new alternative since it might help to improve the throughput of the system, and, therefore, might increase the profitability of the company. The new routing alternative is included in the mathematical model to determine the best product mix of the company as follows:

It is assumed that a batch of P_1 always follows the same routing. In other words, whenever one unit of P_1 is started to be manufactured by using a specific route, all the other parts in that batch must follow the same route. This assumption can be included in the model by using the factor $2000 * Y_1$ instead of the unit-level variable X_1 (Remember that the batch size of P_1 is 2000).

Furthermore, Y_1 can be partitioned into two new integer variables, F_1 and F_2 , such that the summation of F_1 and F_2 is equal to Y_1 . F_1 represents the number of batches of P_1 that follow route 1, and F_2 represents the number of batches of P_1 that follow route 2. The new constraints that are added to the model in Appendix L are given below:

$$70 * 0.2 * 2000 * F_1 + 70 * 0.1 * 2000 * F_2 + 70 * 0.25 * X_2 + 70 * 0.35 * X_3 + 70 * 0.25 * X_4 + 70 * 0.15 * X_5 \leq 1697500$$

$$8 * 0.4 * 2000 * F_1 + 8 * 1.5 * 2000 * F_2 + 8 * 0.6 * X_2 + 8 * 0.5 * X_3 + 8 * 0.5 * X_4 + 8 * 0.7 * X_5 \leq 2000000$$

$$Y_1 = F_1 + F_2$$

F₁ and F₂ are integer variables.

The modified model can be seen in Appendix N. The output of the new model (see Appendix O) shows that the profitability of the AYBEN Company increases by \$39595 to \$1697630 when the activity flexibility alternative for P_1 is taken into account. According to the new model, the company should produce 46000 units of P_1 , 30000 units of P_4 , and 80000 units of P_5 . The automatic machining activity is still the bottleneck activity. However, the whole performance of the system is increased by taking off some load from the bottleneck activity and transferring this load to a non-bottleneck activity (the general machining activity).

The case of the AYBEN Company is very similar to the case of the plant described in Goldratt's book, *The Goal*. One of the bottlenecks of the plant described in *The Goal* was the NCX-10, the most efficient machine of the plant. However, only one machine of this type was not enough to meet the market demand. Therefore, the management of the plant decided to use an old, less effective machine as an alternative route to improve the system. The result was the increased throughput and higher profitability figures [21]. So, it can be said that the key to improve the performance of a system is first to identify the bottleneck, and then use all the other non-bottleneck activities to decrease the workload of the bottleneck.

Although the profitability of the company has increased by considering an activity flexibility option, the automatic machining activity is still the bottleneck (See Appendix O). To improve the performance of the system, assume that the management of the AYBEN Company decided to search for other alternatives and found an outsourcing option for the automatic machining activity of P_5 . It is further assumed that the company

must either outsource the automatic machining activity of P_5 completely, or continue to perform this activity in the company for each unit of P_5 . The purchasing price of this activity for P_5 is \$12 per batch, which is higher than the cost of performing this activity in the company ($= 70\$/\text{mc. hr.} * 0.15 \text{ mc. hr./batch} = 10.5 \$/\text{batch}$). This outsourcing decision can be included in the model as follows:

We can partition X_5 into two new integer variables, J_1 and J_2 , such that the summation of J_1 and J_2 is equal to X_5 .

$$J_1 = \begin{cases} X_5, & \text{if the management decides to perform the automatic machining} \\ & \text{activity of } P_5 \text{ in the company} \\ 0, & \text{if the management decides to outsource the automatic machining} \\ & \text{activity of } P_5 \end{cases}$$

$$J_2 = \begin{cases} X_5, & \text{if the management decides to outsource the automatic machining} \\ & \text{activity of } P_5 \\ 0, & \text{if the management decides to perform the automatic machining} \\ & \text{activity of } P_5 \text{ in the company} \end{cases}$$

Since AYBEN must either outsource the automatic machining activity for P_5 completely, or perform this activity in-house for each P_5 produced, J_1 and J_2 must be mutually exclusive, that is one must be equal to zero when the other is equal to X_5 . This constraint can be included in the model as follows:

Define a new binary variable, K , such that:

$$K = \begin{cases} 0, & \text{if the management decides to perform the automatic machining activity} \\ & \text{of } P_5 \text{ in the company} \\ 1, & \text{if the management decides to outsource the automatic machining} \\ & \text{activity of } P_5 \end{cases}$$

The new constraints that ensure that J_1 and J_2 are mutually exclusive can be written as follows:

$$\begin{aligned} J_1 - 80000 * (1 - K) &\leq 0 \\ J_2 - 80000 * K &\leq 0 \end{aligned}$$

These two constraints also ensure that whatever the decision of AYBEN regarding P_5 will be (outsourcing or performing the activity in the company), the amount of J_1 and J_2 will not exceed the market demand for P_5 .

The variable X_5 , the number of units of P_5 produced in the company, should be replaced by the variable J_1 in the automatic machining activity constraint to guarantee that whenever the automatic machining activity for P_5 is not outsourced, P_5 will use the available capacity of this activity. Accordingly, the automatic machining activity constraint should be modified as follows:

$$70 * 0.2 * 2000 * F_1 + 70 * 0.1 * 2000 * F_2 + 70 * 0.25 * X_2 + 70 * 0.35 * X_3 + 70 * 0.25 * X_4 + 70 * 0.15 * J_1 \leq 1697500$$

The factor “ $-\$12 * J_2$ ” is included in the objective function (See Appendix P) to ensure that AYBEN will pay \$12 to the vendor for each unit of P_5 if the automatic machining activity of this product is decided to be outsourced.

It should also be realized that the outsourcing of the automatic machining activity for P_5 would also affect the consumption amount of other activities by this product. For example, the cost drivers of both the maintenance activity and the automatic machining activity are the numbers of machine hours. If the management of the AYBEN Company decides to outsource the automatic machining activity of P_5 , the consumption amount of the maintenance activity per unit of P_5 will decrease by the amount of the automatic machine activity consumed per unit of P_5 . In other words, P_5 will demand only 0.70 units of the maintenance activity instead of 0.85 units if the automatic machining activity of P_5 is outsourced. This example shows that activities performed in a company are dependent on each other. So, whenever a company is considering outsourcing one or more of its activities as an alternative, the effect of this alternative on the other activities performed in the plant should also be taken into account during the decision process.

Changes in the activity consumption amounts of P_5 if the automatic machining activity of this product is outsourced are given in Table 4.8. This situation can be included in the model as follows:

- The variable X_5 should be replaced with the variables J_1 and J_2 in the first constraint (maintenance activity). The coefficient of J_1 will be equal to the activity charge rate multiplied by the activity consumption amount per unit of P_5 given in the third column of Table 4.8, whereas the coefficient of J_2 will be equal to the activity charge rate multiplied by the activity consumption amount per unit of P_5 given in the fourth column of Table 4.8.
- The variable Y_5 should be replaced with the variables $J_1/1250$ and $J_2/1250$ in the seventh, ninth, and twelfth constraints in the model. The appropriate coefficients for J_1 and J_2 should be included in the model by referring to Table 4.8.

Table 4.8 The activity consumption amounts of per unit P_5 with and without the outsourcing option

Activity Number	Activities	Product 5 (P_5) (without outsourcing)	Product 5 (P_5) (with outsourcing)
Unit- Level Activities			
1	Maintenance	0.85 machine hours/ unit	0.70 machine hours/ unit
2	Automatic Machining	0.15 machine hours/ unit	0.0 machine hours/ unit
3	General Machining	0.7 machine hours/ unit	0.7 machine hours/ unit
4	Assembly	0.125 assembly labor hrs/ unit	0.125 assembly labor hrs/ unit
Batch- Level Activities			
5	Set-up	18 setup labor hours/ batch	14 setup labor hours/ batch
6	Material Handling	130 material moves/ batch	130 material moves/ batch
7	Receiving	9 invoices/ batch	11 invoices/ batch
8	Quality Assurance	180 inspections/ batch	180 inspections/ batch
9	Packing and Shipping	27 shipments/ batch	27 shipments/ batch
10	Production and Inventory Control	26 scheduling labor hrs/ batch	22 scheduling labor hrs/ batch
Product-Sustaining Activities			
11	Engineering	20 ECN/ product	20 ECN/ product
12	Vendor Relations	17 vendors/ product	18 vendors/ product

* Direct labor and material consumption of P_5 is assumed to remain constant with and without the outsourcing decision.

- Since the number of vendors increases by one if the automatic machining activity of P_5 is outsourced, a new factor, $\$10000 * I * K$, should be added to the left-hand side of the fourteenth constraint. Remember that K is a binary variable and is equal to one when the company selects the outsourcing alternative.

The modified model and its output can be seen in Appendix P and Appendix Q, respectively. The output of the new model shows that the best action for the AYBEN Company is to outsource the automatic machining activity of P_5 ($K=1$). The AYBEN Company should produce 166000 units of P_1 , 30000 units of P_4 , and 80000 units of P_5 . If the management gives a decision in favor of the outsourcing alternative, the profitability of the company will increase by $\$1140160$ compared to the previous model in Appendix O.

Now assume that if AYBEN produces more than 50000 units of P_1 , a fixed cost of $\$20000$ will be incurred. If more than 150000 units of P_1 are produced, an extra cost of $\$96000$ will be incurred (See Figure 4.1).

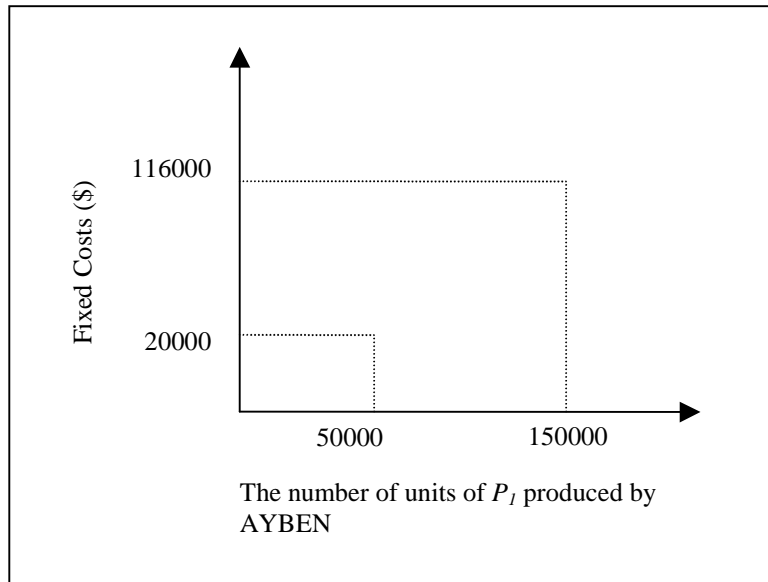


Figure 4.1 The fixed costs associated with P_1 .

This two-time fixed cost associated with P_1 can be included in the model by defining two new binary variables, C_1 and C_2 , such that:

$$C_1 = \begin{cases} 0, & \text{if the number of units of } P_1 \text{ produced by the AYBEN Company} \\ & \text{is less than 50000.} \\ 1, & \text{if the number of units of } P_1 \text{ produced by the AYBEN Company} \\ & \text{exceeds 50000.} \end{cases}$$

$$C_2 = \begin{cases} 0, & \text{if the number of units of } P_1 \text{ produced by the AYBEN Company} \\ & \text{is less than 150000.} \\ 1, & \text{if the number of units of } P_1 \text{ produced by the AYBEN Company} \\ & \text{exceeds 150000.} \end{cases}$$

The three constraints given below guarantee that whenever the unit-level production of P_1 exceeds a certain limit, the associated fixed costs will be incurred.

$$X_1 - 150000 * C_1 \leq 50000$$

$$X_1 - 50000 * C_2 \leq 150000$$

$$C_1 \geq C_2$$

The last constraint ensures that when X_i is greater than 150000 units, the \$20000 fixed cost will still be incurred, as well as the \$96000. Finally, by adding the factor “ $20000 * C_1 + 96000 * C_2$ ” to the objective function, the new model in Appendix R can be obtained.

The output of the new model (Appendix S) suggests producing 150000 units of P_1 , 37000 units of P_4 , and 80000 units of P_5 . The value of the objective function decreased to \$2738075 because of including the fixed costs associated with P_1 in the model. The bottleneck of the system is still the automatic machining activity, however,

the performance of the system increased greatly compared to the first model given in Appendix L. From now on, the management of the company should search for other alternatives that will reduce the workload or increase the capacity of the bottleneck activity. Whenever the automatic machining activity is not a bottleneck any more, management should identify the new bottleneck of the system and should continue to follow the five-step focusing process of TOC described in Chapter 2 to improve the performance of the whole system.

4.3 Case Study III

The purpose of this section is to show that the product-mix decision model used in Case II can be extended to include more than one time period. In order to accomplish this objective, the case of the AYBEN Company is examined herein for the next four quarters.

As mentioned in Case II, the AYBEN Company has five main products, P_1 , P_2 , P_3 , P_4 , and P_5 . The information regarding the batch sizes, and the material and direct labor costs of the five main products are given in Table 4.9. The selling prices and the quarterly demands of each product for the next four quarters are given in Table 4.10. As described in Case II, product 3 and product 4 are mutually exclusive. In each quarter, the AYBEN Company must produce a certain amount of either product. Note that the minimum and maximum amounts that AYBEN can produce from either product 3 or product 4 during each quarter are specified in Table 4.10.

The AYBEN Company performs twelve major overhead activities to produce five products. The information regarding the budgeted rate, the practical quarterly capacity, the cost driver, and the charge rate of each activity are given in Table 4.11. The activity usage of each product for the next four quarters is given in Table 4.12.

Table 4.9 The information regarding the batch sizes, and the material and direct labor costs of the five main products of the AYBEN Company

	Product 1 (P_1)	Product 2 (P_2)	Product 3 (P_3)	Product 4 (P_4)	Product 5 (P_5)
Batch Sizes	2000	1500	1000	1000	1250
Direct Labor Costs (per unit)	\$1.5	\$5.0	\$2.0	\$2.0	\$4.0
Material Costs (per unit)	\$8.0	\$5.0	\$10.0	\$11.0	\$6.0

Table 4.10. The information regarding the quarterly selling prices and demands of the five main products of the AYBEN Company.

	Product 1 (P_1)	Product 2 (P_2)	Product 3 (P_3)	Product 4 (P_4)	Product 5 (P_5)
Quarter 1					
Selling Prices (\$)	53	58	62	60	60
Demands	200,000	150,000	min. 30,000 max. 100,000		80,000
Quarter 2					
Selling Prices (\$)	58	56	60	58	50
Demands	20,000	120,000	min. 12,000 max. 15,000		20,000
Quarter 3					
Selling Prices (\$)	60	52	58	55	48
Demands	200,000	100,000	min. 5,000 max. 7,000		10,000
Quarter 4					
Selling Prices (\$)	58	60	61	58	63
Demands	25,000	160,000	min. 15,000 max. 20,000		100,000

The mathematical model that is used to determine the best product-mix of the AYBEN Company for the next four quarters is given in Appendix T. When building the model, three basic assumptions have been made:

- 1- No inventory is carried from one quarter to the next.
- 2- Whenever a batch of product is started to be manufactured in a specific quarter, that batch has to be completed during that quarter. In other words, the same batch cannot be processed at two different quarters.
- 3- The AYBEN Company must either outsource the automatic machining activity of P_5 completely for the next four quarters or perform this activity in the company. In other words, the company is not allowed to outsource this activity for one quarter and then perform this activity in-house for the next quarter.

Table 4.11 The calculation of charge rates for each overhead activity of the AYBEN Company

Activity Number	Activities	Cost Drivers	Budgeted Rate	Quarterly Practical Capacities	Charge Rates
Unit-Level Activities					
1	Maintenance	No. of machine hours	\$1,500,000	214,286 machine hours	\$7.0/machine hour
2	Automatic Machining	Number of machine hours	\$1,697,500	24,250 machine hours	\$70.0/machine hour
3	General Machining	Number of machine hours	\$2,000,000	250,000 machine hours	\$8.0/machine hour
4	Assembly	Number of assembly labor hours	\$1,050,000	52,500 assembly labor-hours	\$20/assembly labor hour
Batch- Level Activities					
5	Set-up	Number of set-up labor hours	\$150,000	10,000 set-up labor hours	\$15.0/setup labor hour
6	Material Handling	Number of material moves	\$750,000	37,500 material moves	\$20.0/material movement
7	Receiving	Number of invoices	\$170,000	4,250 invoices	\$40.0/invoice
8	Quality Assurance	Number of inspections	\$500,000	50,000 inspection hours	\$10.0/inspection
9	Packing and Shipping	Number of shipments	\$550,000	5,500 shipments	\$100/shipment
10	Production and Inventory Control	Number of scheduling labor hours	\$350,000	8,750 scheduling labor hours	\$40/scheduling labor hour
Product-Sustaining Activities					
11	Engineering	Number of engineering change notices	\$80,000	400 engineering change notices	\$200/engineering change notice
12	Vendor Relations	Number of vendors	\$1,100,000	110 vendors	\$10,000/vendor

Table 4.12 The overhead activity consumption of the five products of the AYBEN Company

Activity Number	Activities	Product 1 (P1)	Product 2 (P2)	Product 3 (P3)	Product 4 (P4)	Product 5 (P5)
Unit- Level Activities						
1	Maintenance	0.6 machine hours/unit	0.85 machine hours/unit	0.85 machine hours/ unit	0.75 machine hours/ unit	0.85 machine hours/ unit
2	Automatic Machining	0.2 machine hours/ unit	0.25 machine hours/ unit	0.35 machine hours/ unit	0.25 machine hours/ unit	0.15 machine hours/ unit
3	General Machining	0.4 machine hours/ unit	0.6 machine hours/ unit	0.5 machine hours/ unit	0.5 machine hours/ unit	0.7 machine hours/ unit
4	Assembly	0.2 assembly labor hrs/ unit	0.15 assembly labor hrs/ unit	0.075 assembly labor hrs/ unit	0.1 assembly labor hrs/ unit	0.125 assembly labor hrs/ unit
Batch- Level Activities						
5	Set-up	32 setup labor hours/ batch	20 setup labor hours/ batch	22 setup labor hours/ batch	25 setup labor hours/ batch	18 setup labor hours/ batch
6	Material Handling	250 material moves/ batch	150 material moves/ batch	90 material moves/ batch	100 material moves/ batch	130 material moves/ batch
7	Receiving	10 invoices/ batch	12 invoices/ batch	8 invoices/ batch	8 invoices/ batch	9 invoices/ batch
8	Quality Assurance	300 inspections/ batch	200 inspections/ batch	150 inspections/ batch	160 inspections/ batch	180 inspections/ batch
9	Packing and Shipping	25 shipments/ batch	30 shipments/ batch	20 shipments/ batch	20 shipments/ batch	27 shipments/ batch
10	Production and Inventory Control	40 scheduling labor hrs/ batch	30 scheduling labor hrs/ batch	20 scheduling labor hrs/ batch	23 scheduling labor hrs/ batch	26 scheduling labor hrs/ batch
Product-Sustaining Activities						
11	Engineering	10 ECN/ product	8 ECN/ product	15 ECN/ product	17 ECN/ product	20 ECN/ product
12	Vendor Relations	20 vendors/ product	25 vendors/ product	15 vendors/ product	18 vendors/ product	17 vendors/ product

The decision variables of the model are described below:

X_{it} : the number of units of product i produced during quarter t , $i= 1, 2, 3, 4, 5$.
 $t= 1, 2, 3, 4$.

Y_{it} : the number of batches of product i produced during quarter t , $i= 1, 2, 3, 4, 5$.
 $t= 1, 2, 3, 4$.

$Z_{it} = \begin{cases} 1, & \text{if product } i \text{ is produced during quarter } t, \\ 0, & \text{if product } i \text{ is not produced during quarter } t, \end{cases}$ $i= 1, 2, 3, 4, 5$.
 $t= 1, 2, 3, 4$.

$F_{1t} =$ The number of batches of P_1 that follow route 1 during quarter t .
 $t= 1, 2, 3, 4$

$F_{2t} =$ The number of batches of P_1 that follow route 2 during quarter t .
 $t= 1, 2, 3, 4$

$J_{1t} = \begin{cases} X_{5t}, & \text{if the management decides to perform the automatic} \\ & \text{machining activity of } P_5 \text{ in the company during quarter } t. \\ 0, & \text{if the management decides to outsource the automatic} \\ & \text{machining activity of } P_5 \text{ during quarter } t. \end{cases}$ $t= 1, 2, 3, 4$.

$$J_{2t} = \begin{cases} X_{5t}, & \text{if the management decides to outsource the automatic} \\ & \text{machining activity of } P_5 \text{ during quarter } t \\ 0, & \text{if the management decides to perform the automatic} \\ & \text{machining activity of } P_5 \text{ in the company during quarter } t \end{cases} \quad t= 1, 2, 3, 4.$$

$$K = \begin{cases} 0, & \text{if the management decides to perform the automatic} \\ & \text{machining activity of } P_5 \text{ in the company} \\ 1, & \text{if the management decides to outsource the automatic} \\ & \text{machining activity of } P_5 \end{cases}$$

$$C_{1t} = \begin{cases} 0, & \text{if the number of units of } P_1 \text{ produced by the AYBEN} \\ & \text{Company during quarter } t \text{ is less than } 50000. \\ 1, & \text{if the number of units of } P_1 \text{ produced by the AYBEN} \\ & \text{Company during quarter } t \text{ exceeds } 50000. \end{cases} \quad t= 1, 2, 3, 4.$$

$$C_{2t} = \begin{cases} 0, & \text{if the number of units of } P_1 \text{ produced by the AYBEN} \\ & \text{Company during quarter } t \text{ is less than } 150000. \\ 1, & \text{if the number of units of } P_1 \text{ produced by the AYBEN} \\ & \text{Company during quarter } t \text{ exceeds } 150000. \end{cases} \quad t= 1, 2, 3, 4.$$

$X_{it}, Y_{it}, F_{1t}, F_{2t}, J_{1t}, J_{2t}$ are integer variables, $Z_{it}, K, C_{1t},$ and C_{2t} are binary variables.

The objective function of the model is to maximize profit for the next four quarters. The profit is calculated by subtracting the left-hand sides of constraints 1- 56, the fixed costs associated with P_j , and the costs of outsourcing the automatic machining activity of P_5 from the revenue. Note that the purchasing price of the automatic machining activity for each unit of P_5 remains constant (\$12) during the next four quarters.

As described in Case II, the activities performed in a company are dependent on each other. The management of the AYBEN Company should consider the effect of outsourcing the automatic machining activity of P_5 on other activities performed in the company when making a decision. This effect can be included in the model by using the entries in the last column of Table 4.13 as the coefficients of the decision variable " J_{2t} " instead of the regular activity consumption amounts of P_5 given in the third column of the same table.

The first fifty-six constraints are capacity-related constraints and are included in the model to guarantee that the amount of activities consumed during a quarter do not exceed the capacity of each activity for that quarter. Constraints 57- 60 ensure that the total number of batches of product 1 produced during a quarter is equal to the summation of the total number of batches of product 1 produced by following route 1 and the total number of batches of the same product produced by following route 2. The information regarding the two routing alternatives for P_j is given in Table 4.14.

Table 4.13 The activity consumption amounts of per unit P_5 with and without the outsourcing option

Activity Number	Activities	Product 5 (P_5) (without outsourcing)	Product 5 (P_5) (with outsourcing)
Unit- Level Activities			
1	Maintenance	0.85 machine hours/ unit	0.70 machine hours/ unit
2	Automatic Machining	0.15 machine hours/ unit	0.0 machine hours/ unit
3	General Machining	0.7 machine hours/ unit	0.7 machine hours/ unit
4	Assembly	0.125 assembly labor hrs/ unit	0.125 assembly labor hrs/ unit
Batch- Level Activities			
5	Set-up	18 setup labor hours/ batch	14 setup labor hours/ batch
6	Material Handling	130 material moves/ batch	130 material moves/ batch
7	Receiving	9 invoices/ batch	11 invoices/ batch
8	Quality Assurance	180 inspections/ batch	180 inspections/ batch
9	Packing and Shipping	27 shipments/ batch	27 shipments/ batch
10	Production and Inventory Control	26 scheduling labor hrs/ batch	22 scheduling labor hrs/ batch
Product-Sustaining Activities			
11	Engineering	20 ECN/ product	20 ECN/ product
12	Vendor Relations	17 vendors/ product	18 vendors/ product

* Direct labor and material consumption of P_5 is assumed to remain constant with and without the outsourcing decision.

Table 4.14 The two routing alternatives for P_1

	Route 1 (old alternative)	Route 2 (new alternative)
Automatic Machining	0.2 hours/ unit	0.1 hours/ unit
General Machining	0.4 hours/ unit	1.5 hours/ unit

Constraints 61 and 62 guarantee that the AYBEN Company must either outsource the automatic machining activity of P_5 completely for the next four quarters or perform this activity in the company. Constraints 63- 66 are used to ensure that the total number of units of P_5 produced during a quarter is equal to the summation of the number of units of P_5 outsourced and the number of units of P_5 produced in-house during the same quarter. Constraints 67- 106 are demand constraints and are included in the model to guarantee that the total number of units of product i produced by AYBEN during a quarter do not exceed the amount demanded during that quarter. Remember that AYBEN does not carry any finished goods inventory. Thus, producing more than the amount demanded during a quarter will only increase the quarterly costs of AYBEN. For this reason, the model does not allow the company to produce more than the amount demanded during a quarter. Constraints 107- 126 are used to define the batch sizes of the five main products of the company. Constraints 127- 146 ensure that whenever a batch of one product is produced during a specific quarter, the relevant product-sustaining activity costs are incurred in the objective function.

The model is solved by using CPLEX. The output of the model is given in Appendix U. By taking the action suggested in Appendix U, the AYBEN Company will earn a total profit of \$8,677,490 at the end of next year. As can be seen from the output of the model, the values of the slacks for constraints 8, 13, 14, 15, and 16 are either equal to zero or very close to zero. This suggests that the automatic machining activity is a bottleneck for all the quarters and the direct labor activity is a bottleneck just for the last quarter.

The values of the decision variables “ J_{1t} ” and “ J_{2t} ” at the optimal solution are summarized below:

$$J_{11} = 0, \quad J_{12} = 0, \quad J_{13} = 0, \quad J_{14} = 0$$

$$J_{21} = 80000, \quad J_{22} = 0, \quad J_{23} = 0, \quad J_{24} = 100000$$

The model suggests that AYBEN should outsource the automatic machining activity of P_5 . The company should outsource the automatic machining activity of 80,000 units of P_5 during the first quarter and 100,000 units of P_5 during the fourth quarter.

The values of the decision variables “ F_{1t} ” and “ F_{2t} ” at the optimal solution are summarized below:

$$F_{11} = 0, \quad F_{12} = 0, \quad F_{13} = 12, \quad F_{14} = 3$$

$$F_{21} = 75, \quad F_{22} = 10, \quad F_{23} = 88, \quad F_{24} = 9$$

The best action for the AYBEN Company is to produce 75 batches of P_1 during the first quarter and 10 batches of P_1 during the second quarter by using the second route. In the third quarter, eighty-eight batches of P_1 should follow route 2, whereas in the fourth quarter nine batches of P_1 should follow this route. As explained in Case II, producing a batch of P_1 by using the second route is more expensive than producing the same product by using the first route. However, the model still suggests producing a major portion of P_1 by using route 2 instead of route 1. This is because the automatic machining activity is the bottleneck of the whole system. Reducing the workload of the bottleneck and using the gained capacity of the bottleneck to produce other products increases the total revenue of the company more than the additional expenses incurred by using a less efficient method to produce P_1 .

This case shows that the model developed in Chapter 3 can be extended to include more than one time period. Companies can use the multi-period product-mix decision model for planning purposes. In other words, by using this model for more than

one period, companies can estimate their potential bottlenecks as early as possible, which will give them enough time to improve the performance of these bottleneck resources before the actual production takes place.

4.4 Case Study IV

The purpose of this section is to demonstrate how the product-mix decision model developed in Chapter 3 can also be used in a lean manufacturing environment. In order to accomplish this objective, a hypothetical case study of GUROTO Company is examined herein.

In this case study, it is assumed that the GUROTO Company has implemented lean manufacturing years ago because of the three main reasons described below:

- The company did not have enough space.
- The company did not have enough money to hold inventory.
- The market demands for the products of the GUROTO Company were relatively small in number, but large in variety. The demand was insufficient to justify mass production.

It is further assumed that the GUROTO Company is now working on a make-to-order basis, that is, the customer order triggers the production. The company starts to build what the customer orders as soon as possible to reduce the lead-time. Consequently, GUROTO does not carry any finished goods inventory.

The GUROTO Company has five main products, P_1 , P_2 , P_3 , P_4 , and P_5 . The selling prices and demands of each product for the next four quarters are given in Table 4.15. The information regarding the batch sizes, and the material and direct labor costs of the five products are given in Table 4.16. The setup costs in the GUROTO Company are very low compared to the other companies in the same industry that are implementing a mass production system. Therefore, the batch sizes of the products produced by GUROTO are one. Having batch sizes of one gives the company an internal flexibility to make multiple models on the same line.

Table 4.15 The information regarding the quarterly selling prices and demands of the five main products of the GUROTO Company.

	Product 1 (P_1)	Product 2 (P_2)	Product 3 (P_3)	Product 4 (P_4)	Product 5 (P_5)
Quarter 1					
Selling Prices (\$)	8,500	14,000	16,000	23,500	27,000
Demands	300	200	200	120	100
Quarter 2					
Selling Prices (\$)	8,000	13,000	15,500	24,000	28,000
Demands	250	130	150	150	125
Quarter 3					
Selling Prices (\$)	8,800	14,300	15,600	21,000	26,000
Demands	330	200	160	80	50
Quarter 4					
Selling Prices (\$)	9,000	14,500	16,500	20,000	23,000
Demands	400	240	220	50	20

Table 4.16 The information regarding the batch sizes, and the material and direct labor costs of the five main products of the GUROTO Company

	Product 1 (P_1)	Product 2 (P_2)	Product 3 (P_3)	Product 4 (P_4)	Product 5 (P_5)
Batch sizes	1	1	1	1	1
Material cost (per unit)	\$1,500	\$3,500	\$3,800	\$7,000	\$7,500
Direct Labor cost (per unit)	\$500	\$1,500	\$1,100	\$1,500	\$1,600

The company performs eleven major overhead activities to produce five products. The information regarding the budgeted rate, the practical quarterly capacity, the cost driver, and the charge rate of each activity are given in Table 4.17. The eleven major

activities of the GUROTO Company can be grouped into three types of activity: the unit-level activities, the batch-level activities, and the product-sustaining activities. However, since the batch sizes all the products produced in the company are one, there is no need to differentiate between the unit-level and the batch-level activities when modeling the system. In other words, we do not need to include an extra variable to the model to represent the number of batches produced in one period. The activity usage of each product is given in Table 4.18.

When we compare the cost structure of the GUROTO Company with those of other companies in the same industry that have not implemented lean manufacturing, the following differences can be observed:

- The preventive maintenance activity in the GUROTO Company has a much higher cost compared to the costs of same activity in companies with similar products. This is because in a lean manufacturing environment keeping material flowing and delivering orders to the customers on-time depends on equipment that is functioning properly. Preventive maintenance is the crucial prerequisite activity to provide highly reliable equipment in a manufacturing system.
- The setup costs of the GUROTO Company are much lower compared to the costs of the same activity in companies with similar products. These low setup costs allow the company to build its products in a one-piece flow in a leveled and mixed production sequence. By using the flexibility in its setup activities, the GUROTO Company gains the advantage of being able to produce a large variety of products.

Table 4.17 The calculation of charge rates for each overhead activity of the GUROTO Company

Activity Number	Activities	Cost Drivers	Budgeted Rate	Practical Capacities	Charge Rates
Unit-Level Activities					
1	Preventive Maintenance	No. of machine hours	\$800,000	133,333 machine hours	\$6.0/ machine hour
2	Automatic Machining	Number of machine hours	\$600,000	15,000 machine hours	\$40.0/ machine hour
3	General Machining	Number of machine hours	\$500,000	100,000 machine hours	\$5.0/ machine hour
4	Assembly	Number of assembly labor hours	\$400,000	50,000 assembly labor-hours	\$8/ assembly labor hour
Batch- Level Activities					
5	Set-up	Number of set-up labor hours	\$300,000	20,000 set-up labor hours	\$15.0/setup labor hour
6	Material Handling	Number of material moves	\$600,000	150,000 material moves	\$4.0/ material movement
7	Receiving	Number of invoices	\$800,000	40,000 invoices	\$20.0/invoice
8	Shipping	Number of shipments	\$200,000	5,500 shipments	\$400/shipment
9	Production and Inventory Control	Number of scheduling labor hours	\$450,000	1,8000 scheduling labor hours	\$25/scheduling labor hour
Product-Sustaining Activities					
10	Engineering	Number of engineering change notices	\$600,000	150 engineering change notices	\$4,000/engineering change notice
11	Vendor Relations	Number of vendors	\$2,000,000	133 vendors	\$15,000/vendor

Table 4.18 The overhead activity consumption of the five products of the GUROTO Company

Activity Number	Activities	Product 1 (P_1)	Product 2 (P_2)	Product 3 (P_3)	Product 4 (P_4)	Product 5 (P_5)
Unit- Level Activities						
1	Preventive Maintenance	120 machine hours/ unit	150 machine hours/ unit	157 machine hours/ unit	182 machine hours/ unit	195 machine hours/ unit
2	Automatic Machining	20 machine hours/ unit	30 machine hours/ unit	33 machine hours/ unit	42 machine hours/ unit	45 machine hours/ unit
3	General Machining	100 machine hours/ unit	120 machine hours/ unit	124 machine hours/ unit	140 machine hours/ unit	150 machine hours/ unit
4	Assembly	25 assembly labor hours/ unit	35 assembly labor hours/ unit	38 assembly labor hours/ unit	45 assembly labor hours/ unit	48 assembly labor hours/ unit
Batch- Level Activities						
5	Set-up	10 setup labor hours/ batch	16 setup labor hours/ batch	18 setup labor hours/ batch	23 setup labor hours/ batch	25 setup labor hours/ batch
6	Material Handling	125 material moves/ batch	140 material moves/ batch	150 material moves/ batch	160 material moves/ batch	165 material moves/ batch
7	Receiving	29 invoices/ batch	36 invoices/ batch	38 invoices/ batch	46 invoices/ batch	50 invoices/ batch
8	Shipping	0.25 shipments/ batch	0.25 shipments/ batch	0.25 shipments/ batch	0.5 shipments/ batch	0.5 shipments/ batch
9	Production and Inventory Control	15 scheduling labor hrs/ batch	20 scheduling labor hrs/ batch	20 scheduling labor hrs/ batch	25 scheduling labor hrs/ batch	25 scheduling labor hrs/ batch
Product-Sustaining Activities						
10	Engineering	20 ECN/ product	22 ECN/ product	20 ECN/ product	23 ECN/ product	25 ECN/ product
11	Vendor Relations	10 vendors/ product	15 vendors/ product	18 vendors/ product	22 vendors/ product	24 vendors/ product

- The costs of the receiving and shipping activities of the GUROTO Company are higher than the costs of these activities in similar companies with mass production systems. For lean manufacturing to work in a company, more frequent receiving and shipping of smaller lots are needed instead of the infrequent larger lots.
- The production and inventory control activities of the GUROTO Company are relatively less costly because the inventory levels are minimized in the company.
- The GUROTO Company builds quality in rather than trying to inspect it in after the fact. The quality control department in GUROTO is an active partner in helping to solve root-cause problems. As a result, unlike many other companies, the costs of this activity are driven by the number of engineering change notices, not by the number of inspections. In GUROTO, the quality control engineering activity is integrated into the manufacturing systems engineering activity and is included in Tables 4.17 and 4.18 under the heading “engineering”.
- The costs of vendor-relations activity are higher in the GUROTO Company compared to the costs of similar activities in other companies in the same industry. This is because suppliers play a crucial role in the success of any lean manufacturing system. For a lean manufacturing system to work, suppliers themselves must also be JIT manufacturers and must be able to deliver high quality parts at specified times. This necessitates GUROTO to have closer relations with its suppliers, which, in turn, increases the costs of vendor-relations activity.

The model is solved by using CPLEX. The output of the model is given in Appendix W. By taking the action suggested in Appendix W, The GUROTO Company will be able to earn \$7,203,503 next year. The bottleneck of the whole system is the automatic machining activity because the value of the slacks in constraints 13- 16 are either zero or very close to zero. From now on, the management of the GUROTO Company should focus on the automatic machining activity to improve the performance of the whole system.

This case shows that the model developed in Chapter 3 can also be used in a lean manufacturing system to determine the best product mix. The key difference that should be taken into account when building the model in a lean environment is that there is no need to differentiate between the unit-level and the batch-level activities in a lean manufacturing system because the batch sizes of all the products in these kinds of systems are assumed to equal one.

Chapter V

CONCLUSIONS and RECOMMENDATIONS

5.1 Conclusions

This thesis demonstrates that using direct labor dollars as the only basis to allocate overhead costs may give misleading information about the profitability of products and may result in poor decisions about the optimal product-mix and bottlenecks of a company. This research proposes that activity-based cost information should be used in determining the best product-mix of a company and develops a new product-mix decision model accordingly. Specifically, the proposed model has the following features that make it distinct from the conventional product-mix problem:

- 1- The conventional product-mix decision model is based on the assumption that there are only two resources, direct labor and material, that may have limited capacities in a system. In other words, the conventional approach ignores the fact that overhead activities performed in a company may also have limited capacities and, therefore, may become bottlenecks just as direct resources. The model developed in this thesis brings a solution to this problem by including capacity-related constraints for indirect resources as well as direct resources in the product-mix problem.
- 2- The overhead activities performed in a company are grouped into three categories, unit-level, batch-level, and product-sustaining level, and are included in the model by using three different kinds of constraints.
- 3- The proposed model can also be used to determine the unit cost of a product under the ABC approach. First, the optimal solution should be determined by using the new model. Then, the unit costs of products under the ABC approach can be calculated by

dividing the total costs allocated to a product during a specific time period by the optimum production level of that product during the same period.

- 4- Since the proposed model uses more accurate cost information than the conventional approach in determining the best product-mix and bottlenecks of a company, it helps management greatly to focus on the right resources in order to improve the performance of the whole system.
- 5- In addition to determining the bottlenecks of a system correctly, the proposed model shows explicitly the amount of slack capacity of each activity available during a specific time period. Knowing the excess capacity that will be available during a specific period ahead of time will help managers to decide whether they can sell this capacity to other companies during that period, and therefore will help to improve the profitability of the company.

Four case studies, all of which are based on hypothetical data, are included in this research to demonstrate the applicability of the proposed product-mix decision model in different manufacturing environments. Based on the findings of the first case study, it can be concluded that using direct labor dollars as the only basis to allocate overhead costs can give misleading information about the profitability of products and can cause companies to manufacture unprofitable products. The management might even use the TOC philosophy based on the information obtained from conventional product-mix decision model and try to improve the capacities of the bottleneck resources in order to increase the throughput. However, increasing the capacities of the bottleneck resources may only cause the company to lose more money because the additional capacities of the bottleneck resources will, most probably, be used to produce more unprofitable products. Therefore, it can be said that using activity-based cost information together with the TOC approach may be critical in determining the optimal product-mix and the bottlenecks of a company.

The second case study illustrates the use of the proposed model in a complex manufacturing environment. In this case study, it was assumed that the management of the AYBEN Company had several alternatives such as activity flexibility and outsourcing that could improve the performance of the system. However, without having a general understanding of the TOC philosophy, management would not be able recognize the values of these alternatives for the company. For example, in the second case study there were two routing alternatives for the company to produce P_1 (see p. 92). If the managers of the AYBEN Company were not knowledgeable enough about the TOC philosophy, they would, most probably, ignore this flexibility option and would continue to manufacture each batch of P_1 by using route 1. This is because the first routing alternative is a more efficient and less costly way to produce P_1 compared to the second routing alternative. Without applying the TOC philosophy, the management of the AYBEN Company would think that the profitability of the company would be increased if they used the most efficient and the least costly routing alternatives to produce products. However, this approach is completely erroneous and can decrease the financial performance of the company significantly because it can cause sub-optimization. Optimizing the way each product is manufactured does not necessarily mean optimizing the performance of the whole company.

Fortunately, the management of the AYBEN Company had a general understanding of the TOC philosophy and considered the activity flexibility alternative seriously after determining the automatic machining activity as the bottleneck of the whole system (See Appendix M). By including this alternative in the proposed model, the management found that using the second routing alternative is more profitable than using the first one whenever the available capacity of the bottleneck operation is insufficient to meet the market demands for other products that do not have this flexibility option. Consequently, this case study shows that using activity-based cost information with the linear programming technique is not enough to increase the profitability of a company. A product-mix decision problem based on activity-based cost information will only give management the necessary information to act on. It is the responsibility of the management to evaluate the output of the model and to apply the principles of the TOC

philosophy in order to improve the profitability of a company. Without using the TOC philosophy, accurate information about the best product-mix and bottlenecks of a company will not be sufficient to take the right actions, and as a result the survival of the company may be in question.

The third case study shows that the model developed in this research can be used for multi-period planning purposes. By extending the model to include more than one time period, companies will be able to estimate their potential bottlenecks and the amount of idle capacities of each non-bottleneck activity ahead of time. This information can give companies a great competitive advantage because they will have enough time to improve the performance of their potential bottlenecks and to search for more profitable usage alternatives of their excess capacities before the actual production takes place.

The fourth case demonstrates that the model developed in this research can also be used in a lean manufacturing environment where the batch sizes of all the products are assumed to equal one. The only difference that should be considered when using the proposed model in a lean environment is that there is no need to differentiate between the unit-level and the batch-level activities in these kinds of systems.

Based on the four case studies presented in this thesis, it can be concluded that using ABC, together with the TOC philosophy and the integer programming technique, is a worthwhile framework for dealing with product-mix problems. Without ABC, managers will not have accurate information regarding the best product-mix and critical bottlenecks of their companies to act on. Without the TOC philosophy, they will not be able to make the right decisions that will improve the profitability figures. However, it should be noted that this conclusion is valid for companies that have diverse products and high overhead costs in relation to direct labor costs. For companies with very few products and low overhead to direct labor cost ratios, using ABC information in determining the best product-mix of a company may not be that critical because traditional costing will not cause major cost distortions in these kinds of environments.

The major difficulty that is expected in a real-life implementation of this research is to collect the data necessary for the proposed product-mix decision model. However, companies that have already implemented ABC can use this new model without spending considerable effort and can benefit greatly from the accurate information obtained.

5.2 Recommendations for Future Research

When building the proposed multi-period product-mix decision model, it was assumed that no inventory is carried from one period to the next. This assumption can be relaxed by modifying the model to include the holding costs of inventory. Furthermore, if any backorders are allowed, the relevant penalty costs must be included in the objective function of the new model.

In this study, to simplify the proposed model, it is assumed that products with an activity flexibility option do not have an outsourcing option and vice versa. Future research is recommended to improve the model by relaxing this assumption. Furthermore, it is also possible for the elements of the sets *PS*, *PS1*, *PS2*, *PS3*, *PF1*, *PF2*, *PO1*, *PO2*, *FC1*, *FC2*, and *FC3* to change by time (see p. 75). For example, a product that has an outsourcing option during a period may not have this option in another period. The proposed model may well be extended to incorporate this feature.

The activity flexibility option is incorporated into the new model by defining two routing alternatives, route 1 and route 2 that include only two activities. The model can be modified to include more than two activities in one route. In addition, future research is needed to include more than one activity flexibility option to the proposed model. Dependencies among the different flexibility options should also be considered while conducting this future research. Furthermore, this study demonstrates how to include an outsourcing alternative for one activity in a product-mix decision problem. Extending the model to include more than one outsourcing option is possible.

This study demonstrates how the profitability of a company can be increased by considering several alternatives such as activity flexibility and outsourcing. However, the

alternative of improving the performance of a system through capital investments is not included in this study. Future research is required to develop systems that will demonstrate the impact of various machinery choices on the performance of a whole system.

All of the four case studies included in Chapter 4 are based on hypothetical data. As a result, the applicability of the proposed model still needs to be confirmed. Therefore, testing the concepts developed in this study on real companies with real data is recommended as a final area of future work.

REFERENCES

- [1] Baxendale, Sidney J. and Mahesh Gupta, "Aligning TOC & ABC for Silkscreen Printing", Management Accounting, April 1998, p.39-44.
- [2] Brausch, John M. and Thomas C. Taylor, "Who is Accounting for the Cost of Capacity?", Management Accounting, February 1997, p.44-50.
- [3] Campi, John P., "It's Not as Easy as ABC", Journal of Cost Management for the Manufacturing v.6, n3, 1992, p.5-11.
- [4] Campbell, Robert J., "Stealing Time with ABC or TOC", Management Accounting, January 1995, p.31-36.
- [5] Campbell, Robert J., Peter Brewer, and Tina Mills, "Designing an Information System Using Activity-Based Costing and the Theory of Constraints", Journal of Cost Management, January-February 1997, p. 16-25.
- [6] Canada, John R., William G. Sullivan, and John A. White, Capital Investment Analysis for Engineering and Management. New Jersey: Prentice Hall Inc., 1996.
- [7] Cokins, Gary, "TOC vs. ABC: Friends or Foes?", APICS 1999 Constraints Management Symposium, Phoenix, March (22) 1999.
- [8] Compton, Ted R. (1996). "Implementing Activity-Based Costing", The CPA Journal, v.66, n.3, 1996, p.20-23.
- [9] Connolly, Tim and Gary Ashworth, "Activity-Based Management: Software Issues and Options", Management Accounting, v.44, n.5, 1994, p.14-16.

[10] Cooper, Robin, “Does Your Company Need a New Cost System?”, Journal of Cost Management, v.6, n.3, 1987, p.45-49.

[11] Cooper, Robin, “The Rise of Activity-Based Costing- Part One: What is an Activity-Based Costing?”, Journal of Cost Management v.7, n.3, 1988, p.45-54.

[12] Cooper, Robin, “The Rise of Activity-Based Costing- Part Two: When Do I Need an Activity Based Cost System?”, Journal of Cost Management v.7, n.4, 1988, p. 41-48.

[13] Cooper, Robin, and Robert S. Kaplan, The Design of Cost Management Systems. New Jersey: Prentice Hall Inc., 1991.

[14] Cooper, Robin and Robert S. Kaplan, “Profit Priorities from Activity-Based Costing”, Harvard Business Review, v.12, n.5, 1991, p.130-135.

[15] Cooper, Robin and Robert S. Kaplan, “Activity-Based Systems: Measuring the Costs of Resource Usage”, Accounting Horizons, September 1992, p.1-13.

[16] Cooper Robin, Robert S. Kaplan, Lawrence S. Maizel, Eileen Morissey, and Ronald M. Oehm, Implementing Activity-Based Cost Management: Moving from Analysis to Action, Montvale, NJ: Institute of Management Accountants, 1992, p.308.

[17] Demmy, Steve, and John Talbott, “Improve Internal Reporting with ABC and TOC”, Management Accounting, November 1998, p. 18-24.

[18] Dilton-Hill, G. Kevin and Ernest Glad, “Managing Capacity”, Journal of Cost Management, Spring 1994, p.32-39.

[19] Galloway, D., and D. Waldron, “Throughput Accounting: The Need for a New Language for Manufacturing”, Management Accounting (UK), November 1988, p.34-35.

[20] Goldratt Eliyahu, What is This Thing called Theory of Constraints and How Should It be Implemented? New York: North River Press Inc., 1990.

[21] Goldratt Eliyahu and Jeff Cox, The Goal: A Process of Ongoing Improvement, North River Press Inc., 1992.

[22] Greenwood, Thomas G., and James M. Reeve, “Activity-Based Cost Management for Continuous Improvement: A Process Design Framework,” Journal of Cost Management, Winter 1992, p.22-40.

[23] Gupta, Mahesh, Sid Baxendale, and Kathy McNamara, “Integrating TOC and ABCM in a Health Care Company”, Journal of Cost Management, July-August 1997, p. 23-33.

[24] Harsh, Mary Frances, “The Impact of Activity Based Costing on Managerial Decisions: An Empirical Analysis” Ph.D. Dissertation, Virginia Polytechnic Institute & State University, Blacksburg, VA, USA, 1993.

[25] Holmen, J. S., “ABC vs. TOC: It’s a Matter of Time,” Management Accounting January 1995, p.37-40.

[26] Ioannou, George and William G. Sullivan, “Use of Activity-Based Costing and Economic Value Analysis for the Justification of Capital Investments in Automated Material Handling Systems”, International Journal of Production Research, v.37, n.9, 1999, p. 2109- 2133.

[27] Johnson, H. T. and Robert S. Kaplan, Relevance Lost: The Rise and Fall of Management Accounting, Boston: Harvard Business School Press, 1987.

[28] Kaplan, Robert S., “Introduction to Activity Based Costing”, NAA Conference, Global Solutions to Global Problems II, Boston, MA, (March 30-31) 1989, p.32-43.

[29] Kee, Robert, “Integrating ABC and the Theory of Constraints to Evaluate Outsourcing Decisions”, Journal of Cost Management v.12, n.1, January-February 1998, p. 24-36.

[30] Klammer, Thomas, Capacity Measurement & Improvement: A Manager’s Guide to Evaluating and Optimizing Capacity Productivity, Chicago: CAM-I, 1996.

[31] Krumwiede, Kip R., “Why It is Tried and How It Succeeds” Management Accounting, April 1998, p.32-38.

[32] Liggett, H. R., J. Trevino, and J. P. Lavelle, “Activity-Based Cost Management Systems in an Advanced Manufacturing Environment” Economic and Financial Justification of Advanced Manufacturing Technologies, H. R. Parsaei, ed. (New York: Elsevier Science Publishers), 1992.

[33] Luebbe, Richard, and Byron Finch, “Theory of Constraints and Linear Programming: a Comparison”, Int. Journal of Production Research, v.30, no.6, 1992, p.1471-1478.

[34] Mabin, V.J. “Using spreadsheet optimisation facilities as a decision aid within the Theory of Constraints framework” Report 9/95, Victoria University of Wellington, Wellington, New Zealand, 1995.

[35] Mabin, V.J. and J. Gibson, “Synergies From Spreadsheet LP Used With the Theory of Constraints – a Case Study”, Journal of Operational Research Society, v.49, 1998, p.918-927.

[36] MacArthur, John B., “Theory of Constraints and Activity-Based Costing: Friends or Foes?” Journal of Cost Management, Summer 1993, p.50-54.

[37] Malik Shadan A., and William G. Sullivan, "Impact of ABC Information on Product Mix and Costing Decisions", IEEE Transactions on Engineering Management v.42, n.2, May 1995, p.171-176.

[38] McNair, C. J., "The Hidden Costs of Capacity", Journal of Cost Management, Spring 1994, p. 12-24.

[39] Morrow, Michael and Martin Hazell, "Activity Mapping For Business Process Redesign," Management Accounting (UK), February 1992, p.36-38.

[40] Nahmias, Steven Production and Operations Analysis Illinois:Irwin Inc, 1993.

[41] Noreen, Eric, Debra Smith, and James T. Mackey, The Theory of Constraints and Its Implications for Management Accounting New Jersey: The North River Press, 1995.

[42] Novin, Adel M., "Applying Overhead: How to Find the Right Bases and Rates" Management Accounting n.3, p.40-43, 1992.

[43] O'Guin, Michael C., The Complete Guide to Activity-Based Costing New Jersey: Prentice Hall, 1991.

[44] Pemberton, Neal R., Logan Arumugam, and Nabil Hassan, "ABM at Dayton Technologies: From Obstacles to Opportunities", Management Accounting v.77, n.9, 1996 p.20-26.

[45] Pryor, "Updating Cost Management: The CAM-I Cost Management System (CMS) Approach" CAM-I Report.

[46] Raffish, N., "How Much Does That Product Really Cost?" Management Accounting v13, n.3, p.36-39, 1991.

[47] Raffish, N., and P. Turney, The CAM-I Glossary of Activity-Based Management Arlington: CAM-I 1991.

[48] Roberts, Michael W. and Katherine J. Silvester, “Why ABC Failed and How It May Yet Succeed?” Journal of Cost Management v.9, n.1, 1996, p.23-35.

[49] Ruchala Linda V., “New, Improved, or Reengineered?” Management Accounting December 1995.

[50] Ruhl, Jack M., “ABM for Continuous Improvement, the Theory of Constraints, and Open Book Management” Journal of Cost Management, Fall 1995.

[51] Salafatinos, Chris, “Integrating The Theory of Constraints and Activity-Based Costing” Journal of Cost Management, Fall 1995, p. 58-66.

[52] Shields, Michael D., Michael A. McEwen, “Implementing Activity-Based Costing Systems Successfully” Journal of Cost Management v.9, n.1, 1996, p.15-22.

[53] Spuede C., E. O. Henke and M. Umble, “Using Activity Analysis to Locate Profitability Drivers” Management Accounting, May 1994, p.43-48.

[54] Stein, Robert E., The Theory of Constraints: Applications in Quality and Manufacturing New York: Marcel Dekker, Inc., 1997.

[55] Stratton, Alan J., “Capacity Management: Back to the Future,” CMA, February 1996, p.15-18.

[56] Sullivan, William G., “A New Paradigm for Engineering Economy” The Engineering Economist v.36, n.3, 1992, p.187-200.

[57] Turney, P. B. B., “What is the Scope of Activity-Based Costing?” Journal of Cost Management v.9, n.4, 1990, p.40-42.

[58] Wagner, H. M., Principles of Operations Research. Englewood Cliffs, NJ: Prentice Hall, 1969.

[59] Weston, Ted, “Locating the Constraint” IIE Solutions May 1999, p.34- 39.