OPTIMAL DESIGN AND OPERATION OF A HYBRID GAS/ELECTRIC CHILLED WATER PLANT

by

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ABSTRACT

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The design of a chilled water plant involves selecting the size and type of chillers to be employed and determining the operating strategy. The types may include both gas engine and electric motor driven chillers. The issues that have to be considered in the selection problem are to incorporate external and internal factors into the decision making. External factors may include the utility rate schedules, the cooling load profile, and the outdoor temperature profile. Internal factors may include the chiller performance characteristics, initial and maintenance costs, and the chiller(s) operating strategy.

A mathematical model representing the chilled water plant design problem is developed. The problem is approached as a mixed integer linear programming problem where non-linear chiller performance curves are transformed into linear constraints through the use of integer variables. The optimization task is to select the best cooling plant configuration and operating strategy to minimize life cycle cost.

A solution procedure is developed which decomposes the optimization problem to reduce extensive computation time. Two case studies are provided to investigate the implementation of the mathematical model.

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TABLE OF CONTENTS

TABLE OF FIGURES

TABLE OF TABLES

NOMENCLATURE

A_t	initial cost parameter in LCC objective function pertaining to chiller of type t, \$/unit
a_1, a_2, a_3, a_4, a_5	piecewise linear coefficients of node variable, non dimensional
AC	annual cost, \$
AEC	annual energy cost, \$
ADC	annual demand charge, \$
AMC	total annual maintenance cost for all chillers, \$
AMC_i	annual maintenance cost for chiller i, \$
$B_{t,d,j}$	maintenance cost parameter in the LCC objective function pertaining to chiller of type t, at load condition j within demand period d, \$/ton-hr
b_i	fixed initial cost for chiller i, \$
$C_{t,d,j}$	full capacity energy cost parameter in the LCC objective function pertaining to chiller of type t, at load condition j within demand period d, \$
$D_{d,j,k}$	the demand at load condition j within demand period d associated with energy resource k, kW
$DC_{d,k}$	the demand charge for demand period d associated with energy resource k, \$
DS_d	maximum power consumption as the distribution demand during demand period d, kW
d	demand interval index, d = 1, …, D, non dimensional
$dist_d$	distribution charge for demand period d, \$/kW
$E_{d,j}$	amount of energy use at load condition j within demand period d, kJ
$EC_{i,d,j,k}$	energy cost associated with energy resource k of chiller i at load condition j within demand period d, \$
$EC_{d,j}$	energy cost at load condition j within demand period d, \$
$e_{d,j,k}$	cost of energy per unit of energy resource k, \$/kWh
FC	total fixed initial cost (FC) for all chillers, \$

$FF_{d,j}$	common plant load fraction at load condition d within demand period d, $0 \leq FF_{d,j} \leq 1$, non dimensional
$FFN_{t,d,j}$	common fractional power consumed by chiller of type t, at load condition j within demand period d, non dimensional
$FN_{t,i,d,j}$	fractional power consumed by chiller i of type t at load condition j within demand period d, non dimensional
$f_{i,d,j}$	fraction of maximum cooling capacity produced by chiller i, operating at load condition j demand period d, $0 \leq f_{i,d,j} \leq 1$, non dimensional
$f_{t,i,d,j}$	fraction of maximum cooling capacity produced by chiller i of type t, operating at load condition j demand period d, $0 \leq f_{t,i,d,j} \leq 1$, non dimensional
g_i	the fixed annual maintenance cost associated with chiller i, $
$H_{d,j}$	electric power use at load condition j within demand period d, kW
$h_{d,j}$	duration of load condition, hr
i	machine index, $i = 1, \ldots, I_t$, non dimensional
I_t	the maximum number of chillers type t to be purchased, non dimensional
j	load condition index, $j = 1, \ldots, J$, non dimensional
k	energy resource index, (k = 1: electric; k = 2: gas), non dimensional
$L_{i,d,j}$	load carried by chiller machine i at load condition j demand period d, kW
LCC	present value life cycle cost, $
N_t	the number of chillers of type t to be purchased, non dimensional
$n_{i,d,j,k}$	ratio of power usage to rate of cooling delivered by chiller i at load condition j demand period d utilizing energy resource k, kW/ton
$n_{t,i,d,j,k}$	ratio of power usage to rate of cooling delivered by chiller i of type t at load condition j demand period d utilizing energy resource k, kW/ton
OC_m	variable operating cost of configuration m , $
OFF_d	off peak power consumption during demand period d, kW
ON_d	on peak power consumption during demand period d, kW
off_d	off peak demand charge for demand period d, $/kW

on_d	on peak demand charge for demand period d, $/kW
p_i	maximum cooling capacity of chiller i, tons refrigeration
p_t	maximum cooling capacity of chiller type t, tons refrigeration
ptc_t	ratio of power to rate of cooling at full load capacity for chiller type t, kW/ton
$R_{1,t,d,j}$, $R_{2,t,d,j}$, $R_{3,t,d,j}$, $R_{4,t,d,j}$, $R_{5,t,i,d,j}$	fractional power usage at 10 %, 40 %, 70 %, and 100% capacity by chiller i of type t at load condition j within demand period d, non dimensional
R	annual discount rate or annual money value difference, non dimensional
T	chiller type index, t = 1, …, T, non dimensional
T	number of chiller types in the selection group, non dimensional
$TL_{d,j}$	total cooling load at load condition j, tons refrigeration
$u_{1,t,i,d,j}$, $u_{2,t,i,d,j}$, $u_{3,t,i,d,j}$, $u_{4,t,i,d,j}$	auxiliary variables for the Special Ordered Sets (SOS) type 2 pertaining to individual chiller i of type t load condition j within demand period d, integer, non dimensional
$u_{1,d,j}$, $u_{2,d,j}$, $u_{3,d,j}$, $u_{4,d,j}$	auxiliary variables for the Special Ordered Sets (SOS) type 2 pertaining to plant common load fraction at load condition j within demand period d, integer, non dimensional
v_i	the variable maintenance cost associated with chiller i, $/ton hr
v_t	the variable maintenance cost associated with chiller type t, $/ton hr
y	year index, y = 1, …, Y, non dimensional
Y	the chiller life time, years
$z_{1,t,i,d,j}$, $z_{2,t,i,d,j}$, $z_{3,t,i,d,j}$,$z_{4,t,i,d,j}$, $z_{2,t,i,d,j}$	auxiliary variables for the node selection of the piecewise linear pertaining to individual chiller i of type t, non dimensional
$z_{1,d,j}$, $z_{2,d,j}$, $z_{3,d,j}$, $z_{4,d,j}$, $z_{5,d,j}$	auxiliary variables for the node selection of the piecewise linear constraints pertaining plant common load fraction at load condition j demand period d, non dimensional

$\gamma_{t,i}$	purchase decision variable pertaining to chiller i of type t, binary integer, non dimensional
$\delta_{t,i,d,j}$	operation decision variable pertaining to chiller i of type t at load condition j within demand period d, binary integer, non dimensional

1. INTRODUCTION

This chapter introduces the background of the problem and states the scope and the organization of the thesis.

1.1. Background

In large commercial and some industrial facilities, the refrigeration and air conditioning plant is one of the primary energy users. With the energy bill representing a significant portion of the operating cost to many facilities, it is natural for the facility owner to attempt to reduce spending for energy. Advances in chiller technology as well as changes in the regulatory and economic environment have created opportunities for reducing energy costs. In order to evaluate these opportunities, building owners and designers need design and optimization tools that can evaluate a variety of complex factors.

Advances in chiller technology include the emergence of engine driven chillers and the development of refrigeration machines with better performance under part load conditions. Refrigeration machines using gas engine driven compressors have been developed in various sizes ranging from 25 tons to 2100 tons to meet the needs of a variety of applications. Both electric motor driven and engine driven chillers have also been improved to operate with a lower ratio of power use to load (kW/ton) when operating at part load as well as at reduced entering condenser water temperatures.

Changes in the regulatory and economic environment include the halt in production of CFC's as the working refrigerant fluid, and deregulation in the electricity market. The phasing out of CFC's as the working refrigerant is mainly driven by environmental considerations. In 1985, the World Meteorological Organization and the United Nations Environment Program (WMO/UNEP) identified CFC's as a contributing factor in the depletion of the ozone. With the adoption of the Montreal Protocol in 1987, it was decided to reduce the use of CFC's by one half by 1998. The Copenhagen Amendments in 1992 scheduled the halt in production of CFC's by the developed countries in 1996. The halt of CFC production in 1996 reduced its supply to the market leading to a significant increase in price. The high cost of CFC's often makes continued operation of old refrigeration machines uneconomical. Thus, it may be attractive to upgrade or replace the refrigeration machines.

The electricity market is undergoing a significant change with the introduction of deregulation in the electric power industry. Prior to deregulation, electricity prices were determined by a single utility company servicing the area in which the customer was located. With the deregulation of electricity, the customer has the option to select among different electric utility rate structures to provide the most economical source of power. This is essentially enabling the customer to obtain the least cost supply of electricity at the expense of increasingly complex decisions associated with energy and energy using equipment.

A typical electricity rate structure established for an industrial or a commercial facility would include both demand charges and energy charges. Furthermore, the demand and energy charges are often based on a schedule with respect to the demand period, day of the week, and hour of the day.

Natural gas is typically priced according to a declining block schedule. With this schedule, a rate is established for the first predetermined block of gas consumption during a demand period. The subsequent consumption of gas will then be charged at a reduced rate. The gas utility typically offers reduced gas rates during summer months.

When planning a new facility or upgrading an existing facility the designer must consider the available energy rate structures as well as cooling requirements, chiller first cost, and operation and maintenance costs.

1.2. Scope of Thesis

The conventional strategy to address the above issues is to procure more than one chiller and to operate the chillers to minimize operating and maintenance costs. The selected chillers may include chillers driven by electric motors or gas engines. In addition, the size of the chillers may also vary. This thesis will develop an optimization approach for selecting and operating chillers to minimize life cycle cost (LCC). The task of minimizing the LCC is approached as a mixed integer linear programming problem. The optimization will incorporate input data for cooling load profiles, chiller cost, chiller performance, and utility cost data. The analysis will be based on the first cost and operating cost, all of which are evaluated at the present time. The option yielding the minimum cost for the entire life cycle will be selected. To accomplish this goal, an optimization problem statement and the associated constraints will be developed and solved numerically to determine the chiller plant configuration and operating strategy that minimizes LCC. Two particular case studies will be presented to illustrate the application of the technique.

The steps undertaken as part of this work include:

1. Development of a mathematical model including the objective function and constraints to represent the chiller selection and operation problem
2. Development of solution approach to the optimization problem
3. Application of the optimization approach to two case studies
4. Evaluation of the approach and recommendations for future developments

1.3. Organization Of Thesis

The thesis is organized into the following chapters:

CHAPTER 1	The background and issues that serve the foundation of the problem are introduced. The objective and scope of the thesis are stated.

CHAPTER 2	A review of literature relevant to the problem is presented.
CHAPTER 3	The problem statement and key assumptions utilized in the development of the problem formulation are stated. The formulation of the mathematical statement representing the problem that includes the objective function and the constraints is presented.
CHAPTER 4	The development of a solution approach to the problem represented by the formulation in Chapter 3 is presented. The solution procedure by which the model may be solved is described.
CHAPTER 5	The implementation of the model is described including a discussion of the validation of the model and the solution procedure, and a description of the solver, platform and interface.
CHAPTER 6	Two case studies are presented. This chapter describes the inputs, and the results of case studies.
CHAPTER 7	The conclusion of the thesis is presented along with recommendations for future modeling of the chiller selection problem.

2. Literature Review

The optimal selection and operation of chillers in a hybrid gas-electric chilled water plant requires the application of optimization techniques as well as an understanding of the performance characteristics of the various types of chillers. The following sections review the literature related to chiller performance characteristics and chilled water system optimization.

2.1. Chiller Performance Characteristics

Chiller performance is typically characterized in terms of the coefficient of performance (COP) which is defined as:

$$\mathrm{COP} \equiv \frac{\text{Cooling effect at the evaporator}}{\text{Rate at which energy is supplied to drive the chiller}}$$

where motive energy may refer to electricity or gas.

Prior work has led to the development of analytical and empirical models that describe chiller performance in terms of key design variables. Since the early developments, the key design variables in chiller models have been expanded significantly up to the present. Allen and Hamilton (1988) identified key design variables incorporated in the simple chiller performance models such as DOE-2 and BLAST. DOE-2 incorporated only one independent variable, i.e., the evaporator cooling load, to determine chiller power consumption. The BLAST model being more complicated than DOE-2 accounts for the exiting condenser and exiting evaporator water temperatures.

Many mathematical models have been developed for steady state chiller operation. Allen and Hamilton (1988) developed a model to determine chiller power use accounting for evaporator and condenser water mass flow rates, entering condenser water temperature

(ECWT), and the entering evaporator water temperature (EEWT). Klein (1992) developed a model that relates the COP to EWCT, EEWT, and the effectiveness of the condenser and evaporator heat exchangers. Ng *et al.* (1997) and Gordon *et al.* (1994) have developed and verified an analytical model that relates the COP to ECWT, EEWT, and the cooling load.

A recent model developed by Browne and Bansal (1998) provides steady state simulation with the capability to account for superheating and sub-cooling in the evaporator and condenser heat exchangers, respectively. This model takes the general parameters such as water mass flow rate, exiting evaporator temperature and ECWT as input.

Based on these models, the critical operating parameters for determining chiller performance are cooling load, condenser water temperature, and evaporator water temperature. Empirical equations describing the performance characteristics of typical chillers as functions of these parameters have been incorporated in chiller modeling software such as YorkCalc developed by York International Co. (1997). This program and others like it provide the capability to calculate annual energy cost incorporating part load performance, weather data, and electric or gas rate structures through the bin temperature method. The use of such software requires the designer to select the type and size of chillers based on trials of cost comparison after determining the chiller operation strategy.

2.2. Optimization of Chilled Water Systems

A chilled water system may be optimized to minimize energy use, first cost, life cycle cost, etc. This section reviews previous work related to the optimization of chilled water systems.

Chiller energy use may be minimized by the proper selection of operating strategy. The maximum COP of a typical chiller does not occur at maximum load but at off-design load or fractional loading. For off-design operating hours, Austin (1991) suggests that multiple chillers be operated at fractional loading to yield the lowest possible power consumption at a given load.

Oetjen (1984) developed an approach to optimizing the operation of multiple chillers using the Lagrange Multiplier method for a given system configuration. The approach determines the fractional loading of each chiller to yield minimum power consumption of a certain system configuration. The best system configuration is determined by comparing the minimum power consumption of a finite number of system configurations based on a given number of chillers.

Chiller energy use may also be reduced by proper selection of operating conditions. Kaya (1991) described an approach to minimize the overall system energy cost by employing a control scheme that minimizes the entering condenser water temperature and maximizes the exit evaporator water temperature while satisfying the cooling load requirement. This approach reflected the trade-off between the decreased cost of compressor operation and the increased cost of pumping due to the adjusted water temperatures.

A systemic approach to determine minimum energy use is suggested by Bellenger *et al*. (1996). The proposed approach includes both the refrigeration cycle and the condenser water system in the analysis. Bellenger maintains that with increasingly efficient chillers, the impact of the auxiliary equipment such as chilled water pumps, condenser water pumps, and cooling tower fans on the overall energy use of the system must be taken into account. The need to include the impact of the auxiliary equipment in the overall energy analysis is due to the modesty of the improvements in pump and fan efficiency.

Beyene (1995) introduced an approach that uses a penalty factor to discourage the operation of chillers at load fractions deviating from the optimum condition. The optimum load fraction is considered to be one that minimizes power usage to deliver a certain cooling load (kW/ton) of a chiller. The difference of the overloaded or underloaded kW/ton to the optimum load fraction multiplied by the number of hours the chiller operates at the overloaded or underloaded condition will represent the additional energy cost charged to the operation of the system. The total additional energy cost throughout the annual operation is aimed to be minimized.

Eppelheimer (1996) describes a strategy that varies the condenser and evaporator flow to achieve system energy efficiency. This method aims at controlling condenser water flow based on the head pressure in order to reduce pumping energy.

Hartman (1996) describes the penalties associated with varying water flow. The penalties include first-cost penalty for employing multiple pumps, part load efficiency penalty for having the bypassed supply chiller water mixed with the return chilled water, and chiller capacity penalty for underloading the chiller at non peak design condition.

As the ECWT is affected by the change in outdoor wet bulb temperature, the chiller COP becomes dependent upon the outdoor wet bulb temperature. Therefore, a system approach considers the chiller performance coupled to the condenser cycle performance, which is related to the performance and size of the cooling tower. Braun *et al.* (1989) devised an effectiveness model in which the correlation between various parameters of the cooling tower and the outdoor temperatures was developed. The availability of the effectiveness model makes it possible to conduct an analysis relating the chiller COP to the outdoor wet bulb temperature.

The determination of the cooling tower size or NTU is in itself an optimization problem. A greater cooling tower size translates into a higher capital cost for the cooling tower although its greater size enables it to deliver a lower entering condenser water temperature or a higher temperature difference. Meyer *et al.* (1994) maintains that from a total (cooling tower – chiller system) life cycle cost point of view, the optimum cooling tower size is determined from the dependency of chiller COP on the entering condenser water temperature.

Life cycle cost analysis has been applied to determine the optimum solution to chiller plant configuration. Arnold *et al*. (1998) applied life cycle cost analysis to hybrid cooling plants in which multiple chillers of different energy sources are to be employed. In this analysis, gas chillers were used during peak electrical demand periods to reduce demand charges. In addition, the author identifies a number of issues that must be considered in the design of a hybrid plant. These issues include the capacity or load distribution among the chillers in operation, acoustics, heat recovery availability, gas engine weight, and engine maintenance. The approach compared life cycle cost limited by the simple payback period for different configurations. The life cycle cost analysis incorporated the installation cost, fuel or utility rate structure, maintenance cost, and load duration curves. However, this approach does not reflect the part-load performance characteristics of the equipment and is limited in its ability to address more complex utility rate structures.

An attempt to develop a methodology for selecting and operating chillers to minimize energy consumption was conducted by Jones (1986). The author establishes weighting factors to base the decision on which chiller configurations are selected. The approach uses life cycle cost at present value incorporating energy rate structure, energy and demand, and capital costs. This method examines the energy use on a total annual basis. The system design takes into account the energy use of the chiller water pump, condenser water pump, and the cooling tower fans. However, the method proposed by the author considered the bin method as the basis of analysis where only total hours of operation and total energy use is taken as part of the analysis.

Thermoeconomic approaches to minimize chiller energy consumption have been described by Tozer (1997), Cammarata *et al.* (1997), and d'Accadia (1998). The thermoeconomic approach makes it possible to determine the balance between the equipment cost and the energy cost through the use of availability analysis for absorption chiller systems.

In general, previous work has primarily focused on the optimal operation of a given chilled water system. The problem of selecting chiller types and sizes to minimize life cycle cost has not been adequately addressed. The goal of the current research is to develop a general approach to the selection and operation of chillers in a hybrid chilled water plant that minimizes life cycle costs while considering equipment operating characteristics and utility rate structures.

2.3. Optimization Techniques and Tools

This section describes the optimization techniques applicable in solving the chiller optimization problem. Mathematical models of chiller performance are typically non-linear with respect to chiller load fraction. In addition, the chillers are typically subjected to an on-off operation scheme during any load condition necessitating integer decision variables. Thus the chiller optimization problem is a non-linear mixed integer programming problem.

An acceptable approach to implement and solve the chiller optimization problem may be to pursue non-linear optimization by first linearizing the problem and solving it with the aid of developing linear constraints to represent the non-linearity. The problem of determining chiller configuration may fall under combinatorial optimization. A formal technique of solving combinatorial optimization problems is to implement the branch and bound technique.

Problems involving a mix of integer and continuous variables are often solved using branch and bound techniques. The branch and bound technique sets the selected integer values and relaxes the remaining integer variables down the tree of branching. The sub problem is then solved as a relaxed linear problem as if the sub problem does not contain integer variables. The solution of the relaxed problem will be the lower bound. Solutions having integer values for the integer variables are taken as the feasible solution. For a minimization problem whenever a feasible solution is found, it will be taken as the upper bound unless it is worse than the previous feasible solution. As the iteration proceeds, all other solutions are compared to the upper bound. The best solution is determined by various criteria such as the tolerance to which a fractional value for the integer variable is considered an integer or by comparison of the best integer feasible solution to the best node cut value. Garfinkel (1979) identified that the branch and bound technique is often able to obtain the best feasible solution early in the iteration process whereas verifying it to be the optimum solution requires extensive iteration count. Therefore, a stopping criterion may be applied to confine the computation time while still approaching near-optimal solutions.

Many commercial programs have been developed for the task of solving linear optimization problem, e.g. CPLEX (Ilog, 1997), MINOPT (Floudas and Schweiger, 1998), and LINGO (Lindo Systems Inc., 1999). These programs employ the branch and bound technique as the method in solving mixed integer continuous linear problems.

The chiller selection problem may also be approached as a constrained non-linear problem. Extensive research has been conducted in the area of non-linear programming (NLP) to solve non-linear optimization problems. Research in non-linear programming include the development of NLP algorithms, the development of deterministic Mixed Integer NLP (MINLP) algorithms for specific types of problems, and the development of heuristic procedures.

Based on the investigation of NLP codes by Schittkowksi (1980), the two most powerful NLP algorithms are those classified as Successive Quadratic Programming (SQP) and Generalized Reduced Gradient Method (GRG) (Reklaitis *et al.*, 1983). These two techniques are capable of solving continuous non-linear problems rigorously. Challenge in the application of NLP to the chiller selection problem is to address the integer decision variables. In addition, Bertsekas (1982), (1995) emphasized that the gradient based algorithms depend on the differentiability of the functions in the objective statement and in the constraints.

The integer decision variables may be applied directly when MINLP algorithms are applied to the chiller selection problem. MINLP algorithms have been developed by Grossman (1990) and Floudas (1995). The algorithms for solving MINLP are incorporated into the MINOPT code using Generalized Bender Decomposition (GBD), Outer Approximation and Variants (OA), and Generalized Cross Decomposition (GCD) (Floudas and Schweiger, 1998). The algorithms and solution techniques developed in MINOPT are applicable to various types of problems and are restrictive to certain platforms.

Heuristic approaches to solving NLP problems include Genetic Algorithms, Simulated Annealing, and TABU algorithms (Goldberg, 1989), (Michalewicz, 1996). Furthermore, Wallace (1996) and Olsommer *et al.* (1999) extended the use of the heuristic approach to become the basis for developing MINLP problems for specific types of problems.

The availability of many optimization techniques enables the task of approaching the goal of the current research conducted from different directions. Thus the problem formulation may take many different forms depending on the optimization technique employed. The current research will approach the problem as a mixed integer linear problem. This approach takes advantage of the availability of commercial software that are capable of solving large numbers of mixed integer variables and use the type of platform available in conducting this research.

3. Problem Formulation

3.1. Overview

Chilled water requirements in large commercial and industrial facilities are typically served by the operation of a central plant. Multiple chillers are often employed in the chilled water plant to meet cooling requirements that may be larger than the capacity of a single chiller machine. The application of multiple chillers offers flexibility to the plant operation in that partial cooling can be provided even if some of the machines are not operational. When all machines are operational, it is possible that only certain machines are operated to meet a particular cooling requirement. The machines in operation may also be operated at part load with lower energy usage.

Multiple chillers are usually configured in parallel. A typical multiple chiller configuration is shown in Figure 1. In this configuration each chiller receives returning water from the facility at the same temperature. The water exiting each chiller evaporator is then mixed and supplied to the facility.

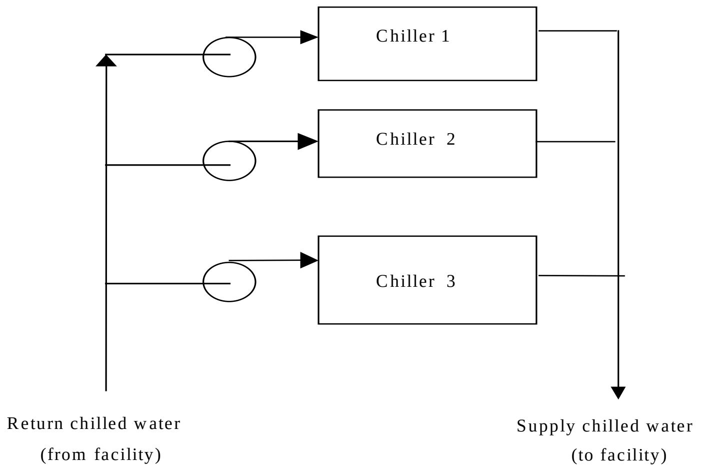

Figure 1. Diagram of multiple chillers in parallel configuration

The problem associated with designing multiple chiller plants is to select the appropriate chiller(s) from a group of chillers available to the designer while considering the proper chiller operating schedule.

In the present work, the selection criterion for the chiller mix is life cycle cost. The life cycle cost will reflect all costs over the entire life of the equipment, including initial, operating, and maintenance costs all of which are evaluated at the present time. A mathematical model and accompanying constraints will be developed to describe the chiller selection and operation problem. The various factors included in the mathematical model are (i) chiller performance with respect to partial load and ECWT, (ii) associated energy and demand charges, and (iii) maintenance costs.

3.2. Major Assumptions

This section describes the major assumptions used in the development of the mathematical model of the optimization problem. The assumptions include:

1. Parallel configuration for multiple chillers
2. Water-cooled condensers
3. Steady-state operation within a load condition
4. Equal design temperature drop across the evaporator for all operating chillers
5. Heat transfer to or from the condenser water piping is negligible
6. Each year consists of D demand periods with each demand period consisting of J load conditions each of which has a duration of h hours. Therefore, the number of load conditions annually amounts to DJ load conditions.
7. Chillers are driven either by gas engines or electric motors.
8. Demand charge applies only to electric motor driven chillers
9. The maintenance cost for each machine is assumed to consist of a fixed cost component and a variable cost that depends on the amount of cooling provided by the chiller
10. All types of chiller have the same life period
11. Interest rate remains constant during the life of the machines

3.3. Chiller Energy and Power Use

The chiller energy use can be approached from two point of views, (i) idealized analytical/theoretical and (ii) actual machine performance curves as given by manufacturer's data.

3.3.1. Idealized Theoretical Performance

A typical diagram of the refrigeration cycle, on which a chiller operates is shown in Figure 2

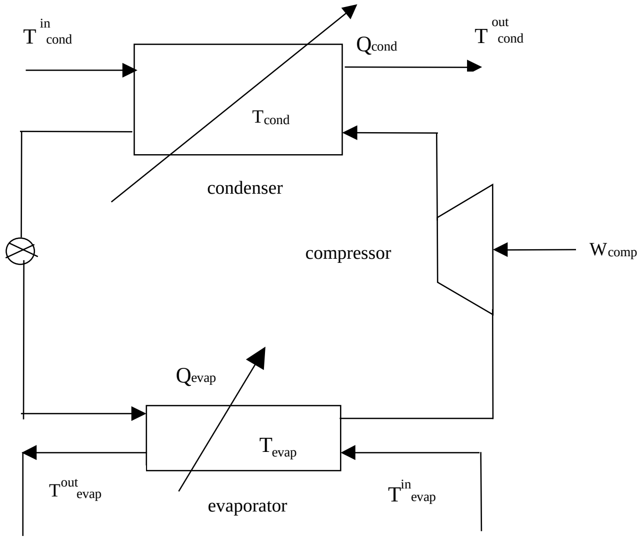

Figure 2. Diagram of a typical refrigeration cycle

Neglecting evaporator superheating and condenser sub-cooling, the cooling load at the evaporator at a particular load condition is given by:

$$\dot{Q}_{evap} = \varepsilon_{evap} \left(\dot{m}C_p\right)_{evap} \left(T^{in}_{evap} - T_{evap}\right) \tag{1}$$

Similarly, the analysis of the condenser leads to:

$$\dot{Q}_{cond} = \varepsilon_{cond} \left(\dot{m}C_p\right)_{cond} \left(T_{cond} - T^{in}_{cond}\right) \tag{2}$$

The coefficient of performance of the chiller, COP is given by:

$$COP = \frac{\dot{Q}_{evap}}{\dot{W}_{comp}} = \frac{\dot{Q}_{evap}}{\dot{Q}_{cond} - \dot{Q}_{evap}} \tag{3}$$

If the chiller is assumed to operate without internal irreversibilities, the COP of the chiller is determined by the temperatures in the evaporator and condenser.

$$COP_{ideal} = \frac{T_{evap}}{T_{cond} - T_{evap}} \tag{4}$$

The expression for the ideal COP in terms of the load temperature (T_{evap}^{in}), sink temperature (T_{cond}^{in}), cooling load ($\dot{Q}_{evap}$), heat exchanger effectiveness, and mass flow rate can be obtained by combining equations (1), (2), (3), and (4) (Klein, 1992) giving

$$COP_{ideal} = \frac{T_{evap}^{in} - \dot{Q}_{evap}\left(\frac{1}{\varepsilon_{evap}\dot{m}_{evap}C_p} + \frac{1}{\varepsilon_{cond}\dot{m}_{cond}C_p}\right)}{T_{cond}^{in} - T_{evap}^{in} + \dot{Q}_{evap}\left(\frac{1}{\varepsilon_{evap}\dot{m}_{evap}C_p} + \frac{1}{\varepsilon_{cond}\dot{m}_{cond}C_p}\right)} \tag{5}$$

The power consumption of the chiller compressor can be obtained by

$$\dot{W}_{comp} = \frac{\dot{Q}_{evap}}{COP(\dot{Q}_{evap}, T_{evap}^{in}, T_{cond}^{in})} \tag{6}$$

where the minimum power use occurs when COP is the highest value.

3.3.2. Actual Performance

A set of typical performance curves for a "water cooled" chiller is given in Figure 3 (York Inc, 1997). In general, the performance curves relate the fraction of full load power use to the cooling load fraction for various values of entering condenser water temperature.

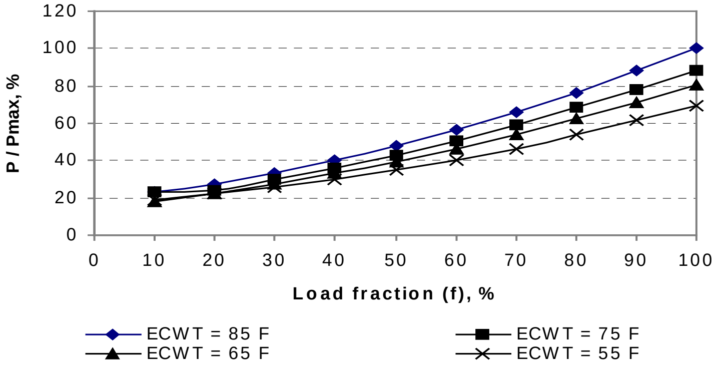

Figure 3. Graph of a typical chiller performance curve

Using this data, the power use may be found as:

$$P(f, ECWT) = \frac{P}{P_{max}}(f, ECWT)\ x \left(\frac{P_{max}}{\text{max capacity}}\right)_{\text{Full Load}} \text{x max capacity} \qquad (7)$$

The chiller power relationship indicated in Figure 3 will result in a non-linear mathematical model for life cycle cost. An alternative approach to maintain linearity in the mathematical modeling is to break the correlation of P(f, ECWT) into segments of piecewise linear continuous functions. The application of piecewise linear continuous functions yields the performance curves to be as shown in Figure 4.

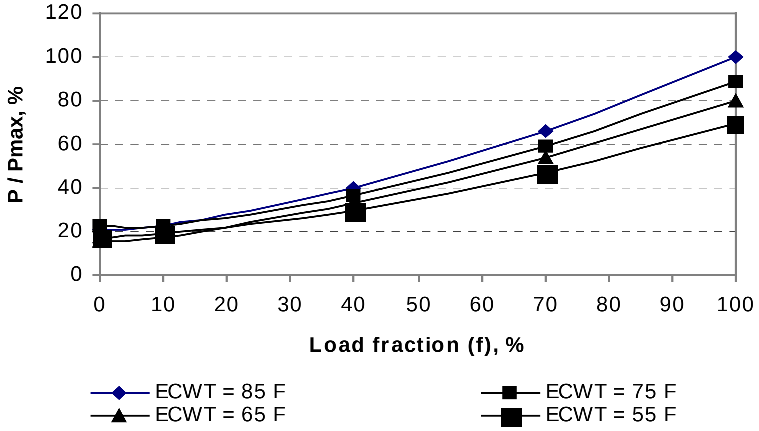

Figure 4. Graph of linearized piecewise continuous chiller performance curve

The application of piecewise linear continuous function assumes linear interpolation within the segments of the curve. Three segments are used for modeling, i.e., 10% to 40%, 40% to 70%, and 70% to 100% load. The piecewise linear function is also extrapolated to 0% load to approximate power usage for load fractions lower than 10% but is discontinuous at 0 % load where the power usage at 0 % load is zero.

Using this approach, the non-linearity inherent in the performance curves is eliminated. However, the penalty for this approach is the generation of more constraints to reflect the nature of the linear continuous functions within the segments. In addition, there exists the need to determine in which segment a particular load fraction belongs. The constraints associated with the piecewise linear continuous functions are described in section 4.2.

3.3.3. Effects of Outdoor Temperature

The outdoor wet bulb temperature affects the performance of the chillers by changing the ECWT. It is assumed that the condenser water loop releases heat to the environment through the use of one or several cooling towers. The ECWT is assumed to be higher than the outdoor wet bulb temperature by a fixed approach of 7 ^{o}F. The lower bound on the condenser water temperature is taken to be 55 ^{o}F.

3.4. Optimization Problem

The optimization problem is approached based on the life cycle cost (LCC) concept evaluated at present time. The motivation for pursuing this approach is the fact that life cycle cost identifies all of the costs inherent to the utilization of the equipment. Following the initial investment or first cost, operating costs are assumed to recur annually. Within each year maintenance, energy, and demand costs are determined for each distinct interval or "load condition".

3.4.1. Designation of Load Conditions

For the purpose of analysis, a year is divided into D demand charge periods. Each demand period is divided into J load conditions. The total cooling requirement that must be supplied by the cooling plant during each load condition is assumed to be known. The number of annual load conditions is D x J load conditions. At each load condition j within a demand period d, the total cooling requirement may be represented by $TL_{d,j}$.

3.4.2. Chiller Load

The total cooling requirement at demand period d and load condition j ($TL_{d,j}$) must be supplied by the chillers. This requires that the sum of all chiller cooling capacities, must at minimum satisfy the total cooling requirement:

$$\sum_{i=1}^{I} L_{i,d,j} \geq TL_{d,j} \tag{8}$$

where: $L_{i,d,j}$ is the load carried by chiller i at load condition j within the demand period d.

The chiller cooling load, $L_{i,d,j}$, can be expressed as a fraction, $f_{i,d,j}$, of the chiller design capacity, p_i :

$$L_{i,d,j} = f_{i,d,j}\ p_i \tag{9}$$

Substituting Equation (9) into Equation (8) yields:

$$\sum_{i=1}^{I} f_{i,d,j}\ p_i \ \geq TL_{d,j} \tag{10}$$

3.4.3. Utility Costs

Electricity or natural gas may be used to power vapor compression chillers. The use of energy resource k by chiller i at load condition j demand period d may be expressed as $E_{i,d,j,k}$:

$$E_{i,d,j,k} = L_{i,d,j}\ h_{d,j}\ n_{i,d,j,k} \tag{11.a}$$

or,

$$E_{i,d,j,k} = f_{i,d,j}\ p_i\ h_{d,j}\ n_{i,d,j,k} \tag{11.b}$$

where :

$h_{d,j}$ is the number of hours represented by a particular load condition

$n_{i,d,j,k}$ ratio of power usage to rate of cooling delivered by chiller i at load condition j demand period d utilizing energy resource k, kW/ton

The variable $n_{i,d,j,k}$ is determined by the chiller type (electrical motor or gas engine driven), and is a function of chiller cooling load ($f_{i,d,j}\ p_i$) and outdoor temperature (T_{wb}) which affects the ECWT.

3.4.3.1. Energy Cost

Introducing the parameter $e_{d,j,k}$ as the unit cost of energy resource k consumed at load condition j demand period d, the cost of energy for the supply of cooling by chiller i at load condition j, demand period d is:

$$EC_{i,d,j,k} = f_{i,d,j}\ p_i\ h_{d,j}\ n_{i,d,j,k}\ e_{d,j,k} \tag{12}$$

The energy cost at load condition j for all chillers:

$$EC_{d,j} = \sum_{i=1}^{I} f_{i,d,j}\ p_i\ h_{d,j}\ n_{i,d,j,k}\ e_{d,j,k} \tag{13}$$

The parameter $e_{d,j,k}$ is dependent on the type of resource consumed (electricity or fuel), and reflects the rate schedule of the utility that varies on the condition of operation hour, i.e., on or off peak hours. It is noted that summation over index k to obtain the energy cost is not required on the basis that chiller i inherently uses only one resource type k.

The annual energy cost is the sum of all energy costs over D demand periods:

$$AEC = \sum_{d=1}^{D} \sum_{j=1}^{J} \sum_{i=1}^{I} f_{i,d,j}\ p_i\ h_{d,j}\ n_{i,d,j,k}\ e_{d,j,k} \tag{14}$$

3.4.3.2. Demand Cost

Within a demand period d, the rate of energy use required to satisfy the cooling load is given by:

$$D_{d,j} = \sum_{i=1}^{I} f_{i,d,j}\ p_i\ n_{i,d,j,k} \tag{15}$$

Typically, only electrical energy use (k=1) is assessed a charge based on the maximum rate of use. This charge is referred to as a demand charge.

An electrical utility company may differentiate the demand charge during on-peak and off-peak hours. The on-peak power usage occurs during certain load conditions j which begin and end according to a schedule established by the electric utility company. In addition, the utility may assess a distribution demand charge. The distribution charge is based on the highest power usage during the demand period d, i.e., for j =1 to J. The total demand charge over all the load conditions J during demand period d becomes :

$$DC_d = \text{distribution charge} + \text{on-peak charge} + \text{off-peak charge} \tag{16}$$

or,

$$DC_d = \text{dist}_d \text{ x } \underset{j=\text{all load cond}}{\text{max of}} \{D_{d,j,k}\} + \text{on}_d \text{ x } \underset{j=\text{on-peak}}{\text{max of}} \{D_{d,j,k}\} + \text{off}_d \text{ x } \underset{j=\text{off-peak}}{\text{max of}} \{D_{d,j,k}\} \tag{17}$$

where:

dist_d is the distribution demand charge assessed by the utility company

on_d is the on-peak demand charge assessed by the utility company

off_d is the off-peak demand charge assessed by the utility company

The general expression for the annual demand charge (ADC) consisting of D demand periods is:

$$ADC = \sum_{d=1}^{D} DC_d = \sum_{d=1}^{D} \left[\text{dist}_{d,k} \text{ x } \underset{j=\text{all load cond}}{\text{max of}} \{D_{d,j,k}\} + \text{on}_{d,k} \text{ x } \underset{j=\text{on-peak}}{\text{max of}} \{D_{d,j,k}\} + \text{off}_{d,k} \text{ x } \underset{j=\text{off-peak}}{\text{max of}} \{D_{d,j,k}\} \right]_{k=1} \tag{18}$$

3.4.4. Annual Maintenance Cost

The maintenance cost for each machine is assumed to consist of a fixed cost component and a variable cost associated with the number of operating hours and the amount of cooling supplied. The annual maintenance cost for chiller i :

$$AMC_i = g_i + \sum_{d=1}^{D} \sum_{j=1}^{J} f_{i,d,j}\, p_i\, h_{d,j}\, v_i \tag{19}$$

where:

g_i is the fixed annual maintenance cost associated with chiller i

v_i is the variable maintenance cost associated with chiller i

The annual maintenance cost for all chillers is:

$$\mathrm{AMC} = \sum_{i=1}^{I} g_i + \sum_{d=1}^{D}\sum_{j=1}^{J}\sum_{i=1}^{I} f_{i,d,j}\, p_i\, h_{d,j}\, v_i \tag{20}$$

3.4.5. Annual Cost

The annual cost will include energy and demand related costs and maintenance costs for machines:

$$\mathrm{AC} = \mathrm{AEC} + \mathrm{ADC} + \mathrm{AMC}$$

$$\begin{aligned}\mathrm{AC} = {} & \sum_{i=1}^{I} g_i + \sum_{d=1}^{D}\sum_{j=1}^{J}\sum_{i=1}^{I} \left\{ f_{i,d,j}\, p_i\, h_{i,d,j}\, \left(v_i + n_{i,d,j,k}\, e_{d,j,k}\right)\right\} + \\ & \sum_{d=1}^{D}\left[\mathrm{dist}_{d,k} \max_{j=\text{all load cond}}\text{of} \left\{D_{d,j,k}\right\} + \mathrm{on}_{d,k} \max_{j=\text{on-peak}}\text{of} \left\{D_{d,j,k}\right\} + \mathrm{off}_{d,k} \max_{j=\text{off-peak}}\text{of} \left\{D_{d,j,k}\right\} \right]_{k=1}\end{aligned} \tag{21}$$

3.4.6. Life Cycle Cost

The LCC is comprised of a fixed initial cost and the recurring annual cost (AC). Summing the present values of all costs over the life of the equipment yields:

$$\mathrm{LCC} = \sum_{i}^{I} b_i + (P/A, r, y)\,\mathrm{AC} \tag{22}$$

where:

b_i	is the cost of each chiller
(P/A,r,y)	is the present worth of an annuity over a number of periods y at an interest rate r

Assuming a constant interest rate, the present worth of an annuity is given by:

$$(P/A, r, y) = \sum_{y=1}^{Y} \frac{1}{(1+r)^y} \tag{23}$$

Combining Equations (21), (22), and (23) yields:

$$\begin{aligned}\mathrm{LCC} = {} & \sum_{i=1}^{I} b_i + \sum_{y=1}^{Y}\frac{1}{(1+r)^y}\left[\sum_{i=1}^{I} g_i + \sum_{d=1}^{D}\sum_{j=1}^{J}\sum_{i=1}^{I} \left\{ f_{i,d,j}\, p_i\, h_{d,j}\, \left(v_i + n_{i,d,j,k}\, e_{d,j,k}\right)\right\} + \right. \\ & \left. \sum_{d=1}^{D}\left[\mathrm{dist}_{d,k} \max_{j=\text{all load cond}}\text{of} \left\{D_{d,j,k}\right\} + \mathrm{on}_{d,k} \max_{j=\text{on-peak}}\text{of} \left\{D_{d,j,k}\right\} + \mathrm{off}_{d,k} \max_{j=\text{off-peak}}\text{of} \left\{D_{d,j,k}\right\} \right]_{k=1}\right]\end{aligned} \tag{24}$$

3.4.7. Chiller Procurement

The objective of the chiller selection problem is to determine the number and type of chillers to be purchased in order to minimize life cycle cost. Chiller types may be selected from a given set of machines consisting of T different types. The maximum number of machines of each type is designated by I_t. The maximum number of machines of each type is determined by the total cooling requirement and the machine capacity. If more than one chiller type is used then the actual number of type t chillers may be less than I_t.

It is assumed that all chillers i = 1, .., I_t are available for purchase. If chiller i of type t is ever to be used during the year then it must be purchased. The purchase or not purchase decision is reflected by the decision variable $\gamma_{t,i}$ (1 = purchase, 0 = no purchase). With this representation, the total number of machines purchased, N_t is given by:

$$N_t = \sum_{i=1}^{I_t} \gamma_{ti} \tag{25}$$

The objective function may now be expressed as:

$$\begin{aligned} \mathrm{LCC} = {} & \sum_{t=1}^{T} b_t N_t + \sum_{y=1}^{Y} \frac{1}{(1+r)^y} \Bigg[\sum_{t=1}^{T} g_t N_t + \sum_{d=1}^{D} \sum_{j=1}^{J} \sum_{t=1}^{T} \sum_{i=1}^{I_t} \left\{ f_{t,i,d,j}\, p_t\, h_{d,j} \left(v_t + n_{t,i,d,j,k}\, e_{d,j,k} \right) \right\} + \\ & \sum_{d=1}^{D} \left[\mathrm{dist}_{d,k} \max_{j = \text{all load cond}} \text{of} \left\{ D_{d,j,k} \right\} + \mathrm{on}_{d,k} \max_{j=\text{on-peak}} \text{of} \left\{ D_{d,j,k} \right\} + \mathrm{off}_{d,k} \max_{j=\text{off-peak}} \text{of} \left\{ D_{d,j,k} \right\} \right]_{k=1} \Bigg] \end{aligned} \tag{26}$$

where the total demand by all chillers is given by:

$$D_{d,j,k} = \sum_{t=1}^{T} \sum_{i=1}^{I_t} f_{t,i,d,j}\, p_t\, n_{t,i,d,j,k} \qquad , \ k = 1 \tag{27}$$

3.4.8. Objective Function Statement

The goal of minimizing the life cycle cost of a chilled water plant may now be expressed as:

Minimize:

$$\mathrm{LCC} = \sum_{t=1}^{T} b_t N_t + \sum_{y=1}^{Y} \frac{1}{(1+r)^y} \Bigg[\sum_{t=1}^{T} g_t N_t + \sum_{d=1}^{D} \sum_{j=1}^{J} \sum_{t=1}^{T} \sum_{i=1}^{I_t} \left\{ f_{t,i,d,j}\ p_t\ h_{d,j}\ \left(v_t + n_{t,i,d,j,k}\ e_{d,j,k} \right) \right\} + \sum_{d=1}^{D} \left[\mathrm{dist}_{d,k} \underset{j=\text{all load cond}}{\text{max of}} \left\{ D_{d,j,k} \right\} + \mathrm{on}_{d,k} \underset{j=\text{on-peak}}{\text{max of}} \left\{ D_{d,j,k} \right\} + \mathrm{off}_{d,k} \underset{j=\text{off - peak}}{\text{max of}} \left\{ D_{d,j,k} \right\} \right]_{k=1} \Bigg] \qquad (26)$$

where:

$$D_{d,j,k} = \sum_{t=1}^{T} \sum_{i=1}^{I_t} f_{t,i,d,j}\ p_t\ n_{t,i,d,j,k} \qquad , k=1 \qquad (27)$$

3.4.9. Problem Simplification

The optimization problem statement given by Equations (26) and (27) can be restated as follows.

Minimize :

$$\mathrm{LCC} = \underbrace{\sum_{t=1}^{T} A_t N_t}_{1^{st}\ \text{term}} + \underbrace{\sum_{t=1}^{T} \left[x\, g_t N_t + \sum_{i=1}^{I_t} \sum_{d=1}^{D} \sum_{j=1}^{J} \left\{ B_{t,d,j}\ f_{t,i,d,j} + C_{t,d,j}\ FN_{t,i,d,j} \right\} \right]}_{2^{nd}\ \text{term}} + \underbrace{x \sum_{d=1}^{D} \mathrm{dist}_{d,k}\ DS_d + x \sum_{d=1}^{D} \mathrm{on}_{d,k}\ ON_d + x \sum_{d=1}^{D} \mathrm{off}_{d,k}\ OFF_d}_{3^{rd}\ \text{term}} \qquad (28)$$

Subject to the constraints as described in section 3.4.10.

where :

$$ON_d = \underset{j=\text{on-peak}}{\text{max of}} \left\{ D_{d,j,k} \right\} \qquad (29.a)$$

$$OFF_d = \underset{j=\text{off - peak}}{\text{max of}} \left\{ D_{d,j,k} \right\} \qquad (29.b)$$

$$DS_d = \underset{j=\text{all load cond}}{\text{max of}} \left\{ D_{d,j,k} \right\} \qquad (29.c)$$

$$x = \sum_{y=1}^{Y} \frac{1}{(1+r)^y} = \left\{ \frac{(1+r)^Y - 1}{r(1+r)^Y} \right\} \qquad (29.d)$$

$$FN_{t,i,d,j} = \frac{f_{t,i,d,j} n_{t,i,d,j}}{ptc_t} \tag{29.d}$$

and the parameters A_t, $B_{t,d,j}$, and $C_{t,d,j}$ represent the following.

$$A_t = b_t \tag{30.a}$$

$$B_{t,d,j} = \sum_{y=1}^{Y} \frac{1}{(1+r)^y} \left(p_t \, h_{d,j} \, v_t\right) = p_t h_{d,j} v_t x \tag{30.b}$$

$$C_{t,d,j} = \sum_{y=1}^{Y} \frac{1}{(1+r)^y} \left(p_t \, h_{d,j} \, e_{d,j,k}\right) = p_t \, h_{d,j} \, e_{d,j,k} \, ptc_t \, x \tag{30.c}$$

Parameters A_t, $B_{t,d,j}$, and $C_{t,d,j}$ can be calculated directly from the problem input data.

3.4.10. Constraints

The objective function given by Equation (28) is subject to the following constraints:

1. The fulfillment of the cooling load demand
2. The procurement constraints
3. The chiller performance constraints
4. The equal load fraction constraint
5. Demand charge constraints
5. Restrictions on the range and type of the decision variables
6. Restrictions on the maximum number of chillers of type t (I_t)

3.4.10.1. Cooling Load Requirement

The fulfillment of the cooling load demand is expressed in the following constraint.

$$\sum_{i=1}^{I} L_{i,d,j} = \sum_{i=1}^{I} f_{i,d,j} \, p_i = \sum_{t=1}^{T} \sum_{i=1}^{I_t} f_{t,i,d,j} \, p_t \geq TL_{d,j} \tag{31}$$

3.4.10.2. Chiller Procurement

The maximum number of chillers of type t, I_t, would be purchased if the entire cooling requirement were met solely with type t chillers. The actual number of type t chillers to be purchased, N_t, is based on the number of type t chillers required by the optimal operating scheme. Chillers of a particular type are numbered i = 1, …, I_t. The procurement decision variable, $\gamma_{t,i}$, introduced in section 3.4.7 indicates whether a chiller is purchased ($\gamma_{t,i} = 1$) or not ($\gamma_{t,i} = 0$).

The procurement decision variable, $\gamma_{t,i}$, is subject to the operation of chiller i type t. If chiller i type t, is never used based on the optimum operating schedule, then it is not purchased (or $\gamma_{ti} = 0$). Otherwise, it is purchased (or $\gamma_{ti} = 1$). To accommodate the above logic the following constraints are added.

To ensure that if machine i type t is not purchased, then it is never operated :

$$\gamma_{t,i} - \delta_{t,i,d,j} \geq 0 \qquad \forall d, j \qquad (32)$$

where $\delta_{t,i,d,j}$ is a decision variable having a value of 1 if a machine is operated during load condition j within demand period d and zero 0 if not.

To ensure that $\gamma_{ti} = 0$ if machine i type t is never operated :

$$\gamma_{ti} \leq \sum_{d=1}^{D}\sum_{j=1}^{J} f_{t,i,d,j} \qquad \forall t, i \qquad (33)$$

3.4.10.3. Equal Load Fraction

Chillers operating in parallel receive return water at the same temperature and are typically controlled to discharge water at the same temperature. Since the cooling load on the chiller is proportional to the temperature difference, chillers in parallel operate at a common load fraction.

The operation of chillers at a common load fraction requires that the model be able to determine which chillers are to be operated among the selected chillers from the selection group. This requirement introduces a binary decision variable, δ (1-0 or operate-do not operate) related to operation at each load condition. The necessary conditions that the equal load fraction constraint must represent are (i) the load fraction of all operating machines must be equal and (ii) the non-operating machines must have the load fraction set to zero which in turn yields the power consumption also to be zero.

The mathematical expression for the statement: if machine i of type t at load condition j, demand period d is not operating then $f_{t,i,d,j} = 0$, otherwise $0 < f_{t,i,d,j} \leq 1$ is as follows.

$$f_{t,i,d,j} \leq \delta_{t,i,d,j} \qquad \forall t,i,d,j \tag{34.a}$$

$$0 \leq f_{t,i,d,j} \leq 1 \qquad \forall t,i,d,j \tag{34.b}$$

$$\delta_{t,i,d,j} = 1,0 \text{ (binary integer)} \tag{34.c}$$

The mathematical expression for the statement: if machine i of type t at load condition j, demand period d is operating then $f_{t,i,d,j}$ must equal to a common load fraction $FF_{d,j}$ is as follows.

$$f_{t,i,d,j} \leq \delta_{t,i,d,j} \qquad \forall t,i,d,j \tag{35.a}$$

$$f_{t,i,d,j} \geq FF_{d,j} \qquad \forall t,i,d,j \tag{35.b}$$

$$FF_{d,j} - f_{t,i,d,j} \leq 1 - \delta_{t,i,d,j} \qquad \forall t,i,d,j \tag{35.c}$$

If $\delta_{t,i,d,j} = 1$ then it is always true that $f_{t,i,d,j} \leq 1$; and $f_{t,i,d,j} \geq FF_{d,j}$ combined with $f_{t,i,d,j} \leq FF_{d,j}$ results in the desired $f_{t,i,d,j} = FF_{d,j}$. If $\delta_{t,i,d,j} = 0$ then it is always true that $f_{t,i,d,j} \geq FF_{d,j} - 1$; and $f_{t,i,d,j} \geq 0$ (from $0 \leq f_{t,i,d,j} \leq 1$) combined with $f_{t,i,d,j} \leq 0$ results in the desired $f_{t,i,d,j} = 0$.

3.4.10.4. Demand Charges

Once the power use for each load condition is known, the demand for a given interval can be found as the maximum power used during the interval.

The electrical demand charge is based on the highest power use within a specific interval of the demand period. The electrical power use at any load condition, $H_{d,j}$, is given by:

$$H_{d,j} = \sum_{t=1}^{T} \sum_{i=1}^{I_t} FN_{t,i,d,j}\, ptc_t\, p_t \tag{36}$$

$$DS_d \geq H_{d,j} , \quad \forall d, j \tag{37}$$

$$ON_d \geq H_{d,j} , \quad \forall d, j = \text{on-peak} \tag{38}$$

$$OFF_d \geq H_{d,j} , \quad \forall d, j = \text{off-peak} \tag{39}$$

In constraints (37) to (39), the decision variables DS_d, ON_d, and OFF_d will only pick the maximum of $H_{d,j}$ within the corresponding d and j domain which are the maximum power, maximum on-peak power, and off-peak power consumption respectively.

3.4.10.5. Bounds

The bounds for the variables are as follows.
Chiller load variables

$0 \leq f_{t,i,d,j} \leq 1$
$0 \leq FF_{d,j} \leq 1$

Chiller performance variables

$0 \leq FN_{t,i,d,j} \leq 1$

Procurement decision variables

$\gamma_{ti} = 0,1$ (binary integer)
$0 \leq N_t \leq I_t$

Operation decision variables

$\delta_{t,i,d,j} = 0,1$ (binary integer)

Restriction on the maximum number of chillers of type t (I_t):

$$I_t = \text{roundup}\left[\frac{\text{max of } \{TL_{d,j}\}}{p_t}\right] \quad , \quad j = 1, \ldots, J; d = 1, \ldots, D \tag{40}$$

The following chapter describes the application of piecewise linear functions to represent the chiller performance curves, and the supplemental constraints. The supplemental constraints include the ranking of the purchase and operation decision variables and establishing lower bounds for maintenance and energy constraints. In addition, the decomposition of the problem and the solution procedure are presented.

4. Solution Approach

The objective of this chapter is to describe the approach for solving the chiller plant optimization problem. The following sections describe the problem characterization, the application of piecewise linear functions representing the chiller performance curves, and the supplemental constraints that aid in solving the problem. The supplemental constraints include the asymmetric constraints for procurement and operation, and the establishment of lower bounds for maintenance and energy costs.

The constraints for ranking the binary decision variables (asymmetric) are provided to limit the combinatorial solution space. In addition, lower bounds for maintenance and energy costs are also incorporated in an attempt to reduce the gap more rapidly. A decomposed model of the full life cycle cost problem is presented along with its solution procedure in an effort to obtain a solution within a reasonable computational time.

4.1. Problem Characterization

The chiller selection problem defined by Equation (28) is a mixed integer-continuous non-linear minimization. The problem involves the following decision variables.

N_t, $t = 1,2 \ldots,T$: integer

$f_{t,i,d,j}$, $0 \leq f_{t,i,,d,j} \leq 1$: continuous

$FN_{t,i,d,j}$, $0 \leq FN_{t,i,,d,j} \leq 1$: continuous

$\gamma_{t,i}$, $\gamma_{t,i} = 0,1$: integer

$\delta_{t,i,d,j}$, $\delta_{t,i,d,j} = 0,1$: integer

The number of continuous decision variables for each $f_{t,i,d,j}$ and $FN_{t,i,d,j}$ is given by $DJ\sum_{t=1}^{T} I_t$. The number of integer decision variables including both $\gamma_{t,I}$ and $\delta_{t,i,d,j}$ is given by $(1+DJ)\sum_{t=1}^{T} I_t$.

The optimization problem includes a MinMax problem (the third term). The first term refers to the initial cost, the second refers to the operation cost, and the third term refers to the demand charge. The operation cost (second term) is subject to the number of machines purchased, N_t in the initial cost (first term). The decision variables $f_{t,i,d,j}$ are common to both the MinMax term (the third term) and the second term.

4.2. Piecewise Linearization of the Chiller Performance Curves

For the purpose of maintaining the objective function as a linear function, the power consumption curves are represented as four piecewise linear functions of load fraction spanning the entire range of load fraction. Each piecewise linear function represents the chiller performance at a different condenser water temperature. It is noted that a larger number of piecewise linear functions will better represent the actual power consumption curves.

To calculate the chiller power consumption at different ECWTs, the chiller performance curve must be selected accordingly. Selection of the performance curve determines the coefficients of the piecewise linear curves and it is conducted prior to entering the optimization problem into the solver.

The chiller performance curves yield values for $n_{t,i,d,j,k}$, the energy used per unit cooling. The energy used is the product of $n_{t,i,d,j}$ and the cooling required, $f_{t,i,d,j}\ p_t$. As the term $f_{t,i,d,j}n_{t,i,d,j,k}$ is a product of decision variables then the product term $FN_{t,i,d,j}$ is linearized.

Treating the terms $FN_{t,i,d,j}$ as a piecewise linear function leads to:

$$FN_{t,i,d,j} = z_{1,t,i,d,j}\, R_{1,t,d,j} + z_{2,t,d,j}\, R_{2,t,d,j} + z_{3,t,i,d,j}\, R_{3,t,d,j} + z_{4,t,i,d,j}\, R_{4,t,d,j} + z_{5,t,i,d,j}\, R_{5,t,d,j} \tag{41}$$

which is subject to the following additional constraints referred to as Specially Ordered Set (SOS) Type 2 constraints (Beale and Tomlin, 1970) :

$$z_{1,t,i,d,j} \leq u_{1,t,i,d,j} \tag{42}$$

$$z_{2,t,i,d,j} \leq u_{1,t,i,d,j} + u_{2,t,i,d,j} \quad (43)$$

$$z_{3,t,i,d,j} \leq u_{2,t,i,d,j} + u_{3,t,i,d,j} \quad (44)$$

$$z_{4,t,i,d,j} \leq u_{3,t,i,d,j} + u_{4,t,i,d,j} \quad (45)$$

$$z_{5,t,i,d,j} \leq u_{4,t,i,d,j} \quad (46)$$

$$u1_{t,i,d,j} + u2_{t,i,d,j} + u3_{t,i,d,j} + u4_{t,i,d,j} = 1 \quad (47)$$

$$z1_{t,i,d,j} + z2_{t,i,d,j} + z3_{t,i,d,j} + z4_{t,i,d,j} + z5_{t,i,d,j} = 1 \quad (48)$$

The load fraction is determined from:

$$f_{t,i,d,j} = a_1\, z_{1,t,i,d,j} + a_2\, z_{2,t,i,d,j} + a_3\, z_{3,t,i,d,j} + a_4\, z_{4,t,i,d,j} + a_5 z_{5,t,i,d,j} \quad (49)$$

and the bounds:

$$z_{1,t,i,d,j},\ z_{2,t,i,d,j},\ z_{3,t,i,d,j},\ z_{4,t,i,d,j},\ z_{5,t,i,d,j} \geq 0$$

$$u_{1,t,i,d,j},\ u_{2,t,i,d,j},\ u_{3,t,i,d,j},\ u_{4,t,i,d,j} = 0,1 \text{ (binary integer)}$$

all of which are $\forall t,i,d,j$

The values of $R_{1,t,d,j}$, $R_{2,t,d,j}$, $R_{3,t,d,j}$, $R_{4,t,d,j}$ and $R_{5,t,d,j}$ are the values of $FN_{t,i,d,j}$ at $a_1 = 0.0$, $a_2 = 0.1$, $a_3 = 0.4$, $a_4 = 0.7$, and $a_5 = 1.0$ fractional load respectively. The subscripts t, d, and j of R refer to the type of chiller, demand period d, and load condition j. The values of R are evaluated at the corresponding ECWT which is subject to demand period d and load condition j. The value of $FN_{t,i,d,j}$ is appropriate to each load condition j demand period d.

For chillers operating at a common load fraction, it is possible to reduce the number of variables in the power usage formulation. The reduction of the number of variables in the formulation is possible due to the fact that all chillers in operation are set to have the same load fraction.

The piecewise linear function is modified to represent all chiller load fractions changing only with respect to the load condition j, demand period d as follows.

$$z_{1,d,j} \leq u_{1,d,j} \qquad \forall d,j \quad (50)$$

$$z_{2,d,j} \leq u_{1,d,j} + u_{2,d,j} \qquad \forall d,j \quad (51)$$

$$z_{3,d,j} \leq u_{2,d,j} + u_{3,d,j} \qquad \forall d,j \quad (52)$$

$$z_{4,d,j} \leq u_{3,d,j} + u_{4,d,j} \qquad \forall d,j \qquad (53)$$

$$z_{5,d,j} \leq u_{4,d,j} \qquad \forall d,j \qquad (54)$$

$$z_{1,d,j} + z_{2,d,j} + z_{3,d,j} + z_{4,d,j} + z_{5,d,j} = 1 \qquad \forall d,j \qquad (55)$$

$$u_{1,d,j} + u_{2,d,j} + u_{3,d,j} + u_{4,d,j} = 1 \qquad \forall d,j \qquad (56)$$

$$a_1 z_{1,d,j} + a_2 z_{2,d,j} + a_3 z_{3,d,j} + a_4 z_{4,d,j} + a_5 z_{5,d,j} = FF_{d,j} \qquad \forall d,j \qquad (57)$$

$$FFN_{t,i,d,j} = z_{1,d,j} R_{1,t,d,j} + z_{2,d,j} R_{2,t,d,j} + z_{3,d,j} R_{3,t,d,j} + z_{4,d,j} R_{4,t,d,j} + z_{5,d,j} R_{5,t,d,j} \qquad \forall t,i,d,j \qquad (58)$$

This representation has 4 segments of piecewise linear function with nodes at $a_1 = 0.0$, $a_2 = 0.1$, $a_3 = 0.4$, $a_4 = 0.7$ and $a_5 = 1.0$. The values of the function at nodes a_1, a_2, a_3, a_4, and a_5 are R_1, R_2, R_3, R_4 and R_5 respectively. When the data regarding R_1 is unavailable the value is obtained by extrapolating the chiller performance curve.

Constraints (59) to (61) govern the power use fraction of chiller i type t. When it is operated, its power use fraction ($FN_{t,i,d,j}$) will equal to the common power use fraction of chiller type t, $FFN_{t,i,d,j}$, otherwise it is set to zero.

$$FFN_{t,i,d,j} \geq FN_{t,i,d,j} \qquad (59)$$

$$FN_{t,i,d,j} \leq \delta_{t,i,d,j} \qquad (60)$$

$$FFN_{t,i,d,j} - FN_{t,i,d,j} \leq 1 - \delta_{t,i,d,j} \qquad (61)$$

If chiller i type t is operated, at load condition j demand period d, then $\delta_{t,i,d,j} = 1$, otherwise $\delta_{t,i,d,j} = 0$. When $\delta_{t,i,d,j} = 1$, all chiller operating loads are set equal to a common load fraction, $FF_{d,j}$ as governed by constraints (35.a), (35.b), and (35.c). This common load fraction, $FF_{d,j}$ then dictates which segment of the piecewise linear function to use. The value of $FF_{d,j}$ sets the value of the variables u_1, u_2, u_3, and u_4 (binary integer variables) and z_1, z_2, z_3, z_4, and z_5 (continues variables) in the constraints as represented by constraints (50) to (57). The value of the common power fraction for chillers of type t ($FFN_{t,i,d,j}$) is set to be the power use fraction ($FN_{t,i,d,j}$) of chiller i type t at load condition j demand period d if operated. The values of the coefficient $R_{1,t,d,j}$, $R_{2,t,d,j}$, $R_{3,t,d,j}$, $R_{4,t,d,j}$, and $R_{5,t,d,j}$ determine the appropriate $FFN_{t,i,d,j}$ according to the type of chiller.

Constraints (59) and (61) dictate that if $\delta_{t,i,d,j} = 1$, then the power consumption for computing the objective function $FN_{t,i,d,j} = FFN_{t,i,d,j}$ while constraint (60) will always be satisfied. If $\delta_{t,i,d,j} = 0$, chiller i type t at load condition j demand period d is not operated. This condition represented by constraint (35.b) with the bound $0 \leq f_{t,i,d,j} \leq 1$ forces the load fraction, $f_{t,i,d,j} = 0$. Also, constraint (60) with the bound $0 \leq FN_{t,i,d,j}$ enforces $FN_{t,i,d,j} = 0$. In this case, constraints (59) and (61) are always satisfied.

This formulation eliminates the need to have integer variables u_1, u_2, etc, for each chiller, i.e. $u_{1,t,i,d,j}$, $u_{2,t,i,d,j}$, etc. Thus, having only $u_{1,d,j}$, $u_{2,d,j}$, etc. may significantly reduce the number of integer variables.

4.3. Asymmetric Constraints

This section describes the constraints developed by Sherali (1999) that rank the procurement decision variables ($\gamma_{t,i}$) and operating decision variables ($\delta_{t,i,d,j}$) thereby limiting the combinatorial space. For a certain type of chiller t, the decision to not purchase a machine of a particular type implies that additional machines of that type should not be considered. Also, at any load condition j within a demand period d, the decision to not operate any machine of type t implies that additional chillers of that type should not be operated.

The following expresses the asymmetric constraints for the purchasing decision variable.

$$\gamma_{t,1} \leq \gamma_{t,2} \leq \ldots \leq \gamma_{t,It} \qquad \forall t \qquad (62)$$

The following expresses the asymmetric constraints for the operating decision variable.

$$\delta_{t,1,d,j} \leq \delta_{t,2,d,j} \leq \ldots \leq \delta_{t,It,d,j} \qquad \forall t,d,j \qquad (63)$$

4.4. Minimum Variable Maintenance and Energy Costs

This section describes the constraints serving as the lower bound for the maintenance and energy costs. The following constraints are constraints for reducing the gap between the best node value and the best feasible solution.

The minimum variable maintenance costs at load condition j demand period d may be approximated according to the following.

$$MC_{d,j} = \sum_{t=1}^{T} \sum_{i=1}^{I_t} B_{t,d,j} \, f_{t,i,d,j} \geq MinMC_{d,j} \qquad \forall d,j \qquad (64)$$

where $MC_{d,j}$ and $MinMC_{d,j}$ are the variable maintenance cost and the minimum variable maintenance cost respectively at load condition j, demand period d.

The minimum energy costs may be approximated according to the following.

$$EC_{d,j} = \sum_{t=1}^{T} \sum_{i=1}^{I_t} FN_{t,i,d,j} \, C_{t,d,j} \geq MinEC_{d,j} \qquad \forall d,j \qquad (65)$$

where $EC_{d,j}$ and $MinEC_{d,j}$ are the energy cost and the minimum energy cost respectively at load condition j, demand period d. The minimum variable maintenance and minimum energy costs are determined by pre-solving sub problems described in section 4.5.

4.5. Minimum Variable Maintenance and Energy Cost Sub Problem

The minimum variable maintenance and energy costs may be used to establish lower bounds to the life cycle cost of a particular configuration being evaluated. The sub problems are solved prior to minimizing the life cycle cost. The values of the solved sub problems determine the minimum variable maintenance and energy costs for each load condition j demand period d.

Both sub problems consider all possible chillers purchased with zero fixed costs and operable. The objective function is to minimize $MC_{d,j}$ and $EC_{d,j}$ $\forall d,j$. All constraints related to the procurement decision are discarded from the sub problems. In addition, the terms representing energy cost, as expressed by Equation (67), or representing variable maintenance cost, as expressed by Equation (66), are discarded from the minimum variable maintenance cost or minimum energy cost sub problems respectively.

$$MC_{d,j} = \sum_{t=1}^{T} \sum_{i=1}^{I_t} B_{t,d,j}\, f_{t,i,d,j} \tag{66}$$

$$EC_{d,j} = \sum_{t=1}^{T} \sum_{i=1}^{I_t} C_{t,d,j}\, FN_{t,i,d,j} \tag{67}$$

The total minimum variable maintenance and energy costs, MinMC and MinEC respectively, are the sum of the minimized $MC_{d,j}$ and $EC_{d,j}$ for all load conditions and demand periods. The values of the minimized $MC_{d,j}$ and $EC_{d,j}$ are stored as $MinMC_{d,j}$ and $MinEC_{d,j}$ for use in the life cycle cost minimizing procedure.

4.6. Decomposed Problem

In order to obtain solutions in a reasonable amount of computational time, the problem is decomposed into two decision problems consisting of plant synthesis and operation. The plant synthesis part relates to identifying all feasible plant configurations supporting the cooling requirement. An enumeration process may be applied to obtain this information based on the plant capacity. The operation problem relates to minimizing the operating costs associated with all feasible configurations being evaluated. The computation time may be further reduced if a lower bound of the LCC of the configuration can be established prior to minimizing the operating costs. The lower bound of the LCC of the configuration is obtained by using the information pertaining to the minimum variable maintenance and minimum energy costs.

The decomposed model evaluates the life cycle cost of only one configuration option at a time and minimizes the operating cost one demand period at a time. To accommodate this idea, the life cycle objective function is modified such that the fixed cost terms are discarded and the operation cost expression are relevant only to the available types of machines selected for the configuration. The fixed cost terms include the initial cost and all the fixed maintenance costs evaluated at present. The constraints pertaining to the decision to purchase a particular type of chiller are discarded.

The expression for the operation cost objective function in the optimization problem pertaining to configuration m (OC_m) follows that of equation (68).

$$OC_m = \sum_{t=1}^{T}\sum_{i=1}^{I_t}\sum_{d=1}^{D}\sum_{j=1}^{J}\left\{B_{t,d,j}\, f_{t,i,d,j} + C_{t,d,j}\, FN_{t,i,d,j}\right\} + x\sum_{d=1}^{D} dist_{d,k}\, DS_d + x\sum_{d=1}^{D} on_{d,k}\, ON_d + x\sum_{d=1}^{D} off_{d,k}\, OFF_d \qquad (68)$$

The constraints related to the decomposed model are similar to those of the full life cycle cost problem. The difference between the two is that all constraints containing the procurement decision variables, $\gamma_{t,i}$, are not included.

4.7. Solution Procedure

This section presents the procedure developed in conjunction with the decomposed model. To represent the full problem, the decomposed problem must evaluate all feasible configurations. However, with the aid of prior knowledge regarding the minimum variable maintenance and minimum energy costs, the number of configurations to be evaluated may be reduced. This reduction of number of configurations to be evaluated is achieved by imposing a cut-off criterion.

This solution procedure is based on minimizing the operating cost for the configuration being evaluated. The operating costs that are being minimized are associated with each and every demand period where the demand periods are independent with respect to the other. Having minimized the operating cost for all demand periods, the life cycle cost of the configuration will be the sum of the fixed cost, which is computed prior to the optimization process, and the operating costs for all demand periods.

Prior to minimizing the life cycle cost, minimum variable maintenance and minimum energy costs are established. After solving the sub problems, all feasible configurations are ranked based on the plant fixed cost in an increasing order. The feasible configurations are those that satisfy the cooling demand for all load conditions. A reference life cycle cost is determined and belongs to the configuration with the least fixed cost. This reference life cycle cost will be compared with and will be updated during the task of minimizing the life cycle cost in the solution procedure.

When given a selection group consisting of T chiller types, there exist a maximum number of chillers associated with a certain type to support the maximum cooling load demand. With the parameter I_t denoting the maximum number of chillers of type T, there exist a maximum number of configurations of chillers, including the configurations that are inadequate for meeting the maximum cooling load demand. The maximum number of chiller configurations can be represented by N_{config} and is given by:

$$N_{config} \leq \prod_{t=1}^{T} (I_t + 1) \qquad (69)$$

where N_{config} is an integer. The value of N_{config} depends on the maximum cooling load and the capacity of each type of chiller, p_t, in the selection group.

The cut-off criterion is based on the lower bound of the life cycle cost. The lower bound is denoted by the initial life cycle cost that represents the sum of the fixed cost of the configuration being evaluated, the minimum variable maintenance cost, and the minimum energy cost. Until the reference life cycle cost is less than the initial life cycle cost, the life cycle cost minimizing procedure proceeds to evaluate the next configuration with a higher fixed cost.

Figures 5 and 6 describe the sequence of activity pertaining to the solution procedure. Prior to minimizing the life cycle cost, the task of establishing lower bounds, as shown by Figure 5, and the task of determining the reference life cycle cost, as shown by Figure 6, are undertaken. Figure 7 shows the task of determining the minimum life cycle cost by comparison with the reference life cycle cost. The configuration being set as the reference life cycle cost is the configuration having the lowest fixed cost. The reference life cycle cost is to be compared with that of the remaining feasible configurations. Whenever the life cycle costs of the current feasible configuration exceed the reference life cycle cost, the current configuration is rejected and the process proceeds to the next feasible configuration. The procedure repeats the previous steps until all feasible configurations with fixed life cycle costs less than the cut-off criteria are examined.

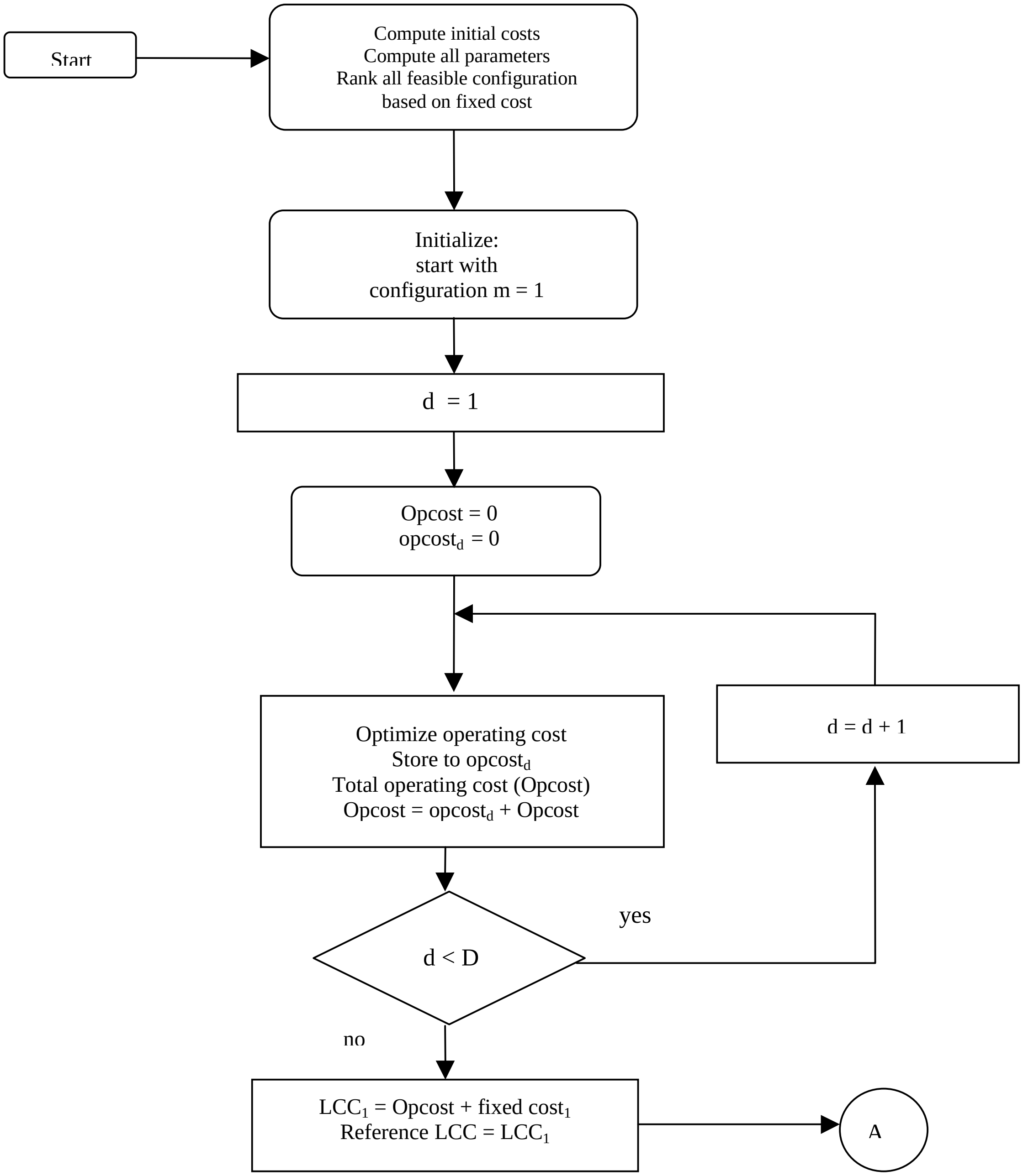

Figure 5. Determination of reference life cycle cost

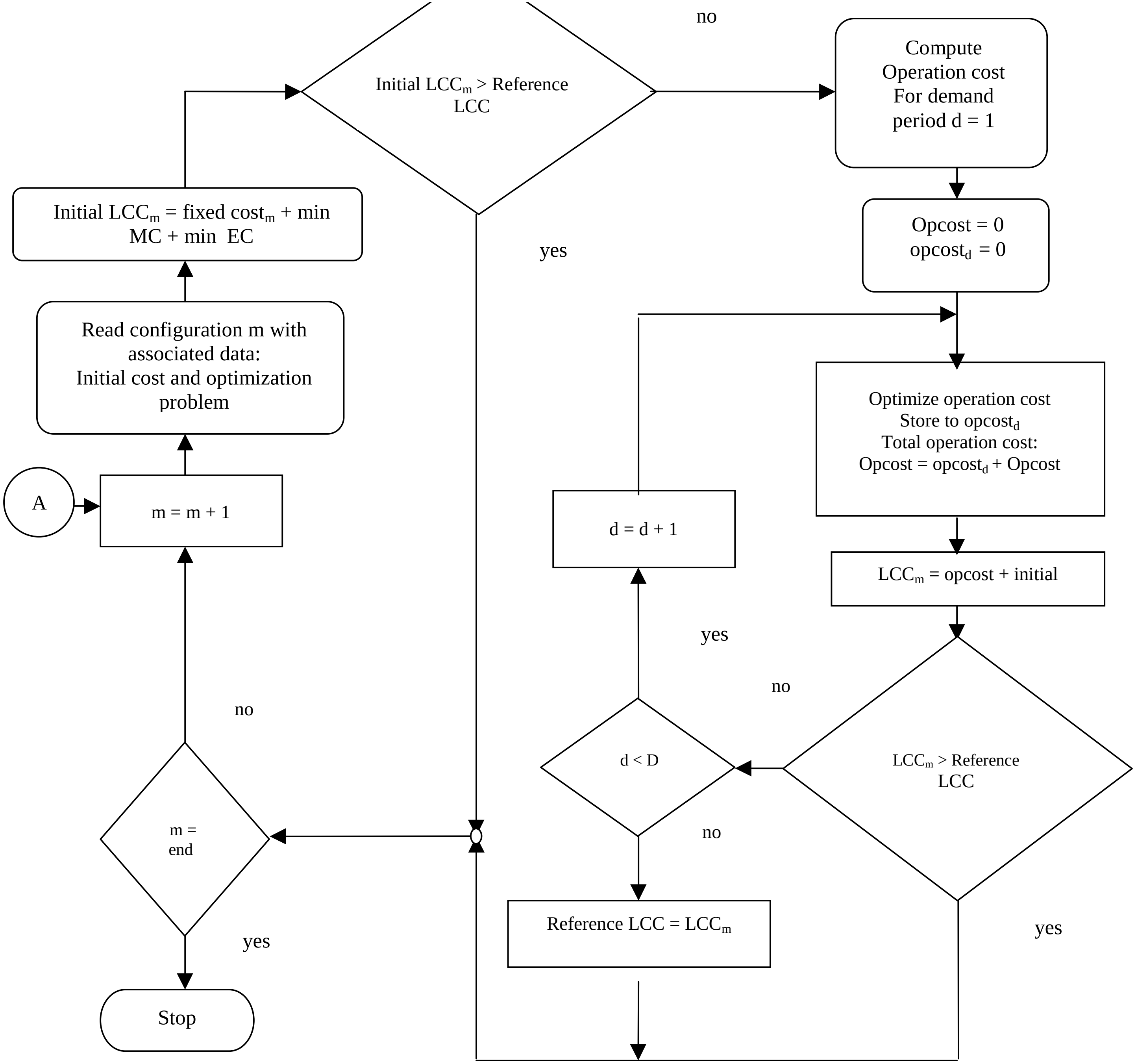

Figure 6. Life cycle minimizing procedure for the decomposed model

4.8. Summary of Problem Representation

This section presents a summary of the problem statements for the full life cycle cost problem and the decomposed problem. The full life cycle cost problem takes the form of a more general problem when compared to the decomposed problem.

4.8.1. Objective Statement

4.8.1.1. Full Life Cycle Cost Problem

The objective statement for the full life cycle cost problem is:

Minimize LCC where

$$\mathrm{LCC} = \sum_{t=1}^{T} A_t N_t + \sum_{t=1}^{T} \left[x\, g_t N_t + \sum_{i=1}^{I_t} \sum_{d=1}^{D} \sum_{j=1}^{J} \left\{ B_{t,d,j}\, f_{t,i,d,j} + C_{t,d,j}\, FN_{t,i,d,j} \right\} \right]$$

$$+\, x \sum_{d=1}^{D} dist_{d,k}\, DS_d + x \sum_{d=1}^{D} on_{d,k}\, ON_d + x \sum_{d=1}^{D} off_{d,k}\, OFF_d$$

and

$$A_t = b_t$$

$$B_{t,d,j} = \sum_{y=1}^{Y} \frac{1}{(1+r)^y} \left(p_t\, h_{d,j}\, v_t \right) = p_t h_{d,j} v_t\, x$$

$$C_{t,d,j} = \sum_{y=1}^{Y} \frac{1}{(1+r)^y} \left(p_t\, h_{d,j}\, e_{d,j,k} \right) = p_t\, h_{d,j}\, e_{d,j,k}\, ptc_t\, x$$

$$x = \sum_{y=1}^{Y} \frac{1}{(1+r)^y} = \left\{ \frac{(1+r)^Y - 1}{r(1+r)^Y} \right\}$$

4.8.1.2. Decomposed Problem

For the decomposed problem, operating costs are minimized for each feasible configuration. The objective statement is minimize OC_m where:

$$OC_m = \sum_{t=1}^{T} \sum_{i=1}^{I_t} \sum_{d=1}^{D} \sum_{j=1}^{J} \left\{ B_{t,d,j}\, f_{t,i,d,j} + C_{t,d,j}\, FN_{t,i,d,j} \right\} + x \sum_{d=1}^{D} dist_{d,k}\, DS_d + x \sum_{d=1}^{D} on_{d,k}\, ON_d + x \sum_{d=1}^{D} off_{d,k}\, OFF_d$$

and the parameters $B_{t,d,j}$, $C_{t,d,j}$, and x are similar to that of the full LCC problem.

4.8.2. Constraints Expression

The following section summarizes constraints for chiller procurement, cooling load, piecewise linearization of the chiller performance curves, demand related charges (distribution, on-peak, and off-peak demand charges), and equal fractional loading.

4.8.2.1. Full Life Cycle Cost Problem

The constraints for fulfilling the cooling load requirement at load condition j demand period d are:

$$\sum_{i=1}^{I} f_{i,d,j}\, p_i = \sum_{t=1}^{T}\sum_{i=1}^{I_t} f_{t,i,d,j}\, p_t \geq TL_{d,j} \qquad \forall d, j$$

The expression for the number of chillers of type t purchased is:

$$N_t = \sum_{i=1}^{I_t} \gamma_{ti} = \gamma_{t,1} + \gamma_{t,2} + \gamma_{t,3} + \ldots + \gamma_{t,I_t} \qquad \forall t$$

The constraints for determining the value of γ_{ti} to ensure that if machine i type t is not purchased, then it is never operated are:

$$\gamma_{t,i} - \delta_{t,i,d,j} \geq 0 \qquad \forall d, j$$

The constraints ensure that $\gamma_{ti} = 1$ if machine i type t is ever operated are:

$$\gamma_{ti} \leq \sum_{d=1}^{D}\sum_{j=1}^{J} f_{t,i,d,j} \qquad \forall t, i$$

$$1 \leq i \leq I_t,\ 1 \leq t \leq T;\ t, i \text{ integer}$$
$$1 \leq d \leq D,\ 1 \leq j \leq J;\ d, j \text{ integer}$$

The constraints for piecewise linear representation of $FN_{t,i,d,j}$ are:

$$z_{1,d,j} \leq u_{1,d,j} \qquad \forall d,j$$
$$z_{2,d,j} \leq u_{1,d,j} + u_{2,d,j} \qquad \forall d,j$$
$$z_{3,d,j} \leq u_{2,d,j} + u_{3,d,j} \qquad \forall d,j$$
$$z_{4,d,j} \leq u_{3,d,j} + u_{4,d,j} \qquad \forall d,j$$
$$z_{5,d,j} \leq u_{4,d,j} \qquad \forall d,j$$
$$\sum_{i=1}^{5} z_{i,d,j} = 1 \qquad \forall d,j$$
$$\sum_{i=1}^{4} u_{i,d,j} = 1 \qquad \forall d,j$$

$$\sum_{i=1}^{5} a_i z_{i,d,j} = FF_{d,j} \qquad \forall d,j$$

$$\sum_{i=1}^{5} R_{i,t,d,j}\, z_{i,d,j} = FFN_{t,i,d,j} \qquad \forall t,i,d,j$$

The constraints for the Demand Charges include

the constraints for Distribution charge

$$DS_d \geq H_{d,j} \qquad \forall d, j$$

the constraints for On-peak Demand charge

$$ON_d \geq H_{d,j} \qquad \forall d, j = \text{on peak}$$

and the constraints for Off-peak Demand charge

$$OFF_d \geq H_{d,j} \qquad \forall d, j = \text{off peak}$$

where

$$H_{d,j} = \sum_{t=1}^{T} \sum_{i=1}^{I_t} FN_{t,i,d,j}\, p_t\, ptc_t \qquad \forall d, j$$

The constraints for equal fractional loading are

$$FF_{d,j} \geq f_{t,i,d,j} \qquad \forall t,i,d,j$$

$$f_{t,i,d,j} \leq \delta_{t,i,d,j} \qquad \forall t,i,d,j$$

$$FF_{d,j} - f_{t,i,d,j} \leq 1 - \delta_{t,i,d,j} \qquad \forall t,i,d,j$$

The constraints for the power fraction are

$$FFN_{t,i,d,j} \geq FN_{t,i,d,j} \qquad \forall t,i,d,j$$

$$FFN_{t,i,d,j} \leq \delta_{t,i,d,j} \qquad \forall t,i,d,j$$

$$FFN_{t,i,d,j} - FN_{t,i,d,j} \leq 1 - \delta_{t,i,d,j} \qquad \forall t,i,d,j$$

The asymmetric constraints for the procurement decision variable are

$$\gamma_{t,1} \leq \gamma_{t,2} \leq \ldots. \leq \gamma_{t,It} \qquad \forall t$$

The asymmetric constraints for the operating decision variable are

$$\delta_{t,1,d,j} \leq \delta_{t,2,d,j} \leq \ldots. \leq \delta_{t,It,d,j} \qquad \forall t,d,j$$

The constraints for minimum maintenance cost are

$$MC_{d,j} = \sum_{t=1}^{T} \sum_{i=1}^{I_t} FN_{t,i,d,j}\, C_{t,d,j} \geq MinMC_{d,j} \qquad \forall d,j$$

The constraints for minimum energy cost are

$$EC_{d,j} = \sum_{t=1}^{T} \sum_{i=1}^{I_t} FN_{t,i,d,j}\, C_{t,d,j} \geq MinEC_{d,j} \qquad \forall d,j$$

4.8.2.2. Decomposed Problem

The constraints associated with the decomposed model are similar to those of the full life cycle problem except that it lacks the constraints pertaining to the procurement decision variable, i.e., the procurement and the procurement asymmetric constraints.

4.8.3. Bounds

The bounds for the variables are as follows:

Chiller load variables

$0 \leq f_{t,i,d,j} \leq 1$

$0 \leq FF_{d,j} \leq 1$

Chiller performance variables

$0 \leq FN_{t,i,d,j} \leq 1$

$0 \leq FFN_{t,i,d,j} \leq 1$

auxiliary variables:

$u1_{d,j}, u2_{d,j}, u3_{d,j}, u4_{d,j} = 0,1$ (binary integer)

$0 \leq z1_{d,j}, z2_{d,j}, z3_{d,j}, z4_{d,j}, z5_{d,j}$

Procurement decision variables

$\gamma_{ti} = 0,1$ (binary integer)

$0 \leq N_t \leq I_t$

Operation decision variables

$\delta_{t,i,d,j} = 0,1$ (binary integer)

5. IMPLEMENTATION

This chapter describes the implementation of the mathematical model. The following sections describe the validation of model and procedure, the solver utilized for optimization and the platform on which the case studies are to be carried out.

5.1. Validation of Model and Procedure

The validation process involves separate calculation of the various parameters using a spreadsheet and using values of variables from program output. The specific validation calculations include:

1. Verifying that the decision variables conform to the constraints.
2. Verifying that the life cycle cost determined by the optimization agrees with summation of initial cost and the operation costs.
3. Verifying that the procedure selects the proper configuration based on the minimum life cycle cost.

5.2. Solver and Platform

The implementation is carried out utilizing a commercial optimization software that employs the branch and bound technique (ILOG, 1997). The solver is capable of handling linear programming problems involving mixed integer-continuous type of variables. The solver provides libraries enabling access from user specific routines or codes. Runs are conducted on an IBM machine with a Pentium II 200 MHz CPU.

5.3. Solver Interface

For the purpose of running the problem using the solution procedure, a user specific code taking advantage of the libraries supplied by CPLEX® is developed. The CPLEX® routines are subroutines handling the optimization tasks within the global program, i.e., minimizing the operation costs of a particular configuration during one demand period. The global problem incorporates the solution procedure described in section 4.7. The input to the global problem takes the form of a data file in a text format. Included in the data file are the information pertaining to the parameters and coefficients required in the mathematical model.

The output from the optimization program is the solution that describes the life cycle cost of the best configuration including its initial and operational costs. The output is a text format that is read for post processing using a spreadsheet program. In applying the decomposed problem, the files containing the solution of each demand period for each chiller configuration are generated separately.

The user is able to specify the stopping criteria for optimality acceptance. The general problem accesses CPLEX to determine the stopping criteria as specified by the user. The stopping criteria may be the relative MIP tolerance gap, the number of iterations performed, or the number of nodes evaluated. A relative MIP tolerance of 5 % and 25,000 iterations are imposed on the sub problems as the stopping criteria.

6. RESULTS

This chapter presents the application of the model and solution procedure developed in the previous chapter to case studies. Included in this chapter are descriptions of the facilities, the electricity and gas rate structures, the chiller selection group, and results of four hypothetical scenarios. The following sections describe the problem, the sources of data, and the output.

6.1. Description of Case Studies

Two facilities are considered as case studies. The facilities are assumed to be located in Atlanta, GA. The outdoor weather data are obtained from the BinMaker^{TM} database. The first facility is a hospital (Industrial Energy Center Study, 1996) with a correlation between cooling load and dry bulb temperature at each load condition j, demand period d taking the following form.

$$[\text{TL (tons)}]_{d,j} = 617.5 + 14.88\,[\,T_{db}\,(\text{F})\,]_{d,j}$$

For this case study there are twelve demand periods corresponding to each month of the year. Each demand period is divided into twelve load conditions each representing a two-hour segment of a typical day. Figure 7 presents the resulting profile of total cooling requirement with respect to the load conditions. The average cooling load for this facility is 81 percent of the maximum cooling load.

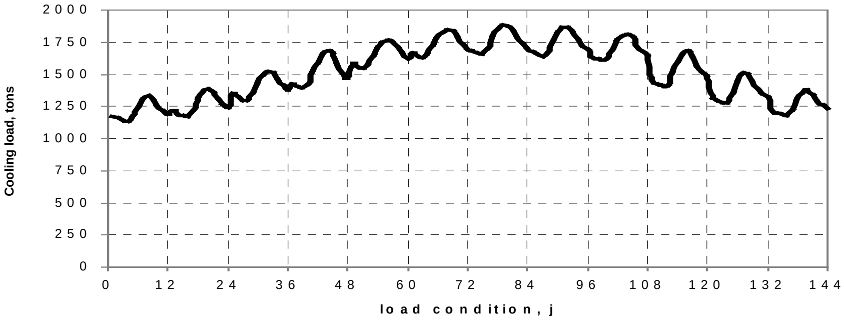

Figure 7. Annual total cooling load demand profile for hospital

The second facility is a hotel for which the cooling requirement at each load condition is determined based on a regression correlation of the cooling load vs. the outside dry bulb temperature developed by Shabo (1992). According to Shabo the cooling requirement is given by:

$$[\text{TL (tons)}] = -495.67 + 15.20\,[\,T_{db}\,(\text{F})\,] \qquad \text{for} \qquad T_{db} < 68.43\ {}^{o}\text{F}$$

$$[\text{TL (tons)}] = -1{,}855.4 + 35.07\,[\,T_{db}\,(\text{F})\,] \qquad \text{for} \qquad T_{db} \geq 68.43\ {}^{o}\text{F}$$

As in the previous case, the year is divided into twelve demand periods, each containing twelve load conditions. Figure 8 presents the resulting profile of total cooling requirement with respect to the load conditions. The average cooling load for this facility is 42 percent of the maximum cooling load.

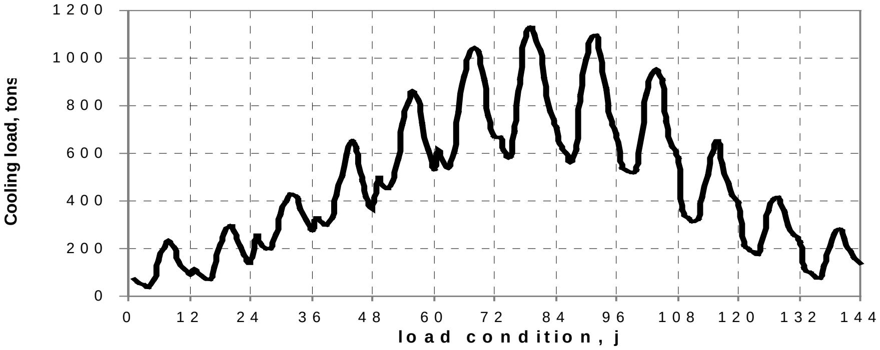

Figure 8. Annual total cooling load demand profile for hotel

For both facilities, the cooling tower approach is assumed to be constant at 7^o F difference between the ECWT and the outdoor wet-bulb temperature. A cut-off temperature at 55^o F is applied below which, the ECWT is maintained at 55^o F.

$$[\ T_{cond}^{in}\ ]_{d,j}\ (F) \qquad = 7 + [\ T_{wb}\ ]_{d,j}\ \ (F)$$

The annual ECWT profile is presented in Figure 9.

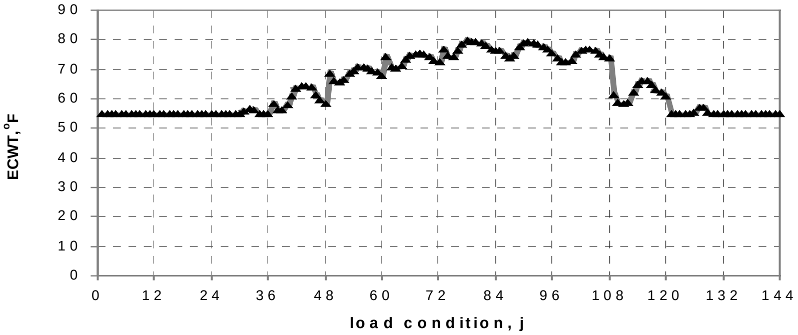

Figure 9. Annual ECWT profile

6.2. Description of Utility Rates and Schedules

The high case electricity rate schedule was obtained from Con Edison, whereas the low electricity rate follows that of Georgia Power. The price of natural gas is quoted from Shell Energy and United Cities gas for the high and low case respectively. The gas price is assumed to be constant within a demand period and is independent with respect to the cumulative consumption.

Table 1 shows the high and low case for the utility rates and schedule. The on-peak hours are set to be the second 4 load condition block, i.e., eight hours from 8 a.m. to 4 p.m. for all demand periods.

Table 1. Utility rate structures and schedule

Utility	Low	High
Electric Demand charge		
Demand Periods : 1,2,3,4,5,10,11,12		
On-peak	8 $/kW	16.47 $/kW [*]
8 am - 4 p.m.		
Off-peak	8 $/kW	0
12 am - 8 am & 4 p.m. - 12 am		
Distribution	0 $/kW	0
Demand Periods : 6,7,8,9		
On-peak	8 $/kW	20.97 $/kW [*]
8 am - 4 p.m.		
Off-peak	8 $/kW	0
12 am - 8 am & 4 p.m. - 12 am		
Distribution	0 $/kW	0
Electric Energy charge		
On -peak	2.53 cents/kWh	5.32 cents/kWh
Off-peak	2.53 cents/kWh	5.32 cents/kWh
Natural Gas energy		
All demand periods	30 cents/therm	39 cents/therm

[*] Demand charge is based on the maximum demand within an entire demand period

6.3. Description of Chiller Selection

This section describes the chiller selection group available to the designer. The information pertaining to the chillers are the performance curves and the associated chiller costs. The performance curves illustrated in Figures 11 and 12 are based on data provided by the chiller manufacturer York Corp for electric motor driven and gas engine driven chillers (York, 1997). The costs associated with the chillers are the initial costs, fixed and variable maintenance costs, and the energy and demand charges. The initial equipment costs for the electric chillers are based on Means Mechanical Cost Data (Means, 1997). Based on quotes from the chiller manufacturer, the initial costs for the gas engine driven chillers are estimated to be $ 600/ton capacity or approximately twice as much as that of the electrical motor driven as estimated by Arnold (1998).

The chiller selection group consists of five types. The chillers of type I are 300 ton electrical motor driven chillers with centrifugal constant speed compressor, type II are 600 ton electrical motor driven chillers with centrifugal constant speed compressor, type III are 200 ton gas engine driven chillers, type IV are 300 ton gas engine driven chillers, and type V are 600 ton gas engine driven chillers. The chiller performance curves are obtained from York. An additional constraint imposed on the selection group is that the maximum number of machines may not exceed 5 which in reality may be due to facility space limitation.

Figures 10 and 11 show the performance curves of the electrical motor driven and the gas engine driven chillers in the selection group of both case studies I and II. Table 2 shows the associated costs of the chillers in the selection group for the case studies.

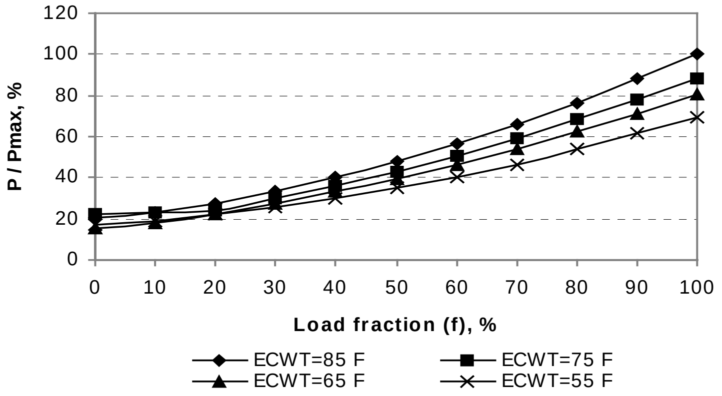

Figure 10. Performance curves for electric motor driven chiller

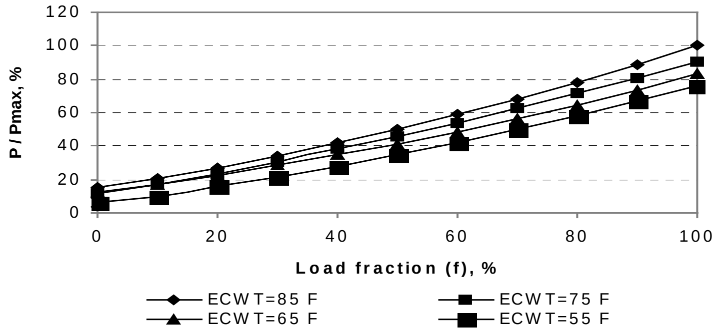

Figure 11. Performance curves for gas engine driven chiller

Table 2. Chiller Cost Data

Chiller Type	Chiller Driver	Chiller Capacity (tons)	Installation Cost ($)	Annual Fixed Maintenance Cost ($/ton)	Variable Maintenance Cost ($/ton-hr)	Power Consumption Ratio at Full Capacity
1	Electric Motor	300	106,000	15	0.00	0.65 kW/ton
2	Electric Motor	600	200,000	10	0.00	0.65 kW/ton
3	Gas Engine	200	120,000	20	0.01	0.26 scm/ton hr *
4	Gas Engine	300	180,000	15	0.01	0.28 scm/ton hr *
5	Gas Engine	600	360,000	10	0.01	0.22 scm/ton hr *

*1 scm is the heating value of one standard cubic meter of gas at 1 atm and 25 °C

The additional assumed data are as follows. The chiller life time operation is assumed to be 25 years with an annual interest rate of 6 percent.

6.4. Termination Criteria

The mathematical model and the solution procedure as developed in chapter 4 is implemented on all case studies. During all minimization sub routines within the solution procedure, the termination criterion is the mixed integer relative tolerance of 5 %. A termination criterion of 25,000 iterations is imposed if extensive computation time is encountered when minimizing a sub problem.

6.5. Results of Case Studies

This section presents the results of the case studies as implemented using the utility rates and schedules and chiller selections as described in the previous sections. Figures 12 to

15 present the minimum life cycle costs of the chiller selections for the four hypothetical scenarios. Tables 4 to 7 present the corresponding chillers selected for the associated scenario.

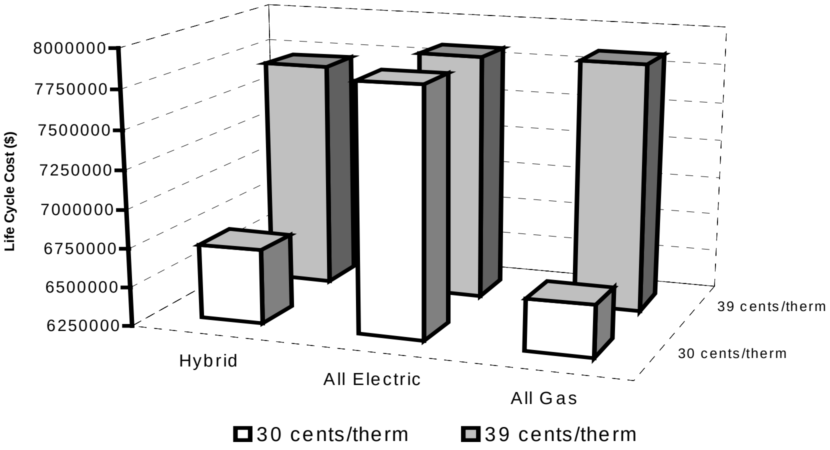

Figure 12. Life cycle costs for hospital with high electricity rates

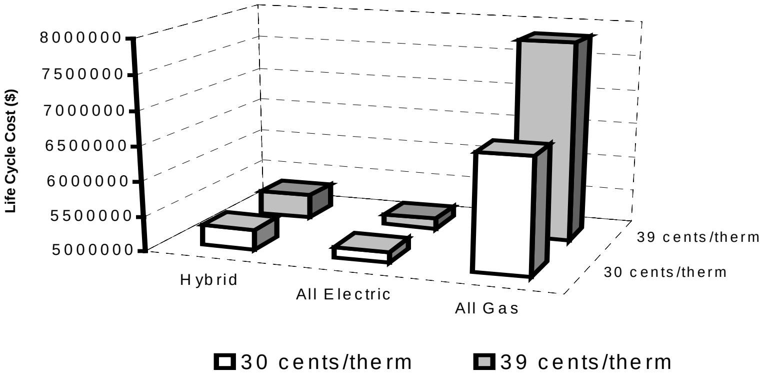

Figure 13. Life cycle costs for hospital with low electricity rates

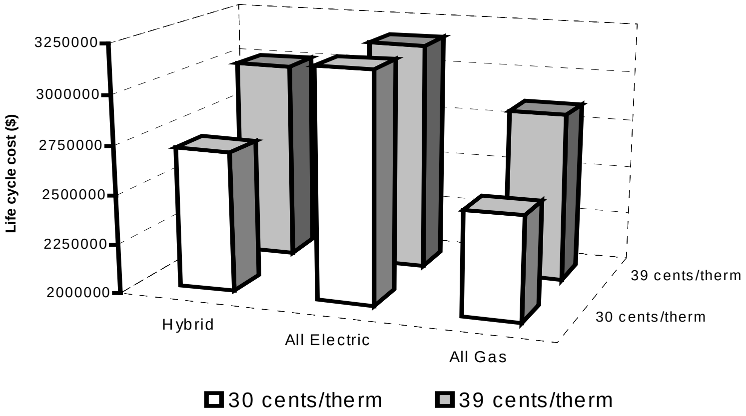

Figure 14. Life cycle costs for hotel with high electricity rates

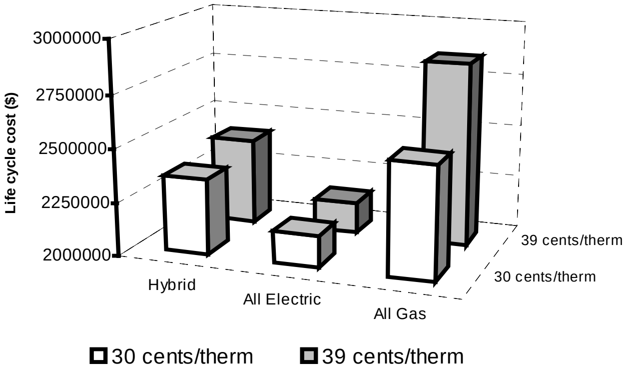

Figure 15. Life cycle costs for hotel with low electricity rates

Table 3. Configuration of plant with best life cycle cost for hospital with high electricity rates

	Hybrid					All Electric					All Gas				
	N_1	N_2	N_3	N_4	N_5	N_1	N_2	N_3	N_4	N_5	N_1	N_2	N_3	N_4	N_5
High Gas	1	0	0	0	3	1	3	0	0	0	0	0	1	0	3
Low Gas	1	0	0	0	3	1	3	0	0	0	0	0	1	0	3

Table 4. Configuration of plant with best life cycle cost for hospital with low electricity rates

	Hybrid					All Electric					All Gas				
	N_1	N_2	N_3	N_4	N_5	N_1	N_2	N_3	N_4	N_5	N_1	N_2	N_3	N_4	N_5
High Gas	0	3	1	0	0	1	3	0	0	0	0	0	1	0	3
Low Gas	0	3	1	0	0	1	3	0	0	0	0	0	1	0	3

Table 5. Configuration of plant with best life cycle cost for hotel with high electricity rates

	Hybrid					All Electric					All Gas				
	N_1	N_2	N_3	N_4	N_5	N_1	N_2	N_3	N_4	N_5	N_1	N_2	N_3	N_4	N_5
High Gas	1	0	0	1	1	2	1	0	0	0	0	0	0	0	2
Low Gas	1	0	0	0	2	2	1	0	0	0	0	0	0	0	2

Table 6. Configuration of plant with best life cycle cost for hotel with low electricity rates

	Hybrid					All Electric					All Gas				
	N_1	N_2	N_3	N_4	N_5	N_1	N_2	N_3	N_4	N_5	N_1	N_2	N_3	N_4	N_5
High Gas	1	1	0	1	0	0	2	0	0	0	0	0	0	0	2
Low Gas	1	1	0	1	0	0	2	0	0	0	0	0	0	0	2

where : N_t represents the number of machines of type t selected

Shading indicates the configuration with the lowest LCC

Excerpts from the operating schedule and a graph of the associated common load fraction are presented in Figures 16 to 21 for the following cases: Figures 16 and 17 for high electrical rate and high gas rate, Figures 18 and 19 for high electrical rate and low gas rate, and Figures 20 and 21 for low electrical rate and both high and low gas rate.

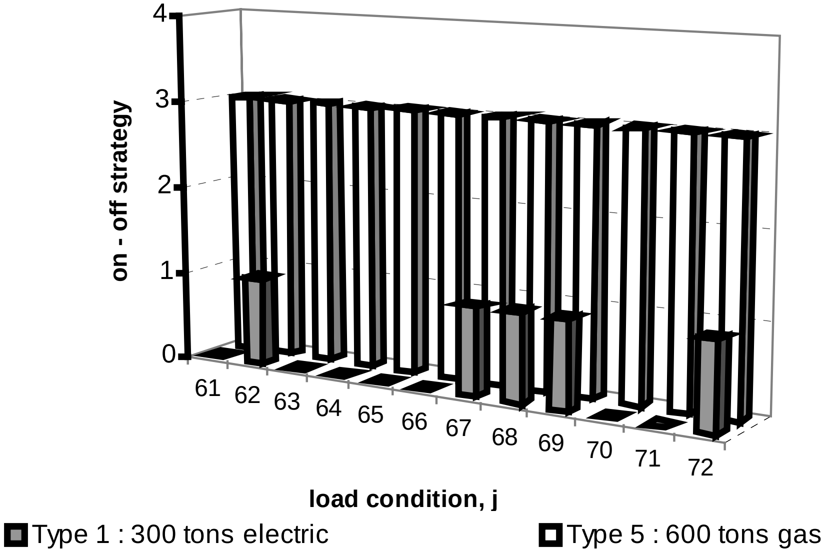

Figure 16. June operation strategy for hybrid chillers configuration for hospital with high electric rate and high gas price

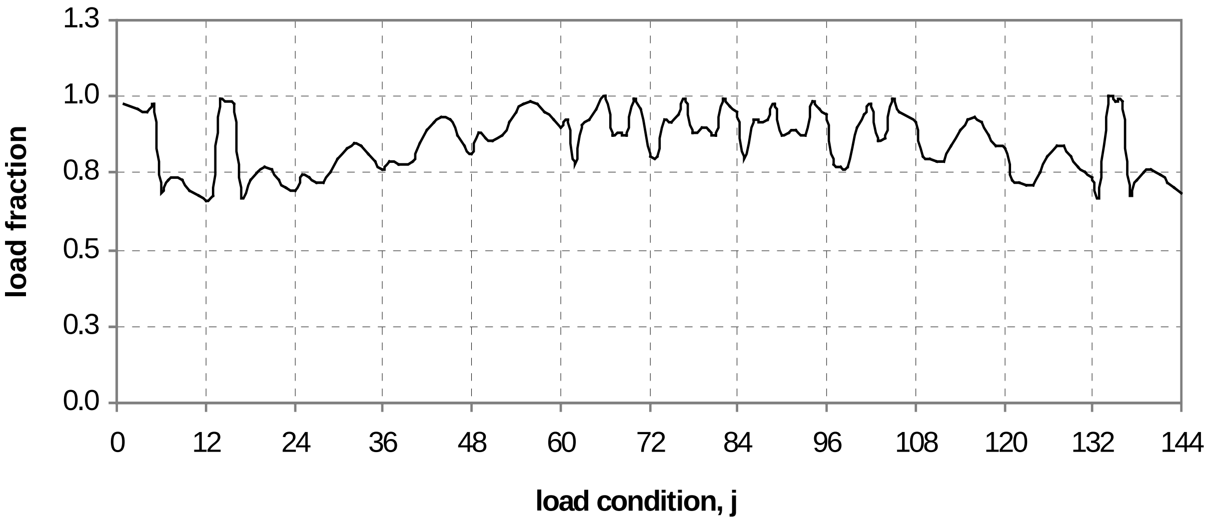

Figure 17. Common load fraction for machines in operation for hybrid configuration for hospital with high electric rate and high gas price

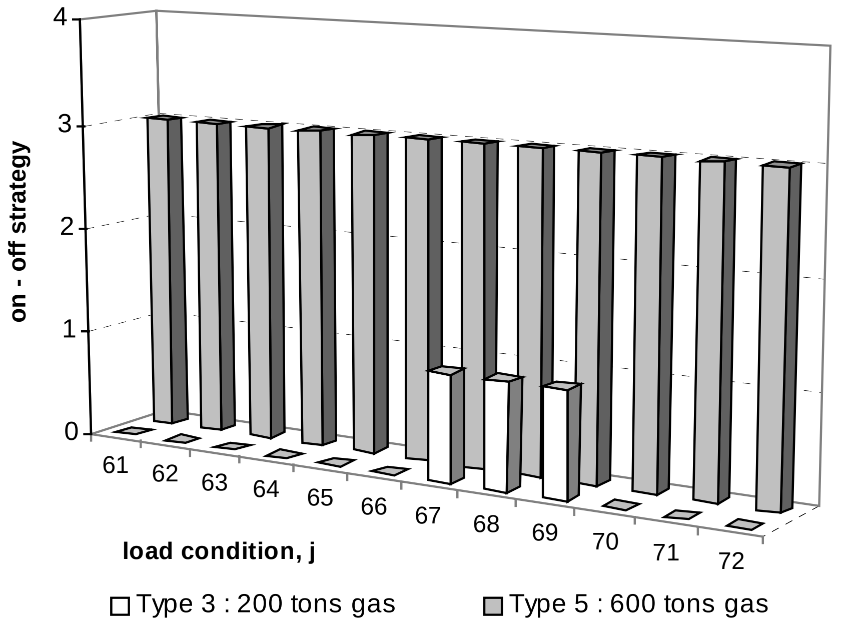

Figure 18. June operation strategy for all gas configuration for hospital with high electric rate and low gas price

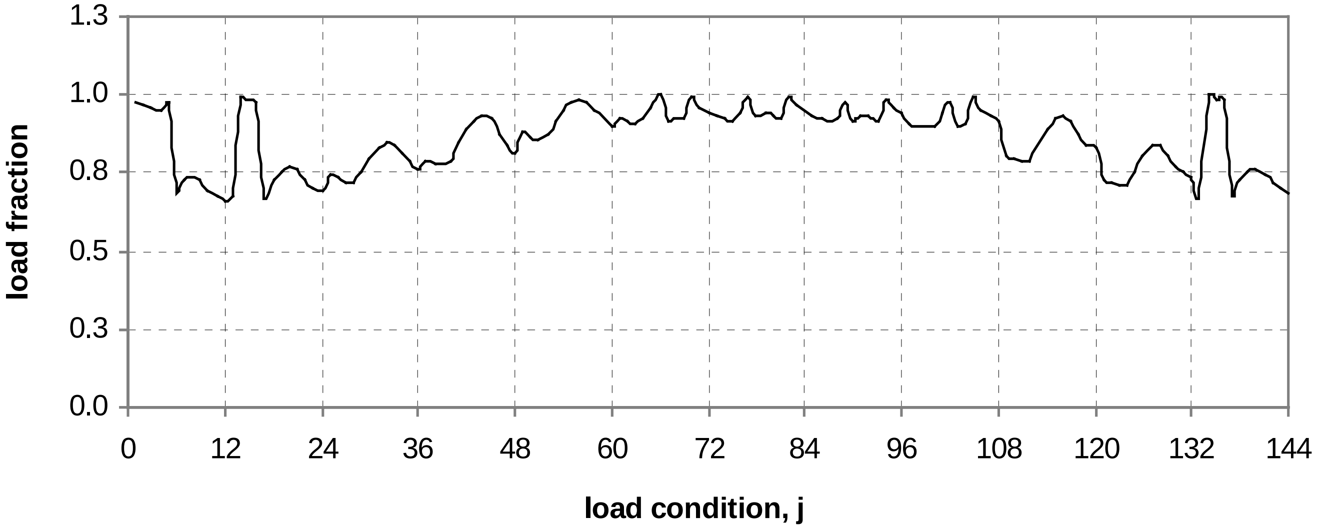

Figure 19. Common load fraction for machines in operation for all gas configuration for hospital with high electric rate and low gas price

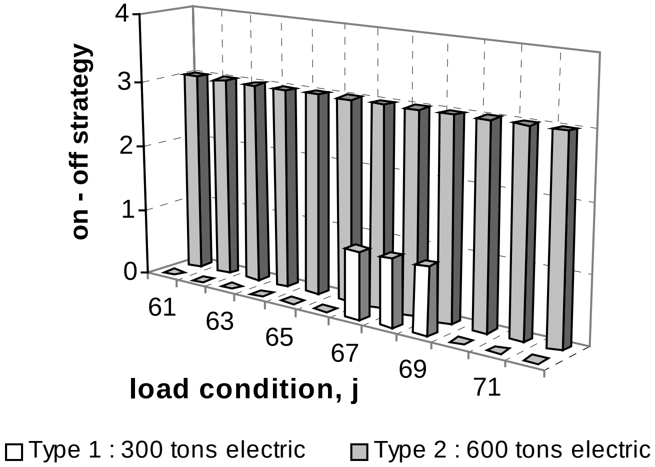

Figure 20. June operation strategy for all electric chillers for hospital with low electric utility rate

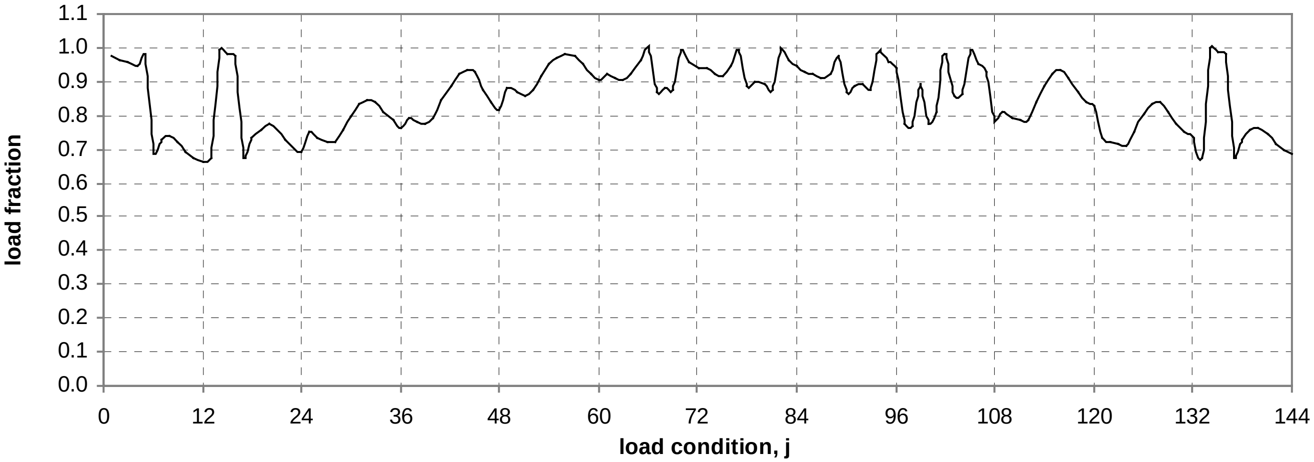

Figure 21. Common load fraction chart for all electric chillers in operation for hospital with low electricity rate

6.6. Discussion of Results

The results obtained from the implementation of the model are the configuration that has minimum life cycle cost, its operation strategy, and the load fraction based on which the chillers may be operated. With this information, a designer may select the configuration that has the lowest life cycle cost and conduct comparisons among other configuration options.

Figures 12 and 13, and Tables 3 and 4 present the life cycle costs and various plant configurations that are most attractive for the hospital case. The selected configurations for various options (all electric, all gas or hybrid plants) do not change when the gas price is between 39 and 30 cents per therm. However, the configuration that has the lowest LCC differs when the gas price is lowered from 39 to 30 cents per therm. In addition, the configuration that is most attractive for the hospital case following a low electricity rate scenario is not affected by that range of gas price.

Figures 14 and 15, and Tables 5 and 6 present the life cycle costs and various plant configurations that are most attractive for the hotel case. The results show that the plant configuration with the lowest LCC does not change for gas prices between 39 and 30 cents per therm. The high electricity rate scenario, as shown by Table 5, shows that for gas prices below 39 cents per therm, an all gas plant is more favorable compared to the other two options. Whereas, the low electricity rate scenario, as shown by Table 6, shows that gas prices between 39 and 30 cents have no effect due to the fact that the lowest LCC is given by an all electric chiller plant. The configuration that changes for gas price between 39 and 30 cents per therm is the hybrid plant and is only for the high electricity rate.

The information pertaining to the operation strategy and load fraction is beneficial to a plant operator. With this information, a plant operator may be able to determine how the chillers are to be operated throughout the corresponding load conditions, i.e., at a certain hour of the day which chiller and to what load fraction is the chiller to be operated.

Figures 16, 18, and 20 show that each configuration has different operation strategy. Similarly, from Figures 18, 20, and 22, the common load fraction are unique to the respective configuration. The uniqueness of the operating strategy and the common load fraction to the respective configuration satisfies the total cooling load requirement at all load conditions. Given the information pertaining to the on-off strategy and the common load fraction, the plant operator is able to operate the plant yielding the lowest operation cost for the respective load conditions.

Based on the examined typical hospital and hotel, all electric chiller plants are favorable when electricity energy may be obtained at 2.53 cents/kWh and demand charges are 8 $/kW for both on-peak and off-peak load condition whereas the gas price are within the 39 to 30 cents/therm range. An all gas plant having gas price below 30 cents/therm may be favorable when electricity must be obtained at both higher energy rate, i.e., 5.32 cents/kWh, and higher demand charges, i.e., 20.97 $/kW for summer and 16.47 $/kW for non summer months. The hybrid plant may be attractive only when the competition among the two source of energy favors the application of such plant through its operation strategy.

7. CONCLUSION AND RECOMMENDATIONS

7.1. Conclusion

A mathematical model defining the chiller selection problem has been developed along with identifying the necessary information as input. The developed approach is capable of selecting chillers from a selection group to minimize life cycle cost subject to given input data. The results of the model may be used by designers who need to select chiller configurations for chilled water plants or by operators who need to determine the operating strategy and schedule of the chillers.

Examination of a typical hospital and hotel using the model reveals that for electricity rates of 2.53 cents/kWh for energy and 8 $/kW for demand compared with gas rates within the range of 39 to 30 cents per therm, all electric plants may be the most attractive. With higher electricity energy and demand rates, and gas prices below 39 cents/therm all gas plants may be more favorable. When the gas price is 39 cents/therm compared with electricity energy rate of 5.32 cents/kWh and demand charge of 20.97 $/kW for summer and of 16.47 $/kW for non-summer months, a hybrid plant may be the most favorable option.

The structure of the model is capable of addressing problems with high numbers of load conditions, a variety of chiller options, and complex rate schedules. The applicability of the model is limited primarily by the availability of optimization techniques capable of addressing a large number of mixed integer-continuous variables in a reasonable computational time. The computational time for each scenario of the case studies implemented on platforms and programs commercially available required approximately 30 hours.

7.2. Recommendations for Future Developments

This section suggests several recommendations for the purpose of improvements in future modeling and solving of the chiller selection problem. The recommendations address both modeling and solution procedure issues.

Recommendations for future work related to the model include:

> *Application to operation optimization*. With the ability to calculate the minimum operating costs, the model is capable of determining the appropriate operating strategy for a given configuration to obtain minimum operating costs during a particular period of time. Therefore, the model may be applied to operation optimization of an existing cooling plant.
>
> *Selection of chillers with different design temperature differences across the evaporator*. This mode of operation will permit unequal loading of the parallel chillers but will lead to irreversibilities due to mixing chilled water of different temperatures.
>
> *Introduce additional decision variables*. This will permit additional aspects of the chiller plant such as evaporator temperature setpoint and cooling tower operation to be incorporated into the optimization.
>
> *Introduce multiphase life cycle cost analysis*. When future plans include the addition or expansion of the plant facility and these activities will be interdependent upon the chiller selection criteria then it may be advantageous to the designer to incorporate the future plans into present investment. Multiphase life cycle cost analysis enables the project to be phased according to the planning of project implementation. Each phase develops its life cycle cost and the total life cycle spanning the entire period of all phases is then to be minimized.

Recommendations for future work related to the solution approach include:

Explore non-linear optimization. The use of non-linear optimization would avoid the need to linearize the chiller performance curves and would eliminate the large number of decision variables associated with the piecewise linear approximation. The issues related with this approach include determining the step size, terminating criteria and formulating the problem such that the discrete decision variables, i.e., purchasing and operating decisions, are incorporated.

Consider dynamic programming. The application of dynamic programming to the resource costs that are dependent on the history of resource consumption may be explored. In practice, variable energy rates exist and are typically established based on a declining block of rate schedule based on its consumption. For example, natural gas is charged at lower prices after passing a certain amount of gas usage and electricity charges are often based on the cumulative energy usage.

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VITAE

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