

Contents

1	Introduction	1
1.1	Past Research	2
1.2	Significance of this Work	4
1.3	Organization of this Thesis	4
2	Wavelet-based Image Compression	5
2.1	Multiresolution Analysis and the Discrete Wavelet Transform	6
2.2	Implementation of the Discrete Wavelet Transform	9
2.2.1	Vanishing Moments of Wavelets	10
2.2.2	DWT of 2-D Signals	11
2.3	Significance Tree Quantization	12
2.3.1	<i>SPIHT</i>	13
2.4	Symmetric Extension	14
3	Multiwavelets and Balancing	16
3.1	Multiwavelet Theory	17
3.1.1	Pre-processing the Input	18
3.1.2	Balanced Multiwavelets	21

3.1.3	The Time-varying Multiwavelet Filter Bank	23
3.2	The 2-D Multiwavelet Decomposition	24
4	Multiwavelet Characteristics	29
4.1	Multiwavelets: Examples	29
4.2	Filter Bank Characteristics	30
4.2.1	Behavior under Shift-variance	31
4.2.2	Magnitude Spectrum Characteristics	36
4.2.3	Group Delay Characteristics	41
4.2.4	Symmetry of Bases and Compact Support	45
4.2.5	Symmetric Extension	45
4.3	Summary of Filter Bank Characteristics	48
5	Performance Analysis and Results	50
5.1	PSNR Results	51
5.1.1	Scalar Wavelets	51
5.1.2	Unbalanced Multiwavelets	52
5.1.3	Balanced Multiwavelets	52
5.2	Subjective Results	58
5.2.1	Scalar Wavelets	59
5.2.2	Unbalanced Multiwavelets	59
5.2.3	Balanced Multiwavelets	59
5.3	Balanced Multiwavelet Compression using <i>HVS</i>	71
5.3.1	Results	73
5.3.1.1	SA ₄ ^b	73

5.3.1.2	Ort ₄ ^b	77
5.3.1.3	Bmw ₈	81
5.3.1.4	Bmw _S	83
5.3.1.5	Bmw ₁₂	85
5.4	Comparing <i>HVS on</i> with <i>JPEG2000</i>	87
5.5	Summary of observations	89
6	Conclusions and Future Work	91
6.1	Conclusions	91
6.2	Directions for Future Research	93
A	Test Images	95

List of Figures

- 2.1 Block diagram of a transform based lossy compression system. 6
- 2.2 Nested spaces for multiresolution analysis. 8
- 2.3 The perfect reconstruction scalar wavelet filter bank 9
- 2.4 Single-level 2-D scalar wavelet decomposition. 12
- 2.5 Pyramid of subbands 13
- 2.6 Hierarchical trees in multi-level decomposition 13
- 2.7 Symmetric extension for an even-tap biorthogonal scalar wavelet 15

- 3.1 The perfect reconstruction multiwavelet filter bank 18
- 3.2 Decomposition using GHM multiwavelet 20
- 3.3 Decomposition using a 2nd order balanced multiwavelet 22
- 3.4 The time-varying multiwavelet filter bank. 23
- 3.5 The 2-D multiwavelet filter bank 25
- 3.6 Single-level multiwavelet decomposition 25
- 3.7 Shuffling scheme. 26
- 3.8 Shuffled single-level unbalanced multiwavelet decomposition. 26
- 3.9 The 2-D balanced multiwavelet filter bank 27
- 3.10 Shuffled single-level balanced multiwavelet decomposition. 28

4.1	SA_4^u bases for even and odd integer shifts.	32
4.2	Ort_4^u bases for even and odd integer shifts.	33
4.3	SA_4^b bases for even and odd integer shifts.	33
4.4	Ort_4^b bases for even and odd integer shifts.	34
4.5	Bmw_8 bases for even and odd integer shifts.	34
4.6	Bmw_5 bases for even and odd integer shifts.	35
4.7	Bmw_{12} bases for even and odd integer shifts.	35
4.8	Magnitude characteristics of SA_4^u filter bank.	37
4.9	Magnitude characteristics of Ort_4^u filters.	37
4.10	Magnitude characteristics of SA_4^b filters.	38
4.11	Magnitude characteristics of Ort_4^b filters	38
4.12	Magnitude characteristics of Bmw_8 filters.	39
4.13	Magnitude characteristics of Bmw_5 filters.	39
4.14	Magnitude characteristics of Bmw_{12} filters.	40
4.15	Lowpass branch output of a balanced multiwavelet with a group delay difference of 2	41
4.16	Group delay characteristics of SA_4^b lowpass filters.	42
4.17	Group delay characteristics of Ort_4^b lowpass filters.	43
4.18	Group delay characteristics of Bmw_8 lowpass filters.	43
4.19	Group delay characteristics of Bmw_5 lowpass filters.	44
4.20	Group delay characteristics of Bmw_{12} lowpass filters.	44
4.21	Symmetric extension for multiwavelets.	47
4.22	Symmetric extension for Bmw_5	48

5.1	Zooming in on the image <i>Barbara</i>	61
5.2	<i>Barbara</i> compressed with D_8 at 64:1	61
5.3	<i>Barbara</i> compressed with $B_{9/7}$ at 64:1	61
5.4	<i>Barbara</i> compressed with $B_{22/14}$ at 64:1	62
5.5	<i>Barbara</i> compressed with SA_4^u at 64:1	62
5.6	<i>Barbara</i> compressed with ORT_4^u at 64:1	62
5.7	<i>Barbara</i> compressed with SA_4^b at 64:1	62
5.8	<i>Barbara</i> compressed with ORT_4^b at 64:1	63
5.9	<i>Barbara</i> compressed with Bmw_8 at 64:1	63
5.10	<i>Barbara</i> compressed with Bmw_S at 64:1	63
5.11	<i>Barbara</i> compressed with Bmw_{12} at 64:1	63
5.12	Zooming in on the image <i>Lighthouse</i>	64
5.13	<i>Lighthouse</i> compressed with D_8 at 64:1	64
5.14	<i>Lighthouse</i> compressed with $B_{9/7}$ at 64:1	64
5.15	<i>Lighthouse</i> compressed with $B_{22/14}$ at 64:1	65
5.16	<i>Lighthouse</i> compressed with SA_4^u at 64:1	65
5.17	<i>Lighthouse</i> compressed with ORT_4^u at 64:1	65
5.18	<i>Lighthouse</i> compressed with SA_4^b at 64:1	65
5.19	<i>Lighthouse</i> compressed with ORT_4^b at 64:1	66
5.20	<i>Lighthouse</i> compressed with Bmw_8 at 64:1	66
5.21	<i>Lighthouse</i> compressed with Bmw_S at 64:1	66
5.22	<i>Lighthouse</i> compressed with Bmw_{12} at 64:1	66
5.23	The original image <i>Yogi</i>	67
5.24	<i>Yogi</i> compressed with D_8 at 64:1	67

5.25	<i>Yogi</i> compressed with $B_{9/7}$ at 64:1	67
5.26	<i>Yogi</i> compressed with $B_{22/14}$ at 64:1	68
5.27	<i>Yogi</i> compressed with SA_4^u at 64:1	68
5.28	<i>Yogi</i> compressed with ORT_4^u at 64:1	68
5.29	<i>Yogi</i> compressed with SA_4^b at 64:1	68
5.30	<i>Yogi</i> compressed with ORT_4^b at 64:1	69
5.31	<i>Yogi</i> compressed with Bmw_8 at 64:1	69
5.32	<i>Yogi</i> compressed with Bmw_S at 64:1	69
5.33	<i>Yogi</i> compressed with Bmw_{12} at 64:1	69
5.34	Contrast sensitivity function	71
5.35	CSF-based mask weights	72
5.36	The <i>HVS on</i> scheme	72
5.37	<i>Barbara</i> at 32:1 using SA_4^b , <i>HVS off</i>	74
5.38	<i>Barbara</i> at 32:1 using SA_4^b , <i>HVS on</i>	74
5.39	Zooming in on <i>Barbara</i> at 32:1 using SA_4^b , <i>HVS off</i>	74
5.40	Zooming in on <i>Barbara</i> at 32:1 using SA_4^b , <i>HVS on</i>	74
5.41	Zooming in on a high frequency region of <i>Barbara</i> at 32:1 using SA_4^b , <i>HVS off</i>	75
5.42	Zooming in on a high frequency region of <i>Barbara</i> at 32:1 using SA_4^b , <i>HVS on</i>	75
5.43	<i>Ruler</i> at 32:1 using SA_4^b , <i>HVS off</i>	75
5.44	<i>Ruler</i> at 32:1 using SA_4^b , <i>HVS on</i>	75
5.45	<i>Lighthouse</i> at 32:1 using Ort_4^b , <i>HVS off</i>	77
5.46	<i>Lighthouse</i> at 32:1 using Ort_4^b , <i>HVS on</i>	77
5.47	Zooming in on <i>Lighthouse</i> at 32:1 using Ort_4^b , <i>HVS off</i>	78
5.48	Zooming in on <i>Lighthouse</i> at 32:1 using Ort_4^b , <i>HVS on</i>	78

5.49	Zooming in again on <i>Lighthouse</i> 32:1 using Ort_4^b , <i>HVS off</i>	78
5.50	Zooming in again on <i>Lighthouse</i> at 32:1 using Ort_4^b , <i>HVS on</i>	78
5.51	<i>Yogi</i> at 32:1 using Ort_4^b , <i>HVS off</i>	79
5.52	<i>Yogi</i> at 32:1 using Ort_4^b , <i>HVS on</i>	79
5.53	Zooming in on <i>Mandrill</i> at 32:1 using Bmw_8 , <i>HVS off</i>	81
5.54	Zooming in on <i>Mandrill</i> at 32:1 using Bmw_8 , <i>HVS on</i>	81
5.55	Zooming in on <i>Lena</i> at 64:1 using Bmw_8 , <i>HVS off</i>	82
5.56	Zooming in on <i>Lena</i> at 64:1 using Bmw_8 , <i>HVS on</i>	82
5.57	<i>Barbara</i> at 32:1 using Bmw_S , <i>HVS off</i>	83
5.58	<i>Barbara</i> at 32:1 using Bmw_S , <i>HVS on</i>	83
5.59	<i>Lighthouse</i> at 32:1 using Bmw_S , <i>HVS off</i>	84
5.60	<i>Lighthouse</i> at 32:1 using Bmw_S , <i>HVS on</i>	84
5.61	<i>Boat</i> at 32:1 using Bmw_{12} , <i>HVS off</i>	85
5.62	<i>Boat</i> at 32:1 using Bmw_{12} , <i>HVS on</i>	85
5.63	<i>Gray21</i> at 64:1 using Bmw_{12} , <i>HVS off</i>	86
5.64	<i>Gray21</i> at 64:1 using Bmw_{12} , <i>HVS on</i>	86
5.65	Original image <i>Lighthouse</i>	87
5.66	<i>Lighthouse</i> at 32:1 using SA_4^b with <i>HVS on</i>	88
5.67	<i>Lighthouse</i> at 32:1 using <i>JPEG2000</i>	88
A.3	Finger.	97
A.4	House.	98
A.5	Lena.	98
A.6	Lighthouse.	98

A.7 Mandrill.	98
A.8 Nitf7.	99
A.9 Peppers.	99
A.10 Gray21.	99
A.11 Ruler.	99
A.12 Satellite.	100
A.13 Yogi.	100

List of Tables

- 4.1 Notation for multiwavelets discussed in this thesis 30
- 4.2 Summary of Wavelet Characteristics. 49
- 5.1 PSNR Results for *Barbara*. 53
- 5.2 PSNR Results for *Boat*. 53
- 5.3 PSNR Results for *Finger*. 53
- 5.4 PSNR Results for *Goldhill*. 54
- 5.5 PSNR Results for *Gray21*. 54
- 5.6 PSNR Results for *House*. 54
- 5.7 PSNR Results for *Lena*. 54
- 5.8 PSNR Results for *Lighthouse*. 55
- 5.9 PSNR Results for *Mandrill*. 55
- 5.10 PSNR Results for *Nitf7*. 55
- 5.11 PSNR Results for *Peppers*. 55
- 5.12 PSNR Results for *Ruler*. 56
- 5.13 PSNR Results for *Satellite*. 56
- 5.14 PSNR Results for *Yogi*. 56
- 5.15 Upper bound on PSNR values for the $Bmws$ multiwavelet. 57

5.16 PSNR Results for SA ₄ ^b with <i>HVS</i> on.	76
5.17 PSNR Results for Ort ₄ ^b with <i>HVS</i> on.	80
5.18 PSNR Results for Bmw ₈ with <i>HVS</i> on.	82
5.19 PSNR Results for Bmw _S with <i>HVS</i> on.	84
5.20 PSNR Results for Bmw ₁₂ with <i>HVS</i> on.	86
A.1 Listing of test images.	96