CHAPTER 4
TREATMENT OF THE DATA

Production data from a manufacturing facility, to be called Manufacture Inc., is used in this research. Model specification issues were addressed both during the data collection at Manufacture Inc. and in the development of the data sets used in the analyses described in Chapter 4. Figure 4-1 shows the major steps in the data treatment. Before describing the collection of data, the basic manufacturing process used by Manufacture Inc., (printed circuit boards) is described in Section 4.1. The consulting work performed for Manufacture Inc. is then discussed in Section 4.2. The data collection and the necessary transformations to that data are described in Section 4.3. These transformations include a Pareto analysis to reduce the number of inputs and normalization to eliminate bias associated with units of measurement. The data sets used in the analysis are then defined in Section 4.3.4.

Section 4.4 presents the results of the statistical analysis. The application of some of the standard methods for the measurement of productive efficiency are described in Section 4.5. Then, Section 4.6 presents the results of the application of the proposed method followed by a comparison of methods in Section 4.7. Supporting material for the data analyses are presented in Appendixes B, C, and D.

4.1 Plant and Manufacturing Processes Description

Manufacture Inc., manufactures Printed Circuit Boards (PCB), also called Printed Wiring Boards (PWBs), using "subtractive processing, in which copper is selectively removed from a Printed Circuit Board to form a circuit. Within this process, however, copper and other metals are also added during plating steps" (EPA, 1995, p. 2-2). The primary production steps are presented in Figure 4-2. These production steps are sequential as shown from the top of Figure 4-2 moving down. The "Strip Solder, Plate Gold, Sample Inspection" step is optional per customer specifications.

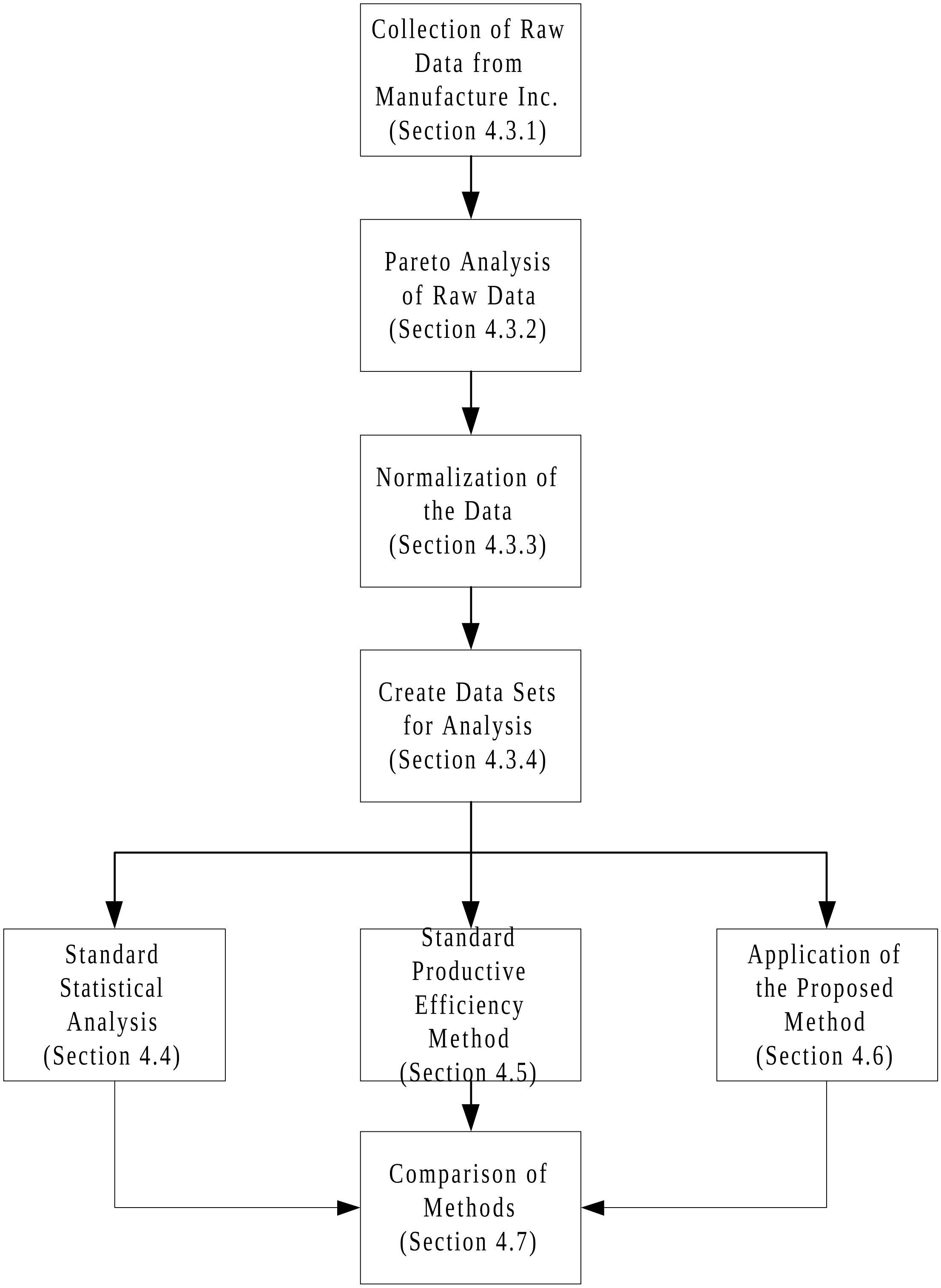

Figure 4-1. Data Collection and Analysis

“[T]he subtractive process begins with copper-clad laminate, composed of a thin copper foil covering both sides of the epoxy-glass core material. The laminate is coated with a sacrificial photopolymer material that acts as a resist in subsequent steps. The resist is photoimaged (exposed/developed) to expose the copper to be removed. The board is then etched, after which the resist material is stripped and disposed in a fabrication waste stream, leaving the desired interconnect pattern in copper on the exposed laminate. In a multilayer structure, each of the inner layers is constructed independently, then laminated together using a B-state epoxy in between each inner layer core to form the overall structure. This process of building multilayers is essentially independent of the number of inner layers laminated together and is used to build PWB structures with any number of layers” (EPA, 1995, p. 2-6).

Each of the major processing steps shown in Figure 4-2 is described below: (Note: Where not otherwise noted this process description is from Manufacture Inc.’s World Wide Web site accessed through the internet.)

Raw Material Inspection: Incoming materials are inspected before being sent to the production process.

Inner Layer Material Prep: The Inner Layer Department starts the manufacturing process. The core material is typically either half ounce or one ounce of copper foil on a fiberglass and epoxy resin base laminate.

Surface Scrub and Photo Resist Lamination: After passing incoming inspection and being approved for use the laminate is processed through a cleaning process after which a dry or photo film is applied to the outside of the cores.

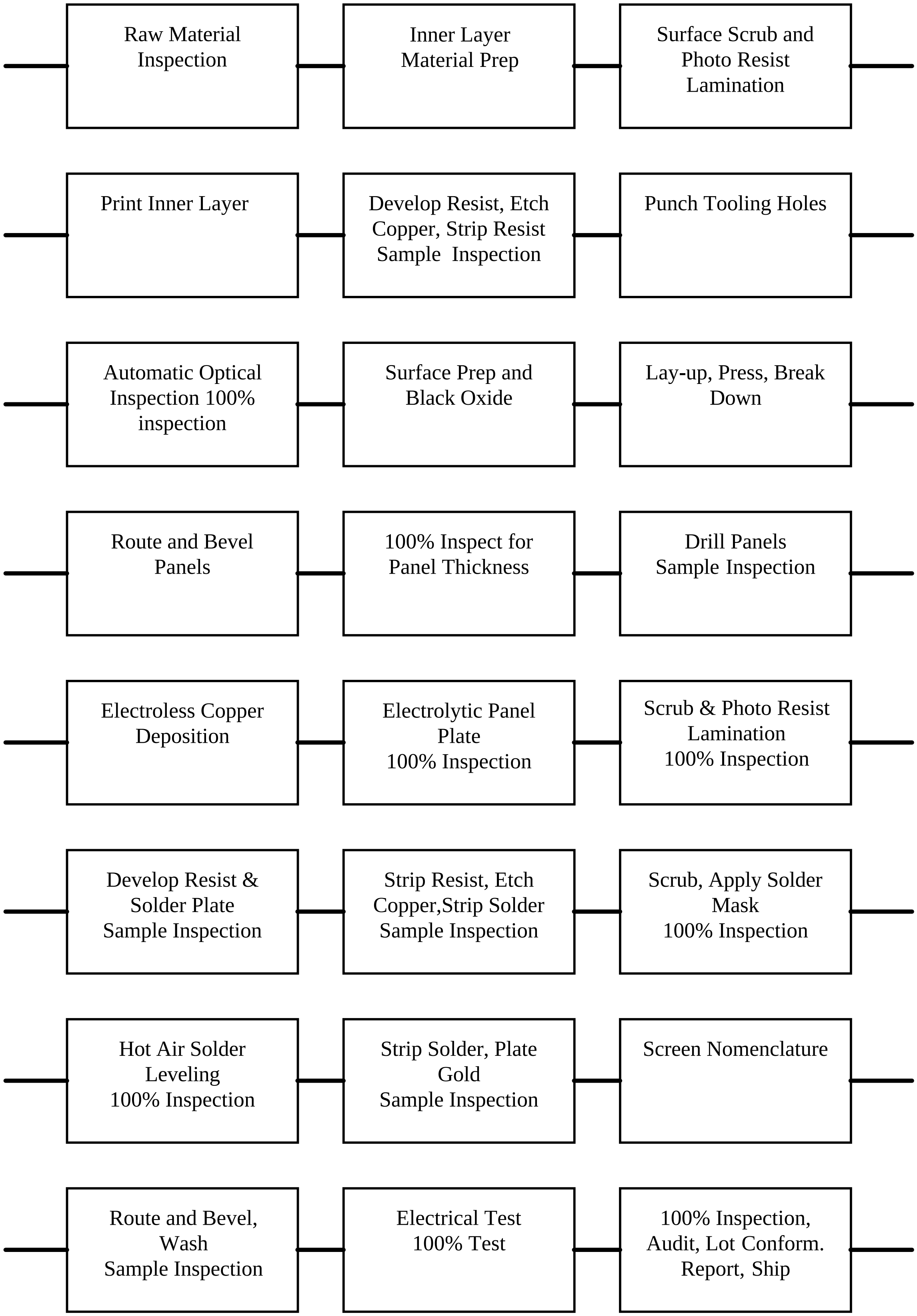

Figure 4-2. Printed Circuit Board Production Process

Print Inner Layer: These cores are then sent into the print units for light exposure. This process places the inner layer image onto the core. "During the 'print' step, photoresist is applied to the surface copper of PWB material. Photoresist is a light sensitive organic coating that can be imaged with a photo-tool and a light source" (EPA, 1995, p. 2-11).

Develop Resist, Etch Copper, Strip Resist (Sample Inspection): The core is then processed through the develop-etch-strip line that develops the film, etches or removes the copper from the non-circuit area and then strips the film off the core. "During the etch step, copper that is not protected by the photoresist (which now becomes the etch-resist), is etched away and only the image of the circuit remains. The photoresist is then stripped, revealing the copper circuit remaining beneath it" (EPA, 1995, p. 2-11).

Punch Tooling Holes: Holes for mounting of the boards during the drilling process are punched.

Automated Optical Inspection 100%: The cores are transported to the inner layer inspection equipment which is comprised of three Visual Inspection Machines and one Inter Trace Electrical Tester. Depending on the complexity of the work, line width and spacing, etc., the cores are either optically or electrically inspected.

Surface Prep and Black Oxide: These cores are racked and processed through the black oxide chemistry, which increases the copper area and provides better laminate bond strength. This process "promotes copper-to-epoxy adhesion in multilayer manufacture" (EPA, 1995, p. 2-16).

Lay-up, Press, Break Down: The cores go into the lay up room where fiberglass pre-preg is added between the cores. Pre-preg is used to separate the circuits on the cores for dielectric reasons and to provide a medium to bond the circuit layers together. Copper foil is added to provide a circuit medium for the panel, top and bottom. The stack-up is then moved into a vacuum-assisted press. Pressing applies pressure and heat under vacuum to bond the multiple layers together creating a panel.

Route and Bevel Panels: After pressing, the material is moved into the finished area where the flashing is removed, the edges are beveled, the tooling holes are spot faced. After 100% inspection for panel thickness the panels move on to the Drill Department.

Drill Panels (Sample Inspection): Computer Numerical Control (CNC) machines are used for drilling both component and via holes. Vias are holes drilled through a PWB for the purposes of layer-to-layer interconnection. After drilling, the inner layer foil extends to the barrel of the hole and is available for interconnections when the hole barrel is metallized" (EPA,. 1995, p. 2-18). These drills are selected for accuracy of small hole drilling. The CNC drill machines all have the drill programs automatically downloaded from the Product Engineering Department. One of the keys to this area is the tool management system that ensures that the proper drill bits are used and that all drill bits are fully inspected before use. The quality of the drilled hole is the key to being able to produce the type of technology demanded by the customer base.

Electroless Copper Deposition: After the drilling process, the panels are processed through a desmear (i.e., hole cleaning) and electroless bath. The desmear process removes resin from the interconnect holes and prepares it for electroless copper. The electroless bath places a small amount of copper on all surfaces of the boards and more importantly, inside of the holes. This thin layer of electroless copper acts as a conductor for the electrolytic copper plating process. Once this seed layer of copper is in place, copper plating can occur. "The electroless copper line typically contributes a significant portion of a PWB shop's overall waste. Water use is high due to the critical rinsing required between nearly all of the process steps. Copper is introduced into the waste water stream due to drag-out from the cleaner conditioner, micro-etch, sulfuric, accelerator, and deposition baths" (EPA, 1995, p. 2-28).

Electrolytic Panel Plate (100% Inspection): After the boards come out of the electroless bath, they immediately go into the electrolytic copper plate bath where 1 millimeter (mil) of copper is plated in the holes. Typically 1.2 mils of copper is also plated on the surface of the copper plate.

Develop Resist & Solder Plate (Sample Inspection): The circuitry image is placed on the outer layer of the panel. The initial step is to place dry film on the panel, a method that is very similar to what is performed in the inner layer area. This film is then exposed using the artwork generated from the Product Engineering data. The negative of the film is exposed to the artwork. The panel moves through the developer which defines the area to be circuitized. The panel is then solder plated. This process places a thin coating of tin/lead over the exposed circuits, which acts as a resistant to the etchant of the next process.

Strip Resist, Etch Copper, Strip Solder (Sample Inspection): The panels are next processed through the strip-etch-strip (SES) line. This machine strips the dry film off of the panel thereby leaving the base copper and the solder plated circuits and holes. The next section of the machine will then etch the copper off of all but the circuits and holes. The final chamber strips off the solder, leaving a finished product that will have copper circuits over the bare fiberglass laminate. The panels are then moved to the solder mask department.

Scrub, Apply Solder Mask (100% Inspection): The majority of Manufacture Inc., product is solder mask over bare copper (SMOBC). When these boards come out of the SES operation, the copper traces and hole walls are exposed. The panels are processed through a quick pumice scrub which is a cleaning action to make sure there is no oxidation of the copper. A solder mask which is usually either a liquid photoimageable (LPI) or a dry film solder mask is applied. The mask, typically green, is coated over the panel. The LPI solder mask is then exposed through artwork received from the photo department. After print, the panel is processed through a developer and then baked. This department also applies the component marking after the surface finish has been completed. Component marking includes all printing required by the customer to identify board features and component placement.

Hot Air Solder Leveling - HASL (100% Inspection): After the solder is applied, hot air knives blow off the residual solder, clean out the holes and level the solder down to the required height. "The purpose of the soldermask is to mask off and insulate physically and electrically those portions of the circuit to which no solder or soldering is required" (EPA, 1995, p. 2-38). Boards are then inspected and passed on to the next station. They are now ready for the application of gold on the fingers, if required. Most PCBs produced have a tin-lead or solder finish on the pads and in the holes. The boards are run through a pre-clean process. Oil and flux are applied and the boards processed through a solder wave.

Strip Solder, Plate Gold (Sample Inspection): The gold is tab plated on a Microplate line. The gold plate process is an in-line conveyorized system that will strip off the solder from the hot air solder leveler to expose the copper on the tabs, plate nickel over the copper, and then plate the gold over the nickel to the thickness specified by the customer. After the gold plating, the boards are then sent to the rout department. "Nickel-gold is a significant alternative finish. Nickel-gold coatings may be electrolytically plated as an etch-resist (a direct substitute for HASL)" (EPA, 1995, p. 2-37).

Screen Nomenclature: Labels as specified by the customer are placed on the boards.

Route and Bevel, Wash (Sample Inspection): In the Route Department, the tool program (from product engineering) is automatically loaded into the rout controllers. These machines rout the panels out to customer specified dimensions, at the same time putting in any type of slots, shapes, palletizing, or other shaping that is required. Other required profiling, whether it is beveling, slotting, etc., is performed before the boards are passed on to the electrical test section. "For most parts, the functions of the surface finish are to prevent copper oxidation, facilitate solderability, and prevent defects during the assembly process" (EPA, 1995, p. 2-37).

Electrical Test (ET) ensures continuity around the holes and surface mount pads. For those jobs that are quick turns, or jobs with small quantities, the boards are put on one of two Probot testing machines. The Probots then test out the circuits and to ensure continuity between all test points. The advantage of using the Probot is to eliminate the time and expense of making a test fixture.

100% Inspection, Audit, Lot Conformation, Report, Ship: Final Inspection visually inspects the boards and ensures that the product meets all of the customer's specifications. The board is dimensionally checked, visually inspected, and reviewed for cosmetic purposes, and double-checked to make sure that the boards meet the requirements of the customer. The boards are then bagged in the final inspection area and forwarded to shipping which will box them according to customer requirements and ship them by the method specified.

Waste Water Treatment and Recycling: Manufacture Inc. has an on-site waste water treatment plant and sends water back to the Publicly Owned Treatment Works well within limits specified under Clean Water Act regulations. Many of the other waste streams are sent back to the suppliers for recycling, but some are sent to disposal. Scrap is sent to a recycler. Sludge from the waste water treatment plant is sent to a recycler.

4.2 Consulting Work Performed for Manufacture Inc.

In exchange for assistance obtaining data the author performed three studies for Manufacture Inc. (Otis, 1996a, 1996b, 1997) and spent a total of about 2 weeks at the facility gathering data and talking with Manufacture Inc. employees. These three studies are not directly part of the research, but did provide useful insight into how the production process functions, how the information systems work, and the limitations of the data.

After a discussion with the company Chief Executive Officer and the head of the maintenance and safety departments it was determined that a more detailed analysis of water use, cost of scrap generation, and bottlenecks in the production process would be useful. All of these issues were being addressed by Manufacture Inc. to some extent. A presentation was made at Manufacture Inc. to review results with plant personnel. These three analysis are each briefly described:

Cost of Water/Water Use Analysis: Water is a major input in most of the production steps at Manufacture Inc. Manufacture Inc. is billed quarterly by the local water authority for water used. There is an on-site treatment plant that returns water to the water authority in better condition than it is received and well below limits imposed by the Clean Water Act. Like most manufacturing facilities many costs are in overhead accounts and are not attributed directly to product. Costs associated with water treatment were obtained from the Accounting Department. The result of this cost analysis was that water was actually about twice as expensive to the company than just the water authority charges. In addition, water flow meters throughout the plant were read on two occasions to perform an initial "water balance" to identify water use patterns. In some cases water in excess of process specifications was being used in production processes. Recycling of water on-site was identified as a potentially attractive option for some of the production steps that are less demanding in terms of water purity.

Bottleneck Identification Analysis: An attempt was made to quantify sources of lost production which are set-up time, maintenance downtime, re-work and scrap generation. Downtime data were obtained from the maintenance management system. Unfortunately the software being used to track work orders was somewhat dated and more importantly was difficult for maintenance personnel to use. As a result, while maintenance work orders were tracked, data on exactly why a production process was down was often not available. Times assigned to down equipment were also not always accurate. The Quality Control system was used to track wasted production from scrap and re-work. This system is excellent with high quality data. Set-up times were not available and so were estimated by walking through each step in the production process and interviewing the operators. These estimates were then reviewed with a production engineer as a "reality check." These set-up time estimates are subject to significant variation and error. Estimates of the maximum production possible for each of the steps in the production process were obtained from the Quality Control system which also tracks production at each major stage in the production process. Taking the maximum possible production estimate and subtracting losses from maintenance downtime, set-up time, scrap, and re-work along with normal production an estimate of idle time for each step in the production process was made. A bottleneck process would have little idle time compared to other processes. An underutilized process would have a lot of idle time relative to other processes. The analysis found that the production process was well balanced within the limits of the analysis imposed by the quality of the data. The drill process was identified as a possible minor bottleneck. Other assessments by Manufacturing Engineering had already shown that production would benefit from Drill process capacity expansion. Manufacture Inc. was waiting for this new equipment to arrive at the time of the final presentation to management in March, 1997.

Cost of Waste Analysis: The Accounting Department was starting an effort to provide cost information more useful for production. A initial and partial attempt was made by the author to assign costs to product. Many (but, not all) overhead cost accounts were apportioned to product at each step in the production process. This then allowed an estimate of the cost of scrap and re-work being generated at each stage in the production process. The detailed analysis required to fully identify cost drivers that would allow all overhead costs to be assigned to product was not performed. The results provided an input to the Accounting Department's ongoing efforts in this area.

4.3 Data Collection and Model Specification

The next four sections review the steps that were performed to create the data sets used in the analyses. These steps are shown in Figure 4-1. After the data was collected and converted to a common weekly basis, a Pareto analysis was performed on the 95 material inputs to reduce the number of inputs enough to make meaningful comparisons based on productive efficiency. Then, the data were normalized. This was done to eliminate the bias that occurs with the use of various units of measurement, i.e., hours, dollars, kilowatt hours, etc. Finally, the data were converted into several data sets appropriate for the types of analyses to be performed.

4.3.1 *Data Collection*

The system being analyzed as part of this research is a single manufacturing facility. The analysis is therefore restricted to data that are typically available to a manufacturing facility. Data were obtained from several departments at Manufacture Inc. for the period October 1, 1994 through July 27, 1996. Weekly direct labor hours for each cost center were obtained from the Safety Department reporting system. Bi-weekly salary hours for the plant were also obtained from the Safety Department reporting system. Monthly materials usage cost was obtained from the accounting department. Weekly throughput, scrap, and rework data for each cost center were obtained from the Quality Control department. Water and electricity use were obtained from Accounting billing records. The data from these various systems were entered into Excel spreadsheets (sometimes by importing electronic files, sometimes manually). All of the data entered into the Excel spreadsheets were then double checked against the data obtained from Manufacture Inc.

The quality of the data varies. Labor hours and data on throughput and scrap production are accurate. Data on rework are calculated by subtracting the scrap rate from defect rate data - this assumes a board is only reworked once which may not be the case.

Materials use data are tracked for accounting purposes and is not directly tied to materials used in production. For example, an input is captured by accounting only when it is removed from inventory. It is not unusual for production units to requisition materials for a number of days of production at a time. Since materials use data are only available in aggregate monthly amounts this type of variation is not excessive, but is enough to affect measurements of efficiency. Also, the data on materials usage are an aggregation. For example, there are many different types of drill bits that are used in the process, but these are reported by accounting as a single input.

Some data were collected weekly, some bi-weekly, and some monthly. To provide a common basis, all of the data were converted to a weekly basis. Data could have also been converted to a monthly basis, but this requires further aggregation and loss of information. Also, months are fiscal months with either 4 or 5 weeks. This further complicates monthly comparisons since the time frame is not the same. The choice of converting data to a weekly basis is also not ideal. Changes in efficiency occurring from week to week may not be detected.

4.3.2 *Pareto Analysis of Material Inputs*

There were 95 material inputs for which data were obtained. All of these inputs in the Pareto analysis have dollars as the unit of measurement. Most of these inputs account for only a small percentage of total input. A Pareto analysis was performed on these 95 inputs to identify those that are the largest contributors to the total material input for the plant. Table B-1 in Appendix B presents the results of this analysis. Fourteen material inputs represent 85% of the total material cost over the period October 1, 1994 through July 27, 1996. The rest of the 95 material inputs are combined into a single 15th input. It is possible to select some time frames for the data for which the Pareto analysis produces a different set of inputs; since some inputs are increasing over time and others are decreasing over time. The entire data set was chosen for the Pareto analysis so that a common basis for comparison over time is obtained.

4.3.3 *Normalization of the Data*

The concept of normalization of data used in statistical cluster analysis is applied to the data. The purpose of normalization is to remove bias from distance measures related to the unit of measurement. Kaufman and Rousseeuw (1990, p. 5) note that "changing the measurement units may even lead one to see a very different clustering structure." It is also noted that the units of measurement are essentially assigning relative weights to the variables.

One way to avoid the problem of unit dependence in the measurement of distance is to standardize input. A method presented by Kaufman and Rousseeuw (1990, pp. 8-9) is to first calculate the average for each variable, calculate a metric of dispersion, and then the standardized score. These formulas for n objects (or production plans) and z variables are:

$$m_f = \frac{1}{n}(x_{1f} + x_{2f} + \ldots + x_{nf}) \qquad \text{(Eq. 4 - 1)}$$

for each f=1,…,z

$$s_f = \frac{1}{n}\{|x_{1f} - m_f| + |x_{2f} - m_f| + \ldots + |x_{nf} - m_f|\} \qquad \text{(Eq. 4 - 2)}$$

$$z_{if} = \frac{x_{if} - m_f}{s_f} \qquad \text{(Eq. 4 - 3)}$$

where i = 1,…n.

The measure of dispersion , s_f , is preferred to the standard deviation since it is more robust in terms of not changing that much if there is one outlying observation. All of the variables are replaced with values of z_{if} which is a unitless measure with a mean of zero and an absolute deviation of 1.0.

All data on inputs and outputs are normalized. Since the resulting data set has negative values (so the calculation of efficiency with standard methods is not possible), the negative number with the largest absolute value was added to all variables to produce a data set with mean of 6.2206.

4.3.4 *Data Sets*

Analysis is performed on a number of variations of the data. Table 4-1 presents the components of the data sets that are used in the analysis. The data are provided in Appendix B. Each of the data sets are discussed in terms of its purpose in the analysis:

Full Set: This data set is representative of the kinds of data obtained from an actual manufacturing process. The data are in sufficient detail to perform analysis related to the performance of a particular manufacturing facility. This is the data set that is the focus of the analysis performed as part of this research.

Full Material Set: The distinction between material and non-material inputs is important in the evaluation of environmental performance as discussed in Chapter 3. A separate data set is created that does not contain non-material inputs (i.e., Salary Hours and Direct Hours).

Full Set - No Undesirable Outputs: Productivity analysis is often performed without considering undesirable outputs. This data set is used to provide a standard of comparison to these standard analyses. The proposed method is applied to this data set.

Seven Input Set: This set is used to apply standard analysis such as, FDH and DEA, that typically use data sets with a high level of aggregation. The 15 material inputs are aggregated into a single material input. Undesirable outputs are treated as inputs since standard methods do not allow direct consideration of undesirable outputs.

Table 4-1. Data Sets

Data Set	**Inputs**		**Outputs**	
	Material Inputs	**Non-Material Inputs**	**Product Outputs**	**Undesirable Outputs**
Full Set	Input 1 Input 2 Input 3 Input 4 Input 5 Input 6	Direct Hours Salary Hours	Boards	Water Scrap Re-Work

Data Set	Inputs		Outputs	
	Material Inputs	**Non-Material Inputs**	**Product Outputs**	**Undesirable Outputs**
	Input 7 Input 8 Input 9 Input 10 Input 11 Input 12 Input 13 Input 14 Input 15 Power			
Full Material Set	Input 1 Input 2 Input 3 Input 4 Input 5 Input 6 Input 7 Input 8 Input 9 Input 10 Input 11 Input 12 Input 13 Input 14 Input 15 Power		Boards	Water Scrap Re-Work
Full Set - No Undesirable Outputs	Input 1 Input 2 Input 3 Input 4 Input 5 Input 6 Input 7 Input 8 Input 9 Input 10 Input 11 Input 12 Input 13 Input 14 Input 15 Power	Direct Hours Salary Hours	Boards	

Data Set	Inputs		Outputs	
	Material Inputs	**Non-Material Inputs**	**Product Outputs**	**Undesirable Outputs**
Seven Input Set	Material (Inputs 1 through 15) Power Water Scrap Re-Work	Direct Hours Salary Hours	Boards	

Inputs and outputs in the Full Data Set are categorized according to the method described in Chapter 3. This categorization requires the identification of the source of inputs such as a "waste" output from another production process (recycled input) and so on. The analysis necessary to identify the sources of the 15 material inputs used in the full data set was not performed. It is noted that many of these material inputs are recycled. This is being performed by the vendors of the various inputs to the production process and is not explicitly tracked by Manufacture Inc. Therefore, the categorizations of the inputs based on source must be assigned since this information is not available. The data set was reviewed and the inputs where significant increases and decreases were occurring over time were selected for ordinal classification. The categorizations of the outputs and inputs are shown in Table 4-2 and Table 4-3, respectively.

Table 4-2. Categorizations of Outputs

Output Category	
Primary Product	Boards
Recycled Output	Scrap ReWork
Waste Discharge	N/A
Permitted Discharge	Water
Non Classified	N/A

Table 4-3. Categorizations of Inputs

Input Category	
Non-Material Inputs	Direct Hours, Salary Hours
Recycled Inputs	Input 1, Input 2, Input 3, Input 4, Input 5
Renewable Inputs	Input 10, Input 12
Non-Renewable Inputs	Power, Input 6
Non Classified	Input 7, Input 8, Input 9, Input 11, Input 13, Input 14, Input 15

4.4 Basic Statistical Analysis of the Normalized Data

Statistical analyses include standard descriptive statistics presented in Section 4.4.1, linear regression presented in Section 4.4.2, autocorrelation results presented in Section 4.4.3, and correlation matrix presented in Section 4.4.4.

4.4.1 *Standard Descriptive Statistics*

The standard descriptive statistics reported in Tables 4-4, 4-5, and 4-6 are produced with Microsoft® Excel using the full data set (1 output, 3 undesirable outputs, and 18 inputs). Figure 4-3 shows the variability of the inputs and outputs. The data for Number of Units Scrapped, Power, and Input 10 have a maximum value well above the average indicating unusual production events. From a data analysis perspective these may perhaps be called outliers. However, almost any production process is going to have periods of off-normal events as well as periods where productivity is unusually high. Some of this is going to be related to the inherent random nature of the production process; but, this also can provide information on the bounds on the production process. Average values are all the same as required by the normalization procedure used.

4.4.2 *Least Squares Regression*

The output of boards may be related to the inputs using standard regression analysis. The first order expression, modified from Neter and Wasserman (1974, p. 215) for this application, is:

$$\text{Output of Boards}_t = a + \beta_1(\text{Scrap})_t + \beta_2(\text{ReWork})_t + \beta_3(\text{Water})_t + \beta_4(\text{Labor Hours})_t + \beta_5(\text{Salary Hours})_t + \beta_6(\text{Input 1})_t \ldots + \beta_{20}(\text{Input 15})_t + \beta_{21}(\text{Power})_t + e_t \quad \text{(Eq. 4-4)}$$

The terms β_k represent the linear regression coefficients with e_t the error (or residual) for the observation at time, t. The expression gives the response for the Output of Boards at time t (also called trial). This model's parameters were estimated using the SAS programming language. This model was also analyzed with the undesirable outputs (scrap, re-work, and water) removed. Table 4-6 shows parameter estimates for the full model as well as results for the t statistic test. A high Prob > |T| means that the parameter may be assumed to be zero and eliminated from the model.

A number of the parameters are negative indicating counter-intuitively that an increase in these inputs will reduce outputs. For "Input 2" this negative value is not significant according to the t statistic as shown in Table 4-6. Other negative value parameters cannot be rejected based on the t statistic. One explanation is that some of the independent variables are not as subject to variation and do not have a strong influence on the dependent variable. This is certainly the case with Salary Hours and Power. Both of these inputs have negative parameters in the linear regression model. Power use tends to be constant since the HVAC, lighting, and other plant systems must be powered regardless of the level of output. Salary Hours also tend to be constant from month to month since most companies, including Manufacture Inc., try to avoid discharging employees based on short term production fluctuations. More detailed outputs from the SAS program are provided in Appendix C.

Table 4-4. Measures of Dispersion for Inputs

Statistic	**Labor Hours**	**Salary Hours**	**In. 1**	**In. 2**	**In. 3**	**In. 4**	**In. 5**	**In. 6**	**In. 7**	**In. 8**	**In. 9**	**In. 10**	**In. 11**	**In. 12**	**In. 13**	**In. 14**	**In. 15**	**Power**
Avg.	6.22	6.22	6.22	6.22	6.22	6.22	6.22	6.22	6.22	6.22	6.22	6.22	6.22	6.22	6.22	6.22	6.22	6.22
Max.	8.18	8.23	9.26	8.92	8.40	9.73	8.01	9.78	9.12	13.36	9.12	13.36	8.12	7.54	9.05	9.43	10.76	19.55
Min.	1.38	3.09	4.58	4.30	2.80	3.99	3.64	3.68	4.24	5.67	4.24	5.67	3.45	4.94	4.04	4.32	3.44	2.71
Std Dev	1.39	1.29	1.22	1.22	1.38	1.27	1.18	1.29	1.19	1.80	1.19	1.80	1.25	1.07	1.22	1.22	1.41	2.12

Table 4-5. Measures of Dispersion for Outputs

Statistic	**Boards**	**Rewrk**	**Scrap**	**Water**
Avg.	6.22	6.22	6.22	6.22
Max.	9.27	9.10	18.08	10.03
Min.	3.58	0.00	4.06	4.11
Std Dev	1.25	1.33	1.65	1.40

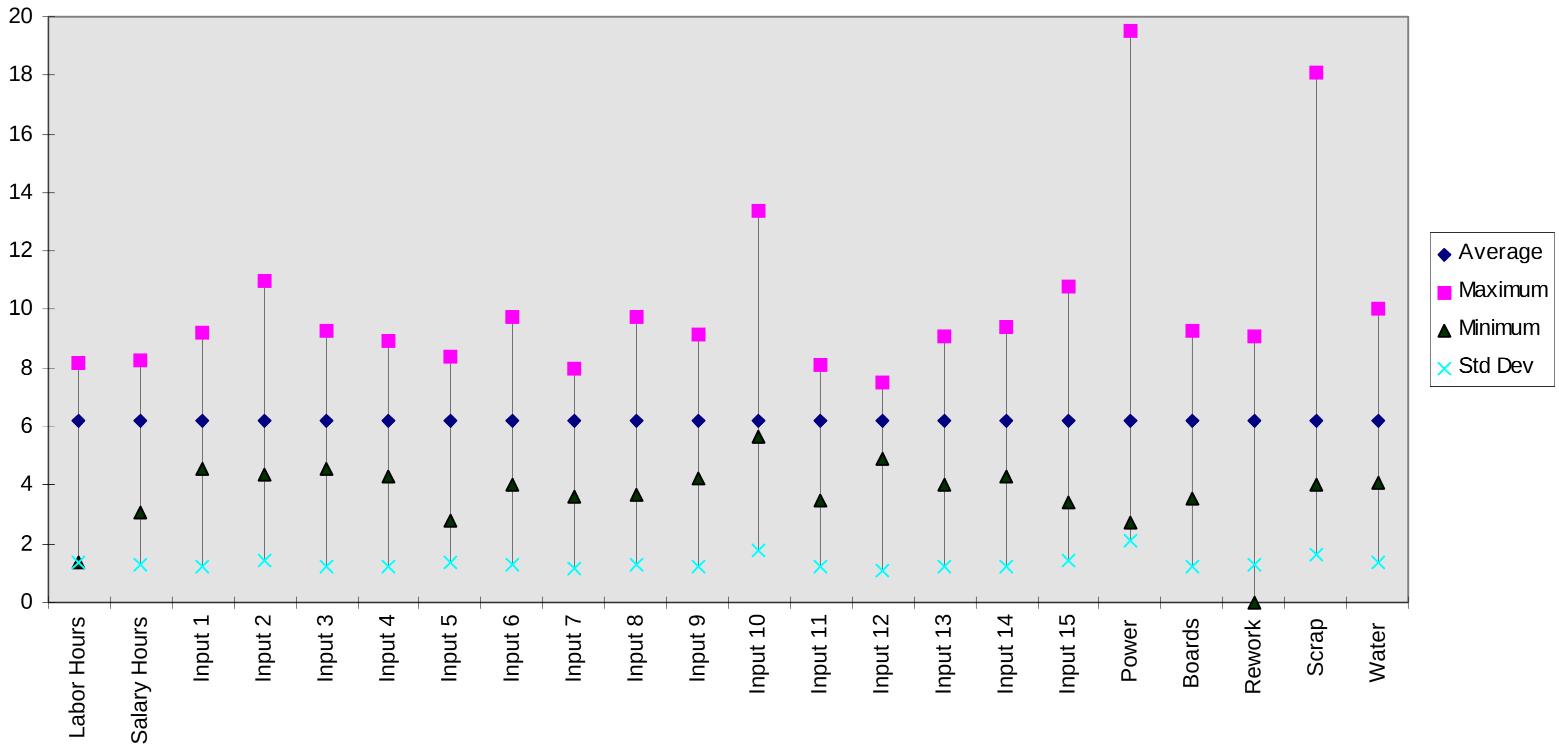

Figure 4-3. Measures of Dispersion for Inputs and Outputs

Table 4-6: Parameter Estimates for the Full Model

Dependent Variable is Number of Boards Output	**Parameter**	**T for Ho**	**Prob > T**
INTERCEPT	0.168	0.082	0.93
Scrap	0.249	2.86	0.01
Re-work	0.073	0.72	0.47
Water	0.221	1.86	0.07
Labor Hours	0.217	2.1	0.04
Salary Hours	-0.205	-1.82	0.07
Input 1	0.133	0.44	0.66
Input 2	-0.002	-0.01	0.99
Input 3	-0.124	-0.56	0.58
Input 4	0.107	0.57	0.57
Input 5	0.070	0.35	0.72
Input 6	0.097	0.47	0.64
Input 7	0.108	0.60	0.55
Input 8	0.067	0.46	0.65
Input 9	0.281	1.13	0.26
Input 10	-0.106	-1.12	0.27
Input 11	0.004	0.02	0.99
Input 12	0.137	0.50	0.62
Input 13	-0.113	-0.42	0.68
Input 14	0.113	0.63	0.53
Input 15	-0.227	-1.46	0.15
Power	-.126	-2.14	0.03
Value of α is 0.05			

The t statistic indicates that Input 11 as well as Input 2 may be removed from the model. These inputs are certainly required to produce the output. However, they appear to not strongly influence the level of production. There is no apparent reason that these inputs do not have a significant influence on output.

The R-square value is used as a general indicator of the fit of a model. This is also called the coefficient of multiple determination. "It measures the proportionate reduction of total variation in Y [dependent variable] associated with the use of the set X [independent variables]" (Neter and Wasserman, 1974, p. 228). In other words the R-square value is roughly the percentage of the variation in the dependent variable explained by the model. The higher (maximum is 1.0) the R-square value the better in terms of the model explaining the response of the dependent variable. The R-square value for the full model is 0.5573 indicating a not particularly good fit to the data. This not particularly good fit of the model is surprising since all material inputs along with labor inputs are part of the model. There are other factors not considered in the regression model that have a significant affect on output.

4.4.3 *Autocorrelation*

The basic regression model assumes that the random error terms e_t are either uncorrelated random variables or independent normal random variables. (Neter and Wasserman, p. 352, 1974). When error terms are in fact correlated over time this is called autocorrelated or serially correlated. Where a first order regression equation is being used the Durbin-Watson test for first order autocorrelation may be applied. This test was requested as part of the SAS program that is shown in Appendix D with the results shown in Appendix C. The SAS Durbin-Watson statistic assumes that time t values are only influenced by time t-1 values in the time series.

The D value for the full model shown in Equation 4-4 is 2.519. The lower and upper limit critical values are estimated from Neter and Wasserman (1974) to be 1.27 and

2.08. Since the upper value of 2.08 is well below the D value of 2.519 it can be concluded that autocorrelation is not occurring. Values for residuals are reported in Appendix C for each of the 100 production plans. Residuals are consistently above zero indicating that there are differences from one time period to the next. There are not extended periods when the residual is either positive or negative. Residuals shift from positive to negative and from negative to positive in the time series.

The conclusion is that there is a high degree of independence from one time period to the next for the model in Equation 4-4. In the Manufacture Inc. production process there is in fact a high degree of flexibility in terms of what products are scheduled and how much product is produced. Material inputs (Input 1 through Input 15) similarly increase or decrease from period to period as output fluctuates. The results of the Durbin-Watson test are consistent with the intuitive expectation that a production process like Manufacture Inc's must respond from day to day and week to week to changing demand and specifications.

4.4.4 *Correlation*

Table 4-7 and Table 4-8 show the correlation matrix for the model in Equation 4-4. The 22 X 22 matrix is presented in two parts. The SAS option used to generate this matrix excludes the intercept since this is not of interest in the analysis. Because of the large number of inputs this table is presented in two parts. Only the Pearson correlation coefficient is reported. The more detailed SAS output is provided in Appendix C.

As would be expected many of the inputs and outputs are highly correlated. This analysis is at the plant-wide level. There are some conclusions that can be drawn from the correlation matrix in Table 4-7 and 4-8. Output is highly and positively correlated with scrap production (0.48) as expected. While output is positively correlated with water (0.19), this is not as strong a correlation as might be expected for such a fundamental part of the production process. The indication is that water use can be more strictly controlled.

Direct labor hours are also highly and positively correlated with output (0.45) while salary hours are not nearly as strongly correlated (0.22). This is to be expected since increases and decreases in salary employment is typically not possible from a month to month basis. Manufacturing work on the floor can be demanding and so Manufacture Inc. does have significant turnover of direct labor allowing this input to more closely track output.

The material inputs to the production process (Input 1 through Input 15) are in general positively correlated. There are, however, some anomalous results. In some cases one material input is negatively correlated with another. Since the product being produced is subject to significant variation as well as some variations in the process, such as gold plating vs. no gold plating, some of this negative covariance reflects substitutions. One material input is increased while another is decreased depending on the product being produced. In a review of the raw data where the identity of materials is shown (not reported due to confidentiality concerns of Manufacture Inc.), at least some of this negative correlation does in fact seem to be due to product variations.

4.5 Application of Standard Methods for the Measurement of Efficiency

Standard methods are applied to the data as a means of comparison with the proposed method. The first of these three methods differ in their assumptions concerning returns to scale, disposability, and convexity (C.A. Knox Lovell, 1993). Returns to scale may be constant or variable and are specified by a constraint on the linear programming formulation. Constant returns to scale are achieved if the constraint $\lambda_k \geq 0$ as part of the formulation (refer to Equation 2-3 for the full linear programming formulation). The value of λ_k is solved for each production plan to provide the fraction of inputs and outputs that produce the optimal (i.e., highest efficiency) value. This is the original Data Envelopment Analysis method developed by Charnes, Cooper, and Rhodes (1978) CCR. Constant returns to scale means that an increase in inputs of say 20% for a production plan results in an increase in outputs by 20%. If the constraint is instead

Table 4-7. Covariance Matrix for the Full Regression Model - Part 1

	Output	**Scrap**	**Re-Work**	**Water**	**Labor Hours**	**Salary Hours**	**Input 1**	**Input 2**	**Input 3**	**Input 4**	**Input 5**
Output	1.00	0.48	-0.4	0.19	0.45	0.22	0.38	0.34	0.39	0.25	0.46
Scrap	0.48	1.00	-0.47	0.09	0.39	0.31	0.32	0.23	0.23	0.17	0.27
Re-work	-.04	-0.47	1.00	0.12	0.12	-0.01	-0.14	-0.06	-0.04	-0.08	-0.15
Water	0.19	0.09	0.12	1.00	0.22	0.19	0.16	0.19	0.12	0.29	0.07
Labor Hours	0.45	0.39	0.12	0.22	1.00	0.65	0.47	0.42	0.43	0.39	0.42
Salary Hours	0.22	0.31	-0.01	0.19	0.65	1.00	0.51	0.49	0.44	0.43	0.43
Input 1	0.38	0.32	-0.14	0.16	0.47	0.51	1.00	0.73	0.84	0.59	0.63
Input 2	0.34	0.23	-0.06	0.19	0.42	0.49	0.73	1.00	0.73	0.54	0.70
Input 3	0.39	0.23	-0.04	0.11	0.43	0.44	0.84	0.73	1.00	0.44	0.72
Input 4	0.25	0.17	-.08	0.29	0.39	0.43	0.58	0.54	0.44	1.00	0.51
Input 5	0.46	0.27	-0.15	0.07	0.42	0.43	0.63	0.70	0.72	0.51	1.00
Input 6	0.27	0.12	-0.05	-0.22	0.01	-0.11	-0.05	-.21	0.05	-0.27	0.18

	Output	**Scrap**	**Re-Work**	**Water**	**Labor Hours**	**Salary Hours**	**Input 1**	**Input 2**	**Input 3**	**Input 4**	**Input 5**
Input 7	0.52	0.32	-0.06	0.31	0.43	0.38	0.58	0.61	0.61	0.60	0.70
Input 8	0.42	0.35	-.23	-0.09	0.29	0.23	0.41	0.38	0.31	0.25	0.46
Input 9	0.24	-0.05	0.04	0.16	0.26	0.28	0.21	0.32	0.35	0.27	0.44
Input 10	-0.04	0.16	-0.13	0.46	0.15	0.19	0.12	0.02	-0.08	0.33	0.04
Input 11	0.27	0.13	0.023	0.37	0.38	0.35	0.55	0.45	0.41	0.70	0.43
Input 12	0.34	0.38	-0.12	-0.03	0.37	0.38	0.71	0.67	0.65	0.39	0.60
Input 13	0.31	0.08	-0.06	0.03	0.22	0.17	0.41	0.26	0.44	0.49	0.61
Input 14	0.14	-0.12	0.14	-0.07	-0.04	-0.12	-0.23	-0.11	0.04	-0.05	0.25
Input 15	-0.04	-0.13	0.02	0.43	0.14	0.21	-0.01	0.11	0.03	0.13	-0.07
Power	-0.07	0.016	-0.02	0.03	0.09	0.08	0.17	0.24	0.17	0.34	0.25

Table 4-8. Covariance Matrix for the Full Regression Model - Part 2

	Input 6	**Input 7**	**Input 8**	**Input 9**	**Input 10**	**Input 11**	**Input 12**	**Input 13**	**Input 14**	**Input 15**	**Power**
Output	0.27	0.52	0.42	0.24	-0.04	0.27	0.34	0.31	0.14	-0.05	-0.7
Scrap	0.12	0.32	0.35	-0.05	0.16	0.13	0.38	0.08	-0.12	-0.13	0.02
Re-work	-0.05	-0.06	-0.23	0.04	-0.13	0.02	-0.12	-0.06	0.14	0.02	-0.02
Water	-0.22	0.31	-0.09	0.162	0.46	0.37	-0.03	0.03	-0.07	0.43	0.03
Labor Hours	0.01	0.43	0.29	0.26	0.15	0.38	0.37	0.22	-0.04	0.14	0.09
Salary Hours	-0.11	0.37	0.23	0.28	0.19	0.35	0.38	0.17	-0.12	0.21	0.08
Input 1	-0.05	0.58	0.41	0.21	0.12	0.55	0.71	0.41	-0.23	-0.01	0.17
Input 2	-0.21	0.61	0.38	0.32	0.02	0.45	0.67	0.26	-0.11	0.11	0.24
Input 3	0.05	0.61	0.31	0.35	-0.08	0.41	0.65	0.44	0.04	0.03	0.17
Input 4	-0.27	0.60	0.25	0.27	0.33	0.70	0.39	0.49	-0.04	0.13	0.34
Input 5	0.18	0.70	0.46	0.44	0.04	0.43	0.59	0.61	0.25	-0.07	0.25
Input 6	1.00	0.17	0.57	0.20	-0.20	-0.45	0.02	0.18	0.11	-0.12	-0.10
Input 7	0.17	1.00	0.51	0.53	-0.01	0.53	0.43	0.54	0.22	0.20	0.29
Input 8	0.57	0.51	1.00	0.34	0.00	0.01	0.38	0.26	-0.07	0.08	0.03

	Input 6	Input 7	Input 8	Input 9	Input 10	Input 11	Input 12	Input 13	Input 14	Input 15	Power
Input 9	0.20	0.53	0.34	1.00	-0.17	0.30	-0.14	0.60	0.52	0.62	0.20
Input 10	-0.20	-0.01	0.00	-0.17	1.00	0.23	0.23	-0.20	-0.30	0.08	0.10
Input 11	-0.45	0.53	0.01	0.30	0.23	1.00	0.34	0.37	0.10	0.15	0.20
Input 12	0.02	0.43	0.37	-0.14	0.23	0.34	1.00	-0.00	-0.35	-0.34	0.19
Input 13	0.18	0.54	0.26	0.60	-0.20	0.37	-0.00	1.00	0.57	0.04	0.29
Input 14	0.11	0.22	-0.06	0.52	-0.30	0.10	-0.35	0.57	1.00	0.12	-0.01
Input 15	-0.12	0.20	0.08	0.62	0.08	0.14	-0.34	0.04	0.12	1.00	0.08
Power	-0.10	0.29	0.03	0.20	0.10	0.20	0.19	0.29	-0.01	-0.08	1.00

$\sum_{k=1}^{K} \lambda_k \geq 0$, then variable returns to scale are assumed. This formulation was developed by Banker, Charnes, and Cooper (1984) (BCC).

The CCR and BCC approach both assume convexity. Relaxing this assumption results in the method known as Free Disposal Hull (FDH) developed by Deprins, Simar, and Tulkens (1984). The constraint, $\lambda_k \geq 0$, becomes $\lambda_k \in \{0,1\}$ (C.A. Knox Lovell, 1993). This has the result of converting the FDH model to a mixed integer programming problem. FDH formulations assume variable returns to scale.

The CCR, BCC, and FDH models all assume strong disposability. "Strong disposability refers to the ability to dispose of an unwanted commodity with no private cost. Weak disposability refers to the ability to dispose of an unwanted commodity at positive private cost (Färe, Grosskopf, Lovell, 1994, p. 38)." The standard CCR formulation may be modified to assume weak disposability by replacing the constraints in the standard CCR formulation (refer to Equation 2-3) (C.A. Knox Lovell, 1993):

$$-\sum_{k=1}^{K} u_{jk}\lambda_k + u_{j0}z_0 \leq 0; \qquad j = 1,\ldots,J$$

$$\sum_{k=1}^{K} x_{ik}\lambda_k \leq x_{i0}; \qquad i = 1,\ldots,I$$

with

$$-\sum_{k=1}^{K} u_{jk}\lambda_k + u_{j0}z_0 = 0; \qquad j = 1,\ldots,J$$

$$\sum_{k=1}^{K} x_{ik}\lambda_k = x_{i0}; \qquad i = 1,\ldots,I$$

The standard notation from Appendix A is being used with x as inputs, i, and u as outputs, j. There are k production plans. This assumption of weak disposability is not widely applied.

The Benchmark Correspondence, developed by Tulkens and Vanden Eeckaut (1995), does away entirely with the frontier and compares data directly. This method is based on the definition of dominance (all inputs less than or equal to a reference and all outputs greater than or equal to a reference with at least one input less than the reference or one output greater than the reference).

Standard analysis techniques are applied to the first 50 production plans and second 50 production plans. However, the data sets used for the proposed method cannot be used for analyses with standard methods. With 15 inputs the standard methods simply assign all production plans an efficiency score of 1.0. In order to reduce the data set the 15 material inputs were combined and then normalized. The undesirable outputs of water, scrap, and re-work are considered to be inputs in these standard methods (since undesirable outputs are not allowed). This reduced data set contains all of the inputs and outputs of the full data set, except the 15 material inputs are aggregated.

The program IDEAS is used to do the CCR and BCC analysis. A MATHEMATICA program developed by Girod (1996) is used to produce measures of efficiency using the FDH method. The Benchmark Correspondence results are produced using a MATHEMATICA program developed as part of this research. Details are provided in Appendix D.

When calculating measures of productive efficiency using frontier methods like CCR, BCC, and FDH either input reducing or output increasing measures may be calculated. Input reducing measures reduce inputs as necessary to find the efficient combination of inputs on the frontier. The efficiency measure is then the ratio of the distance to the actual production plan to the distance to the frontier production plan. An efficient production plan is already on the frontier and so has an efficiency score of 1.0. An inefficient production plan is using more inputs than are necessary to produce the output and therefore has an efficiency score of less than 1.0. Similarly, output increasing measures are based on the amount of additional output required to move from the actual production plan to the frontier production plan. An efficiency production plan for output increasing measures also has a score of 1.0 since it is on the production frontier. An inefficient production plan has less output than production plans on the frontier and is given a score of greater than 1.0.

The Benchmark Correspondence method results place all production plans for the first and the second 50 sets of production plans in the dominance indifferent set. Therefore, there are no measures of progress or regress since there are no production plans that are dominating or dominant. The inapplicability of the Benchmark Correspondence method to most manufacturing systems is caused by a number of features. First, most manufacturing systems most of the time are fairly stable in terms of productive efficiency. For example, progress will occur if all inputs decrease or stay the same and all outputs increase or stay the same; with an increase in at least one input or output. This is less likely to occur where there are many inputs and outputs. Also, where data are not being used at a highly aggregated level there are often substitutions occurring among inputs. If one input is increasing and another is decreasing then this is by definition going to be considered dominance indifferent. Lastly, there is random variation in any production system. Inputs and outputs are increasing and decreasing daily and weekly reducing the possibility of a production plan dominating all production plans in the dominance indifferent set.

Table 4-9 shows input reducing measures and output increasing measures using the BCC model and the second 50 production plans as an example of how these measures compare. These are simply the 50 production plans representing the second year of the two years of production data obtained from Manufacture Inc. The relative order of the production plans in terms of efficiency is not the same for input reducing and output increasing measures. For example, Production Plan 56 is the second most inefficient production plan (only one production plan, 99, is less efficient) for output increasing measures with a score of 1.703. This same production plan is however more efficient than 26 of the other production plans based on input reducing measures. Different conclusions concerning the efficiency of a production plan may be drawn depending on whether input reducing or output increasing measures are used.

Table 4-9. Comparison of Input Reducing and Output Increasing Measures of Efficiency

Production Plan	**BCC - Input Reducing**	**BCC - Output Increasing**
51	1	1
52	0.9764544	1.119926
53	0.9717013	1.454033
54	1	1
55	0.9748614	1.377352
56	0.9761311	1.702254
57	0.9798521	1.071382
58	0.9835095	1.057956
59	0.9682176	1.182616
60	0.9623738	1.202546
61	0.9683356	1.049073
62	0.9704658	1.173124
63	0.967883	1.28121
64	1	1
65	1	1
66	1	1
67	0.9932119	1.14475

Production Plan	BCC - Input Reducing	BCC - Output Increasing
68	1	1
69	1	1
70	1	1
71	1	1
72	1	1
73	1	1
74	1	1
75	0.935061	1.284801
76	1	1
77	0.974444	1.323429
78	0.9286256	1.460376
79	0.9286256	1.391142
80	0.9286256	1.380743
81	0.9285541	1.238956
82	0.9201738	1.606976
83	0.9201738	1.241389
84	0.9435111	1.183318
85	0.9144892	1.235467
86	0.8818044	1.474123
87	0.8822426	1.450051
88	0.8848655	1.398064
89	0.898744	1.088142
90	0.9350508	1.203234
91	0.8600221	1.229512
92	0.8600221	1.464588
93	0.891354	1.110481
94	0.8707644	1.388432

Production Plan	BCC - Input Reducing	BCC - Output Increasing
95	0.8855163	1.356684
96	0.9199818	1.045946
97	1	1
98	0.9400107	1.577253
99	0.8443441	2.120515
100	0.9559996	1.035813

The objective of reducing environmental impact is more consistent with input reducing measures - i.e., less input for the same output. In addition, the model being used is considering undesirable outputs as inputs that are desirable to decrease. Therefore, the remaining analysis using standard methods is focused on input reducing measures. Table 4-10 and Table 4-11 present input reducing measures for the first and second 50 production plans for the CCR, BCC, and FDH methods as a means of comparison among these three methods. There are an enormous number of variations on DEA. The intent is to provide results from some standard approaches that may be compared with the proposed method.

Table 4-10. Input Reducing Efficiency Scores for Standard Methods - First 50 Production Plans

Production Plan	CCR-Input Reducing	FDH - Input Reducing	BCC - Input Reducing
1	0.7890612	1	1
2	0.608376	1	1
3	1	1	1
4	0.7221845	1	1
5	0.8304963	1	1
6	0.7116467	1	0.8876312

Production Plan	CCR-Input Reducing	FDH - Input Reducing	BCC - Input Reducing
7	0.8879172	1	0.9617639
8	1	1	1
9	0.9031517	1	1
10	0.6977322	1	1
11	0.8066895	1	1
12	0.9030957	1	1
13	0.6518214	1	0.9801406
14	0.6323183	1	1
15	0.8747265	1	1
16	0.7926959	1	1
17	0.9717563	1	1
18	1	1	1
19	0.7052743	1	1
20	1	1	1
21	0.7149166	0.983901	0.932709
22	1	1	1
23	0.8856878	0.987304	0.9454329
24	0.7908216	0.983901	0.8670858
25	0.8456674	1	0.8724999
26	0.9522638	1	0.9524004
27	0.7846134	0.948833	0.9025946
28	0.9900236	1	1
29	0.981993	1	1
30	0.8168268	1	0.8816709
31	0.9050996	1	0.905861
32	0.7354833	1	0.9923176
33	0.810554	1	0.9023627

Production Plan	CCR-Input Reducing	FDH - Input Reducing	BCC - Input Reducing
34	0.9926627	1	1
35	1	1	1
36	0.8248304	0.941756	0.8250236
37	0.8973777	1	0.9375119
38	0.7746461	1	0.8302258
39	0.6502342	1	1
40	0.9882448	1	1
41	0.8586574	1	1
42	1	1	1
43	1	1	1
44	1	1	1
45	1	1	1
46	0.8988133	1	1
47	0.8501288	1	1
48	1	1	1
49	0.7845059	1	1
50	0.7785512	1	1

Table 4-11. Input Reducing Efficiency Scores for Standard Methods - Second 50 Production Plans

Production Plan	CCR-Input Reducing	FDH - Input Reducing	BCC - Input Reducing
51	0.8580005	1	1
52	0.8905033	1	0.9764544
53	0.6871758	1	0.9717013
54	1	1	1

Production Plan	CCR-Input Reducing	FDH - Input Reducing	BCC - Input Reducing
55	0.7247756	1	0.9748614
56	0.5812896	0.983490451	0.9761311
57	0.9287692	1	0.9798521
58	0.8949729	1	0.9835095
59	0.8367425	0.968217568	0.9682176
60	0.828225	0.96237379	0.9623738
61	0.9501593	1	0.9683356
62	0.7954581	1	0.9704658
63	0.6697492	1	0.967883
64	1	1	1
65	1	1	1
66	0.9316208	1	1
67	0.8019882	1	0.9932119
68	0.8610764	1	1
69	1	1	1
70	1	1	1
71	0.6340484	1	1
72	0.6629969	1	1
73	0.7457328	1	1
74	0.7069153	1	1
75	0.7707596	1	0.935061
76	1	1	1
77	0.7556129	1	0.974444
78	0.6808145	0.928625641	0.9286256
79	0.7121929	0.956000318	0.9286256
80	0.7183671	0.928625641	0.9286256
81	0.791585	1	0.9285541

Production Plan	CCR-Input Reducing	FDH - Input Reducing	BCC - Input Reducing
82	0.6047437	0.920173742	0.9201738
83	0.7799658	0.920173742	0.9201738
84	0.8278466	0.999775069	0.9435111
85	0.7737271	0.914489239	0.9144892
86	0.6575919	0.881883187	0.8818044
87	0.6795272	0.914735162	0.8822426
88	0.7048202	0.994968239	0.8848655
89	0.8779364	1	0.898744
90	0.8263466	1	0.9350508
91	0.7692686	0.869260655	0.8600221
92	0.6539946	0.860022091	0.8600221
93	0.8772125	0.971519748	0.891354
94	0.7018526	0.899370881	0.8707644
95	0.6717159	0.887421718	0.8855163
96	0.9113842	1	0.9199818
97	0.7881416	1	1
98	0.6249091	0.949236519	0.9400107
99	0.4682582	0.84936303	0.8443441
100	0.9413902	1	0.9559996

From inspection of the results reported in Table 4-10 and Table 4-11 it can be seen that there are significant differences between the efficiency scores for these methods. Conclusions concerning the relative efficiency of production plans can be quite different depending on which of these three methods is used. The BCC model is selected for additional analysis because the assumption of variable returns to scale is more appropriate than the constant returns to scale assumption of CCR and because standard linear programming procedures may be used. The slacks for the BCC input reducing model applied to the first and second 50 production plans are reported in

Appendix C. Although the efficiency scores differ significantly for different methods there is general agreement that the production process is efficient most of the time with occasional periods of decreased efficiency. The second 50 production plans do exhibit more inefficiency than the first 50 production plans. One possible explanation is the increasing complexity of the product being produced and the increasing amount of small batch orders.

The slacks are the amount of a particular input that is in excess of those required to place the production plan on the frontier. All production plans with an efficiency score of 1.0 do not have any slacks for the BCC model since these are defined to be optimum production plans in terms of technical efficiency. While the slacks produced are somewhat informative, the extent of aggregation means that they have little meaning at the operational level. For example, material inputs show a slack, but this input is so highly aggregated that the slacks reported cannot be related to the production process.

4.6 Application of the Proposed Method

A program in the programming language MATHEMATICA was developed by the author to partition the data set according to the set definitions and to calculate metrics of performance. This program is in Appendix D. A characteristic of the proposed method is the generation of large amounts of information and computational intensiveness. The number of computations increases rapidly as production plans are added to the set to be evaluated. The computer being used has 16 MB of memory and a 75 mHz Pentium I processor. Running a data set of 10 production plans takes about 1 minute of processing time. Running a data set of 25 production plans takes about 20 minutes of processing time. Running a data set of 50 production plans takes about 12 hours or processing time. To allow runs to be performed in a reasonable amount of time the data set was divided into two sets of 50 production plans rather than evaluating the entire data set of 100 production plans together. Each of the 50 production plan data sets is about 1 year of data (50 weeks).

Partitioning of the time series into two parts has other advantages. In general, using long periods of historical data when a production process is undergoing modifications of various sorts may result in production plans that no longer relate to the production process being used as the bases for performance measures. The use of two separate data sets also allows a more thorough evaluation of the proposed and existing methods since two separate example data sets are now being evaluated. The partitioning of the data set in half is valid in terms of testing and evaluating the proposed and existing methods. When applied to perform actual evaluations to be used in decision making the issue of where to segment the time series data would have to be addressed.

The next four sections report the results of the analysis using the proposed method. First, the meaning of set memberships is reviewed in Section 4.6.1. Counts of set membership for each production plan t are presented in Section 4.6.2. Then in Section 4.6.3 representative production plans at t=50 and t=96 are used to illustrate a more detailed analysis based on exactly which production plans are assigned to the sets. Distance measures are then discussed in Section 4.6.4 and Section 4.6.5. Finally, the results of the analysis with the material inputs only (i.e., salary hours and direct hours removed from the data set) are reviewed in Section 4.6.6

4.6.1 *Meaning of Set Memberships*

As previously discussed, the algorithm used in the Benchmark Correspondence method results in different production plans being assigned to the sets D_d, D_g, and D_i even though the set definitions are the same. The inclusion of undesirable outputs results in the sets TE (Technically Dominating-Preferred Environmentally), TI (Technically Dominated-Non Preferred Environmentally), and TU (Technically and Environmentally Dominance Indifferent). Consideration of changes in input and output mix relative to a defined ordinal classification of inputs and outputs results in the sets SE (Substitution Environmental Performance - Dominating), and SI (Substitution Environmental Performance - Dominated).

There are many different combinations of set memberships that are possible. Various combinations of set memberships can be used to analyze performance. Table 4-12 summarizes the possibilities and their meaning. Table 4-12 is divided into three parts corresponding to a production plan being placed in either set D_g, D_d, or D_i. Given membership is one of these three sets, the possible set memberships in the sets TE, TI, TU, SE, and SI are shown. The meaning of the set memberships is briefly stated.

If a production plan is a member of the set D_g then it cannot be a member of the set TI. This is because by definition membership in the set D_g requires that the production plan dominates in both inputs and outputs. If the production plan also dominates in terms of undesirable outputs it is a member of the set TE. Otherwise, the production plan must be a member of the set TU. By definition only those production plans that are part of the set TU may be further categorized into the sets SE or SI. Similarly, any production plan that is dominated in inputs and outputs and is, therefore, placed in the set D_d cannot be a member of the set TE. A production plan that is dominated in outputs and inputs and also dominated in undesirable outputs is part of the set TI. Production plans that are part of the set D_i cannot be part of the sets TE and TI since by since by definition inputs or outputs are both greater than and less than the reference production plan. Production plans in D_i must therefore be part of the set TU. These same production plans may, however, be members of either SE or SI.

There is one unusual condition that is not considered in Table 4-12. If a production plan is the same in inputs and outputs as the reference production plan, then it is part of the set D_i . If undesirable outputs are greater than or equal to a reference production plan with at least one undesirable output greater than the reference production plan, then the production plan is in set TI. Similarly, if undesirable outputs

Table 4-12. Possible Set Memberships for Production Plans

D_g	**Environmental Performance Sets**					
	TE	**TI**	**TU**	**SE**	**SI**	**Meaning**
	X					Improved productive efficiency with undesirable outputs at least not increasing. Therefore, this is also an improvement in environmental performance.
			X			Improved productive efficiency that is neutral in terms of change in environmental performance.
			X	X		Improved productive efficiency that is also an improvement in environmental performance through a change in the input or output mix.
			X		X	Improved productive efficiency that is a decrease in environmental performance through changes in the input or output mix.
D_d	**Environmental Performance Sets**					
	TE	**TI**	**TU**	**SE**	**SI**	**Meaning**
		X				Reduction in productive efficiency that is a reduction in environmental performance.
			X			Reduction in productive efficiency that is neutral in terms of environmental performance.
			X	X		Reduction in productive efficiency that is an improvement in environmental performance through changes in the input or output mix.
			X		X	Reduction in productive efficiency that is a decrease in environmental performance through changes in the input or output mix.
D_i	**Environmental Performance Sets**					
	TE	**TI**	**TU**	**SE**	**SI**	**Meaning**
			X			Changes in Productive Efficiency and Environmental Performance are not detected.
			X	X		Improvement in Environmental Performance through changes in input or output mix.
			X		X	Reduction in Environmental Performance through changes in input or output mix.

are less than or equal to a reference production plan with at least one undesirable output less than the reference production plan, then the production plan is in set TE.

The meaning of the set memberships shown in Table 4-12 are discussed further for each of the three parts of the table corresponding to membership in the sets D_g, D_d , or D_i. A production plan that is both a member of the set D_g and TE is an improvement in both productive efficiency and environmental performance. This production plan has improved productive efficiency (produces the same product with less inputs or produces more product with the same inputs or both) without increasing undesirable outputs. Pollution prevention activities (e.g., increasing energy efficiency, increasing conversion in a chemical process, reducing scrap production) would produce this kind of set membership. If a company just implements end-of-pipe pollution controls then the production plan would not have set membership in D_g since inputs like capital and labor are being increased without any benefit in terms of increased product output. However, if sufficient improvements are made in the production process through pollution prevention then also implementing end-of-pipe pollution controls may allow this set membership. In other words, memberships in the sets D_g and TE do not necessarily distinguish whether or not environmental compliance is being achieved through pollution prevention or a combination of pollution prevention and end-of-pipe controls.

If a production plan is a member of the set D_g and TU then this production plan is an improvement in productive efficiency that at least does not result in a deterioration of environmental performance. Any production plan that is a member of the set TU may exhibit changes in the input or output mix that are improvements in environmental performance. This is indicated by set membership in the set D_g as well as the sets TU and SE. By definition this production plan has inputs that are not greater than the reference and product outputs that are not less then the reference for it to be in set D_g. Therefore, membership in the set TU requires that some undesirable outputs are greater than the reference production plan. Membership in the sets D_g, TU, and SE may also represent pollution prevention activities as well as end-of-pipe treatment.

Given membership in the set D_d a production plan is dominated and is a decrease in productive efficiency. A reduction in productive efficiency is also a decline in environmental performance in the sense that more inputs are required to produce the same output. Such a production plan may also be a member of the set TI indicating a further decline in environmental performance. Such a production plan not only is using more inputs to produce a product, it also is generating more undesirable outputs. Production plans in the sets D_d as well as TU may also have changes in input or output mix placing them in the sets SE or SI.

Given membership in the set D_i a production plan must also be in the set TU (with the exception as previously noted). This is because by definition inputs are both greater than and less than a reference production plan; or product outputs are both greater than and less than a reference production plan which places such a production plan in the set TU regardless of what is occurring with undesirable outputs. Production plans may be classified in the sets SE or SI by changes in the input or undesirable output mix.

4.6.2 *Set Counts*

Set counts represent an overall measure of performance. Figures 4-4, 4-5, 4-6 and 4-7 show the set counts using the full data set (18 inputs, 3 undesirable outputs, and 1 output). The count data that the Figures are based on are shown in Appendix C. The figures provide an overall view of performance. These counts are reported in two sets of 50 - the first 50 production plans and then the next 50 production plans. For each production plan at time t all previous production plans are assigned to a set based on the production plan at time t as a reference.

The first two figures show the counts for the sets D_d (number of production plans before time t that are dominated by the production at time t) and D_g (number of production plans before time t that dominate the production plan at time t). If a lot of production plans are dominating (assigned to D_g) then this indicates the production plan at time t represents a reduction in productive efficiency compared to these previous production plans. If a lot of production plans are dominated D_d then this indicates the production plan at time t represents an improvement in productive efficiency over these previous production plans. The counts for the set D_d are assigned negative values for charting purposes.

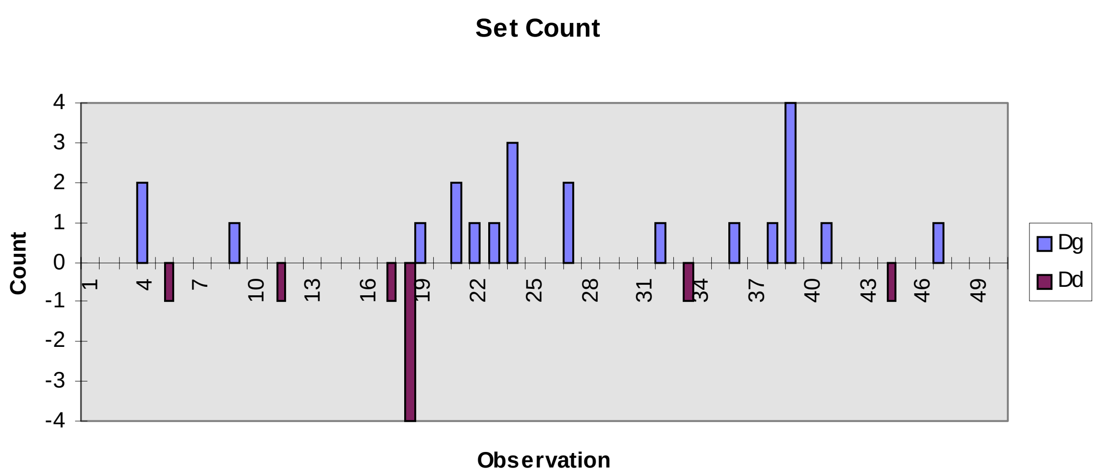

Figure 4-4. Set Counts for D_g and D_d for the First 50 Production Plans

There can be seasonal variations in production. At Manufacture Inc. production tends to be less for the last couple of weeks in December, the first week in January, and the first week in July. This corresponds to observations 17, 18, 19, 45, 69, 70, 71, and 97 (refer to Appendix B for a listing of week ending dates).

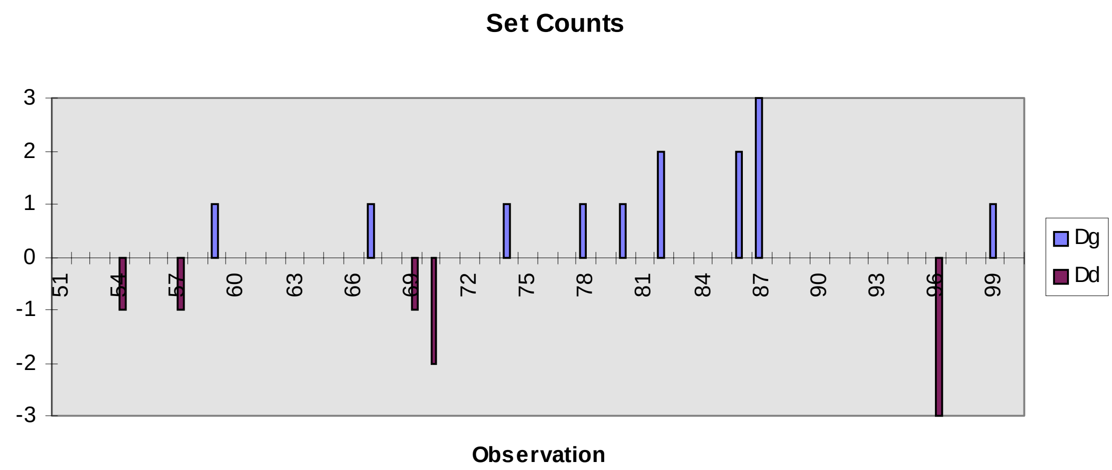

Figure 4-5. Set Counts for D_g and D_d for the Second 50 Production Plans

The counts for set membership in D_g and D_d are low - less than four. This indicates that that there is not a strong trend to either reducing or improving productive efficiency. If there was such a trend then a large number of the production plans before time t would be classified as either in D_g (if there was a decreasing productive efficiency trend) or D_d (if there was an increasing productive efficiency trend). The fact that there are counts in D_d and D_g indicates that there is variation in the productive efficiency of the process. For 18 inputs and one output to be classed as dominant or dominated would seem to require more than just random variation.

For the first 50 and the second 50 production plans there are no production plans assigned to either TE or TI. The additional requirement that the undesirable outputs must dominate for membership in the set TE or that the undesirable outputs must be dominated for membership in the set TI results in all production plans being dominance indifferent. It is also noted that in the correlation analysis reported in Section 4.4 the undesirable output of scrap is positively correlated with the product output. This means that if product increases the undesirable output of scrap also tends to increase. The result is that membership in the sets TI and TE are even more unlikely than would be expected just from the addition of undesirable outputs to the dominance criteria.

Figures 4-6 and 4-7 show the counts for the sets SE and SI. There is substantial change in the input or output mix occurring in the plant over time. The fact that set counts in SE and SI occur for production plans that are close to each other in the time series indicates that there is significant variation in the production process. The counts in Figures 4.6 and 4.7 are also high enough in many instances to indicate that there has been a change in the input/output mix that changes environmental performance. The first 50 production plans in Figure 4.6 show that there is a tendency for environmental performance (in terms of changes in input or output mix) to improve. The second 50 production plans in Figure 4-7 show a tendency for environmental performance to be reduced.

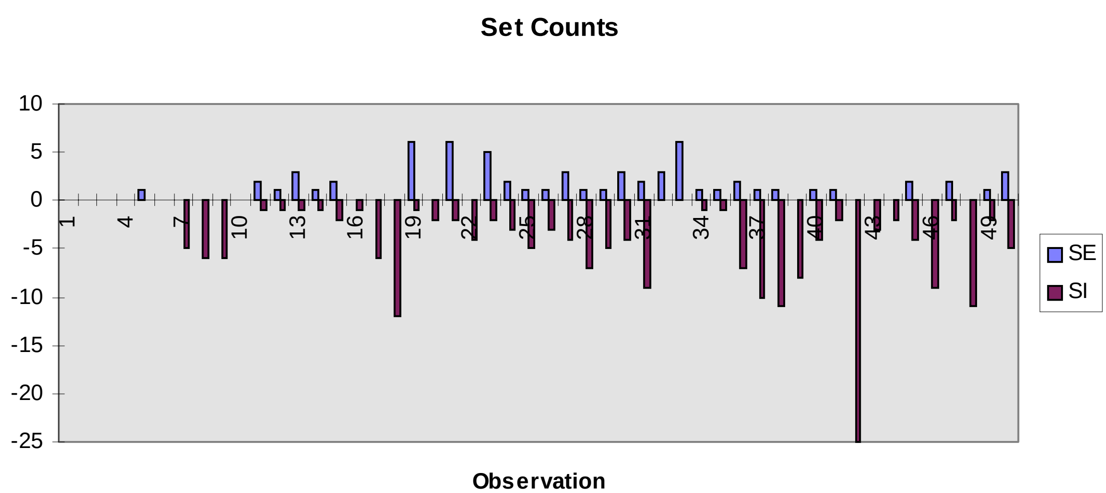

Figure 4-6. Set Counts for SE and SI for the First 50 Production Plans

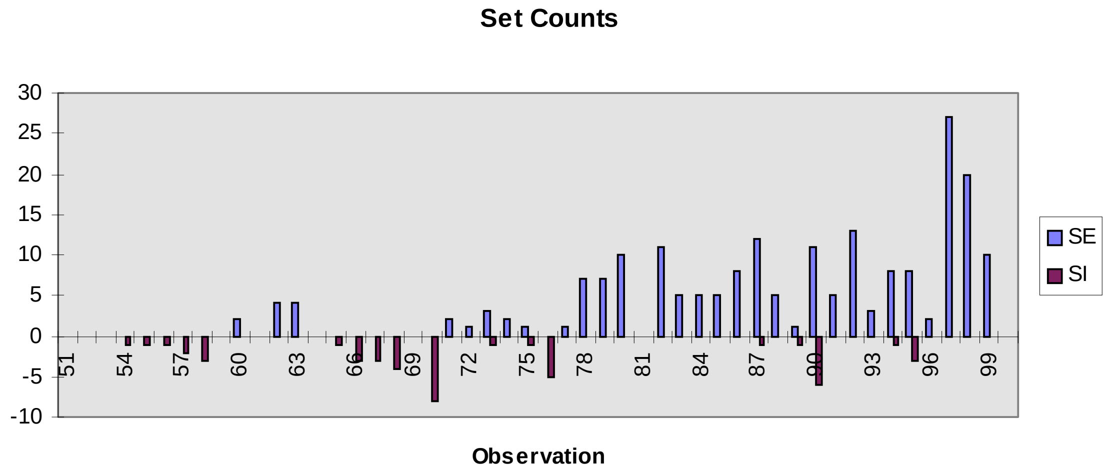

Figure 4-7. Set Counts for SE and SI for the Second 50 Production Plans

4.6.3 *Assignment of Production plans to Sets*

Counts provide a means of characterizing entire sets. More specific information on exactly which production plans are members of particular sets is also generated by the MATHEMATICA program. This information is used to evaluate current plant performance relative to past performance. As with counts, the data set is divided into the first 50 production plans and the second 50 production plans which each correspond to a about one year of data. These results are shown in Table 4-13 and Table 4-14 for the 50^{th} and 96^{th} production plan.

All production plans before the particular production plan at time t are evaluated in terms of their set memberships and distance measures. Therefore, the 50^{th} production plan and the 100^{th} production plan result in all previous production plans in the two sets being evaluated. In the first set of 50 production plans the 50^{th} production plan is selected to illustrate the method. Unfortunately, in the second set of 50 production plans the 97^{th}, 98^{th}, 99^{th}, and 100^{th} production plans do not have many of the previous production plans assigned to sets. Therefore, in the second 50 production plan set the

96th production plan is selected as providing sufficient data to illustrate the approach. Where a "1" is the entry in the table this indicates the production plan is a member of this set. Otherwise, a "0" is the entry and indicates the production plan is not a member of the set. Production plans that are members of sets other than D_i or TU are shaded.

For the 50th production plan all previous production plans are dominance indifferent and are therefore assigned to the sets D_i and TU as shown in Table 4-13. However, there are a number of instances where the change in the substitution mix of inputs or outputs results in a change in environmental performance. Production plans 38, 48 and 49 are Substitution Efficient (SE) compared to the reference production plan at t = 50. The two production plans before the 50th production plan indicates that there may be undesirable change in the input/output mix occurring (since the 48th and 49th production plans have environmentally preferable input or output mix). The production plans 2, 10, 11, and 39 are classed as SI indicating that the production plan at t=50 is more substitution efficient than these production plans in terms of input or output mix.

Table 4-13. Membership in Sets for the 50th Production plan

	Membership in a Set for Production plan 50							
obs.	D_g	D_d	D_i	TE	TI	TU	SE	SI
1	0	0	1	0	0	1	0	0
2	0	0	1	0	0	1	0	1
3	0	0	1	0	0	1	0	0
4	0	0	1	0	0	1	0	0
5	0	0	1	0	0	1	0	0
6	0	0	1	0	0	1	0	0
7	0	0	1	0	0	1	0	0
8	0	0	1	0	0	1	0	0

	Membership in a Set for Production plan 50							
obs.	D_g	D_d	D_i	TE	TI	TU	SE	SI
9	0	0	1	0	0	1	0	0
10	0	0	1	0	0	1	0	1
11	0	0	1	0	0	1	0	1
12	0	0	1	0	0	1	0	0
13	0	0	1	0	0	1	0	0
14	0	0	1	0	0	1	0	1
15	0	0	1	0	0	1	0	0
16	0	0	1	0	0	1	0	0
17	0	0	1	0	0	1	0	0
18	0	0	1	0	0	1	0	0
19	0	0	1	0	0	1	0	0
20	0	0	1	0	0	1	0	0
21	0	0	1	0	0	1	0	0
22	0	0	1	0	0	1	0	0
23	0	0	1	0	0	1	0	0
24	0	0	1	0	0	1	0	0
25	0	0	1	0	0	1	0	0
26	0	0	1	0	0	1	0	0
26	0	0	1	0	0	1	0	0
28	0	0	1	0	0	1	0	0
29	0	0	1	0	0	1	0	0
30	0	0	1	0	0	1	0	0
31	0	0	1	0	0	1	0	0
32	0	0	1	0	0	1	0	0
33	0	0	1	0	0	1	0	0
34	0	0	1	0	0	1	0	0

	Membership in a Set for Production plan 50							
obs.	D_g	D_d	D_i	TE	TI	TU	SE	SI
35	0	0	1	0	0	1	0	0
36	0	0	1	0	0	1	0	0
37	0	0	1	0	0	1	0	0
38	0	0	1	0	0	1	1	0
39	0	0	1	0	0	1	0	1
40	0	0	1	0	0	1	0	0
41	0	0	1	0	0	1	0	0
42	0	0	1	0	0	1	0	0
43	0	0	1	0	0	1	0	0
44	0	0	1	0	0	1	0	0
45	0	0	1	0	0	1	0	0
46	0	0	1	0	0	1	0	0
47	0	0	1	0	0	1	0	0
48	0	0	1	0	0	1	1	0
49	0	0	1	0	0	1	1	0

Table 4-14 shows the set memberships where the production plan at t=96 is the reference. For the 96th production plan, there are two instances where a previous production plan is in the set SE; the 64th and 76th production plans. The 93rd, 94th, and 95th production plans are all part of the set D_d indicating that these production plans are dominated by the reference production plan at t=96. This indicates that productive efficiency in increasing over the last few production plans.

Table 4-14: Membership in Sets for the 96th Production plan

	Membership in a Set for Production Plan 96							
Obs.	D_g	D_d	D_i	TE	TI	TU	SE	SI
51	0	0	1	0	0	1	0	0
52	0	0	1	0	0	1	0	0
53	0	0	1	0	0	1	0	0
54	0	0	1	0	0	1	0	0
55	0	0	1	0	0	1	0	0
56	0	0	1	0	0	1	0	0
57	0	0	1	0	0	1	0	0
58	0	0	1	0	0	1	0	0
59	0	0	1	0	0	1	0	0
60	0	0	1	0	0	1	0	0
61	0	0	1	0	0	1	0	0
62	0	0	1	0	0	1	0	0
63	0	0	1	0	0	1	0	0
64	0	0	1	0	0	1	1	0
65	0	0	1	0	0	1	0	0
66	0	0	1	0	0	1	0	0
67	0	0	1	0	0	1	0	0
68	0	0	1	0	0	1	0	0
69	0	0	1	0	0	1	0	0
70	0	0	1	0	0	1	0	0
71	0	0	1	0	0	1	0	0
72	0	0	1	0	0	1	0	0
73	0	0	1	0	0	1	0	0
74	0	0	1	0	0	1	0	0

	Membership in a Set for Production Plan 96							
Obs.	D_g	D_d	D_i	TE	TI	TU	SE	SI
75	0	0	1	0	0	1	0	0
76	0	0	1	0	0	1	1	0
77	0	0	1	0	0	1	0	0
78	0	0	1	0	0	1	0	0
79	0	0	1	0	0	1	0	0
80	0	0	1	0	0	1	0	0
81	0	0	1	0	0	1	0	0
82	0	0	1	0	0	1	0	0
83	0	0	1	0	0	1	0	0
84	0	0	1	0	0	1	0	0
85	0	0	1	0	0	1	0	0
86	0	0	1	0	0	1	0	0
87	0	0	1	0	0	1	0	0
88	0	0	1	0	0	1	0	0
89	0	0	1	0	0	1	0	0
90	0	0	1	0	0	1	0	0
91	0	0	1	0	0	1	0	0
92	0	0	1	0	0	1	0	0
93	0	1	0	0	0	1	0	0
94	0	1	0	0	0	1	0	0
95	0	1	0	0	0	1	0	0

In order to more fully describe the dominance based method the following tables present the inputs and outputs that are compared to determine set memberships. The undesirable outputs are not included in the determination of the memberships in the sets D_g, D_d, and D_i. Only the inputs that are part of the ordinal rankings along with all of the outputs are used to determine membership in the sets SE and SI. All inputs and outputs are used to determine membership in the sets TE, TI, and TU. Set memberships are first discussed for the 50th production plan followed by discussion of the 96th production plan. Table 4-15 and Table 4-16 present the data for the production plan at t=50. There are no production plans that are part of the sets D_g, D_d, TE, or TI.

Table 4-15. Outputs with Production Plan 50 as the Reference

Outputs with Production plan 50 as the Reference				
	Output	Undesirable Outputs		
		Recycled Outputs Scrap and Re-Work		Permitted Discharge Water
2	3.58	4.06	5.66	7.06
10	4.30	5.49	5.50	5.35
11	5.03	6.30	6.57	5.35
14	3.89	5.66	5.73	5.35
38	5.94	6.48	5.75	10.03
39	4.27	6.09	5.93	4.11
48	6.62	5.52	5.77	4.11
49	5.08	5.88	4.89	4.11
50	5.13	5.74	5.80	4.11

Production plans 2, 10, 11, and 14 are all classified as substitution dominated, SI. These production plans have inputs that are less than those for production plan 50, but also have product output that is less than that for production plan 50. As a result, production plans 2, 10, 11, and 14 are placed in the dominance indifferent sets D_i and TU. Production plans 2, 10 and 14 are part of the set SI since the Permitted Output is greater than that for production plan 50 while the other outputs are less than those for production plan 50. This is an unfavorable change in the output mix based on the ordinal ranking criteria for judging environmental performance. Production plan 11 is even more unfavorable in terms of the output mix compared to production plan 50. For production plan 11, both the Permitted Output and the Recycled Outputs are greater than those for production plan 50 while the product output is less.

Table 4-16. Inputs that are Part of Ordinal Preferences - Production Plan 50 Reference

Inputs with Production Plan 50 as the Reference											
	Non-Material Inputs		Recycled Inputs					Renewable Inputs		Non-Renewable Inputs	
	L. Hrs	S. Hrs	#1	#2	#3	#4	#5	#10	#12	#6	Power
2	3.54	4.09	4.60	4.38	4.69	4.62	2.80	5.67	4.94	5.41	5.43
10	5.81	6.75	4.82	5.45	4.62	4.50	3.84	5.67	4.94	5.96	3.49
11	5.45	6.75	4.82	5.45	4.62	4.50	3.84	5.67	4.94	5.96	3.49
14	5.84	3.44	4.60	4.55	4.58	4.82	4.68	5.67	4.94	5.74	9.63
38	6.91	7.36	5.96	6.22	6.58	6.55	6.66	5.67	4.94	5.27	5.73
39	6.16	7.36	5.96	6.22	6.58	6.55	6.66	5.67	4.94	5.27	5.73
48	6.97	7.35	5.84	5.66	5.92	5.83	6.27	5.67	6.97	7.42	2.71
49	6.91	7.35	6.45	7.25	6.67	8.03	7.21	5.67	7.54	6.14	16.33
50	6.59	7.26	6.45	7.25	6.67	8.03	7.21	5.67	7.54	6.14	19.55

Production plan 38 is in the set SE because the Non-Material inputs, Salary Hours and Direct Hours, are greater than those for production plan 50 while the other inputs are less than or equal to those for production plan 50. Production plan 39 is in the set SI because Product Outputs are less than those for production plan 50 while undesirable outputs are greater than or equal to those for production plan 50. Production plan 48 is in the set SE because Product Outputs are greater than those for production plan 50 while undesirable outputs are less than or equal to those for production plan 50. Production plan 49 is in the set SE because the Non-Material inputs, Salary Hours and Direct Hours, are greater than those for production plan 50 while the other inputs are less than or equal to those for production plan 50.

The inputs and outputs for the production plans that when compared to the production plan at t=96 are classified in sets other than dominance indifferent sets are shown in Table 4-17 and 4-18. Since some of the production plans are classified in the set D_g the inputs that are not part of the ordinal ranking are also compared and are shown in Table 4-19.

Table 4-17. Outputs with Production Plan 96 as the Reference

Outputs with Production Plan 96 as the Reference				
	Output	Undesirable Outputs		
		Recycled Outputs Scrap and Re-Work		Permitted Discharge Water
64	9.27	8.95	7.03	6.23
76	8.80	6.98	5.50	5.72
93	7.52	7.87	4.58	8.30
94	6.43	6.69	6.18	8.30
95	6.77	6.73	8.93	8.30
96	8.19	7.05	5.69	8.30

Table 4-18. Inputs that are Part of the Ordinal Preferences - Production Plan 96 Reference

Inputs with Production Plan 96 as the Reference											
	Non-Material Inputs		Recycled Inputs					Renewable Inputs		Non-Renewable Inputs	
	L. Hrs	S. Hrs	#1	#2	#3	#4	#5	#10	#12	#6	Power
64	7.60	3.90	5.68	6.26	5.73	6.39	6.49	5.67	6.98	5.37	6.04
76	7.29	6.97	8.39	6.82	7.45	8.92	6.83	5.67	6.45	4.82	6.45
93	7.85	6.24	7.07	6.56	5.31	7.75	6.46	10.51	7.02	7.00	6.97
94	7.76	8.23	7.07	6.56	5.31	7.75	6.46	10.51	7.02	7.00	6.89
95	8.18	8.23	7.07	6.56	5.31	7.75	6.46	10.51	7.02	7.00	6.77
96	7.70	4.94	7.07	6.56	5.31	7.75	6.46	10.51	7.02	7.00	6.77

Table 4-19. Inputs, Not Ranked, with Production Plan 96 as the Reference

	Inputs Not Part of the Ordinal Ranking						
	#7	**#8**	**#9**	**#11**	**#13**	**#14**	**#15**
93	6.92	8.04	6.16	7.8197	5.7602	4.3206	7.0828
94	6.92	8.04	6.16	7.8197	5.7602	4.3206	7.0828
95	6.92	8.04	6.16	7.8197	5.7602	4.3206	7.0828
96	6.92	8.04	6.16	7.8197	5.7602	4.3206	7.0828

Production plan 76 has a greater output than production plan 96 with all undesirable outputs less than those for Production plan 96. This puts production plan 76 in the set SE. Production plan 64 has the Output and Recycled Outputs greater than those for production plan 96 while the Permitted Output is less. This puts production plan 64 in the set SE.

The production plans 93, 94, and 95 are all dominated by production plan 96. Inspection of Table 4-17 shows that production plan 96 has more product than production plans 93, 94, and 95. The undesirable outputs are not considered in the partition of the production plans for the sets, D_d, D_g, and D_i. It can be seen that there are some undesirable outputs for the production plans 93, 94, and 95 that are greater than those for production plan 96. This is why these production plans are not also assigned to the set TE. All of the inputs for production plan 96 are less than or equal to (with at least one less than) those for the production plans 93, 94, and 95.

4.6.4 *Measurement of Euclidean Distance*

Where a production plan is not dominance indifferent (i.e., is a member of the set D_g, D_d, TE, or TI) or is a member of the set SE or SI the magnitude of the difference between that production plan and the reference production plan is estimated by the euclidean distance. The distance to the reference production plan (production plans at 50 and 96) are given for inputs, desirable outputs, and undesirable outputs in Table 4-20 and Table 4-21.

As would be expected the production plans that are closest to the 50th production plan in the time series tend to have shorter distance since there is some collinearity in the data. While there is an overall trend for the input distance to decrease (indicating decreasing inputs) there is no clear trend over these production plans in Table 4-20 for the output or for undesirable outputs. For production plans 38 and 39 it is clear that there is a change in the undesirable outputs that has resulted in production plan 38 being SE while the next production plan, 39, is SI. For production plans 48 and 49 there is change occurring in the input mix since there is a significant change from 48 to 49 in terms of the euclidean distance.

The distance measures in Table 4-21 indicate that the production plans at times 64 and 76 differ significantly in inputs from the reference production plan of 96. This is consistent with the criteria for assignment to the set SE that requires significant variation on the input mix. The distance for the Output does not vary a lot for the production plans in Table 4-21. Undesirable outputs do vary from the reference production plan at time t=96.

Distance measures are useful in quantifying how production plans differ rather than just their membership in a set. It is clear from the variations in the distance measures that production plans that are being assigned to the same set differ significantly in their input or output mix. Manufacture Inc., like any manufacturing facility, must often contend with varying demand for its product. The product produced by Manufacture Inc. is also constantly changing is terms of its requirements. Many small lots of printed circuit boards are produced to customer specification adding additional variability to inputs and outputs. Therefore, from one week to the next inputs and outputs can vary. Given the small number of production plans that are being placed in the defined sets it appears that Manufacture Inc., is in a period of fairly stable production but with significant week to week variation.

Table 4-20. Euclidean Distance to Production Plan 50

50th Production plan Reference		**Euclidean Distance**		
Production plan	**Set Membership**	**Input**	**Output**	**Undesirable Output**
2	SI	17.2977	1.5464	3.40392
10	SI	18.2935	0.8283	1.30789
11	SI	18.3124	0.0997	1.56444
14	SI	13.0865	1.2379	1.2504
38	SE	15.5019	0.8135	5.96507
39	SI	15.5042	0.8545	0.366434
48	SE	17.734	1.494	0.220386
49	SE	3.23526	0.0463	0.91738

Table 4-21. Euclidean Distance to Production Plan 96

96th Production plan Reference		**Euclidean Distance**		
Production plan	**Set Membership**	**Input**	**Output**	**Undesirable Output**
64	SE	8.04069	1.0752	3.11119
76	SE	7.24044	0.6119	2.58237
93	D_d	1.3214	0.6744	1.38273
94	D_d	3.28628	1.7612	0.606223
95	D_d	3.31818	1.4215	3.25785

4.6.5 *Measurement of Average Euclidean Distance*

The average Euclidean distance provides a means of summarizing data and also filtering-out some of the normal variation that occurs with almost any production process. This average distance may be calculated for the reference production plan to all production plans in some set such as D_g, D_d, TE, TI, SE, and SI. Since there are only a small number of production plans that are ever assigned to the sets D_g or D_d an average distance is not useful. There are no production plans assigned to the sets TE or TI so an average distance cannot be calculated. In the case of the sets SE and SI there are instances where a fairly large number of production plans are assigned. An average distance does provide a useful summary as well as another means of discerning whether of not there is a tread or real change occurring.

The example that will be used is the set SE for the production plans at 96 (2 previous production plans in set SE), 97 (27 previous production plans in set SE), 98 (20 previous production plans in set SE), and 99 (10 previous production plans in set SE). Table 4-22 shows the distances to each of the production plans that are part of the set SE along with the average. The results do not show a trend. The output value for production plan 96 is low compared to the other production plans, but there are only 2 production plans in the set SE so definite conclusions can not be made.

Table 4-22. Average Distance from Reference to Production plans in Set SE

Production Plan (Count)	**Average for Inputs**	**Average for Output**	**Average for Undesirable Outputs**
96 (2)	7.640565	0.84355	2.84678
97 (27)	10.8774	2.899326	3.484044
98 (20)	9.75577	2.13284	2.827032
99 (10)	10.05604	1.99207	2.546853

4.6.6 *Results from the Materials Input Only Data Set*

Any production plan that is classed in a set D_g, D_d, TE, or TI with non-material inputs (i.e., salary hours and direct hours) will in almost all cases also be part of these sets with the non-material inputs removed. The only exception will occur if all of the inputs, except non-material inputs, are equal and the outputs are equal. In this case, it is the non-material inputs that are the deciding factor in placing a production plan in a set. When these inputs (labor hours and salary hours) are removed it is more likely that a production plan will be moved from the set D_i or TU to the set D_g, D_d, TE, or TI.

Table 4-23 and Table 4-24 show how the counts differ where non-material inputs are excluded for the first 50 and the second 50 production plans. The number in each cell represents the additional production plans that are placed in the set when non-material inputs are removed. For the data analyzed there were no instances where the elimination of the non-material inputs resulted in the removal of a production plan from the sets D_d or D_g.

Table 4-23. Additional Set Counts for the First 50 Production plans

t	D_g	D_d
1	0	0
2	1	0
3	0	0
4	0	0
5	1	0
6	0	0
7	0	1
8	0	2
9	0	1

t	D_g	D_d
10	0	0
11	0	0
12	0	2
13	3	0
14	0	0
15	0	1
16	0	1
17	0	2
18	0	0
19	1	0
20	0	0
21	1	0
22	0	1
23	1	0
24	0	0
25	0	0
26	0	0
26	0	0
28	0	1
29	0	2
30	0	1
31	0	2
32	1	0
33	0	0
34	0	2
35	0	3
36	0	0
37	0	1

t	D_g	D_d
38	0	0
39	1	0
40	0	0
41	0	0
42	0	2
43	0	2
44	0	3
45	0	0
46	0	1
47	0	1
48	0	3
49	0	0
50	0	0

Table 4-24. Additional Set Counts for the Second 50 Production plans

t	D_g	D_d
51	0	0
52	0	1
53	0	0
54	0	0
55	0	1
56	0	0
57	0	1
58	0	0
59	0	0

t	D_g	D_d
60	0	0
61	0	1
62	0	0
63	0	0
64	0	2
65	1	0
66	0	0
67	0	0
68	0	1
69	0	2
70	1	0
71	0	0
72	1	0
73	0	0
74	0	0
75	0	0
76	0	1
77	2	0
78	2	0
79	0	0
80	0	0
81	0	0
82	1	0
83	1	1
84	0	0
85	0	0
86	0	0
87	0	0

t	D_g	D_d
88	0	0
89	0	0
90	1	1
91	1	0
92	0	0
93	0	1
94	0	1
95	0	2
96	0	1
97	0	0
98	0	0
99	0	0
100	0	1

For the first 50 production plans shown in Table 4-23, additional production plans assigned to set D_g are close in the time series to additional production plans assigned to set D_d. This indicates significant week to week variability in the production process. However, starting at about production plans 38 there does appear to be a trend towards increasing or sustained membership in the set D_d. The indication is that there may be an improvement in terms of productive efficiency related to material inputs. For the second 50 production plans shown in Table 4-24, there are again instances where there are additional production plans assigned to sets D_d and D_g that are close in the time series. There does appear to be a bit of trend towards the end of the time series for additional production plans to be assigned to the set D_d indicating increased productive efficiency. Note that these set counts are with the production plan at time t as the reference. Membership in the set D_d indicates previous production plans that are dominated by the production plan at time t. While membership in the set D_g indicates previous production plans that dominate the production plan at time t.

4.7 Comparison of Standard Methods to the Proposed Method

The results from the method developed as part of this research differs substantially from standard methods. The existing method that is most similar is the Benchmark Correspondence method which also compares production plans directly rather than using a mathematically defined frontier as the reference. However, the Benchmark Correspondence method places all of the production plans in the data set into the dominance indifferent set. This means that the method did not detect any progress or regress in the production plans over time and, therefore, did not produce any data that may be used for decision making.

The standard methods of CCR, BCC, and FDH did produce results. However, these methods are not consistent among themselves in terms of the relative efficiency of production plans. Conclusions drawn concerning the productive efficiency of Manufacture Inc. may very well differ depending upon which of these three methods is applied. In the case of the BCC method the slacks produced by the linear programming solution procedure are also reported in Appendix C and provide more detailed information on the reasons for inefficiency. However, all three of these standard methods require relatively small numbers of inputs or outputs to produce meaningful results. It was necessary to aggregate the 15 material inputs into a single material input. As a result conclusions that can be drawn about the operational level of a manufacturing facility are limited.

There are some general conclusions that can be drawn from the results produced by the CCR, BCC, and FDH methods. These are:

- The production process goes through periodic episodes of relatively less efficiency.

- There are five periods when the CCR and BCC (and sometimes the FDH method as well) show inefficiency to be occurring. These are production plans 23-27, 30-33, 36-38, 55-63, and 77-100.

Given the limitations of these methods when inefficiency occurs identifying any causal factors in the production process is difficult.

The proposed method does generate more information on operational level behavior than standard methods. Since inputs do not have to be aggregated as much for the proposed method there is the possibility of, in some cases, identifying sources of inefficiency. And, the proposed method does more explicitly consider undesirable outputs so that performance in reducing these outputs may be tracked.

The standard methods are further compared for the production plans 50 and 96. For production plan 50 the FDH and the BCC methods give an efficiency of 1.0. This means that the production plan is on the production frontier and is as efficient as possible. The CCR method provides an efficiency of 0.779 indicating that improvements can be made in this production plan. The proposed method assigns all previous production plans 1 through 49 to the sets D_i and TU when compared to the production plan 50. So, like the BCC and the FDH methods, the proposed method indicates this production plan is “efficient.” Additional information can be obtained from the CCR linear programming formulation as slacks. These slacks for production plan 50 are shown in Table 4-25. The slacks indicate that Power is the primary cause of inefficiency given the large slack value. A review of the data in Table 4-16 shows that indeed the power usage for production plan 50 is unusually high. In the proposed method a single input or output that is high will not necessarily be detected as inefficiency since the dominance criteria compare all inputs and outputs. However, this kind of information was detected as part of the statistical analysis reported in Section 4.4.

For production plan 96 the FDH method has an efficiency score of 1.0. However, the CCR and the BCC methods have efficiency scores of 0.9114 and 0.92, respectively. This indicates some inefficiency for production plan 96. The proposed methods shows three previous production plans at t=93, 94 and 95 that are placed in the set D_d . When the inputs of labor hours and salary hours are removed the production plan at t=92 is also added to the set D_d by the proposed method. This indicates that these production plans are less efficient than production plan 96. Referring to Table 4-11, the four previous production plans are all indicated to be less efficient than production plan 96 by the CCR, BCC, and FDH methods. The results from the proposed method are generally consistent with standard methods. The selection of the most efficient production plans may not be the same for the proposed method and standard methods; but, then the CCR, BCC, and FDH methods are not consistent.

Table 4-25. Slacks for 50th Production Plan Using CCR Method

Direct Hrs	Salary Hrs	Material	Power	Rework	Scrap	Water Use
0	0.701	1.011	12.908	0.271	0	0

Additional information may be obtained from the slacks. These are reported for the BCC and CCR method in Table 4-26. Referring to Table 4-17 and Table 4-18 it can be seen that for production plan 96 Labor Hours (7.7) and Water Use (6.77) are greater than the average value of 6.22 as shown in Table 4-4 and Table 4-5.

Production plan 96 did have three previous production plans (93, 94, 95) placed in the set D_d so that distance measures are calculated. The greater the distance measure the greater the difference between the reference production plan at 96 and the production plans that are dominated. Based on these distance measures (refer to Table 4-21) production plans can be ordered by their relative performance. For inputs, production plan 93 has the better performance with production plans 94 and 95 about the same. For outputs, production plan 93 has the better performance with production plans 94 and 95 about the same. For undesirable outputs, production plan 95 has the worst performance followed by production plan 93 and then production plan 94. The standard methods of CCR, FDH, and BCC all show production plan 93 as being the highest productive efficiency of the three production plans. The standard methods differ on which of the two production plans 94 and 95 is next in productive efficiency. These results are consistent with the proposed method results.

Table 4-26. Slacks for 96th Production Plan Using CCR and BCC Methods

	Labor Hrs	Salary Hrs	Material	Power	Rework	Scrap	Water Use
BCC	2.13	0.00	0.42	0.11	0.00	0.00	1.79
CCR	2.05	0	0.27	0	0	0	1.68

The proposed method also assigns production plans to the sets SE or SI if the input/output mix changes in a way that is environmentally desirable or undesirable. This sort of analysis is not performed by the standard methods that have been applied. The proposed method in general does provide more detailed information. In combination with statistical analysis the proposed method could potentially be used to make operational level decisions. Possible extensions to the proposed method are discussed in Chapter 5.