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THE CORRELATION BETWEEN THE SPECIAL THEORY OF RELATIVITY AND SUBSONIC COMPRESSIBLE FLUID MECHANICS

by

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SYMBOLS

V = free-stream velocity

a = velocity of light

f = frequency of sound wave

L = wave length for sound

c = velocity of sound

F = observer in F-frame of reference

x = distance along the horizontal axis of F

y = distance along the vertical axis of F'

t = time units of F frame of reference

F' = observer in F'-frame of reference

x' = distance along the horizontal axis of F'

y' = distance along the vertical axis of F'

t' = time units of F'-frame of reference

D = chord of airfoil

S = length of streamline element

L' = lift per unit span for incompressible flow

C_l = section lift coefficient

q = dynamic pressure in pounds per sq. ft.

$\lambda = (1 - (V/c)^2)^{-\frac{1}{2}}$

u = velocity component parallel to x-axis

v = velocity component parallel to y-axis

$(P)_{comp.}$ = pressure coefficient for compressible flow

$(P)_i$ = pressure coefficient for incompressible flow

Δp = difference in pressure on upper and lower surfaces

SYMBOLS - continued

C_N = normal pressure coefficient

C_c = chordwise pressure coefficient

d = chord for pressure distribution

t = thickness of airfoil

NACA = National Advisory Committee for Aeronautics

ρ_0 = actual mass-density of the fluid

ρ = apparent mass-density of the fluid

V = velocity of fluid particle relative to frame of reference

F_N = normal pressure force

ds = straight-line segment of a body surface in direction of flow (unit thickness spanwise)

p = static pressure in the free-stream

p_1 = local static pressure (as at airfoil surface)

M = free-stream Mach number (V/c)

M_1 = local Mach number $(V/c)_1$

l = linear dimension of a body defined by the Reynold's Number

μ = coefficient of viscosity

INTRODUCTION

The need for accurate knowledge of compressible fluids moving at high speeds has been responsible for the rapid development of the science of compressible fluid mechanics. The advent of jet propulsion meant the availability of higher aircraft speeds than had ever been possible with the conventional reciprocating engine. As of a consequence, there was an urgent need for a valid theoretical approach to high speed flight phenomena in the field of aeronautical science.

The first scientific investigations revealed that in the field of fluid mechanics which dealt with compressible gases such as air, there existed four realms of study. That is, if a body, initially at rest in air, is brought up to a supersonic velocity, i.e., a velocity greater than the speed of sound, in the process of going through different speeds, the body will experience forces which follow four entirely different sets of laws. Each of the four laws are valid only in specific speed ranges as follows:

1. From 0-76 miles per hour (Mach number, $M = 0.1$) the forces follow classical laws of incompressible fluid mechanics. Mach number is the ratio of the velocity of the body to the velocity of sound in the fluid, i.e., $M=V/c$.

2. From 76 miles per hour up to 760 miles per hour (i.e., for Mach numbers $.1 < M < 1$) the fluid forces follow the so-called subsonic compressible flow theory.

3. Precisely at the speed of sound ($M = 1$) which is known as the "Sonic Barrier" or the Transonic region, the forces follow another set of laws which may be called the Transonic flow theory.

4. For speeds above the speed of sound ($M > 1$) the forces generated follow the so-called supersonic flow theory.

These four realms of study were discovered only after years of extensive fundamental work including both theoretical and experimental investigation. It was found experimentally that the incompressible fluid laws predicted the forces on a body with satisfactory accuracy as long as the speeds were below 76 miles per hour. However, for subsonic speeds above 76 miles per hour, the incompressible fluid theory failed to predict the forces experienced by the body. A theoretical investigation in dimensional analysis supported by experiment explains the discrepancies for these two speed ranges. It was proved that the forces on the body in the range, $0 \leq M < .1$, are a function of the Reynold's Number ($\rho V l / \mu$) and for speeds $.1 < M < 1$, the forces are a function of the Mach number, $M = V/c$. From these facts it is apparent why no one set of physical laws could be made compatible for these two regimes of flow; Reynolds Number is the ratio of inertia forces to viscous forces while Mach number is the ratio of inertia forces to elastic forces.

The experimental results for the low-speed range (0 to 76 mph) are in excellent agreement with the incompressible fluid theory. Also the supersonic theory has been shown by experiment to predict accurately the forces in that range ($M > 1$). Unfortunately, in the subsonic compressible flow range ($.1 < M < 1$) the experimental results do not agree with the present subsonic compressible flow theories. Since, also, there has been no valid theoretical approach to the Transonic regime, only the first and last speed ranges have adequately been formulated.

It is the purpose of this thesis to introduce a new theory of subsonic compressible flow which will accurately predict the fluid forces in this range ($.7 < M < 1$). With this new contribution to the science of compressible fluid mechanics the gap between the low-speed laws and the transonic region will be filled. Although it is not the purpose of this thesis, it is believed that this new approach may even be able to explain partially the phenomenon that occurs in this transonic region since part of the flow is subsonic.

In the development of the theory an attempt will be made to apply the concepts of Einstein's Special Theory of Relativity to compressible fluid flow. The relativistic idea that the speed of light is the limiting speed is replaced by a restricted universe, called the Sound-Space, where the speed of sound is the imposed limit. Results of the Lorentz transformation will be used to relate the space-and-time-units between the compressible and incompressible spaces. Thus a relation between the incompressible and compressible fluid theories will be established.

The concept of "local" reference frames is then to be introduced which will explain why the present classical theories are, at best, only approximations to the solution. It will also be clear, bythis analysis, why the relativistic approach can be considered to be an exact solution to the problem.

It should be stated that the two most successful approximate solutions (the Prandtl-Glauert and the Karman-Tsien relations) are attempts to solve the classical partial differential equation of motion by the so-called "small perturbation method" and the hodograph method.

The results of these two methods along with those of the Sound-Space Theory will be compared with various experimental data to show the greater accuracy of the relativistic approach.

Perhaps the most important fundamental relations which are derived by the Sound-Space Theory are those concerned with the mass, momentum and energy relations between the compressible and incompressible spaces. These relations which will be found are in the familiar Newtonian form, but with remarkable difference; the difference being an apparent increase in mass with increase in velocity. The simplicity of the results of the Sound-Space Theory are rather surprising. In a word, it may be said that the compressibility effects at any point on a body are solely a function of the Mach number at that point i.e., a function of the local Mach number. (See equation 19.5).

It is believed that the results of the Sound-Space Theory, in accurately predicting the compressible distribution over a body, will eliminate many costly wind tunnel testing programs. At the same time, designers in the field of compressible fluid mechanics will have available a reliable analytical tool with great flexibility.

In order to maintain complete originality in the analysis to follow, repetition of derivations from other work will be strictly avoided. Wherever a result is used that is not original with this thesis, it will be properly noted and the reference fully credited.

1. THE SPECIAL THEORY OF RELATIVITY ANALOGY

Certain physical concepts associated with compressible fluid flow can be better understood by an analogy of this flow to Einstein's Special Theory of Relativity. More specifically, the relativistic idea, that the velocity of light is the limit for the relative velocities of two Galilean frames of reference, is replaced by a realm in which the velocity of sound is the limiting case. The results are similar to the original Einstein conclusions but the boundary conditions are changed to meet the needs of the aerodynamic considerations.

It does not seem appropriate to go into the detail mathematical discussion of the Special Theory of Relativity since the analogy will adequately cover the necessary parts applicable to the objective.

Moving frames of reference in acoustics, using the Doppler effect, have been discussed in consideration of the concept of absolute space.[1] The conclusions were that an acoustic analogy would not hold since the medium carrying the sound (the air) provides a special co-ordinate frame to which all motion is to be referred. The essential, and very important, difference in this acoustic analogy and the present work with sound-space is that, in the concept of the sound-space, there is no absolute frame of reference.

2. CONCEPT OF THE SOUND-SPACE

Analogous to the form of the Special Theory of Relativity three hypotheses are postulated:

I. A frame of reference which is made up of fixed particles in a rigid body and which is isotropic with respect to sound experiments will be called a sonic frame. (Isotropic means the same velocity of propagation in all directions.)

II. Two sonic frames of reference are completely equivalent for sound experiments within certain boundary conditions. (See Hypothesis III)

III. Any two sonic frames of reference have, relative to one another, a uniform velocity of translation where the relative velocity is less than the velocity of sound.

This sound-space is filled with a certain hypothetical "medium" which is isotropic and has the identical properties of air with respect to propagation of sound at standard conditions. The reason for introducing this medium is that sound is to be propagated and it is desired to have its speed at standard air conditions.

As might first be suspected, there is no real loss of generality with the original relativity idea when Sound-Space is substituted for the remote vacuum. That is, by carefully avoiding the idea that there is an <u>absolute</u> frame of reference from which to view the two sonic frames, results similar to the original relativity theory will be derived. Of course, the results found will be valid only within the range of the boundary conditions (III) as are the results of the relativity theory valid only for relative speeds less than the speed of light.

3. OLD AND NEW CONCEPTS OF TIME AND SPACE

Up to the time of Einstein's Theory of Relativity the habitual mode of thinking followed these lines:

3.1) The time between two events is independent of the condition of motion of the frame of reference.

3.2) The distance between two points of a rigid frame is independent of the motion of the frame of reference.

These statements which are the conclusions of Newtonian mechanics are supported by the idea that there is only <u>one</u> absolute time. Newton defined time accordingly:

"Absolute, true and mathematical time flows in virtue of its own nature uniformly and without reference to any external object."

Einstein, however, pointed out[2] that no physical means exist at present whereby the <u>absolute</u> time can be singled out. Accepting this idea, it is necessary to deal with time-units and corresponding space-units which depend upon the motion of the observer.

For an understanding of the analogy, the reader must keep in mind that each of the observers in the two frames of reference have their own time-and-space-units which depend upon the motion of the observer. Of utmost importance is the idea that the two frames of reference cannot be viewed from any absolute frame. It is granted that the time-and-space-units thus employed are ficticious but in the study of actual fluid flow they suffice to give a truthful description of certain phenomena. Despite the ficticious nature of the different time-space systems used by observers under different conditions of motion, these systems are powerful tools for the study of fluid phenomena.

4. WAVELENGTH, FREQUENCY AND VELOCITY RELATIONS FOR SOUND

If a tuning-fork makes 320 complete vibrations in one second, it is said to have a frequency of "320 cycles per sec". When this fork is vibrating, it is sending out into the surrounding medium one cycle of a sine wave for each vibration of the fork, i.e., when 320 cycles of a sine wave are emitted per second, the frequency, f, is 320 cycles per second. After the fork has vibrated for one second, the front of the first wave will have traveled a distance of about 1,120 ft. from the source (velocity of sound in air is 1,120 ft. per sec.). The fork has emitted 320 whole waves in one second and these 320 waves lined up, end to end, will just fill the space between the first wave and the source, a distance of 1,120 ft. divided by 320 or 3.5 ft. The relation is

$$c = fL$$

5. MECHANICAL PRINCIPLES FOR SOUND WAVES

Rayleigh[3] stated that within wide limits, the velocity of sound is independent of its pitch (frequency). If this were not true, a quick piece of music would be heard at a little distance hopelessly confused and discordant. Suppose an observer stands at a distance from two tuning-forks, one of which is of high pitch and one of low pitch. If both of these forks are struck simultaneously the sound waves emitted will reach him at the same instant, i.e., the velocity of a sound wave is independent of the frequency.

The pitch (frequency) of a sound is liable to modification when the source and the recipient are in relative motion. It may be stated, for instance, that an observer approaching a source will meet waves with a higher pitch (frequency) than if there were no relative motion between them. Likewise, an observer moving away from the source would meet waves of lower frequency than if there were no relative motion between the source and the observer.

Corresponding results occur when the source is in motion either toward or away from the observer, the alteration of frequency depending only on the relative motion in the line of hearing.[4] If the source and the observer move with the same velocity there is no alternation of frequency whether the medium is in motion or not. This principle of alterration of <u>pitch</u> by relative motion was first found by Doppler in the study of light and is called the "Doppler Effect".[5] There has been some confusion between two perfectly distinct cases: that in which there is a relative motion of the source and observer, and that in which the medium is in motion while the source and the observer are at rest. In the latter case the circumstances are <u>mechanically</u> the same as if the medium were at rest and the source and observer had a common motion, and therefore, by Doppler's principle, no change of pitch is to be expected.[4] (It is assumed that in all cases the medium has the properties of homogeneous standard air with respect to propagation of sound waves.)

6. THE MECHANICAL PRINCIPLE OF RELATIVITY IN SOUND-SPACE

An accepted principle of physics is called the Mechanical Principle of Relativity and may be stated as follows:[6]

> "The laws of mechanics remain unchanged in form for any transformation from one co-ordinate system to another which has a uniform translatory motion relative to the first."

The preceding discussion leads immediately to the Mechanical Principle of Relativity for sound. That is, in each of the cases stated above, the observer may experience a change of frequency but, relative to the observer, the velocity of the sound is a constant value in all cases. If this were not true the statement that the velocity of sound is independent of the frequency would be violated. This startling fact may prompt the skeptical reader to state that the original frequency of the source is the basis for measurement of the velocity and any other observed frequency would correspond to a different velocity of sound. The error in this reasoning can be brought to light by the question: Is there an absolute value of the frequency or is the so-called original frequency just another relative value? By the Einstein idea, that there is no absolute time-and-space-units, then the latter is true, i.e., the value of the frequency is relative and not an absolute value because no absolute reference exists.

Another example should clarify the principle of relativity. Suppose an observer in his reference frame observes a sound of one frequency and after that he receives another sound of some other frequency. Suppose the idea, that there is no absolute frame, is violated for the moment, in order to see that the source and observer are fixed with respect to each other. Then, as explained before, the two sounds of different frequency would travel with the same velocity and the observer would measure each of them to have the same velocity with respect to his reference system. Now fulfill rigorously the Einstein idea of no absolute frame and take the only possible point of view: that of the observer in his reference frame. Assume that he has no knowledge of the origin of the sources of the two sound waves he has received. How is it possible for him to distinguish whether the two sources are fixed with respect to him or moving away from or toward him? For that matter, is he able to assure himself that there are two sources of sound or just one source either fixed the first time and moved the second time or vice-versa? The answer is, of course, obvious. There would be no experiment he could carry out that would enlighten him further. In any case he would measure the two sounds to have the same velocity relative to him.

7. NECESSARY CONDITIONS FOR COMPATIBILITY IN SOUND-SPACE

It is seen that, according to the Newtonian modes of thought (3.1) and (3.2), the Mechanical Principle of Relativity for sound-space is incompatible. For example, suppose that a stationary source of sound is placed in the middle of a highway and is sounded just as two automobles approach the source. Suppose auto A is moving with uniform velocity V and auto B is moving in the opposite direction with uniform velocity U. The air is assumed to be standard and for simplicity to be at rest with respect to the source. To an observer on the ground the velocity of sound is c; to the observer in auto A the forward sound wave has the velocity c + V, the rear sound wave a velocity c - V; to the observer in auto B, the forward sound wave to him has a velocity c + U, the rear sound wave a velocity c - U.

According to the Mechanical Principle of Relativity for Sound-Space it should be that

$$c = c - V = c + V = c - U = c + U$$

since the law of transmission of sound in standard air, which is a natural law, must be the same for the observers in the automobiles as for the stationary observer on the ground.

The Sound-Space principle (hypotheses II) may be made compatible by denouncing the Newtonian concepts (3.1) and

(3.2). This means the assumptions are that the time-and-space-intervals are different in the two reference systems. This assumption will lead to a logical theory which is analogous to the Special Theory of Relativity.

8. LIMITING CONDITIONS

For the analogy, let F and F' be the observers in the two sonic frames of reference. Instead of a flash of light being the event observed and recorded by F and F', the velocity of sound is to be measured. Thus, analogous to the relativity theory, the passage of a sound is to be measured by both observers in their separate frames of reference and from these separate histories the Lorentz transformation is used to obtain results similar to the relativity predictions.

It should be stated forcibly that the results to be obtained will be valid only where the velocity is less than the velocity of sound, i.e., the results to follow will only be applicable, at most, to subsonic regimes of flow.

9. RESULTS OF THE LORENTZ TRANSFORMATION

In a manner identical to the Special Theory of Relativity[7], the Lorentz transformation is used to relate the units of length and time between two sonic reference frames. Figure I depicts the two sonic reference frames F and F' moving with a constant velocity V relative to each other.

In the analogy the means of communication between the two frames of reference is restricted to sound. Thus a sound must be mutually observed by both F and F' in order that the Lorentz transformation may be applied. By hypothesis the relative velocity between the two systems F and F' must be less than the speed of sound. This limiting relative velocity is obvious in view of the postulated means of communication in the sonic space, i.e., the event, otherwise, could not possibly be observed by both F and F'.

It is important to note that the velocity of sound in the hopothetical Sound-Space holds the same position as does the velocity of light in the Theory of Relativity: that of a universal constant.

Thus, the application of the Lorentz transformation for the observers F and F' in the sonic space gives the relations between the time and space-units as[7]

9.1 $$x' = \frac{1}{\sqrt{1 - (V/c)^2}}(x - Vt)$$

or $$x' = \lambda(x - Vt)$$

9.2 $$x = \frac{1}{\sqrt{1 - (V/c)^2}}(x' + Vt')$$

or $$x = \lambda(x' + Vt')$$

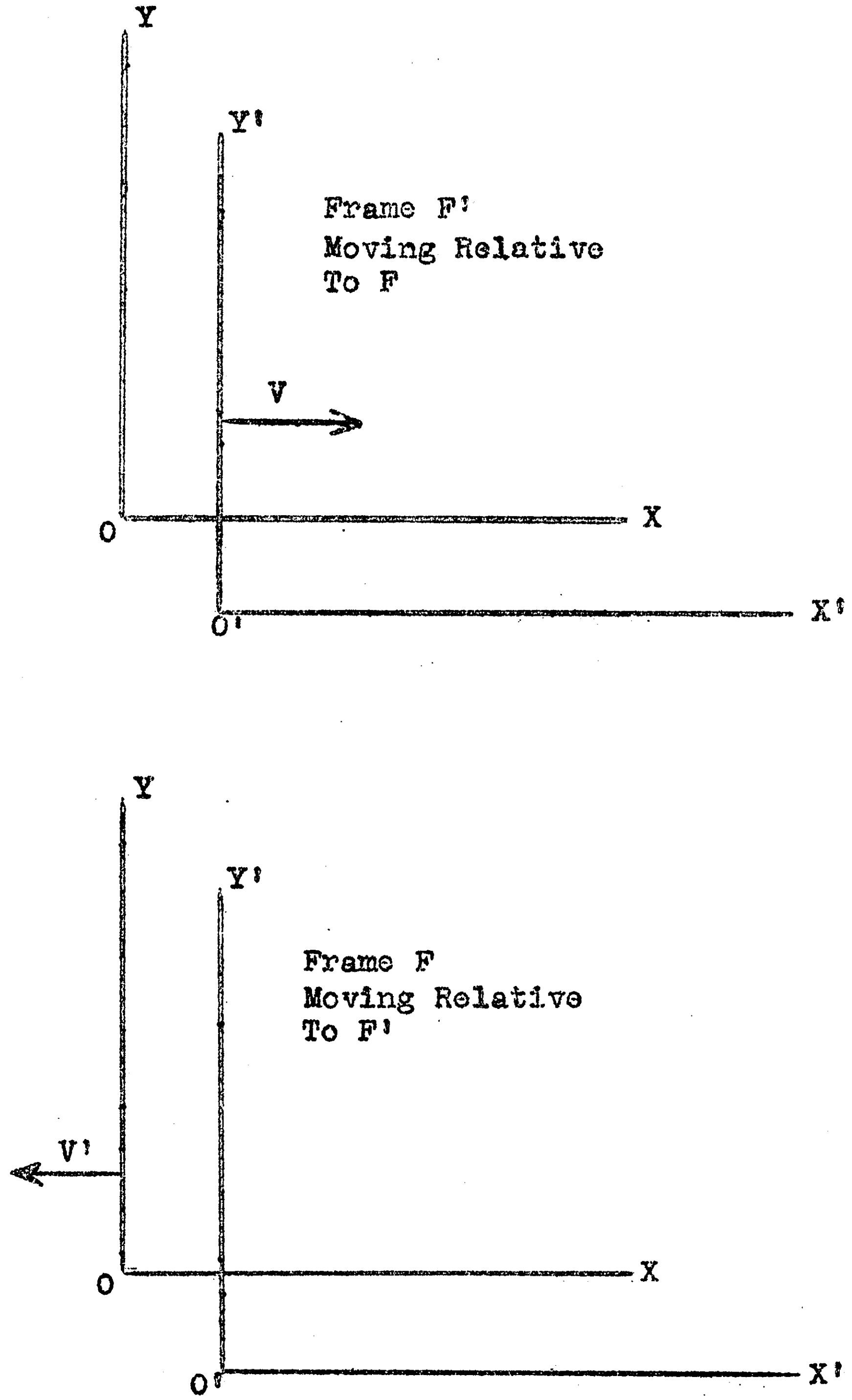


Fig I.Relative Velocities Between Observers F and F'.

where c = velocity of sound

$$\lambda = \frac{1}{\sqrt{1 - (V/c)^2}}$$

x = length in F-Frame in direction of velocity

x' = length in F'-Frame in direction of velocity

V = relative velocity between the F and F'-Frames

t = time in F-Frame

t' = time in F'-Frame

10. INTERPRETATION OF THE RESULTS

Results of experiments in real compressible fluid flow indicate clearly that there exists two entirely different regimes of flow. That is, at speeds below the speed of sound in the fluid, a body immersed in the fluid experiences certain characteristic effects. However, when the relative velocity between the body and fluid reaches the magnitude of the speed of sound in the fluid, a transition occurs such that an entirely different type of flow exists for speeds greater than the sonic speed. This transition from one type of flow (subsonic-compressible) to a radically different type (supersonic) which occurs at the speed of sound in the fluid, has prompted experimenters to call it the critical velocity.

In view of these facts, it seems obvious that the next logical step is to attempt application of results of section 9, obtained by use of the Lorentz transformation for the Sound-Space, to compressible fluid flow.

Proceeding with this idea, consider the case of an airfoil moving through a compressible gas (air) and fix one of the sonic frames of reference in the airfoil and fix the other in the fluid. For convenience, assume the frame of F' attached to the airfoil and the frame of F fixed in the fluid. See Figure II. To the observer in the F'-frame (fixed in the airfoil) the length of the chord of the airfoil is

$$D' = x_b' - x_a'$$

But to the observer with reference axis fixed in the fluid (F-frame) the apparent length to him at any instant is

$$D = x_b - x_a$$

According to equation (9.1)

10.1
$$x_b' = \lambda (x_b - Vt)$$
$$x_a' = \lambda (x_a - Vt)$$

Substituting these values in the expression of D' the relation of the chord length becomes

10.2 $$D' = \lambda D \qquad \text{or} \qquad D = D' \sqrt{1 - (V/c)^2}$$

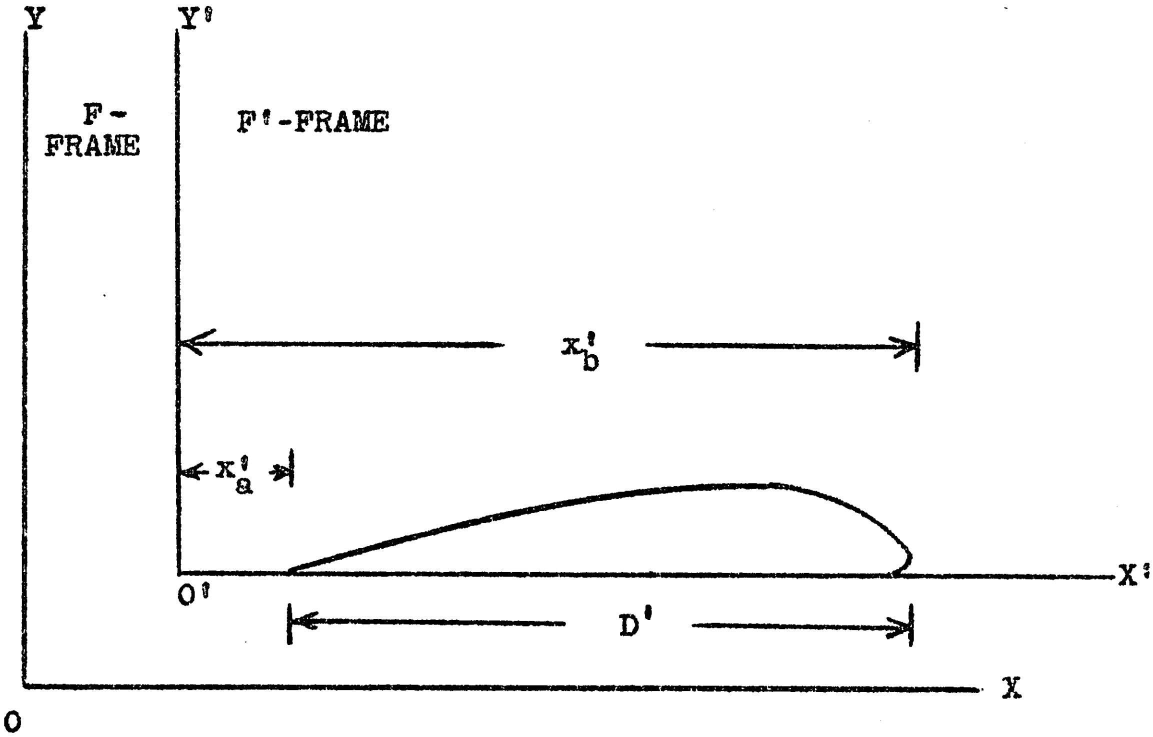


Fig II. Apparent Length of Chord To F Fixed in Fluid.

This analysis reveals that, to the observer whose reference axis is fixed in the fluid, the chord of the airfoil appears to be contracted by the amount

$$D'(1-(1 - (V/c)^2)^{-\frac{1}{2}})$$

(where V/c = Mach Number)

It might be expected that since, to the observer fixed in the fluid, the airfoil seems to be shortened, that conversely, to the observer fixed in the airfoil, the streamlines appear stretched. This is not the case.

To the observer in the F-frame (fixed in the fluid) the segment of the streamline is

$$S = x_a - x_b$$

Figure III. But to the observer fixed in the airfoil the streamline is of length

$$S' = x'_a - x'_b$$

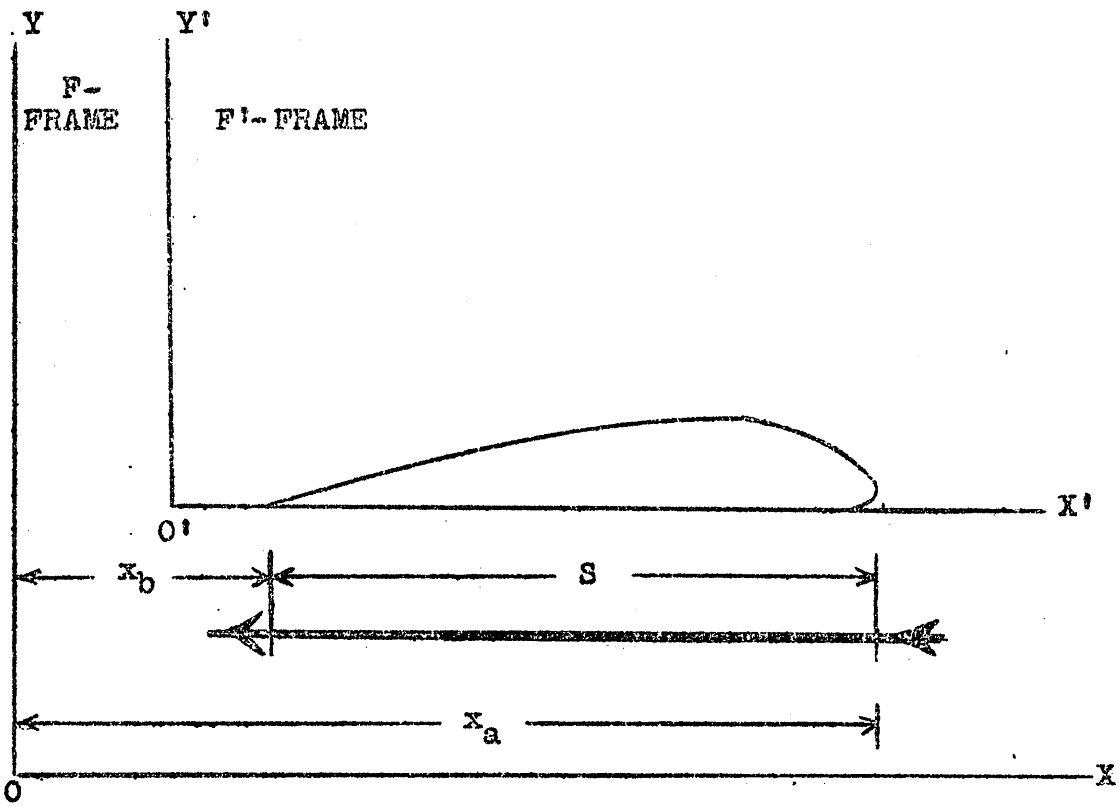


Fig III. Apparent Length of Streamline To F' Fixed in Airfoil.

10.3 $x_a = \lambda(x'_a + Vt')$

and $x_b = \lambda(x'_b + Vt')$ and also $t = t'\sqrt{1 - (V/c)^2}$

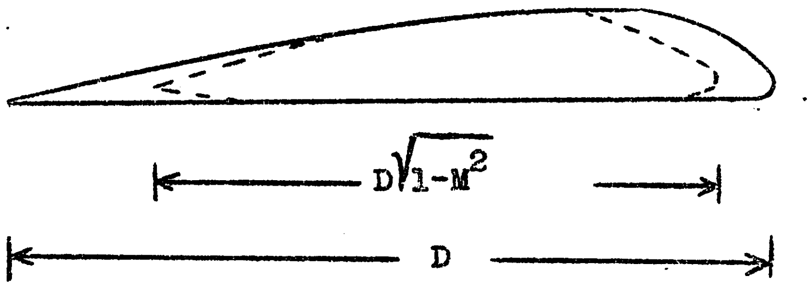
therefore $S = \lambda(x'_a - x'_b)$

or $S = \lambda S'$

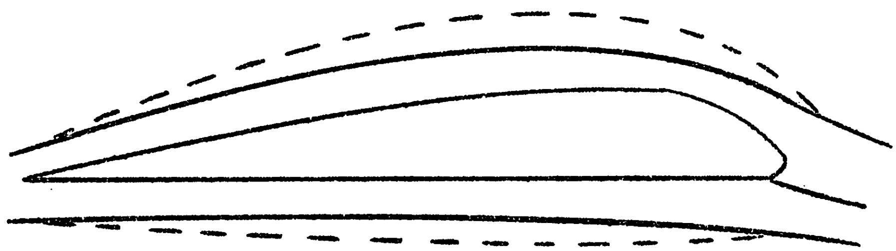
then $S' = S\sqrt{1 - (V/c)^2}$

Therefore, to the observer fixed in the airfoil, the streamlines appear to him to be contracted also by the same parameter. See Figure IV.

These results are analogous to the results of the Special Theory of Relativity as found by Einstein. However, it should be noted that in the case of the observer in the fluid the "overall" length of the airfoil appeared shortened. It is to be shown that, if the above analysis were completed at this point, the results would be only an approximation, no better than those derived by the perturbation method of Glauert. In other words, the airfoil used in the above analysis should have been a flat plate moving at an angle of attack of zero degrees in order for the analysis to be an exact solution. Thus only if the body is a flat plate airfoil at zero angle of attack could the velocity over the entire body be a constant value: the velocity of the free-stream. This is the key point in the exact solution to follow.



Apparent Contraction To F



Apparent Contraction To F'

FigIV. Contractions Observed By F And F'.

In order to show that the first results of the Sound-Space Theory are the same as those found by Glauert and Prandtl a comparison will be made. It will be clear immediately why existing theories are only crude approximate solutions.

11. COMPARISON WITH THE PRANDTL-GLAUERT RELATIONS

Ia. H. Glauert[8] and L. Prandtl, in an entirely different analysis, found the same result as above (apparent contraction of the chord) and explained that the solution is valid only when the Mach Number is less than unity, i.e., for speeds below the speed of sound.

b. This is clearly the case in the analogy since the Lorentz transformation is not valid for the relative velocity greater than the speed of sound (i.e. $V \geq c$).

IIa. Glauert stated that for the compressible fluid case it is necessary to increase the strength of the circulation by the factor $(1-(V/c)^2)^{-\frac{1}{2}}$. He explained that the distortion of the system is necessary near the airfoil but the flow at infinity is unaltered and therefore the lift is the same in both cases (compressible and incompressible). He further explained that the chord of the airfoil in the compressible fluid is less than in the perfect fluid, and hence for a given airfoil the lift in a compressible fluid is increased by the factor $(1-(V/c)^2)^{-\frac{1}{2}}$

b. To interpret the analogy, examine the lift equation for incompressible flow:

$$L' = C_1 \quad q(\text{chord}) = (\text{lift per unit span}).$$

It is assumed that the lift imparted to the fluid by the airfoil is of the same magnitude as for the incompressible case but is produced by an airfoil of apparently smaller chord. Therefore the lift for the compressible case is

$$L' = C_l \, q(\text{chord}\sqrt{1 - (V/c)^2})$$

Then, since the chord length is actually not shortened but affects the forces as if it actually does contract, the lift is multiplied by the factor $(1 - (V/c)^2)^{-\frac{1}{2}}$. It is important to note that the result of the shortened chord was for <u>two-dimensional</u> flow, i.e., the Lorentz transformation was restricted to the x-direction.

This increase in lift implied that the lift coefficient for the incompressible case (two-dimensional flow) is multiplied by λ for the compressible case:

11.1 $$(C_l)_{comp} = \lambda (C_l)_i$$

where $(C_l)_{comp}$ = lift coefficient (compressible)

$(C_l)_i$ = lift coefficient (incompressible)

By definition of the slope of the lift curve for incompressible flow, $dC_l/d\alpha$ it is clear that the slope of the lift curve will be multiplied by the factor λ.

IIIa. As pointed out by Glauert and others,[9] these effects on the lift coefficient are only approximations. In the derivation of this relation, Glauert assumed that the u-component (parallel to the x-axis) is exactly equal to the free-stream velocity. It is obvious that this would only be true for flat plate airfoils at zero angle of attack.

IIIb. It is seen immediately that this is the case for the Sound-Space theory. That is, the basic assumption was that the two reference frames have a <u>uniform</u> relative velocity. For any real airfoil, of course, there is a variation in velocity over the airfoil and this would mean that an infinite number of F and F' frames exist over the surface. See Figure V.

Each infinitestimal fluid element over the airfoil is effected by the "local" value of λ . Any success of the first approximation in predicting the effect of compressible flows is that the value, V, is a good average of the flow over the airfoil (at small angles of attack).

12. LOCAL REFERENCE FRAMES

The smooth curved surfaces of the airfoil in Figure V explains the difficulties in arriving at more than an approximation to the solution for compressible flows. An airfoil of simpler construction will bring out other factors and suggest a more versatile attack to the problem.

Consider the sharp nose supersonic-type airfoil as shown in Figure VI. For a closer approximation it seems necessary to consider the effect of several "elementary"

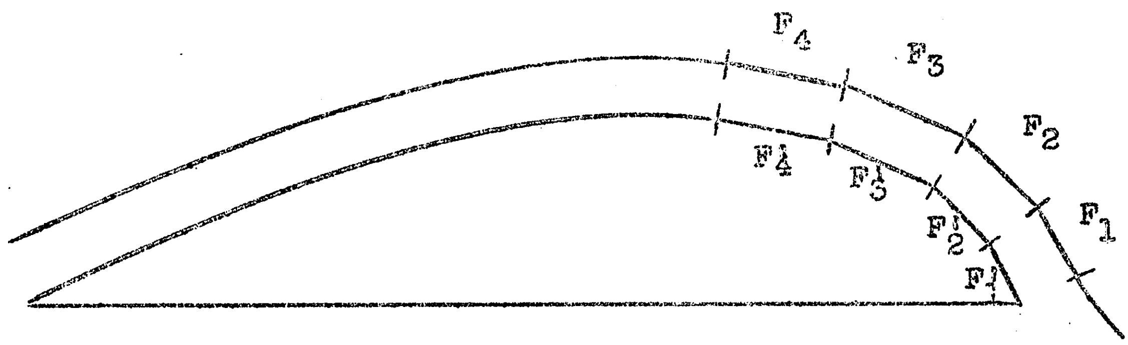


Fig V. Local Reference Frames For Conventional Airfoil.

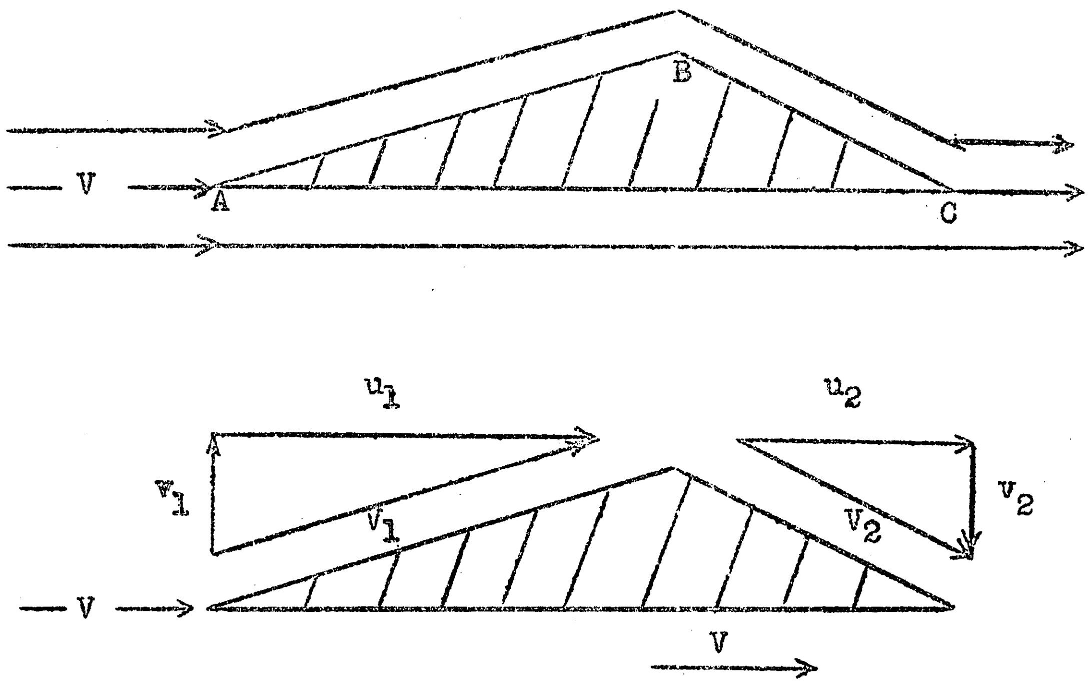


Fig VI. Airfoil With Three Elementary Reference Frames.

frames of reference such as AB, BC, AND AC. That is, over each of these segments there is a difference in the "local" velocity and therefore the separate contribution of each "local" frame must be taken into account.

The local frame AC would not differ from the first approximation since the value of the velocity over the lower surface is identical to the free-stream. However it is obvious, from conventional airfoil theory, that the contributions of the normal force (lift) will be greater for the local frames AB and BC since the velocity over these segments are necessarily greater than the free-stream velocity. Thus, the "local" Mach number will be greater than the free-stream Mach number and, hence, the effect will be an increase of "local" lift coefficient at AB and BC. It is clear, from these considerations, why equation (11.1) is only a conservative approximation to the resulting lift coefficient, i.e., it appears that in similar cases for conventional airfoils, the lift coefficient calculated by equation (11.1) is underestimated, (provided steady flow exists throughout). The determination of the actual lift coefficient would depend upon the accuracy with which the velocities over the local frames are predicted.

It was shown in the first approximation that the "overall" or average effect of compressibility is an apparent shortening of the chord only, i.e., there could be no change in thickness of the airfoil. That there is actually a rather sizable "perturbation" velocity, v, is seen on inspection of Figure VI. Reference frames inclined as are AB and BC will be shortened by the factor λ^{-1} as usual. However, since they are inclined, not only is the component parallel to the x-axis (chord) shortened by the factor $(1 - (u_1/c)^2)^{\frac{1}{2}}$ but also the component parallel to the y-axis (thickness) shortened by the factor $(1 - (v_1/c)^2)^{\frac{1}{2}}$. This means, that for an airfoil of non-zero camber, the effect would also be noteworthy, depthwise, i.e., by the same reasoning as in the first approximation, it appears that there would be an increase in the "pressure-drag" coefficient. Thus, a closer inspection of the problem has revealed the necessity for an adjustment for the lift coefficient and also predicts an increase of drag.

Considerable attention has been given to this apparent shortening of the dimensions of the body[10]. In these attempts at an approximate solution, the velocity potential was used to find a transformation equation between the compressible and incompressible planes. It was found that if there was circulation about the incompressible body, i.e., if the body experienced a lifting force, the corresponding compressible body could not be closed in the transformation[10]. In order to close the body in the transformation, i.e., that the body surface have no discontinuities, it was necessary that the circulation be zero. This means that the body cannot have a lifting force acting on it if corresponding shapes are to be found by the conformal transformation method. This result might have been anticipated since the first investigation by Glauert was only valid for an infinitely thin flat-plate airfoil acting at zero angle of attack. In any case, the treatment of the problem on the basis of body contractions alone did reveal an approximate solution to the problems of subsonic compressibility effects.

The body contractions found by the Sound-Space Theory were thoroughly investigated with the purpose of finding a body in the compressible plane that corresponded to a certain incompressible body shape.

The results of these investigations showed that the difficulty of finding a body in the compressible plane that would close up in the transformation was again present. It was possible to find the corresponding compressible body if and only if there was no circulation around the incompressible body. As in the other attempts, it was realized that this was only a trivial solution. The problem of finding an exact solution for a lifting body appeared to be founded on amore important fundamental relation than mere body contractions. Thus the apparent body contractions were abandoned and a search made for a more direct solution.

13. CONSIDERATION OF PRESSURE-COEFFICIENT

The refinement of the problem, by use of the local reference frames, has added to the complexity of the problem. To attack the problem more directly, it seems logical to consider the solution from the pressure distribution over the surface.

It is known from incompressible flow that the normal and chordwise force components are directly proportional to the pressure coefficient (dimensionless). Then the general relation between the compressible and incompressible force coefficient is

13.1 $$(P)_{comp} = (P)_i \lambda$$

where $(P)_{comp}$ = pressure coefficient (compressible)

$(P)_i$ = pressure coefficient (incompressible)

Caution should be taken in defining the normal and chordwise coefficients, since the thickness of the airfoil is not contracted by the same amount as the chord. That is, if the normal and chordwise coefficients are defined as

13.2 $$C_N = \frac{1}{C}\int \frac{\Delta p}{q}\, dC = \frac{1}{C}\int (P)_{comp}\, dC$$

13.3 $$C_c = \frac{1}{z}\int \frac{\Delta p}{q}\, dz = \frac{1}{z}\int (P)_{comp}\, dz$$

where C = chord

z = thickness

the area under the curve of the pressure coefficient plotted normal to the chord should be divided by the chord to find C_N. Likewise, the area under the curve of the pressure coefficient plotted normal to the thickness, z, should be divided by the thickness to produce C_c. By following this procedure, the varying effect of the perturbation velocities appears to be taken into account automatically.

14. RELATIONS OF MASS, MOMENTUM AND ENERGY

The relations of mass, momentum, and energy of fluid particles in compressible flow can be derived by an analysis similar to the relativity theory.[11] Results of the derivations for these relations reveal that relativistic mechanics are entirely different from Newtonian Mechanics. The relations are listed as follows:

14.1 Relative mass (density) $= \rho = \dfrac{\rho_0}{\sqrt{1 - (V/c)^2}}$

14.2 Relative momentum $= \rho V = \dfrac{\rho_0 V}{\sqrt{1 - (V/c)^2}}$

14.3 Relative energy $= KE = (\rho - \rho_0)c^2 = \dfrac{\rho_0 c^2}{\sqrt{1 - (V/c)^2}} - \rho_0 c^2$

Where ρ_0 = actual mass-density of the fluid particles

V = velocity of the particle relative to the frame of reference

ρ = apparent mass of the fluid particle

In equation (14.1) it is seen that the apparent mass of the fluid (or airfoil) is multiplied by the same factor λ. From the aerodynamic considerations, it appears that the effect of an apparent increase in mass of the fluid is quite important. This important relation appears in equation (14.2) where it is seen that the equation for momentum is in the Newtonian form but with remarkable difference. The difference is that the mass of the fluid particles are <u>not</u> constant but vary with the relative speed of the particle and the airfoil, i.e., the momentum is multiplied by the factor λ. The effect of the energy relations can be shown by the binomial expansion of $(1 - (V/c)^2)^{-\frac{1}{2}}$ and is

$$KE = \rho_0\left[\frac{1}{2} V^2 + \frac{3}{8}\frac{V^4}{c^2} + \ldots\right]$$

15. LIMITS OF APPLICATION

From the relations above it appears that the theory predicts infinite values of mass, momentum and energy at a Mach number of one. It must be remembered, however, that the equations derived are valid only for Mach numbers less than unity.

It is possible that an airfoil will have "critical" points (where the local velocity of sound is reached) before the airfoil as a whole reached sonic speeds. In these cases the relations found by the relativity theory again break down.

Even under the conditions of subsonic flows, an airfoil moving through a real viscous fluid might not experience steady flow conditions over the entire surface; turbulent flow regimes can be expected. The sonic theory in its present form is valid only for <u>steady flow</u>

conditions and would not predict accurately the actions of turbulent flows which undoubtedly occur over the rear portions of bodies in flight.

16. THREE DIMENSIONAL CASE

The Lorentz transformation can be developed for three-dimensional flow. If the flow over a finite wing were such that there was no lateral flow component, i.e., only flow in the x and y directions, the sonic theory would predict no compressibility effects spanwise. However, for a wing a finite aspect ratio there are lateral flow components caused by wing vortices. Qualitatively, it would appear that, since the circulation is increased by the factor, λ, an increase in induced drag could be expected with a corresponding reduction of "effective" aspect ratio. It may be that these effects would not be as great as one might, at first, be tempted to suggest. It does seem feasible, however, that with the apparent increase of mass, momentum, and energy good approximations can be made for downwash and vortex effects.

The transformation formula for the three dimensional case would be

$$\bar{r}' = \begin{pmatrix} \frac{1}{\sqrt{1-M_x^2}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{1-M_y^2}} & 0 \\ 0 & 0 & \frac{1}{\sqrt{1-M_z^2}} \end{pmatrix} \bar{r}$$

17. SUBSONIC PRESSURE DISTRIBUTION

One of the major problems in subsonic compressible flow is the reliable prediction of the pressure distribution over the surface of a body. From the pressure distribution the lift and pressure drag may be found as explained in 13.

After deriving the method of prediction of subsonic pressure distributions, a step-by-step procedure for the required calculations will be given. In order to show the ability of this new theory to predict a pressure distribution with greater accuracy and simplicity than other existing methods, comparative plots against experimental data will be made.

It should be particularly noted that the following method of predicting pressure distributions is not valid over a surface where the local Mach number is unity or greater.

18. BASIC CONSIDERATIONS

The foregoing theory of the Sound-Space has been introduced involving the relations of space and time between two reference frames (see equation 10.3). It will be convenient now to give the two reference frames F and F' the names compressible and incompressible planes, respectively. This is done in order to utilize the mass-density relations 14.1 with a minimum of confusion.

The relativistic equations for mass, momentum and energy of the air (equations 14.1, 14.2, 14.3) have now been found as are body contractions (equation 10.2) which are in the form of the Prandtl-Glauert contractions but are basically different. Although the expression for the body contraction is of the Prandtl-Glauert form, it is importantly different because the theory showed that there are <u>local</u> contractions which are a function of the <u>local</u> Mach number rather than the free-stream Mach number.

The task at hand is to derive an exact relation between the compressible and incompressible pressure coefficients. The concept of "local reference frames"[12] (see section 12) now enables one to predict the <u>local</u> pressure coefficients on the surface of a body at any given free-stream Mach number less than the critical provided the incompressible ($M = 0$) pressure distribution is known. In calculating the pressure distribution for a desired free-stream Mach number, there cannot be any point where the local Mach number of unity has been reached since the expression for the local compressible pressure coefficient is undefined at that point.

The Prandtl-Glauert and the Karman-Tsien relations[13] between the compressible and incompressible pressure coefficients have been used to calculate the compressibility effects for given free-stream Mach numbers for an NACA airfoil (NACA-4412) for which reliable experimental pressure distributions were known. These calculated results have been plotted along with those predicted by the Sound-Space theory and it appears that the latter predicts the experimental pressure distribution with greater accuracy.

The simplicity of the Sound-Space calculation in predicting the pressure distribution over a body and the closer agreement with experiment, particularly over the nose of the airfoil and especially over the entire lower surface, make this method of solution desirable.

19. THEORETICAL CONSIDERATIONS

Preliminary results of the Sound-Space Theory show that the apparent contraction of a straight-line portion (local reference frame) of a surface in a steady flow is of the magnitude (Figure VII)

19.1 $$(ds)_{comp} = (ds)_i \sqrt{1 - (V/c)_1^2}$$

The time relations between the compressible and incompressible plane are also found in the form:

19.2 $$(t)_{comp} = (t)_i \sqrt{1 - (V/c)_1^2}$$

These relations of space and time between the compressible and incompressible planes are then used in the same manner as in the Special Theory of Relativity[7] to find the relations of mass-density of the air:

19.3 $$(\rho)_{comp} = \frac{(\rho)_i}{\sqrt{1 - (V/c)_1^2}}$$

$$(ds)_{comp} \cong (ds)_1 \sqrt{1-(V/c)_1^2}$$

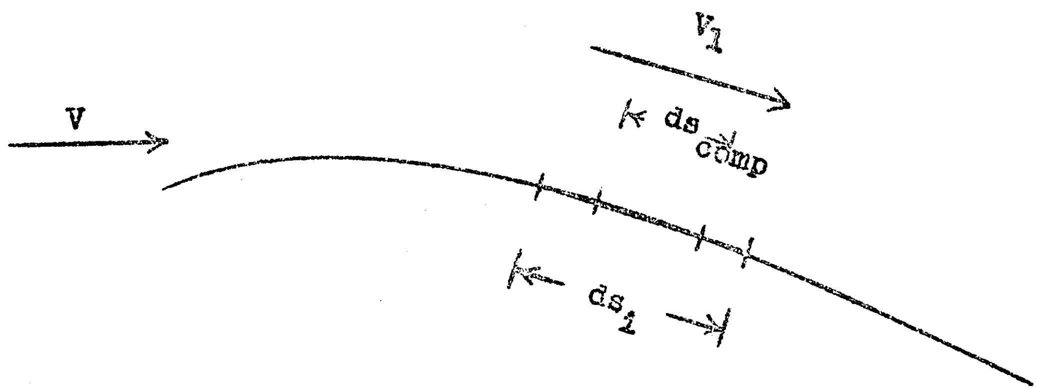


Fig VII. Apparent Contraction of a Straight-Line Portion of a Surface.

This expression reveals that the apparent mass-density in the compressible case is greater than that for the incompressible case by the factor $(1 - (V/c)_1^2)^{-\frac{1}{2}}$. It should be noted that this means that over a body, such as an airfoil, where the local Mach number, $(V/c)_1$, is different for different points along the surface, the magnitude of the apparent mass-density is different.

Obviously, the apparent contraction of a surface element as expressed in equation(19.1)is likewise a function of local Mach number. This is the essential and important difference in the results of the Sound-Space as compared to the Prandtl-Glauert relations with respect to contractions. It should be noted that equation 19.3 is new since only by using the relativistic equations 19.1 and 19.2 could this relation between the compressible and incompressible planes be derived.

The local density relation 19.3 can now be used to find the relation between the pressure forces on an infinitesinal surface element, ds, for a compressible and an incompressible fluid. For the same fluid element, ds, and the velocity over this element the same in both the compressible and incompressible case, the pressure forces have, according to equation (19.3), the relation

19.4 $$(dF_N)_{comp} = \frac{(dF_N)_i}{\sqrt{1 - (V/c)_i^2}}$$

This means that the pressure force on an elemental surface is proportional to the dynamic pressure and, since the velocities are the same for both cases, the relation between pressure forces is the same as for the density relations.

The local pressure forces are, of course, directly proportional to the local pressure coefficients corresponding to the compressible and incompressible case and therefore, the pressure coefficient relation may be written as

19.5 $$(P_{comp})_1 = \frac{(P_i)_1}{\sqrt{1 - (V/c)_1^2}}$$

The relation 19.5 then becomes the simple tool with which the pressure distribution for a given free-stream Mach number (below the critical Mach number) can be predicted.

A question may arise at this point as to why the mass-density relations were used instead of the contraction equation 19.1 as is done in the classical solution of Glauert. Actually the result could not have been derived by the classical methods. The Sound-Space results 19.1 and 19.2 are not only new but the density relations allow a more straightforward solution. The reason for this is that the apparent contractions are merely a step to deriving the more significant relations of mass-density.

20. INCOMPRESSIBLE PRESSURE DISTRIBUTION

To calculate the pressure distribution over a body for a given free-stream Mach number, using equation 19.5 it is necessary to have the experimental pressure distribution for a free-stream Mach number as near to zero as possible, (a Mach number of the order of 0.06 is considered satisfactory). In this case the local pressure coefficients for small angles of attack (usually in the range

-3° to +3°) are sufficiently low enough to consider them to be unaffected by compressibility, ie, all local pressure coefficients will be incompressible coefficients, $(P_i)_i$. To insure the best accuracy for calculating the pressure distribution at higher angles of attack, it is necessary to secure experimental pressure distributions at a free-stream Mach number low enough to reduce local compressibility effects to a minimum. It is believed that accurate results can be obtained if the maximum local Mach number is less than 0.2.

It has been shown[14] that the effect of Reynolds Number on the maximum negative pressures at low speeds ($M = .06$) are of no importance for Reynolds Numbers below 2×10^6.

21. METHOD OF PREDICTING COMPRESSIBLE PRESSURE DISTRIBUTION

The method of calculating the desired pressure distribution over a body is given in step form as follows:

(1) Calculate the maximum local Mach number on the surface of the body from the low-speed experimental pressure distribution to ascertain that no appreciable compressibility effects are present. A maximum local Mach number less than 0.2 is considered satisfactory. The value of the local Mach number can be obtained from the equation[15]

21.1 $$(P_i)_1 = \frac{2}{\gamma M^2} \left[\left(\frac{1 + \frac{\gamma - 1}{2} M^2}{1 + \frac{\gamma - 1}{2} M_1^2} \right)^{\gamma/\gamma - 1} - 1 \right]$$

where $(P_i)_1$ = local incompressible pressure coefficient

M = free-stream Mach number

M_1 = local Mach number corresponding to pressure coefficient

γ = 1.405 (for air)

(2) Choose the free-stream Mach number, less than the critical, for which the pressure distribution is desired and set up equation 21.1. For example, suppose the pressure distribution is to be calculated for a free-stream Mach number of $M = 0.534$. Then equation 21.1 is

21.2 $$M_1^2 = 5 \left[\frac{1.057}{(0.1995(P_i)_1 + 1)^{0.286}} - 1 \right]$$

Pick off successive values of the local low-speed pressure coefficients, $(P_i)_1$, over the upper and lower surfaces of the body at sufficiently close intervals to allow a smooth plot and calculate the local Mach number squared, $(V/c)_1^2$.

(3) Calculate the corresponding local compressible pressure coefficients by substituting the values of the local incompressible (low-speed) pressure

coefficients together with the corresponding local Mach number squared into equation 19.5.

(4) In calculating a sufficiently large number of points over the surface of the body, particularly over the nose-section, a better defined pressure distribution can be plotted and any irregularities can be quickly identified as probable calculation errors.

A tabular method of calculation may be set up but it is considered to be of no particular advantage due to the simplicity of the calculation.

22. DISCUSSION

An inspection of Figure VIII reveals the accuracy of the Sound-Space method outlined above in predicting the maximum negative pressure coefficient of the NACA 4412 airfoil at -2° angle of attack. The theory predicts exactly the experimental results up to a free-stream Mach number of 0.3. For higher free-stream Mach numbers the Sound-Space Theory predicts slightly higher maximum negative pressure coefficients than the experimental values show. This is to be expected since the theory assumes steady flow throughout and in reality there were local shock disturbances as the critical free-stream Mach number was approached. In Figure IX the theory predicts less accurately for the lower free-stream Mach numbers but for the higher free-stream Mach numbers (between 0.5 and 0.6) the predicted values fall within the experimental scatter for the lower surface. It

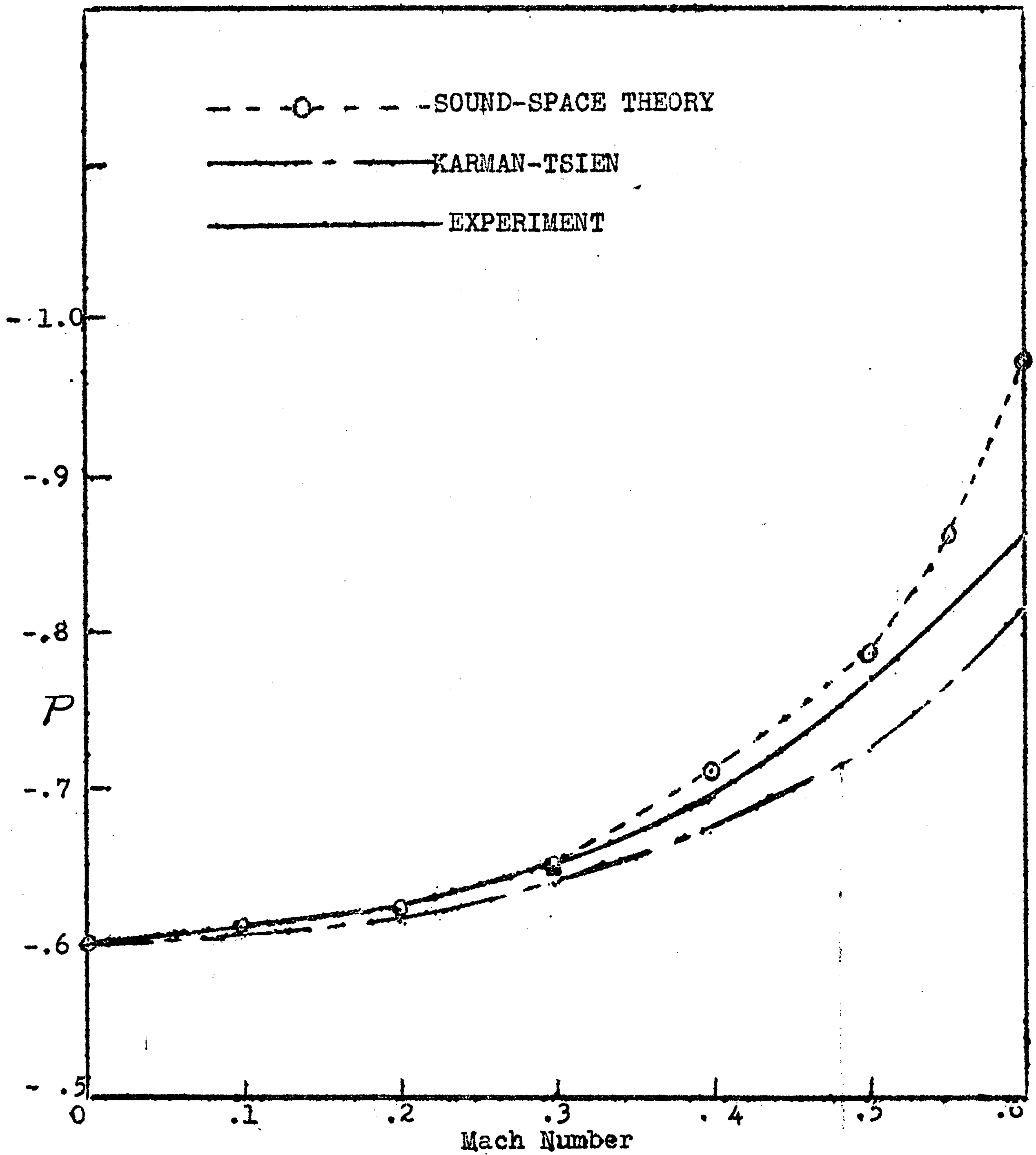


Fig. VIII. Pressure Coefficient vs. Free-Stream Mach Number
NACA Airfoil, P at 30% Chord, Upper Surface, α=-2°.

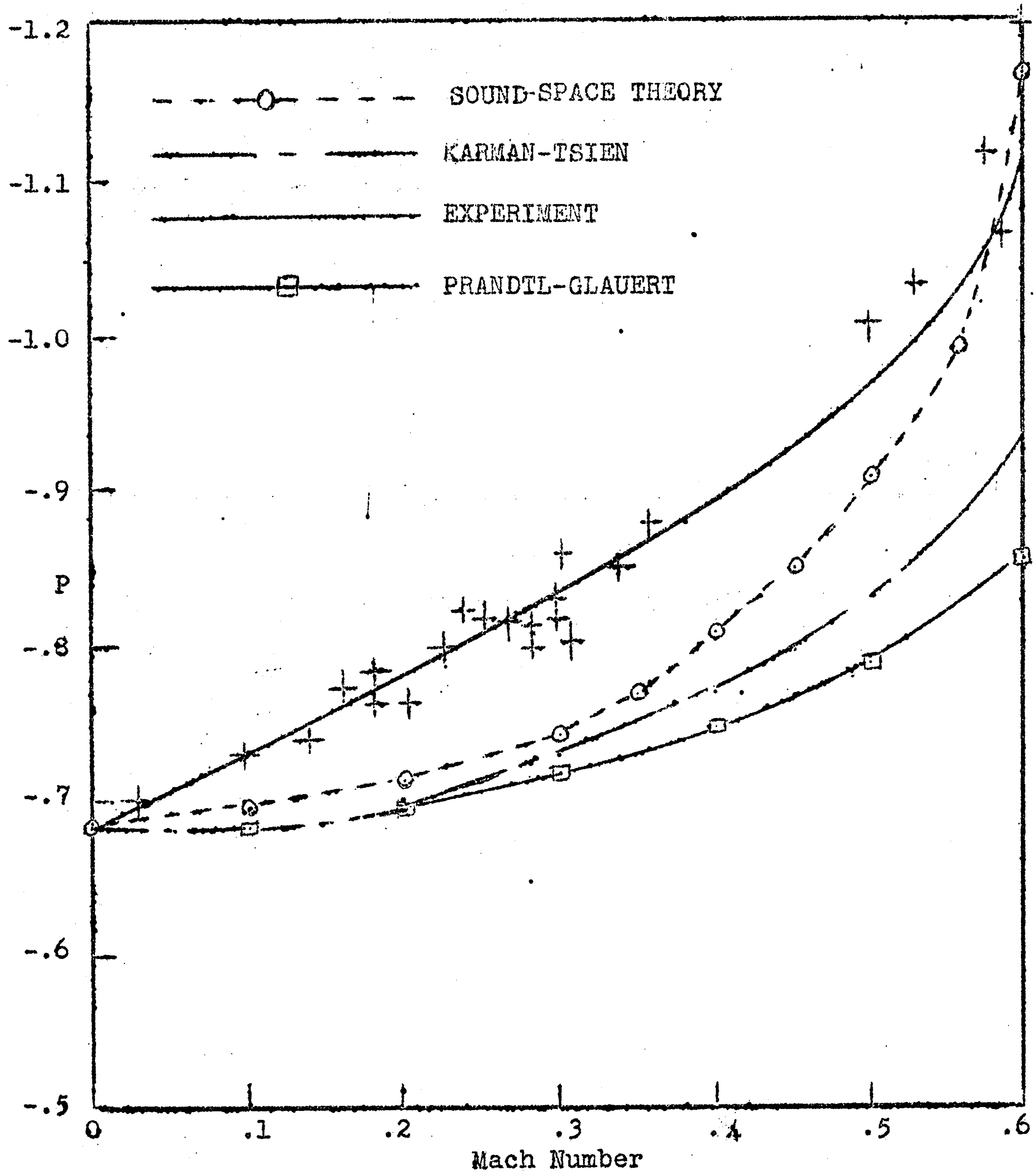


Fig. IX Pressure Coefficient vs. Free-Stream Mach Number
NACA 4412 Airfoil, P at 30% Chord, Lower Surface, α = - 2 °

is clear that the Sound-Space results are in better agreement with the experimental values than those predicted by the Karman-Tsien method, particularly for the lower surface. To further demonstrate the accuracy of the theory Figures X and XI are shown for the same airfoil at an angle of attack of 0°15'. In general the theory appears to predict closer to the experimental values for incompressible pressure coefficients that are relatively large. It is believed that this is the reason for the excellent agreement with experiment over the nose-section of the airfoil where the local pressure coefficients are large even for the incompressible case.

Complete pressure distributions for the NACA 4412 airfoil have been calculated for a series of free-stream Mach numbers by the Sound-Space method as outlined above (Figures XII, XIII and XIV). The low-speed experimental data, from which these pressure distributions were calculated was for a free-stream Mach number of 0.1.[16] An examination of the calculated pressure distribution for a free-stream Mach number of 0.498, Figure XII, shows almost complete agreement with the experimental values. The calculated values disagree slightly for both the upper and lower surface in the region from 15 percent to 30 percent chord positions. For the free-stream Mach numbers of 0.534, Figure XIII and 0.603, Figure XIV, the calculated values in this region appear to overestimate the effects of compressibility on the upper surface and underestimate

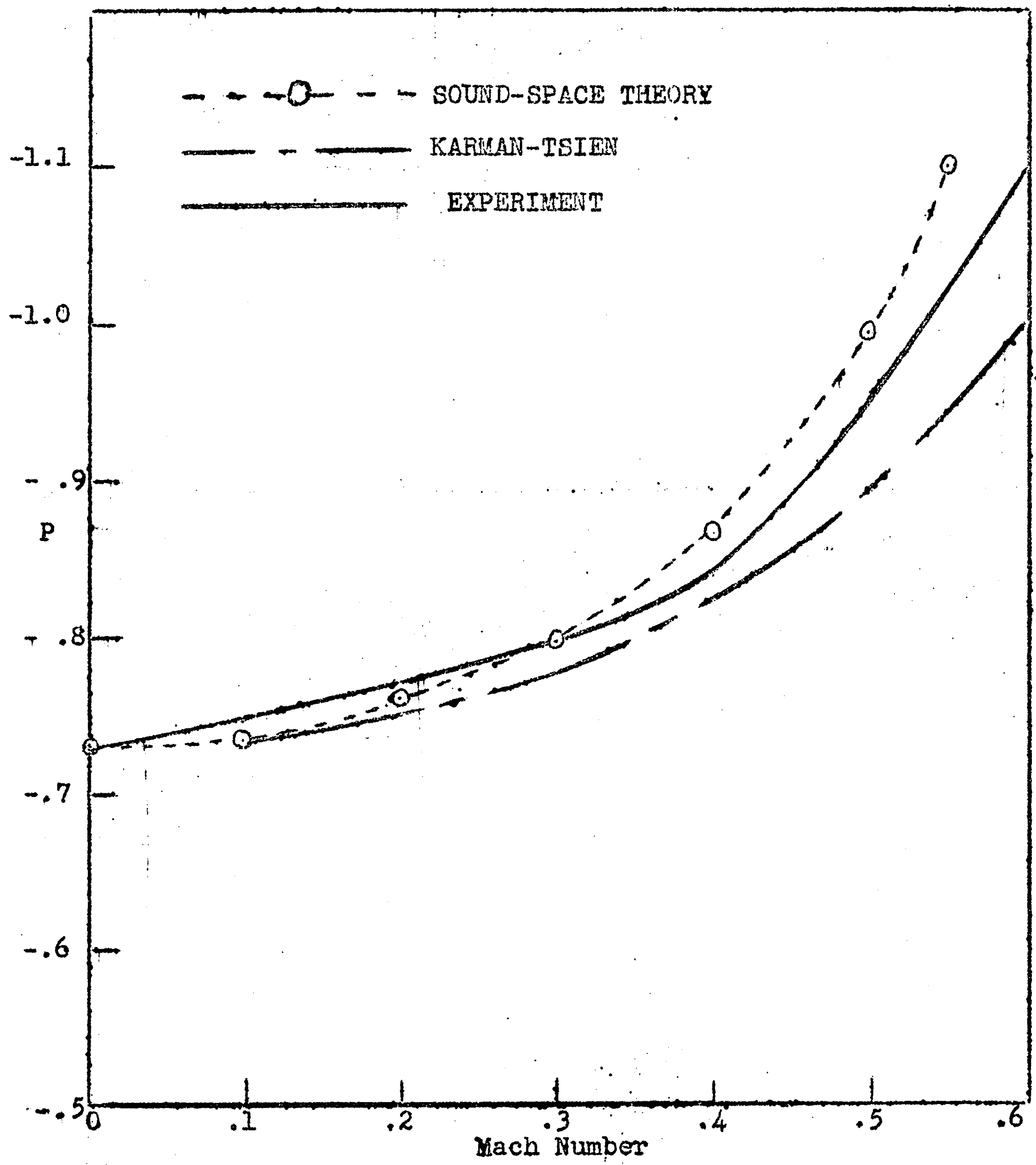


Fig. X Pressure Coefficient vs. Free- Stream Mach Number
NACA 4412 Airfoil, P at 30% Chord, Upper Surface =0°15'

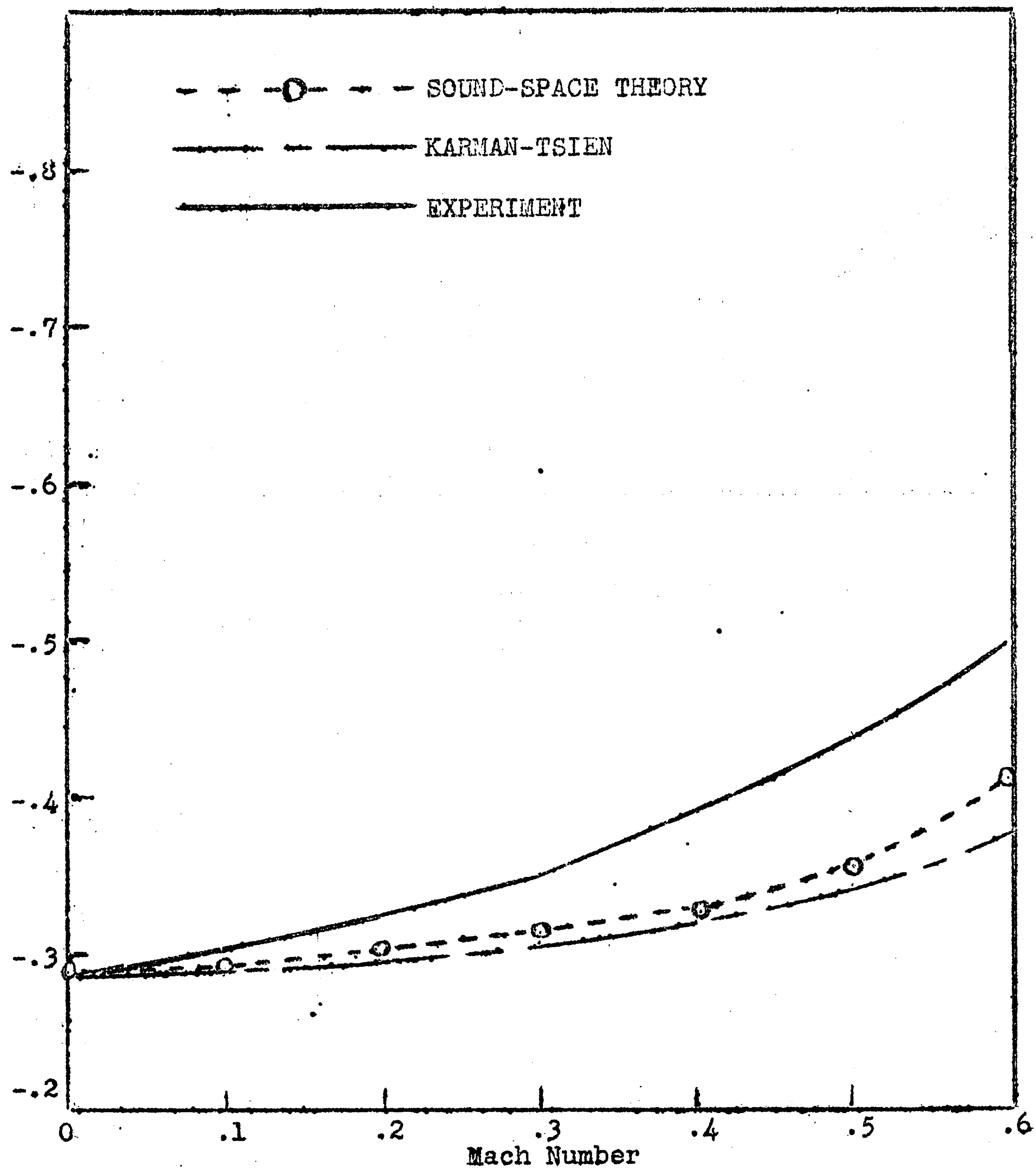


Fig. XI Pressure Coefficient vs. Free-Stream Mach Number
NACA 4412 Airfoil, P at 30% Chord, Lower Surface
$\alpha = 0° 15'$

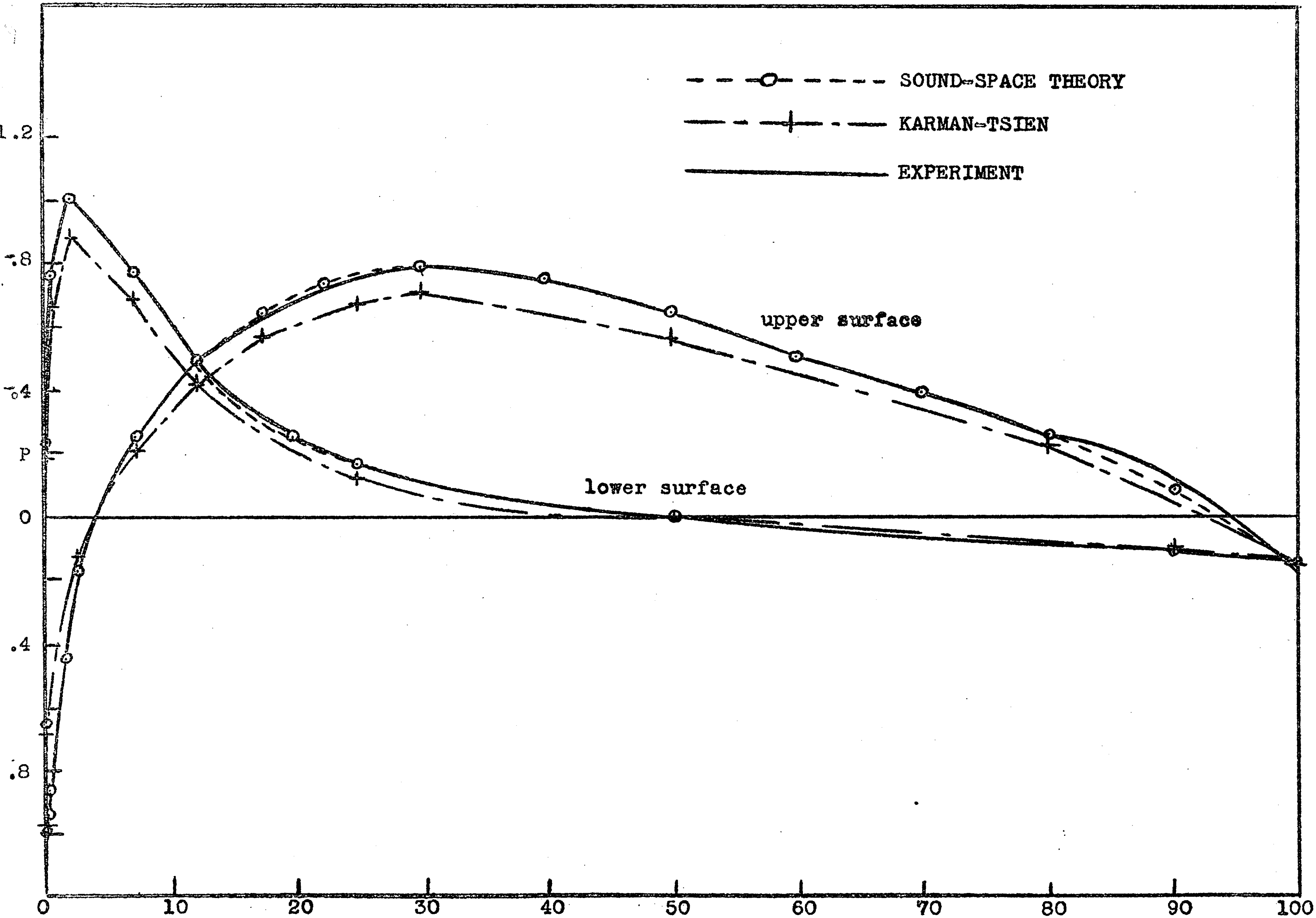
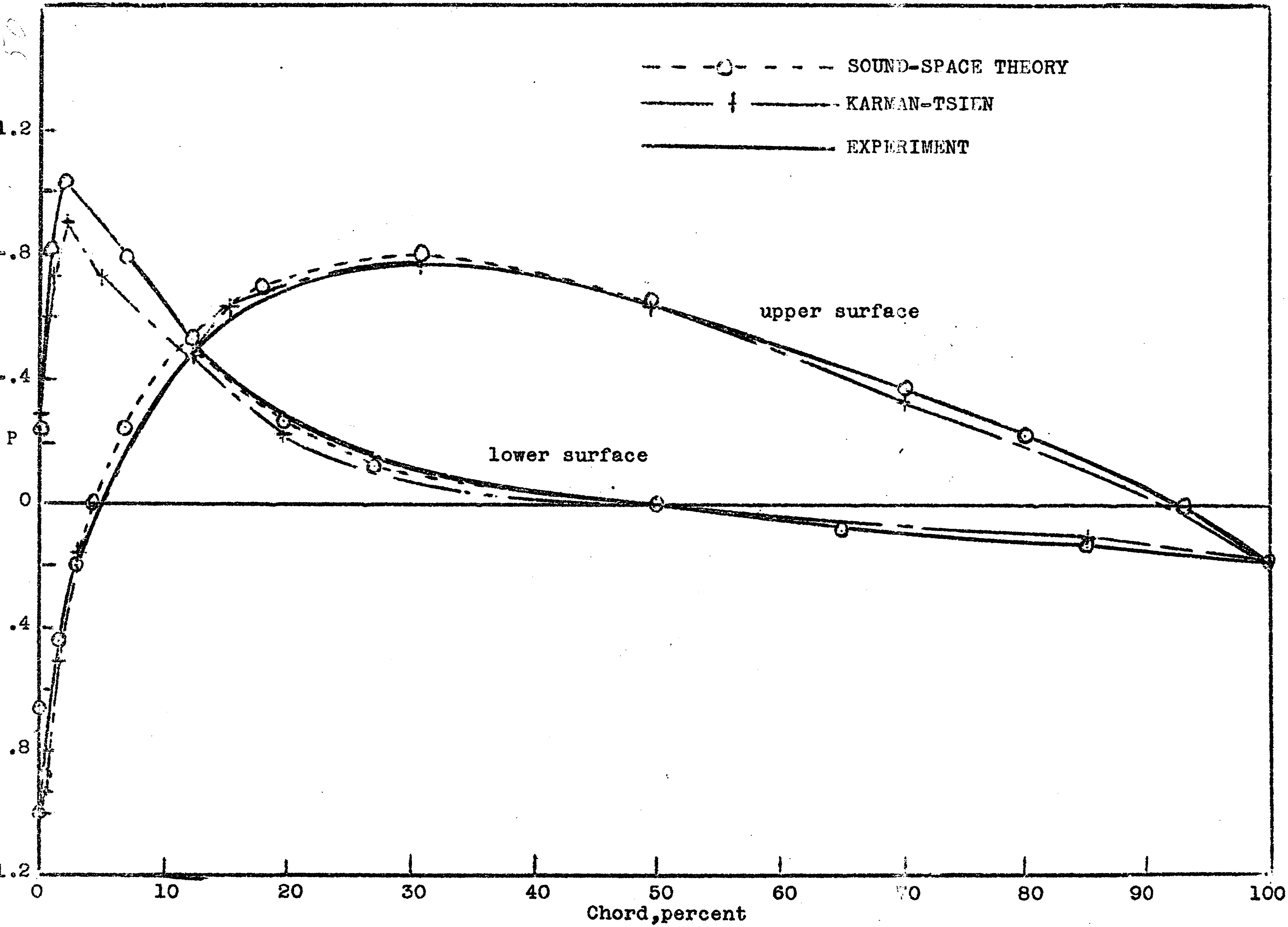


Fig XII. Comparison of Sound-Space Theory with Experimental Pressure Distribution for NACA 4412 Airfoil, α=2 degrees, M 0.498



FigXIII.Comparison of Sound-Space Theory with Experimental Pressure Distribution for

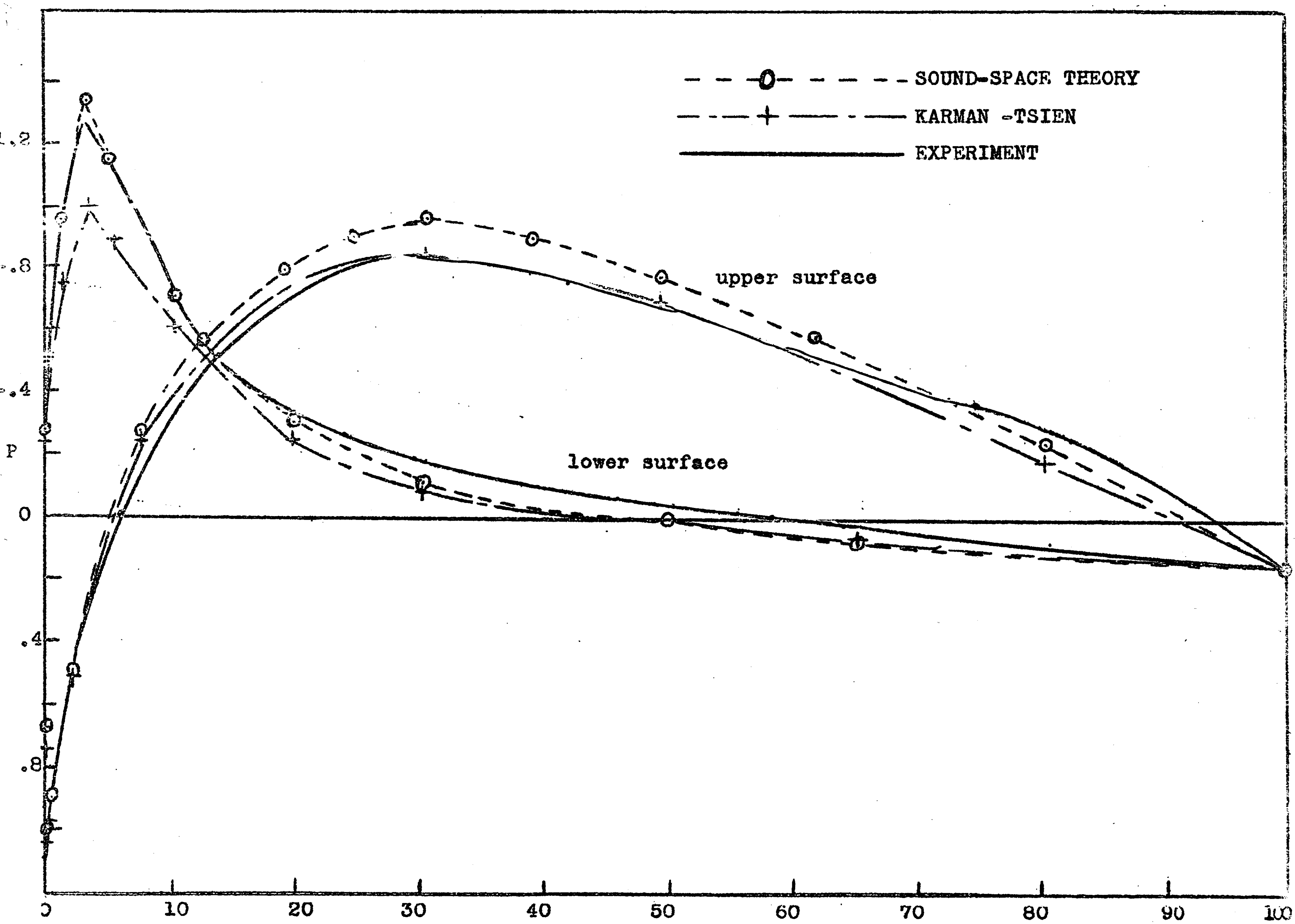


Fig XIX Comparison of Sound-Space Theory with Experimental Pressure Distribution for them on the lower. In both cases however, there is considerably better agreement with experiment over the nose portion for the Sound-Space Theory than for other methods.

CONCLUSIONS

It is of utmost importance to note that the Sound-Space approach to subsonic compressibility effects is an exact solution to the problem. That is, equation 19.5 is an exact relation between the compressible and incompressible local pressure coefficients. Nowhere in the preceding analysis was it necessary to make simplifying assumptions such as small perturbations, etc. It is true that the Sound-Space Theory assumes steady reversible-adiabatic flow and consequently it is to be expected that there will be discrepancies in the calculated and experimental values.

The validity of the physical principles upon which the Sound-Space Theory was founded and the consequent exact solution to the problem of subsonic compressibility effects, should signify an important advance to the better understanding of compressible fluid phenomena.

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16. Stack, Op. Cit., p. 5, Fig. 3.