

# A New Adaptive Array of Vibration Sensors

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(ABSTRACT)

The sensing technique described in this dissertation produces modal coordinates for monitoring and active control of structural vibration. The sensor array is constructed from strain-sensing segments. The segment outputs are transformed into modal coordinates by a sensor gain matrix.

An adaptive algorithm for computing the sensor gain matrix with minimal knowledge of the structure's modal properties is proposed. It is shown that the sensor gain matrix is the modal matrix of the segment output correlation matrix. This modal matrix is computed using new algorithms based on Jacobi rotations. The procedure is relatively simple and can be performed gradually to keep computation requirements low.

The sensor system can also identify the mode shapes of the structure in real time using Lagrange polynomial interpolation formula.

An experiment is done with an array of piezoelectric polyvinylidene fluoride (PVDF) film segments on a beam to obtain the segment outputs. The results from the experiment are used to verify a computer simulation routine. Then a series of simulations are done to test the adaptive modal sensing algorithms. Simulation results verify that the sensor gain matrix obtained by the adaptive algorithm transforms the segment outputs into modal coordinates.

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