COMPARISON OF DESIGN METHODS FOR SHEAR IN REINFORCED CONCRETE BEAMS

by

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Thesis submitted to the Faculty of the

Virginia Polytechnic Institute and State University

in partial fulfillment of the requirements for the degree of

Master of Science

in

Civil Engineering

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April 1992

Blacksburg, Virginia

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(ABSTRACT)

There are two basic methods used to analyze and design reinforced concrete members for shear, the American Concrete Institute Code method (used in the United States) and the truss model method (used in different European Codes and in the Canadian Codes). The ACI Code method is a semi-empirical solution since it builds on fitting actual test results to the analytical mechanism method. Although it will lead to safe results, the ACI method lacks a physical model to represent the actual forces in the beam. For this reason, attention is increasingly being given to develop a mechanical-mathematical model to show the actual behavior of the beam failing in shear. The truss model theory provides a more promising way to treat shear since it can model the structural action in the beam. It was first proposed by Ritter and Mörsh at the turn of the twentieth century. The concept has been extended by recent work of Lampert and Thürlimann (1971), Collins and Mitchell (1980), MacGregor (1988), and others.

The purpose of this study is to compare the ACI method and the truss model method for the design of reinforced concrete slender and deep beams.

The results of this study suggest that the truss model is better suited for the design of deep reinforced concrete beams because it models the dominant

mechanism that happens in the beam which is the force transfer from load to reaction by the direct compression struts. For slender reinforced concrete beams, either method is suitable for design.

ACKNOWLEDGEMENTS

The author would like to express his gratitude to Dr Richard M. Barker for his help and guidance throughout this study especially for carefully reviewing the manuscript. Thanks are also due to Dr S. M. Holzer and D. A. Garst for serving on the committee. The author is grateful to all his friends, especially Jacem Tissaoui and Moncef Sellami for their assistance. Finally, the author appreciates the moral support and encouragement of his family while pursuing his college studies.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF TABLES

CHAPTER 1

INTRODUCTION

1.1 Shear Stresses in Concrete Beams

In elastic homogeneous beams, where stresses are proportional to strains, two types of stresses occur, bending and shear. They can be found using the following well-known expressions [Gere and Timoshenko, 1984]:

$$f = \frac{M\,c}{I} \qquad (1.1)$$

$$v = \frac{V\,Q}{I\,t} \qquad (1.2)$$

Where f is the bending stress, M is the bending moment, c is the distance from the neutral axis to the point where the stress is being calculated, I is the moment of inertia, v is the shear stress, V is the shear force, Q is the statical moment about the neutral axis of the area between the surface and the point at which shearing stresses are to be determined, and t is the thickness of the considered section. The effect of shear stresses can be visualized by loading a laminated beam as shown in Figure 1.1. The beam is formed by gluing two rectangular pieces. If the glue is strong enough, the beam will deform as a single unit, as shown in Figure 1.1.a. However, if the glue is weak, the two pieces will slide relative to each other along their contact surface, as shown in Figure 1.1.b. This means that there are horizontal stresses acting on the contact surface. These stresses are shown in Figure 1.1.c. In general, these

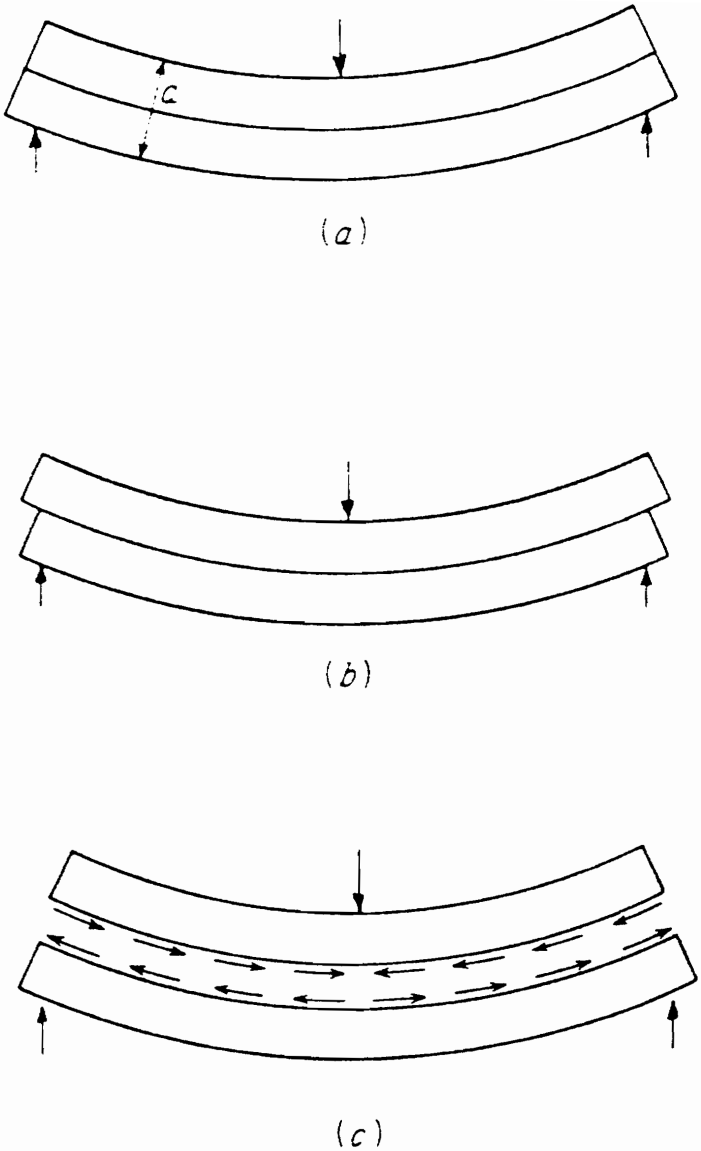


Figure 1.1

Shear stresses in homogeneous rectangular beams

(Nilson and Winter, 1991)

stresses occur in horizontal planes in any beam. They are called horizontal shear stresses and their intensity differs with the distance from the neutral axis. Using equilibrium, one can show that shear stresses of the same magnitude as the horizontal stresses will occur in any vertical plane of the beam (this is true because if there were only horizontal stresses, spinning would occur).

If we isolate a small square element located at the neutral axis, the only stresses that act on that piece are shear stresses, as shown in Figure 1.2.b. However shear stresses as well as bending stresses will act on any piece that is located neither at the neutral axis nor at the outer edges, as shown in Figure 1.2.d. These stresses combine into inclined compressive and tensile stresses, called principal stresses, which can be determined from the following expression [Gere and Timoshenko, 1984]:

$$t = \frac{f}{2} \pm \sqrt{\frac{f^2}{4} + v^2} \qquad (1.3)$$

Where t are the principal stresses. The direction of these principal stresses with respect to the beam's axis can be found using the following equation:

$$\tan 2\alpha = 2\,\frac{v}{f} \qquad (1.4)$$

At different positions along the beam the magnitude of v and f change, and therefore the directions of the principal stresses change. At the neutral axis, the principal stresses will be located at 45° angle with the horizontal, as

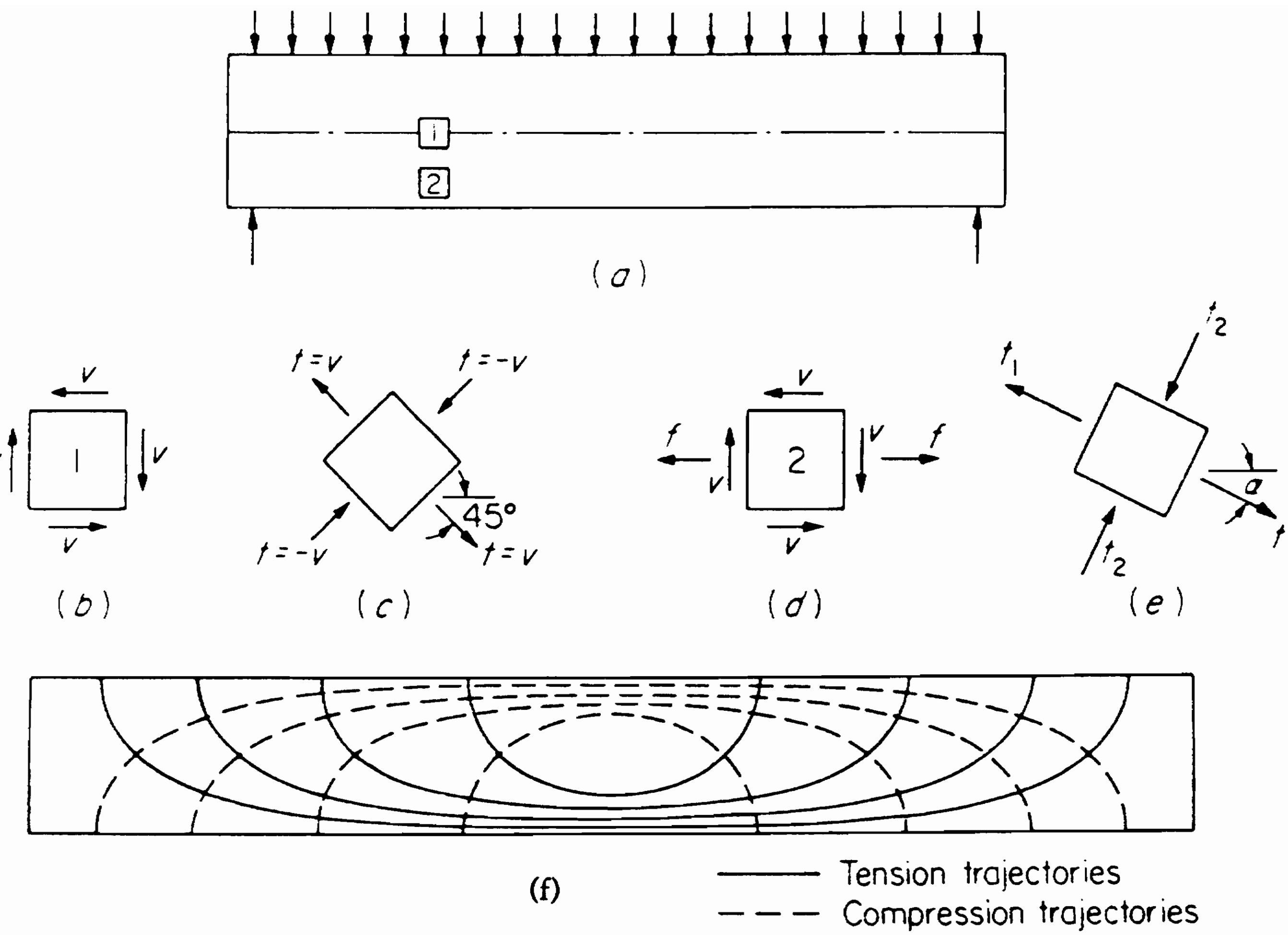


Figure 1.2

Stress trajectories in homogeneous rectangular beams

(Nilson and Winter, 1991)

shown in Figure 1.2.c. Figure 1.2.f shows the orientation of the principal stresses in a uniformly loaded rectangular beam. One can conclude that tension stresses (which are important since concrete has a low tensile strength) of various inclinations and magnitudes resulting from shear alone (at the neutral axis) or from the combination of shear and bending exist in all parts of a beam. Therefore a shear failure in a concrete beam results from this inclined tension, which is often referred to as diagonal tension.

1.2 Literature Review

Ritter (1899) and Mörsh (1902) proposed the concept of the truss model to treat shear problems in reinforced concrete beams. They assumed that a cracked reinforced concrete beam acts as a truss composed of two parallel longitudinal chords, diagonal concrete struts, and transverse steel ties. This truss analogy is shown in Figure 1.3. They also assumed that the concrete struts are inclined at 45° to the steel bars. Mörsh (1922) explained this choice for the angle of inclination of the diagonals by :
"We have to comment with regards to practical application that it is absolutely impossible to mathematically determine the slope of the secondary inclined cracks according to which one can design the stirrups. For practical purposes one has to make a possibly unfavorable assumption for the slope θ and therefore, with $\tan 2\theta = \infty$, we arrive at our usual calculation for stirrups which presumes θ equal 45°. Originally this was derived from the initial shear cracks which actually exhibit this slope."

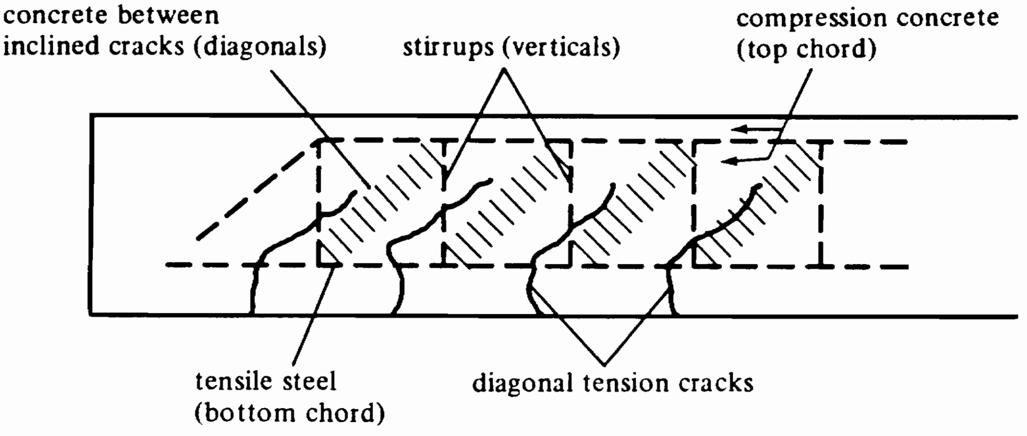


Figure 1.3

Truss analogy (McCormac, 1986)

Experience showed that the results of the 45° truss model were quite conservative especially for beams with small amounts of web reinforcement. For this reason, in the American Concrete Institute Code, there is an empirical correction term to the 45° truss equation. This added shear capacity, known as the "concrete contribution", is equal to the shear at the commencement of diagonal cracking.

Lampert and Thürliman (1971) found out that the angle of inclination of the concrete struts may deviate from 45°. Based on their experimental work on beams in torsion, they came up with some empirical limits on this angle.

Collins and Mitchell (1980) used compatibility equations to determine the angle of inclination of the diagonals. They assumed that this angle coincide with the angle of inclination of the principal compression stress and strain. Their theory was known as the compression field theory. The limits for the angle can be simplified to:

$$\theta_{min} = 10 + 110\left(\frac{Vu}{\phi f_c' b_w jd}\right) \text{degrees} \qquad (1.5.a)$$

$$\theta_{max} = 90 - \theta_{min} \qquad \text{degrees} \qquad (1.5.b)$$

Where V_u is the factored shear force, b_w is the web width, jd is the lever arm, f_c' is the concrete compressive strength, and ϕ is the strength reduction factor.

1.3 Purpose and Organization

The purpose of this study is to compare the ACI Code method and the Truss Model Theory in the design of slender and deep reinforced concrete beams for shear. This comparison will be based on design examples. Chapter Two contains the ACI Code provisions for designing slender reinforced concrete beams. The procedure and design examples will be included. Chapter Three consists of a description of the Truss Model Theory for slender beams. The examples used in Chapter Two will be reworked using the Truss Model Theory. Chapters Four and Five will be concerned with the design of deep reinforced concrete beams using the ACI Code and the Truss Model Theory, respectively. Chapter Six contains an evaluation of the two methods based on a comparison of the results obtained from the design examples.

CHAPTER 2

DESIGN OF SLENDER R/C BEAMS FOR SHEAR - ACI CODE

2.1 Types of Web Reinforcement

Web reinforcement usually takes the form of stirrups, spaced at varying intervals along the axis of the beam, as shown in Figure 2.1a. The most common stirrups are U shaped (Figure 2.1b). Other types of stirrups are shown in Figures 2.1c, 2.1d, 2.1e (multiple stirrups which are more desirable for wide beams), and 2.1f.

Bars, called hangers, are placed on the compression side of beams to support the stirrups. The stirrups are passed around the tensile steel and are hooked around the hangers. In most cases, special anchorage must be provided. Section 2.6 is a summary of the ACI Code requirements for anchorage of stirrups.

In some cases, shear reinforcement may be provided by bending up a part of the tensile steel where it is no longer needed to resist flexural tension. This is illustrated in Figure 2.1g.

2.2 ACI Code Basic Equation for Shear Design

In the ACI Code (article 11.1.1), the basic design equation of cross sections subject to shear is:

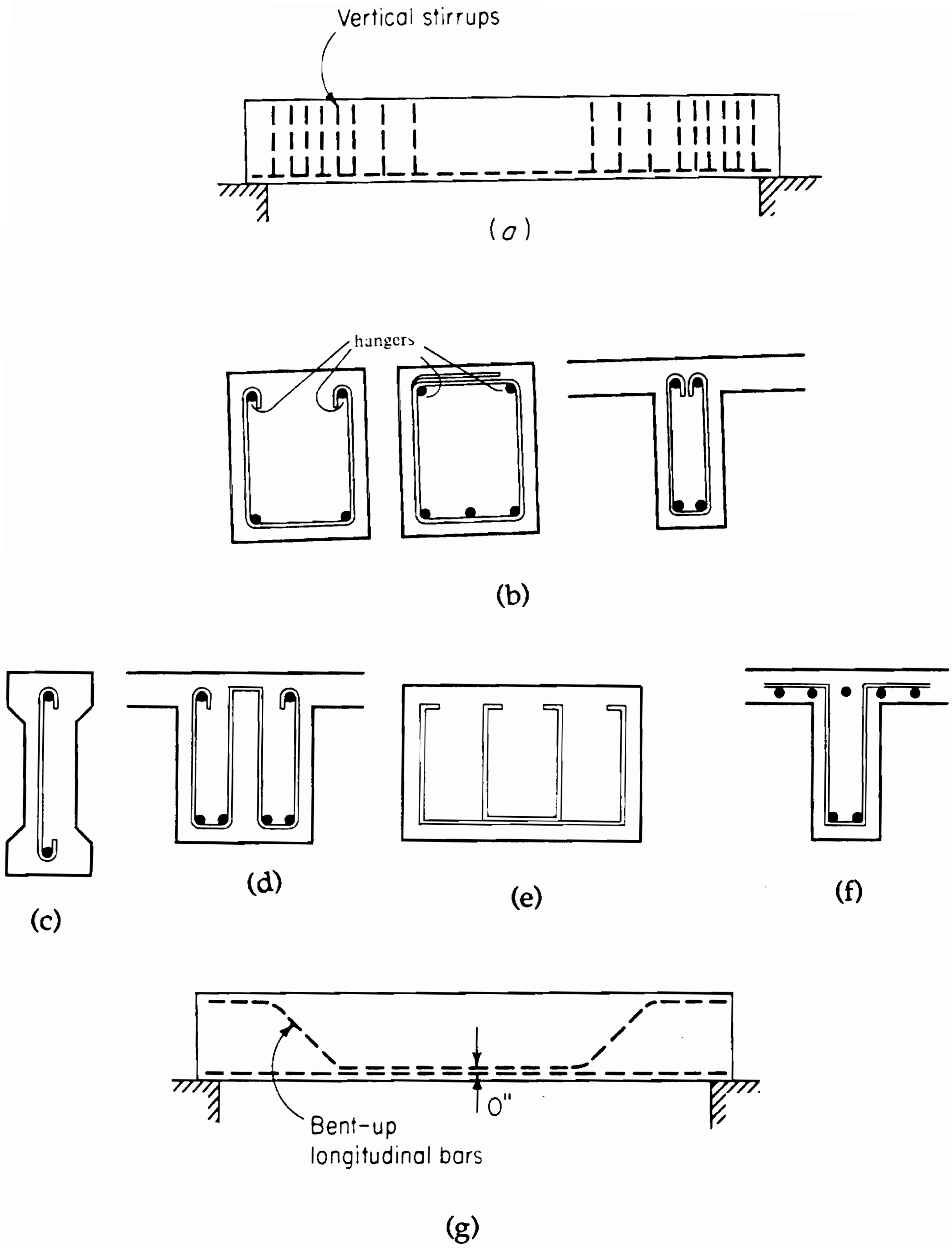


Figure 2.1

Types of web reinforcement

$$V_u \leq \Phi V_n \qquad (2.1)$$

where V_u is the factored shear force at the section considered, V_n is the nominal shear strength, and Φ is the strength reduction factor which is equal to 0.85.

The nominal shear strength is equal to the sum of the contributions of the concrete V_c and the web steel V_s. This is written in equation form as:

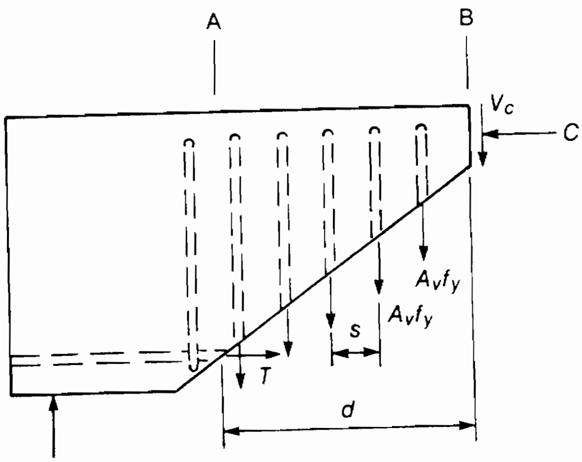
$$V_n = V_c + V_s \qquad (2.2)$$

Figure 2.2a shows a free body diagram of a section located between the end of a beam and an inclined crack. Assuming that the stirrups yield at failure, the shear resisted by these stirrups is:

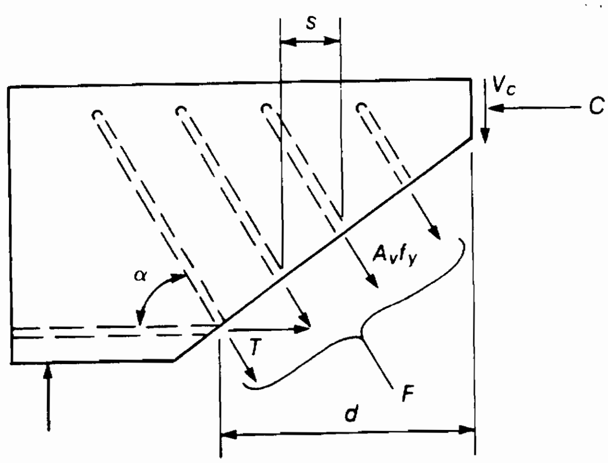
$$V_s = \frac{A_v f_y d}{s} \qquad (2.3)$$

Where A_v is the steel area of each stirrup (this means that if U stirrups are used, then A_v is equal to twice the cross sectional area of the bar. For stirrups such as those in Figure 2.1e, A_v is equal to four times the area of the bar), fy is the steel yield strength, d is the beam depth, and s is the stirrups spacing. If the stirrups are placed as shown in Figure 2.2b, the shear resisted by these inclined stirrups is:

$$V_s = A_v f_y (\sin\alpha + \cos\alpha) \frac{d}{s} \qquad (2.4)$$



(a) Vertical stirrups.



(b) Inclined stirrups.

Figure 2.2

Shear resisted by stirrups (MacGregor, 1992)

2.3 Shear Strength Provided by the Concrete

Figure 2.3 shows the shear capacity of simply supported beams without stirrups as a function of the steel ratio. For beams developing shear failure (steel ratio between 0.0075 and 0.025), the shear strength is approximately:

$$V_c = 2\sqrt{f'_c}\, b_w\, d \tag{2.5}$$

One can note from the graph that Equation 2.5 overestimates V_c for beams with small steel percentage (ACI-ASCE committee 426, 1978).

A more exact equation to determine the concrete contribution to shear strength is given by:

$$V_c = (1.9\sqrt{f'_c} + 2500\frac{\rho_w V_u d}{M_u})\, b_w\, d \le 3.5\sqrt{f'_c}\ b_w\, d\ ;\ \frac{V_u d}{M_u} \le 1.0 \tag{2.6}$$

Where ρ_w is the longitudinal tensile steel ratio, V_u and M_u are the factored shear force and bending moment at the considered section, and b_w and d are the section dimensions in inches. Figure 2.4 shows the correlation of Equation (2.6) with test results. Since the quantities ρ_w , V_u and M_u change along the span of the beam, the use of Equation (2.6) is usually limited to computerized design. Therefore Equation (2.5) is the most commonly used one in determining the concrete contribution.

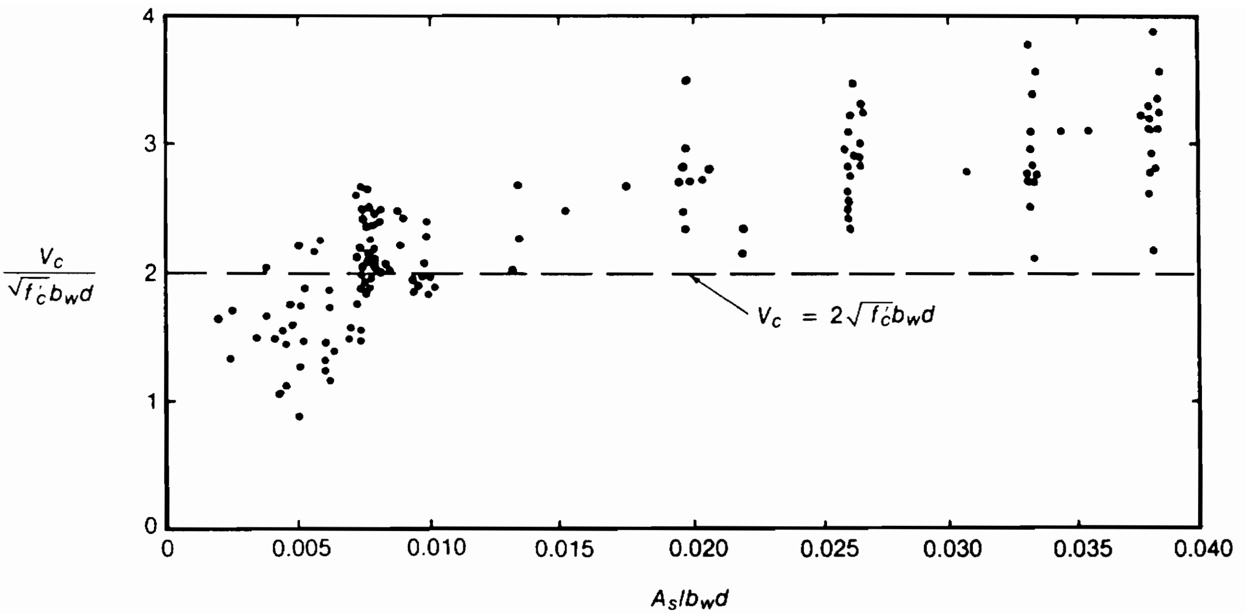


Figure 2.3

Shear capacity of simply supported beams without stirrups (MacGregor, 1992)

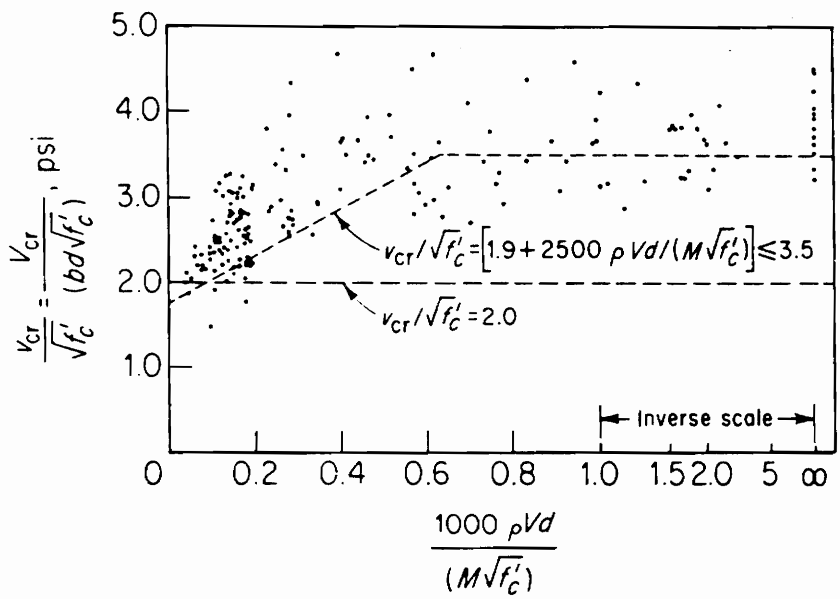


Figure 2.4

Correlation of Eq. 2.6 with test results (Nilson and Winter, 1991)

Equations (2.5) and (2.6) were based on test results obtained using beams with concrete compressive strength between 3000 and 5000 psi. These equations overestimate the concrete contribution to shear strength for beams constructed using high strength concrete with f'_c above 6000 psi [Elzanaty and Nilson 1986; Mphonde and Franz 1984; Ahmed, Khaloo, and Proveda 1986]. Therefore, there is an upper limit of 100 psi to use for $\sqrt{f'_c}$ in Eqs. (2.5) and (2.6) [ACI Code 11.1.2]. However, if the minimum amount of web reinforcement is increased, values of $\sqrt{f'_c}$ greater than 100 psi may be used in those equations.

2.4 Minimum Web Reinforcement

If the value of ΦVc calculated using either Equation (2.5) or (2.6) is greater than the total factored shear force, then, theoretically there is no need for web reinforcement. However, ACI Code 11.5.5 requires some minimum web reinforcement. This minimum is taken as:

$$A_v = 50 \frac{b_w s}{f_y} \tag{2.7}$$

Where s is the longitudinal spacing of web reinforcement in in., f_y is the yield strength of web steel in psi, and A_v is the total cross-sectional area of web steel in in^2. This minimum web reinforcement must be satisfied unless V_u is one-half or less of the design shear strength ΦVc provided by the concrete. One may feel that the use of this minimum web reinforcement is unnecessary.

However, studies of earthquake damage in recent years have shown large amounts of shear damage occurring in reinforced concrete structures, and it is thought that the use of this minimum web reinforcement will improve the resistance of such structures to seismic forces. Actually, many designers think that the minimum web reinforcement should be used throughout the whole beam [McCormac, 1986].

2.5 Design of Web Reinforcement

The design of web reinforcement using the ACI Code provisions is based on the equations:

$$V_u \leq \Phi V_c + \Phi \frac{A_v f_y d}{s} \quad \text{; for vertical stirrups} \qquad (2.8)$$

$$V_u \leq \Phi V_c + \Phi A_v f_y (\sin\alpha + \cos\alpha) \frac{d}{s} \quad \text{; for inclined stirrups} \qquad (2.9)$$

Since there are standard stirrup sizes (usually in the range from numbers 3 to 5), one should first select a trial steel area A_v based on the stirrup size. Then, spacing of the stirrups can be found using the following equations:

$$s = \frac{\Phi A_v f_y d}{V_u - \Phi V_c} \quad \text{; for vertical stirrups} \qquad (2.10)$$

$$s = \frac{\Phi A_v f_y d (\sin\alpha + \cos\alpha)}{V_u - \Phi V_c} \quad \text{; for inclined stirrups} \qquad (2.11)$$

The ACI Code does not allow a diagonal 45° crack to run a vertical distance of more than d/2 without being intercepted by a stirrup. Therefore ACI section 11.5.4.1 sets the maximum spacing of vertical stirrups as the smaller of d/2 or 24 in. For inclined stirrups, ACI Code Section 11.5.4.2 states: "Inclined stirrups and bent longitudinal reinforcement shall be so spaced that every 45° line, extending toward the reaction from mid-depth of member d/2 to longitudinal tension reinforcement, shall be crossed by at least one line of shear reinforcement". These spacing requirements are shown in Figure 2.5. If V_S exceeds $4\sqrt{f_c'}\ b_W\ d$, the maximum allowable stirrup spacing are reduced to the smaller of d/4 and 12 in. for vertical stirrups. This is because smaller spacing gives narrower inclined cracks and provides better anchorage for the lower ends of the compression diagonals.

2.6 Anchorage Requirements

When developing Equations (2.3) and (2.4), it was assumed that the stirrups will yield at ultimate. For that to be true, the stirrups have to be well anchored. The ACI Code requirements for anchorage are illustrated in Figure 2.6 and are summarized as follows [ACI 318-89]:

a) Section 12.13.1- "Web reinforcement shall carried as close to compression and tension surfaces of members as cover requirements and proximity of other reinforcement will permit."

b) Section12.13.2- "Ends of single leg, simple U-, or multiple U-stirrups shall be anchored by one of the following means:

- For #5 bar and D31 wire, and smaller, and for #6, #7, and #8 bars with

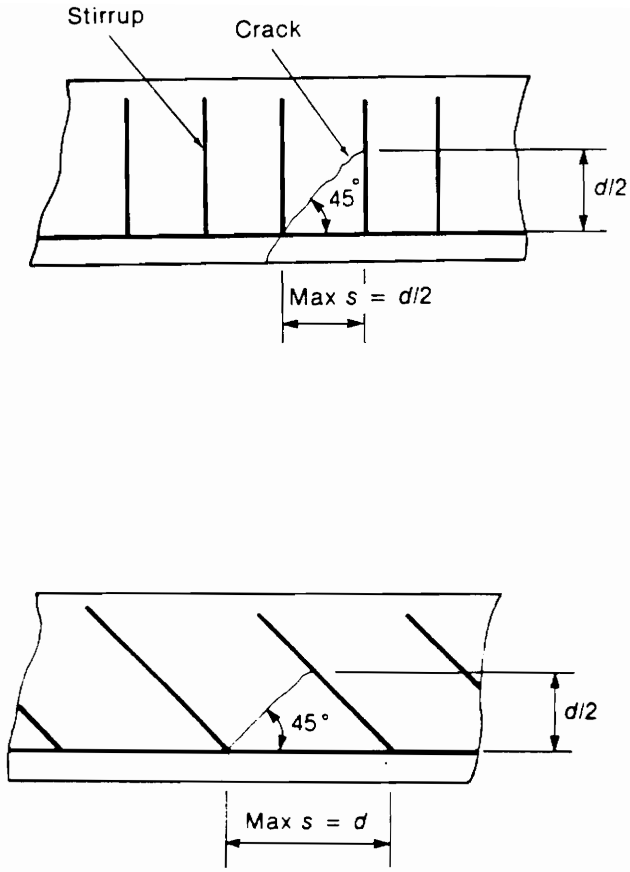


Figure 2.5

Maximum spacing of stirrups

(MacGregor, 1992)

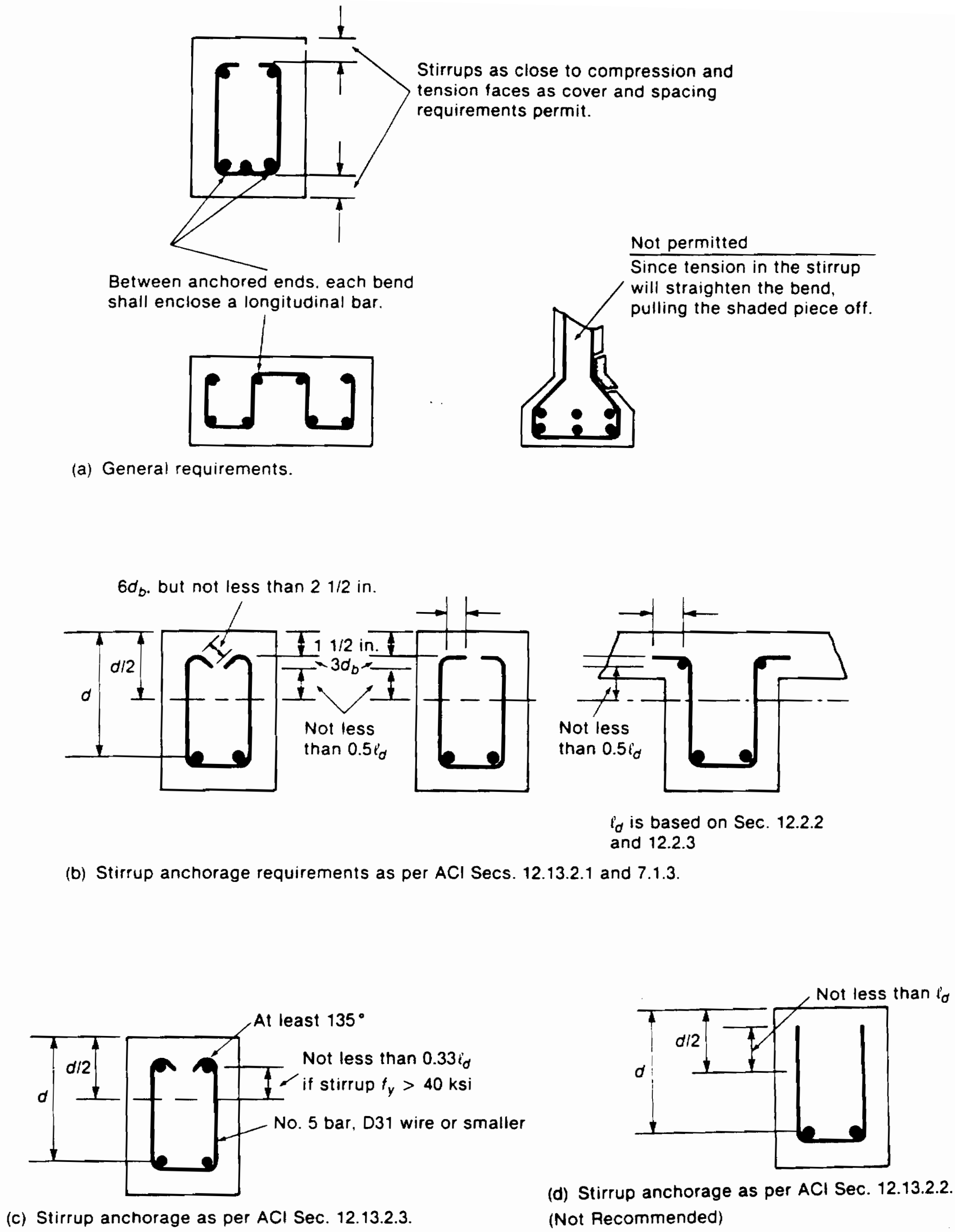


Figure 2.6

Anchorage requirements (MacGregor, 1992)

f_y of 40000 psi or less, a standard hook around longitudinal reinforcement (12.13.2.1).

- For #6, #7, and #8 stirrups with f_y greater than 40000 psi, a standard hook around a longitudinal bar plus an embedment between mid-height of the member and the outside end of the hook equal to or greater than $0.014 d_b f_y / \sqrt{f'_c}$ (12.13.2.2).
- For each leg of welded plain wire fabric forming simple U-stirrups, either :

  (a) Two longitudinal wires spaced at a 2 in.spacing along the member at the top of the U.

  (b) One longitudinal wire located not more than d/4 from the compression face and a second wire closer to the compression face and spaced not less than 2 in. from the first wire. The second wire may be located on the stirrup leg beyond a bend, or on a bend with an inside diameter of bent not less than $8d_b$ (12.13.3)."

2.7 Summary and Design Examples

Table 2.1 illustrates the steps involved in designing a slender reinforced concrete beam for shear using the ACI Code procedure. These steps will be followed in the next three design examples.

Table 2.1
Summary of steps for designing vertical stirrups
(McCormac, 1986)

Is shear reinforcing necessary ?	ACI section
1. Draw V_u diagram	
2. Calculate V_u at a distance d from support	11.1.3.1
3. Calculate ΦV_c	11.3.1.1
4. Stirrups are needed if $V_u > 0.5\,\Phi V_c$	11.5.5.1
Design of stirrups	
1. Calculate theoretical stirrup spacing ; $s = \frac{\Phi A_v f_y d}{V_s}$	11.5.6.2
2. Determine maximum spacing to provide minimum area of shear reinforcement; $s = \frac{A_v f_y}{50\, b_w}$	11.5.5.3
3. Compute maximum spacing: $\frac{d}{2} < 24$ in., if $V_s < 4\sqrt{f'_c}\, b_w d$	11.5.4.1
4. Compute maximum spacing; $\frac{d}{4} < 12$ in., if $V_s > 4\sqrt{f'_c}\, b_w d$	11.5.4.3
5. V_s may not be $> 8\sqrt{f'_c}\, b_w d$	11.5.6.8
6. Minimum practical spacing = approximately 3 or 4 in.	

2.7.1 Design Example 1

A simply supported beam with a rectangular cross-section (Figure 2.7a) carries a dead load of 1.75 k/ft (including its self weight) and a live load of 2 k/ft . Design this beam using the ACI Code procedure.

f'_c = 3500 psi, f_y = 60000 psi, L = 30 ft, b_w = 15 in, d = 28.5 in.

$w_u = 1.4\ w_D + 1.7\ w_L = 5.85$ k/ft (Figure 2.7b)

Is shear reinforcing necessary ?

1. Figure 2.7c shows the factored shear force diagram (V_u diagram)

2. V_u at a distance d from the support is equal to:

$$87.75 \times \frac{(15 - 2.375)}{15} = 73.86 \text{ k.}$$

3. ΦV_c is equal to:

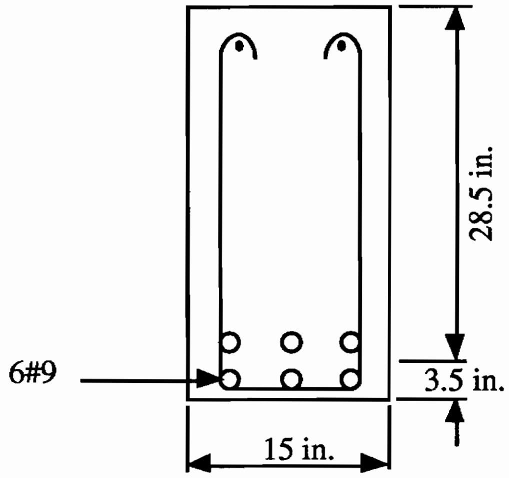
$$0.85 \times 2 \times \sqrt{3500} \times 15 \times 28.5 \ / \ 1000 = 43.0 \text{ k}$$

4. $V_u = 73.86 \text{ k} > 1/2\ \Phi V_c$, therefore stirrups are required

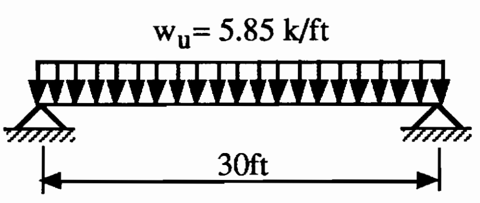
Design of stirrups

1. Theoretical spacing of #3 U-shaped stirrups ($A_v = 2 \times 0.11 = 0.22$), Grade 60, is equal to:

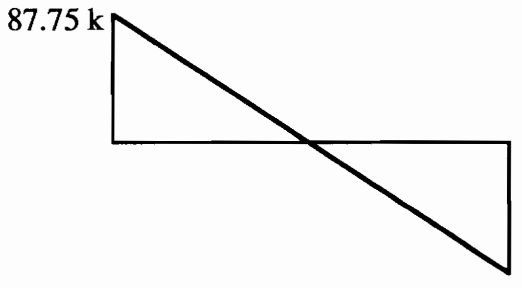
$$s = \frac{\Phi\ A_v\ f_y\ d}{V_u - \Phi V_c}$$



(a)



(b)



(c)

Figure 2.7

Design example 1

$$= \frac{0.85 \times 0.22 \times 60000 \times 28.5}{(73.86 - 43.0) \times 1000}$$

$= 10.4$ in.

2. Maximum spacing to provide minimum area of web reinforcement:

$$s = \frac{A_v f_y}{50 b_w}$$

$$= \frac{0.22 \times 60000}{50 \times 15}$$

$= 17.6$ in.

3. $V_s = 30.86\ k < 4 \times \sqrt{3500} \times 15 \times 28.5\ /1000 = 101.2\ k$

   Therefore maximum spacing is d/2 = 14.25 in., say 14in.

4. Find point where spacing is equal to 14 in.

   At that point, the shear force is: $V_u = \Phi V_c + \Phi V_s$

$$= 43 + \frac{0.85 \times 0.22 \times 60 \times 28.5}{14}$$

$= 65.84\ k$

   This shear force occurs at: $x_1 = 15\ \frac{(87.75 - 65.84)}{87.75}$

$= 3.8$ ft

$= 46$ in. (from support)

5. Find point where stirrups are no longer needed:

   Point where $V_u = 1/2\ \Phi Vc$ is: $x_2 = 15\ \frac{(87.75 - 21.5)}{87.75}$

$= 11.3$ ft

$= 136$ in. (from support)

6. Summary of design:

Since the theoretical spacing is equal to 10.5 in., we need to place the first stirrup at 5 in. from the support.

Use #3, Grade 60, stirrups as follows:

1 @ 5 in. (5 in)

4 @ 10.5 in. (5 + 42 = 47 in.)

7 @ 14 in. (47 + 98 = 145 in.)

Total number of stirrups in the whole beam is: 2 x 12 = 24.

Figure 2.8 shows these stirrups in the left end of the beam.

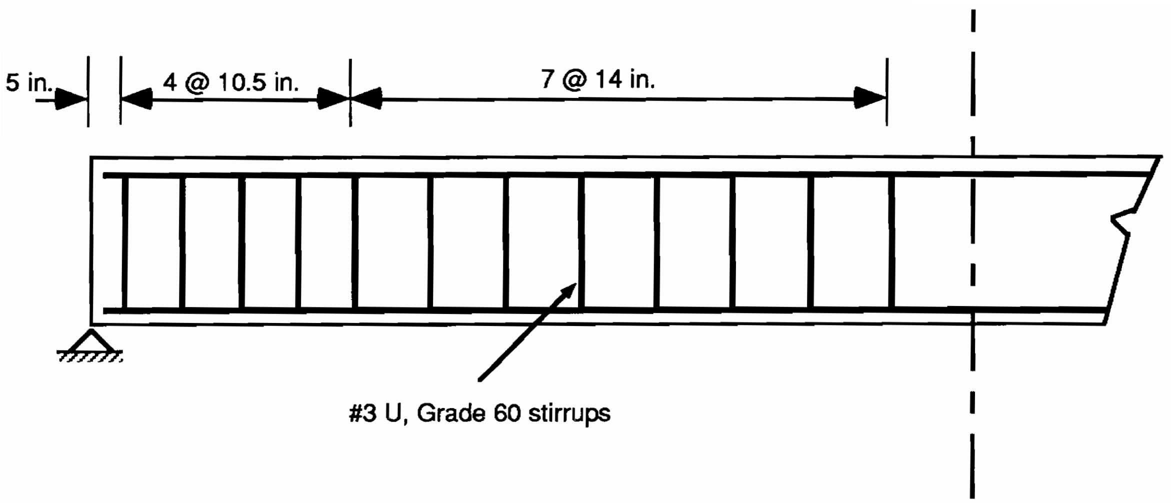


Figure 2.8

Stirrups in beam of example 1

2.7.2 Design Example 2

A simply supported T beam (Figure 2.9a) carries a dead load of 1.00 k/ft (including its self weight) and a live load of 2 k/ft. Design this beam using the ACI Code procedure.

f'_c = 3000 psi, f_y = 40000 psi, L = 20 ft, b_w = 12 in., d = 17.5 in.

$w_u = 1.4\, w_D + 1.7\, w_L = 4.8$ k/ft (Figure 2.9b)

Is shear reinforcing necessary ?

1. Figure 2.9c shows the factored shear force diagram (V_u diagram)
2. Vu at a distance d from the support is equal to:

$$48 \times \frac{(10 - 1.46)}{10} = 41 \text{ k.}$$

3. ΦV_c is equal to:

$$0.85 \times 2 \times \sqrt{3000} \times 12 \times 17.5 \,/\, 1000 = 19.6 \text{ k.}$$

4. $V_u = 41 \text{ k} > 1/2\, \Phi V_c$, therefore stirrups are required.

Design of stirrups

1. Theoretical spacing of #3 U-shaped stirrups (A_v = 0.22), Grade 40, is equal to:

$$s = \frac{\Phi\, A_v\, f_y\, d}{V_u - \Phi V_c}$$

$$= \frac{0.85 \times 0.22 \times 40000 \times 17.5}{(41 - 19.6) \times 1000} = 6.11 \text{ in.}$$

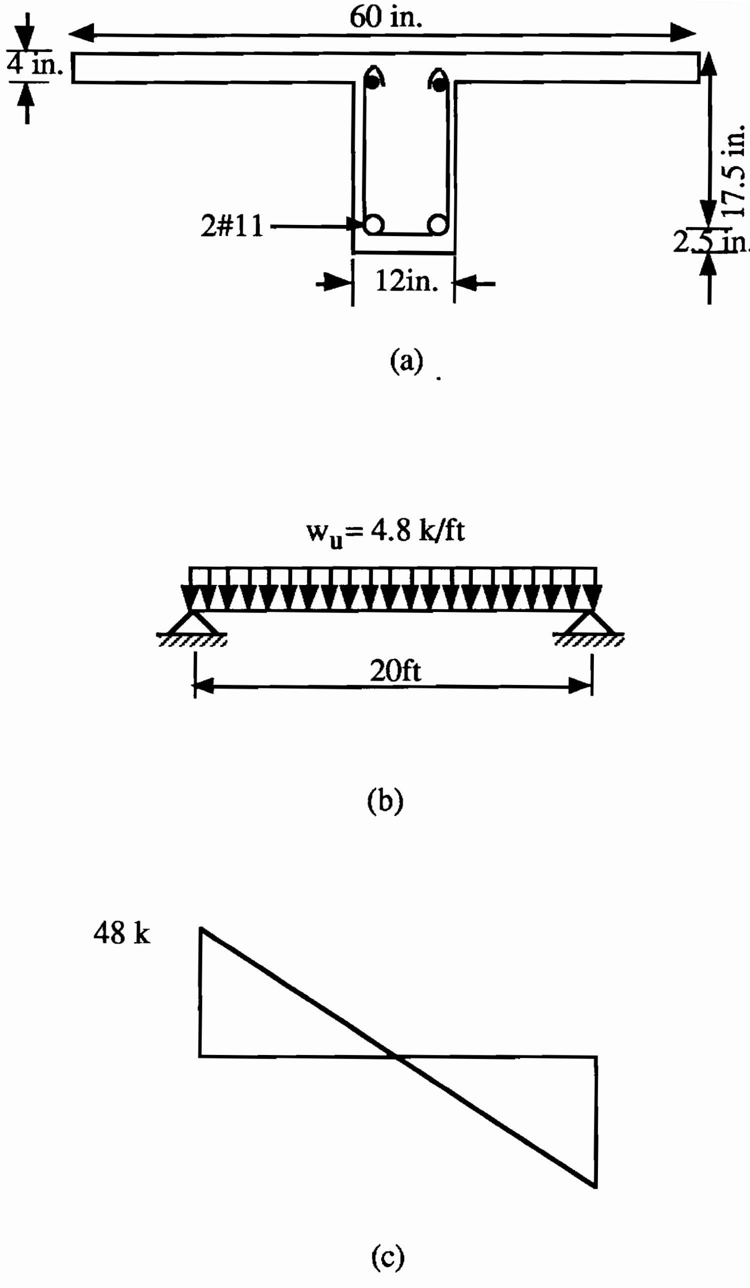


Figure 2.9

Design example 2

2. Maximum spacing to provide minimum area of web reinforcement:

$$s = \frac{A_v f_y}{50 b_w}$$

$$= \frac{0.22 \times 40000}{50 \times 12}$$

$$= 14.7 \text{ in.}$$

3. $V_s = 21.4 \text{ k} < 4 \times \sqrt{3000} \times 12 \times 17.5 / 1000 = 46 \text{ k}.$

   Therefore maximum spacing is d/2 = 8.75 in., say 8.5 in.

4. Find point where spacing is equal to 8.5in.

   At that point, the shear force is: $V_u = \Phi Vc + \Phi\ V_s$

$$= 19.6 + \frac{0.85 \times 0.22 \times 40 \times 17.5}{8.5}$$

$$= 35 \text{ k}$$

   This shear force occurs at: $x_1 = 10 \frac{(48 - 35)}{48}$

$$= 2.7 \text{ ft}$$

$$= 32.5 \text{ in. (from support)}$$

5. Find point where stirrups are no longer needed:

   Point where $V_u = 1/2\ \Phi V_c$ is: $x_2 = 10 \frac{(48 - 9.8)}{48}$

$$= 8 \text{ ft}$$

$$= 96 \text{ in. (from support)}$$

6. Summary of design:

Since the theoretical spacing is equal to 6 in., we need to place the first stirrup at 3in. from the support.

Use #3, Grade 40, stirrup as follows

1 @ 3 in. (3in.)

5 @ 6 in. (3 + 30 = 33 in.)

8 @ 8.5 in. (33 + 68 = 101 in.)

Total number of stirrups in the whole beam is = 2 x 14 = 28

Figure 2.10 shows these stirrups in the left end of the beam.

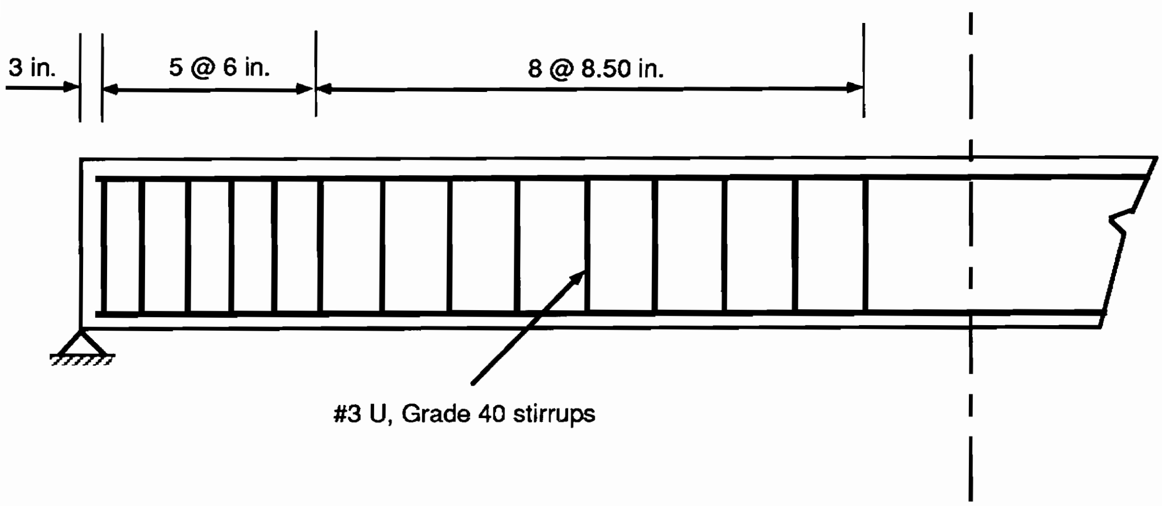


Figure 2.10

Stirrups in beam of example 2

2.7.3 Design Example 3

A simply supported T-beam (Figure 2.11a) carries a dead load of 0.7 k/ft (factored self weight), Two concentrated factored live loads of 40 k each, and two concentrated factored moments of 240 k-ft at the left support and 180 k-ft at the right support (Figure 2.11b). Design this beam for shear using the ACI Code procedure.

$f'_c = 3000$ psi, $f_y = 40000$ psi, $b_w = 12$ in., $d = 17.5$ in.

<u>Is shear reinforcing necessary ?</u>

1. Figure 2.11c shows the factored shear force diagram (V_u diagram)
2. V_u at a distance d from the support is equal to:

$$45.5 + \left(\frac{10 - 1.46}{10}\right) \times (52.5 - 45.5) = 51.5 \text{ k.}$$

3. ΦV_c is equal to:

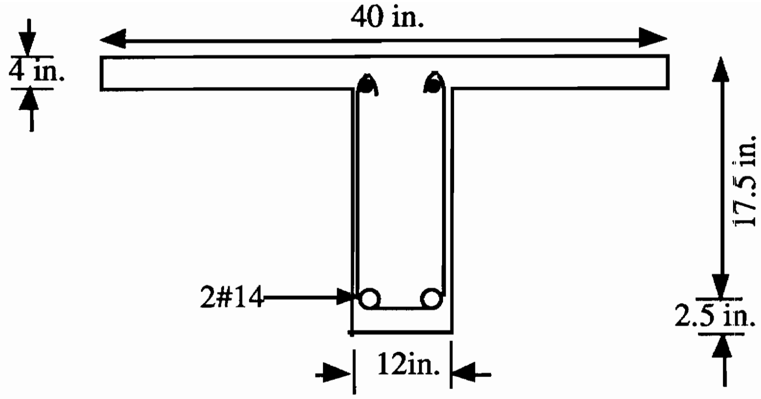
$$0.85 \times 2 \times \sqrt{3000} \times 12 \times 17.5 \,/\, 1000 = 19.6 \text{ k.}$$

4. $V_u = 51.5$ k > 1/2 ΦV_c, therefore stirrups are required

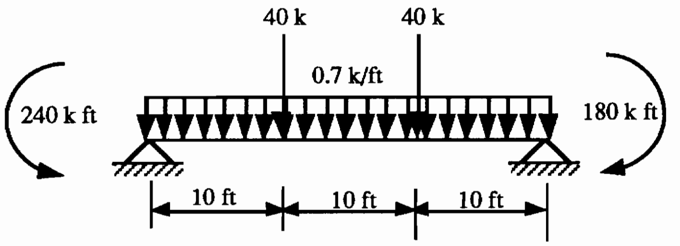
<u>Design of stirrups</u>

1. Theoretical spacing of #3 U-shaped stirrups (A_v = 0.22), Grade 40, is equal to:

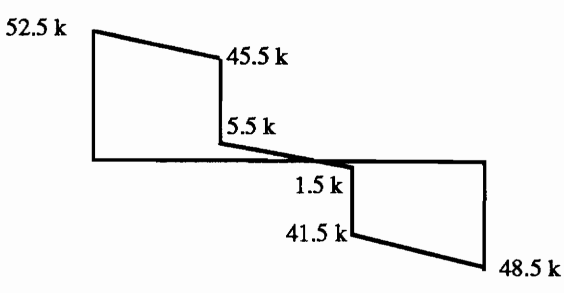
$$s = \frac{\Phi\, A_v\, f_y\, d}{V_u - \Phi V_c}$$



(a)



(b)



(c)

Figure 2.11

Design example 3

$$= \frac{0.85 \times 0.22 \times 40000 \times 17.5}{(51.5 - 19.6) \times 1000}$$

$= 4.1$ in.

2. Maximum spacing to provide minimum area of web reinforcement:

$$s = \frac{A_v f_y}{50 b_w}$$

$$= \frac{0.22 \times 40000}{50 \times 12}$$

$= 14.7$ in.

3. $V_s = 31.9 \text{ k} < 4 \times \sqrt{3000} \times 12 \times 17.5 / 1000 = 46 \text{ k}$.

   Therefore maximum spacing is $d/2 = 8.75$ in.

4. $1/2\ \Phi V_c = 9.8$ k. Since, in the second span of the beam, the shear force is less than 9.8 k, there is no need to place stirrups in this region.

5. In the first span, the shear force varies from 45.5 k to 52.5 k, therefore we will use the 4 in. spacing throughout the whole span. The same spacing will be used for the third span.

6. Summary of design:

   Use #3, Grade 40 stirrups at 4 in. in the first and third spans of the beam.

   Total number of stirrups is 60.

   Figure 2.12 shows these stirrups in the whole beam.

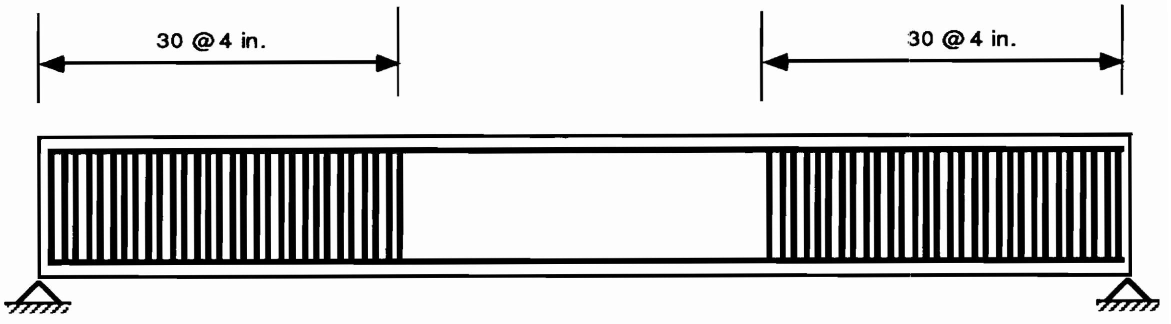


Figure 2.12

Stirrups in beam of example 3

CHAPTER 3

DESIGN OF SLENDER R/C BEAMS FOR SHEAR-TRUSS MODEL THEORY

3.1 Beam Action and Arch Action

The relationship between shear and bar force can be expressed as [Park and Paulay, 1975]:

$$V = \frac{d(T\,jd)}{dx} \tag{3.1}$$

or

$$V = \frac{d(T)}{dx} jd + \frac{d(jd)}{dx} T \tag{3.2}$$

Where T is the tension force in the steel bar, jd is the lever arm, and V is the shear force. In normal elastic beam theory, the lever arm, jd, is assumed to be constant. This means that the quantity d(jd)/dx is equal to zero and therefore Equation (3.2) becomes:

$$V = \frac{d(T)}{dx} jd \tag{3.3}$$

The quantity d(T)/dx is the shear flow across any horizontal plane between the steel bar and the compression zone of the beam. This shear flow is shown in Figure 3.1. If this shear flow exists in any region of the beam, then we say

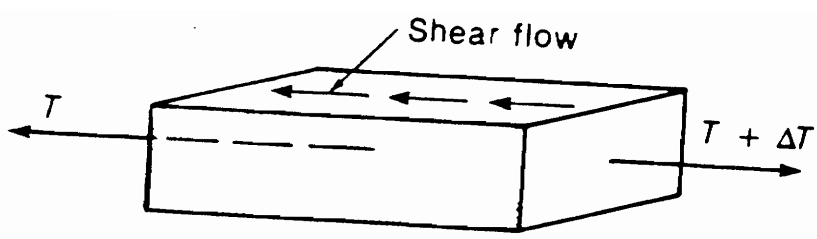


Figure 3.1

Shear flow in a beam (MacGregor, 1992)

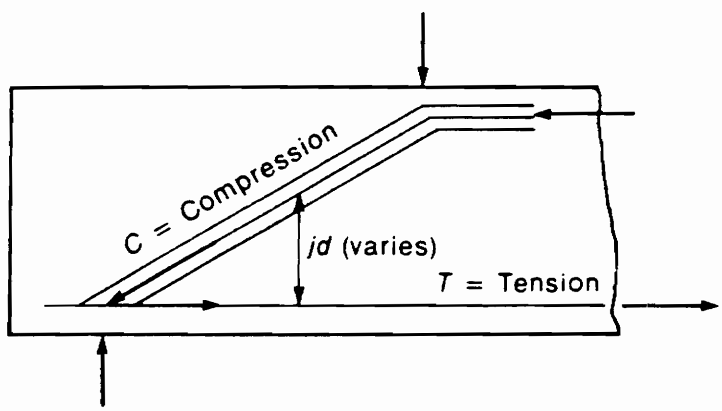


Figure 3.2

Arch action in a beam (Macgregor, 1992)

that region is displaying beam action and it is referred to as a B-region (B means beam action).

If the shear flow cannot be transmitted, then the quantity d(T)/dx is equal to zero and therefore the shear force has to be equal to:

$$V = \frac{d(jd)}{dx} T \tag{3.4}$$

This means that the lever arm, jd, is no longer constant. The shear flow can be prevented due to either an inclined crack extending from the load to the reaction or to the steel being unbonded (bond stresses between steel and concrete disappear). The regions in which the shear flow is prevented are called D-regions (D means discontinuity or disturbance). In these regions, the shear is transferred by arch action as shown in Figure 3.2. Figure 3.3 shows examples of D-regions and B-regions. The D-regions extend one member depth each way from concentrated loads, reactions, or sudden change in section (holes for example) and direction [Schlaich, Schaefer, and Jennewein, 1987]. The B-regions are those regions between the D-regions.

3.2 Truss Model Theory

Figure 3.4a shows a beam with inclined cracks. The forces acting on this beam are tensile forces in the bottom flange and in the stirrups and compressive forces in the top flange and the concrete diagonals (between the cracks). These forces are replaced by an analogous truss as shown in Figure 3.4b. This simple truss has been formed by lumping all of the stirrups cut by

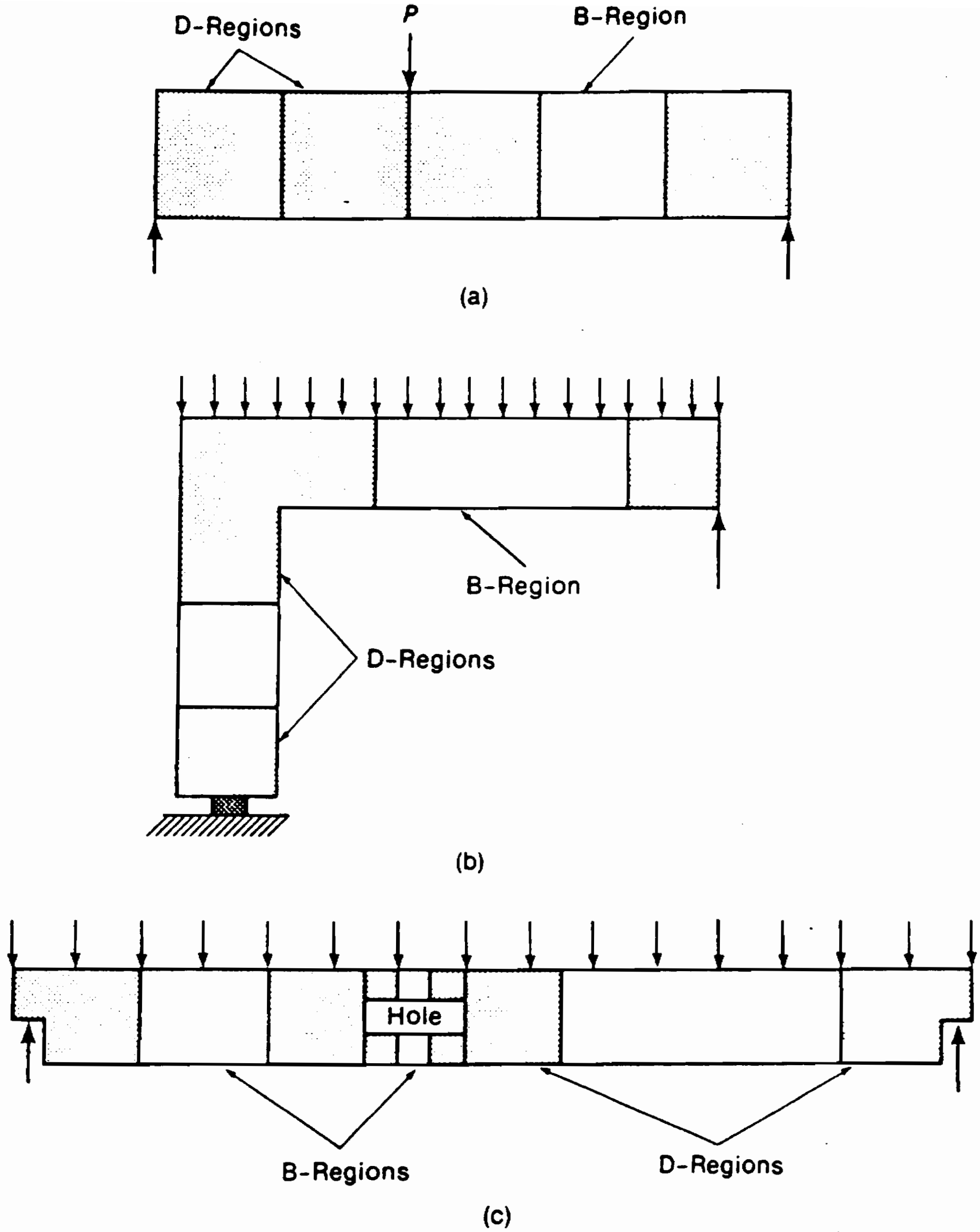
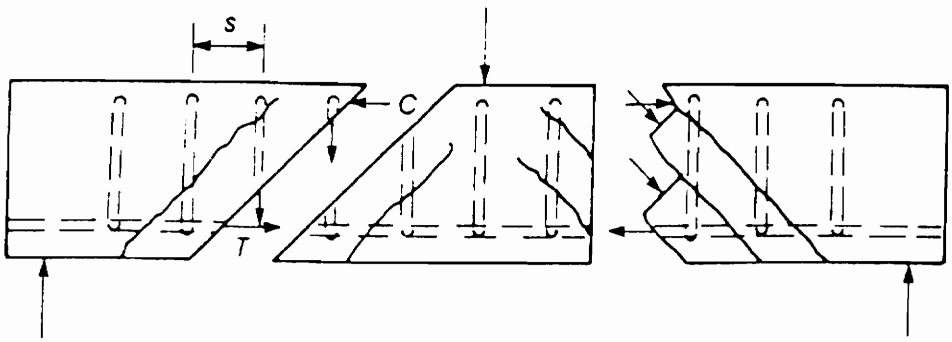
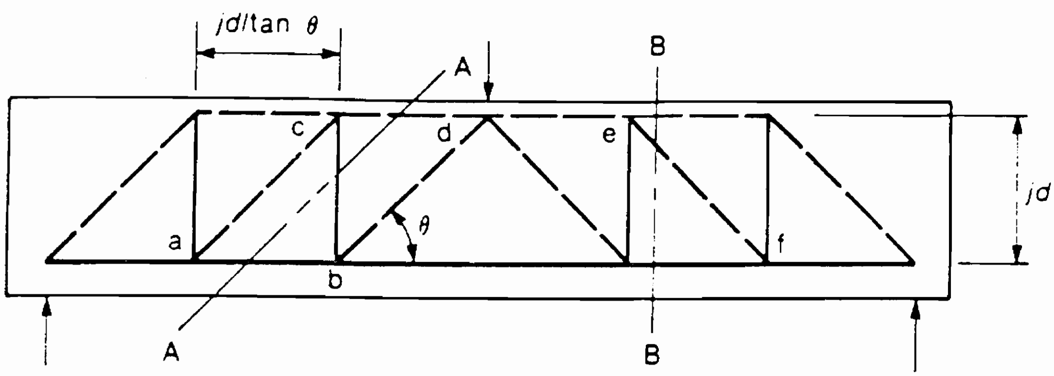


Figure 3.3

B-regions and D-regions (MacGregor, 1992)



(a) Internal forces in a cracked beam.



(b) Pin-jointed truss.

Figure 3.4

Truss analogy (MacGregor, 1992)

section A-A into one vertical bar b-c and all the diagonal concrete members cut by section B-B into one diagonal member e-f. In constructing this truss model, it is assumed that the cracks are at angle θ to the horizontal, all the shear is resisted by stirrups, and that shear and flexural failure occur simultaneously. The following analysis problem will illustrate these assumptions and show what the Truss Model Theory is.

Figure 3.5 shows a truss model for a beam subjected to three concentrated loads. Since it is assumed that all the stirrups have yielded, the vertical force carried by each stirrup is $A_v f_y$ = (0.22) x (60) = 8.8 k. The moment at midspan is found to be 316.8 k ft. Assuming that the lever arm, jd, is equal to 20 in., the compression and tension forces C and T at midspan are 316.8/20 = 190.08 k. The vertical applied load of 52.8 k (at midspan) must be transmitted by diagonal compression struts to enough stirrups to equilibrate it. Since each stirrup can resist a vertical force of 8.8 k, six stirrups are required to transmit 52.8 K. Since the beam is symmetric, the vertical applied load of 52.8 K will be transmitted by three diagonals from each side ($U_{12}L_{11}$, $U_{12}L_{10}$, $U_{12}L_9$, $U_{12}L_{13}$, $U_{12}L_{14}$, and $U_{12}L_{15}$) to joints L_9, L_{10}, L_{11}, L_{13}, L_{14}, and L_{15}. Figures 3.6 and 3.7 show the equilibrium of the bottom and top chord joints of the beam, respectively. For joint L_{11} (for example), we have a horizontal tension force (tension in the longitudinal reinforcement) of 190.08 k, a vertical tension force (tension in the stirrup) of 8.8 k. Therefore, the vertical component of the diagonal must be 8.8 k. From the slope of $L_{11}U_{12}$, we find it must have a horizontal component of 5.28 k. Summing the horizontal forces, we find that the tension force between L_{11} and L_{10} is 190.08 - 5.28 = 184.8K. This process is repeated for all the remaining joints.

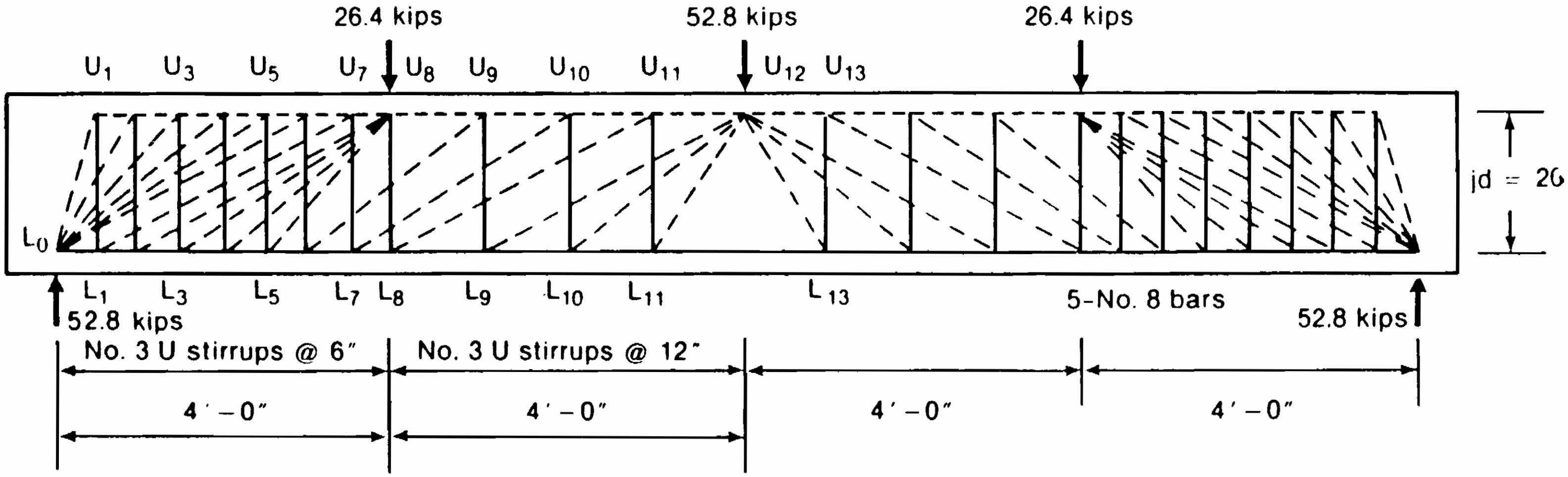


Figure 3.5

Truss model of the analysis problem (MacGregor, 1992)

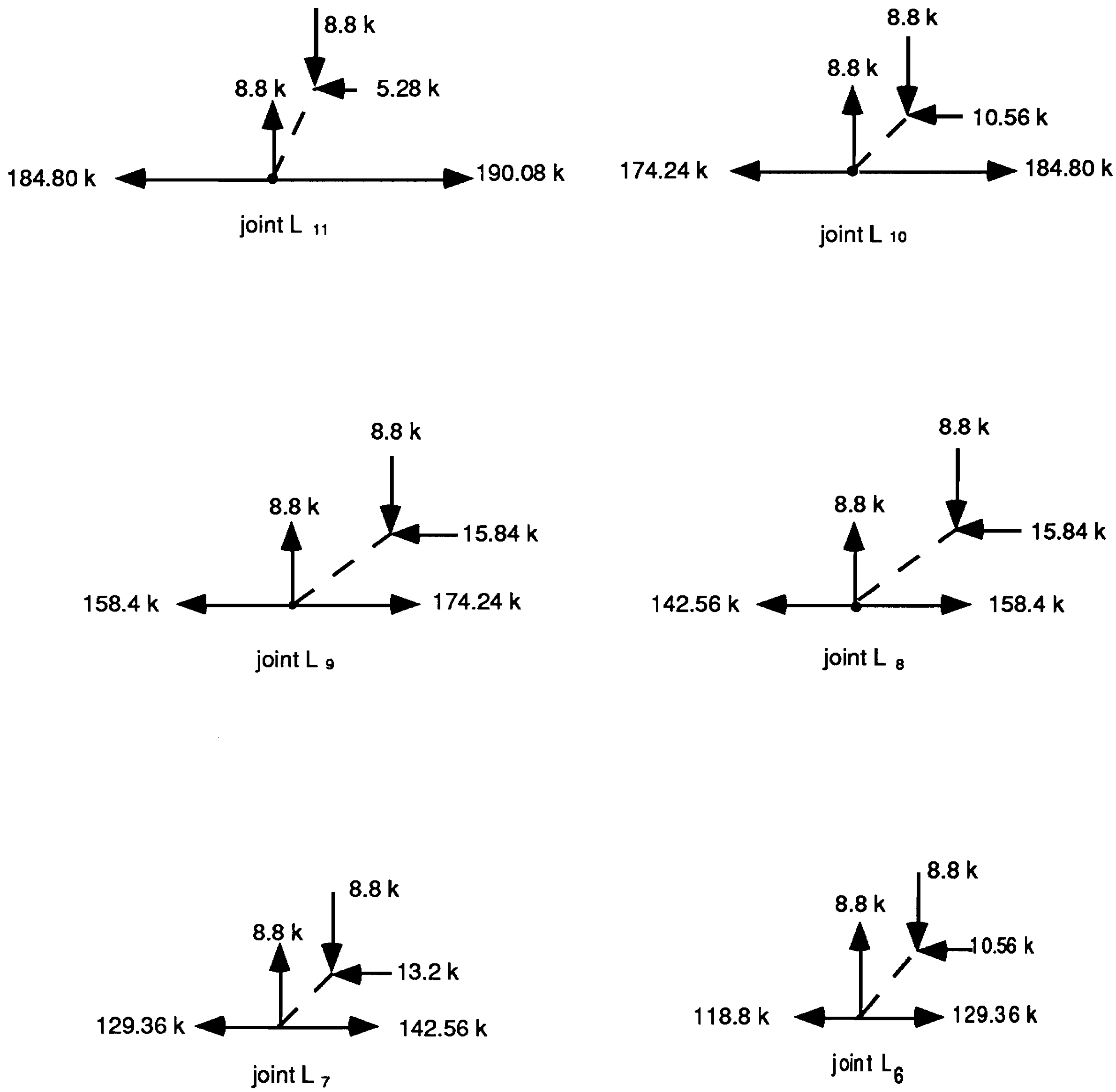


Figure 3.6

Equilibrium of the bottom chord joints

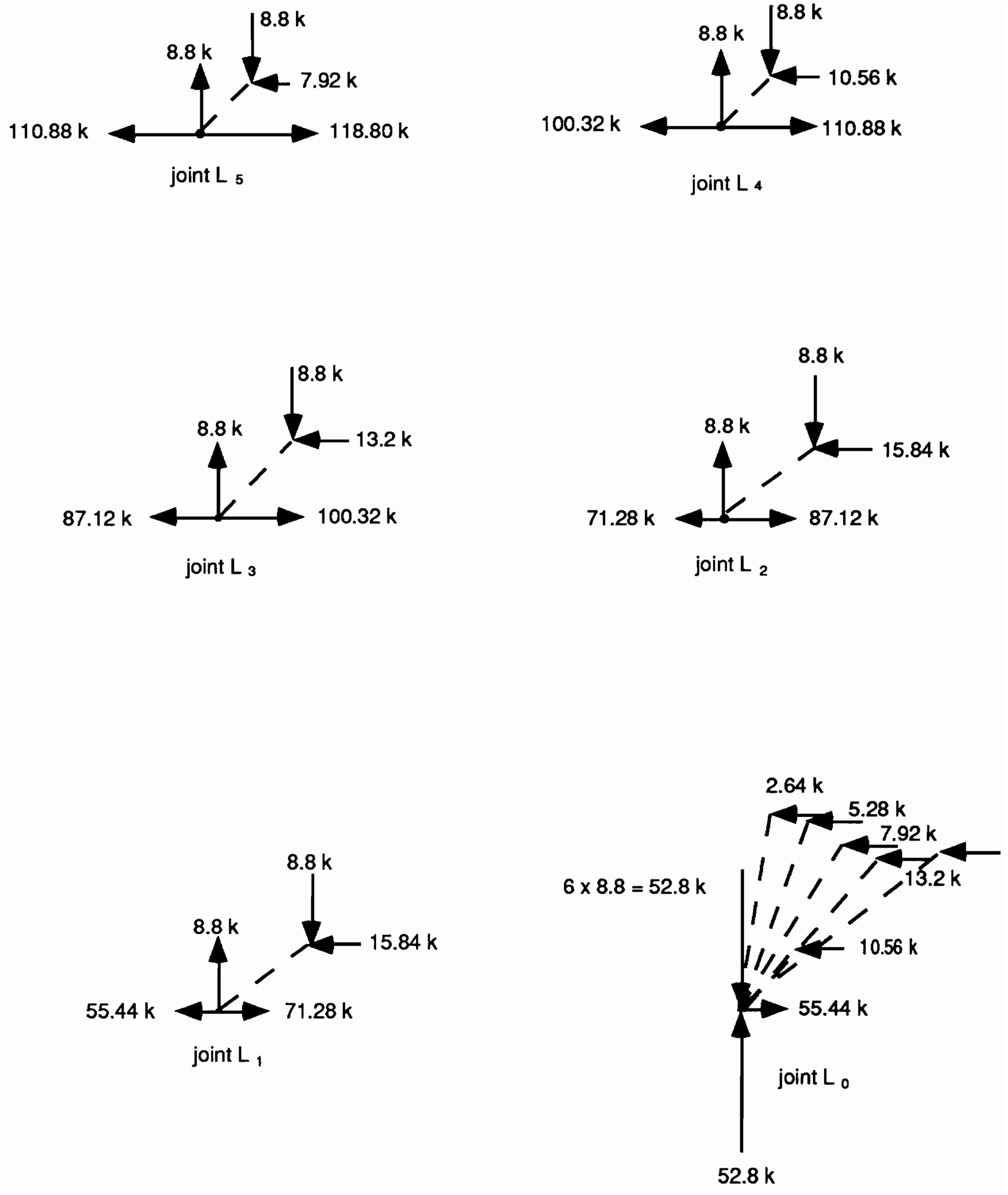


Figure 3.6 (continued)

Equilibrium of the bottom chord joints

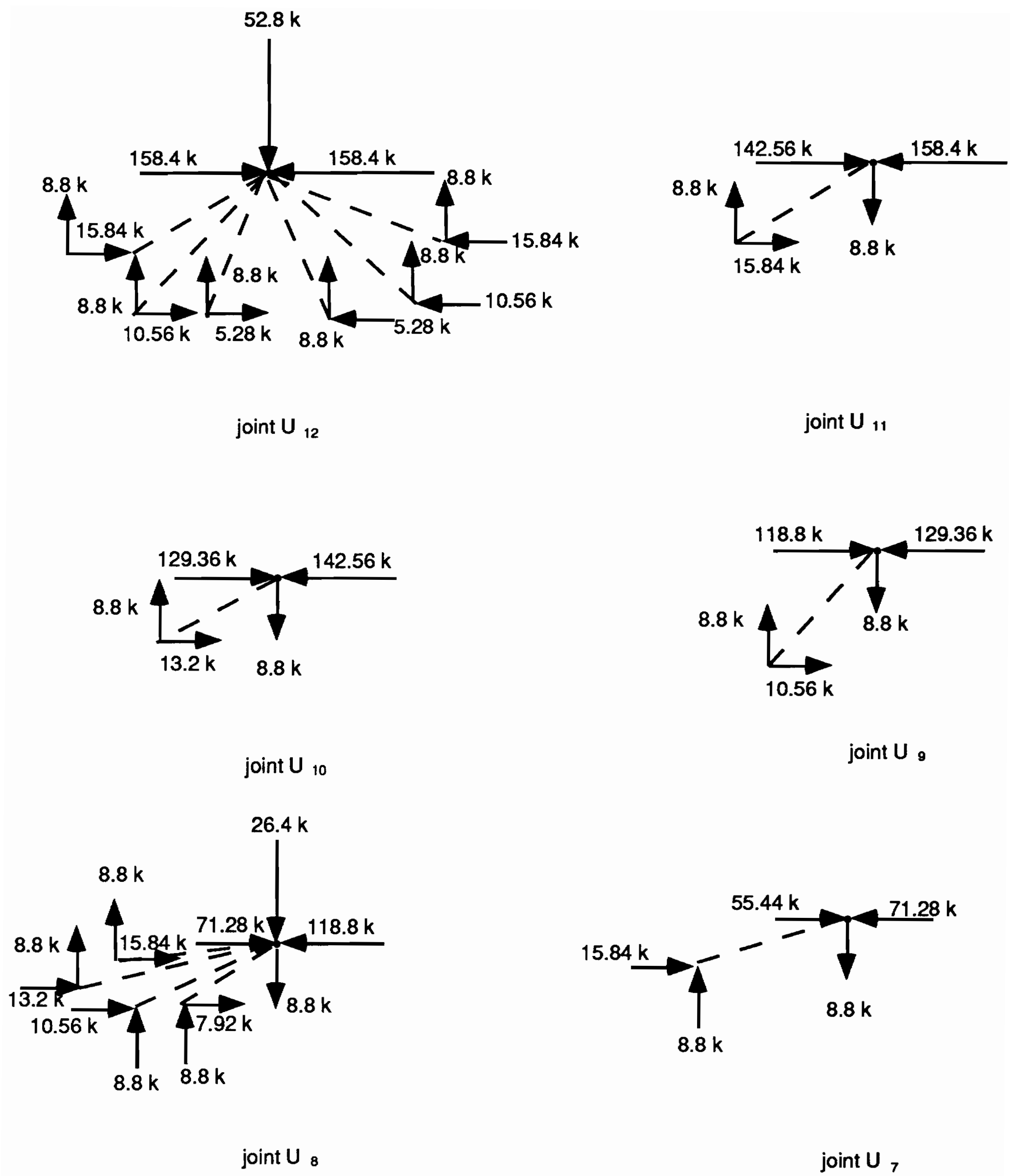


Figure 3.7

Equilibrium of the top chord joints

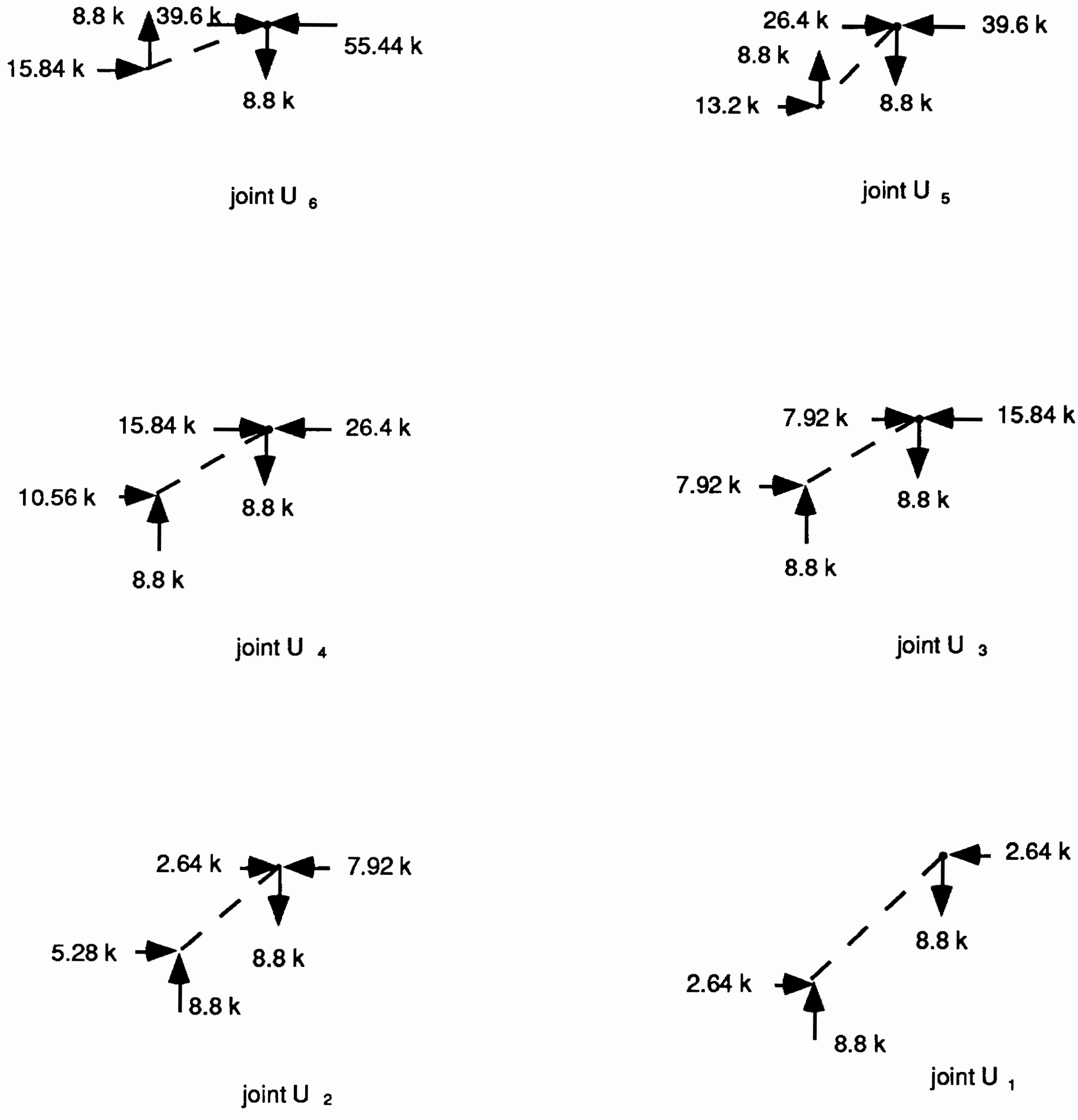
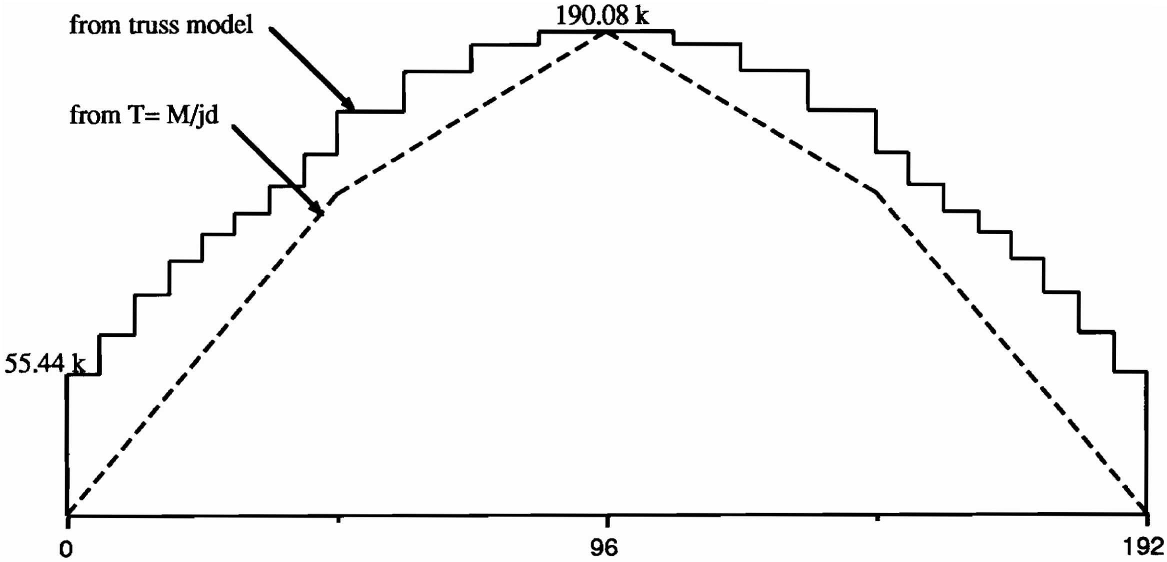


Figure 3.7 (continued)

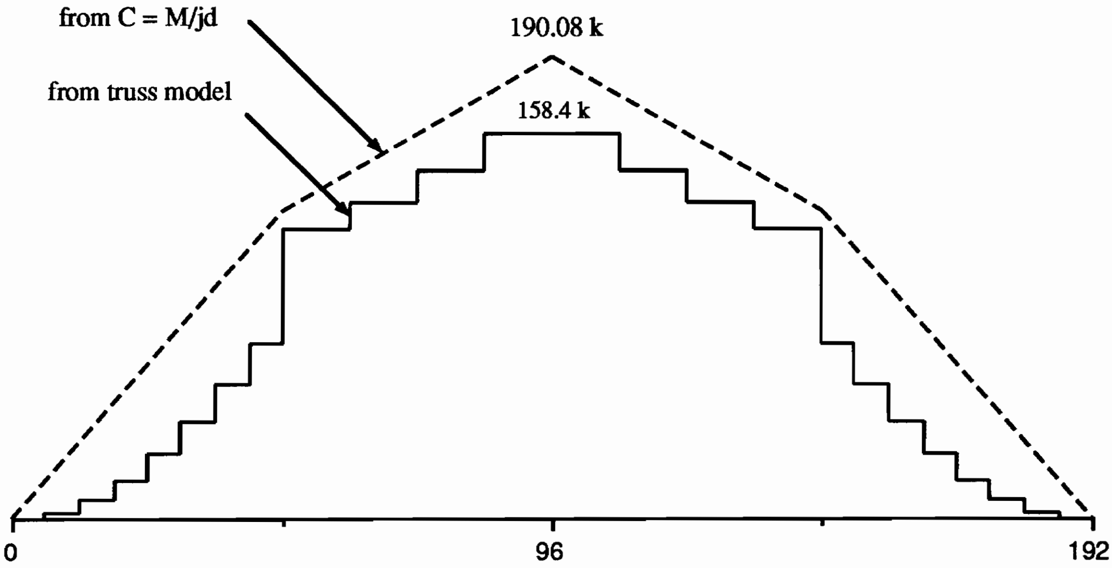
Equilibrium of the top chord joints

The compressions diagonals, L_9U_{12}, $L_{10}U_{12}$, $L_{11}U_{12}$, $L_{13}U_{12}$, $L_{14}U_{12}$, and $L_{15}U_{12}$ form what we call a compression fan. We have other compression fans originating at the two other concentrated loads and at the two reactions. Between the compression fans there is what we call a compression field (diagonal L_1U_7, for example). The angle θ of the compression field is determined by the number of stirrups required to resist the vertical loads in the compression fans. The compression fan is a D-region while the compression field is a B-region.

Figure 3.8 shows the distribution of forces in the bottom (tension) and top (compression) flanges of the beam. If we compare this distribution with the one due to flexure (obtained by setting C or T equal to M/jd and shown in Figure 3.8 by the dashed lines), we note that in the compression side, the force is less than C = M/jd while in the tension side the force is larger than T = M/jd. This means that the presence of inclined cracks has increased the tension reinforcement at all points in the shear span except in the region of maximum moment (where the tension force is the same as computed from T = M/jd = 190.08 k). At the support, the tension in the reinforcement is found to be 54.44 k. Therefore, we need to anchor the longitudinal bars to resist this force even though the moment at this point is zero. The ACI Code does not explicitly treat this increase in the tension force due to shear. However, Section 12.10.3 states: " Reinforcement shall extend beyond the point where it is no longer required to resist flexure for a distance equal to the effective depth of member or $12d_b$, whichever is greater, except for simple spans and at free end of cantilevers."



(a) Tension in longitudinal reinforcement



(b) Compression in bottom flange

Figure 3.8

Distribution of forces

3.3 Internal Forces in the Truss Model

Figure 3.9a shows the free body diagram of the part of a beam from the compression field cut by a section parallel to the concrete diagonals. The horizontal projection of this section is equal to $jd/\tan\theta$ and therefore the number of stirrups this section cuts is $jd/(s \tan\theta)$. Since it is assumed that the stirrups will yield at failure, the force in one stirrup is $A_v f_y$. It is also assumed that the shear force is resisted only by stirrups. This means that we must have:

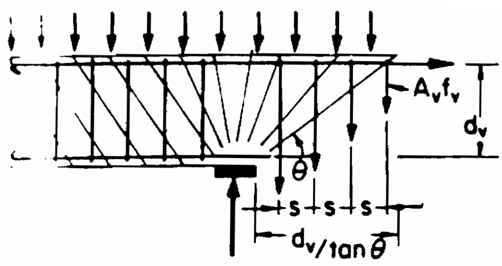
$$V = A_v \, f_y \left(\frac{jd}{s \tan\theta}\right) \tag{3.5}$$

Therefore, the force in each stirrup is:

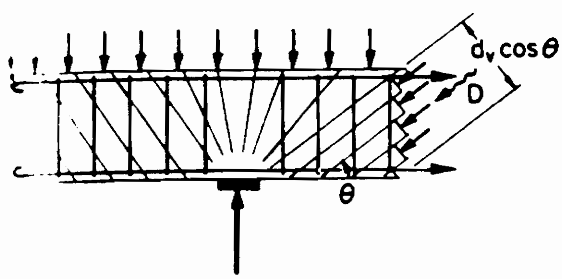
$$A_v f_y = \frac{V \, s \tan\theta}{jd} \tag{3.6}$$

Figure 3.9b shows a free body diagram of the part of a beam cut by a vertical section. Since the force in the stirrups is the shear force (V), the vertical component of the diagonal force has to be equal to this shear force to equilibrate it. Therefore, the force in the concrete diagonal is:

$$D = \frac{V}{\sin\theta} \tag{3.7}$$



(a)



(b)

Figure 3.9

Internal forces in the truss model (Collins and Mitchell, 1980)

The area the concrete diagonal is (jd cosθ) b_w. Hence, the compression stress in the diagonal is:

$$f_d = \frac{V}{b_w \, jd \cos\theta \sin\theta} \tag{3.8}$$

Using trigonometric identities, Equation (3.8) is equivalent to:

$$f_d = \frac{V}{b_w \, jd} \left(\tan\theta + \frac{1}{\tan\theta}\right) \tag{3.9}$$

If this stress is large (when the web is thin and b_w is small), it may lead to a crushing of the web as shown in Figure 3.10.

Since the force in the diagonals is V/sinθ, it must have a horizontal component of V/tanθ. Figure 3.11 shows the resolution of these internal forces. This means that the shear force, V, has been replaced by a diagonal compression force, D, and an axial tension force T_v equal to:

$$T_v = \frac{V}{\tan\theta} \tag{3.10}$$

Mitchell and Collins (1980) assumed that the shear stress is constant over the height of the beam and therefore the resultant of D and T_v act at mid-height. This assumption will divide T_v into two components: one acting at the top chord and of magnitude V/2tanθ, the other one is acting at the bottom flange and having the same magnitude (V/2tanθ). This explains the decrease in the compression force at the top flange and the increase in the

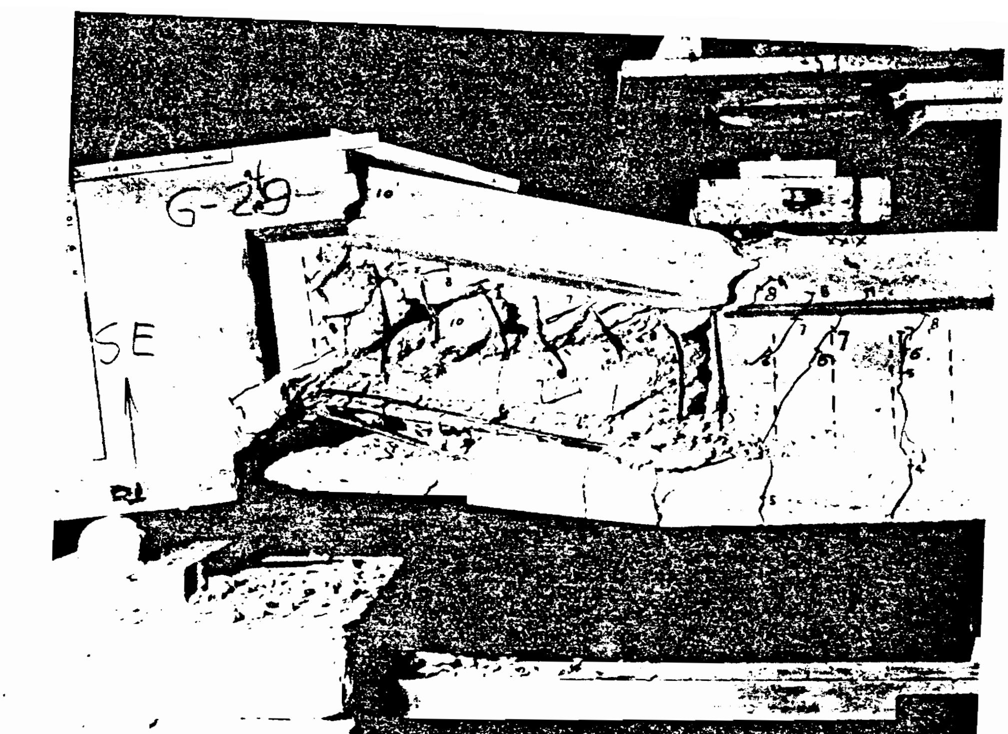


Figure 3.10

Web crushing failure (MacGregor, 1992)

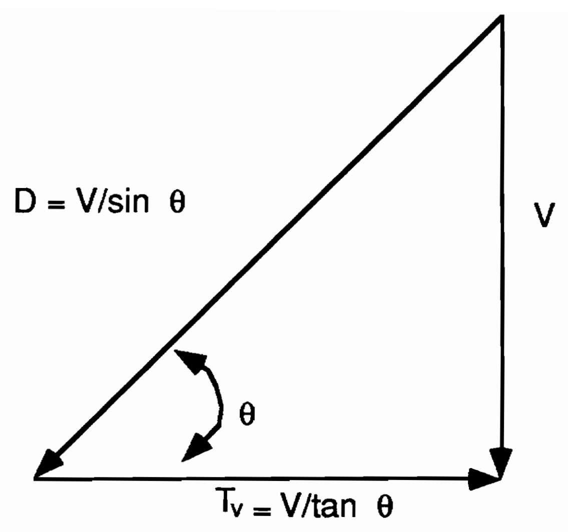


Figure 3.11

Resolution of internal forces

tension force in the longitudinal steel in the analysis problem. Another thing to note is that when the shear force is zero, there would be no increase in the tension force (because $T_v = V/\tan\theta = 0$, if $V = 0$). This would happen at the point of maximum moment which agrees with the analysis problem, too.

3.4 Design Values of θ

In the Swiss Code, the allowable value of θ is taken to be in the range from 26° to 64° (which is equivalent to $0.5 < \tan\theta < 2$). In the European Concrete Committee Model Code, the range of θ is between 31° and 59° (which means that $3/5 < \tan\theta < 5/3$). Mitchell and Collins (1980) used compatibility analysis to derive limits for the angle θ. These limits can be expressed as:

$$10 + \frac{35\,\tau_n / f_c'}{(0.42 - 50\varepsilon_l)} < \theta\ (\text{deg}) < 80 - \frac{35\,\tau_n / f_c'}{(0.42 - 65\varepsilon_t)} \tag{3.11}$$

Where τ_n is the nominal shear stress which is equal to $V_n/b_w jd$, ε_l is the longitudinal steel strain, and ε_t is the transversal (web reinforcement) steel strain. Therefore, in designing a beam for shear, the nominal shear stress would first be determined. Based on the yield strength of the steel used for reinforcement (either longitudinal or transversal), we can determine ε_l and ε_t. Equation (3.11) could then be used to find the range of the possible values of θ. If the lower limit is found to be higher than the upper one, then the beam section is inadequate. Usually, a value of θ closer to the lower limit is chosen. This would decrease the number of stirrups, but would increase the compression stresses in the beam and would increase Tv (which would lead to a bigger shift in the moment diagram).

3.5 Design procedure

After choosing an appropriate value of θ, the spacing of the stirrups can be determined from the following equations:

$$s = \frac{\Phi A_v f_y jd}{V_u \tan\theta} \quad ; \text{For vertical stirrups} \qquad (3.12)$$

$$s = \frac{\Phi A_v f_y jd}{V_u} \left(\frac{\sin\alpha}{\tan\theta} + \cos\alpha\right) \quad ; \text{For inclined stirrups} \qquad (3.13)$$

The stirrup spacing, however, cannot be larger than $jd/3\tan\theta$. This is because large stirrup spacing would make the diagonal stresses concentrate at the stirrup locations which may result in premature diagonal crushing. This is illustrated in Figure 3.12

Figure 3.13 shows the design procedure used for uniformly loaded beams (where shear is not constant over the length of the beam). We can note that the shears for which the stirrups are to be designed are found by shifting the shear force diagram a distance of $jd/2\tan\theta$ towards the support (this is shown by a dashed line in Figure 3.13). Over the length $jd/\tan\theta$, equilibrium will be satisfied if the stirrups are designed for the average shear force over this length (which is the lowest value of V_u within this length). The following design examples will illustrate this design procedure.

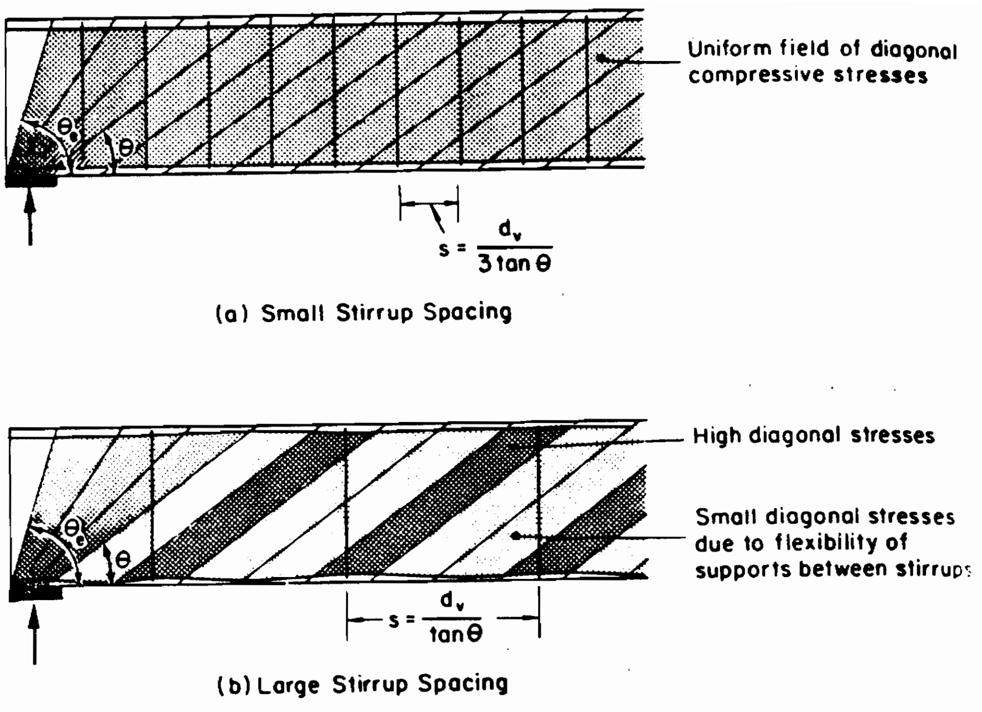


Figure 3.12

Maximum stirrup spacing (Mitchell and Collins, 1980)

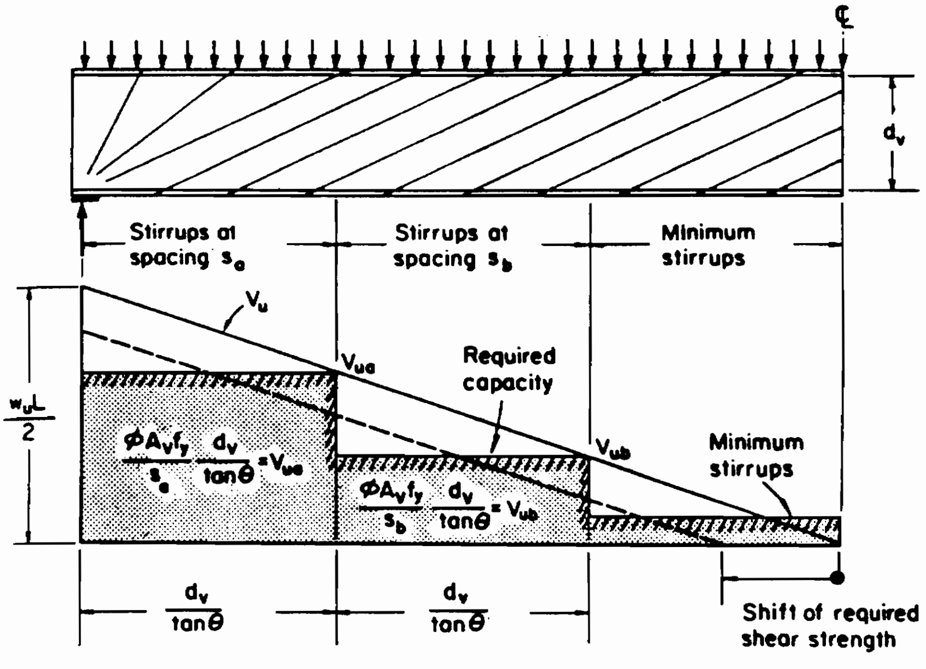


Figure 3.13

Design procedure for uniformly loaded beams (Mitchell and Collins, 1980)

3.6 Design Examples

3.6.1 Design Example 1

Example 1 of Chapter 2 will be reworked using the Truss Model Procedure (refer to Figure 2.7).

Determine jd:

$A_s f_y = 6 \times 60 = 360k = 0.85 \times 3.5 \times 15 \times a$, therefore a = 8.07 in.

Thus jd = d - a/2 = 24.46 in.

Take jd = 24.5 in.

1. $\tau_n = \dfrac{V_n}{b_w\, jd}$

$= \dfrac{V_u}{\Phi\, b_w\, jd}$

$= \dfrac{87.75}{0.85 \times 15 \times 24.5}$

$= 0.281$ ksi

$\tau_n/f_c^{'} = 0.281/3.5 = 0.08$

2. $\varepsilon_l = \varepsilon_t = \dfrac{60}{29000} = 2.07\ 10^{-3}$

3.

$$10 + \frac{35\,\tau_n / f_c^{'}}{(0.42 - 50\varepsilon_l)} < \theta\ (\text{deg}) < 80 - \frac{35\,\tau_n / f_c^{'}}{(0.42 - 65\varepsilon_t)}$$

Therefore;

$$18.9^\circ < \theta < 68.6^\circ$$

Hence, the section is adequate

Choose $\theta = 20^\circ$

4. $jd/\tan\theta = 24.5/\tan 20^\circ = 67.3$ in.
5. $s_{max} = jd/3\tan\theta = 24.5/\ 3\tan 20^\circ = 22.5$ in.
6. $$s_1 = \frac{\Phi\ A_v\ f_y\ jd}{V_{u1}\ \tan\theta}$$

$V_{u1} = 87.75 - (5.85 \times 67.3)/12 = 54.94$ kips.

Therefore, for #3 U-shaped stirrups ($A_v = 0.22$), Grade 60, the spacing s_1 is equal to:

$$s_1 = \frac{0.85 \times 0.22 \times 60 \times 24.5}{54.94 \times \tan 20^\circ}$$

$= 13.7$, say $s_1 = 13.5$ in.

7. $$s_2 = \frac{\Phi\ A_v\ f_y\ jd}{V_{u2}\ \tan\theta}$$

$V_{u2} = 87.75 - (5.85 \times 134.6)/12 = 22.13$ kips.

$$s2 = \frac{0.85 \times 0.22 \times 60 \times 24.5}{22.13 \times \tan 20^\circ} = 34 \text{ in.}$$

Since s_2 is greater than s_{max}, use s_2 = 22.5 in.

8. Summary of design

Use #3 U-shaped stirrups, Grade 60, as follows

1 @ 6 in. (6 in.)

5 @ 13.5 in. (6 + 67.5 = 73.5 in.)

4 @ 22.5 in. (73.5 + 90 = 163.5 in.)

Total number of stirrups is: 2 x 10 = 20

Figure 3.14 shows these stirrups in the left end of the beam.

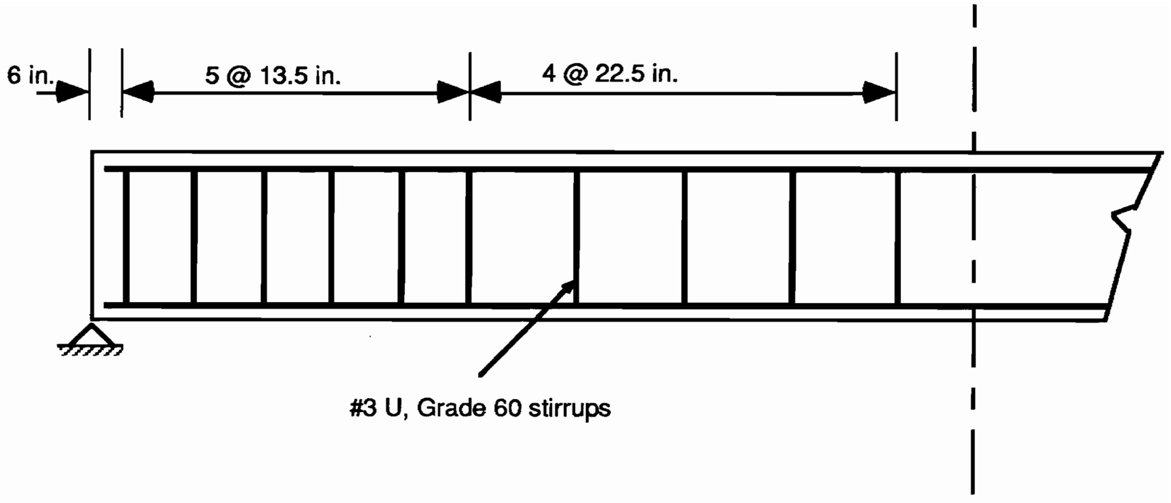


Figure 3.14

Stirrups in beam of example 1

3.6.2 Design Example 2

Example 2 of Chapter 2 will be reworked using the Truss Model Procedure (refer to Figure 2.9).

Determine jd:

$A_s f_y = 3.12 \times 60 = 187.2\ k = 0.85 \times 3.0 \times A_c$, therefore $A_c = 73.4\ in^2$.

$a = 73.4/60 = 1.22$ in.

Thus $jd = d - a/2 = 16.88$ in.

Take $jd = 16.9$ in.

1. $$\tau_n = \frac{V_n}{b_w\, jd}$$

$$= \frac{V_u}{\Phi\, b_w\, jd}$$

$$= \frac{48}{0.85 \times 12 \times 16.9}$$

$$= 0.278 \text{ ksi}$$

$$\tau_n/f'_c = 0.278/3.0 = 0.0927$$

2. $$\varepsilon_l = \frac{60}{29000} = 2.07\ 10^{-3}$$

$$\varepsilon_t = \frac{40}{29000} = 1.38\ 10^{-3}$$

3.

$$10 + \frac{35\, \tau_n / f'_c}{(0.42 - 50\varepsilon_l)} < \theta \text{ (deg)} < 80 - \frac{35\, \tau_n / f'_c}{(0.42 - 65\varepsilon_t)}$$

Therefore;

$$20.3° < \theta < 70.2°$$

Hence, the section is adequate

Choose $\theta = 22°$

4. $jd/\tan\theta = 16.9/\tan 22° = 41.8$ in.
5. $s_{max} = jd/3\tan\theta = 16.9/\ 3\tan 22° = 14$ in.

6. $$s_1 = \frac{\Phi\ A_v\ f_y\ jd}{V_{u1}\ \tan\theta}$$

$V_{u1} = 48 - (4.8 \times 41.8)/12 = 31.28$ kips.

Therefore, for #3 U-shaped stirrups ($A_v = 0.22$), Grade 40, the spacing s_1 is equal to:

$$s_1 = \frac{0.85 \times 0.22 \times 40 \times 16.9}{31.28 \times \tan 22°}$$

$$= 10 \text{ in.}$$

7. $$s_2 = \frac{\Phi\ A_v\ f_y\ jd}{V_{u2}\ \tan\theta}$$

$V_{u2} = 48 - (4.8 \times 83.6)/12 = 14.56$ kips.

$$s2 = \frac{0.85 \times 0.22 \times 40 \times 16.9}{14.56 \times \tan 22°} = 21.5 \text{ in.}$$

Since s_2 is greater than s_{max}, use s_2 = 14 in.

8. Summary of design

   Use #3 U-shaped stirrups, Grade 40, as follows

   1 @ 5 in. (5 in.)

   4 @ 10 in. (5 + 40 = 45 in.)

   5 @ 14 in. (45 + 70 = 115 in.)

   Total number of stirrups is: 2 x 10 = 20

   Figure 3.15 shows these stirrups in the left end of the beam.

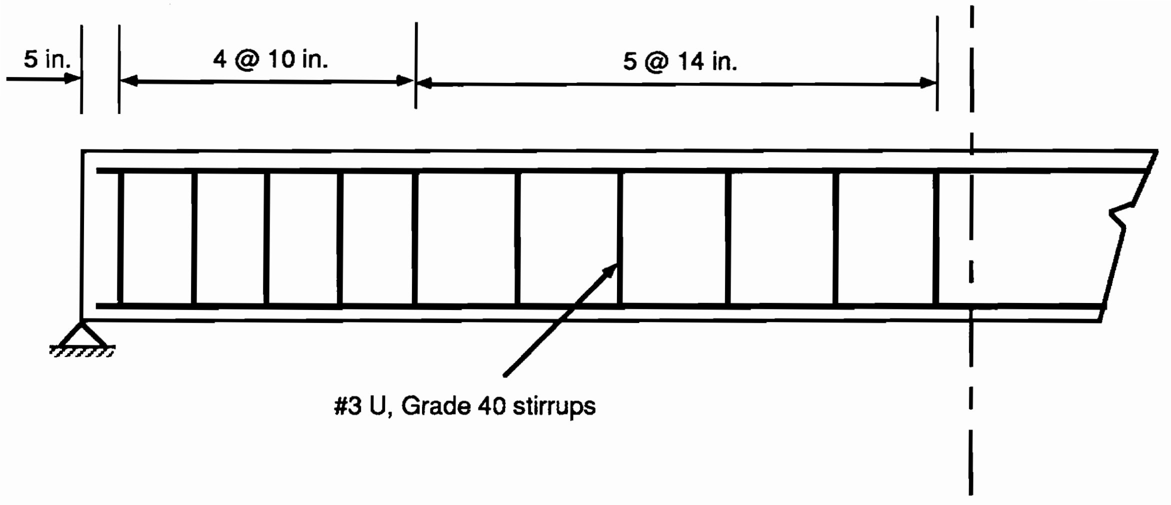


Figure 3.15

Stirrups in beam of example 2

3.6.3 Design Example 3

Example 3 of Chapter 2 will be reworked using the Truss Model Procedure (refer to Figure 2.11).

Determine jd:

$A_s f_y = 4.5 \times 60 = 270$ k $= 0.85 \times 3.0 \times A_c$, therefore $A_c = 105.9$ in^2.

a = 105.9/40 = 2.65 in.

Thus jd = d - a/2 = 16.18 in.

Take jd = 16.2 in.

1. $$\tau_n = \frac{V_n}{b_w\, jd}$$

$$= \frac{V_u}{\Phi\, b_w\, jd}$$

$$= \frac{52.5}{0.85 \times 12 \times 16.2}$$

$$= 0.318 \text{ ksi}$$

$$\tau_n / f'_c = 0.318/3.0 = 0.106$$

2. $$\varepsilon_l = \frac{60}{29000} = 2.07\ 10^{-3}$$

$$\varepsilon_t = \frac{40}{29000} = 1.38\ 10^{-3}$$

3.

$$10 + \frac{35\,\tau_n / f'_c}{(0.42 - 50\varepsilon_l)} < \theta\,(\text{deg}) < 80 - \frac{35\,\tau_n / f'_c}{(0.42 - 65\varepsilon_t)}$$

Therefore;

$$21.8° < \theta < 68.8°$$

Hence, the section is adequate

Choose $\theta = 25°$

4. $jd/\tan\theta = 16.2/\tan 25° = 34.7$ in.
5. $s_{max} = jd/3\tan\theta = 16.2/\ 3\tan 25° = 11.6$ in., say 12 in.

6. $s_1 = \dfrac{\Phi\ A_v\ f_y\ jd}{V_{u1}\ \tan\theta}$

$V_{u1} = 52.5 - (0.7 \times 34.7)/12 = 50.5$ kips.

Therefore, for #3 U-shaped stirrups ($A_v = 0.22$), Grade 40, the spacing s_1 is equal to:

$$s_1 = \frac{0.85 \times 0.22 \times 40 \times 16.2}{50.5 \times \tan 25°}$$

$= 5.1$ in.,say 5 in.

7. $s_2 = \dfrac{\Phi\ A_v\ f_y\ jd}{V_{u2}\ \tan\theta}$

$V_{u2} = 52.5 - (.7 \times 69.4)/12 = 48.5$ kips.

$$s2 = \frac{0.85 \times 0.22 \times 40 \times 16.2}{48.5 \times \tan 25°} = 5.4 \text{ in., say 5 in.}$$

Therefore, use s = 5 in. in the first and third span of the beam and s = 12 in.

in the second span of the beam.

8. Summary of design

   Use #3 U-shaped stirrups, Grade 40, as follows

   24 @ 5 in. (120 in.)

   10 @ 12 in. (120 + 120 = 240 in.)

   23 @ 5 in. (240 + 115 = 355 in.)

   Total number of stirrups is: 57

   Figure 3.16 shows these stirrups in the whole beam.

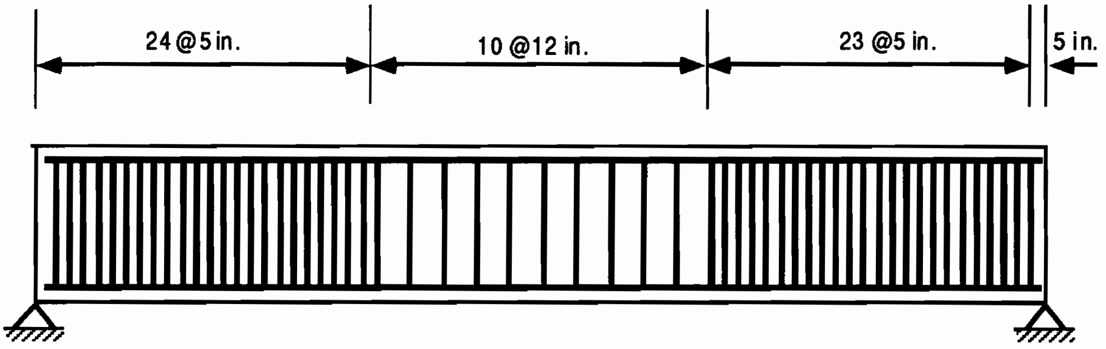


Figure 3.16

Stirrups in beam of example 3

CHAPTER 4

DESIGN OF DEEP R/C BEAMS FOR SHEAR-ACI CODE

4.1 Definition

For shear consideration, a deep beam is one for which l_n/d is less than 5, where l_n is defined as the clear span of a beam measured from face to face of supports (ACI Section 11.8.1 states: "...members with l_n/d less than 5 that are loaded on one face and supported on the opposite face so that compression struts can develop between the loads and the support."). MacGregor (1991) defines more accurately deep beams as: "A deep beam is a beam in which a significant amount of the load is carried to the supports by a compression thrust joining the load and the reaction. This occurs if a concentrated load acts closer than about 2d to the supports, or for uniformly loaded beams with a span-to-depth ratio, l_n/d, less than about 4 or 5.". Deep beams are used as transfer girders as shown in Figures 4.1(single span) and 4.2 (three-span), floor diaphragms, and shear walls.

4.2 General Remarks on the Behavior of Deep Beams

Finite element analysis and two-dimensional elasticity show that the plane section theory does not apply for deep beams. In fact, high shear stresses cause the warping of the cross section [dePaiva and Siess (1965)]. Shear strength of reinforced concrete deep beams is greater than that predicted for slender beams (Equations 2.5 and 2.6). For deep beams, the load is

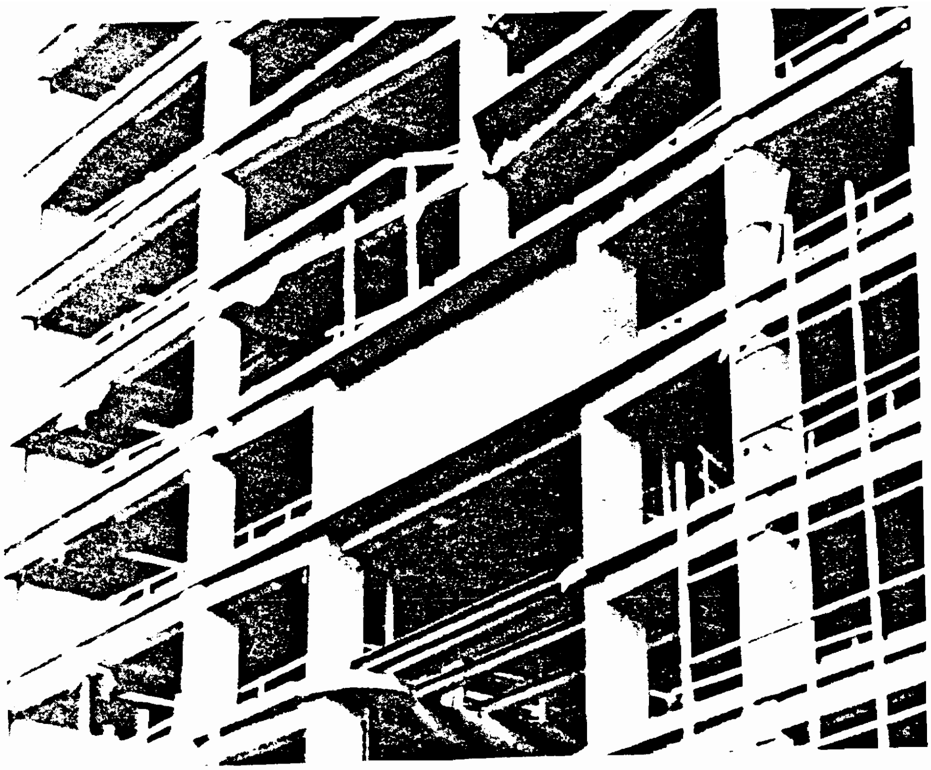


Figure 4.1

Single-span deep beam (MacGregor, 1992)

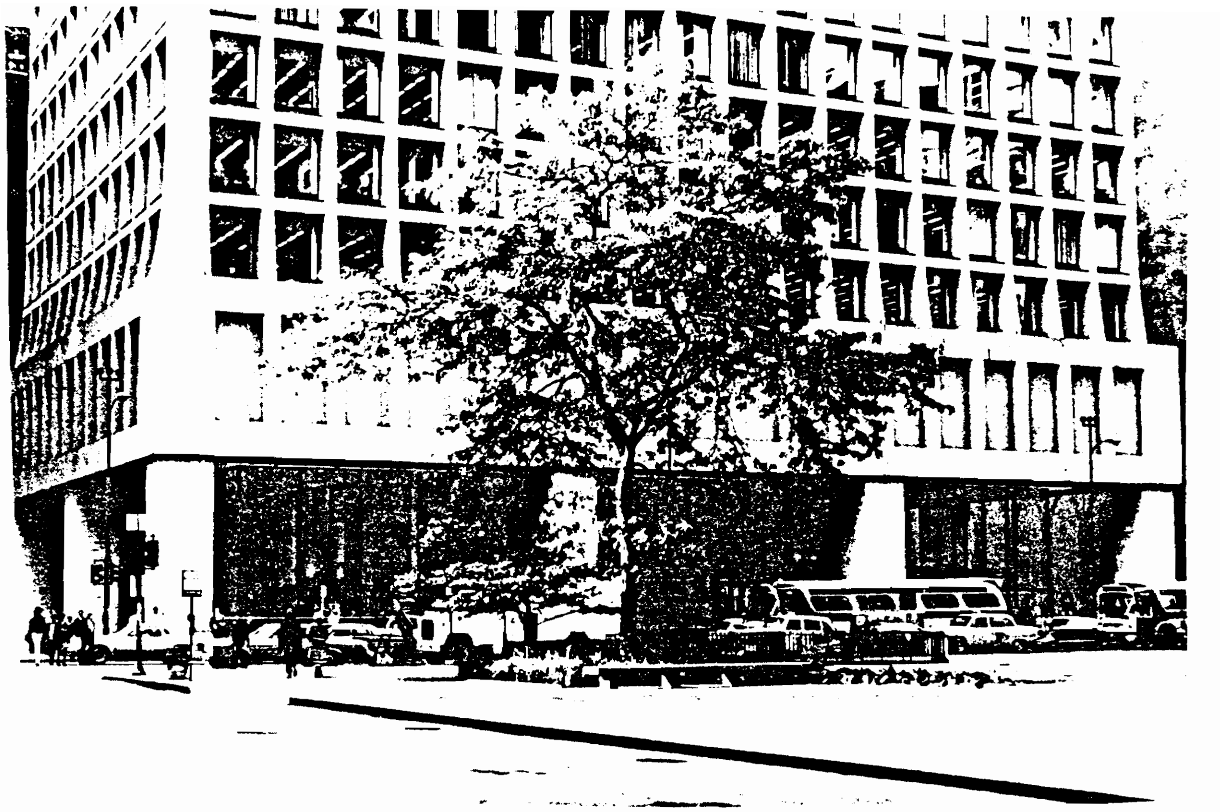


Figure 4.2

Three-span deep beam (MacGregor, 1992)

transferred directly from the point where it acts to the supports by diagonal compression struts. This is illustrated in Figure 4.3 for a deep beam with two concentrated loads. In general, when diagonal cracking occurs in deep beams, it will be steeper than 45°. This is due to the orientation of the principal stresses (stress trajectories) as shown in Figure 4.4. For this reason, horizontal as well as vertical web reinforcement is used. The horizontal web steel is used mainly because it acts in the direction perpendicular to the diagonal crack [Crist (1966)]. Figure 4.5 shows the distribution of web reinforcement in deep beams.

4.3 ACI Code Provisions

Similar to slender beams, the basic design equation for shear is:

$$V_u \leq \Phi V_n \tag{4.1}$$

Where Φ is equal to 0.85 and V_n is the nominal shear strength which is equal to :

$$V_n = V_c + V_s \tag{4.2}$$

Figure 4.6 shows the variation of the maximum permissible V_n as a function of the parameter l_n/d. In equation form, the maximum permissible nominal shear strength is taken to be:

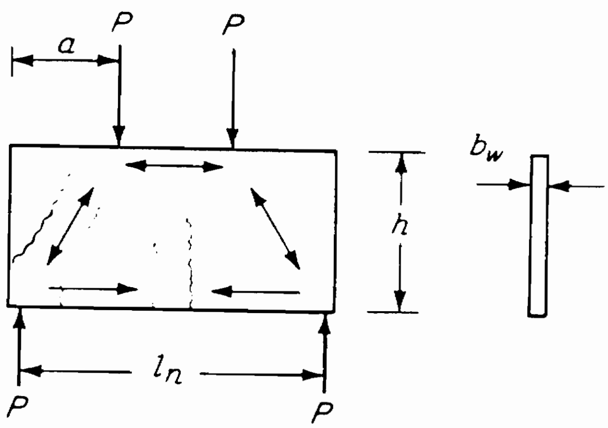


Figure 4.3

Internal forces in a deep beam with two concentrated loads

(Nilson and Winter, 1991)

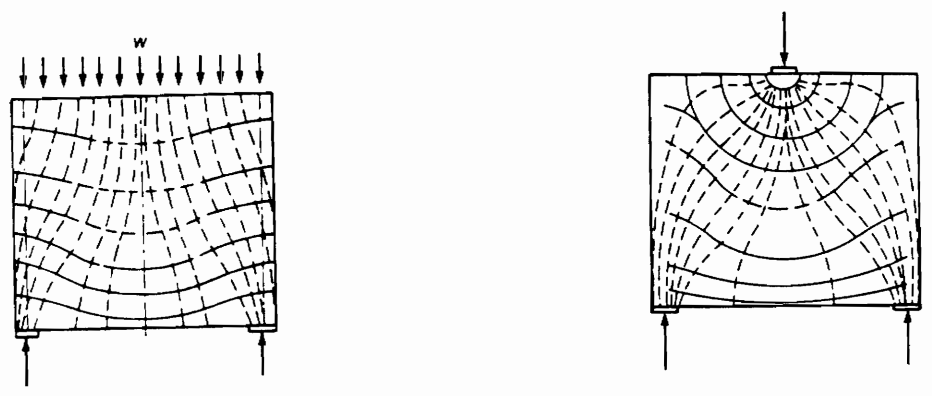


Figure 4.4

Stress trajectories in deep beams (MacGregor, 1992)

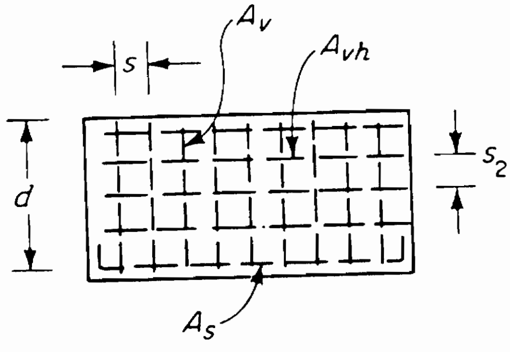


Figure 4.5

Web reinforcement in deep beams (Nilson and Winter, 1991)

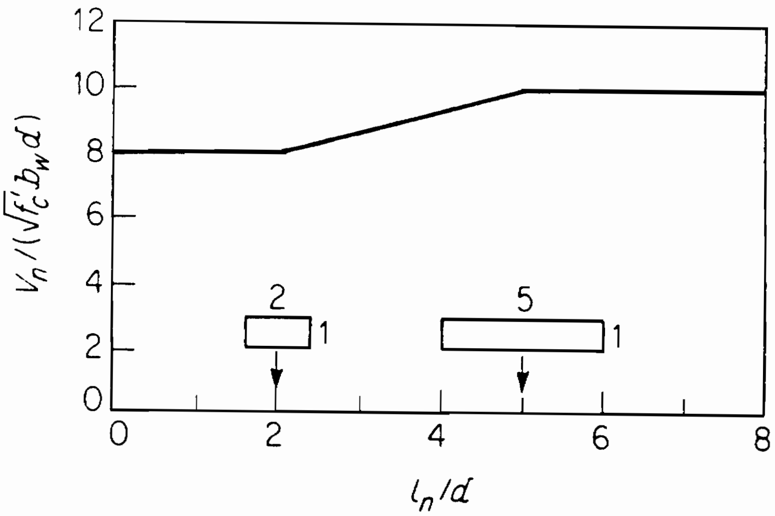


Figure 4.6

Maximum permissible nominal shear strength for deep beams

(Nilson and Winter, 1991)

$$V_n = 8\sqrt{f'_c}\ b_w d \qquad \text{For} \qquad l_n/d < 2 \tag{4.3}$$

$$V_n = \frac{2}{3}(10 + \frac{l_n}{d})\sqrt{f'_c}\ b_w d \qquad \text{For } 2 \leq l_n/d \leq 5 \tag{4.4}$$

$$V_n = 10\sqrt{f'_c}\ b_w d \qquad \text{For} \qquad l_n/d > 5 \tag{4.5}$$

The shear strength provided by the concrete is taken to be:

$$V_c = (3.5 - 2.5\frac{M_u}{V_u d})\,(1.9\sqrt{f'_c} + 2500\,\frac{\rho_w V_u d}{M_u})\,b_w d \leq 6\sqrt{f'_c}\ b_w d \tag{4.6}$$

where $(3.5 - 2.5\frac{Mu}{Vu\,d}) \leq 2.5$

Hence Equation 4.6 is the same as Equation 2.6 (concrete contribution for slender beams) increased by the multiplier $(3.5 - 2.5\frac{Mu}{Vu\,d})$. Figure 4.7 shows the variation of this multiplier as a function of the quantity $(M_u/V_u\ d)$.

M_u and V_u used in Equation 4.6 are the factored bending moment and shear force at the critical section, respectively. For deep beams, the critical section is taken to be at a distance 0.15 l_n from the face of supports for uniformly loaded beams and 0.5a (a is the distance from the face of the support to the concentrated load) for beams with concentrated loads. In either case, the critical distance is not to exceed a distance equal to the beam depth, d, from the support face.

When the factored shear force V_u exceeds the shear strength provided by the concrete ΦV_c, shear reinforcement must be used. The contribution of the

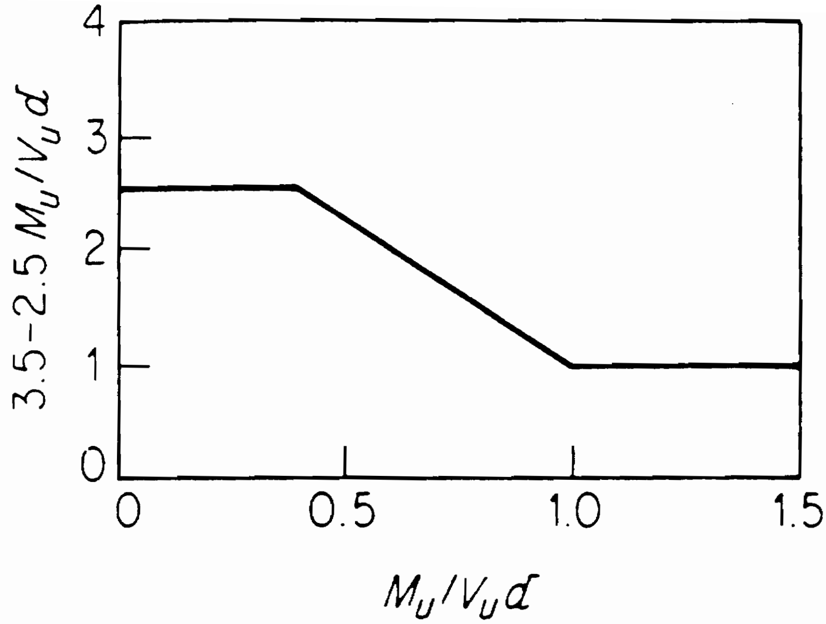


Figure 4.7

Nominal shear strength multiplier for deep beams (Nilson and Winter, 1991) web steel V_s is found from the following equation:

$$V_s = \left[\frac{A_v}{s}\left(\frac{1 + l_n/d}{12}\right) + \frac{A_{vh}}{s_2}\left(\frac{11 - l_n/d}{12}\right)\right] f_y d \qquad (4.7)$$

Where A_v is the web area provided by each vertical stirrup, s is the spacing of the vertical stirrups, A_{vh} is the area provided by each horizontal stirrup, and s_2 is the spacing of the horizontal stirrups.

Combining Equations 4.1, 4.2, and 4.7 we obtain the shear reinforcement design equation for deep beams:

$$\frac{A_v}{s}\left(\frac{1 + l_n/d}{12}\right) + \frac{A_{vh}}{s_2}\left(\frac{11 - l_n/d}{12}\right) = \frac{V_u - \Phi V_c}{\Phi f_y d} \qquad (4.8)$$

The area A_v must not be less than $0.0015 b_w s$ and s must not exceed d/5 or 18in. The area A_{vh} must not be less than $0.0025 b_w s_2$ and s_2 must not exceed d/3 or 18in. Figure 4.8 shows the design details for simply supported deep beams. Figure 4.9 shows the variation of the two coefficients used in Equation 4.7 ($(1 + l_n/d)/12$ and $(11 - l_n/d)/12$) with respect to the parameter l_n/d. One should note that for deep beams, the horizontal steel is dominantly effective. Therefore, in design, it is more efficient to add web steel in the form of horizontal bars while satisfying the minimum requirements for vertical web steel. The following design example will illustrate the steps involved in designing a reinforced concrete deep beam for shear using the ACI Code provisions.

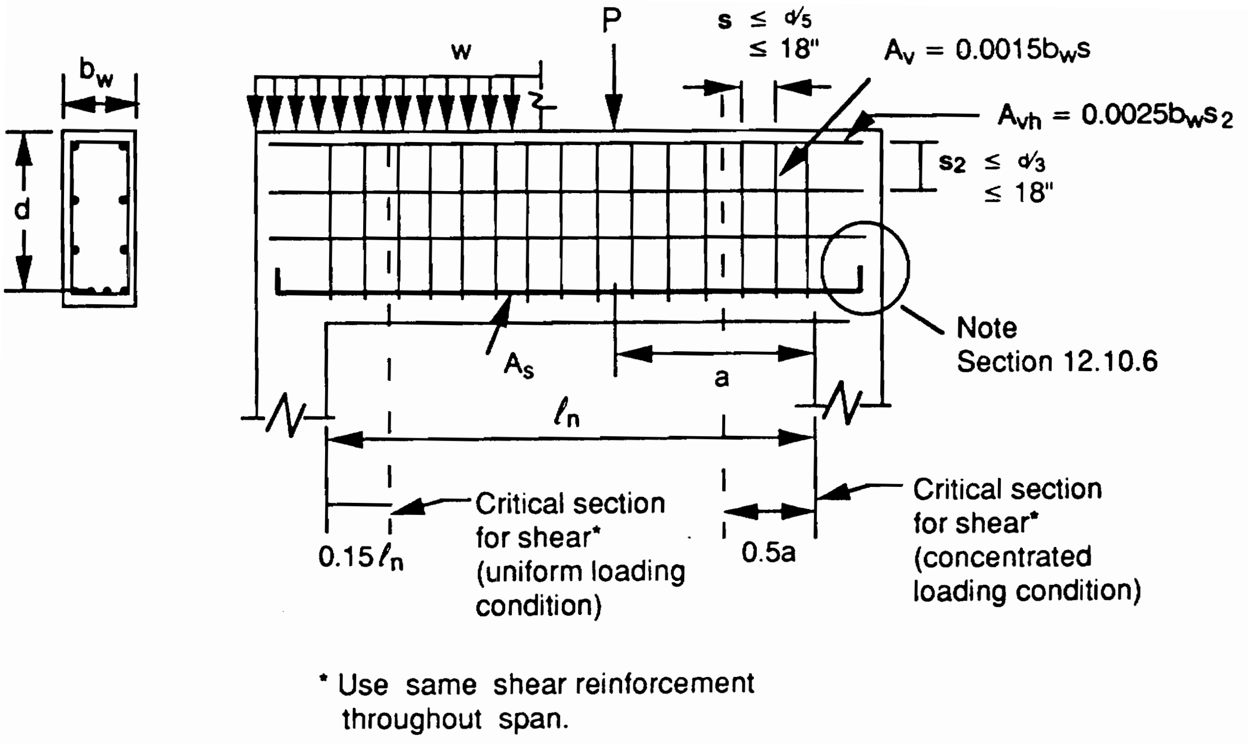


Figure 4.8

Design details for simply supported deep beams (PCA Notes, 1990)

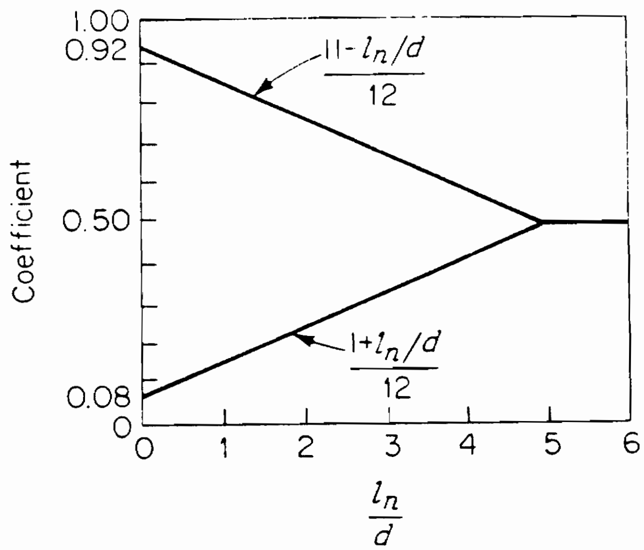


Figure 4.9

Variation of vertical and horizontal web reinforcement coefficients

(Nilson and Winter, 1991)

4.4 Design Example

A transfer girder is to carry one column with a factored load of 2500 kips. A factored distributed load of 10 kips/ft (including the self weight of the beam) is applied along the top of the girder. The general arrangement is shown in Figure 4.10. For geometric reasons, The girder width is 2 ft and its total depth is 12 ft (Figure 4.11). Use fy = 60,000 psi and f'_c = 5,000 psi.

1. Determine l_n/d:

   $l_n/d = (36 - 2) \times 12/136 = 3 < 5 \Rightarrow$ beam must be considered as a deep beam.

2. Determine V_u and M_u at the critical section:

   The critical section is located at 0.5 a = 0.5 x (18 - 1) = 8.5 ft from the support face or 9.5 ft from the support centerline.

   Figure 4.12a shows the factored shear force diagram and Figure 4.12b shows the factored bending moment diagram.

   At the critical section:

   $$V_u = 1430 - 10 \times 9.5 = 1335 \text{ kips.}$$

   $$M_u = \left(\frac{1430 + 1335}{2}\right) \times 9.5 = 13133.75 \text{ kip-ft}$$

3. Determine $V_{n,max}$:

$$V_{n,max} = \frac{2}{3}\left(10 + \frac{l_n}{d}\right)\sqrt{f'_c}\; b_w d$$

$$= \frac{2}{3}(10 + 3) \times \sqrt{5000} \times 24 \times 136/1000 = 2000 \text{ kips.}$$

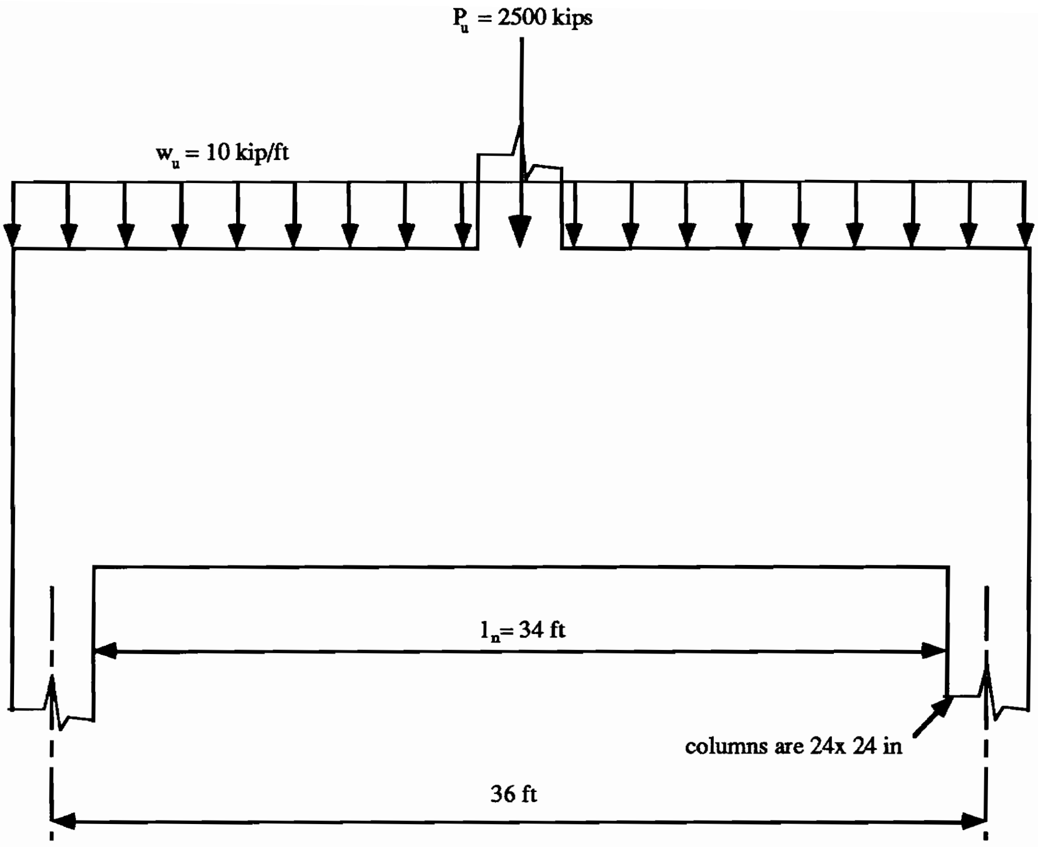


Figure 4.10

Beam arrangement for design example

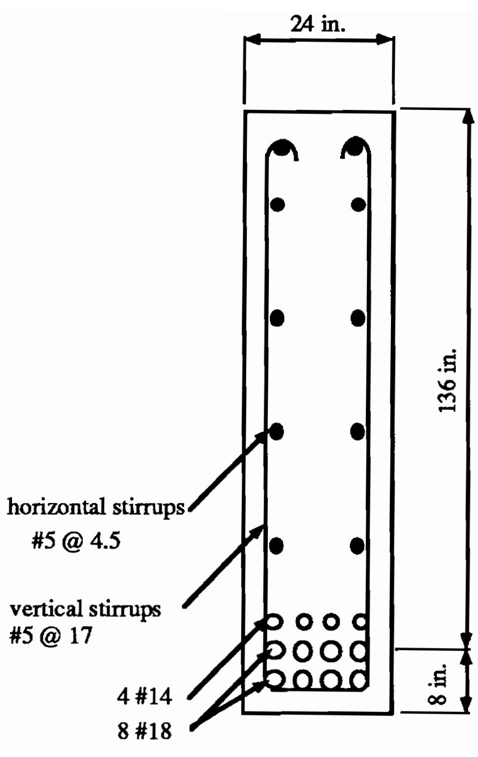


Figure 4.11

Beam cross-section for design example

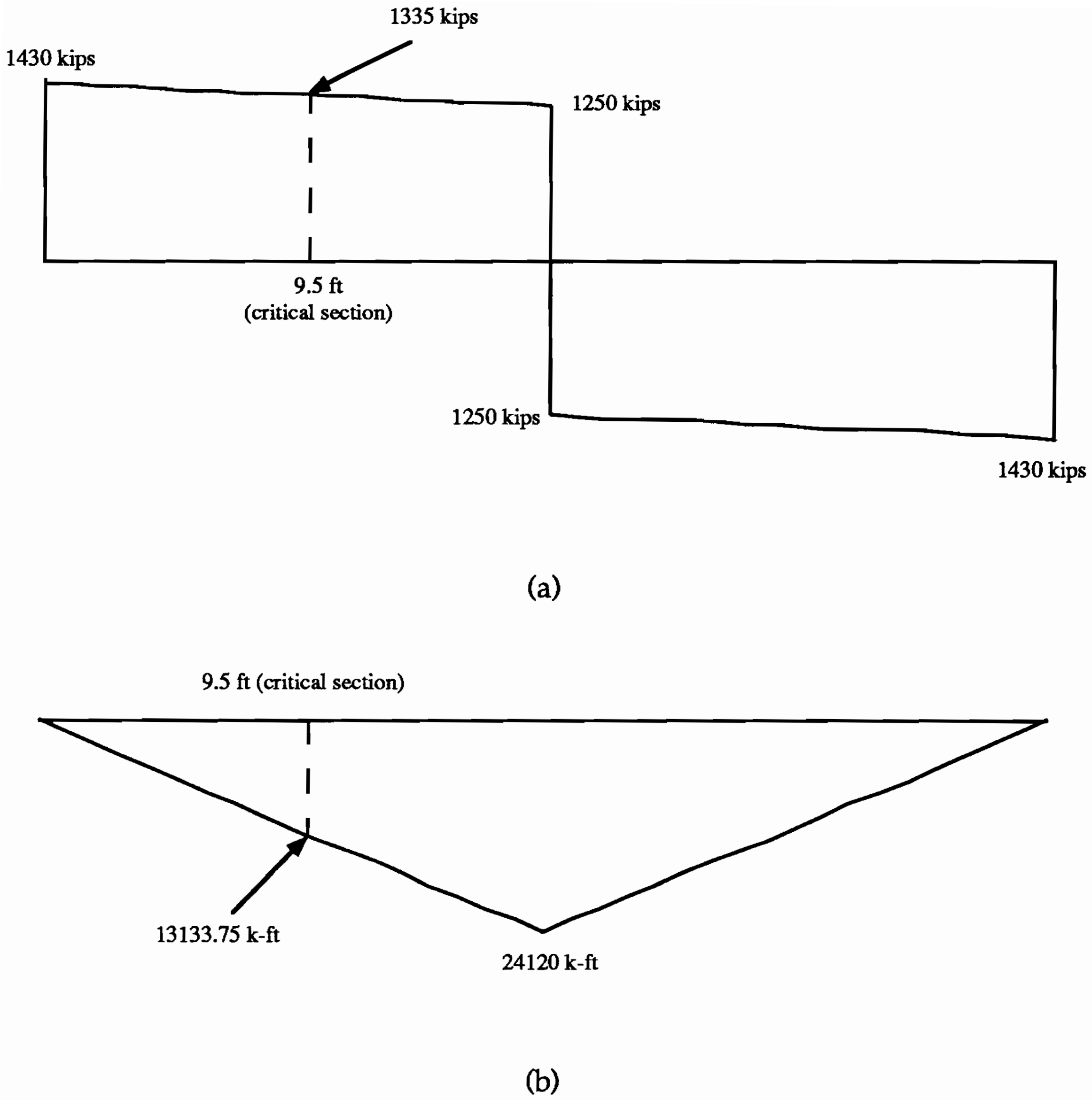


Figure 4.12

(a) Shear force diagram; (b) Bending moment diagram

$\Phi V_{n,max} = 0.85 \times 2000 = 1700 \text{ kips} > V_u$ ok.

4. Determine $M_u/(V_u\, d)$:

$$\frac{M_u}{V_u\, d} = \frac{13133.75 \times 12}{1335 \times 136}$$

$$= 0.868$$

$$3.5 - 2.5\frac{M_u}{V_u\, d} = 3.5 - 2.5 \times 0.868 = 1.33 < 2.5 \quad \text{ok.}$$

5. Determine V_c:

$$V_c = (3.5 - 2.5\frac{M_u}{V_u\, d})\ (1.9\sqrt{f_c'} + 2500\frac{\rho_w V_u\, d}{M_u})\ b_w\, d$$

$$\rho_w = \frac{11 \times 4}{136 \times 24} = 0.01348$$

$$Vc = 1.33 \times (1.9 \times \sqrt{5000} + 2500 \times \frac{0.01348}{0.868}) \times 24 \times 136/1000$$

$$= 751.8 \text{ kips} < 6\sqrt{f_c'}\ \ b_w\, d = 1385 \text{ kips} \quad \text{ok.}$$

6. Shear design equation (for Grade 60 steel):

$$\frac{A_v}{s}\left(\frac{1 + l_n/d}{12}\right) + \frac{A_{vh}}{s_2}\left(\frac{11 - l_n/d}{12}\right) = \frac{V_u - \Phi V_c}{\Phi\, f_y\, d}$$

$$\frac{A_v}{s}\left(\frac{1 + 3}{12}\right) + \frac{A_{vh}}{s_2}\left(\frac{11 - 3}{12}\right) = \frac{1335 - 0.85 \times 751.8}{0.85 \times 60 \times 136}$$

$$0.333\frac{A_v}{s} + 0.667\frac{A_{vh}}{s_2} = 0.100$$

From the design equation, we note that the coefficient for the horizontal web reinforcement is twice the one for vertical stirrups. Therefore, we will design the vertical stirrups based on the minimum required.

Maximum spacing of vertical stirrups is not to exceed d/5 = 136/5 = 27 in., or 18 in., which controls. The spacing of vertical stirrups based on the minimum required A_v is:

$$s = \frac{A_v}{0.0015 \times 24}$$

For #5 U-shaped stirrups, $A_v = 2 \times 0.31 = 0.62\ in^2$, therefore

$$s = \frac{0.62}{0.0015 \times 24}$$

$$= 17.22 \text{ in., say } 17 \text{ in.}$$

This is below the maximum permitted spacing of 18 in. Use #5, Grade 60 vertical stirrups at 17 in.

The spacing of the horizontal bars must not exceed d/3 = 136/3 = 45 in., or 18 in., which controls.

The spacing based on the minimum required A_{vh} and using #5 U-shaped, Grade 60, stirrups is:

$$s_2 = \frac{A_{vh}}{0.0025\, b_w}$$

$$s_2 = \frac{0.62}{0.0025 \times 24}$$

$$= 10.3 \text{ in.}$$

Based on the design equation, we have:

$$0.667 \frac{A_{vh}}{s_2} = 0.100 - 0.333 \times \frac{0.62}{17} = 0.088$$

Therefore the spacing of the horizontal bars is:

$$s_2 = \frac{0.667 \times 0.62}{0.088}$$

$$= 4.7 \text{ in., which controls}$$

Hence, use #5 U-shaped, Grade 60, stirrups at 4.5 in.

The arrangement of the web steel is summarized in Figure 4.13

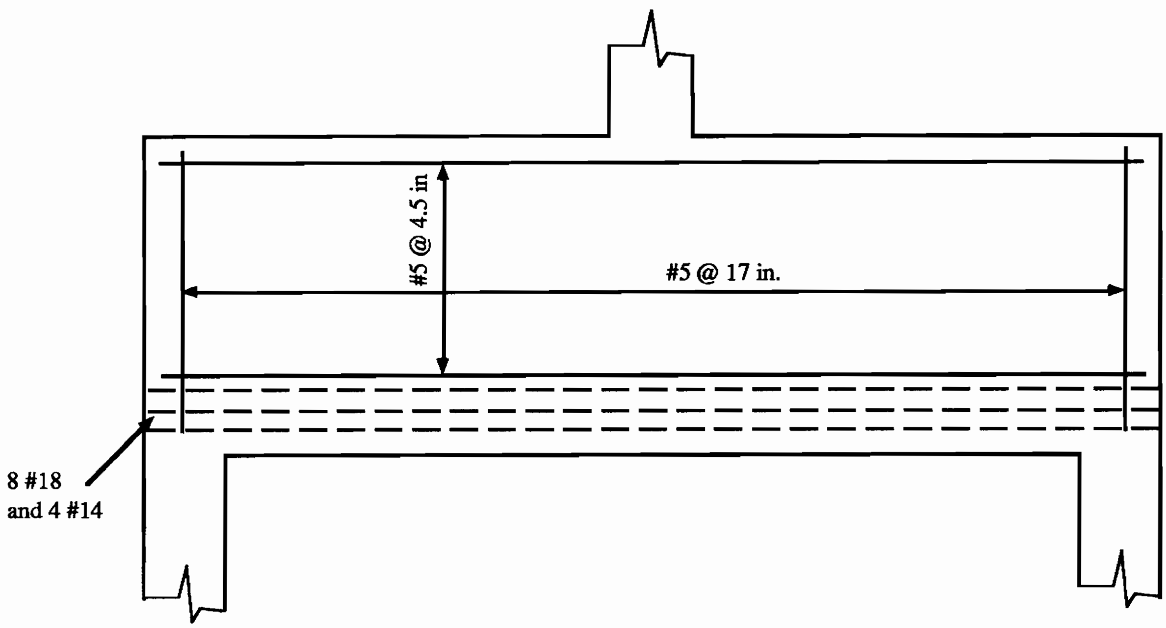


Figure 4.13

Reinforcement in the design example beam

CHAPTER 5

DESIGN OF DEEP R/C BEAMS-TRUSS MODEL THEORY

5.1 Components of the Truss Model

In Chapter 2, the concept of the truss model was presented and used to design slender reinforced concrete beams. The truss consisted of compression struts (concrete diagonals), stirrups, and longitudinal chords (tension chord formed by the longitudinal steel reinforcement and compression chord formed by the compression zone of the beam). Shear is transmitted by compression struts which form a compression fan under concentrated loads and the supports. Between the compression fans there is what we have called a compression field. For deep beams, in addition to these components, there are two other important components: the major compression diagonal and the truss node.

Figure 5.1 shows a deep beam with a concentrated load, P. This load is resisted by two major inclined compression diagonals (shown in the figure by light shaded area). Major compression diagonals occur if the compression fan zones overlap. This means that there will be no compression field. The compression struts are stressed to the concrete effective compressive strength $f_{ce} = \lambda f_c'$, where λ is an "efficiency factor" (which will be discussed later).

Truss nodes (shown in Figure 5.1 by dark shaded area) are wedges of concrete which have both in-plane principle stresses equal to f_{ce}. Therefore, Mohr's circle of the nodes for the in-plane stresses is represented by a point. For that reason nodes are usually referred to as "hydrostatic elements".

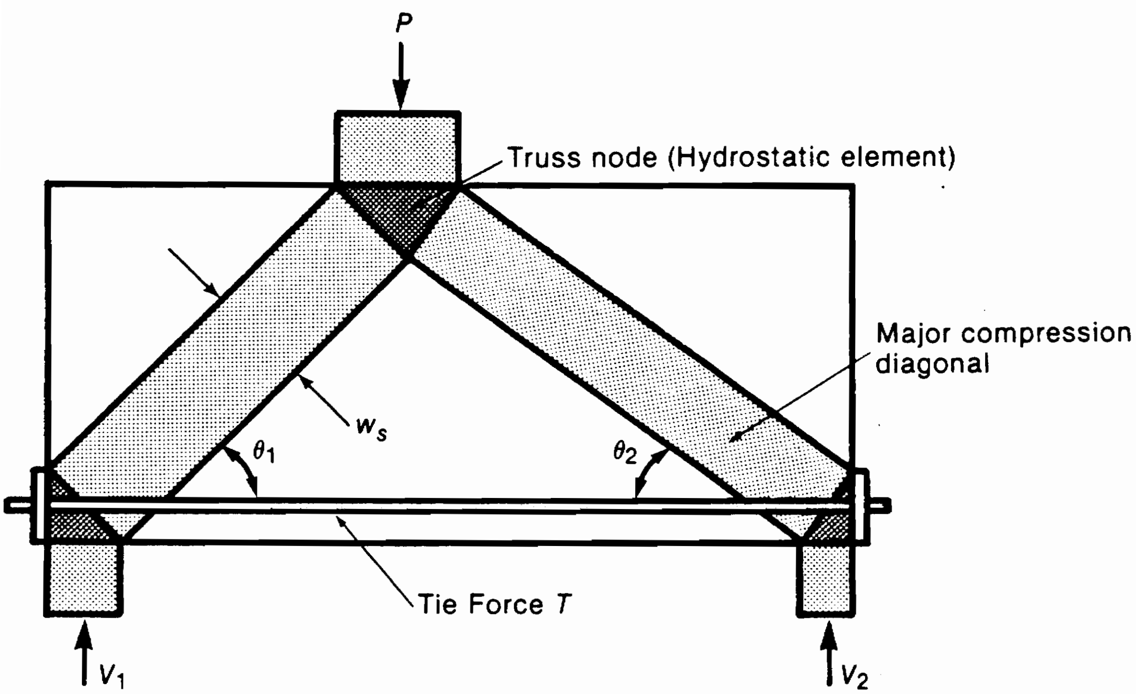


Figure 5.1

Components of the truss model in a deep beam (MacGregor, 1992)

Nodes may have any polygonal shape.

5.2 Assumptions

The use of the truss model to design deep beams requires:

(1) Equilibrium of the truss must be satisfied.

(2) The concrete does not resist any tension. It only resists compression. The concrete compressive members are stressed to the concrete effective compressive stress, f_{ce}.

(3) Tensile forces are resisted by the steel.

(4) Failure of the truss occurs in one of three ways:

(a) Flexural steel (called tie in Figure 5.1) could yield.

(b) The stress in the struts exceeds f_{ce} which causes the crushing of the concrete diagonals.

(c) A node failure if the stress exceeds f_{ce}.

(5) The centroid of each truss member and the lines of action of all externally applied loads must coincide. This will limit the size of the major compression diagonal as illustrated in Figure 5.2. The node of Figure 5.2a has been redrawn in Figure 5.2b with a decrease in the cover of the flexural steel. For the axes of the members to meet, the major compression strut must be smaller. This means it can resist a smaller compression force.

5.3 Effective Concrete Strength

The concrete diagonals and the nodes are assumed to be stressed to the

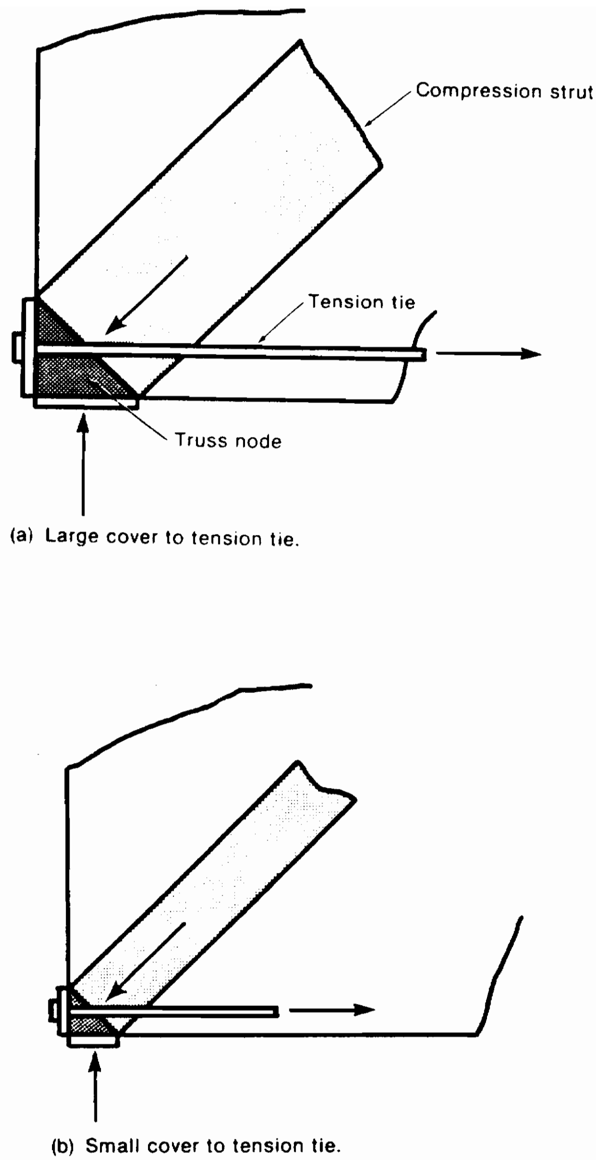


Figure 5.2

Effect of the cover on the size of the major compression strut

(MacGregor, 1992)

effective concrete compressive strength $f_{ce} = \lambda f'_c$, where λ is an "efficiency factor". The "Comité International du Béton" (1978) set a value of 0.6 for λ which means that the concrete effective compressive strength is $f_{ce} = 0.6 f'_c$. Nielsen (1978) has proposed:

$$f_{ce} = (0.7 - \frac{f'_c}{29000})\, f'_c \qquad \text{(psi)} \qquad (5.1)$$

$$f_{ce} = (0.7 - \frac{f'_c}{200})\, f'_c \qquad \text{(MPa)} \qquad (5.2)$$

Ramirez (1984) has suggested:

$$f_{ce} = 30\sqrt{f'_c} \qquad \text{(psi)} \qquad (5.3)$$

$$f_{ce} = 2.5\sqrt{f'_c} \qquad \text{(MPa)} \qquad (5.4)$$

Based on their experimental work, Collins and Mitchell (1980) and Vecchio and Collins (1982) related the effective concrete strength (f_{ce}) to the principal tensile strain, ε_l, perpendicular to the direction of the principal compressive stress (figure 5.3a). They came up with the following relationship:

$$f_{ce} = \frac{f'_c}{0.8 + 170\,\varepsilon_l} \qquad (5.5)$$

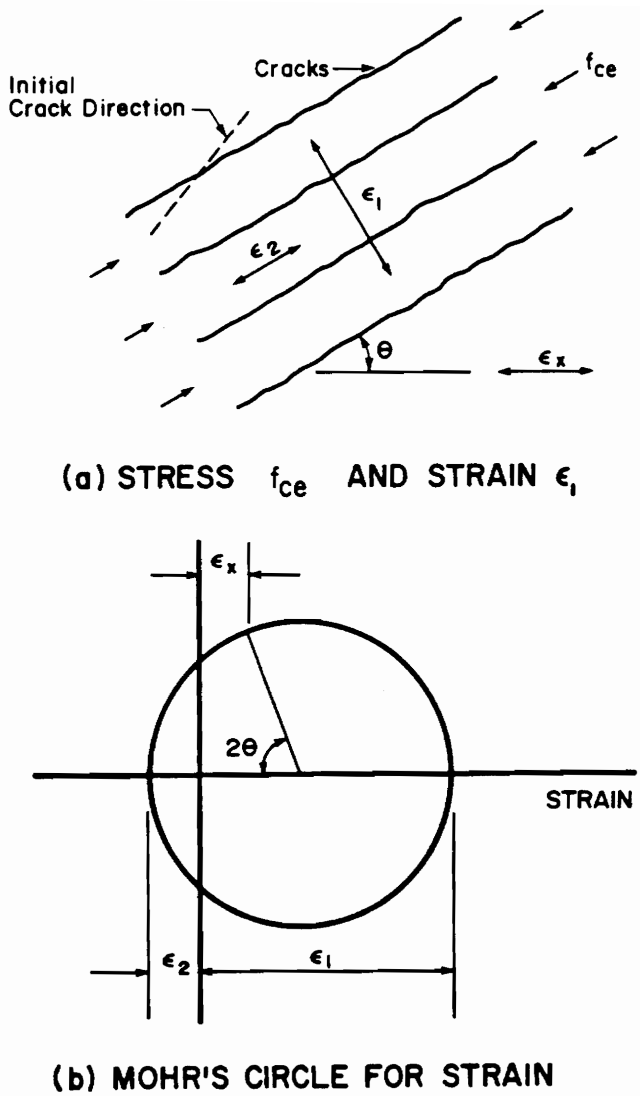


Figure 5.3

Strains in a cracked web (Collins and Mitchell, 1980)

Figure 5.3b shows Mohr's circle for strain relating the strain in a direction parallel to the axis of the beam (ε_x), the maximum strain occurring in the direction of the compressive stress fce (ε_2), and the angle θ. The strain ε_2 corresponds to the highest point in a concrete compression stress-strain curve. This is taken as 0.002. Mitchell and Collins took the same value (0.002) for the strain ε_x. They found out that when θ varies from 0° to 45°, λ varies almost linearly from 0 to 0.55. Then λ goes back to 0 as θ goes to 90°.

Table 5.1 gives the recommended values of the effective compressive strength, f_{ce}.

5.4 Design Procedure

The design of a deep beam using the truss model theory consists of drawing to scale a proper truss that will resist the applied loads. The concrete diagonals and the nodes are designed for the stress fce. Several iterations may be required before we draw a final truss that fits within the beam and has adequate cover.

Figure 5.4 shows a truss model for a simply supported beam with a concentrated load. The shear is resisted by the major compression diagonal (V_c) and the stirrups (V_s). The vertical force in each stirrup is found based on the assumption that the stirrups have yielded. Therefore, the vertical component in each of the small concrete diagonals must be equal to the force in each stirrup for the joint to be in equilibrium. Knowing the total shear force V_n, the shear resisted by the major concrete diagonal is equal to $V_c = V_n - V_s$. From the geometery of the truss, one can determine the slope of the

Table 5.1

Effective concrete strength, f_{ce} (MacGregor, 1992)

Structural Member	f_{ce}
Truss node	
Joints bounded by compressive struts and bearing areas	$0.85f'_c$
Joints anchoring one tension tie	$0.65f'_c$
Joints anchoring tension ties in more than one direction	$0.50f'_c$
Isolated compression struts in deep beams or D-regions	$0.5f'_c$
Severely cracked webs of slender beams	
$\theta = 30°$	$0.25f'_c$
$\theta = 45°$	$0.45f'_c$

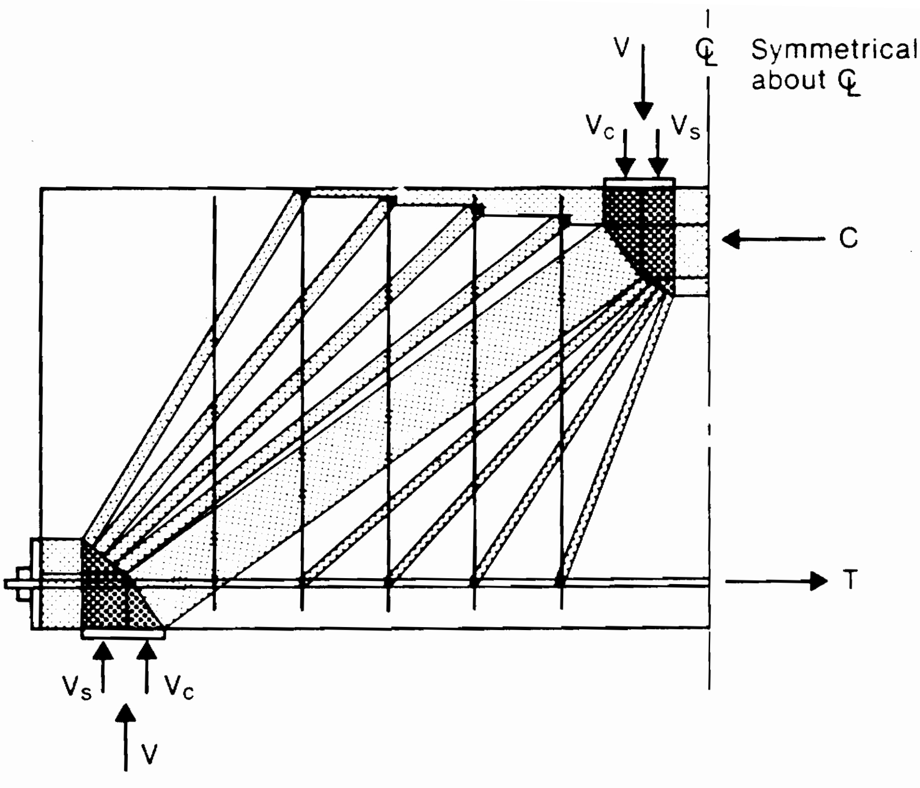


Figure 5.4

Truss model for a simply supported beam with a concentrated load

(MacGregor, 1992)

diagonals (including the major diagonal) and therefore their compression forces. Assuming that the stress in the diagonals is equal to f_{ce}, we can determine the width of the diagonals (the width of the major diagonal is referred to as w_s as shown in Figure 5.1). After determining the width of all the diagonals, we can refine the layout of the struts. This technique is illustrated in the following design example.

5.5 Design Example

The design example of Chapter 4 will be reworked using the Truss Model Theory. Use $f_{ce} = 0.6 f_c'$ and $25° < \theta < 65°$ with fy = 60,000 psi and $f_c' = 5{,}000$ psi.

1. Compute V_u:

   V_u diagram is shown on Figure 4.12a.

2. Draw idealized truss:

   To simplify the drawing, we shall idealize the uniform loading as a series of 20 k concentrated loads at 2 ft on centers. Similarly, idealize the stirrups as being 2 ft on centers and compute the stirrup force per 2-ft interval. We will then select the spacing of the #5 U-shaped, Grade 60, stirrups that will give a stirrup capacity at least equal to the stirrup force.

   A final truss is shown in Figure 5.5. This truss was obtained after several iterations.

3. $V_u/\Phi = 1430/0.85 = 1682$ k.

   Assume #5 stirrups at 12 ft on centers. Although stirrups will be

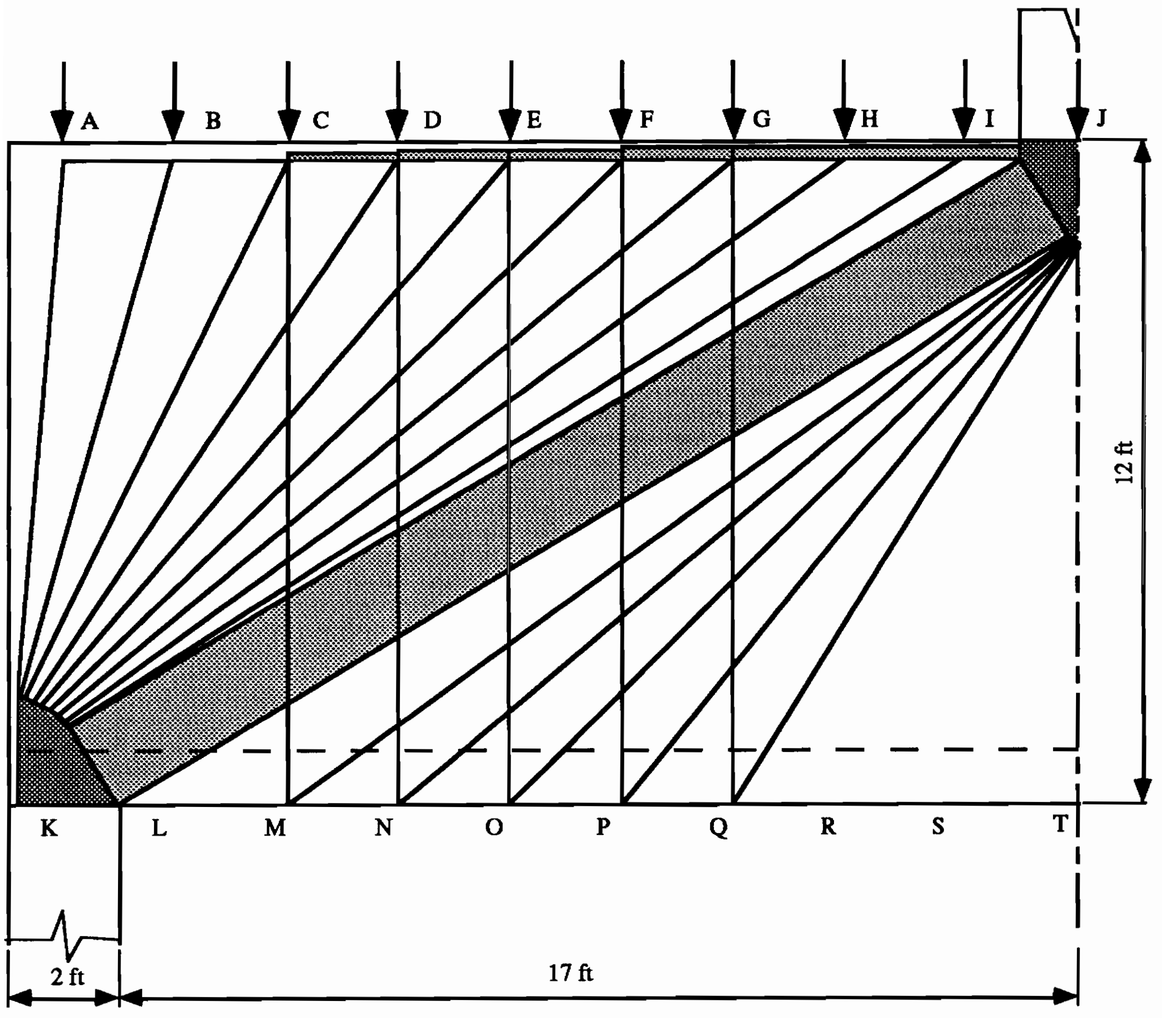


Figure 5.5

Truss model of the design example

provided at JT, IS, and HR, these will be disregarded in the calculations since they are loaded by compression struts steeper than 65°. Stirrups AK and BL will also be disregarded since they interfere with the main compression diagonal. This will leave us with 5 stirrups. For a first trial, try #5 stirrups at 12 in. The capacity of these stirrups for each 2 ft spacing is equal to:

$$2\ A_v\ f_y = 2 \times 0.62 \times 60 = 74.4 \text{ kips}$$

The total shear transmitted by the stirrups is:

$$V_s = 5 \times 74.4 = 372 \text{ k}$$

Thus the stirrups transmit 372/1682 = 22% of the whole shear. This is below the desired minimum range of 25 to 35%. Therefore, try #5 stirrups at 6 in. The capacity of the stirrups for each 2 ft spacing is now equal to:

$$4\ A_v\ f_y = 4 \times 0.62 \times 60 = 148.8 \text{ kips}$$

The total shear transmitted by the stirrups is now increased to:

$$V_s = 5 \times 148.8 = 744 \text{ k}$$

This represents 744/1682 = 44% of the whole shear which is acceptable.

4. Width of struts:

* Width of the major compression strut

The shear transmitted by the stirrups is: $V_s = 744$ k

The shear transmitted by the major compression diagonal is:

$$V_c = 1250/0.85 - 744 = 726 \text{ k}$$

Slope (measured from the sketch) of the major compression diagonal (JK) is 32°.

Force in the major compression diagonal is:

$$D = 726/\sin 32° = 1370 \text{ k}$$

Width of the major compression diagonal is:

$$w_s = \frac{D}{f_{ce}\, b_w} \qquad (5.6)$$

$$= \frac{1370}{0.6 \times 5 \times 24} = 19.0 \text{ in.}$$

The widths of the other struts are shown in table 5.2.

At this point it is possible to refine the layout of the struts. The length of node K is measured from the sketch to be 20 in. Therefore, the centroid of the flexural steel should be located at mid-height of the hydrostatic element (this means that d = 144 - 10 = 134 in.). Figure 5.6 shows node K.

5. Compute the vertical and horizontal components of the strut forces:

The horizontal and vertical forces in each of the diagonal struts are calculated in Table 5.3. The slope of the struts are measured from the final drawing of the truss. The vertical force component in a strut is the sum of the stirrup force and the applied load. The horizontal component is equal to the vertical component divided by the tangent of the angle of the strut.

6. Compute the forces in the lower chord:

The force in the lower chord between K and M is (refer to Table 5.3):

T_{KM} = (2.1 + 6.7 + 87.8 + 116.2 + 149.8 + 178.4 +212.8 + 32.3 + 36.2) + 1116.8

= 1984.8 k

The equilibrium of joint M is shown in Figure 5.7. From that figure, we can calculate the tension force between M and N to be:

T_{MN} = 1984.8 + 204.8 = 2188.9 k

Table 5.4 shows the forces in the lower chord.

Table 5.2

Widths of compression struts

Strut	Vertical force (kips)	Slope (deg)	Compressive force, D (kips)	Width (in.)
IK	20/0.85 = 23.5	33	43.1	0.60
HK	23.5	36	40	0.56
GK	23.5+148.8=172.3	39	273.8	3.80
FK	172.3	44	248	3.40
EK	172.3	49	228.3	3.20
DK	172.3	56	207.8	2.90
CK	172.3	63	193.4	2.70
BK	23.5	74	24.4	0.34
AK	23.5	85	23.6	0.33

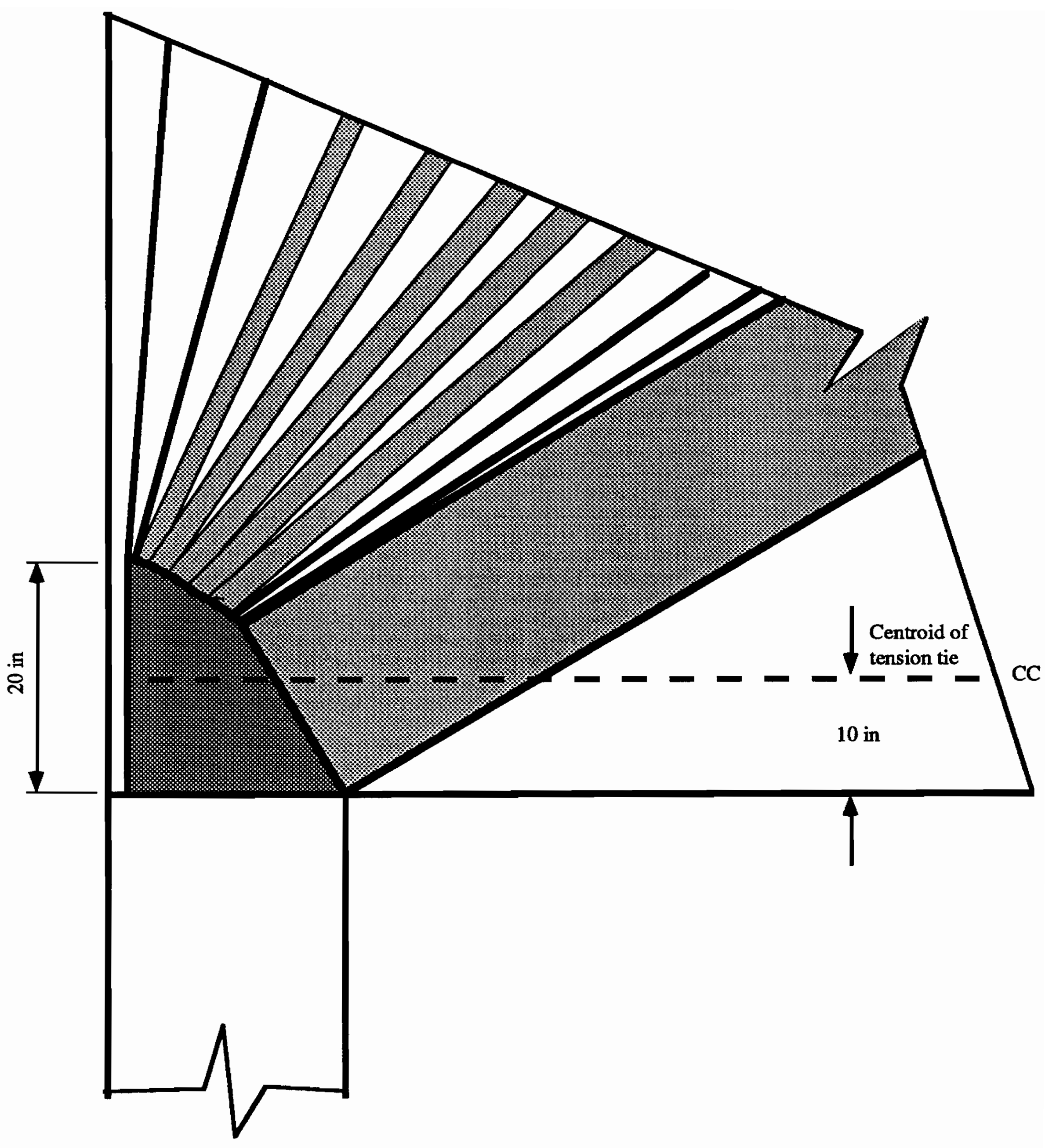


Figure 5.6

Node K of the design example

Table 5.3

Strut forces

		Force Components	
Strut	Slope (deg)	vertical (k)	horizontal (k)
AK	85	23.5	2.1
BK	74	23.5	6.7
CK	63	148.8+23.5 = 172.3	87.8
DK	56	172.3	116.2
EK	49	172.3	149.8
FK	44	172.3	178.4
GK	39	172.3	212.8
HK	36	23.5	32.3
IK	33	23.5	36.2
JK	32	726.0	1161.8
JM	36	148.8	204.8
JN	41	148.8	171.2
JO	35	148.8	148.8
JP	51	148.8	120.5
JQ	59	148.8	89.4

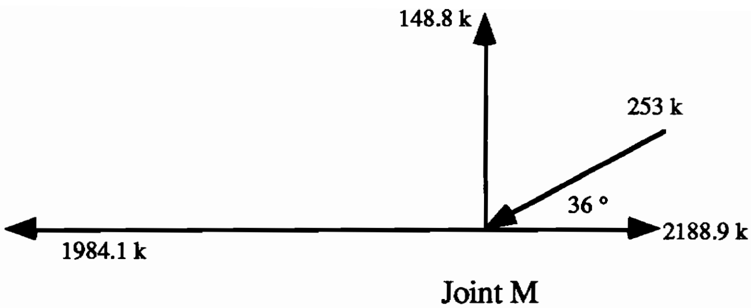


Figure 5.7

Equilibrium of joint M

Table 5.4

Forces in bottom chord

	Force added	Force in chord
KM	0	1984.1
MN	204.8	2188.9
NO	171.2	2360.1
OP	148.8	2508.9
PQ	120.5	2629.4
QT	89.4	2718.8

As a check, if jd = 10.1 ft (measured from the drawing), then the moment at mid-span is equal to (2718.8) x (10.1) = 27460 k-ft. According to Figure 4.12b, M_u/Φ at mid-span is 24120/0.9 = 26800 k-ft. This is close to the value found from the truss model (the error is a result of the sketches). Therefore, we can design for the longitudinal steel using the truss model.

7. Select horizontal web reinforcement:

Horizontal web reinforcement is not necessary as part of the truss. Use the minimum required by the ACI Code. For #5 U-shaped, Grade 60, stirrups, the minimum spacing is:

$$s_2 = \frac{A_{vh}}{0.0025bw} = \frac{0.62}{0.0025 \times 24} = 10 \text{ in.}$$

8. Summary of design

Use #5 U-shaped, Grade 60, vertical stirrups at 6 in. and #5 U-shaped, Grade 60, horizontal stirrups at 10 in. Figure 5.8 shows the stirrup distribution in the beam.

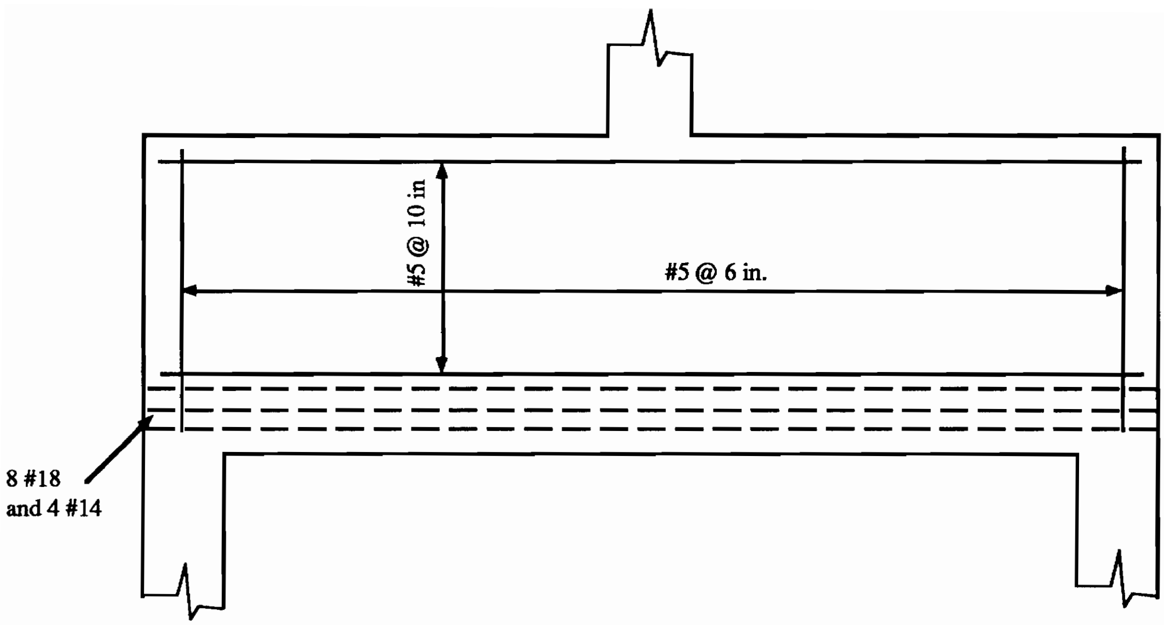


Figure 5.8

Stirrups in design example beam

CHAPTER 6

CONCLUSION AND RECOMMENDATIONS

Comparison of Results

Table 6.1 presents the results obtained for the design examples of slender beams using ACI Code provisions and the truss model theory. Both design procedures gave close results (if we compare the total number of stirrups used in the different beams). However, the ACI Code method was more conservative (especially for design example 2).

Table 6.2 is a summary of results for the deep beam design example using both procedure. The results were quite different with the horizontal and vertical spacings being reversed. The spacing of vertical stirrups obtained by the truss model is much smaller than that obtained from the ACI Code. On the other hand, the spacing of horizontal stirrups found by the ACI Code method is smaller than that found from the truss model theory. This is due to the fact that the ACI Code uses the minimum required amount of vertical web reinforcement and the truss model procedure uses the minimum required required amount of horizontal web reinforcement.

6.2 Comparison of Methods

For slender beams, both design methods are simple and consist of different easy to follow steps. The ACI Code procedure is a semi-empirical solution since it combines some test results with the equations obtained from the 45°

Table 6.1

Results of the slender beam design examples

Method	Example 1 #3 U shaped, Grade 60, stirrups	Example 2 #3 U shaped, Grade 40, stirrups	Example 3 #3 U shaped, Grade 40, stirrups
ACI-Code	1 @ 5 in. 4 @ 10.5 in. 7 @ 14 in. **Total:** 22	1 @ 3 in. 5 @ 6 in. 8 @ 8.5 in. **Total:** 28	30 @ 4 in. (first span) 30 @ 4 in. (first span) **Total:** 60
Truss Model	1 @ 6 in. 5 @ 13.5 in. 4 @ 22.5 in. **Total:** 20	1 @ 5 in. 4 @ 10 in. 5 @ 14 in. **Total:** 20	24 @ 5 in. 10 @ 12 in. 23 @ 5 in. **Total:** 57

Table 6.2

Results of the deep beam design example

Method	#5 U shaped, Grade 60, stirrups
ACI-Code	Vertical spacing = 17 in. Horizontal spacing = 4.5 in.
Truss Model	Vertical spacing = 6 in. Horizontal spacing = 10 in.

truss model. This means that the ACI Code method lacks a physical model to describe the actual behavior of the beam subjected to shear combined with bending. For slender beams, the truss model theory is not the best solution since it does not consider the shear transfer (by dowel action of the main steel, aggregate interlock along the diagonal crack, and shear in the uncracked concrete beyond the end of the crack) in the slender beam before the formation of the inclined cracks.

For deep beams, the truss model theory is complicated since it involves several iterations with different sketches. However, it is more reliable than the ACI Code since it models the dominant mechanism that happens in a deep beam which is the force transfer from load to reactions by the direct compression struts.

6.3 Recommendations

The truss model theory is recommended for use in designing deep reinforced concrete beams. For slender beams, either the ACI Code provisions or the truss model theory can be used.

Further work can be done by comparing the ACI Code provisions and the truss model theory for designing reinforced concrete beams for torsion.

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