

# A Multistage Stochastic Programming Approach to the Optimal Surveillance and Control of the Emerald Ash Borer in Cities

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**Abstract.** Emerald ash borer (EAB), a wood-boring insect native to Asia and invading North America, has killed untold millions of high-value ash trees that shade streets, homes, and parks and caused significant economic damage in cities of the United States. Local actions to reduce damage include surveillance to find EAB and control to slow its spread. We present a multistage stochastic mixed-integer programming (M-SMIP) model for the optimization of surveillance, treatment, and removal of ash trees in cities. Decision-dependent uncertainty is modeled by representing surveillance decisions and the realizations of the uncertain infestation parameter contingent on surveillance as branches in the M-SMIP scenario tree. The objective is to allocate resources to surveillance and control over space and time to maximize public benefits. We develop a new cutting-plane algorithm to strengthen the M-SMIP formulation and facilitate an optimal solution. We calibrate and validate our model of ash dynamics using seven years of observational data and apply the optimization model to a possible infestation in Burnsville, Minnesota. Proposed cutting planes improve the solution time by an average of seven times over solving the original M-SMIP model without cutting planes. Our comparative analysis shows that the M-SMIP model outperforms six different heuristic approaches proposed for the management of EAB. Results from optimally solving our M-SMIP model imply that under a belief of infestation, it is critical to apply surveillance immediately to locate EAB and then prioritize treatment of minimally infested trees followed by removal of highly infested trees. **Summary of Contributions:** Emerald ash borer (EAB) is one of the most damaging invasive species ever to reach the United States, damaging millions of ash trees. Much of the economic impact of EAB occurs in cities, where high-value ash trees grow in abundance along streets and in yards and parks. This paper addresses the joint optimization of surveillance and control of the emerald ash borer invasion, which is a novel application for the INFORMS society because, to our knowledge, this specific problem of EAB management has not been published before in any OR/MS journals. We develop a new multi-stage stochastic mixed-integer programming (MSS-MIP) formulation, and we apply our model to surveillance and control of EAB in cities. Our MSS-MIP model aims to help city managers maximize the net benefits of their healthy ash trees by determining the optimal timing and target population for surveying, treating, and removing infested ash trees while taking into account the spatio-temporal stochastic growth of the EAB infestation. We develop a new cutting plane methodology motivated by our problem, which could also be applied to other stochastic MIPs. Our cutting plane approach provides significant computational benefit in solving the problem. Specifically, proposed cutting planes improve the solution time by an average of seven times over solving the original M-SMIP model without cutting planes. We calibrate and validate our model using seven years of ash infestation observations in forests near Toledo, Ohio. We then apply our model to an urban forest in Burnsville, Minnesota, that is threatened by EAB. Our results provide insights into the optimal timing and location of EAB surveillance and control strategies.

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## 1. Introduction

We present a new multistage stochastic programming model to address an important environmental problem—the surveillance and control of invasive species—and we develop effective solution algorithms to tackle the computational difficulty of this highly challenging optimization problem. In this section, we first provide motivation and background regarding the problem class of interest, discuss related prior work, and provide a summary of our key contributions and results.

### 1.1. Motivation and Background

Invasive species are nonnative species whose introduction does or is likely to cause economic or environmental harm or harm to human health (National Invasive Species Council 2016). Invasive species can adversely affect agricultural (Office of Technology Assessment 1993), aquatic (Lovell et al. 2006), forested (Aukema et al. 2011), and rangeland (Duncan et al. 2004) ecosystems. Every year, the damage of invasive species costs governments, industries, and private citizens billions of dollars (Pimentel et al. 2005). For example, nonnative wood- and phloem-boring insects that damage urban forests in the United States caused nearly \$1.7 billion in local government expenditures and approximately \$830 million in lost residential property values each year (Aukema et al. 2011). Many other effects of invasive species are not easily monetized (e.g., increased disease spread and water pollution) yet have profound impacts on human well-being (Pejchar and Mooney 2009).

Measures to address the adverse effects of invasive species depend on the stage of the invasion and include prevention (keep invasive species from entering a new ecosystem), surveillance (find established populations of invaders), and control (minimize their spread and adverse effects). In general, the invasive species management problem is to allocate resources among different measures (prevention, surveillance, and control) over space and time, with the objective of minimizing the economic and environmental damage, as well as the cost of management. Optimization models have been developed for many aspects of this general problem (for a detailed discussion of such models, see the review by Büyüktaktakın and Haight 2018).

Most former optimization studies in invasive species management have been published in domain journals. Those studies focus on a specific application to generate managerial insights and provide simplistic models from an optimization perspective (see the reviews by Billionnet 2013 and Büyüktaktakın and Haight 2018). Some of the pioneering work in the area has presented linear programming (LP) models that are convex by nature and thus could easily be solved by LP solvers (Hof 1998, Hof and Bevers 2000). There are very few studies that present more complex

nonconvex formulations and their analysis in operations research/management science journals (Büyüktaktakın et al. 2014, Kibiş and Büyüktaktakın 2017), yet those studies are deterministic and are only limited to control measures. Consequently, research from an operations research perspective is needed to formulate realistic models that involve multiple dimensions of invasive species management, combine surveillance and control measures, and develop algorithms to tackle the computational difficulty of such complex models.

In this study, we address the cost-effective allocation of resources to invasive species management from a complex stochastic optimization point of view. Our model handles the biological, economic, and stochastic components of invasive species management in one mathematical formulation. In particular, we present a multistage stochastic mixed-integer programming (M-SMIP) formulation to study the joint optimization of surveillance and control decisions associated with invasive species management. The high degree of nonlinearity of the model is avoided by embedding surveillance decisions into the stochastic scenario tree and further linearizing the nonlinear maximum infestation capacity constraints. We develop new cutting planes to solve the M-SMIP formulation and obtain an optimal solution. Use of the stochastic optimization approach is demonstrated on a realistic case study that is supported by field-based observational data.

In invasive species management, surveillance measures include placing detection devices or inspecting plots to locate newly established populations and track their spread. Models in the surveillance domain concentrate on the optimal location and intensity of surveillance measures assuming that the location of the invader is unknown and that control measures are automatically applied to eradicate or slow the spread of found populations (Mehta et al. 2007, Bogich et al. 2008, Baxter and Possingham 2011, Homans and Horie 2011, Epanchin-Niell et al. 2012). Control measures include mechanically removing individuals of the species or its host, applying chemical treatments (biocides or toxicants), and using biological control (use of other living organisms to suppress invasive species). Optimization models in the control domain assume that the location of an invader is known and address the question of where, when, and how intensively control measures should be applied (Hof 1998, Blackwood et al. 2010, Büyüktaktakın et al. 2011, Epanchin-Niell and Wilen 2012, Kovacs et al. 2014, Büyüktaktakın et al. 2015, Kibiş and Büyüktaktakın 2017). Only a few models address the joint optimization of surveillance and control measures (Horie et al. 2013, Yemshanov et al. 2017). They are limited to short (two-period) time horizons

and have not fully captured the biological complexity of the problem, including the classification of the host population with respect to health conditions and the transition of one disease class into another in a multistage stochastic program as we propose in this study. Former studies on surveillance and control have mainly aimed to obtain general inferences about management strategies given a specific application while keeping the mathematical model as simple as possible. The search and control study of Onal et al. (2019) presents an integrated simulation-optimization framework that simulates the growth of the invader and uses it as an input into the optimization model. The authors simulate growth through multiple years over a landscape similar to discrete reaction-diffusion models (Kibiş and Büyüktaktın 2019). However, their study is limited in that the optimization model is solved using a rolling-horizon fashion after obtaining each run of the simulation model.

We address the limitations mentioned earlier in an M-SMIP model of invasive species surveillance and control and demonstrate the use of our M-SMIP model on the realistic case of emerald ash borer (EAB) management. EAB is one of the most damaging invasive species ever to reach the United States. EAB is a wood-boring insect, native to Asia, that was accidentally introduced into North America in the early 1990s (Herms and McCullough 2014). Although the pathway and vector for introduction are unknown, EAB was probably imported via crating, pallets, or dunnage made from infested ash wood. Since its discovery near Detroit in 2002, EAB has spread to 30 states and three Canadian provinces (as of March 1, 2018) and killed untold millions of ash trees. It is increasingly likely that EAB will functionally extirpate one of North America's most widely distributed tree genera, with devastating economic and ecological impacts (Haight et al. (2009), Herms and McCullough 2014).

Much of the economic impact of EAB occurs in cities, where high-value ash trees were planted in abundance along streets and in yards and parks during the last five decades (Poland and McCullough 2006). Kovacs et al. (2010) modeled the spread of EAB in the United States from 2009 to 2019 and estimated that the discounted cost to homeowners and local governments of treating or removing the affected urban trees would be \$10.7 billion and twice that if ash in adjacent suburban communities were included. To slow ash mortality and reduce damage, city governments are developing EAB management plans, including surveillance to discover the location of infestations in early stages, treatment of ash trees with systemic insecticides to kill any EAB adults or larvae present, and preemptive removal of infested ash trees to reduce population growth and spread (Herms and McCullough 2014).

In the invasive species literature, “prevention” usually refers to activities that take place at a regional, national, or international scale that prevent the movement and establishment of an invasive species into a new area (Büyüktaktın and Haight 2018). Prevention activities include quarantines against the movement of materials that might harbor the species, inspection of shipments at ports of entry, treatment of shipments (e.g., fumigation), and so on. The main methods of preventing the spread of EAB are preventing people from accidentally moving EAB around. Quarantines (implemented at the national or state level) make it against the law to move ash wood or logs that have not been treated to kill EAB larvae. Awareness campaigns (implemented at national, state, or local levels) educate people about the hazards of moving potentially infested ash firewood. Although prevention is important, those prevention activities are beyond the scope of this paper. The types of actions (surveillance and treatment) considered in our paper are completely controlled at the local level and would occur once prevention has failed and EAB has arrived in a city. However, because we are concerned about slowing the spread of an established invader into new neighborhoods of a city, we could count surveillance and control activities as preventing the spread of invaders within a city and to potential neighbor cities.

The objective of our M-SMIP model is to maximize the benefits of healthy ash trees in a city by determining the optimal surveillance, treatment, and removal of ash over space, time, and tree infestation level with uncertainty about infestation growth and constraints on the budget. Within each landscape unit, we model the population dynamics of ash using a stage-structured formulation in which trees are classified by the infestation level. The level of tree infestation is important because it affects the visibility of the infestation, the spread of EAB to neighboring trees, and the efficacy of insecticide treatment. Trees move between classes based on infestation growth and ash treatment and removal decisions. We handle uncertainty in infestation growth with a scenario tree where each stage corresponds to a time period and growth is modeled using scenarios with certain probabilities. Furthermore, surveillance decisions are integrated into uncertain infestation growth scenarios to form a hybrid scenario tree with two types of decisions at each node: whether surveillance is applied and the number of trees that are treated and removed at each infestation level. At the beginning of each stage, we assume that the decision maker has a belief about the number of infested trees, which may be updated with surveillance. If surveillance is applied, the decision maker may then select the number of trees to treat and remove in each infestation level based on the actual numbers of infested trees that are found with surveillance. Therefore, our model differs from typical

stochastic programming models by requiring surveillance to find the actual values of the uncertain variables before making control decisions.

## 1.2. M-SMIP Model and Decision-Dependent Uncertainty

M-SMIP models are among the most difficult to solve. Although there are many solution strategies for two-stage stochastic integer programs, the progress on multistage stochastic integer programming and, in particular, mixed-integer programming is limited (Ahmed, 2010, Birge and Louveaux 2011, van Ackooij et al. 2017). The core difficulty with the M-SMIP model is the presence of both integer and continuous variables, which leads to a nonconvex and non-continuous feasible region that is not amenable to direct decomposition.

Most solution approaches to M-SMIP models consider the extensive form of the problem and then relax the coupling constraints to decompose it into multiple scenario-based subproblems. For example, Carøe and Schultz (1999) describe a dual-scenario decomposition where the nonanticipative constraints are subjected to Lagrangian relaxation. The resulting Lagrangian dual contains a separable minimization, which reduces to solving several single-scenario linear integer problems. The solution of the dual provides a lower bound for the original primal problem, and heuristic methods are used to obtain upper bounds from the dual solution. The procedure is embedded in a branch-and-bound scheme, which is finite if all decision variables are discrete. Decomposition algorithms, such as Lagrangian and Dantzig–Wolfe decompositions, have been used successfully to solve various classes of multistage stochastic integer problems (Nowak and Römisich 2000, Römisich and Schultz 2001, Ahmed et al. 2003, Birge and Louveaux 2011). Other decomposition algorithms for solving M-SMIP problems include column generation (Lulli and Sen 2004), nested Benders decomposition (Parpas and Rustem 2007) and stochastic dual dynamic programming (SDDP; Heitsch and Römisich 2003, Shapiro 2011), and decomposition-based heuristics such as progressive hedging (Watson and Woodruff 2011, Gul et al. 2015). Zou et al. (2019) propose a stochastic dual dynamic integer programming (SDDiP) algorithm and Lagrangian cuts for solving M-SMIP problems with binary variables. The authors report encouraging results for solving M-SMIP problems using the SDDiP approach. Other approaches on multistage stochastic programming focus on stochastic bounding and sampling (Norkin et al. 1998, Kleywegt et al. 2002, Shapiro 2003).

Cutting-plane algorithms for multistage stochastic integer programming involve Benders cuts (Sherali and Fraticelli 2002), specialized branch-and-cut algorithms (Sen and Sherali 2006, Luedtke 2014), and

stochastic extensions of formerly defined inequalities such as knapsack covers, tree inequalities, and lift-and-project cuts (Carøe and Tind 1997, Guan et al. 2009, Jiang et al. 2016) to solve a deterministic equivalent of the multistage stochastic integer or mixed-integer programs. Former literature on cutting-plane approaches for M-SMIP problems usually extend cutting planes proposed for the deterministic mixed-integer programming problem and the convex linear relaxation of the stochastic problem to the case of M-SMIP problems (Guan et al. 2009). Thus, the development of new cutting planes for M-SMIP problems is a growing research area.

Typical stochastic programs model exogenous uncertainty or decision-independent uncertainty, where realizations of the uncertain parameters in the scenario tree do not depend on decisions. Recently, stochastic programs with endogenous uncertainty or decision-dependent uncertainty, in which optimization decisions determine the times when the uncertainties in some of the parameters will be resolved, have received increased attention (Jonsbråten et al. 1998, Goel and Grossmann 2006, Boland et al. 2008, Solak et al. 2010).

Our M-SMIP model incorporates endogenous uncertainty in which the realization of random parameters is contingent on some of the decision variables (e.g., surveillance actions). Our study is different than earlier studies on decision-dependent uncertainty in that we represent surveillance decisions and the corresponding uncertain parameter realizations as binary branches in a multistage stochastic scenario tree. To avoid the nonlinearity in the formulation, we embed the binary surveillance decisions into the scenario tree rather than explicitly representing them as decision variables in the formulation. Because the proposed M-SMIP model is distinguished from earlier studies by its computational complexity, we present new cutting-plane approaches that take advantage of the stage structure inherent in the formulation. Our computational results have proven that those cutting planes are quite effective in solving the proposed model and have the potential to be useful for other similar M-SMIP problems.

## 1.3. Key Contributions and Results

The key contributions of our study to theory and application are as follows. The main methodological contribution of our paper is to provide a new M-SMIP formulation and scenario tree representation for endogenous stochastic programming models where realization of the uncertain parameter depends on decision variables in the optimization model. Traditional multistage stochastic programming models assume exogenous uncertainty where the actual values of random variables are determined at the end of each stage. Our stochastic programming scenario tree is

different by requiring a surveillance action to determine the actual values of the random variables during each stage. If surveillance is not applied, then random variables are set to their expectations. Different from the earlier work on modeling decision-dependent uncertainty in stochastic programs, we represent surveillance decisions and realization of the random parameter as binary branches in a multistage stochastic scenario tree. The representation of surveillance decisions as branches in the tree also helps to avoid the high-degree nonlinearities in the mathematical model.

Another core methodological contribution of this paper is that we develop a new cutting-plane methodology that also can be applied to other similar M-SMIP models. The solution algorithm receives the values of the complicating binary variables by solving a preprocessing model under the worst-case scenario. Using logical arguments, the values of some binary variables are then fed into the original M-SMIP model. Our cuts significantly reduce the solution time and thus improve the solvability of the original M-SMIP model.

The main application contribution of our paper is to present the first M-SMIP formulation in the invasive species literature. This M-SMIP model jointly optimizes surveillance and control actions over space and time with uncertainty about infestation spread. The model incorporates infestation scenarios with corresponding probabilities to find the best surveillance and control strategy. In contrast to former studies, surveillance tracks actual infestation levels but does not imply control. Another distinguishing aspect of this study is that control measures are contingent on the outcome of surveillance and are undertaken only if they are cost-effective and within budget.

Proposed formulation is also different in that we formulate a stage-structured model that captures the dynamics of the host population with different levels of the EAB infestation. Each stage represents a cluster of ash tree population with respect to its health condition, as opposed to defining it as the age of an invasive plant (Büyüktaşkın et al. 2015). Our formulation allows the detectability of an infestation and the efficacy of treatment to depend on the level of host infestation. These features are important for EAB management because EAB may infest host trees for one or more years before the damage is visible, and chemical treatment is effective only for trees with low to moderate levels of infestation. To our knowledge, such a complex stochastic optimization model that tracks the host population with its health stages at a spatial and temporal level has not been proposed before in the invasive species and EAB management literature (Kovacs et al. 2014, Büyüktaşkın and Haight 2018).

We calibrate and validate our model of ash tree dynamics using seven years of EAB infestation observations in northern Ohio. We then apply the model to a possible infestation in the city of Burnsville, Minnesota, to provide insights into optimal EAB surveillance and control strategies.

Our approach leads to several important results as summarized. First, our new cutting-plane algorithm improves the solution time by an average of seven times over all instances solved compared with solving the original M-SMIP model without the cutting planes. The improvement in the solution time becomes apparent as the size of the problem increases. In particular, whereas the original model cannot be solved for a six-by-six gridded landscape in a four-year time horizon, the algorithm enables us to solve the model for a six-by-six gridded landscape in a five-year time horizon under various budgets.

Second, the model provides valuable rules of thumb for surveillance and control of an EAB infestation. In almost all infestation and budget scenarios, it was never optimal to delay surveillance. Thus, under a belief of possible infestation, it is critical to apply surveillance immediately and follow up with the treatment of the first- and second-level infested trees and removal of third-level infested and dead trees. In addition, surveillance and treatment or removal actions should mainly focus on the spatial locations where the infestation has started. Also, treatment priority is given to trees with second-level (midlevel) infestation, followed by trees with first-level infestation. Remaining funds are used to remove third-level (highly) infested trees, followed by dead trees. Specifically, if the budget is not sufficient, the decision maker may need to let some highly infested trees die in favor of saving low- and midlevel infested trees and preventing new infestations.

Third, even though it is possible to derive general inferences based on model results, determination of the precise locations, timing, and amount of the surveyed, treated, and removed trees requires the use of a complex mathematical model that integrates multiple dimensions of the EAB infestation and its uncertainty into a stochastic optimization formulation.

Fourth, a comparative analysis of our M-SMIP model with six different heuristics highlights that significant benefits are obtained by using the complex M-SMIP model as opposed to the simple approaches currently used by the cities of Minneapolis and St. Paul for the management of EAB. Those results also emphasize the importance of surveillance before using treatment or removal to maximize benefits from ash trees and reduce management costs.

The structure of the remaining paper is as follows. In Section 2, we define the problem and present the M-SMIP model. In Section 3, we present new cutting

planes for the M-SMIP model. In Section 4, we discuss model calibration and validation based on infestation observations in Ohio, and we present the model application to a possible case of EAB invasion in the city of Burnsville, Minnesota. Section 5 follows with computational results that illustrate the efficiency of the solution algorithm and provide management implications. Section 6 presents a discussion and offers directions for future research.

## 2. Optimization Model

### 2.1. Problem Definition

Suppose that many large and stately trees—including ash trees—are an important part of our city. We need a plan for surveillance of the ash population and control of EAB if it is present to maximize the benefits of healthy ash trees while staying within an available city budget.

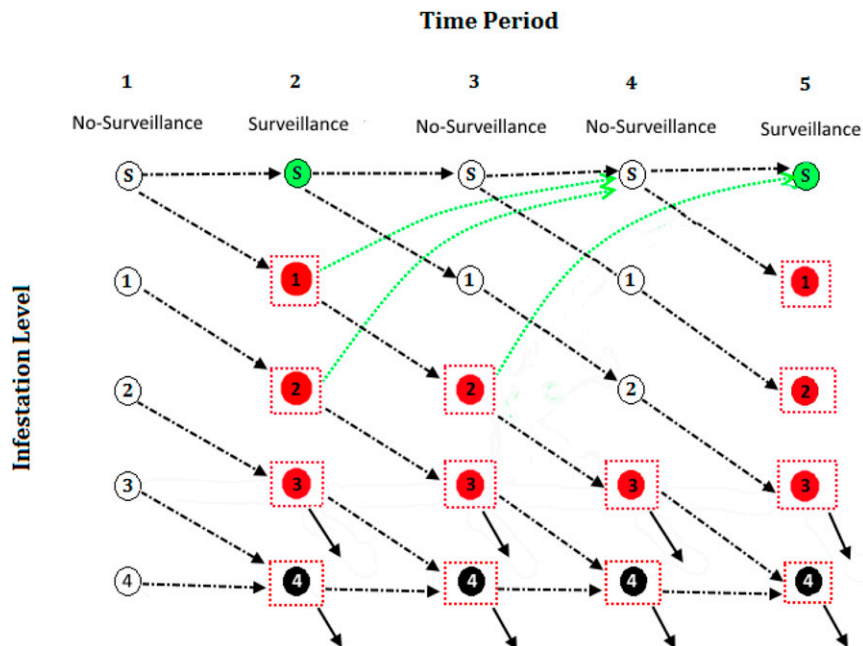
To address this problem, we created a stage-structured model of the ash population dynamics. We first divide the city into neighborhoods and further divide the ash trees in each neighborhood into two categories: susceptible trees that are prone to infestation and infested trees. Infested trees are further divided into four classes based on their infestation levels. The population of trees in a neighborhood is fully described by the numbers of trees in the susceptible and infested tree classes. Trees in each infestation class yield EAB adults that may infest nearby susceptible trees, and then the trees transition to the next infestation level in the following period until reaching the maximum infestation level when they die. The model

is stochastic because the number of newly infested trees is a random variable that depends on the total number of EAB adults produced in the neighborhood and adjacent neighborhoods.

Management actions vary by tree infestation class. The two lowest levels of infestation are not visible without surveillance, which involves inspection for larvae inside the tree’s bark. Trees in these classes that are found infested may be treated with an insecticide that kills the infestation and prevents further infestation for two years. Trees in the third level have visible damage and are irreversibly infested. Those trees may be removed to reduce EAB spread. Trees in the fourth level are assumed to be dead and must be removed because of their hazard. Without surveillance, neither treatment nor removal is applied because infested trees are not found without inspection. Thus, there is only one strategy in the case of no surveillance: no action. In the case of surveillance, there are three possible strategies: (1) no action, (2) treatment, and (3) removal.

We represent the progression of an EAB infestation over five years by the network in Figure 1, where S represents susceptible (healthy) trees, and 1, 2, 3, and 4 correspond to tree infestation levels. Susceptible class in periods two and five represent the actual numbers of susceptible trees; infestation classes 1, 2, and 3 with dotted squares represent the actual numbers of infested trees in the corresponding infestation levels for which treatment or removal can be applied; infestation class 4 with dotted squares represent the level-four infested trees, which are dead

**Figure 1.** (Color online) An Illustrative Example of a Possible Transition of EAB Infestation Classes Among Ash Trees Over Five Years Under a Specific Surveillance Regime

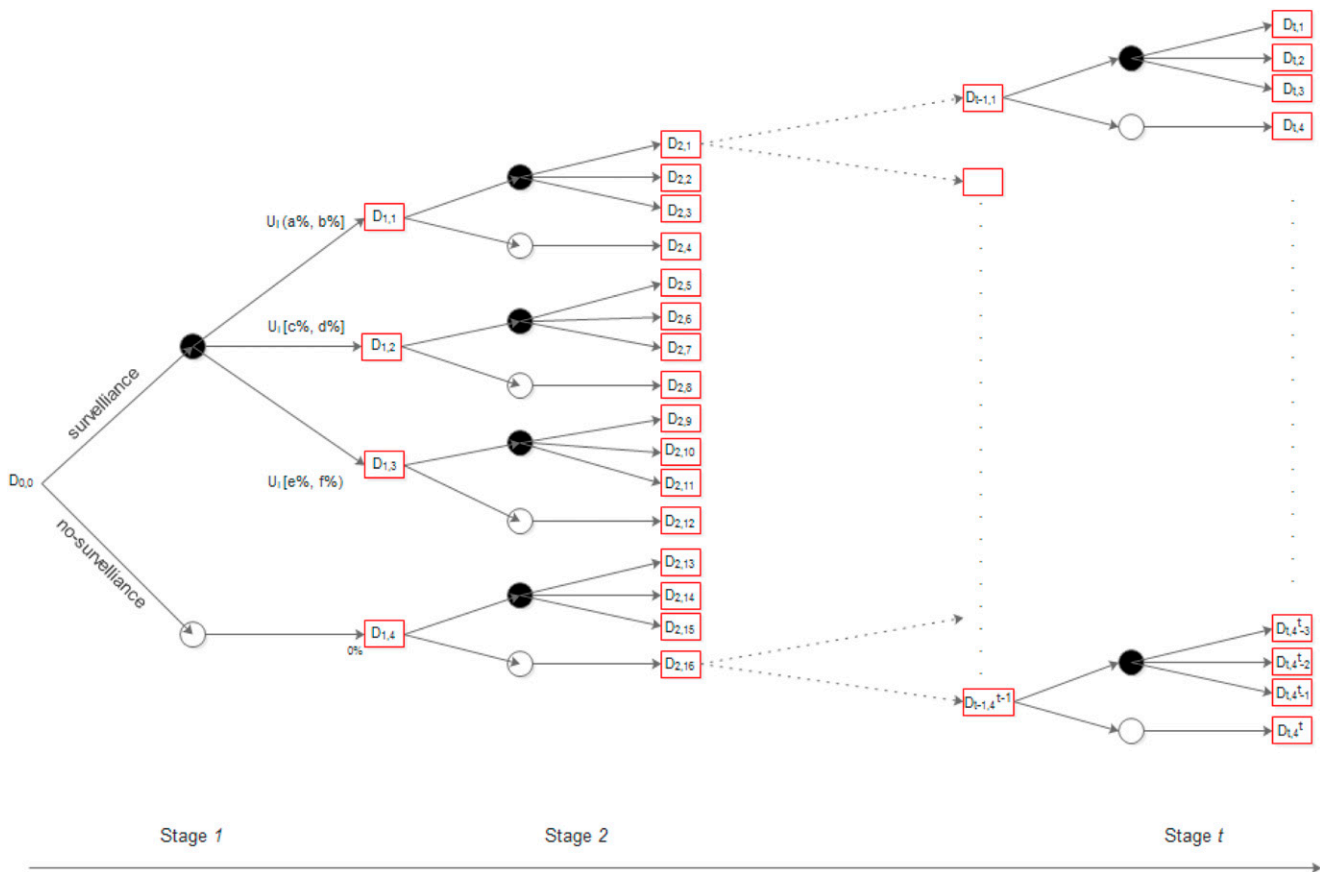


and do not impact susceptible trees; and finally, all non-squared nodes in periods one, three, and four represent the infestation classes in which the actual number of trees is not known because there is no surveillance. Moreover, dashed lines represent the period-to-period transition of trees from one infestation level into another, solid lines from infestation level 3 to infestation level 4 represent the removed trees, curved dotted lines from period two to four and period three to five represent treated trees that become susceptible two periods later.

The network in Figure 1 shows an example transition that may occur when surveillance takes place in periods 2 and 5. In period 1, the extent of the initial infestation is not known, and the manager has beliefs about the number of susceptible trees and the number of trees in each infestation class. Those beliefs are used to move trees to the next period’s infestation classes, including susceptible trees that become infested in period 2 because of the infestation spread. Once surveillance is applied in period 2, the actual number of trees in each infestation class is known, and trees in infestation classes 1 and 2 may be treated, whereas

trees in infestation class 3 may be removed, based on the available budget. In period 3, surveillance is not applied, and the numbers susceptible trees and newly infested trees (level 1) become beliefs because of uncertainty in infestation spread. Hence, treatment will not be applied to infested trees in class 1 because their location is not known. By contrast, treatment can be applied to trees in class 2, and removal can be applied to trees in classes 3 and 4 because the number of trees in those classes was learned in the previous period when those trees were in infestation classes 1, 2, and 3, respectively. Similarly, in year 4, removal can be applied to only class 3 and 4 infestations because corresponding infestation levels were determined with surveillance in the second year. Note that remaining trees in classes 3 and 4 transition to class 4 and remain in that infestation class unless they are removed. Finally, because surveillance is applied in period 5, the actual numbers of trees in all infestation classes are known, and treatment or removal can be applied to all trees based on the available budget. The details and notation of the optimization model are explained in the following section.

Figure 2. (Color online) Representation of Decision Tree in Multistage Form



Note. Here  $U_i [a\%, b\%]$  represents the uniformly distributed percentage change in the estimated infestation level based on the realization of uncertainty at each stage for uncertainty outcome  $i = \text{high, medium, and low}$ .

## 2.2. M-SMIP Model

**2.2.1. Scenario Tree.** We develop a scenario tree to represent the surveillance decisions and stochastic spread over time (Figure 2). Black and white nodes represent surveillance and no-surveillance decisions in a given stage, respectively. Outgoing arcs from circle nodes represent possible realizations of the beliefs about the number of trees in susceptible and infested classes. The percentage change in the belief of infestation level as a result of the surveillance action is determined by a uniform distribution  $U_i[a\%, b\%]$ , which denotes a number drawn uniformly from the interval  $[a\%, b\%]$  for each infestation realization case  $i$  (= low, medium, and high). Each node  $D$  represents the allowed treatment and removal decisions taken in the corresponding stage. Each black node yields three possible realizations as a result of the surveillance action, and each white node yields a single belief of infestation as a result of the lack of surveillance. Therefore, we assume four possible outcomes (three realizations based on surveillance and one belief of infestation based on no surveillance) from each decision node. Consequently,  $4^t$  scenarios are generated by the end of stage  $t$ .

Each path of the trees from stage 1 to the final stage of the planning horizon (stage  $t$  in Figure 2) represents a scenario. We describe a scenario as a combination of surveillance decisions and realizations of infestations. Thus, all possible binary surveillance decisions over all stages are embedded on a multistage stochastic scenario tree of infestation realizations. For example, for a three-stage problem, a possible infestation realization of low (L), medium (M), and high (H) in stages 1, 2, and 3, respectively, could be defined by two scenarios based on the surveillance action: no surveillance (NS) in the first stage followed by surveillance (S) in stages 2 and 3 [(NS, S, S), (L, M, H)] and surveillance in all three stages [(S, S, S), (L, M, H)]. Note that under no surveillance, the infestation realization is assumed to be low and stays a belief.

**2.2.1.1. Notation.** Notation for indices and sets includes the following:

- Index  $i$  for site, where  $\mathfrak{I}$  is the set of all sites ( $i \in \mathfrak{I}, \mathfrak{I} = \{1, \dots, \bar{\mathfrak{I}}\}$ );
- Index  $k$  for infestation level, where  $K$  is the set of infestation levels ( $k \in K, K = \{1, 2, \dots, n - 1, n\}$ );
- Index  $t$  for time period, where  $T$  is the set of time periods ( $t \in T, T = \{1, \dots, \bar{T}\}$ );
- Index  $j$  for neighboring site, where  $\Theta_i$  is the set of neighboring sites of site  $i$  ( $j \in \Theta_i$ ); and
- Index  $s$  for scenario, where  $\Xi$  is the set of scenarios in the scenario tree ( $s \in \Xi, \Xi = \{1, \dots, \bar{\Xi}\}$ ).

Notation for parameters includes the following:

- $\pi_s$  = probability for scenario  $s$ ;
- $c_1$  = cost of surveillance;

- $c_2$  = cost of treatment;
- $c_3$  = cost of removal;
- $\alpha$  = monetary value of each susceptible tree;
- $\vartheta_k$  = penalty value assigned to each infested tree at infestation level  $k = n$  and  $n - 1$ ;
- $r_k$  = impact rate of each infested tree at infestation level  $k$  within site  $i$ , that is, number of new infestations per infested tree at level  $k$ ;
- $\tau$  = discount rate;
- $\delta_t$  = discount factor at time  $t$ , which is equal to  $1/(1 + \tau)^t$ ;
- $\Psi_s$  = budget for scenario  $s$ ;
- $\theta_k$  = infestation impact of  $k$ th-level infested trees in neighboring site  $j$ ;
- $p_{j \rightarrow i}$  = probability of infestation spread from site  $j$  to  $i$ ;
- $\beta_{iks}^t$  = percentage change in belief of infestation for site  $i$ , infestation level  $k$ , at time  $t$ , for scenario  $s$ ;
- $\bar{N}_i$  = initial number of tree population at site  $i$ ; and
- $\bar{I}_{ik}$  = initial number of infested tree population at each infestation level  $k$  at site  $i$ .

Notation for binary decision variables fixed in the discrete scenario tree includes the following:

$$x_{ks}^t = \begin{cases} 1 & \text{if surveillance is applied to infestation} \\ & \text{level } k \text{ at time } t \text{ for scenario } s \\ 0 & \text{otherwise} \end{cases}$$

Notation for decision variables includes the following:

- $N_{is}^t$  = total number of trees at site  $i$  at time  $t$  for scenario  $s$ ;
- $S_{is}^t$  = number of susceptible trees at site  $i$  at time  $t$  for scenario  $s$ ;
- $\tilde{I}_{iks}^t$  = believed number of infested trees at site  $i$  at time  $t$  at infestation level  $k$  for scenario  $s$  before surveillance;
- $\hat{I}_{iks}^t$  = transition number of infested trees at site  $i$  at time  $t$  at infestation level  $k$  for scenario  $s$  after surveillance and before taking the maximum number of tree population that could be infested into account;
- $I_{iks}^t$  = actual number of infested trees at site  $i$  at time  $t$  at infestation level  $k$  for scenario  $s$  after surveillance with consideration of total tree population;
- $V_{iks}^t$  = number of treated trees at site  $i$  at time  $t$  at infestation level  $k$  for scenario  $s$ ;
- $R_{iks}^t$  = number of removed trees at site  $i$  at time  $t$  at infestation level  $k$  for scenario  $s$ ;
- $H_{iks}^t$  = number of trees surveyed at site  $i$  at time period  $t$  at infestation level  $k$  for scenario  $s$ ; and
- $Q_{iks}^t$  = number of infested trees remaining after treatment and removal at site  $i$  at time  $t$  at infestation level  $k$  for scenario  $s$ .

Notation for linearization variables includes the following:

$$u_{iks}^t = \begin{cases} 1, & \text{if transition population is assigned to} \\ & \text{infestation level } k \text{ at site } i \text{ at time } t, \\ 0 & \text{otherwise.} \end{cases}$$



**2.2.2. Mathematical Model.** A general M-SMIP model is formulated using the scenario tree shown in Figure 2. Based on the notation presented in Section 2.2.1, the M-SMIP model is described as follows.

**2.2.2.1. Budget Constraint.** It is assumed that while trees in the  $k = 1, \dots, n - 2$  infestation classes can be saved with treatment, the high-level infestation in  $k = n - 1$  and  $n$  is irreversible, whereas  $k = n - 1$  still poses a threat to the environment, and  $k = n$  represents dead trees. Therefore, although treatment could be applied to infestation levels  $k = 1, \dots, n - 2$ , highly infested and dead trees should be removed to prevent further damage based on the available budget. Therefore, the budget constraint is formulated as

$$c_1 \sum_{t \in \bar{T}} \sum_{i \in \bar{S}} H_{is}^t + c_2 \sum_{t \in \bar{T}} \sum_{i \in \bar{S}} \sum_{k=1}^{n-2} V_{iks}^t + c_3 \sum_{t \in \bar{T}} \sum_{i \in \bar{S}} \sum_{k=n-1}^n R_{iks}^t \leq \Psi_s, \quad \forall s, \quad (1)$$

which ensures that surveillance, treatment, and removal decisions are restricted by the available budget for each scenario  $s \in \bar{S}$ . In the budget constraint,  $H_{is}^t$  represents the number of trees that are surveyed and is formulated as

$$H_{iks}^t = \left( S_{is}^t + \sum_{k=1}^n I_{iks}^t \right) x_{ks}^t, \quad \forall s, i, k, \quad t = 1, \quad (1a)$$

$$H_{iks}^t = S_{is}^{t-1} x_{ks}^t + \sum_{k \in K} \left[ \left( \prod_{l=1}^k \left( 1 - x_{ks}^{\max\{t-l, 1\}} \right) \right) x_{ks}^t I_{iks}^{t-1} \right], \quad \forall s, i, k, \quad t = 2 \dots \bar{T}. \quad (1b)$$

Equation (1a) ensures that the number of surveyed trees in the initial period is either zero or equals the number of susceptible plus infected trees based on surveillance decisions given in the initial period. Note that surveillance decisions  $x_{ks}^t$  are not formulated as variables; instead, they are embedded into the scenario tree as binary parameters to prevent the non-linearity that would result in Equations (1a) and (1b).

Similarly, Equation (1b) determines the number of surveyed trees at period  $t$  based on surveillance; however, this equation takes into account all surveillance efforts performed in previous  $k$  time periods. Let  $t'$  ( $t' \leq t$ ) represent the number of periods until surveillance is applied in period  $t$ ; then the number of susceptible trees and the numbers of trees in the first  $k = t'$  infestation level will be unknown and remain as a belief. If surveillance is performed in period  $t$  after  $t' \leq n$  periods (where  $n$  is the highest infestation level), then trees in the susceptible cluster and in the first  $k = t'$  infestation level will be surveyed

in time period  $t$ . Note that surveillance is not necessary for infestation levels  $k = t' + 1, \dots, n$  because the number of trees at these infestation levels was determined  $t'$  periods previously, at the latest surveillance application. By contrast, if  $t' \geq n$  and surveillance are applied, then susceptible trees and all infested trees will be surveyed in a given landscape.

**2.2.2.2. Total Population.** The tree population that can become infested at a given site  $i$  is reduced by the removal of highly infested trees. Furthermore, trees that are treated in a given period are safe from infestation for the following two years and can become infested two years later. Therefore, the total population that is under EAB threat in site  $i$  and scenario  $s$  is formulated as

$$N_{is}^{t+1} = N_{is}^t - \sum_{k=1}^{n-2} V_{iks}^t - \sum_{k=n-1}^n R_{iks}^t, \quad \forall s, i, \quad t = 1, \quad (2)$$

$$N_{is}^{t+1} = N_{is}^t - \sum_{k=1}^{n-2} V_{iks}^t - \sum_{k=n-1}^n R_{iks}^t + \sum_{k=1}^{n-2} V_{iks}^{t-1}, \quad \forall s, i, \quad t = 2, \dots, \bar{T} - 1. \quad (3)$$

**2.2.2.3. Transition Infestation Level.** Surveillance finds the actual number of trees at each infestation level. The believed (expected) number of infested trees for a particular infestation level  $k$ ,  $\tilde{I}_{iks}^t$ , may change (increase, remain the same, or decrease) after the surveillance, or it may not be updated if surveillance is not applied. Therefore, the number of infested trees for each infestation level  $k$  is formulated as

$$\tilde{I}_{iks}^t = \tilde{I}_{iks}^{t-1} \cdot \left( 1 + x_{ks}^t \beta_{iks}^t \right), \quad \forall s, t, i, k, \quad (4)$$

where  $\tilde{I}_{iks}^t$  represents the transition number of infested trees at each infestation level  $k$  after surveillance before considering the maximum number of tree population that could be infested in a given site. Note that the term  $\beta_{iks}^t$ , the percent change in the belief of infestation when the infestation is determined by surveillance, also includes the uncertainty in the effectiveness of the surveillance.

**2.2.2.3. Carrying Capacity Constraints and Actual Infestation Level.** The carrying capacity limitation (e.g., maximum number of trees that could be infested in site  $i$ ) implies that the actual number of infested trees at level  $k$  cannot exceed the number of remaining trees that can be infested. As a result, if trees at level  $k + 1, \dots, n$  infest the entire population, then (lower) infestation levels  $k, k - 1, \dots, 1$  do not exist at

the given site. Therefore, the actual number of infested trees in infestation level  $k$  after considering the total tree population is formulated using the following non-linear equation:

$$I_{iks}^t = \min \left( N_{is}^t - \sum_{d=\min(k+1,n)}^n I_{ids}^t, \ddot{I}_{iks}^t \right), \quad \forall s, t, i, k, \quad (5)$$

where the actual number of trees in an infestation level  $k$  is assigned the minimum value between the transition population level and the remaining population after  $k+1, \dots, n$  levels of infested trees populate a given site. Note that Equation (5) is nonlinear and can be equivalently represented with the following linear inequalities:

$$I_{iks}^t \leq \ddot{I}_{iks}^t, \quad (5a)$$

$$I_{iks}^t \leq N_{is}^t - \sum_{d=\min(k+1,n)}^n I_{ids}^t, \quad (5b)$$

$$\ddot{I}_{iks}^t - I_{iks}^t \leq \bar{N}_i \cdot (1 - u_{iks}^t), \quad (5c)$$

$$\left( N_{is}^t - \sum_{d=\min(k+1,n)}^n I_{ids}^t \right) - I_{iks}^t \leq \bar{N}_i \cdot u_{iks}^t, \quad (5d)$$

where  $u_{iks}^t$  is a binary variable that is defined to determine the  $k$ th level infested-tree population, and  $\bar{N}_i$  is the initial total tree population in a given site. Note that although Equations (5a) and (5b) provide an upper bound on the actual number of infested trees, Equations (5c) and (5d) provide a lower bound by activating the binary variable  $u_{iks}^t$ . If the transition population level is less than the remaining capacity ( $\ddot{I}_{iks}^t \leq N_{is}^t - \sum_{d=\min(k+1,n)}^n I_{ids}^t$ ), then  $u_{iks}^t = 1$ , or vice versa. Therefore, the linearization equations ((5a)–(5d)) ensure that the equality  $I_{iks}^t = \min(N_{is}^t - \sum_{d=\min(k+1,n)}^n I_{ids}^t, \ddot{I}_{iks}^t)$  given in Equation (5) holds.

**2.2.2.4. Susceptible (Healthy) Tree Population.** Furthermore, the number of susceptible trees equals the total population less the total number of infested trees. Therefore, the susceptible tree population is formulated as

$$S_{is}^t = N_{is}^t - \sum_{k=1}^n I_{iks}^t, \quad \forall s, t, i. \quad (6)$$

**2.2.2.5. Number of Treated and Removed Trees.** Treatment or removal can be applied to infestation level  $k$  if surveillance has been applied at least once in the last  $k$  time periods including the current time period. Furthermore, the number of treated or removed trees

cannot exceed the number of trees at the same infestation level. Therefore, the number of treated or removed trees is formulated as

$$V_{iks}^t \leq I_{iks}^t \cdot \sum_{a=\max[t-k+1,1]}^t x_s^a, \quad \forall s, t, i, k=1, \dots, n-2, \quad (7)$$

$$R_{iks}^t \leq I_{iks}^t \cdot \sum_{a=\max[t-k+1,1]}^t x_s^a, \quad \forall s, t, i, k=n-1, n. \quad (8)$$

**2.2.2.6. Believed (Expected) Number of Infested Trees.**

At each time period, susceptible trees in a given site can become infested by the impact of infested trees that are not treated within the site and by the spread of infestation from surrounding sites  $j \in \Theta^i$ . Therefore, the believed number of newly infested trees ( $k=1$ ) at time  $t+1$  is formulated as

$$\begin{aligned} \tilde{I}_{i1s}^{t+1} = & \sum_{g=1}^n Q_{igs}^t \cdot r_g \\ & + \sum_{g=1}^n \sum_{j \in \Theta^i} Q_{jgs}^t \cdot \theta_g \cdot p_{j \rightarrow i}, \quad \forall s, i, t=1, \dots, \bar{T}-1, \end{aligned} \quad (9)$$

where  $Q_{igs}^t$  represents infested but untreated or unremoved trees at site  $i$ , infestation level  $g$ , at time  $t$  under scenario  $s$  and is formulated as

$$Q_{igs}^t = \begin{cases} I_{igs}^t - V_{igs}^t, & g=1, \dots, n-2, \\ I_{igs}^t - R_{is}^t, & g=n-1, n, \end{cases} \quad \forall s, t, i,$$

and  $p_{j \rightarrow i}$  represents the probability of infestation spread from neighboring site  $j$  to site  $i$ .

Furthermore, infested but untreated trees will transition to the upper infestation level as a belief in the following period. Therefore, the believed number of infested trees for infestation level  $k=2, \dots, n-1$  is formulated as

$$\begin{aligned} \tilde{I}_{iks}^{t+1} = & I_{i(k-1)s}^t - V_{i(k-1)s}^t, \quad \forall s, i, t=1, \dots, \bar{T}-1, \\ & k=2, \dots, n-2. \end{aligned} \quad (10)$$

In addition, once the infested trees reach the highest infestation level  $k=n$ , they remain in the  $n$ th level unless they are removed from the population. Therefore, the believed number of infested trees at level  $n$  in a given period consists of the unremoved trees of level  $n-1$  and  $n$  in the previous period. Therefore, the believed number of infested trees for level  $n$  is formulated as

$$\begin{aligned} \tilde{I}_{iks}^{t+1} = & \left( I_{i(k-1)s}^t - R_{i(k-1)s}^t \right) + \left( I_{iks}^t - R_{iks}^t \right), \\ & \forall s, i, t=1, \dots, \bar{T}-1, k=n-1, n. \end{aligned} \quad (11)$$

Finally, the initial total population and initial belief of infestation levels are defined as

$$N_{is}^1 = \bar{N}_i, \quad \forall s, i, \quad (12)$$

$$\tilde{I}_{iks}^1 = \bar{I}_{ik}, \quad \forall s, i, k, \quad (13)$$

$$N_{is}^t, S_{is}^t, I_{iks}^t, \tilde{I}_{iks}^t, \ddot{I}_{iks}^t, V_{iks}^t, R_{iks}^t \geq 0, \quad u_{iks}^t \in \{0, 1\}, \quad \forall s, t, i, k. \quad (14)$$

**2.2.2.7. Nonanticipativity Constraints.** The multistage problem requires that scenarios that share the same history up to a stage  $t$  in the scenario tree must also share the same decisions up to that stage. This can be achieved by introducing a set of nonanticipativity constraints so that the tree structure shown in Figure 2 can be embodied fully in the model. Defining  $s_t$  as the realization of scenario  $s \in \Xi$  up to stage  $t$ , Equations (15) represent the nonanticipativity constraints, which ensure that, for any stage  $t$ , decision variables that correspond to scenarios  $s$  and  $s'$ , which are indistinguishable up to stage  $t$ , should be equal and are given as follows:

$$\begin{aligned} N_{is}^t &= N_{is'}^t, S_{is}^t = S_{is'}^t, I_{iks}^t = I_{iks'}^t, \tilde{I}_{iks}^t \\ &= \tilde{I}_{iks'}^t, \ddot{I}_{iks}^t = \ddot{I}_{iks'}^t, V_{iks}^t = V_{iks'}^t, R_{iks}^t = R_{iks'}^t, u_{iks}^t = u_{iks'}^t, \\ &\forall s = s' \in \Xi, \quad \text{such that } s_t = s'_t, \quad \forall t, i, k. \end{aligned} \quad (15)$$

**2.2.2.8. Objective Function.** The objective function of this model is to maximize the net benefits of susceptible (healthy) ash trees while penalizing trees that are subject to removal over the entire landscape and planning horizon. Therefore, the objective is formulated as

$$\max \sum_{s \in \Xi} \pi_s \left( \sum_{t \in T} \delta_t \sum_{i \in \mathcal{I}} \left( \alpha S_{is}^t - \sum_{k=n-1}^n \vartheta_k I_{iks}^t \right) \right). \quad (16)$$

### 3. New Cutting Planes for Linearization Variables

In this section, we discuss methods of generating cutting planes for the proposed model to tackle the computational difficulty of the presented multistage stochastic model. The M-SMIP models are among the most difficult-to-solve problems. To solve the problem to optimality, we develop new cutting planes by studying the binary linearization variables and their values under the worst-case scenario. Specifically, we present two types of cutting planes called *remaining capacity cuts* and *transition population cuts*. Remaining capacity cuts are valid inequalities, whereas transition population cuts are specific type of cuts, which may cut off integer solutions but do not cut off the optimal solution. Both cutting planes are added to the model a priori before solving it with CPLEX (IMB 2017).

#### 3.1. Cutting Planes for Remaining Capacity

As discussed in Section 2.2.2, the actual number of infested trees in infestation level  $k$  receives the minimum value between the transition population and the remaining population after higher infestation levels populate a given site (Equation (5)). Therefore, if infestation level  $k$  can only occupy the remaining population ( $u_{iks}^t = 0$ ), then this implies that the given site has been totally infested by infestation levels  $k$  and higher ( $k + 1, \dots, n$ ), and no space is left for infestation levels lower than  $k$  ( $1, 2, \dots, k - 1$ ). Therefore, if  $u_{iks}^t$  is equal to zero, then the lower infestation levels  $k - 1, \dots, 1$  are assigned zero ( $I_{iks}^t = 0$ ), implying that  $u_{i(k-1)s}^t$  is also equal to zero. By contrast, if infestation level  $k$  is assigned the transition population ( $u_{iks}^t = 1$ ), then lower infestation levels  $k - 1, \dots, 1$  can be assigned either the transition or remaining population; that is,  $u_{i(k-1)s}^t$  is either zero or one. Thus, to represent this relationship between the  $u_{iks}^t$  and  $u_{i(k-1)s}^t$  variables, we derive remaining capacity cutting planes (RC cuts) as follows:

$$u_{i(k-1)s}^t \leq u_{iks}^t, \quad \forall s, t, i, k. \quad (17)$$

#### 3.2. Preprocessing Algorithm and Transition Population Cutting Planes

Our goal is to derive additional cuts that define the transition population, namely transition population cutting planes (TP cuts). TP cuts are specific cuts that may cut off an integer solution but do not cut off the optimal solution. We begin the cut-generation procedure by choosing the scenario that leads to the minimum economic benefit from a susceptible tree population (worst-case scenario). To choose the worst-case scenario  $s_{\text{worst}} \in \Xi$ , we consider all cases in which we perform surveillance and identify the scenario that gives a high infestation realization in each stage. Then we formulate a new subproblem M-SMIP<sup>s<sub>worst</sub></sup>, where the constraints (1)–(14) and objective function (16) are only defined for  $s_{\text{worst}} \in \Xi$ . The M-SMIP<sup>s<sub>worst</sub></sup> problem is then solved, and the value of the  $u$ -variables  $\tilde{u}_{iks_{\text{worst}}}^t$  are extracted from the solution.

We then define a subprocedure to determine the first time period  $t'$  under which the maximum infestation population is reached ( $\tilde{u}_{iks_{\text{worst}}}^{t'} = 0$ ). In this subprocedure, time index  $t$  is initially set to 0, and  $t'$  is set to  $\bar{T} + 1$ . In a while loop, we perform a number of steps until  $t' = t$  or  $t = \bar{T}$ . In each step of the while loop, increase  $t$  by 1 and check to see if  $\tilde{u}_{iks_{\text{worst}}}^t = 0$ . If  $\tilde{u}_{iks_{\text{worst}}}^t = 0$ , then set  $t' = t$  and exit the while loop and end the procedure. Otherwise, if  $\tilde{u}_{iks_{\text{worst}}}^t = 1, t' \neq t$ , and thus a cut ( $u_{iks}^t = 1$ ) is added into the M-SMIP model for each scenario  $s \in \Xi$ , site  $i \in \mathcal{I}$ , infestation level  $k \in K$ . Because  $t'$  is the earliest period when the population of infestation level  $k$  reaches the maximum allowable population under the worst-case scenario, the

transition population can be assigned to infestation level  $k$  ( $u_{iks}^t = 1$ ) for each time period where  $t < t'$  for all scenarios. Once  $t$  is equal to  $t'$  or  $\bar{T}$ , the procedure ends.

The cut-generation routine to derive TP cuts is defined in the following procedure:

**Procedure** Routine to generate TP cutting planes

- 1: **Preprocessing Model** (M-SMIP<sup>s<sub>worst</sub></sup>)
- 2: Initialization. Choose worst-case scenario  $s_{\text{worst}} \in \Xi$
- 3: Set  $V_{iks_{\text{worst}}}^t \leftarrow 0$ . Set  $R_{iks_{\text{worst}}}^t \leftarrow 0$ .
- 4: Solve preprocessing model
- 5: **end Preprocessing**
- 6: Extract  $u_{iks_{\text{worst}}}^t$  values from solution of M-SMIP<sup>s<sub>worst</sub></sup>
- 7: Set  $\tilde{u}_{iks_{\text{worst}}}^t \leftarrow u_{iks_{\text{worst}}}^t$ .
- 8: **Generating TP cuts for M-SMIP Model**
- 9: Set  $t \leftarrow 0$  and  $t' \leftarrow \bar{T} + 1$ ;
- 10: **while** ( $t < t'$ ) **do**
- 11: Set  $t \leftarrow t + 1$
- 12: **if**  $\tilde{u}_{iks_{\text{worst}}}^t = 0$
- 13: **then**
- 14: Set  $t' \leftarrow t$ ; **end if**; **end while**
- 15: **else if**  $\tilde{u}_{iks_{\text{worst}}}^t = 1$
- 16: **then**
- 17: **for each scenario**  $s \in \Xi$ , site  $i \in \mathfrak{J}$ , infestation level  $k \in K$
- 18: Set  $u_{iks}^t \leftarrow 1$ , i.e., add  $u_{iks}^t = 1$  to the M-SMIP
- 19: **end for**
- 20:
- 21: **end if**
- 22: **end while**
- 23: **End** Generating TP cuts for M-SMIP model

The algorithm given in this procedure helps us to derive TP cutting planes in the following form:

$$u_{iks}^t = 1, \quad \forall s, i, k, \quad t < t'. \quad (18)$$

TP cuts in Equation (18) ensure that infestation level  $k$  is set to its transition population for all periods less than  $t'$ .

It is easy to see that the worst objective value is observed when we solve M-SMIP<sup>s<sub>worst</sub></sup>. This is because treatment and removal are set to zero in M-SMIP<sup>s<sub>worst</sub></sup>, so the related constraints ((7) and (8)) are inactive. The remaining set of constraints serves as population dynamics equations, that is, flow balance constraints. Equations (4) and (5) ensure that the higher the infestation realization is, the larger the infested ash tree population will be, which, in turn, leads to the highest infestation possible and thus the worst objective function value. Once the M-SMIP<sup>s<sub>worst</sub></sup> problem is solved, we can determine the first period  $t'$  under which the remaining population is assigned to infestation level  $k$  ( $u_{iks_{\text{worst}}}^{t'} = 0$ ). Because  $t'$  is the earliest

period when the population of infestation level  $k$  reaches the maximum allowable population under the worst-case scenario, the transition population can be assigned to infestation level  $k$  ( $u_{iks}^t = 1$ ) for all scenarios  $s \in \Xi$  and time periods  $t \in T$  where  $t < t'$ , that is, maximum population cannot be reached for all other scenarios before period  $t'$ .

## 4. Parameter Calibration, Model Validation, and Application

In our model of ash population dynamics, we modeled EAB spread using impact parameters  $r_k$  and  $\theta_k$  for the number of newly infested trees in a given plot per infested tree in level  $k$ ,  $k = 1, 2, 3$ , in the plot and in neighboring plots, respectively. We calibrated those impact parameters and validated model projections using EAB infestation data collected by Knight et al. (2013) and Flower et al. (2013a) from ten sites near Toledo, Ohio, from 2005 to 2011 (see e-Companion S). Each site includes three to six 400-m<sup>2</sup> circular plots composed primarily of ash trees. The data include number of ash trees by canopy health class, rated visually on a scale of one to five, where one is a tree with a healthy canopy (healthy, full foliage), five is a dead canopy (no foliage), and two to four are stages of canopy decline based on thinning of foliage and dieback of branches (Smith 2006, Knight et al. 2014). The ratings are easy to apply in the field, and the ratings correspond to EAB larval densities in trees (Flower et al. 2013b). The five canopy health classes also correspond to the five tree classes in our model (susceptible, low, medium, high infestation, and dead trees). Although individual trees in the plots sometimes improved in health or declined by multiple categories within a year, the mean data at the plot level generally fit our model assumption of a yearly decline by one health category.

The length of a period is set to one year because of the biology of emerald ash borer, which typically has one generation per year (although sometimes it takes two years to complete its life cycle), and the biology of ash trees, whose phenology is based on a yearly cycle. The actions of surveillance (one to five tree ratings June to August when tree canopy is leafed out) and treatment (spring or early summer) are also applied on a yearly cycle.

### 4.1. Parameter Calibration

We calibrated the impact parameters using one site with three plots in Maumee Bay State Park (see e-Companion S). We assumed that the plots were configured in a  $2 \times 2$  gridded landscape so that each plot had two neighboring plots. We further assumed that the impact rates of infested trees within the plot were the same as the impact rates of infested trees in

**Figure 3.** (Color online) Representation of Validation Sites in Toledo, Ohio

neighboring plots (i.e.,  $r_k = \theta_k$ ,  $k = 1, 2, 3$ ) with a plot-to-plot spread probability of 0.125. Then we used trial and error to choose values of the impact parameters so that the projections of the cumulative number of dead trees over time fit the observed number of dead trees summed across plots. After several computational experiments, impact rates were estimated as 0.18, 0.25, and 0.32 for trees in the low-, medium-, and high-infestation levels, respectively. Figures showing the observed and predicted cumulative number of dead trees over time for the three plots and the overall site provide visual evidence of a good fit (see e-Companion S). A paired  $t$ -test comparing the observed and predicted number of newly dead trees each year across all three plots showed that there is no statistical difference between the observed and predicted number of dead trees at the 5% significance level over the seven-year period (see e-Companion S).

#### 4.2. Model Validation

After calibrating the impact parameters, we used the other nine sites of EAB infestation observations for model validation, as shown in Figure 3. Each site had three plots; thus, a  $2 \times 2$  gridded landscape was generated for each site to represent EAB spread between plots in a given site. Visual comparisons of the predicted and observed cumulative number of dead trees show a good fit over the seven years (Figure 4; see e-Companion S). A paired  $t$ -test for the comparison of yearly predicted and observed numbers of dead trees in each of the nine sites demonstrates that there is no statistical difference between predicted and observed numbers of dead trees at the 5% significance level in each plot (see e-Companion S). Based on the statistical results and visual comparisons, we conclude that the proposed model of ash dynamics subject to an EAB infestation provides a

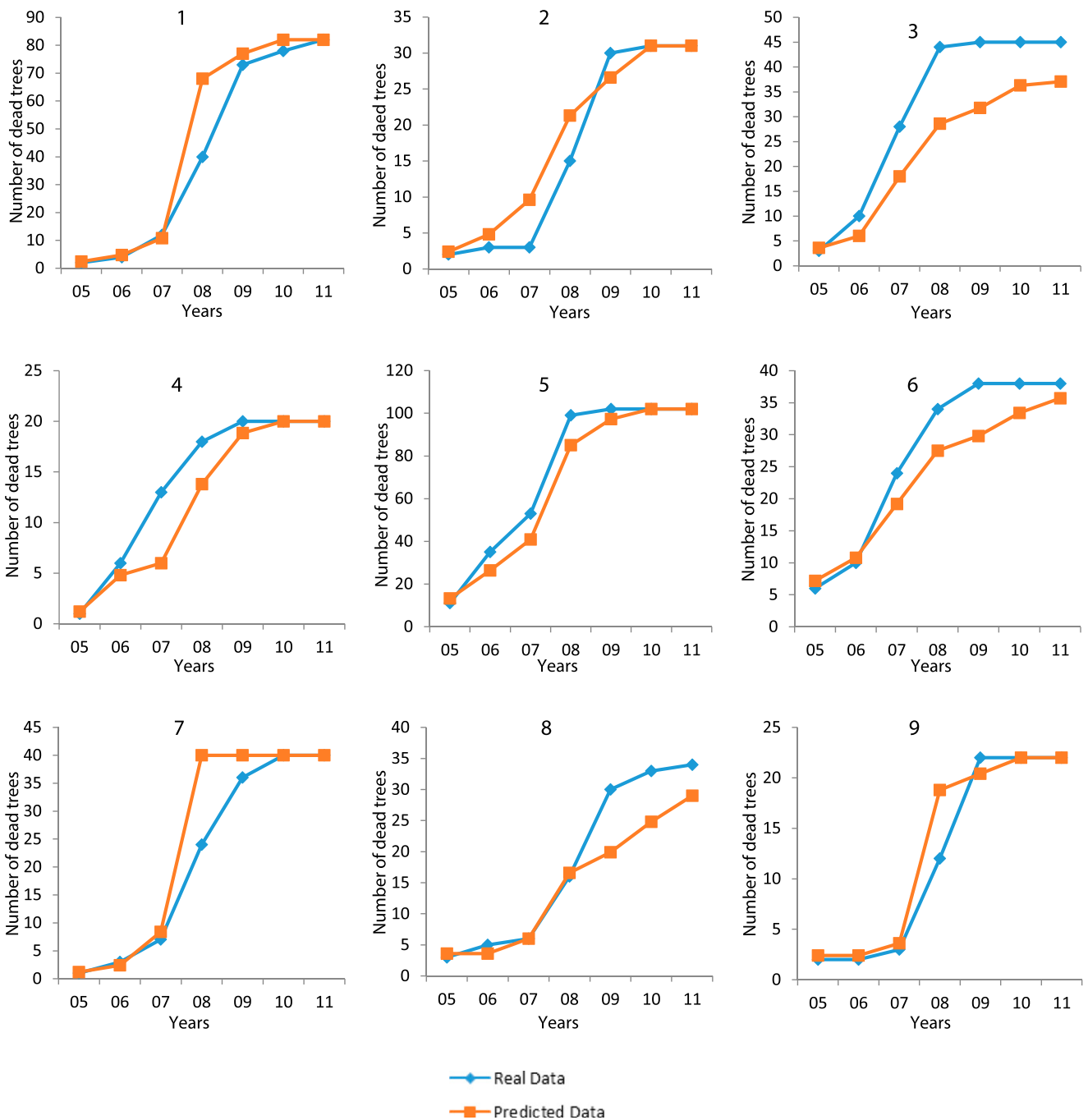
valid estimation of the number of dead trees over a seven-year time horizon.

#### 4.3. Model Application

We applied our surveillance and control optimization model to a population of ash trees in the city of Burnsville, Minnesota (Figure 5). The study area is divided into 23 square sites, each  $0.276 \times 0.276$  miles (48.74 acres) in size. Each site has public ash trees, which are marked as red dots in Figure 5, and private ash trees, which were estimated using ash density information from the City of Burnsville. The estimated number and spatial distribution of public and private trees in the study area is shown in Figure 6.

Table 1 shows each model parameter, its symbol, unit, case-study value, and reference. The impact rates of each infested tree (number of newly infested trees per infested tree) at infestation levels 1, 2, 3, and 4 are 18%, 25%, 32%, and 0%, respectively, based on calibration exercise. According to these parameter values, trees with increasing levels of infestation pose an increasing threat to susceptible trees, except for those that are dead. Furthermore, each tree with infestation level  $k$  transitions to the next infestation class in the following period until it reaches the dead-tree cluster. Although managers have beliefs about numbers of trees in each infestation level, the actual numbers might be different because of variations in EAB population growth and spread caused by unpredictable changes in weather or transportation of infested wood. Therefore, surveillance is necessary to determine the actual number of infested trees (refer to Figure 1 for details about surveillance). Accordingly, we assume that the belief about the number of trees by infestation level might change by 0%, +20%, and +40% (low, medium, and high realization, respectively) with probability 0.4, 0.3, and 0.3, respectively, after

**Figure 4.** (Color online) Comparison of Real Data and Predicted Results Using the Optimization Model for Infestation Level 4 Over a Seven-Year Period for Sites 1–9



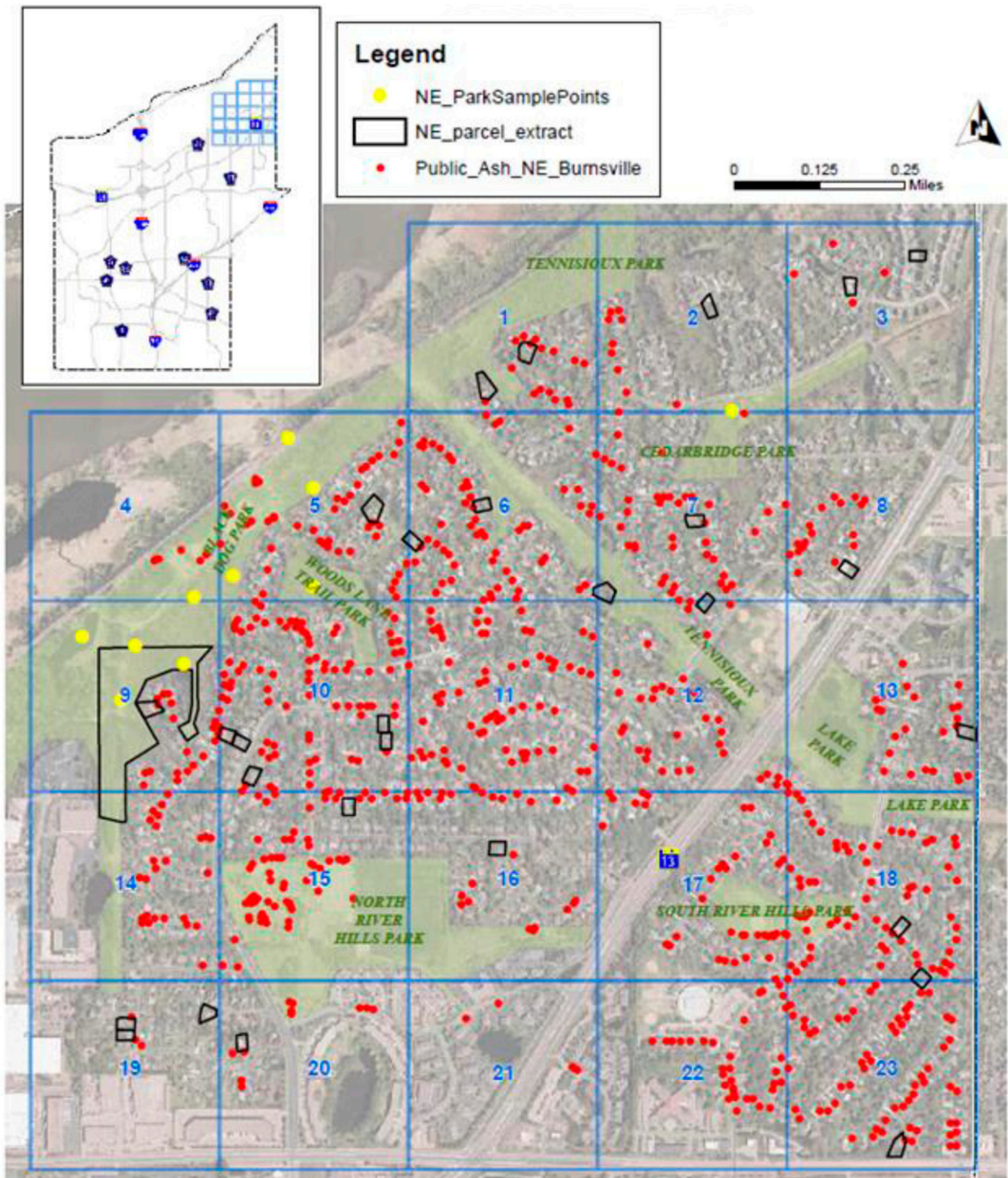
surveillance (see three branches emanating from each black node in Figure 2). If surveillance is not applied, then the belief about the number of trees by infestation class does not change, with a probability of 1, although trees do move up one infestation class.

For our paper, we assume that surveillance involves visual inspection of individual ash trees, where an inspector looks for woodpecker damage on the branches or trunk, dead foliage, and signs of larvae inside the bark (by chopping the bark away from spots

on the trunk or branches). Based on the outcomes of the visual inspection, the inspector rates the trees on the one to four rating scale to define the tree infestation class. We assumed that an inspector spends about 30 minutes per tree. With an intern wage of \$20/hour, inspection costs \$10 per tree.

The standard treatment for controlling EAB involves injecting a systemic insecticide into the base of the tree. An insecticide with emamectin benzoate as an active ingredient is shown to prevent colonization

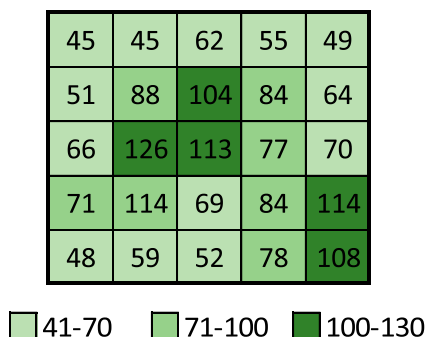
**Figure 5.** (Color online) Boulevard and Park Ash Tree Locations within the North and South River Hills Neighborhoods of Burnsville, Minnesota



and injury from EAB for two years (McCullough et al. 2009, McCullough and Mercader 2012). We use an insecticide treatment cost of \$120/tree, which is es-

timated by Kovacs et al. (2010) using the EAB cost calculator for Indiana (<http://www.entm.purdue.edu/EAB/>). Because different types of insecticides could

**Figure 6.** (Color online) Public and Private Tree Population in Burnsville, Minnesota



have different costs, the value of the cost parameter in the mathematical model could be adjusted easily to accommodate the cost of different insecticide treatment options.

We assume that insecticide treatment may be applied to trees in infestation levels 1 and 2 only. The insecticide kills all the larvae and adults present in the tree and prevents a new infestation for two years (McCullough et al. 2009). Trees in infestation levels 3 and 4 are subject to removal. The cost of tree removal is \$700 per tree. The objective function maximizes the discounted benefits of susceptible (healthy) ash trees (\$54 per healthy tree per year) minus the discounted cost of third- and fourth-level infested trees (−\$50 per tree per year) subject to a total budget constraint that ranges from \$100,000 to \$450,000 over a five-year horizon. The real discount rate is 2%.

## 5. Results

Results demonstrate the computational effectiveness of the new cutting planes and provide insights

regarding optimal surveillance, treatment, and removal strategies for controlling EAB invasions. The objective of the experiments regarding the case-study problem discussed in Section 4 is to investigate the best timing of surveillance when an EAB infestation is expected, the optimal spatiotemporal allocation of budget among surveillance, treatment and removal decisions, and the best prioritization strategy to manage ash trees with different infestation classes.

The multistage stochastic model presented in Section 1 was solved using CPLEX 12.7 (IBM 2017) on a desktop computer running with an Intel i7 central processing unit (CPU) and 64.0 GB of memory. A time limitation of 24 hours with 1% gap limit was imposed for solving the test instances. In most instances, the solution gaps were much tighter than the imposed limit. Selected results chosen from interesting problem configurations are reported in this section.

### 5.1. Computational Results for Cutting Planes

We compared the computational performance of the M-SMIP model with and without the RC and TC cuts given in Equations (17) and (18) for different time horizons (three, four, and five periods) and landscape sizes (2 × 2 to 10 × 10 sites) under an ample budget. The initial number of ash trees per cell in each of the test landscapes was generated using the mean (75.8) and standard deviation (24.5) of the number of ash trees per cell from the Burnsville, Minnesota, data (Figure 6). We assumed that 30%, 20%, 10%, and 0% of the trees in each cell have infestation levels of one, two, three, and four, respectively. A budget level of \$8,000 per cell was used for testing three- and four-period instances, and it is increased three times to

**Table 1.** Parameters of Surveillance and Control Optimization Model

Model parameter	Symbol	Unit	Case-study value	Footnote
Impact rate of each infested tree at infestation level $k$	$r_k$	—	18%, 25%, 32%, 0%	a
Infestation impact of $k$ th-level infested trees in neighboring site $j$	$\theta_k$	—	18%, 25%, 32%, 0%	a
Percentage change in belief of infestation	$\beta_k$	—	0%, 20%, 40%	—
Probability assigned to change in the belief	$p$	—	0.4, 0.3, 0.3	—
Probability for scenario $s$	$\pi_s$	—	(0,1)	—
Surveillance cost	$c_1$	\$/tree	10	b
Treatment cost	$c_2$	\$/tree	120	c
Removal cost	$c_3$	\$/tree	700	b
Monetary value of each susceptible tree	$\alpha$	\$/tree/year	+54	b
Penalty value assigned to each highly infested tree	$\delta_k$	\$/tree/year	−50	b
Budget	$\Psi_s$	\$/scenario	\$100,000–\$450,000	—
Probability of infestation spread from site $j$ to $i$	$p_{j \rightarrow i}$	—	0.125	—
Discount rate	$\tau$	—	2%	d

<sup>a</sup>Calibrated and validated using data from Knight et al. (2013) and Flower et al. (2013a).

<sup>b</sup>Expert opinion.

<sup>c</sup>Kovacs et al. (2014).

<sup>d</sup>Federal Reserve Economic Data (2015).



**Table 2.** Comparison of Solution Performances

No. of periods	Size	Original model				Original Model + cuts					
		CPU time	CPLEX cuts	Gap (%)	Objective	CPU time	CPLE × cuts	RC cuts	TC cuts	Gap (%)	Objective
3	2 × 2	4	0	0.37	10,821	4	0	2,304	2,916	0.31	10,821
	3 × 3	4	1,092	0.61	25,439	4	0	5,184	6,336	0.33	25,439
	4 × 4	4	2,634	0.66	51,421	2	0	9,216	11,264	0.05	51,421
	5 × 5	6	5,618	0.58	73,489	2	0	14,400	17,664	0.08	73,489
	6 × 6	8	8,624	0.65	107,291	2	0	20,736	25,344	0.09	107,291
	7 × 7	12	15,931	0.69	150,846	2	0	28,224	34,560	0.11	150,846
	8 × 8	14	19,109	0.98	197,605	2	0	36,864	45,184	0.12	197,605
	9 × 9	20	23,696	0.86	243,184	3	0	46,656	57,216	0.12	243,184
10 × 10	27	28,698	0.80	297,254	4	0	57,600	70,464	0.11	297,254	
Average solution time		11				3					
4	2 × 2	17	9,187	0.99	16,240	10	0	12,288	13,312	0.00	16,240
	3 × 3	118	0	0.02	38,069	7	0	27,648	29,952	0.52	38,069
	4 × 4	276	0	0.89	76,244	13	7,125	49,152	53,248	0.70	76,226
	5 × 5	6,484	0	0.03	109,560	21	2,211	76,800	83,712	0.98	109,515
	6 × 6	—	—	—	—	558	30,354	110,592	119,808	0.09	160,011
	7 × 7	—	—	—	—	167	39,647	150,528	163,584	0.07	224,176
	8 × 8	31,247	714	0.01	293,803	1,608	49,750	196,608	214,016	0.12	293,697
	9 × 9	36,491	6,893	0.08	362,286	2,584	62,036	248,832	271,104	0.07	362,259
10 × 10	7,507	5,361	0.06	443,035	3,843	77,115	307,200	333,312	0.10	442,942	
Average solution time		11,734				979					
5	2 × 2	146	0	0.60	22,069	31	2,132	61,440	41,984	0.27	22,070
	3 × 3	1,090	65,274	0.79	52,215	803	362	138,240	96,256	0.19	52,215
	4 × 4	40,281	202,283	0.78	105,145	6,296	65,987	245,760	168,960	0.10	105,682
	5 × 5	14,897	326,329	0.99	149,995	3,702	0	384,000	258,048	0.31	149,995
	6 × 6	—	—	—	—	40,435	94,029	552,960	373,760	0.72	222,097
	7 × 7	—	—	—	—	—	—	—	—	—	—
Average solution time		14,104				2,708					

Note. —, instances unsolved because of memory problems.

solve five-period instances. All instance data files used in Section 5.1 are presented in the e-companion.

We found that the RC and TP cuts obtained with the preprocessing algorithm significantly reduced the solution time of the test cases (Table 2). The results show that RC and TP cuts improved the average CPU solution times by an approximate factor of four for three-period instances and twelve for four-period instances. The improvement in solution times becomes more apparent as the size of the problem increases temporally and spatially. Note that for some instances (e.g., four periods with 6 × 6 and 7 × 7 sites and five periods with 6 × 6 and larger sites), CPLEX (IMB 2017) fails to find a solution without the proposed cuts. Because of the complexity and size of the original model with or without cuts, five-period instances with landscape sizes 7 × 7 and larger were not solvable with the current state-of-the-art solver because of memory problems; thus, we only present results for instances from 2 × 2 to 6 × 6 sites. When the results for five-period instances with 2 × 2 to 5 × 5 sites are averaged, the cutting planes improved the solution time by a factor of 5.2 over the model without cuts. The five-period instances are the hardest to

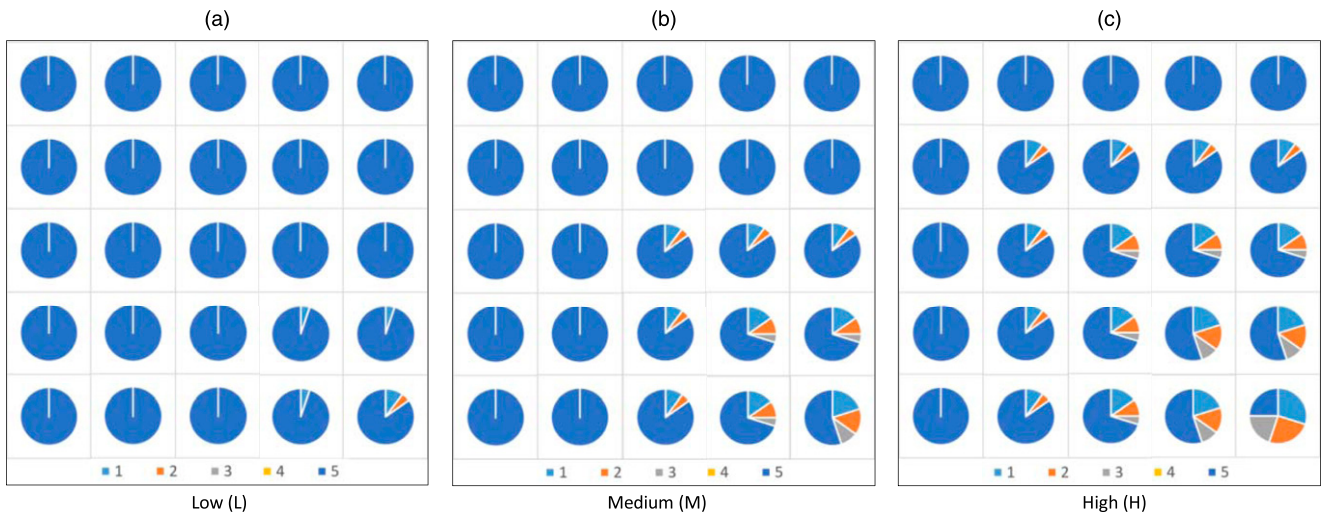
solve, resulting in an average of 14,104 CPU seconds of solution time without cuts and 2,708 CPU seconds of solution time with the RC and TP cuts over the first four instances. We anticipate that the benefit of using our cuts would be even higher if the instances with memory problems were solved.

**5.2. Optimal Management**

We computed optimal management policies for the 5 × 5 cell Burnsville landscape (Figure 6) over a five-year horizon. Because we do not know the location of infested trees, we generated three different initial infestation cases—high (H), medium (M), and low (L)—each of which started in the southeast corner of the landscape. The ratios of trees among different infestation levels in each of the three cases are shown in Figure 7.

Using a budget constraint of \$100,000, we computed optimal solutions for 4<sup>5</sup> = 1,024 infestation realization scenarios (see the scenario tree in Figure 2) for each of the three initial infestation cases. All instance data files used in the rest of this paper are presented in the e-companion.

**Figure 7.** (Color online) Possible (a) Low, (b) Medium, and (c) High Initial Infestation Cases with Location and Ratio of Infestation Levels 1–4 and Susceptible Trees (5) in Boulevard of the City of Burnsville, Minnesota

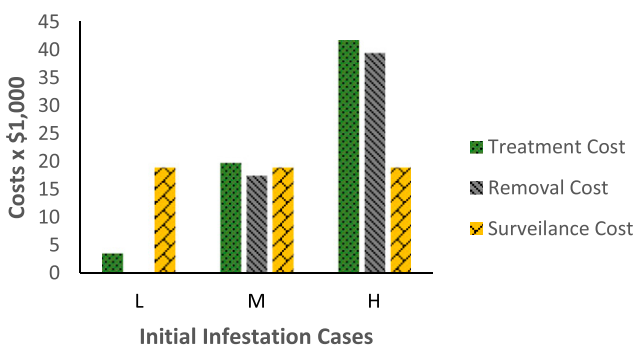


Although the optimal budget allocation among surveillance, treatment, and removal varies by scenario, the scenario with the highest net benefit for each initial infestation case is scenario 768, which applies surveillance only in the first period. Figure 8 shows the costs of treatment, removal, and surveillance for the low-infestation realization over the five consecutive years (L-L-L-L-L) for the three initial infestation cases. The surveillance cost is the same for all three cases because surveillance is applied to all trees regardless of their infestation level. The treatment and removal costs increase from low to high initial infestation.

The spatial distribution of the optimal number of surveyed, treated, and removed trees for the first year of scenario 768 is given in Figure 9. With the \$100,000 budget, all trees are surveyed, whereas treatments and removals take place in the southeast corner of the landscape.

**5.2.1. Sensitivity Analysis on the Location of Initial Infestation.** To perform sensitivity analysis on the

**Figure 8.** (Color online) Treatment, Removal, and Surveillance Cost for the First Year of Low (L), Medium (M), and High (H) Initial Infestation Cases with \$100,000 Budget Under Scenario 768



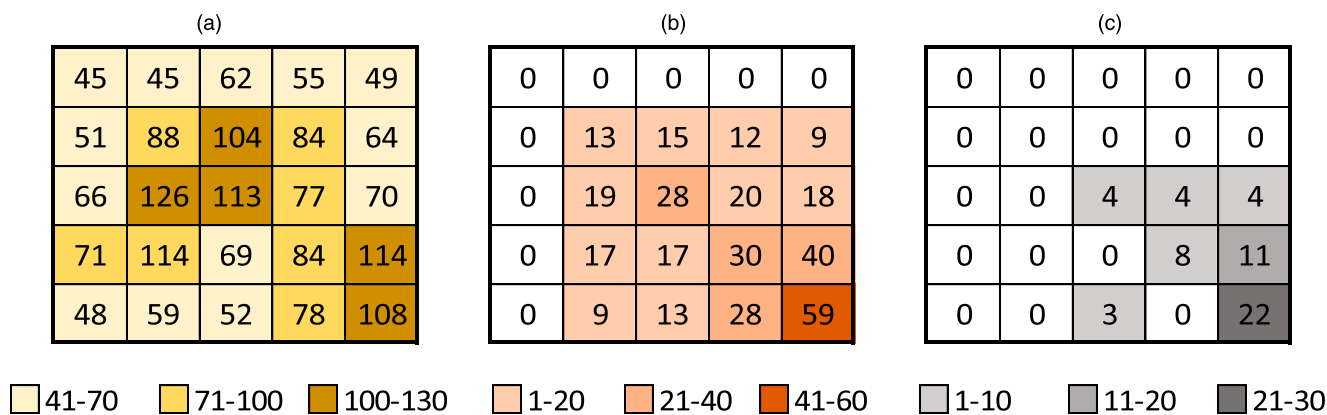
impact of the location of the initial infestation, we generated four more cases—L-1, L-2, M-1, and H-1; L-1 represents the low-infestation (L) case where infested trees are moved to the northwest corner of the landscape from the southeast corner, L-2 represents the low-infestation (L) case where infested trees are equally distributed to the northwest and southeast corners of the landscape, M-1 represents the medium-infestation (M) case where infested trees are centered in the landscape, and H-1 represents the high-infestation (H) case where infested trees are again moved to the northwest corner of the landscape from the southeast corner.

We compare the L, M, and H cases with the four new cases L-1, L-2, M-1, and H-1 in terms of the objective function value and the expected cost allocation among treatment, removal, and surveillance over all scenarios. We observe that for low and medium cases, neither the objective function value nor any of the expected costs change with respect to the change in the infestation location considered in L-1, L-2, and M-1 cases. By contrast, the H-1 case objective value is slightly lower than the objective value of the H case (\$336,081 versus \$337,333) with higher treatment cost (\$35,367 versus \$34,944) and lower removal cost (\$5,163 versus \$5,418). Factoring the surveillance cost, the total costs of management are not different in the H and H-1 cases. However, the location of treatment and removal is sensitive to locations of infestation, and the optimal management focuses on the initially infested locations in all cases, as expected.

**5.3. Comparison with Heuristics**

We developed three EAB management heuristics based on a study of stakeholder preferences for EAB management in Twin Cities communities (Dunens et al. 2011).

**Figure 9.** (Color online) Number and Location of (a) Surveyed, (b) Treated, and (c) Removed Trees for High Initial Infestation Case Under Scenario 768



Each heuristic is based on premises or assumptions about managers’ ability to slow EAB spread and save ash trees. In addition to the three types of management strategies discussed earlier, we added three more heuristic strategies that are based on solving the M-SMIP model for only one specific scenario and fixing the solution for all other scenarios that involve surveillance: the worst-case, best-case, and expected-case scenarios. The six heuristic strategies that are used to compare with the M-SMIP optimization model are explained in detail.

**H1.** Staged removal of as many ash trees as possible under a fixed budget. For example, remove 20% of the public ash trees each year for five years. This strategy corresponds to the “doom and gloom” premise where EAB spread cannot be stopped and represents the approach taken by the city of Minneapolis. In our implementation, we considered removing 20% of the randomly selected public and private ash trees each year for five years without performing any surveillance effort.

**H2.** Monitor and remove ash trees when they are found to be infested. This strategy corresponds with the premise that EAB is not present yet, but it would be terrible if EAB became established. This is the management strategy taken by St. Paul. Note that insecticide treatment is not a part of this strategy. In our implementation, we considered performing surveillance, which is followed by removing 20% of the third-level infested public and private ash trees each year for five years.

**H3.** Random treatment of susceptible trees each year. For example, randomly select 20% of the susceptible public and private ash trees for treatment each year. McCullough and Mercader (2012) found using simulation models that annual treatment of 20% of ash trees annually protected 99% of trees after 10 years, and the cumulative costs of treatment were substantially

lower than the costs of removing dead or severely declining ash trees. This strategy corresponds with the premise that the problem is manageable and deemphasizes surveillance in favor of proactive chemical treatment. The strategy represents what Burnsville is currently doing, actively treating its public trees, which compose about 20% of the ash trees in the city, and not spending any money on surveillance. In our implementation, we considered treating 20% of the randomly selected public and private ash trees each year for five years without performing any surveillance effort.

**H4.** Worst-case scenario strategy. Solving the M-SMIP model only for the worst-case scenario, that is, high infestation realization in each period over five years (H-H-H-H-H), and applying this solution to all other scenarios with a five-year surveillance regime in the optimization model. This strategy applies surveillance each year but foresees that only the worst possible outcome will happen in the future.

**H5.** Best-case scenario strategy. Solving the M-SMIP model only for the best-case scenario, that is, low infestation realization in each period over five years (L-L-L-L-L), and applying this solution to all other scenarios with a five-year surveillance regime in the optimization model. This strategy applies surveillance each year but assumes that only the best possible outcome will happen in the future.

**H6.** Expected scenario strategy. Solving the M-SMIP model only for the expected scenario, that is, medium infestation realization in each period over five years (M-M-M-M-M), and applying this solution to all other scenarios with a five-year surveillance regime in the optimization model. This strategy applies surveillance each year but expects that a medium-level outcome will happen in the future.

Because dead trees do not impact the infestation, and their removal is taken care of by another department

**Table 3.** Comparison of the M-SMIP Optimization Model 1–16 with Six Different Heuristic Strategies

Infestation level	Economic valuation	Heuristic strategy						Optimization
		H1: Staged removal	H2: Monitor and remove	H3: Random treatment	H4: Worst-case scenario	H5: Best-case scenario	H6: Expected scenario	M-SMIP model
Low case	Objective value (\$)	318,005	477,617	411,361	476,136	476,131	484,877	484,877
	Surveillance cost (\$)	0	69,899	0	69,863	69,863	70,533	70,533
	Removal cost (\$)	886,754	363,294	0	6,710	6,692	1,861	1,861
	Treatment cost (\$)	0	0	196,821	554	553	4,113	4,113
	Total cost (\$)	886,754	433,193	196,821	77,127	77,107	76,507	76,507
	Total net benefit (\$) <sup>a</sup>	-568,749	44,424	214,540	399,009	399,024	408,370	408,370
	Improvement over heuristic (%) <sup>b</sup>	239.3	89.1	47.5	2.3	2.3	0.0	
Medium case	Objective value (\$)	236,564	399,428	331,616	385,089	385,048	385,089	437,207
	Surveillance cost (\$)	0	63,688	0	63,296	63,295	63,296	67,063
	Removal cost (\$)	850,448	489,262	0	43,517	43,243	43,527	32,568
	Treatment cost (\$)	0	0	200,269	2,250	2,247	2,250	21,925
	Total cost (\$)	850,448	552,950	200,269	109,063	108,784	109,073	121,555
	Total net benefit (\$) <sup>a</sup>	-613,884	-153,522	131,347	276,026	276,264	276,016	315,652
	Improvement over heuristic (%) <sup>b</sup>	294.5	148.6	58.4	12.6	12.5	12.6	
High case	Objective value (\$)	129,337	293,320	222,087	262,638	261,487	260,935	369,431
	Surveillance cost (\$)	0	55,236	0	54,522	54,495	54,491	62,107
	Removal cost (\$)	799,607	657,608	0	94,335	89,064	89,004	80,749
	Treatment cost (\$)	0	0	204,616	4,513	4,857	5,016	45,563
	Total cost (\$)	799,607	712,844	204,616	153,370	148,416	148,510	188,419
	Total net benefit (\$) <sup>a</sup>	-670,270	-419,524	17,471	109,268	113,071	112,425	181,012
	Improvement over heuristic (%) <sup>b</sup>	470.3	331.8	90.3	39.6	37.5	37.9	
Overall average	Objective value (\$)	227,969	390,122	321,688	374,621	374,222	376,967	430,505
	Surveillance cost (\$)	0	62,941	0	62,560	62,551	62,773	66,568
	Removal cost (\$)	845,603	503,388	0	48,187	46,333	44,797	38,392
	Treatment cost (\$)	0	0	200,569	2,439	2,552	3,793	23,867
	Total cost (\$)	845,603	566,329	200,569	113,186	111,436	111,363	128,827
	Total net benefit (\$) <sup>a</sup>	-617,634	-176,207	121,119	261,435	262,786	265,604	301,678
	Improvement over heuristic (%) <sup>b</sup>	334.7	189.8	65.4	18.2	17.4	16.8	

<sup>a</sup>Total net benefit (\$) = objective value – total cost.

<sup>b</sup>Improvement over heuristic (%) = (total net benefit<sup>M-SMIP</sup> – total net benefit<sup>Heuristic</sup>)/total net benefit<sup>M-SMIP</sup> × 100.

within the city, we do not consider the removal of dead trees in any of the heuristics and the optimization model for this computational analysis.

Table 3 compares the results for the six heuristic strategies (H1–H6), with the solution of the M-SMIP optimization model under low, medium, and high initial infestation cases defined in Section 5.2 over a budget of \$1.5 million. Here we also present the average results over all three cases in the last row block of the table referred as the “Overall average.” Specifically, we compare results in terms of the objective value defined in (16) (total expected benefits from ash trees), surveillance cost, removal cost, treatment cost, total cost (including the sum of surveillance, removal, and treatment costs), total net benefit (which is equal to the objective value minus total cost), and improvement over heuristic (which is the percent improvement

achieved by the M-SMIP optimization model) in terms of the total net benefits over each heuristic strategy considered. All instances in Table 3 were solved within 150 CPU seconds either with heuristic strategies or with the optimization model.

Clearly, the M-SMIP optimization model provides the best objective value, that is, total expected benefits from ash trees compared with the other six heuristic strategies. Comparing the total net benefits, we also observe that the M-SMIP optimization model gives significantly better solutions than any other heuristic strategy. When looking at the average improvement over heuristic (%), the best heuristic strategy after the M-SMIP optimization is H6, the expected scenario strategy, followed by H5, H4, H3, H2, and H1, respectively. However, the M-SMIP model improves the H6 strategy by 16.8% on average. The improvement

of the M-SMIP model over heuristics increases as the initial infestation scenario moves from low to high infestation.

The M-SMIP model improves net benefits over the H3 (random treatment) strategy, which is the management approach taken by Burnsville, by 65.4% on average. The average improvement by the M-SMIP model over the H1 (staged removal) and H2 (monitor and remove) strategies is 334.7% and 189.8%, respectively. The main drawback of heuristics H1 and H2 is the high cost of removal, which leads to negative total net benefits in most cases. Treatment is also costly; however, treating 20% of the ash trees each without surveillance (H3 case) performs better than the H1 and H2 strategies, consistent with the results of McCullough and Mercader (2012). The performance of the optimal M-SMIP strategies relative to the performance of the heuristic strategies highlights the importance of surveillance, which focuses our limited budget on managing only infested trees to reduce our removal or treatment costs rather than randomly selecting trees to treat or remove.

By contrast, heuristic strategies H4, H5, and H6 assume that only one particular scenario (worst, best, or expected) will happen in the future, and thus each represents a deterministic version of the M-SMIP model. Based on the improvement over heuristic results, we observe that a multistage stochastic model is clearly superior to its deterministic version, improving the deterministic results by 16.8%–18.2% on average.

#### 5.4. Effect of Surveillance Timing on Net Benefits of Ash Trees

The timing of surveillance affects the net benefits of management. Although surveillance is applied to the entire landscape, the number of surveyed trees depends on the size of the initial infestation and the timing of the former surveillance (see Figure 1 for a detailed explanation). To examine the effects of surveillance decisions, we computed optimal solutions for the Burnsville landscape under the three initial infestation cases with a budget of \$95,000. Scenario 768, which applies surveillance only in the first period, maximized net benefits in all three cases. We compared the net benefits and costs associated with scenario 768 with the net benefits and costs of scenarios that have the same infestation realizations over time (L-L-L-L-L) but differ in their surveillance regime (704, 688, 684, and 683). Surveillance is applied in the first two, three, four, and five periods for scenarios 704, 688, 684, and 683, respectively. Net benefits decrease as the number of surveillance periods increases for all three levels of initial infestation (Figure 10(a)), suggesting that surveillance should be applied only in the first year of the planning

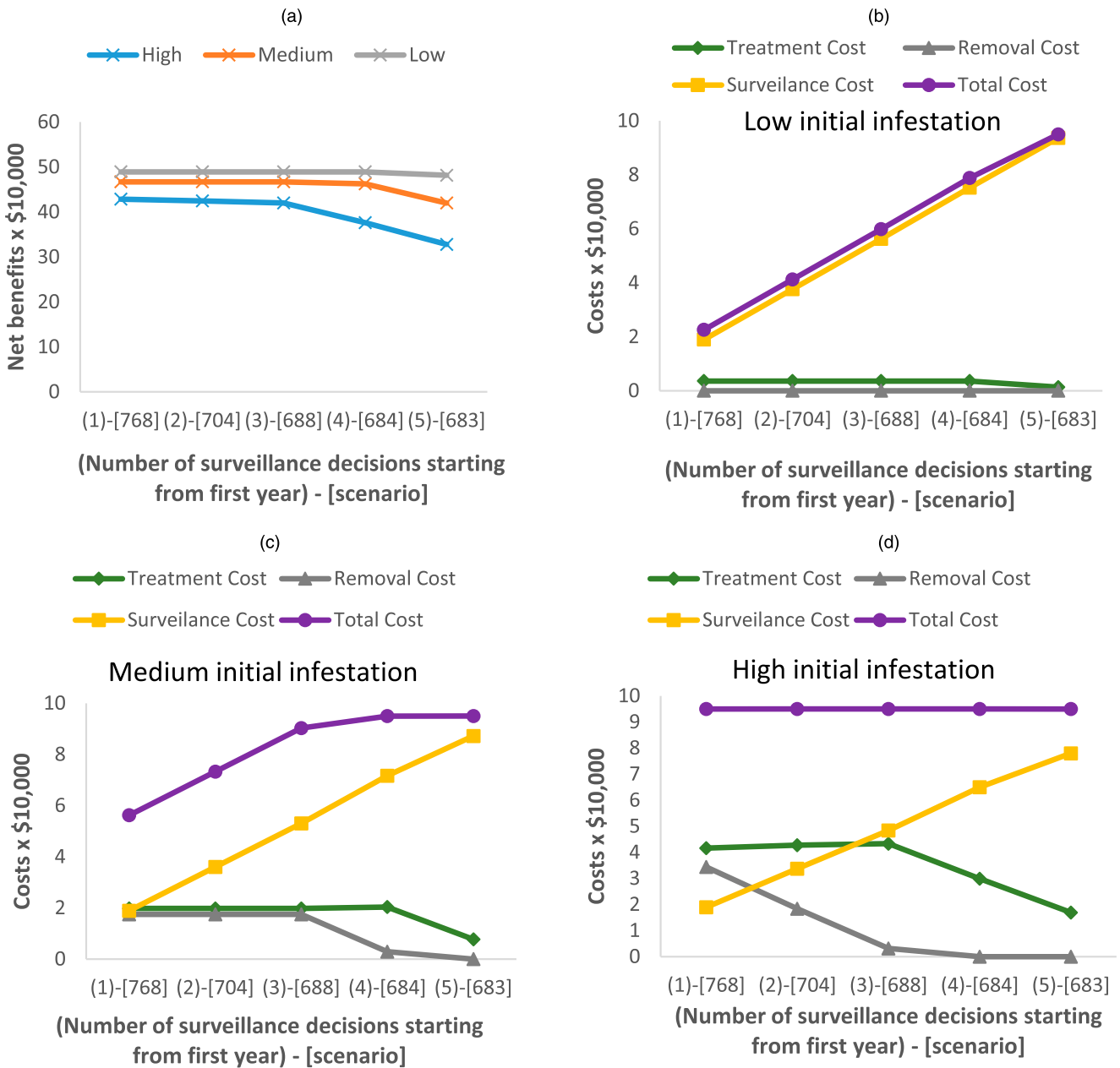
horizon. The reason is that surveillance in the first year is sufficient to detect infested trees, which are subsequently treated or removed with the given budget allocation. Increasing the number of surveillance periods takes more of the budget and reduces the amount that can be spent on treatment and removal to slow ash mortality. In the low-infestation case (Figure 10(b)), there is no removal cost under all scenarios because the initial infestation is low, and the budget is big enough to treat all the trees in the first year, resulting in no dead trees in following years. For moderate and high initial infestation cases (Figure 10, (c) and (d)), removal and treatment costs are higher than the low initial infestation case when surveillance takes place in the first year (scenario 768). Then removal and treatment costs decrease as surveillance is repeated consecutively over multiple years because more of the budget is needed to cover the cost of surveillance. We also notice that total cost increases for the low and medium initial infestation cases (Figure 10, (b) and (c)) because of the increasing cost of repeated surveillance. By contrast, the total cost remains constant for the high initial infestation case because there are more infested trees, which consume most of the budget for treatment and the remaining budget for removal of the highly infested trees.

We also investigated the loss in net benefits when surveillance is delayed over one-, two-, and three-year periods. Figure 11 shows the effects of delay using scenarios that have low-infestation realizations over the five consecutive years (L-L-L-L-L) for the three initial infestation cases. The loss in net benefits increases exponentially as surveillance is delayed in time. Further, the loss increases as the initial infestation increases from low to high.

#### 5.5. Rules of Thumb for Treatment and Removal

We examined how priorities for treatment and removal changed under different budgets. We solved the optimization problem for the Burnsville landscape, the high initial infestation case, and budgets increasing from \$90,000 to \$120,000. Optimal treatments and removals over time for scenario 768, which maximizes net benefits for L-L-L-L-L (all low realizations over the first five periods), are shown in Figure 12 using budgets of \$90,000 and \$120,000. Under both budgets, surveillance takes place in year 1, and priority is given to treating all trees in infestation levels 1 and 2. The treatments prevent the movement of second-level trees to the third level in the following period. Once all the first- and second-level infested trees are treated, then the remaining budget is allocated to removing the third-level infested trees. As the budget increases from \$90,000 to \$120,000, all the highly infested trees are removed. All infested trees are

**Figure 10.** (Color online) (a) Net Benefits and Costs for (b) Low, (c) Medium, and (d) High Initial Infestation Cases with \$95,000 Budget Given Surveillance Decisions in Consecutive Years Starting from Year 1



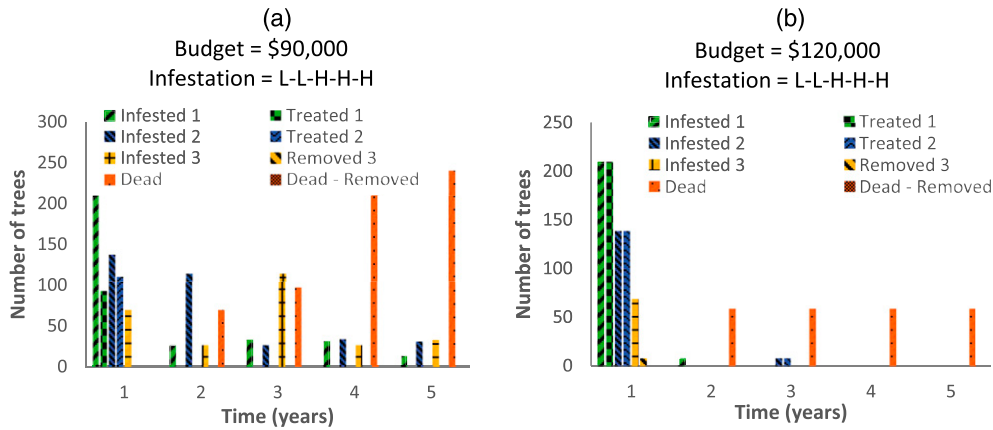
either treated or removed in the first period, and surveillance is not required in the following periods.

Note that it is crucial to initially treat trees in the second infestation level followed by trees in the first infestation level because this prevents trees from transitioning into upper infestation classes where they may have more impact on susceptible trees. Furthermore, although the third infestation level poses the highest threat to susceptible trees in the first period, they will transition to dead trees and will no longer spread the infestation. Therefore, treating lightly infested clusters is given priority to removing highly infested trees.

We also examined the treatment and removal priorities for the optimal solution (scenario 705) when the infestation realization is L-L-H-H-H (Figure 13). With a budget of \$90,000, all the funds are spent in year 1 for surveillance and treatment of first- and second-level infested trees, which significantly reduces the size of the infestation and threat to susceptible trees. As a result, the number of infested trees remains small through the five-year period, whereas the number of dead trees increases rapidly. When the budget increases to \$120,000, all the first- and second-level infested trees are treated, whereas a small number of the third-level infested trees are removed in



**Figure 13.** (Color online) Total Number of Infested, Treated, and Removed Trees Over Time for Scenario 705 with Budgets of (a) \$90,000 and (b) \$120,000 and Infestation Realization L-L-H-H-H



These insights gained from our modeling experience address two quandaries facing managers: whether to invest in surveillance and to treat healthy looking ash trees. Managers are using these insights to inform their surveillance and control strategies because the alternative of conducting field experiments to gain insights would be very expensive and time-consuming.

Unless the EAB population is cleared from all ash trees, which is a very difficult goal to achieve, the growth and dispersal of the species continue until all ash trees are infested or killed. In multistage stochastic programming, we may observe end-of-horizon effects on the optimal policy, which are distortions in the model decisions because the model has a finite or limited horizon, whereas the real invasive species problem usually has an infinite period or a much longer horizon than the considered one. For example, it is possible that the initially invaded areas may not be totally cleared of infestation with the given budget in a five-year time horizon; however, the available budget is generally consumed within five years because of the consideration of a finite planning period. As opposed to considering five years, if we formulate a 10-year horizon in the model, we would

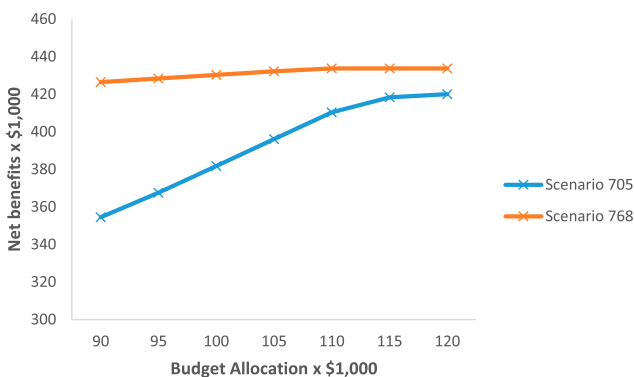
expect that not all funds are consumed by the end of year 5 and that some of the budget is left to treat future infestations between years 6 and 10. However, no matter what the infestation case and planning horizon are, it is still crucial to immediately address an anticipated infestation by performing surveillance promptly followed by treatment or removal. Thus, even if the planning horizon is extended to 10 years, we expect that most of the budget is consumed for surveillance and treatment in the initial few years.

Surveillance is not perfect. With visual surveillance, false negatives can happen much more than false positives, which are rare. Data are not available on the detection rates, and we are not aware of any studies that estimate the detection rate (probability of detecting an infestation when it is present) associated with visual surveillance. Our guess is that the detection rate is less than 100% for trees in infestation level 1 because very low densities of larval infestation do not cause visible signs of canopy dieback or woodpecker damage.

Whereas the lack of data on detection rates is a real limitation, in a future study, the efficacy of surveillance could be handled by incorporating a detection rate parameter into the model. The detection rate could be defined as the percentage of infested trees that are correctly identified as infested after surveillance (e.g., 80% for trees in class 1 and 90% for trees in class 2). The imperfectness of surveillance could also impact the number of infested trees that are treated and removed correctly. Thus, a future extension of the model can incorporate the detection parameter into Equations (9)–(11) to reflect the imperfectness of the surveillance, treatment, and removal applications under detection uncertainty.

The number of constraints and variables increase exponentially as the problem size increases spatially and temporarily. We use a desktop computer with an 8-core/64-GB random-access memory/Windows

**Figure 14.** (Color online) Net Benefits for Different Budget Allocations for High Initial Infestation for Scenarios 768 and 705





operating system to solve the proposed M-SMIP model. The proposed model could be solved for up to a  $6 \times 6$  gridded landscape over a five-year period (~4.3 million constraints) with ample budget allocation, and the problem size was reduced to a  $5 \times 5$  gridded landscape (~3 million constraints) to solve the problem with a tight budget allocation. Therefore, although the solution time is highly improved with the proposed algorithm, different algorithms could be developed to solve the proposed model more efficiently.

The unique model structure with efficient constraints significantly reduces the size and complexity of the problem. We initially fixed the discretized binary variables to represent all possible surveillance decisions in the model. Therefore, we prevent nonlinearities in the model, especially in Equations (1a), (1b), and (4), thus facilitating solution performance by considering all possible surveillance decisions. We further reduce the size of the problem by providing a surveillance decision for the entire landscape instead of for each site. However, the elegant formulation of Equations (1a) and (1b) by clustering infestation classes and tracking each infested tree prevents unnecessary surveillance of infested trees at each cell over a planning horizon. Therefore, discretizing the surveillance decisions makes the proposed model of practical importance.

Our numerical results indicate that the preprocessing algorithm significantly improves the solution time. The integration of cuts obtained from the preprocessing problem into the model reduces the size of the branch-and-bound tree and tightens the upper bound of the problem. This, in turn, improves the solution time by up to an average of seven times compared with solving the original model. The improvement in the solution time is more apparent for larger instances because of the increased number of sites in a given landscape and thus the increased number of constraints and variables.

Additional work could focus on developing new decomposition methods to solve the proposed model for bigger instances. Furthermore, the discretized binary surveillance parameters could be replaced with binary decisions at the expense of increasing the problem complexity and thus the solution time.

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