A Model for Determining Leakage in Water Distribution Systems

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(ABSTRACT)

Leaks in pipe networks cause significant problems for utilities and water users in terms of lost revenue and interrupted service. In many cities the leakage is as high as forty percent. A water audit is carried out to assess system-wide leakage. However, to detect leakage at the level of a pipeline, a physical measurement technique is generally employed. For large cities the distribution piping length amounts to a few thousand miles. Therefore, the physical measurements can become tedious and expensive. In this thesis it is assumed that a spatial distribution of leakage can be estimated at nodes based on a water audit bookkeeping scheme. A mathematical formulation consisting of continuity, energy (headloss), pressure-dependent demands and/or leakage, and flow direction preservation equations are utilized to distribute demand flows and leakage among pipes. The leakage is attributed to the formation of corrosion holes. Based upon the extent of corrosion, the leakage flow arriving at a particular node is apportioned among all pipes that are converging at that node. Therefore, the formulation presented in this thesis captures the two essential elements behind leakage, namely, pressure driven flow distribution and the vulnerability of pipes to corrosion.

The proposed formulation allows utilities to be more proactive in identifying leakage prone districts within the water distribution system. An understanding of the pressure-dependent leakage in the system is helpful when performing a water audit and in developing strategies for

leak repair programs. Restoring the full capacity of the water distribution system will greatly increase the reliability of the system, thereby benefiting local utilities and water users.

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Chapter 1

Introduction to Losses in Water Distribution Systems

1.1 Introduction to water supply systems

Water is an entity that many people take for granted. Turning on the faucet and expecting water to flow out freely is a relatively new convenience that is essential to our modern lifestyle. However, most people have no idea of what actually goes into making this a reality. The method by which water goes from source to tap is an extensive process involving several components and numerous potential trouble spots. The following list itemizes the main parts of a water supply system:

1. Source development/Supply;
2. Water treatment;
3. Pumping;
4. Transmission and Storage;
5. Distribution Pipe Network;
6. User.

The water supply system in a community provides potable water to customers by first treating raw water using various unit processes, pumping and transmitting treated water to the community, storing the water, and finally distributing the water to the customer. Each of these components is necessary for the efficient realization of the end result, which is to provide potable water at adequate pressure and flow at the demand locations. Each of the six elements listed above is essential to the overall scheme of delivering water to the customers. However, each step in the water supply system is subject to problems and water losses, which could affect the overall reliability of the system in accomplishing its goal.

1.2 Water distribution systems

In the water industry, certain terms must be defined to understand the technical terms used in this thesis. The term, “water supply system,” is the broad term that encompasses all six of the items listed on the previous page. The term, “water distribution system,” is a narrower phrase that includes items 3-6 of that list. The water distribution system includes pumping, transmission and storage, and the distribution pipe network, as well as all other appurtenances, such as valves, hydrants, and service connections. These appurtenances are necessary to deliver potable water to customers within the confines of the municipality. This thesis will focus on the backbone of the water distribution system, namely, the distribution piping network.

The purpose of a municipal water distribution system is to deliver potable water to a community to meet domestic, industrial, commercial, and fire flow demands. Under normal operating conditions, the system is required to supply customers with an adequate flow of good quality water under sufficient pressure. Deficient conditions occur in a system when one or more of the flow, pressure, or quality requirements are not met.

There are several possible causes of deficient conditions in a water distribution system. A water distribution system comprises several different components including water mains, storage tanks, pump stations, valves, hydrants, meters, and service connections. Failure in any one of these devices may lead to deficient conditions in some portion of the network. Water mains are the backbone of the entire system and therefore have the greatest effect on the overall efficiency of the network. As such, deficient conditions stemming from water main leaks and failures will be the emphasis of this thesis.

1.3 Leaks in distribution piping networks

Leaks or breaks in water mains pose significant problems for utilities. Pipe failures contribute to significant losses of potable water that, in turn, translate into lost revenue. Significant water losses in a system may cause premature development of new water sources at significant cost. In addition, water leaks may result in increased groundwater levels, damage to adjacent property, and possible contamination of the water supply. Municipal utilities recognize these problems; however, time and money often preclude them from completely solving the problem of leakage in the system.

To handle the problems associated with leaks in pipe networks this thesis presents a mathematical formulation for analyzing water distribution systems under deficient operational conditions. This formulation is intended to determine the magnitude of flow being lost from the pipe network due to leakage. In addition, this thesis will present methods by which leakage flows can be allocated among pipes to determine which pipes have the most urgent need of rehabilitation or replacement.

By determining the location and magnitude of leaks in the system, municipalities can prioritize repairs to the pipe network. By focusing on the locations with the greatest loss of water, time and money can be saved by fixing the largest leaks first. Restoring the full capacity of the water distribution system will greatly benefit local utilities and water users.

1.4 Thesis organization

This chapter has introduced the essential elements of a water supply system, along with an introduction to leakage loss, which is the main focus of this thesis. Leaks cost utilities a tremendous amount of money each year and reduce considerably the reliability of service to customers. The objective of this research is to help utilities identify leakage losses in pipe networks.

The thesis is organized as follows. Chapter 2 presents a full discussion of the assessment, causes, and consequences of leaks. Chapter 3 describes leak detection methods. Chapter 4 discusses the distinction between planning and operational problems. Chapter 5 presents a mathematical formulation covering pressure-dependent flows (operational problem) and leakage allocation among nodes. Chapter 6 provides a methodology to identify leakage-prone pipes due to corrosion and apportion leakage among stricken pipes. Chapter 7 contains an example application of a test problem from the literature. Chapter 8 contains the application to a subnetwork of the Blacksburg water distribution system. Chapter 9 offers the summary of the entire research and discusses directions for future research.

Chapter 2

Assessment, Causes, and Consequences of Leakage in Water Distribution Systems

2.1 Introduction

Closed conduit water distribution systems normally operate under pressure. The water in the system follows the path of least resistance, especially if that path leads out of the system. The pipe network of a water distribution system is designed to deliver water to specific withdrawal points with adequate flow and pressure. These releases of water from the network at designated withdrawal locations are necessary to provide service to customers and to provide for municipal uses. Normal demands for domestic and commercial purposes are usually metered to keep track of the amount of water used.

Although certain locations are specified for water to leave the system, water may also leave at unwanted points. The first step in understanding losses is to identify certain terms used in the water industry. The American Water Works Association has identified three major categories of "losses" in a water supply system. These categories are (AWWA, 1987)

1. Accounted-for losses,
2. Real losses, and
3. Unaccounted-for losses.

2.1.1 Accounted-for losses

Water leaving the treatment plant and entering the distribution system (called water produced) is a valuable commodity for utilities and water authorities. Their goal is to maximize the monetary return on the water produced and distributed. Money is returned to utilities by billing consumers according to how much water they use based on meter readings.

Water meters are placed at service connections to monitor the amount of water that a billable customer uses. Meters may also be placed on service connections to non-paying customers who put the water to beneficial use. Non-billable customers include municipal users and the fire station. All water that is metered, whether it be sold or unsold, can be identified and quantified by the utility. Accounted-for losses occur at metered locations within the water distribution system. All metered water leaving the pipe network is termed accounted-for losses.

2.1.2 Real losses

The ultimate goal of a water utility should be to know the amount of water gathered at the source and be able to track its use throughout the water supply system. Unfortunately, a large percentage of water entering the water supply system is neither metered nor put to a beneficial use. Water that falls into this category is called “real losses.” Real losses cannot be tracked by a utility.

Figure 2.1 illustrates how water is distributed in a typical water supply system. Leakage is the main culprit with regard to real losses in a water supply system accounting for approximately 14% of the total supply. Table 2.1 identifies the types of real losses for each component of the water supply system.

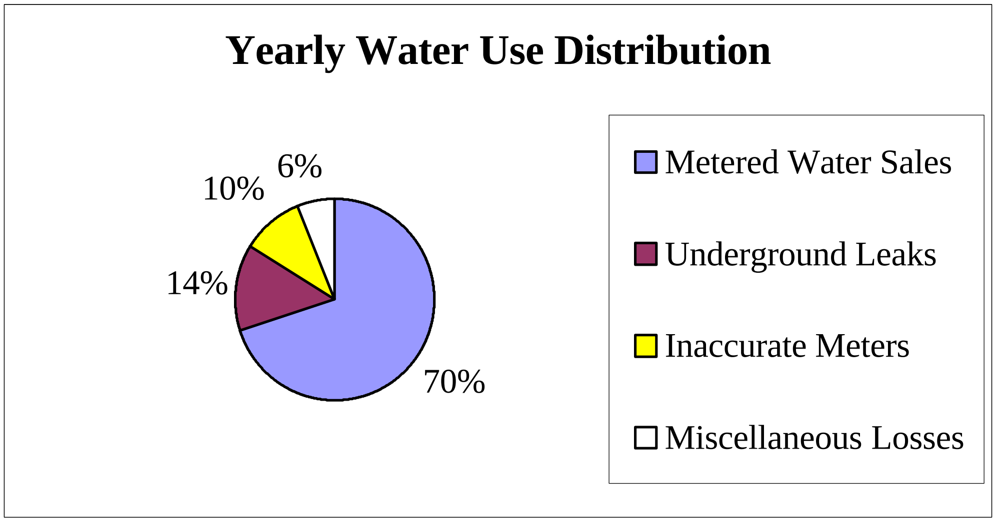

Figure 2.1: Where is water used? (Smith, 1986)

Table 2.1: Examples of real losses in a water supply system (AWWA, 1987)

Water Supply Component	Real Losses from each component
Source	Leaks in mains running from the source to the treatment plant
Treatment Plant	1. Excessive backwashing of the filters 2. Leaks in the reservoirs 3. Reservoir overflows
Transmission	Leaks in transmission lines
Storage	1. Leaks from storage tanks 2. Storage tank overflows
Distribution piping network	1. Leaks in service connections 2. Leaks or breaks in water mains
User	1. Excessive uses 2. Inappropriate uses 3. Plumbing leaks (toilets, faucets, etc.)

2.1.3 Unaccounted-for losses

The third type of loss in a water supply system is termed “unaccounted-for losses.” Unaccounted-for losses are losses from the system that are put to beneficial use. However, these beneficial uses are either not metered or are underregistered due to meter errors. The largest source of unaccounted-for losses is municipal use. Recently there has been a large push for municipalities to meter all uses even if they are not billable. However, the initial investment deters government agencies from implementing this suggestion. For examples of unaccounted-for losses please see Table 2.2.

Table 2.2: Examples of unaccounted-for losses

Unaccounted-for Losses	Types of losses
Unmetered Uses	1. Fire-fighting 2. Illegal users 3. Lawn watering in municipal parks 4. Main flushing 5. Municipal uses 6. New construction projects 7. Street cleaning 8. Unmetered public buildings 9. Unmetered service connections
Meter Underregistration	1. Accounting errors 2. Inaccurate commercial, industrial, or residential meters 3. Inaccurate production meters 4. Recording errors 5. Unmetered service connections

Many municipal uses are not metered, such as fire fighting and main flushing. Defective water meters may under-register actual water use. Therefore, much of the water entering the system becomes unaccounted-for losses.

2.2 Revenue vs. Non-revenue producing water

For utilities, the bottom line is whether the water they distribute is able to produce revenue. The scope of the problem faced by utilities is twofold: (Wallace, 1987)

1. “All of the water purchased or produced by a water utility or department does not reach its intended destination," (i.e. underground leaks or illegal connections);
2. “Some of the water that does arrive at its intended destination is never paid for,” (i.e. meter underregistration or municipal uses).

The following outline illustrates the various uses of water according to monetary factors.

Outline of Revenue vs. Non-Revenue Producing Water

I. Revenue Producing Water
  A. Metered water sales (70% of total water produced)
II. Non-revenue Producing Water
  A. Underground Leaks (14% of total water produced)
  B. Meter Underregistration due to inaccurate meters (10% of total water produced)
  C. Miscellaneous Losses (6% of total water produced)
    1. Non-billable Municipal Uses
    2. Illegal connections
    3. Unmetered users

Within a water distribution system there are two types of uses, revenue producing and non-revenue producing. Those uses of water that produce revenue are those that are metered and billed. Metered uses are those for which the specific quantity of water leaving the system at a given location can be measured and recorded by means of a water meter. Revenue producing water goes to supply residential, commercial, and industrial uses. Municipal uses are often unmetered and, even if they are metered, they do not produce revenue for the utility since it is for public uses unless the supply is from a private company. As shown in Figure 2.1, the bulk of water produced (70%) does bring a return on investment in the form of metered water sales. However, 30% of the water produced does not bring in any revenue for the utility. The largest non-revenue producing use is underground leakage in water mains, which accounts for approximately 14% of the total water produced. The second most troubling part of the water supply system is the problem of inaccurate meter readings, which account for 10% of the water produced.

Determining the amount of real losses and unaccounted-for losses in the system is the key to determining whether the pipe network is functioning adequately in terms of reliability and service. If the amount of real losses exceeds 15%, then remedial measures should be taken to restore the system to its full capacity by repairing the leakage in the system. The two main rehabilitation techniques involve repairing leaks in the pipe network and fixing meters. This thesis will focus on techniques to find leakage in the system since it is the main source of total water losses in a water distribution system.

2.3 Causes of pipe leaks and breaks

Now that we have identified pipe leakage as the main source of water losses in a water distribution system, the focus now shifts to how those leaks got there in the first place. Leaks in pipe networks result from a variety of situations. In order to be able to fix the leaks, one must have an understanding of the causes of leaks so that they can be prevented in the future and repaired accordingly in the present. Leaks or breaks in water mains may occur at various points along the line. Leaks can occur at joints, valves, service connections, or in the pipe itself.

Leaks at pipe joints "may occur when mechanical joints are improperly tightened or gaskets are displaced" (AWWA, 1987). Service connections are also a common source of leaks. Leaks at valves are not a significant source of leakage in a network. Leakage in water mains “can be caused by ruptures due to cracking, crushing, or longitudinal splitting” (AWWA, 1987). Pipes may also be subject to corrosion that cause perforations in the pipe wall. These corrosion holes often “occur in bunches” leading to significant water losses. (AWWA, 1987)

The main cause of leaks in water mains is external corrosion. “Corrosion is the root, if not the immediate, cause of most breaks in metal pipes. Metals tend to want to return to their ore state” (Walski, 1984). Pipes in wet, humid soils are more susceptible to external corrosion than pipes in drier soils. Low redox potential and low resistivity in wet soils lead to significant corrosion problems. Although corrosion is a natural process, stray electrical currents may contribute significantly to corrosion rates in metal pipes. As will be discussed later in Chapter 6, the problem of pitting, which causes corrosion holes, must be taken into account when modeling leakage in a pipe network.

Not only does corrosion actually cause holes to form in pipe walls; corrosion also debilitates the strength of the pipe to the point that other problems may cause significant damage. “It has been recognized … by water utility personnel that the majority of the breaks occur at location where the pipe wall has been weakened. Such weakening is the result of graphitic corrosion of cast iron and, although the actual failure may be due to stress, corrosion can be shown to be the real cause” (Fitzgerald, 1968). Often external corrosion is the underlying factor in most leak or break situations.

Although corrosion is the most critical factor in pipe leakage, several other factors work either with corrosion, or separately, to cause leaks or breaks in water mains. These factors include physical pipe characteristics, soil type and behavior, pressure in the system, installation procedures, etc. Table 2.3 lists several of the important factors that contribute to the formation of leaks.

Table 2.3: Factors affecting leaks/breaks in pipe networks
(Moyer, 1985) and (Walski, 1984)

Physical pipe characteristics	1. Age of the pipe 2. Corrosion (metal pipes) 3. Diameter 4. Pipe material
Soil Behavior	1. Expansive soils 2. Freezing soil around the pipe 3. Frost penetration 4. Landslides 5. Settlement (due to loading on the soil) 6. Subsidence (pumping water in the area)
Operation & Maintenance	1. High pressure zones 2. Improper tapping of service connections 3. Main flushing 4. New construction in the vicinity of a pipe altering the loads that a pipe must bear. 5. Water hammer
Installation of a new main	1. Manufacturer defects in pipe 2. Pipe damaged in shipment 3. Pipe laid improperly 4. Poor backfill and bedding 5. Poor joining methods
External Conditions	1. Contact with other structures 2. Construction equipment driving over mains 3. Excavation and external damage 4. Heavy traffic loads causing vibrations 5. Highly corrosive soil conditions 6. Stray electric current
Water Conditions	Aggressiveness Pressure (excessive pressures in the network) Temperature

The risk of pipe failure increases with the age of the pipe because corrosion is able to damage the pipe. Since corrosion is often the cause of leaks in pipes, either directly or indirectly, it is important to note that certain materials are more resistant to corrosion than others. Plastic pipes are much more resistant to corrosion than cast or ductile iron (Beaudet et al., 1985). Often, many different factors work in conjunction with corrosion to cause a water main leak or rupture. Table 2.4 lists several different types of pipe leaks and their causes.

Table 2.4: Types of leaks
(AWWA, 1987) and (Walski, 1984)

Type of leak	Cause	Repair Mechanism
Circumferential	1. Excessive loading 2. Poor bedding	Replace pipe section and restore bedding
Longitudinal split	1. Pipe fatigue 2. Water hammer	Sleeve
Puncture/Hole	1. Corrosion (Pitting) 2. Rocks or clay mounds in the bedding	Patch
Full break	1. Corrosion (Graphitization)	Replace pipe or section of pipe.
Joint	1. Corrosion 2. Displaced gaskets 3. Expansive soils 4. Improperly tightened joints 5. Overloading 6. Subsidence	Seal the joint
Service Connection	Excessive pressure	Replace connection

2.4 Consequences of leaks in pipe networks

Leaks or pipe breaks can pose tremendous problems to a municipal water supply system. Table 2.5 illustrates the significance of the water loss problem in representative water distribution systems throughout the world. There are several prominent reasons that will be discussed in this section concerning the reasons why local governments should be concerned with the loss of water from a pipe network.

Table 2.5: Leakage losses in various cities (Khadam et al., 1991)

City	Water Loss due to leakage (%)	Year
Manila, Philippines	51	1983
Bangkok, Thailand	49	1983
Boston, Massachusetts	43	1980
Bombay, India	33	1970-80
Hong Kong	30	1983
Springfield, Illinois	25	1984
Stockholm, Sweden	19	1986

Leaks or breaks in water mains pose significant problems to utilities. Pipe failures contribute to significant losses of potable water that in turn translates into lost revenue and lost resources. The main problems associated with leakage in a community fall into two categories: financial and environmental. Municipal utilities recognize these problems; however, time and money often preclude them from completely solving the problem of leakage in the system.

As shown in Table 2.5, water leakage accounts for a significant amount of the water in a system. This lost water costs municipalities millions of dollars to recover. In Ar-Riyadh, Saudi Arabia the cost of potable water lost due to leakage has been estimated to be about $50 million/year (Khadam et al., 1991). In addition to this tremendous economic loss, the city has also experienced increased groundwater levels due to water leaking into the underlying soil.

2.4.1 Financial consequences of leakage

In the most simple of terms, lost water is lost revenue. When water is lost due to leakage several consequences are incurred on utilities or agricultural users. Leakage in a pipe network causes the cost of pumping, treating, and distributing water to be unnecessarily higher than it should be. In the treatment process, energy and chemicals are used to treat the necessary amount of water. However, when there is leakage in the system, more of the resources are being used than are necessary at a significant cost. The extra water that must be produced not only increases costs, but there is no return on this extra investment since leakage flows do not produce revenue. The utility will shift the cost to the consumer by dividing the total cost among the paying customers because no utility can be run at a loss.

Often, water losses cause municipalities to prematurely develop alternative water sources. The cost to initiate and develop water projects is extremely expensive. By accounting for and returning lost flow to the system, the need to find new supplies of water can be delayed for several years. Often the cost to rehabilitate the system will outweigh the costs associated with water resource development. This is especially true in arid regions of the world such as the western United States and the Middle East. Water resources in these areas are precious and in short supply. Any effort to conserve water and eliminate waste in a water distribution system will provide great benefits in terms of delaying further development of water resources.

In areas where water is abundant, the problem may not be in developing new water sources; it may be in the need for expanding the existing water treatment plant. Premature expansion of the water treatment plant may incur unnecessary costs that could be avoided simply by increasing the capacity of the distribution system by leakage reduction.

Another important problem is that of damage to private property caused by leaks in a network or pipe breaks. Often the municipality will compensate private property owners for damages caused due to a pipe failure. However, if the city is not legally bound to cover the costs, then the private citizen must either seek relief from his insurance agency, or cover the costs personally. Legal implications carry a heavy financial burden in the case of damage to private property.

There are also social costs that occur due to a lack of, or no, water flowing to certain parts of the network. Business revenue losses occur due to interrupted service at restaurants, water-related industries, etc. Hospitals, schools, and other public buildings cannot operate when there is no water. Traffic interruption due to water damage and related intangible losses incur social, as well as, financial losses on a community.

As leakage water exits a pipe, the water erodes the bedding material around the pipe. Eventually, the water leaving the pipe will undermine the foundation of a roadway or sidewalk by creating an underground cavity. This will cause the roadbed or overlying property to sink. The cost to fix the problem once the roadway caves in is significant and time consuming.

Leakage harms the overall reliability of the water distribution system. Water pressure may decrease, or flow may be shut-off completely, in the zone surrounding the leak thereby inconveniencing customers who have come to expect adequate pressure and flow. When this event occurs, customers may seek other methods of obtaining their water supply. Also, water lost from the system due to leakage can neither be bought nor sold. This loss in revenue is significant and causes consumer water prices to be higher than they should be. The cost of leakage to private and public entities is substantial.

2.4.2 Environmental consequences of leakage

Obviously, the most significant consequence of leakage is financial; however, the environmental consequences must be addressed as well. As water leaks from distribution pipes, it has to go somewhere. The water may travel to one of three locations:

1. The groundwater;
2. The ground surface;
3. Existing storm, sanitary, or combined sewers.

First, as leakage flows seep down into the groundwater, the water table may increase to unwanted levels. As water flows into either sanitary or combined sewers, this water flows to the wastewater treatment plant. This extra flow increases the necessary capacity of the wastewater treatment plant thereby incurring greater treatment costs and possible expansion of the plant. The increase in flows in combined sewers may contribute to combined sewer overflows (CSOs) which can greatly harm receiving waters.

The main environmental problem with leaks in pipe networks is the fact that any opening in a pipe allows potential contaminants to enter the water supply through the leak opening. Unwanted sediment and chemicals may enter the pipe network at an opening in the pipe thereby causing a significant water quality problem and potentially hazardous situation to the community.

2.5 Benefits of leak reduction

There are several benefits of instituting a leak detection and repair program. The most obvious benefits are financial, but there are several other reasons why leakage should be controlled in a water distribution system. Chapter 3 will focus on how to perform a leak detection and repair program, as well as detail the current leak detection techniques.

2.5.1 Benefits of a leakage detection and repair program

As utilities begin to understand the magnitude of the problems facing them due to leakage, they will want to make a concerted effort to reduce leakage levels in their water distribution system. The following list contains the numerous benefits that come from locating and repairing leaks in pipe networks: (Moyer, 1985)

1. Avoided costs for treatment plant and distribution system expansion;
2. Increased knowledge of the water distribution system;
3. Reduced risk of water contamination;
4. Increased fire-fighting capability;
5. Less wear and tear on pumps, treatment plants, and distribution systems since there is less water going into the system;
6. Less property damage resulting in fewer insurance claims or lawsuits;
7. Enhanced public relations through more efficient service to customers;
8. Delayed expansion of treatment plant and distribution system;
9. More efficient leak repair, resulting in fewer "surprise" leaks or breaks;
10. Less overtime required of utility workers for fixing leaks at odd hours;

11. Reduced flow to wastewater treatment plants due to less inflow into sanitary or combined sewers;
12. Improved overall environmental quality;
13. Increased revenue to the utility.

By determining the location and magnitude of leaks in a water distribution system, municipalities can prioritize repairs to the network. By focusing on the locations with the greatest loss of water, time and money can be saved by fixing the largest leaks first. Restoring the full capacity of the water distribution system will not only save utilities large amounts of money, but will also improve the reliability and quality of service to their customers.

Chapter 3

Leak Detection Techniques

3.1 Water audit

Once utilities realize that they are losing precious water and revenue due to real losses and unaccounted-for losses, it is essential that a water audit take place to assess the overall state of the water distribution system. The method by which a water distribution system can be evaluated is through the implementation of what is called a "water audit." "Analyzing the results of a water audit may lead to the development of programs that increase the accuracy of water meters, locate visible and nonvisible leaks, and track previously unmetered uses" (Jensen, 1987). The analysis of a water audit allows a utility to assess the needs of a distribution system and implement a well-defined leak detection program.

"A water audit identifies how much water is lost and what that loss costs the utility. Records and system-control equipment, such as water meters, are thoroughly checked for accuracy. The overall goal of the audit is to help the utility select and implement programs to reduce the distribution system losses" (AWWA M36, 1990). Just as in any business, an audit helps to determine assets (the water supply) and liabilities (the amount of water used). A water audit is a way in which all of the water produced by the utility can be accounted for by determining what happens to the water once it actually enters the distribution system.

The American Water Works Association (AWWA) has produced an excellent manual that clearly details all aspects of the water audit process. No attempt will be made here to thoroughly reproduce this document, however, this chapter will give a good introduction to water audits and their role in fixing leakage problems in a water distribution system. If more detailed information is required please refer to AWWA Manual M36 (AWWA M36, 1990).

3.1.1 Conducting a water audit

The following section will discuss the steps involved in conducting a water audit. As explained above, no attempt will be made to describe each step in detail but rather a general outline will be given of the steps involved. A water audit is divided into three main categories:

1. Before the audit;
2. The audit;
3. After the audit.

3.1.1.1 Before the audit

The key to a successful water audit is the preparation taken prior to commencing the actual audit. The first step is to establish a worksheet whereby information gathered during the audit can be entered and reviewed. A sample worksheet for gathering all of the information is provided in AWWA Manual M36. The next step is to set the length of the study period. The AWWA manual recommends that a period of 12 months be studied in order to account for seasonal fluctuations in water use. The final step prior to beginning the audit is to determine what unit of measure will be used. It is preferable to measure water use as a volume rather than as a flowrate. The unit of ac-ft is a common unit of water volume measurement.

In summary, the steps, which must be completed prior to the audit, are:

1. Establish a worksheet;
2. Set a study period;

3. Choose an official unit of measure.

3.1.1.2 The audit

The actual water audit is divided into five specific tasks. The ultimate goal of these tasks is to balance the inflows and outflows and determine potential problem areas within the distribution system.

The first task in performing a water audit is to measure the total supply of water that enters the distribution system. Each of the inflow sources and locations must be identified. The inflow must be measured for the entire study period. Hopefully the inflow location contains an accurate meter. If there are any inaccurate meters or storage then the inflow values must be adjusted accordingly.

The second task involves measuring the authorized, metered use. Each metered use must be identified and accounted for. Once all metered uses are known then the values of the meter readings are taken for the study period. A sampling of meters should be taken to check for accuracy. Once again, if there are any inaccurate meters then the values will have to be adjusted to account for the inaccuracies.

The next task is to calculate the authorized unmetered uses in the system. Unmetered uses often take the form of municipal uses such as fire fighting, street cleaning, landscaping, and main flushing. Identify all of the unmetered authorized uses and then estimate the volume of water used by each of these categories.

The fourth task in performing a water audit involves calculating the water losses from the system. These losses account for all other unmetered uses in the system. These may be real or unaccounted-for losses. These losses may be illegal connections, discovered leaks, other leaks, or accounting errors. During the audit, many leaks and illegal connections will be found, but many will not. Therefore, it is necessary to make some kind of estimate of these unknown losses by using the water audit worksheet. It is important to note that the difference between the measured supply and measured metered uses must equal the calculated unmetered uses and losses.

The final task in the audit is to analyze the results by focusing on recoverable leakage. By following the itemized worksheet, this task involves figuring out the value of recoverable leakage, determining the cost of finding and fixing the leaks, and calculating the cost of conducting a leak detection and repair program.

In summary, the following steps are involved in performing a water audit:

1) Measure the Supply
    a) Identify and map sources
    b) Measure the water from each source
    c) Adjust figures for total supply
2) Measure Authorized Metered Use
    a) Identify metered uses
    b) Measure metered use
    c) Adjust figures for metered use
3) Calculate Authorized Unmetered Use
    a) Identify unmetered uses
    b) Estimate unmetered use
4) Calculate Water Losses
    a) Identify potential water losses

    b) Estimate losses by type

5) Analyze Audit Results
    a) Identify recoverable leakage
    b) Figure the value of recoverable leakage
    c) Figure the cost of recovering leakage
    d) Calculate the cost of leak detection.

3.1.1.3 After the audit

The work does not stop once the audit is completed. This is when the analysis turns into action as the decisions are made to implement a leak detection program. The following steps should be followed after the audit has been completed:

1. Analyze the value of losses and corrective measures;
2. Evaluate the potential corrective measures;
3. Update the audit;
4. Update the master plan.

The key to this process is to continue the maintenance and audit schedule once the audit has been completed. In order to have an effective and reliable water distribution system, it is essential to have an on-going maintenance program and have periodic water audits. It is also important to carry out the leak detection program. To be effective, a leak detection program must follow a water audit. The leak detection program will focus on the problem zones identified during the water audit.

3.2 Leak detection techniques

The following is an example of the severe losses that can occur due to a small invisible leak in a water distribution system. The importance of implementing a leak detection program to find leaks in pipe networks is exemplified in the following example.

> "City of Beverly Hills: An 8-in. diameter steel main with a pressure of 80 psi had a leak from a 1/2-in. hole in the top of the main. No evidence of leakage was visible. Flow from the leak was calculated to be 53 gpm. The water lost from this leak for the estimated two-year duration was 171 ac-ft. The value of the water lost was calculated to be $43,000" (AWWA M36, 1990).

Once it is known that there is leakage occurring and the problem areas have been identified, utilities must now focus on pinpointing the exact location of leaks. The problem with locating leaks is that most leaks are underground and invisible to the naked eye. Since it is a difficult process to find invisible leaks, utilities often turn to companies that specialize in leak detection. As such, commercial leak detection is a large industry in the United States. Once leaks have been pinpointed in the system, either by a commercial operation or by the utility itself, the utility then goes in and repairs the problems.

Currently, the most widely used technique for pinpointing leaks in a network is the acoustic sounding method. Other methods have been tested but are not widely accepted yet in the United States. Other leak detection methods include "gas injection and detection, aerial infrared photography, and monitoring pressures while a foam plug is forced through the main" (AWWA, 1987).

The following sections will discuss various methods by which leaks can be identified and pinpointed in a pipe network.

3.2.1 Flow measurements

3.2.1.1 Zone flow measurements

This leak detection method is not meant to pinpoint individual leaks, but rather its main purpose is to determine whether a certain portion of the water distribution system suffers from severe leakage problems. The zone flow measurement may be done during a water audit or can be administered separately as an initial step in a leak detection program. The following steps should be followed in performing a zone flow measurement program:

1. Select a certain section of the pipe network that needs to be investigated;
2. Close valves to allow water flow through a single line;
3. The usage rate is estimated for this line and then compared to the actual flowrate to determine whether the actual flow is higher than the estimated flow;
4. A high flowrate indicates that there is a possible leakage problem in this zone of the network.

3.2.1.2 Net night flow measurements

Measuring the net night flow (NNF) employs a similar technique as the zone measurement. The NNF is “the difference between the minimum night flow and an assumed water consumption by properties at night” (Al-Dhowalia and Shammas, 1991). The key is to isolate a single water main that supplies a specific geographic zone or a district. Measure the flowrate through the main for a one-hour period. Then use the following equation to determine the leakage loss in that zone of the network.

Leakage Loss = (minimum hourly night flow) – (measured industrial night flow demand) –
(estimated other customers assessed night flow demand) (3.1)

3.2.2 Physical leak detection methods

The easiest and quickest way of finding leaks in a pipe network is by physical/visual inspection of the system. The methods of physical leak detection are:

1. Identifying visible leaks
2. Customer complaints
3. Increased storm/sanitary/combined sewer flows.

3.2.2.1 Visible leaks

Leaks can be identified as water comes up to the ground surface. Leaks are often identified as water is found ponding on the surface. Even though water ponding on the surface could be produced by a natural spring, often if the visible water is near a water main one can be fairly certain that the leak is coming from the distribution system. Visible leaks usually allow for easy identification of the leak location, however, in some cases the location of ponding water may be some distance away from the actual leak location.

In order for leakage to emerge at the ground surface, a cavity is first eroded in the underlying soil. The water follows the path of least resistance to emerge at the surface. To locate the leak one would assume that the location of ponding water would be down gradient from the leak location. Once the leak emerges at the surface, the leak may have been occurring for quite some time thereby causing significant erosion of the underlying soil. A leak that surfaces can cause severe damage to roads, sidewalks, and even private property.

3.2.2.2 Customer complaints

Even though underground leaks cannot be identified by visual inspection, they may be identified by a noticeable decrease in pressure and flow by customers. Therefore, another important physical method of locating leaks is simply by taking complaint calls from customers who notice a significant drop in water flow or pressure at their household taps. However, if only one customer files a complaint then the leak is probably in the service connection connecting that person's house to the main line. If several people call in from the same area of the network then there is a high likelihood that there is a problem with the main line. Customers may also report locations where water has surfaced or locations of significantly higher vegetative growth overlying leaking water.

3.2.2.3 Increased storm/sanitary/combined sewer flows

Water leaking from a water distribution system may not necessarily emerge at the surface, but rather will find another outlet point. The leaking water may flow into the storm/sanitary sewer system, underground channels/cavities, or even old abandoned pipes. Therefore, another physical method of locating leaks is to monitor significant increases in the flowrate of clean water in both sanitary and storm sewer systems. Significant flow in storm sewers during dry weather periods could indicate leakage from the water distribution system; however, it could also indicate seepage from the groundwater into the storm sewer. The wastewater treatment plant may also notice increased flows due to leakage into sanitary sewers or combined sewers. Even though one may determine that leakage is occurring with this method, it may be difficult to determine where the leakage is actually emanating from.

3.2.3 Acoustic methods

Traditionally, leaks in pipe networks have been located through listening methods, also called sonic or acoustic methods. As water exits a pipe through a leak a unique sound is emitted depending on the size of the leak and the pipe material. “Electronic leak detectors pick up vibrations created by the water escaping from the pipe (or striking the material around the leak) and make them audible to the human ear" (Smith, 1994).

> "As high pressure water is forced out through the leak, it loses energy to the pipe wall and to the surrounding soil area. This energy creates sound waves in the audible range, which can be picked up by electronic transducers … The sound waves are then evaluated by an individual trained in leakage detection who can determine the exact location and relative size of the leak" (Heim, 1979).

Special listening devices, either geophones or hydrophones, are used to listen for the vibrations caused by water leaving the pipe. Geophones are placed on the ground surface over the pipe, whereas hydrophones are placed directly in contact with an element of the distribution system, such as a fire hydrant (Male et al., 1985). Both the hydrophones and geophones are attached to a headset so that the operator can listen for the leak noises.

By placing the geophone on the ground surface, noises are transmitted to the headset. The trained leak detection expert must then interpret the noises to determine where the actual leak is located. The following section will explain how this is to be done.

3.2.3.1 Noises emitted by a pipe leak

As water leaves the pipe through a leak there are three distinct noises that are produced which aid in pinpointing the leak location. These three noises are as follows: (Heim, 1979)

1. The loudest sound occurs at the point where the water exits through the orifice. This high-intensity sound is sent down along the wall of the pipe and has a frequency in the range of 500-800 Hz.
2. The water leaving the pipe shoots out away from the pipe and impacts the surrounding soil and bedding. The noise made by the water impacting the soil is much quieter than the first noise with a frequency range of 20-250 Hz.
3. After the water collides with the soil it forms a cavity, or hole, in the soil around the leak location. The escaping water impacts the soil and then circulates around in this cavity.

Even though the second and third noises are much quieter than the first one, they serve an important purpose in pinpointing leaks. These two distinct leak noises are not transmitted along the pipe wall, but rather they are confined to the area of the leak. The fact that all three leak noises occur at the leak location causes the noise generated at the leak to be the loudest and most noticeable, thereby aiding in pinpointing the leak location.

Initial detection of leaks is made by bringing the hydrophone in contact with a system component such as a valve, meter, or fire hydrant. This will confirm whether or not the given pipe has a leak in it. Once there is confirmation of a leak in the pipe, then the geophone equipment can be moved along the ground surface over the pipe to pinpoint the exact location of the leak. The leak is then located by listening for the point where the leak sound has the strongest intensity.

3.2.3.2 Factors affecting leak sound intensity

There are several factors that may affect the intensity of the noises emitted by a pipe leak. These inhibiting factors may either enhance or diminish the noise thereby changing the face of the leak detection survey. The following is a list of inhibiting leak noise factors:

1. Pipe material: Metallic pipe materials convey sound much further along the pipe wall than plastic piping. "Sonic leak detection is not as effective on plastic piping because the sounds do no transmit very far - often only a few feet" (Leauber, 1997).
2. Gaskets: Gaskets are used at pipe joint connections to seal the joint and prevent water from escaping. Gaskets are made out of a rubber material, which will dampen any sound waves being carried along the pipe wall. Therefore, each pipe section should be checked separately to pinpoint the exact location of a leak.
3. Minimum pressure: "A minimum pressure of 20-25 psi is required to create an acoustic energy level" (Smith, 1994).
4. Depth of cover: Obviously, the deeper the pipe the more difficult it will be to detect the sound of the leak due to the muffling effect of the overlying soil.
5. Soil type: Heavier, denser soils, such as clay dampen the sound from the leak making it more difficult to detect. Loose soils, such as sand, allow for easier detection of leak noises.
6. Surface cover: When the electronic leak detector is used on the surface over the pipe, the surface cover becomes an important issue. "Sod tends to insulate and muffle sounds, while asphalt and concrete are good resonators and give a uniform sounding surface" (Heim, 1979).
7. Leak configuration: The shape of the leak (circular, rectangular, etc.), and the size of the leak, will alter the frequency and intensity of the leakage sound. A larger orifice area will emit a much deeper sound than a small hole (Heim, 1979).

8. Water table conditions: “When a pipe is under water, the sound propagation will be reduced along the pipe wall” (Smith, 1986).

3.2.3.3 Dealing with background noises

One of the major inhibitors of accurate sonic leak detection is background noise. Heavy background noise includes; traffic, air conditioning units, and construction equipment. All of these noises will make it difficult to detect a leak because the leak may either be the same frequency as another sound, or else the leak noise will be drowned out by all the other background noises. "Electronic amplifiers are now used to enhance the sound quality by filtering out unwanted distracting sounds and increasing the acoustic intensity of noisy leaks" (Dumbleton, 1996). Even though advances in technology have made leak detection devices more reliable, it is still best to perform an acoustic leak detection survey at night when there is less traffic and fewer competing background noises.

It is also important when listening for leaks to mark the location of valves (including pressure-reducing valves), tees, and elbows before attempting to pinpoint a leak on a certain line. "Reducers, tees, and elbows in the line also create frequencies which often produce sounds like those from actual leaks" (Smith, 1994). These elements should be mapped so that the leak detection operator can distinguish between leakage noises and other similar sounds.

3.2.4 Leak correlators

Leak correlation technology works in conjunction with acoustic leak surveys to pinpoint the exact location of leaks. The leak correlation equipment is used in a two-phase process. First, the acoustic survey is performed to locate possible pipes in which leaks are located. Then the leak correlator is used on the targeted pipe to pinpoint the exact leak location.

Leak correlator technology works in the following way: (Jensen, 1987; AWWA, 1987)

1. Listening devices, or sensors, are attached to the pipe at two locations, on opposite sides of the suspected leak.
2. Soundwaves from the sensors are transmitted to the computer.
3. Operators must enter the following data into the computer: the distance between the sensors, the pipe material, the pipe diameter, and the current flowrate in the pipe.
4. Since the sensors are at different distances from the leak (if one exists), the sound will arrive at them at different times. The equipment delays the sound from one sensor until the two sounds match; this correlation indicates the existence of a leak.
5. The computer then computes the distances to the leak from each listening device.
6. The operator now knows the exact location of the leak; excavation and repairs can easily be performed now.

3.2.5 Tracer gas surveys

In certain situations it may be infeasible to perform an acoustic leak detection survey due to extreme background noise or when plastic pipes are involved. The use of tracer gases is one alternative to sonic leak detection, however it is extremely expensive. Therefore, the cost of a tracer gas survey may be prohibitive. Table 3.1 lists the available tracer gas survey methods.

Table 3.1: Gases used in tracer gas leak detection surveys (Heim, 1979)

Tracer Gas	Advantages	Disadvantages
Nitrous Oxide with infrared detection	1. It is water soluble 2. The pipe does not have to be dewatered prior to use 3. An infrared detector can spot nitrous oxide very easily	1. Nitrous oxide is heavier than air 2. Test holes must be checked at the depth of the pipe
10% helium – 90 % air	1. Helium is a very small, light molecule 2. It is detectable by thermal conductivity or comparative sonics	1. The pipe must be dewatered prior to use
Methane-nitrogen	Methane is lighter than air No test holes are required Flame ionization can detect methane in the low parts-per-million range	1. The pipe must be dewatered prior to use
Methane-argon	1. Methane can be detected by flame ionization 2. Argon serves as a confirming gas that can be detected by comparative sonics	1. Natural gas surveys may interfere

3.2.6 Other leak detection techniques

3.2.6.1 Infrared photography

This method of leak detection is useful when "ground temperature is lower than that of potable water. In this method, large leaks may put enough heat into the surrounding soil to raise its temperature" (Jensen, 1987). The infrared photography is then able to identify this thermal difference at the leak location. The drawback to this method is the need for highly "sophisticated equipment and trained personnel" (Jensen, 1987).

3.2.6.2 Miniprobe sensors

These sensors are put directly into a pipe with a suspected leak. The sensors are equipped with small radio transmitters. "A surface sensor monitors the movement of the probe, which flows to location of a major leak" (Jensen, 1987). This technology can be useful when large leaks are suspected in plastic pipes.

3.2.6.3 Pressure sensors

This method involves installing pressure and flow measuring devices at various nodes within the water distribution system. When a large leak or break occurs, the sensors will be able to show a large drop in pressure thereby indicating a possible leak. This method is similar to the customer complaint method, but it may provide a quicker turn around time.

3.3 Discussion

This chapter has presented the methodology for conducting a water audit. A water audit is an essential part of good water supply management and should be implemented by all water utilities. This chapter also presents several different methods for identifying and pinpointing leaks in pipe networks. The methods for pinpointing leaks are time-consuming and often very expensive due to a lack of prior knowledge about which pipes are potentially leaking.

This thesis presents a mathematical formulation in Chapter 5 by which leaking pipes can be identified and, once the pipes are identified, the leakage loss flowrate can be calculated. This formulation can enhance and augment the work done in a water audit by providing more detailed information to utilities on which pipes to focus on for rehabilitation and repair. Traditional leak detection techniques would still be necessary to pinpoint the exact location of pipe leaks; however, the pinpointing process could be done more efficiently with the prior knowledge obtained using the formulation presented in this thesis.

Chapter 4

Planning vs. Operational Problems

4.1 Steady-state water distribution system analysis

The steady-state analysis of water distribution systems falls into two basic categories:

1. Analysis of networks with a "specified" demand pattern. (Planning/Design Problem)
2. Analysis of networks with an "uncontrolled" demand pattern. (Operational Problem)

4.2 Pipe network planning and design

When designing a new water distribution system, engineers and city planners estimate the demand flowrate necessary at each outflow location. They base demand flowrate estimates on projected population growth, land use patterns, zoning ordinances, and fire flow requirements. Next, engineers lay out the looped pipe network to supply water to each of the demand locations. The next step in designing the pipe network entails sizing the storage tanks and distribution pipes to satisfy the necessary pressure and flowrate requirements in each section of the community. Finally, the network is analyzed to ensure that the network meets the minimum (and maximum) pressure requirements, as well as flowrate requirements. The design process is a trial and error procedure because the network must be redesigned if the minimum requirements are not met.

The following list summarizes the planning process involved in designing a water distribution system:

1. Determine demands and pressure requirements.
2. Layout the pipe network.
3. Determine pipe material to be used (roughness).

4. Determine storage tank volume and pipe diameters.
5. Analyze the network, checking for adequate flow and pressure at all demand points within the network.
6. Repeat steps if minimum requirements are not met.

4.2.1 Analysis of pipe networks with specified demand patterns

The analysis of a pipe network entails using one of several available methods to determine the flow in each pipe and the pressure head at each node. Before the invention and proliferation of computers, the analysis of pipe networks was an extremely tedious and time-consuming process because of the numerous hand calculations. The following hand calculation methods are:

- Linear Theory Method
- Newton-Raphson Method
- Hardy-Cross Method.

Each of these calculation methods assumes that the demands are known. The method then uses two basic sets of equations to analyze the network:

1. Continuity at each node must be satisfied,
2. The friction headloss around a loop is equal to zero using the energy equation.

In recent years, with the increase in computer usage, several software programs have been developed that analyze pipe network problems very efficiently. However, just as with the hand calculation methods, all of the demands must be known before running the analysis. Each of the computer programs is able to solve for two items: the flowrate in each pipe and the pressure distribution in the network. The most common computer software packages currently in use for pipe network analysis are:

- EPANET
- CyberNet
- KYPIPE.

Each of these programs performs steady-state analysis, but they may also do extended period simulations, mainly for chlorine propagation. The main problem with these software packages is that they are only as good as the demand estimates that are input into them. For the design problem, demands have to be estimated to get the initial pipe sizes.

4.3 Operational analysis of water distribution systems

Once the water distribution system is built and the network is in operation, the analysis of the system becomes much different. The analysis of the operational problem is much more difficult due to the numerous unknowns at any given time. In the operational problem, the demands are unknown at any given moment, as well as the flows and pressure heads. Since the demands are unspecified in the operational problem, all of the aforementioned analysis methods are inadequate to solve the problem. Also, the leakage cannot be readily modeled using these current methods.

When a water distribution system is in actual operation, the demands are constantly fluctuating based on use. The demands are “uncontrolled” in the sense that it is impossible for them to be specified. The outlet may have some kind of flow restriction device to limit the outflow to the desired value, but in reality there is no way to specify the demands under operational conditions. In the past few years, several researchers have begun to understand and attempt to solve the operational problem by using various equations for uncontrolled demands.

4.3.1 Literature Review: Analysis of pipe networks with uncontrolled demands

Germanopoulos (1985) was one of the first people to attempt to tackle the operational problem of uncontrolled demands in pipe networks. He included pressure-dependent demand and leakage terms into a pipe network formulation model. The model accounted for the fact that resulting outflow from either demand or leak locations is directly related to the pressure in the pipe. Germanopolous used the two standard pipe network equations: continuity and energy. He also included two other sets of equations, an exponential relationship between residual head and demand outflow, and then an equation for leakage losses in a given pipe based on the average service pressure. Wagner et al. (1988) and Chandapillai (1991) provided a similar formulation except that they suggested a parabolic relationship between the nodal head and the outflow. The drawback to these formulations is that they all impose an upper limit on the outflow at a given node based on the estimated nodal demand.

Reddy and Elango (1989) contribute a more realistic interpretation of pressure-dependent outflow by assuming that the outflows are completely uncontrolled. There is no upper limit imposed on the outflow and the resulting equation is similar to the orifice equation. This interpretation seems to capture the full extent of pressure-dependent outflow because it allows the outflow to be completely controlled by the pressure head. “Thus the actual flow can also be significantly higher than the demand, when the residual pressure at a node is more than the design head for which the outflow equals the demand” (Reddy and Elango, 1989). There is no need to estimate peak demands. The outflow is based solely upon hydraulic considerations, namely the pressure at the node and the outlet characteristics, such as orifice area and headloss coefficient.

The formulation presented by Reddy and Elango (1989) for uncontrolled pressure-dependent outflow will be adopted in this thesis to model demands in the operational problem. The orifice equation thus becomes the third essential equation, along with continuity and energy, in analyzing pipe networks under operational conditions. Chapter 5 will discuss ways in which pressure-dependent demand and leakage terms can be incorporated into a mathematical formulation.

Chapter 5

Problem Formulation

5.1 Overview of water distribution systems

The purpose of a water supply system is to deliver an appropriate quantity of potable water, at adequate pressure, to all demand locations (including fire demand) in the system. As discussed in Chapter 1, a typical water supply system is made up of the following elements:

(i) *The supply* may be either a surface water source such as a lake or river, or a groundwater source containing a system of wells;

(ii) *The treatment plant* removes impurities, including bacteria, from the raw water pumped in from the source with the aid of various physical and chemical processes. To further protect it from the elements in the distribution system pipes, the water is disinfected with chemicals such as chlorine, and as an additional measure to aid in consumers' health other chemicals such as fluoride may be added;

(iii) *The water distribution system*, made up of pumping stations, storage tanks, and pressure conduits, distributes the treated water to the demand locations. The focus of this chapter is this third element, namely the distribution system.

The water supply system serving Blacksburg, Virginia will be used to illustrate the function of each of these components. The focus is then shifted entirely to the distribution system piping. The emphasis is laid on the leakage in the pipes. The various physical and acoustic methods for leak detection were discussed previously in Chapter 3. In this chapter a mathematical formulation is presented that yields leak flows through each pipe. A "leak flow" is defined as the contribution of flow in each pipe towards downstream nodal leakage. It is emphasized that the leak flow through a pipe does not imply that that amount is actually leaving the pipe as leakage. The actual leakage from a given pipe is found either by directly solving for pipe leakage based on corrosion hole information, or by using a second step that apportions leakage to pipes that converge on a leaking node.

5.1.1 Pumping stations

A pumping station is necessary at any point in the system where an increase in pressure head is needed to provide adequate service. Pumps are often necessary at the following locations: higher elevations in the system, adjacent to water tanks, or at supply points to the system.

In the Blacksburg system, the raw water source is the New River. Raw water is pumped from the New River to the Blacksburg-Christiansburg-VPI Water Authority treatment plant where the water is treated. After treatment, the potable water is fed to the Blacksburg water distribution system as shown in Figure 5.1. The supply point to the Blacksburg pipe network is a pumping station located on Route 460 (behind the Dairy Queen). The pumps at this pumping station are controlled at the water treatment plant. A schematic of this process by which water goes from the New River to the Blacksburg booster pumping station is shown in Figure 5.1.

As opposed to the planning stage demand configuration in which specified demands are assigned to nodal points, in the operational stage the demands are not known. Then the question arises, how do we satisfy an unknown demand which must be provided for. Clearly it is very difficult to keep the pumps running at different flow rates in cohort with the varying demand. The other strategy is to withdraw from storage tanks. In the Blacksburg case that is exactly what is being done. The role of the pumps at the treatment plant and at the Blacksburg pumping station is to maintain the tanks near full condition. Only a fluctuation of 5-ft. from the full tank is allowed. (The water level in the tank is always kept within the range of 25’-30’). The treatment plant engineers monitor the tank levels at two-hour intervals. If the tank water level drops down close to 25-ft., then the pumps are turned on.

Figure 5.1 shows a schematic of the water supply system serving the Blacksburg and Christiansburg area. From the treatment plant, the water is pumped into the High Head Tank, water flows by gravity from this tank to the Highway 114 Booster station where the water is pumped through two 16" pipes to the two Merrimac tanks. The Merrimac tanks feed water to the Blacksburg booster station on Route 460. This pumping station has three single speed pumps that feed water into the Blacksburg pipe network. Once the water enters the network, it flows through the network to satisfy the demands. Excess water flows into the main Blacksburg storage tank (See Figure 5.1). This main storage tank in Blacksburg is monitored at the treatment plant to make sure that the water level stays within the set limits. As mentioned earlier, the pumps at the Blacksburg booster station are controlled in order to keep the water level in the main storage tank between 25'-30'.

There are other storage tanks and pumping stations within the Blacksburg water distribution system, but they are not monitored at the treatment plant. A pump station is located at Laurel Ridge to pump water into a 30,000-gallon storage tank that supports the Laurel Ridge water system customers. The Allegheny pump station at Harding Avenue boosts pressure in the Allegheny Heights portion of the system, and the Highland Park pump station fills the Highland Park tanks and the North Main Street tank.

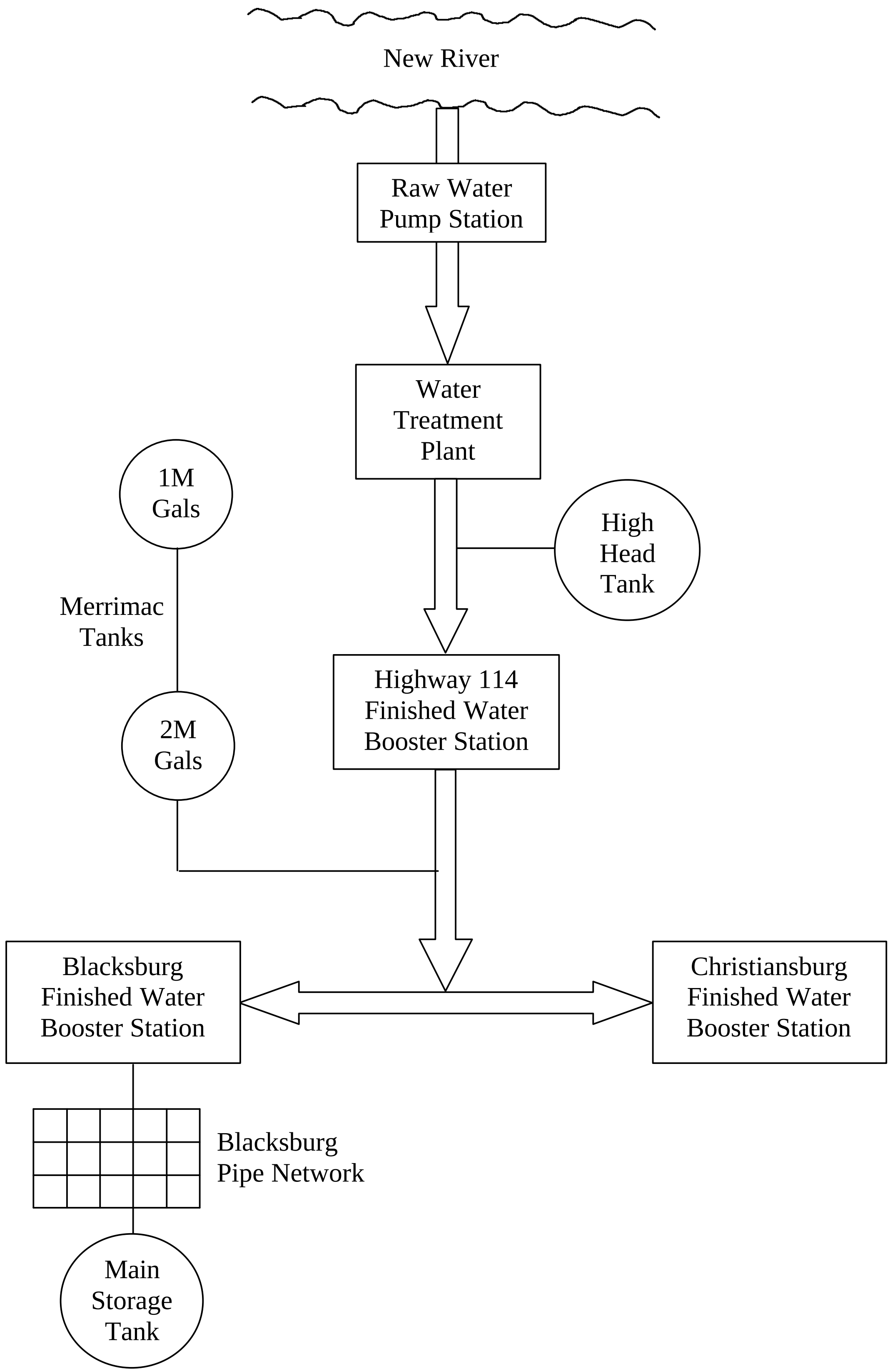

Figure 5.1: Schematic of the Blacksburg-Christiansburg-VPI water supply system.

5.1.2 Distribution storage

There are several roles that storage tanks play in a water distribution system. The most important role is obviously to provide adequate water supply to cope with the varying demand and to provide some emergency capacity during pump outages. Storage tanks are also helpful in optimizing pumping and equalizing pressure in the system.

In the Blacksburg network, the main storage tank serves an important role as it tells the engineering staff at the treatment plant how much water needs to be fed into the system. It shifts the burden of operating pumps to cope with an erratic demand, to a simple operation of keeping the tank within at near full condition. The head established by water in the tank is used to feed the network. Blacksburg also has four other secondary storage tanks to serve the storage requirements for the system.

5.1.3 Distribution piping

The backbone of any water distribution system is the pipe network itself. The pipe network, which is normally designed in a loop configuration to provide redundancy, conveys water to all points in the system. The pipe network is divided into individual sections having a constant diameter. These sections of constant diameter are known as pipes or links. A given pipe may contain bends or valves, but does not change diameter. In this formulation, the intersection of two or more pipes is called a junction or "node." A node may also be a storage tank or supply location. Pipes are made or various types of materials such as cast iron, ductile iron, welded steel, wood, concrete, and plastic. The headloss through a given pipe section is dependent on the material roughness, in addition to the pipe geometry and flow properties.

Distribution pipes are normally laid underneath curbs or sidewalks in a city to serve adjacent homes, businesses and fire demands. The main line runs under the street with smaller pipes, or “laterals”, coming off the main pipe to supply the demands. Usually, before a new street is graded and paved, the pipe will be laid underneath the proposed road location. The pipe is laid by digging a trench in the ground about 5-6 feet deep. The pipe is laid in 15-20 foot sections on bedding, which can either be the native soil or some other bedding material. Pipe sections are carefully connected using gaskets. The pipe is then covered up with soil again subjecting it to the underground soil environment.

Even though leaks can occur at valves, hydrants, etc., the majority of water losses in a water distribution system occur within the pipe network itself. When leaks or breaks occur in a given pipe, significant losses of water can occur, thereby compromising the reliability of the system.

To formulate the problem of locating and quantifying leaks in pipe networks, a mathematical program is devised containing linear and nonlinear equations. Traditionally, when solving pipe network problems, two types of equations are written, namely continuity and energy (headloss). However, in this formulation other sets of equations will be added to model the pressure-dependent demand and leakage terms.

5.2 Continuity equation

Traditional analysis of pipe network problems has involved the determination of flows and heads in the network based on known peak demands. However, in actual operation of the system, the demands are unknown at any given time. A typical concern not addressed in specifying certain flow and minimum head is whether such a head can sustain that specified demand. It is common experience that when the pressure is low there is not much flow at the faucet. However, hydraulics books traditionally have chosen to ignore the link between the specified flow and the head at the corresponding node. In this thesis, the continuity equation is written by taking the pressure dependence of flow into consideration; also, leakage along a pipeline is accommodated.

5.2.1 Continuity equation for demand

In a typical pipe network, a distributed set of flows to various households is aggregated at a single point, called a node. To carry the flow to the furthest destination point at a required minimum pressure, typically 20-psi, we must also impose a pressure requirement at the node. Therefore, one may visualize each node to represent a subdivision, representing the aggregated flow demands of the entire subdivision and the pressure head needed to support the distribution of flows within the subdivision. In this formulation we will assume that all demands are concentrated at the nodes. The demands can be modeled as unknowns based on the pressure and orifice area using the orifice equation. However, demands may also be estimated when there is insufficient information to implement the orifice equation.

Assuming that the flow is steady and incompressible, the continuity equation must be satisfied at each node. The flow that enters the system ultimately leaves the system at known pressure-dependent demand locations. The continuity equation is written in the following form:

$$\Sigma(\text{Flows entering node } i) - \Sigma(\text{Flows leaving node } i) = \text{Demand } (+) \text{ and Supply } (-) \qquad (5.1)$$

or

$$\sum Q_{ji} - \sum Q_{ij} = D_i - S_i \qquad \text{(for each node } i\text{)} \qquad (5.2)$$

Where, Q_{ij} = Pipe flowrate from node i (head node) to node j (tail node).

D_i = Flow leaving node i as either a pressure-dependent, or estimated, demand.

S_i = Flowrate entering the node i as either a fixed, or varied, supply.

5.2.2 Continuity equation for the combined demand and leakage

Leakage occurs along the length of a pipe at locations where the pipe wall has been ruptured, either due to corrosion holes or by other means. To handle leakage flow leaving the pipe network, this formulation includes a second set of continuity equations. This second set of continuity equations handles leakage that is lumped at the nodes where leaking pipes converge.

Previous formulations, by Pudar and Liggett (1992) and others, have made the assumption that leaks are lumped at the nodes, with no attempt being made to allocate leakage among the incoming pipes. (An incoming pipe is defined as a pipe in which the flow is towards, rather than away from, the leaking node). The present formulation improves upon the previous formulations in the following two ways. First, guidance is provided on how to initially lump leakage at the nodes in the continuity equation. Second, an apportionment scheme is provided to assign leakage losses to specific pipes, either directly by applying a modified orifice equation to corroded pipes, or indirectly as a second step that involves apportioning nodal leakage among converging pipes. This formulation presents three different methods for writing the second set of continuity equations.

5.2.2.1 Method #1: Nodal leakage, with subsequent allocation among pipes

Recent formulations lump all of the leakage at the node, but they do not allocate the leakage among the pipes. In Method #1, an explicit nodal leakage magnitude, L_i, is first computed based on the orifice equation using a distinct nodal head. Next, this computed magnitude of nodal leakage, L_i, is allocated among pipes using a weighted average method to determine the actual magnitude of leakage, L_{ji}, being lost from each incoming pipe. This method assumes that the only pipes in a network that are leaking are those that converge on a leaking node. If there is no leakage at a given node then none of the incoming pipes will receive a leakage allocation.

Therefore, Method #1 is a two-step process. First, leakage is lumped at the nodes where leaking pipes converge using the orifice equation. Secondly, after solving the formulation, the resultant nodal leakage is allocated among incoming pipes using an allocation scheme based on the leak flow in each of those pipes. Chapter 7 gives a clear example of how this is to be done.

This method involves giving an actual orifice area at a node for leakage that is being lost from converging pipes. This method has limitations however because the orifice area for a leak is normally not readily known. Chapter 8 will discuss a technique to determine the orifice area based on the analysis of corrosion hole formation in the pipe wall.

The continuity equation for the combined demand and leakage in this Method #1 is given by the following equation:

$$\sum Qp_{ji} - \sum Qp_{ij} = L_i + D_i - Sp_i \qquad \text{(for each node } i\text{)} \qquad (5.3)$$

Where, Qp_{ji} = Total pipe flow entering node i for the pipe connecting nodes j and i.

Qp_{ij} = Total pipe flow leaving node i.

L_i = Leakage flow at node i for incoming pipes based on the orifice equation.

D_i = Flow leaving node i as either a pressure-dependent, or estimated, demand.

Sp_i = New supply flow necessary to satisfy leakage in the system. (Note: If the supply is fixed then this term will be the same as S_i.)

The variable Qp_{ij} denotes the sum of demand and leakage flowrates as opposed to Q_{ij} which solely caters to the demand, D_i, at node *i*. The difference between Qp_{ij} and Q_{ij} gives the flow carried by each pipe towards downstream nodal leakage, what was defined previously as the "leak flow." The leak flow for each pipe is then used to determine the actual leakage losses, L_{ji}, from each pipe using a second step. This procedure for allocating nodal leakage among pipes using a weighted average scheme is embodied in the following equation:

$$L_{ji} = \frac{L_i \left| Qp_{ji} - Q_{ji} \right|}{\sum \left| Qp_{ji} - Q_{ji} \right|} \tag{5.4}$$

Where, L_{ji} = Leakage loss flowrate from the pipe that connects node *j* and node *i*.

L_i = Leakage flow at node *i*.

Qp_{ji} = Total pipe flow from node *j* to node *i*.

Q_{ji} = Pipe flowrate from node *j* to node *i*.

5.2.2.2 Method #2: Direct allocation of leakage among pipes

Leakage can also be allocated among pipes directly using a one-step process. As will be discussed in Chapter 6, corrosion holes occur along the length of a pipe. These holes allow water to escape at various points throughout the length of the pipe. The amount of leakage occurring along a pipe length can be determined using the average of the head node and tail node pressures. The orifice equation can be modified to include the average nodal pressure rather than just the pressure at one specific node. Also, the equivalent orifice area of the corrosion holes can be determined using equations. By using the average nodal pressure, and equivalent corrosion hole orifice area, the orifice equation can be solved directly for leakage losses, L_{ij}, in each pipe. Method #2 will be effectively illustrated in Chapter 8.

Since this formulation includes equations for pipe leakage, the second step involving allocation of nodal leakage is not necessary. However, the pipe leakage does need to be apportioned among nodes in the continuity equation. This is done by dividing half of the pipe leakage, to both of the nodes, on either end of the leaking pipe.

Method #2 is the most direct way of solving for leakage losses in pipes. The following continuity equation illustrates that half of the leakage leaving a corroded pipe is allocated to the upstream node and the other half of the leakage amount is allocated to the downstream node. To lump the entire pipe leakage to the downstream node is not feasible since the flow direction is unknown. To accommodate flow reversal, the leakage is divided equally among both ends of the pipe as illustrated in the following equation:

$$\sum Qp_{ji} - \sum Qp_{ij} = \sum\left(\frac{L_{ij}}{2}\right) + \sum\left(\frac{L_{ji}}{2}\right) + D_i - Sp_i \qquad \text{(for each node } i\text{)} \qquad (5.5)$$

Where, Qp_{ij} = Total pipe flow (including flow to satisfy leakage) from node i to node j.

L_{ij} = Leakage loss flowrate from the pipe that connects node i and node j.

D_i = Flow leaving node i as either a pressure-dependent, or estimated, demand.

Sp_i = New supply flowrate necessary to satisfy leakage in the system.

5.2.2.3 Method #3: Leakage expressed as a percentage, with subsequent allocation among pipes

Recent formulations expect the user to specify the magnitude of leakage at nodes, but no guidance is given to ascertain the leakage. As discussed in Chapter 3, a water audit is a critical step in assessing the welfare of a water distribution system. In conducting a water audit, the aggregate leakage is assessed by noting the difference between the amount of water leaving the water treatment plant and the sum of the metered consumption and an estimated consumption for unmetered services. After conducting a water audit, a utility should have a good idea of the percentage of water that is being lost due to leakage in the system. For some cities the percent leakage can be as high as 50% as noted in Table 2.5. For Blacksburg leakage losses amount to approximately 10-15% of the total supply, based on discussion with the town engineer (Formica, 1998).

Assuming the consumption is fully satisfied, then the treatment plant must transmit an amount equal to the full consumption and the percentage leakage. Therefore, at each node, [(1+Leakage expressed as a fraction) * Demand], must be satisfied to accommodate the full consumption. While the 0.15 (15%) leakage is a system-wide averaged leakage amount, it may be spatially distributed among nodes, either uniformly or in a skewed form. The orifice equation is not used to model leakage in this method; rather leakage is accounted for merely by increasing nodal demands by a certain percentage.

As determined by the water audit, certain zones within the distribution system may be leaking at a higher rate due to corrosive soils, high-pressure zones, etc. These zones could be adjusted by increasing or decreasing the percent leakage accordingly. Some nodes may not be assigned any leakage if they are not susceptible to corrosion. Therefore, to account for these increased losses due to leakage the continuity equation involving percent leakage may be written as the following:

$$\sum Qp_{ji} - \sum Qp_{ij} = \left[1 + \left(\frac{\%Leakage}{100}\right)\right] * D_i - Sp_i \qquad \text{(for each node } i\text{)} \qquad (5.6)$$

Where, Qp_{ij} = Total pipe flow (including flow to satisfy leakage) between nodes *i* and *j*.

%Leakage = The percentage of flow being lost due to leakage in that section of the pipe network.

D_i = Flow leaving node *i* as either a pressure-dependent, or estimated, demand.

Sp_i = New supply flowrate necessary to satisfy leakage in the system.

The second step involves allocating the leakage among pipes. After the formulation is solved, a weighted average is used to allocate leakage among incoming pipes at a leaking node. The leakage losses in each pipe, L_{ij}, are computed using the difference between Qp_{ij} and Q_{ij} as shown in Equation (5.4). Method #3 will be illustrated in Chapter 8.

5.3 Pressure-dependent outflow

As mentioned earlier in Chapter 4, under operational conditions the demands are not known. The amount of flow withdrawn at a faucet depends upon the energy head available, which in turn is dictated by the street level energy head. This type of demand is called a pressure-dependent demand. Leakage may also be modeled as a pressure-dependent outflow as mentioned in Methods #1 and 2 above.

5.3.1 Pressure-dependent nodal demand

To properly simulate the operational problem, pressure-dependent nodal demands are modeled using the orifice equation. This equation is written as follows:

$$D_i = c_d A_{Di} \sqrt{2gH_i} \tag{5.7}$$

Where, D_i = Flow leaving node i as a pressure-dependent demand.

c_d = Coefficient of discharge for flow through an orifice.

A_{Di} = Area of the orifice opening for demand at node i.

g = Acceleration due to gravity.

H_i = Pressure head at node i.

5.3.2 Pressure-dependent leakage concentrated at the node

When leakage losses are lumped at a node and then allocated among converging pipes, as discussed in Section 5.2.2.1, the following equation may be used to model the lumped nodal leakage:

$$L_i = c_d A_{Li} \sqrt{2gH_i} \tag{5.8}$$

Where, L_i = Leakage flowrate at node i.

c_d = Coefficient of discharge for flow through an orifice.

A_{Li} = Area of the orifice opening for leakage at node i.

g = Acceleration due to gravity.

H_i = Pressure head at node i.

The orifice is assumed to be sharp-edged for an orifice leak opening. In this case a value of 0.61 is used for the discharge coefficient, c_d.

5.3.3 Pressure-dependent leakage along the pipe length

Under operational conditions, leakage actually occurs along the length of a pipe and is not withdrawn at nodes as aggregated demands. One may approximate the leakage flow leaving a given pipe, L_{ij}, as pressure-dependent flow with the average of the adjacent nodes' pressures utilized. This method for directly solving for leakage was discussed in Section 5.2.2.2. The following equation is used to solve directly for the actual leakage losses from a given pipe:

$$L_{ij} = c_d A_{Lij} \sqrt{2g\left(\frac{H_i + H_j}{2}\right)} \tag{5.9}$$

Where, L_{ij} = Leakage flowrate occurring in the pipe that connects node i and node j.

c_d = Coefficient of discharge for flow through an orifice.

A_{Lij} = Area of the orifice opening for leakage along pipe ij.

g = Acceleration due to gravity.

H_i = Pressure head at node i.

5.4 Energy equation

In the Blacksburg water distribution system, the pipe network is composed of pipes made from a variety of materials, the most common being cast iron. Each pipe material has an associated pipe roughness coefficient that is used to determine the friction factor for that pipe.

The friction factor also includes the flow characteristics through the Reynold's number. The energy equation is used to determine the headloss along the length of a pipe.

5.4.1 Darcy-Weisbach equation

The Darcy-Weisbach equation can be utilized to relate the flowrate to the headloss in a pipe:

$$h_f = f_{ij} \frac{l_{ij}}{d_{ij}} \frac{V_{ij}^2}{2g} \tag{5.10}$$

Where, h_f = Headloss in the pipe due to friction.

f_{ij} = Friction factor based on pipe material and Reynold's number.

l_{ij} = Length of pipe that connects nodes i and j.

d_{ij} = Diameter of pipe that connects nodes i and j.

V_{ij} = Velocity in pipe connecting nodes i and j.

However, Equation (5.10) is difficult to work with in analyzing pipe networks. The Darcy-Weisbach equation can be rewritten in the following form as explained below:

$$(H_i + E_i) - (H_j + E_j) = r_{ij} Qp_{ij} \left| Qp_{ij} \right| \tag{5.11}$$

Where, H_i = Pressure head at node i.

E_i = Elevation at node i.

r_{ij} = Resistance coefficient for the pipe connecting node i and node j.

Qp_{ij} = Total pipe flow (including flow to satisfy leakage) between nodes i and j.

In writing the equations to analyze a pipe network, it is necessary to assume a flow direction initially. However, after solving the problem it may turn out that a wrong direction was assumed. The absolute value sign in Equation (5.11) is necessary to allow for flow-reversal in the solution. A negative sign in the solution for Qp_{ij} indicates that the wrong direction was initially assumed and that the flow is actually in the opposite direction.

The resistance coefficient in the above equation can be written in the following form:

$$r_{ij} = \frac{8 f_{ij} l_{ij}}{\pi^2 g d_{ij}^5} \tag{5.12}$$

Where, f_{ij} = Darcy-Weisbach friction factor for the pipe that connects nodes i and j.

l_{ij} = Length of pipe that connects nodes i and j.

g = Acceleration due to gravity

d_{ij} = Diameter of pipe that connects nodes i and j.

5.4.2 Hazen-Williams equation

In addition to the Darcy-Weisbach equation, the Hazen-Williams equation can also be used to relate headloss to flow. The following is the general Hazen-Williams equation:

$$h_f = r_{ij} Qp_{ij}^{1.852} \tag{5.13}$$

or

$$(H_i + E_i) - (H_j + E_j) = r_{ij} Qp_{ij} \left| Qp_{ij} \right|^{0.852} \tag{5.14}$$

Where, H_i = Pressure head at node i.

E_i = Elevation at node i.

r_{ij} = Resistance coefficient for the pipe connecting nodes *i* and *j*.

Qp_{ij} = Total pipe flow (including flow to satisfy leakage) between nodes *i* and *j*.

The resistance coefficient in Equation 5.13 is written as follows:

English units:

$$r_{ij} = \frac{4.727(l_{ij})}{C_{ij}^{1.852} d_{ij}^{4.8704}} \tag{5.15}$$

SI units:

$$r_{ij} = \frac{10.675(l_{ij})}{C_{ij}^{1.852} d_{ij}^{4.8704}} \tag{5.16}$$

Where, r_{ij} = Resistance coefficient for the pipe that connects nodes i and j.

l_{ij} = Length of pipe that connects nodes *i* and *j*.

C_{ij} = Hazen-Williams coefficient for the pipe that connects nodes *i* and *j*.

d_{ij} = Diameter of pipe that connects nodes *i* and *j*.

5.5 Flow direction constraints

One important issue that should be dealt with is the inability of the continuity equations to ensure that both Q_{ij} and Qp_{ij} have the same flow direction. The following two constraints impose this requirement. The first one simply makes sure that magnitude-wise Qp_{ij} is greater than or equal to Q_{ij}. The second one demands that both must follow the same direction. The magnitude constraint is given by,

$$\left|Qp_{ij}\right| \geq \left|Q_{ij}\right| \tag{5.17}$$

and the flow direction is maintained by,

$$Qp_{ij}Q_{ij} - \left|Qp_{ij}\right|\left|Q_{ij}\right| = 0 \tag{5.18}$$

5.6 Solvability of the demand-leakage problem

Let there be P pipes and J nodes. Table 5.1 lists the unknowns and the available equations. There are 3(P+J) unknowns and 2P+4J equations. Because P>J, we have more unknowns than independent equations.

Table 5.1: Unknowns and available equations

Variable	Description	No. of Variables	Variable and Type of Equation	No. of Equations
Qp_{ij}	Total flow	P	Qp_{ij}, Continuity	J
Q_{ij}	Demand flow	P	Q_{ij}, Continuity	J
A_{Lij}	Orifice area for leakage along pipe	P	L_{ij}, Pressure-dependent leakage along pipe	P
A_{Li}	Orifice area for leakage at a node	J	L_i, Pressure-dependent leakage at a node	J
A_{Di}	Orifice area for demand at a node	J	D_i, Pressure-dependent demand at a node	J
H_i	Pressure head	J	H_i - H_j, Headloss	P

If some values for *A* (orifice area) are specified, then the flows (leakage and/or demand) dictated by the heads, H_i must be compatible with continuity; otherwise, the solution will become infeasible. But, note that when the *A* values are specified no new equations are added; these merely become some compatibility restrictions with the continuity and the energy loss equations. The solution obtained with the aid of Qp_{ij} and Q_{ij} continuity and the energy constraints should correspond to the results of the pressure-dependent equation. Under this circumstance we have 2P+J unknowns and P+2J equations consisting of more unknowns than equations.

The resulting system of equations is solved with an objective function maximizing the leakages in pipes. It is assumed that flow will choose the path of least resistance to the nearest corrosion point rather than to the downstream node. A test problem from the literature is solved in Chapter 7 to illustrate the applicability of the formulation.

5.7 Formulation summary

Each of the three sets of equations: continuity, energy, and pressure-dependent flow can be applied to a water distribution system. In summary, the steps involved in the formulation are:

1. Write the demand continuity equation at each node, assuming no losses from the system.
2. Write the combined demand-leakage continuity equation at each node in the appropriate form (Use Method # 1, 2, or 3).
3. Write the pressure-dependent flow (orifice) equation for each nodal demand or leak, if necessary.
4. Write the energy (headloss) equation for each pipe connecting two nodes, using the combined demand-leakage flows.
5. Write the constraints to preserve the same flow direction for the demand flow and the combined demand-leakage flow for each pipe.

6. Obtain leakage and flows by maximizing the leakage in the system.
7. Allocate leakage among pipes using a weighted average scheme, if necessary.

5.8 Discussion

It is observed that the formulation given in this chapter yields demand and leakage flows at the nodes and through the pipes. However, with regard to leakage flows, whether a leak occurs within a particular pipe or the pipe is a non-leaking contributor to leakage in downstream pipes is not identified. This question is answered in Chapter 6; namely, how one can determine whether or not a pipe is leaking.

Chapter 6

Corrosion of Pipes in Water Distribution Systems

6.1 Definition of corrosion

The problem of leakage in water distribution systems is prevalent in cities and towns throughout the world. Lost revenue and inefficient distribution of potable water plagues utilities and communities. The main cause of all these headaches, especially in locations served by metallic pipes, stems from one common source: *Corrosion*. "Corrosion is the root, if not the immediate, cause of most breaks in metal pipes" (Walski, 1984). Corrosion weakens unprotected pipes, leaving them vulnerable to an array of problems that could potentially damage or completely break the pipes.

Corrosion is "an electrochemical reaction that deteriorates a metal or an alloy" (AWWA M27, 1987). We know from the 2nd Law of Thermodynamics that everything is attempting to return to its chaotic or natural state. Therefore, corrosion is simply a natural process by which refined metals desire to return to their natural state.

In the environment metals can be found in their stable, oxidized (corroded) form. To produce a useful product, these stable metals must be refined to render the metal into its elemental or unoxidized form. However, after metals are refined they desire to return to their natural, oxidized state. The process by which refined metals return to their natural form is called corrosion. It is inevitable that corrosion will occur, however, the rate at which it occurs is dependent on several different environmental factors in which the metal is subjected such as water chemistry, soil type, electrical conditions, and air quality.

6.1.1 Graphitization

Graphitization is one of the actual culprits in the corrosion problem. Graphitization is the process by which “corrosion pulls iron out of pipes, leaving carbon behind. The pipe may look virtually the same, but it will not be nearly as strong” (Walski, 1984). Graphitization robs the pipe of its iron strength. Even though the pipe may appear to be fully intact and strong, the corrosion process has severely weakened the pipe. “It has been recognized by water utility personnel that the majority of the breaks occur at locations where the pipe wall has been weakened. Such weakening is the result of graphitic corrosion (graphitization) and, although the actual failure may be due to stress, corrosion can be shown to be the real cause” (Fitzgerald, 1968).

6.1.2 Pitting

Whereas graphitization weakens the pipe, pitting literally forms holes in the pipe. Pitting is the process by which corrosion holes form in pipe walls. This thesis focuses on pitting, rather than graphitization, as a major source of leakage in pipe networks. Whereas graphitization weakens large portions of a pipe, pitting is easier to quantify since it is a "localized attack" (Wranglen, 1985). Pitting is a significant cause of leakage in pipes, especially pipes that do not have corrosion protection.

Over time, metallic pipes in contact with soil begin to form what are called pits, meaning that the pipe wall begins to corrode at a specific point. "In order for a pitting attack to occur on a metal, a certain minimum corrosion potential, the so-called pitting or break-through potential, must be attained" (Wranglen, 1985). Once a pit has formed on the outer pipe wall, the pit continues to grow over time until it has penetrated through the entire wall thickness. "The rate with which a pit increases in depth with time in the soil under given conditions obeys an exponential equation of the following form:

$$P = kt^n \tag{6.1}$$

Where P is the depth of the deepest pit at time t, and k and n are constants" (Wranglen, 1985).

Eventually the pit depth equals the wall thickness and a leak results. Later in this chapter, equations will be given which can be used to determine which pipes are susceptible to pitting and eventual leakage.

6.1.3 Corrosion processes

Corrosion of a pipe may occur either internally or externally. The water flowing through the pipe coming in contact with the pipe wall causes internal corrosion. External corrosion of a pipe is due to the pipe coming in contact with soil and groundwater, or stray electrical current. Three different types of corrosion cause pipe leakage problems:

1. Internal corrosion;
2. External galvanic corrosion;
3. External electrolysis.

This thesis will focus exclusively on external corrosion. Modern water treatment facilities are able to treat water to a level where internal corrosion is not a major factor. The main culprit in corrosion of metal pipes is external corrosion. Therefore the next section will focus on external corrosion.

6.2 External corrosion

Pipes buried in soil are subject to external corrosion due to contact with soil and water. The contact on the outer pipe wall may eventually cause corrosion holes to penetrate through the entire wall thickness causing leakage to occur due to pitting. Otherwise the reduction in strength may cause the pipe to fail.

Certain factors cause the corrosion rate to increase and leave the pipe susceptible to graphitization. The following environmental factors are more conducive to corrosion: (AWWA M27, 1987)

1. Dissimilar metals;
2. Soil variances;
3. Naturally corrosive soils;
4. Environmental contamination.

6.2.1 Dissimilar metals

"Different metals or alloys in contact with each other and with a common media, such as water or soil" may increase the rate of corrosion in metallic pipes (AWWA M27, 1987). The most common example of this occurs at service connections, where water is withdrawn from the main line to serve a household connection. For example, the water main is ductile iron, the corporation stop is made of brass, and the lateral is copper. These dissimilar metal in contact with each other form what is called a bimetal couple, which may lead to corrosion of the water main. The actual process by which a bimetal couple causes corrosion will be discussed in a later section.

6.2.2 Soil variances

Pipes that pass through two or more completely different soil types are more susceptible to corrosion than those pipes laid in a uniform soil type. Nonuniformity in soil type is another prime situation in which corrosion can occur.

6.2.3 Naturally corrosive soils

Certain soil types are more corrosive than others based on their characteristics. Certain soil types abundantly exhibit these characteristics. Soils found in swamps or bogs, including peat soils or alkali soils are examples of naturally corrosive soils. The several indicators for corrosive soils, however, "the principal indicator for the potential corrosive environment for a buried cast iron main is soil resistivity" (Karaa and Marks, 1990). Other corrosive indicators include:

1. Low electrical resistivity
2. Low redox potential
3. Low pH (below 5.0)
4. Low oxygen
5. Saturated soils (poorly-drained soils)
6. Sulfate-reducing bacteria
7. Organic contamination (dead vegetation, leaves, refuse)
8. Nonuniform soil characteristics.

6.2.4 Environmental contamination

Environmental contamination may lead to increased corrosion rates in metallic pipes. "The current practice of heavy use of deicing salts on streets may be a source of future underground corrosion" (AWWA M27, 1987). Also, "sites where chemical contamination occurs, such as refuse dumps, landfills, and industrial waste disposal areas, may cause deterioration of water utility materials" (AWWA M27, 1987).

6.3 Overview of corrosion cells

A corrosion cell is the term used to describe the location where the corrosion process occurs. All four of these elements must be present for corrosion to occur. The elements that form a corrosion cell are: (Colson and Moriber, 1997)

1. *Anode* – an electrode (metal) which gives up electrons (oxidation) and where corrosion takes place.
2. *Cathode* – the protected electrode which receives electrons (reduction) and where corrosion is greatly reduced or eliminated.
3. *Electrolyte* – the environmental medium (typically soil or water) surrounding the anode and cathode which provides a path for current flow.
4. *Return current path* – the electrical connection between the anode and cathode (the pipe wall in our case).

A sample corrosion cell illustrating the function of the four elements within the cell is shown in Figure 6.1.

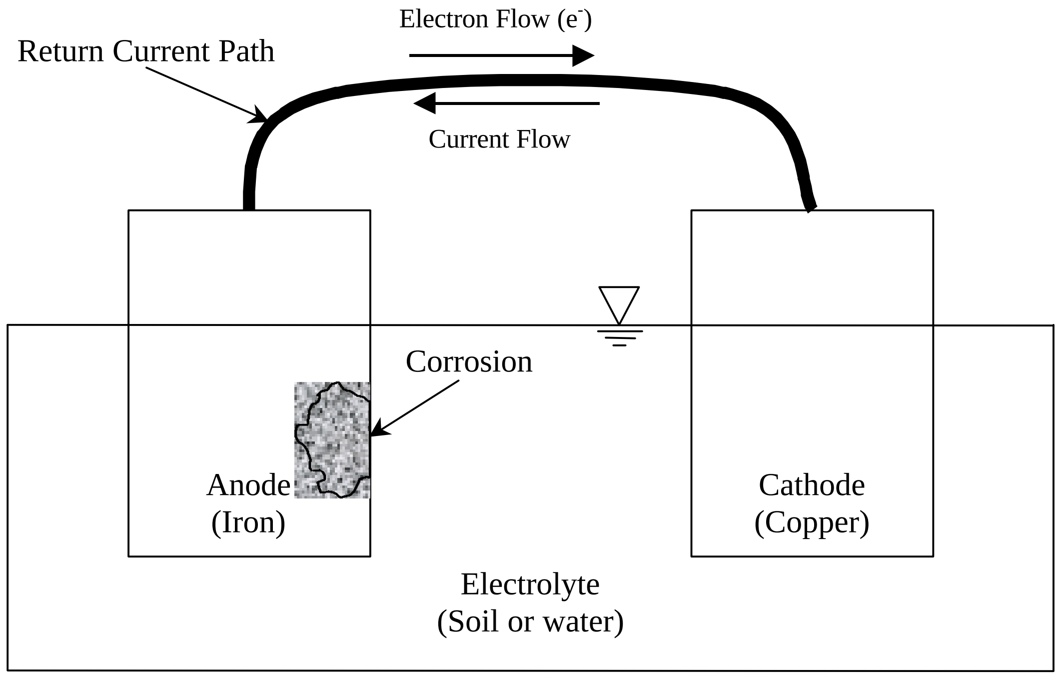

Figure 6.1: The four basic elements of a corrosion cell.
Adapted from (AWWA M27, 1987)

6.3.1 Galvanic corrosion

The first main type of external corrosion is called galvanic corrosion. “Galvanic corrosion is in effect a battery. Current flows from the anode (corroding) area to the cathode (protected) area because of the potential difference between them” (Fitzgerald, 1968). In order for galvanic corrosion to occur, a corrosion cell must exist as shown in Figure 6.1.

6.3.1.1 Dissimilar metals

When the anode and cathode are different metals then corrosion can occur very readily. As mentioned in Section 6.2.1, a bimetal couple forms an automatic corrosion cell when placed in contact with soil or water. "The more active metal (e.g., steel pipe) becomes anodic and corrodes with respect to the cathodic connecting metal (e.g., brass valve)" (Colson and Moriber, 1997). The two dissimilar metals serve as the anode and cathode respectively, the soil serves as the electrolyte, and the pipe wall acts as the return current path.

6.3.1.2 Nonuniform soil conditions

When the anode and cathode are made of the same material (i.e., a single pipe), a second type of corrosion cell can form. For a single piece of metal, corrosion will occur when the electrolyte is nonuniform. Nonuniform electrolytic conditions lead to a difference in electrical potential, thereby causing electrons to transfer within the corrosion cell.

As mentioned in Section 6.2.2, a pipe passing through two or more different soil types provides a situation where a corrosion cell can form. The uniform pipe material can serve as both an anode and cathode as it passes through different soil types. The metal will then corrode in that portion of the pipe that is serving as the anode.

6.3.2 Electrolysis

The second major type of external corrosion is called electrolysis. Electrolytic corrosion is "caused by the discharge of stray direct current from the surface of a buried metal" (Fitzgerald, 1968). Examples of stray direct current include: (Fitzgerald, 1968; Walski, 1984)

- Grounding electrical devices to water mains;
- Electrified railways;
- Cathodic protection of nearby gas mains or other structures
- Welding or plating operations.

For electrolysis to cause corrosion, the four basic elements of a corrosion cell are necessary. However, the main difference is that "the corrosion reaction is driven by a direct-current source originating outside the cell" (AWWA M27, 1987). The stray direct current must contact the return current path then feed into the cathode thereby driving the process. Stray direct current can lead to severe corrosion of metallic pipes.

6.4 Evaluating corrosion potential and subsequent leakage problems

As previously mentioned, a corrosion cell contains four basic elements; an anode, a cathode, an electrolyte, and a path for return current. There are two types of corrosion cells:

1. Dissimilar metals in contact with each other surrounded by a uniform electrolyte;
2. A single piece of metal passing through nonuniform electrolytic conditions.

Therefore, "the electrolyte is a key variable in determining where and to what extent corrosion will occur" (AWWA M27, 1987). The soil characteristics determine whether or not the soil will perform well as an electrolyte. As mentioned above in Section 6.2.3, some the attributes that determine whether a soil will serve well as an electrolyte are low resistivity, low redox potential, low pH, saturated conditions, low oxygen content, etc. The determination of whether a certain soil will cause corrosion is essential to the effective management of a water distribution system.

Engineering field tests can be done to determine the potential corrosive action of the soil by measuring each of the soil characteristics listed in Section 6.2.3. In soil surveys published by the U. S. Department of Agriculture, each soil type in a given county is listed with a corresponding risk of corrosion to various materials (i.e. steel and concrete). The risk of corrosion is classified as either: low, moderate, or high. Using the information presented in the soil survey, engineers may determine whether or not pipes buried in a certain soil type have a high, moderate, or low risk of corrosion. This information can be very helpful in planning rehabilitation and replacement programs for the water distribution system, especially for pipes that do not have corrosion control.

This soil corrosion information will be incorporated into the formulation presented in this thesis. Pipes buried in high-risk corrosive soils have a higher potential for leakage than pipes buried in low-risk soils. The following section quantifies this risk by using corrosion equations.

6.4.1 Corrosion due to pitting

As mentioned earlier in this chapter, pitting is a major cause of leakage in unprotected pipes. The corrosion process causes pits to form in pipe walls and eventually the pit will penetrate through the entire wall thickness. To determine whether a pipe is leaking due to pitting, the following equation presented by Rossum (1969) can be used to find the time it takes for a pit to penetrate the entire wall of the pipe. If the pipe has been in the ground longer than the pitting time, then there will be leaks in the pipe.

$$t_L = \frac{r}{10 - pH}\left(\frac{z}{K_n K_a}\right)^{1/n}\left(\frac{1}{A_S}\right)^{a/n} \tag{6.2}$$

Where, t_L = Time for a pit to penetrate through the wall thickness (years).

r = Soil resistivity (ohm-cm).

pH = Measured pH of the soil.

z = Pipe wall thickness (mils).

K_n = A constant based on soil conditions.

K_a = Relative pit depth based on pipe material.

n = A constant based on soil aeration.

A_S = Exposed surface area of the pipe (ft^2).

a = A constant depending on the soil and the metal.

If there are leaks in the pipe, the next step is to figure out how many potential corrosion holes there are in the pipe length. The following equation given by Rossum (1969) can be used to determine the number of leaks in a given pipe. Rossum actually used L as the variable for the number of leaks; however, N will be used in this thesis. The number of leaks, N, by time t is,

$$N = A_S \left(\frac{K_n K_a}{z} \right)^{1/a} \left(\frac{t(10 - pH)}{r} \right)^{n/a} \tag{6.3}$$

Where, N = Number of leaks along the entire pipe length.

A_S = Exposed surface area of the pipe (ft^2).

K_n = A constant based on soil conditions.

K_a = Relative pit depth based on pipe material.

z = Pipe wall thickness (mils).

a = A constant depending on the soil and the metal.

t = Time of exposure (years).

pH = Measured pH of the soil.

r = Soil resistivity (ohm-cm).

n = A constant based on soil aeration.

The Mt. Tabor subnetwork will be used to illustrate the process by which corrosion pitting can be taken into account in the leakage analysis. The pipe network is overlaid onto the soil survey map in order to identify the soil type surrounding each buried pipe. The risk of corrosion is identified and leakage rates are calculated using the mathematical formulation given in Chapter 5. When the number of pipe leaks is known, an equivalent orifice area can be determined for the pipe. Leakage flow exiting the pipe can be solved for directly by using the average pressure method in the Equation (5.9).

6.4.2 Flow apportionment among leaking pipes

Equation (6.2) determines which pipes have corrosion pitting that has fully penetrated the pipe wall. Once the pipes with corrosion holes are identified, flows can be proportionately allocated among these pipes with the aid of Equation (6.3). Equation (6.3) yields the number of fully developed corrosion pits as a function of surface area. The number of pits can then be converted into an equivalent orifice area for use in the orifice equation. The leakage flow allocation is made based on:

1. Whether the pipe has fully developed corrosion holes,
2. The number of fully developed pits for a given surface area.

By solving Equations (6.2) and (6.3), we can readily distinguish the leaking pipes from the non-leaking pipes. By knowing which pipes are leaking, leakage flows can be solved for directly using Equation (5.9), or leakage losses can be lumped at nodes connecting leaking pipes. In Chapter 8 this scheme is applied to the Blacksburg water distribution system.

Chapter 7

Comparison with the Pudar-Liggett Paper

7.1 The Pudar-Liggett paper

In performing the literature review for this thesis, several papers were found which discussed the analysis of leakage in pipe networks. One of the most pertinent and influential papers related to this topic is entitled “Leaks in Pipe Networks”, by Ranko S. Pudar and James A. Liggett (Pudar and Liggett, 1992). Pudar and Liggett present a mathematical formulation (hereafter known as "the P-L formulation") that is able to determine leakage flows in a pipe network. The P-L formulation simplifies the analysis of the problem by concentrating leakage at the nodes. However, as mentioned earlier in this thesis, the real problem comes in trying to determine which pipes are leaking and need to be replaced. Simply knowing the amount of leakage flow being lost from the pipe network does not give enough information. Nodal leakage must also be allocated among pipes to truly solve the operational problem. Once pipe leakage is known, sound engineering decisions can be made concerning pipe rehabilitation or replacement.

The formulation presented in Chapter 5 of this thesis (hereafter known as "the S-L formulation") can solve for leakage concentrated at nodes and it can also allocate leakage among pipes based on the value of nodal leakage. The S-L formulation improves upon the P-L formulation in that it can go a step further by allocating leakage among pipes rather than simply concentrating leakage at the nodes. (Please refer to Method #1 of Section 5.2.2.1). To illustrate this point, the pipe network solved by Pudar and Liggett is re-solved using the S-L formulation to yield a more detailed solution.

7.2 Pudar-Liggett formulation

Before solving the pipe network problem it is important to understand the P-L formulation. This section details the formulation given by Pudar and Liggett in their paper (Pudar and Liggett, 1992). Their formulation consists of three different sets of equations:

1. The continuity equation at each node;
2. The energy equation, or headloss, through each pipe;
3. Head-dependent outflow based on the orifice equation.

7.2.1 Continuity equation (P-L formulation)

The continuity equation is as follows:

$$\sum_{pipes} Q_{k_{ij}} = D_i + q_i^l \tag{7.1}$$

Where, k_{ij} = The number of the pipe that lies between nodes i and j.

D_i = Demand (known outflow) at the node (which may be negative).

$q^l{}_i$ = Leak at the node.

Q_{kij} = Flow in pipe k_{ij} that is positive from node i to node j.

7.2.2 Energy equation (P-L formulation)

The equation for headloss along a given pipe length is as follows:

$$p_i - p_j = K_{k_{ij}} \left| Q_{k_{ij}} \right|^{n-1} Q_{k_{ij}} - g\left(E_i - E_j\right) \tag{7.2}$$

Where, p = Pressure

γ = Specific weight of the fluid.

K = A coefficient of resistance and includes the pipe diameter.

n = The exponent to be applied to the velocity in the friction equation.

E = Node elevation.

7.2.3 Orifice equation (P-L formulation)

The orifice equation for pressure-dependent outflow is given by the following formula:

$$q_i^l = C_{oi} A_i^l \sqrt{\frac{2gp_i}{g}} \tag{7.3}$$

Where, C_{oi} = Orifice coefficient.

A^l_i = Equivalent orifice area.

g = Acceleration due to gravity.

The above equations are written for each element of the pipe network and solved using the Levenberg-Marquardt method. "The primary solution criterion is the minimization of the difference - in the least-squares sense - of the measured and calculated heads" (Pudar and Liggett, 1992). Therefore, this method is based on some knowledge of existing pressure heads in the system. This required knowledge of some pressures and flows is also a drawback to the P-L formulation.

7.3 The Pudar-Liggett problem

Pudar and Liggett's paper demonstrates the usefulness of the P-L formulation by analyzing the pipe network shown in Figure 7.1 with seven nodes and eleven pipes. Certain constraints are placed on the network as outlined in the paper and reproduced here. The flowrates entering nodes 5 and 6 are fixed. (Fixed flowrates such as these could indicate single speed pumps continually feeding water into the network.) Node 1 is a fixed-grade node, meaning that the pressure head remains constant at this node. The incoming flowrate at node 1 is not fixed but rather depends on the continuity of flow in the system. Node 3 is the only demand point in the system and the outflow at this point is pressure-dependent. Nodes 2, 4, and 7 are identified as possible leak nodes; therefore equivalent orifice areas are given for these nodes. The Hazen-Williams coefficient is assumed to be 120 for all pipes in the network.

In order to verify the effectiveness of the S-L formulation, the same pipe network problem is solved. Table 7.1 gives the pipe network data and the value of the coefficients used in solving the problem. Since the problem uses the Hazen-Williams equation in SI units, the resistance coefficients were found using Equation (5.16).

Table 7.1: Pudar-Liggett pipe network data

Pipe #	Head node – Tail node	Pipe Length (m)	Pipe Diameter (mm)	Hazen-Williams C_{HW}	Resistance Coefficient
1	1-2	305	250	120	392.91
2	1-7	215	250	120	276.97
3	7-2	215	200	120	821.16
4	2-4	305	200	120	1164.90
5	2-3	215	250	120	276.97
6	4-3	215	250	120	276.97
7	7-4	215	250	120	276.97
8	6-4	305	250	120	392.91
9	5-4	215	200	120	821.16
10	6-5	215	150	120	3333.71
11	6-7	215	150	120	3333.71

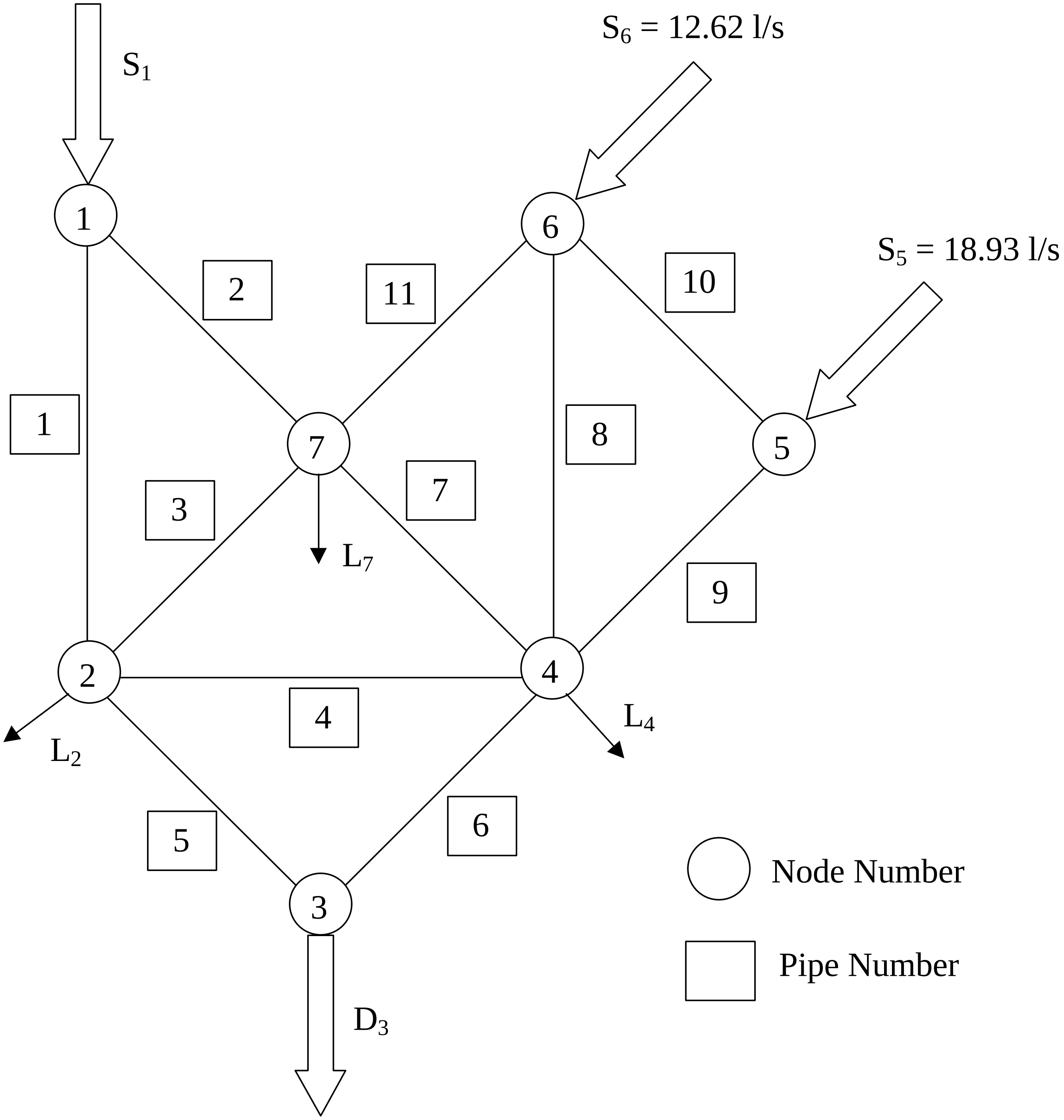

Figure 7.1: Pipe network given in the Pudar-Ligget paper

7.3.1 Solution of pipe flow problems in terms of nodal heads

Pudar and Liggett substitute Equations (7.2) and (7.3) for Q_{kij} and q_i^l respectively to obtain a set of head-dependent equations given by the following equation, from Equation (7.1):

$$\sum_{pipes} \mathrm{sgn}(h_i - h_j)\left|\frac{h_i - h_j}{K'_{kij}}\right|^{1/n} = D_i^c + C_{oi}^d A_i^d \sqrt{2gh_i} + C_{oi}^l A_i^l \sqrt{2gh_i} \quad \text{(For } i = 1, 2, \ldots, J\text{)} \tag{7.4}$$

Where, sgn(x) = sign function = x/|x| for x ≠ 0 and = 0 for x = 0.

J = number of nodes.

C_{oi}^d, C_{oi}^l = Orifice coefficient for demand and leakage respectively.

A_i^d, A_i^l = Orifice areas for demand and leakage respectively.

$h_i = p_i/\gamma$ = hydraulic head at node i.

$K'_{kij} = K_{kij}/\gamma$ = resistance coefficient.

If all supplies and demands are known, then one of the J continuity equations will be redundant.

7.3.2 Even-determined and over-determined problem

Pudar and Liggett assume that there are only (J-1) independent equations when using Equation (7.4). In solving their example problem (Pudar and Liggett, 1992, page 1039), they suggest an even-determined case in which heads h_3, h_5, and h_6 are given, but A_2^l, A_4^l, and A_7^l and h_2, h_4, and h_7 are unknowns (J-1 = 6 equations of Equation (7.4) in the six unknowns). They also suggest an over-determined case in which all h_1, h_2, …, h_7 are known and A_2^l, A_4^l, and A_7^l are to be solved for, from Equation (7.4).

In this thesis, the set of equations formed by using Equation (7.4) is solved directly using the values of A_2^l, A_4^l, A_7^l, and A_3^d provided by Pudar and Liggett. These values are, A_2^l = 0.00232 m^2, A_4^l = 0.00232 m^2, A_7^l = 0.0000929 m^2, and A_3^d = 0.005415 m^2. The six equations in Equation (7.4) are solved using LINGO for the six unknowns h_2, h_3, …, h_7 for the four cases involving zero, one (at node 2), two (at nodes 2 and 4), and three (at nodes (2, 4, and 7) leaks. A comparison of results is available in Tables 7.2 and 7.3. The respective formulations and outputs are provided in Appendix B and Appendix C.

Table 7.2: Comparison of pressure head values at each node in meters.

	No Leaks			One Leak @ Node 2			Two Leaks @ Nodes 2 and 4			Three Leaks @ Nodes 2, 4, and 7		
Node	P-L Paper	P-L LINGO	S-L LINGO	P-L Paper	P-L LINGO	S-L LINGO	P-L Paper	P-L LINGO	S-L LINGO	P-L Paper	P-L LINGO	S-L LINGO
1	30.48	30.48	30.48	30.48	30.48	30.48	30.48	30.48	30.48	30.48	30.48	30.48
2	29.22	29.13	29.13	27.59	27.40	27.40	25.92	25.64	25.64	25.87	25.58	25.58
3	27.67	27.48	27.48	26.26	25.99	25.99	24.45	24.09	24.09	24.39	24.03	24.03
4	29.22	29.13	29.13	27.89	27.72	27.72	25.74	25.46	25.46	25.67	25.39	25.39
5	29.55	29.49	29.49	28.26	28.12	28.12	26.17	25.92	25.92	26.10	25.85	25.85
6	29.47	29.40	29.40	28.20	28.06	28.06	26.15	25.90	25.90	26.09	25.83	25.83
7	29.56	29.50	29.50	28.51	28.38	28.38	27.01	26.80	26.80	26.93	26.71	26.71

Table 7.3: Comparison of inflows (the negative sign indicates an outflow) at each node in liters/second.

	No Leaks			One Leak @ Node 2			Two Leaks @ Nodes 2 and 4			Three Leaks @ Nodes 2, 4, and 7		
Node	P-L Paper	P-L LINGO	S-L LINGO	P-L Paper	P-L LINGO	S-L LINGO	P-L Paper	P-L LINGO	S-L LINGO	P-L Paper	P-L LINGO	S-L LINGO
1	94.64	94.20	94.20	145.46	144.53	144.53	191.68	190.07	190.07	193.61	191.91	191.91
2	0	0	0	-54.07	-53.79	-53.79	-52.41	-52.04	-52.04	-52.35	-51.98	-51.98
3	-126.18	-125.75	-125.75	-122.93	-122.29	-122.29	-118.61	-117.73	-117.73	-118.61	-117.58	-117.58
4	0	0	0	0	0	0	-52.22	-51.85	-51.85	-52.03	-51.78	-51.78
5	18.93	18.93	18.93	18.93	18.93	18.93	18.93	18.93	18.93	18.93	18.93	18.93
6	12.62	12.62	12.62	12.62	12.62	12.62	12.62	12.62	12.62	12.62	12.62	12.62
7	0	0	0	0	0	0	0	0	0	-2.30	-2.13	-2.13

7.3.3 Under-determined problem

Pudar and Liggett solve the example network in Figure 7.1 keeping A_2^l, A_4^l, and A_7^l and h_2, h_3, …, h_7 as unknowns with the available six equations from Equation (7.4). To force the solution in a correct direction, they assume either one or two of the h_2, h_3, …, h_7 are known. Clearly, there are more unknowns than the available six equations. To obtain a preferred solution an objective function must be chosen. They use: $\mathrm{sqrt}[(A_2^l)^2 + (A_4^l)^2 + (A_7^l)^2]$ for no leaks; and $\mathrm{sqrt}[(A_4^l)^2 + (A_7^l)^2]$ only if node 2 is leaking and therefore, driving the leakage flows at nodes 4 and 7 to zero by making $A_4^l + A_7^l$ zero. The solutions by LINGO for the under-determined case are given in Tables 7.4 and 7.5 for the different cases resulting from the known h_i values. The LINGO input and output for each case is given in Appendix B.

Table 7.4: Results of Under-determined Calculations for One Measurement

(a) One Leak (in Node 2 of Possibilities 2, 4, and 7)						
One head measurement	L_2		L_4		L_7	
	P-L paper	P-L LINGO	P-L paper	P-L LINGO	P-L paper	P-L LINGO
Even-determined problem	54.07	53.79	0	0	0	0
Measurement at node 3	19.24	53.79	21.20	0	22.90	0
Measurement at node 5	14.76	53.78	20.38	0	15.39	0
Measurement at node 6	15.33	53.78	21.20	0	15.96	0

(b) Two leaks (in Nodes 2 and 4 of Possibilities 2, 4, and 7)						
One head measurement	L_2		L_4		L_7	
	P-L paper	P-L LINGO	P-L paper	P-L LINGO	P-L paper	P-L LINGO
Even-determined problem	52.41	52.04	52.22	51.85	0	0
Measurement at node 3	37.35	20.43	41.77	72.64	33.56	10.21
Measurement at node 5	31.54	120.38	43.53	0	33.75	0
Measurement at node 6	31.29	75.48	42.71	31.39	34.13	1.11

Note: All nodal leaks (L_i) are expressed in units of liters per second.

Table 7.5: Results of Under-determined Calculations for Two Measurements

(a) One Leak (in Node 2 of Possibilities 2, 4, and 7)

Two head measurements	L_2		L_4		L_7	
	P-L paper	P-L LINGO	P-L paper	P-L LINGO	P-L paper	P-L LINGO
Even-determined problem	54.07	53.79	0	0	0	0
Measurements at nodes 3 and 5	55.90	53.78	-2.40	0	2.65	0
Measurements at nodes 5 and 6	15.14	51.48	25.68	1.19	6.37	0.56
Measurements at nodes 3 and 6	56.59	53.78	1.01	0	-2.40	0

(b) Two leaks (in Nodes 2 and 4 of Possibilities 2, 4, and 7)

Two head measurements	L_2		L_4		L_7	
	P-L paper	P-L LINGO	P-L paper	P-L LINGO	P-L paper	P-L LINGO
Even-determined problem	52.41	52.04	52.22	51.85	0	0
Measurements at nodes 3 and 5	53.29	50.36	32.37	29.68	27.51	31.30
Measurements at nodes 5 and 6	32.74	52.01	65.68	51.87	1.64	0
Measurements at nodes 3 and 6	56.91	61.43	33.06	12.89	21.77	38.84

Note: All nodal leaks (L_i) are expressed in units of liters per second.

7.4 LINGO input

As outlined in Chapter 5, the S-L formulation involves several different sets of equations. The S-L formulation was applied to the Pudar and Liggett problem. The following equations illustrate the use of the S-L formulation by giving the equations used for each set.

The following pages illustrate how the mathematical formulation given in Method #1 of Chapter 5 is applied to the Pudar-Liggett problem. This nonlinear program can be input into the LINGO software program and solved. Appendix A is a primer intended to introduce the novice user to the LINGO program. This primer shows the format necessary for inputting a mathematical model into LINGO. Appendix C gives the actual LINGO input for the equations on the following pages.

Objective Function

$$\text{Maximize: } (Qp_{12}-Q_{12})^2 + (Qp_{17}-Q_{17})^2 + (Qp_{72}-Q_{72})^2 + (Qp_{24}-Q_{24})^2 + (Qp_{23}-Q_{23})^2 + (Qp_{43}-Q_{43})^2 + (Qp_{74}-Q_{74})^2 + (Qp_{64}-Q_{64})^2 + (Qp_{54}-Q_{54})^2 + (Qp_{65}-Q_{65})^2 + (Qp_{67}-Q_{67})^2$$

Set 1: Continuity for demand written at each node using Equation (5.2)

1) $-Q_{12} - Q_{17} = -S_1$ (Node 1)
2) $Q_{12} + Q_{72} - Q_{24} - Q_{23} = 0$ (Node 2)
3) $Q_{23} + Q_{43} = D_3$ (Node 3)
4) $Q_{54} + Q_{64} + Q_{74} + Q_{24} - Q_{43} = 0$ (Node 4)
5) $Q_{65} - Q_{54} = -S_5$ (Node 5)
6) $-Q_{64} - Q_{65} - Q_{67} = -S_6$ (Node 6)
7) $Q_{17} + Q_{67} - Q_{72} - Q_{74} = 0$ (Node 7)

Set 2: Continuity equation for combined demand and leakage using Equation (5.3)

1) $-Qp_{12} - Qp_{17} = -Sp_1$ (Node 1)
2) $Qp_{12} + Qp_{72} - Qp_{24} - Qp_{23} = L_2$ (Node 2)
3) $Qp_{23} + Qp_{43} = D_3$ (Node 3)
4) $Qp_{54} + Qp_{64} + Qp_{74} + Qp_{24} - Qp_{43} = L_4$ (Node 4)
5) $Qp_{65} - Qp_{54} = -S_5$ (Node 5)
6) $-Qp_{64} - Qp_{65} - Qp_{67} = -S_6$ (Node 6)
7) $Qp_{17} + Qp_{67} - Qp_{72} - Qp_{74} = L_7$ (Node 7)
8) $Sp_1 = S_1 + L_2 + L_4 + L_7$ (Mass balance of total inflows and outflows)

Set 3: Head Loss equations (including leakage flows) using Equation (5.14)

1) $H_1 - H_2 = r_{12}Qp_{12}|Qp_{12}|^{0.852}$ (Pipe 1)
2) $H_1 - H_7 = r_{17}Qp_{17}|Qp_{17}|^{0.852}$ (Pipe 2)
3) $H_7 - H_2 = r_{72}Qp_{72}|Qp_{72}|^{0.852}$ (Pipe 3)
4) $H_2 - H_4 = r_{24}Qp_{24}|Qp_{24}|^{0.852}$ (Pipe 4)
5) $H_2 - H_3 = r_{23}Qp_{23}|Qp_{23}|^{0.852}$ (Pipe 5)
6) $H_4 - H_3 = r_{43}Qp_{43}|Qp_{43}|^{0.852}$ (Pipe 6)
7) $H_7 - H_4 = r_{74}Qp_{74}|Qp_{74}|^{0.852}$ (Pipe 7)
8) $H_6 - H_4 = r_{64}Qp_{64}|Qp_{64}|^{0.852}$ (Pipe 8)
9) $H_5 - H_4 = r_{54}Qp_{54}|Qp_{54}|^{0.852}$ (Pipe 9)
10) $H_6 - H_5 = r_{65}Qp_{65}|Qp_{65}|^{0.852}$ (Pipe 10)
11) $H_6 - H_7 = r_{67}Qp_{67}|Qp_{67}|^{0.852}$ (Pipe 11)

Set 4: Pressure-Dependent Demand and Leakage equations using Equation (5.8)

1) $L_2 = c_d A_2 (2gH_2)^{0.5}$ (Leak @ node 2)
2) $D_3 = c_d A_3 (2gH_3)^{0.5}$ (Demand @ node 3)
3) $L_4 = c_d A_4 (2gH_4)^{0.5}$ (Leak @ node 4)
4) $L_7 = c_d A_7 (2gH_7)^{0.5}$ (Leak @ node 7)

Set 5: Constraint on leakage flows using Equation (5.16)

1) $|Qp_{12}| \geq |Q_{12}|$ (Pipe 1)
2) $|Qp_{17}| \geq |Q_{17}|$ (Pipe 2)
3) $|Qp_{72}| \geq |Q_{72}|$ (Pipe 3)
4) $|Qp_{24}| \geq |Q_{24}|$ (Pipe 4)
5) $|Qp_{23}| \geq |Q_{23}|$ (Pipe 5)
6) $|Qp_{43}| \geq |Q_{43}|$ (Pipe 6)
7) $|Qp_{74}| \geq |Q_{74}|$ (Pipe 7)
8) $|Qp_{64}| \geq |Q_{64}|$ (Pipe 8)
9) $|Qp_{54}| \geq |Q_{54}|$ (Pipe 9)
10) $|Qp_{65}| \geq |Q_{65}|$ (Pipe 10)
11) $|Qp_{67}| \geq |Q_{67}|$ (Pipe 11)

Set 6: Set flow direction for leakage and normal flows using Equation (5.17)

1) $Qp_{12}*Q_{12} - |Qp_{12}|*|Q_{12}| = 0$ (Pipe 1)
2) $Qp_{17}*Q_{17} - |Qp_{17}|*|Q_{17}| = 0$ (Pipe 2)
3) $Qp_{72}*Q_{72} - |Qp_{72}|*|Q_{72}| = 0$ (Pipe 3)
4) $Qp_{24}*Q_{24} - |Qp_{24}|*|Q_{24}| = 0$ (Pipe 4)
5) $Qp_{23}*Q_{23} - |Qp_{23}|*|Q_{23}| = 0$ (Pipe 5)
6) $Qp_{43}*Q_{43} - |Qp_{43}|*|Q_{43}| = 0$ (Pipe 6)
7) $Qp_{74}*Q_{74} - |Qp_{74}|*|Q_{74}| = 0$ (Pipe 7)
8) $Qp_{64}*Q_{64} - |Qp_{64}|*|Q_{64}| = 0$ (Pipe 8)
9) $Qp_{54}*Q_{54} - |Qp_{54}|*|Q_{54}| = 0$ (Pipe 9)
10) $Qp_{65}*Q_{65} - |Qp_{65}|*|Q_{65}| = 0$ (Pipe 10)
11) $Qp_{67}*Q_{67} - |Qp_{67}|*|Q_{67}| = 0$ (Pipe 11)

The following values are known, (as given in the P-L paper):

$H_1 = 30.48$ m (Head at the fixed-grade node)
$Q_5 = 0.01893$ m^3/s (Constant inflow at Node 5)
$Q_6 = 0.01262$ m^3/s (Constant inflow at Node 6)
$g = 9.81$ m/s^2 (Acceleration due to gravity)
$C = 1$ (Orifice coefficient)
$A_2 = 0.00232$ m^2 (Equivalent orifice area for leakage at Node 2)
$A_4 = 0.00232$ m^2 (Equivalent orifice area for leakage at Node 4)
$A_7 = 0.0000929$ m^2 (Equivalent orifice area for leakage at Node 7)
$A_3 = 0.005415$ m^2 (Orifice opening at Node 3 for pressure-dependent demand)

In “Leaks in Pipe Networks,” (Pudar and Liggett, 1992) four different situations were analyzed to determine the leakage flows.

1. No-leak condition, the only outflow from the network is located at node 3 which is a demand location;
2. One leak located at node 2;
3. Two leaks located at nodes 2 and 4;
4. Three leaks located at nodes 2, 4, and 7.

7.4.1 LINGO runs

The system of linear and non-linear equations given above was solved using the LINGO software program for each of the four situations. Appendix C contains the LINGO input files and corresponding output for each of the four cases using the S-L formulation. Tables 7.6 - 7.9 give the results for each case as solved by LINGO. These tables also give the results of the allocation of nodal leakage to the converging pipes. Some of the results for these four cases were given previously in Tables 7.2 and 7.3.

Table 7.6: Resulting flowrates and leakage for the S-L formulation (zero leaks)

			Zero Leaks			
Pipe #	Head node	Tail node	Qp_{ij}	Q_{ij}	$Qp_{ij}-Q_{ij}$	L_{ij}
1	1	2	0.04677	0.04677	0	0
2	1	7	0.04743	0.04743	0	0
3	7	2	0.01569	0.01569	0	0
4	2	4	-0.00039	-0.00039	0	0
5	2	3	0.06285	0.06285	0	0
6	4	3	0.06289	0.06289	0	0
7	7	4	0.02813	0.02813	0	0
8	6	4	0.01968	0.01968	0	0
9	5	4	0.01547	0.01547	0	0
10	6	5	-0.00346	-0.00346	0	0
11	6	7	-0.00361	-0.00361	0	0

Note: All flowrate values (Q and L) are in cubic meters per second.

Table 7.7: Resulting flowrates and leakage for the S-L formulation (one leak)

			One Leak			
Pipe #	Head node	Tail node	Qp_{ij}	Q_{ij}	$Qp_{ij}-Q_{ij}$	L_{ij}
1	1	2	0.07296	0.01917	0.05379	0.05379
2	1	7	0.07157	0.07157	0	0
3	7	2	0.02647	0.02647	0	0
4	2	4	-0.01206	-0.01206	0	0
5	2	3	0.05771	0.05771	0	0
6	4	3	0.06458	0.06458	0	0
7	7	4	0.03831	0.03831	0	0
8	6	4	0.02212	0.02212	0	0
9	5	4	0.01621	0.01621	0	0
10	6	5	-0.00272	-0.00272	0	0
11	6	7	-0.00678	-0.00678	0	0

Note: All flowrate values (Q and L) are in cubic meters per second.

Table 7.8: Resulting flowrates and leakage for the S-L formulation (two leaks)

			Two Leaks			
Pipe #	Head node	Tail node	Qp_{ij}	Q_{ij}	Qp_{ij}-Q_{ij}	L_{ij}
1	1	2	0.09307	0.04103	0.05204	0.05204
2	1	7	0.09700	0.04515	0.05185	0
3	7	2	0.02886	0.02886	0	0
4	2	4	0.00898	0.00898	0	0
5	2	3	0.06092	0.06092	0	0
6	4	3	0.05681	0.05681	0	0
7	7	4	0.05632	0.00447	0.05185	0.05185
8	6	4	0.02575	0.02575	0	0
9	5	4	0.01761	0.01761	0	0
10	6	5	-0.00132	-0.00132	0	0
11	6	7	-0.01181	-0.01181	0	0

Note: All flowrate values (Q and L) are in cubic meters per second.

Table 7.9: Resulting flowrates and leakage for the S-L formulation (three leaks)

			Three Leaks			
Pipe #	Head node	Tail node	Qp_{ij}	Q_{ij}	Qp_{ij}-Q_{ij}	L_{ij}
1	1	2	0.09368	0.07019	0.02349	0.02349
2	1	7	0.09823	0.01584	0.08239	0.00213
3	7	2	0.02849	0	0.02849	0.02849
4	2	4	0.00923	0.00923	0	0
5	2	3	0.06096	0.06096	0	0
6	4	3	0.05662	0.05662	0	0
7	7	4	0.05591	0.00414	0.05177	0.05177
8	6	4	0.02567	0.02567	0	0
9	5	4	0.01758	0.01758	0	0
10	6	5	-0.00135	-0.00135	0	0
11	6	7	-0.0117	-0.0117	0	0

Note: All flowrate values (Q and L) are in cubic meters per second.

7.5 Key differences between the P-L and S-L formulations

By referring to Tables 7.4 and 7.5 it is readily seen that the S-L formulation subsumes the P-L formulation. From Tables 7.2 and 7.3 related to the P-L formulation it is seen that the following drawbacks apply:

1. There is no formal mechanism for assigning nodal leakages.
2. There is no formal procedure to identify the leaking pipes.
3. There is no formal apportionment of leakage amounts among pipes.
4. There is a need for a special computer program as opposed to a general purpose software package such as LINGO.

The S-L formulation overcomes the above drawbacks in the following manner:

1. A water audit based nodal leakage assignment is suggested. If a system is losing 10% of its total water supply to leakage, a proportional nodal increase among nodes is suggested. Also, a pressure-dependent leak can be accommodated within the same S-L formulation.
2. A formal procedure in terms of corrosion prone pipes is offered to identify leaking pipes. Chapter 8 provides elaborate details of this corrosion based procedure.
3. The S-L formulation identifies the leakage potential as the difference of Qp_{ij} and Q_{ij}. This leakage potential is later converted into actual amounts of leakage based on the susceptibility to corrosion.
4. By using LINGO, improved formulations such as the S-L formulation can be directly and easily incorporated. There is no need for new computer codes.

Chapter 8

Application to the Blacksburg Pipe Network

8.1 Overview of the Blacksburg pipe network

The Blacksburg water distribution system will be used to illustrate the effectiveness of the mathematical formulation presented in the previous chapter. The Blacksburg water distribution system consists of 1,116 junctions and 646,742 feet (122.5 miles) of pipe in 1346 pipes throughout the system. The system serves approximately 35,000 people living within the Town of Blacksburg.

The original pipe network was laid out using cast iron pipes. Currently many of the original or older water mains are being replaced with ductile iron pipes with a minimum diameter of 8 inches to provide necessary fire flow. Leakage in the Blacksburg pipe network accounts for approximately 10% of the total supply to the system.

8.2 Subdivision within the Blacksburg pipe network

A subdivision of the Blacksburg pipe network will be used to illustrate the practicality of the mathematical formulation presented in Chapter 5. Since the entire network is so large a representative subnetwork will be utilized. The work in choosing the subnetwork was done in conjunction with Shivaram Subramanian, a Ph.D. candidate in Industrial and Systems Engineering. The following criteria were used (in order of importance) to choose a representative subnetwork within the Blacksburg system to analyze.

1. The subnetwork had to have a single pipe connecting the subdivision to the rest of the network. (i.e. Single source)

2. The size of the subnetwork had to be substantial enough to perform the analysis yet small enough not to be overwhelming.

In the Blacksburg network there are two subdivisions which have a single source. The first subnetwork in southwest Blacksburg was too large for the scope of this work. The second subnetwork located in northwest Blacksburg was an appropriate size, plus there were no pumps or storage tanks to deal with. The source for this subnetwork is located at the junction of North Main St. and Mt. Tabor Road. Since Mt. Tabor is the main road, the subnetwork will hereafter be termed the "Mt. Tabor subnetwork."

The Town of Blacksburg Engineering Department provided the node and pipe data for the entire Blacksburg pipe network, as well as an AutoCAD file showing the location of all pipes in the network. The full set of data and the map were then scrutinized to pick out the appropriate pipe and nodal data for the Mt. Tabor subnetwork.

Several dead end lines were observed in the subdivision that feed cul-de-sacs and other offshoots. Therefore, to further simplify the process, the Mt. Tabor subnetwork was further reduced to include only the main lines. The main lines of this subnetwork run beneath the following streets: Mt. Tabor Road, Woodbine Drive, Lombardi Drive, Cottonwood Drive, and a portion of Scott Drive. A drawing of the pipe network serving the Mt. Tabor area of Blacksburg is given in Figure 8.1. Tables 8.1 and 8.2 give the pipe and nodal data for the Mt. Tabor subnetwork. The resistance coefficients in Table 8.1 were determined using Equation (5.15).

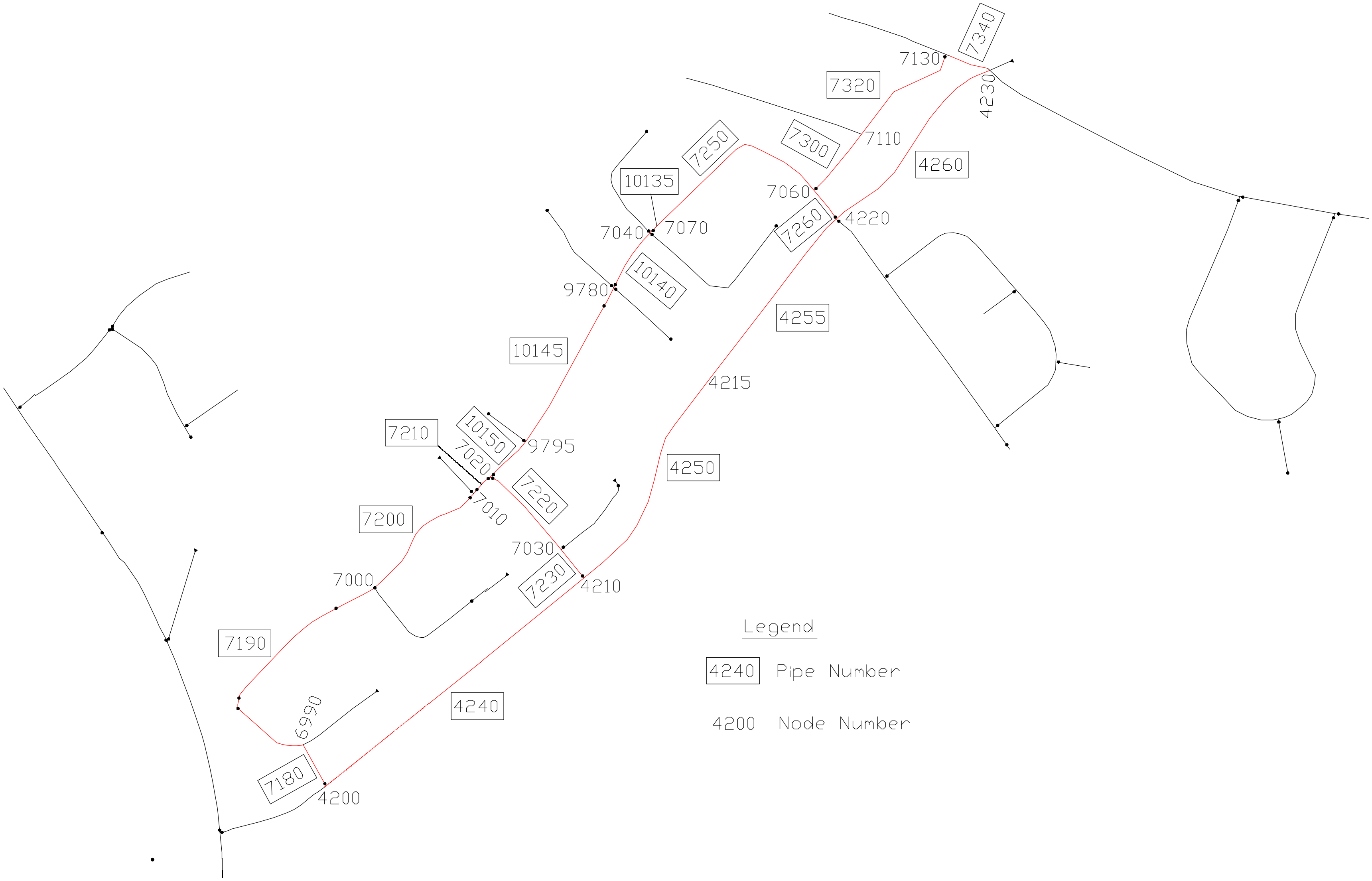

Figure 8.1: Mt. Tabor subnetwork
(The main pipes used in the analysis are numbered accordingly.)

Table 8.1: Pipe data for the Mt. Tabor subnetwork

Pipe No.	Diameter (in.)	Head-node	Tail-node	Length (ft.)	C_{HW}	Resistance Coefficient
4240	10	4200	4210	1363	120	2.2084
4250	10	4210	4215	991	120	1.6057
4255	10	4215	4220	841	120	1.3626
4260	10	4220	4230	888	120	1.4388
7180	8	4200	6990	194	120	0.9319
7190	8	6990	7000	1098	120	5.2745
7200	8	7000	7010	578	120	2.7765
7210	8	7010	7020	95	120	0.4564
7220	8	7030	7020	419	120	2.0128
7230	8	4210	7030	151	120	0.7254
7250	8	7070	7060	823	120	3.9535
7260	8	7060	4220	155	120	0.7466
7300	8	7110	7060	303	120	1.4555
7320	8	7130	7110	499	120	2.3971
7340	8	4230	7130	189	120	0.9079
10135	6	7040	7070	59	120	1.1506
10140	6	9780	7040	271	120	5.2850
10145	6	9795	9780	730	120	14.2364
10150	6	7020	9795	208	120	4.0564

Table 8.2: Node data for the Mt. Tabor subnetwork.

Node No.	X-Coord.	Y-Coord.	Demand (gpm)	Demand (cfs)	Elevation (ft.)
4200	10927345.04	3623273.19	0.58	0.00129	2163.0
4210	10928406.26	3624128.82	2.11	0.00470	2141.0
4215	10928922.31	3624945.78	0	0	2110.0
4220	10929443.86	3625605.40	5.39	0.01201	2132.0
4230	10930068.21	3626214.90	9.03	0.02012	2121.0
6990	10927251.27	3623442.65	3.84	0.00856	2153.5
7000	10927545.34	3624089.39	5.96	0.01328	2141.5
7010	10927952.22	3624475.26	1.73	0.00385	2129.0
7020	10928019.51	3624542.31	0.58	0.00129	2127.0
7030	10928310.47	3624244.96	2.70	0.00602	2127.0
7040	10928677.81	3625546.86	4.42	0.00985	2109.5
7060	10929350.82	3625723.07	0.77	0.00172	2139.0
7070	10928718.72	3625589.17	2.11	0.00470	2110.0
7110	10929546.37	3625954.10	3.65	0.00813	2144.5
7130	10929894.15	3626281.70	0.38	0.00085	2116.0
9780	10928527.81	3625323.10	4.04	0.00900	2099.5
9795	10928165.78	3624689.36	1.92	0.00428	2120.0

8.3 Soil survey overlay

Once the subnetwork is chosen, the next step is to overlay the pipe network onto the corresponding section of the Montgomery County soil survey (Creggar et al., 1985). By overlaying the network onto the soil survey, the soil type within which each pipe is buried can be determined. The soil type for each pipe is noted, including incidences where a pipe passes through more than one soil type. Table 8.3 gives each pipe in the subnetwork and the corresponding soil types that they are buried in.

Table 8.3: Soil types in which each pipe is buried.

Pipe No.	Head node	Tail node	Soil Types that each pipe passes through
4240	4200	4210	11B, 16C, 16B, 16C
4250	4210	4215	16B, 16C, 11B, 8D
4255	4215	4220	11B, 16B
4260	4220	4230	16B, 11B, 16C
7180	4200	6990	11B
7190	6990	7000	11B, 12C
7200	7000	7010	12C
7210	7010	7020	12C
7220	7030	7020	11B, 12C
7230	4210	7030	16B
7250	7070	7060	13C
7260	7060	4220	16B
7300	7110	7060	16B
7320	7130	7110	11B, 12C
7340	4230	7130	16C
10135	7040	7070	12C
10140	9780	7040	12C
10145	9795	9780	12C, 11B
10150	7020	9795	12C

Key
8D – Caneyville-Opequon-Rock outcrop complex (silt loam), 7 - 25 percent slopes, well-drained.
11B – Duffield-Ernest complex (silt loam), 2 to 7 percent slopes, well-drained.
12C – Frederick and Vertrees silt loams, 7 to 15 percent slopes, well-drained.
13C – Frederick and Vertrees cherty silt loams, 7 to 15 percent slopes, well-drained.
16B – Groseclose and Poplimento soils (silt loam), 2 to 7 percent slopes, well-drained.
16C – Groseclose and Poplimento soils (silt loam), 7 to 15 percent slopes, well-drained.

8.4 Analysis of corrosion potential

In the Montgomery County soil survey, each soil type is listed with a corresponding risk of corrosion to buried concrete and steel. The risk of corrosion to each buried material is listed as either low, moderate, or high. This classification can give a general idea of the risk involved; however, a more sophisticated method is necessary to determine the actual risk of corrosion pitting for a given pipe.

First, Equation (6.2) is used to calculate the time it takes for the first corrosion hole to penetrate the wall thickness. Next, Equation (6.3) is used to determine the number of corrosion holes due to pitting that will occur in a given pipe. Knowing these two items, the mathematical formulation presented in Chapter 5 can be applied to the subnetwork to locate pipes in the system that are leaking and calculate the leakage losses from the system.

8.5 Corrosion equations

The corrosion equations given in Chapter 6, namely Equations (6.2) and (6.3), can be used to determine which pipes are leaking and how many corrosion holes have formed due to pitting. The results may then be used in the mathematical formulation to determine leakage flows from the network.

There are five main categories that need to be addressed for use in the corrosion equations according to the research done by John R. Rossum, namely: (Rossum, 1969)

1. Aeration of the soil;
2. Resistivity;
3. Pipe thickness;

4. Surface Area of exposed pipe;
5. Time of exposure.

8.5.1 Aeration of the soil

Three variables in the corrosion equations are affected by the soil aeration, namely n, K_n, and pH. The variables n and K_n are both constants based on the soil aeration. The pH is a measure of the acidity or alkalinity of the soil. The pH scale ranges from 0-14 with a pH of 7 being neutral. Soils with pH values below 7 are termed acidic and soils with pH values above 7 are termed alkali soils. To determine the values of these three variables, the aeration (poor, fair, or good) of the soil must be determined.

Rossum took much of the information concerning soil aeration from the work of Romanoff (1957). The aeration of a given soil "depends largely on drainage, which is indicated by topographic features, average height of the water table, texture of the soil, etc." (Romanoff, 1957). Romanoff discusses the aeration of various soil types.

Soils classified as having *good aeration* include coarse sands or sandy loams, light-textured silt loams, and porous loams. "The soils of *fair aeration* might be considered well-drained soils in an agricultural sense, that is, no artificial drainage would be required. These soils are generally silt loams or sandy loams" (Romanoff, 1957). Finally, the soils of *poor aeration* are "as a class, soils of heavy texture (clay loams and clays), which would require artificial drainage if used for growing crops" (Romanoff, 1957).

All of the soils within the Mt. Tabor subnetwork are of the silt loam type and they are all well-drained, according to the Montgomery County soil survey. Therefore, all the soils would be classified as being of "fair" aeration under the definitions given by Romanoff. Once the degree of aeration has been determined, the values of the variables n and K_n are found using Table 3 on page 308 of Rossum's paper (1969). The pH of each soil type is given in the Montgomery County soil survey. Table 8.4 gives the values of these variables for each soil type.

Table 8.4: Aeration variables

Soil Type	Aeration	n	K_n	pH
8D	Fair	1/3	222	4.5 - 7.3
11B	Fair	1/3	222	5.1 - 6.5
12C	Fair	1/3	222	4.5 - 5.5
13C	Fair	1/3	222	4.5 - 5.5
16B	Fair	1/3	222	5.1 - 6.5
16C	Fair	1/3	222	5.1 - 6.5

The problem with the pH values given in the soil survey is that a range is given. Therefore, some interpretation is necessary to come up with a single pH value. The lowest pH value (most acidic) is the worst case scenario. The lowest pH value gives the shortest time to the first leak. For pipes that pass through more than one soil type, the pH value will be averaged.

8.5.2 Soil resistivity

The soil resistivity measures the potential of a given soil to serve as an electrolyte. As discussed in Chapter 6, an electrolyte is one of the essential elements necessary for a corrosion cell to form. "Resistivity, reported in ohm-centimeters (Ω-cm), represents the average of the electrical resistances across each cubic centimeter of soil in a given volume" (AWWA M27, 1987). The lower the value of resistivity, the better the soil will serve as an electrolyte and, therefore, the soil has more potential to cause corrosive pitting in metallic pipes.

Soil resistivity is directly related to soil moisture content. As moisture content increases, the soil resistivity decreases. A chart developed by the Philadelphia Water Department

(Philadelphia Water Dept., 1985) shows the relationship between soil moisture and soil resistivity for ten different soil types. Assuming the worst case of 30% moisture content, the corresponding resistivity for silt loam soils is approximately 4,000 ohm-cm. The variable used by Rossum to represent soil resistivity is r.

8.5.3 Pipe wall thickness

The pipe wall thickness, z, is a critical variable in analyzing corrosion potential. The wall thickness dictates the amount of time that it will take the corrosion pit to go through the wall. Table 8.5 gives nominal wall thicknesses for cast iron pipes.

Table 8.5: Nominal wall thicknesses (Philadelphia Water Dept., 1985)

Diameter	6"	8"	12"	24"
Wall Thickness	0.25"	0.43"	0.49"	0.56"

The units for wall thickness in Equations (6.2) and (6.3) are "mils" which is 1/1000". There are two other variables related to material and wall thickness. These variables are a and K_a. The variable, a, refers to experimental data for various materials. K_a is the relative pit depth based on the material. A table giving values for both of these variables is on page 308 of Rossum's paper (1969). Table 8.6 lists each pipe and the corresponding wall thicknesses to be used in solving the corrosion equations as well as the corresponding values for the two other variables.

Table 8.6: Pipe wall thicknesses

Pipe No.	Diameter, d (in.)	Wall Thickness (in.)	Wall Thickness, *z* (mils)	Value of "a" to be used on structures	Relative pit depth, K_a
4240	10	0.46	460	0.22	1.40
4250	10	0.46	460	0.22	1.40
4255	10	0.46	460	0.22	1.40
4260	10	0.46	460	0.22	1.40
7180	8	0.43	430	0.22	1.40
7190	8	0.43	430	0.22	1.40
7200	8	0.43	430	0.22	1.40
7210	8	0.43	430	0.22	1.40
7220	8	0.43	430	0.22	1.40
7230	8	0.43	430	0.22	1.40
7250	8	0.43	430	0.22	1.40
7260	8	0.43	430	0.22	1.40
7300	8	0.43	430	0.22	1.40
7320	8	0.43	430	0.22	1.40
7340	8	0.43	430	0.22	1.40
10135	6	0.25	250	0.22	1.40
10140	6	0.25	250	0.22	1.40
10145	6	0.25	250	0.22	1.40
10150	6	0.25	250	0.22	1.40

8.5.4 Exposed surface area

The total external surface area of a pipe is given by the following equation:

$$A_T = (2pR_{ij})*l_{ij} \tag{8.1}$$

Where, A_T = Total surface area of the outer pipe wall.

R_{ij} = Radius of the pipe that connects nodes *i* and *j*.

l_{ij} = Length of the pipe that connects nodes *i* and *j*.

Equation (8.1) gives the total surface area of the pipe. However, the actual exposed surface area that is susceptible to corrosion is not 100% of the surface area. If a pipe is covered with some type of corrosion protection then one can assume that perhaps 1% of the pipe will be exposed to the soil due to inconsistencies in corrosion control coverage. Thus the value of the variable, V, in the following equation would be 0.01.

$$A_S = V * \left(2pR_{ij}\right) * l_{ij} \tag{8.2}$$

Where, A_S = Surface area of the outer pipe wall that is susceptible to corrosion.

V = Vulnerability index ($0 < V < 1$).

R_{ij} = Radius of the pipe that connects nodes i and j.

l_{ij} = Length of the pipe that connects nodes i and j.

If a given pipe is completely unprotected then one could assume that the full surface area would be used in the equation. However, the concept of "vulnerability index" is introduced in this thesis to accommodate the situation of unprotected pipes. In Rossum's paper, he only discussed the case of protected pipes.

To assume that the full pipe length is susceptible to corrosion is not accurate since certain sections of the pipe will not corrode. As mentioned in Chapter 6, certain conditions control the corrosion process. The "vulnerability index" is based on the soil conditions, pipe conditions, stray current conditions etc. Therefore, only a certain percentage of the pipe is susceptible to corrosion pitting. Unfortunately, this concept of the vulnerability index is not fully developed in this thesis. Further research needs to be done to quantify this vulnerability index.

Since no data is currently available to determine the vulnerability index, one can assume that on average, 20% of the total surface area of unprotected pipes is susceptible to corrosion. Therefore the value of V in Equation (8.2) would be 0.2. Table 8.7 contains the data for the pipe surface area calculations.

Table 8.7: Surface area data

Pipe No.	Diameter, d_{ij} (in.)	Length, l_{ij} (ft.)	Total Surface Area, A_T (ft^2)	Vulnerability Index, V	Susceptible Surface Area, A_S (ft^2)
4240	10	1363	3568.33	0.20	713.67
4250	10	991	2594.43	0.20	518.89
4255	10	841	2201.73	0.20	440.35
4260	10	888	2324.78	0.20	464.96
7180	8	194	406.31	0.20	81.26
7190	8	1098	2299.65	0.20	459.93
7200	8	578	1210.56	0.20	242.11
7210	8	95	198.97	0.20	39.79
7220	8	419	877.55	0.20	175.51
7230	8	151	316.25	0.20	63.25
7250	8	823	1723.69	0.20	344.74
7260	8	155	324.63	0.20	64.93
7300	8	303	634.60	0.20	126.92
7320	8	499	1045.10	0.20	209.02
7340	8	189	395.84	0.20	79.17
10135	6	59	92.68	0.20	18.54
10140	6	271	425.69	0.20	85.14
10145	6	730	1146.68	0.20	229.34
10150	6	208	326.73	0.20	65.35

8.5.5 Time of exposure

The time of exposure, t, refers to the amount of time that a pipe has been buried in the ground. This data should be kept and recorded by local utilities. However, Blacksburg has not kept up-to-date records of this information. Therefore, a conservative estimate of the time of exposure for the pipes in the Mt. Tabor subnetwork is 35 years.

8.6 Corrosion calculations

Now that each of the variables is known, Equations (6.2) and (6.3) can be utilized to determine the corrosion potential of the subnetwork, including equivalent orifice areas. Equation (6.2) is used to determine the time that it will take for the corrosion pit to penetrate through the full wall thickness. Equation (6.3) is used to find the number of leaks that will have occurred during the life of the pipe. The answer found by using Equation (6.3) must be rounded down to the nearest integer value, the number of full perforations. Equation (8.3) can then be utilized to determine the equivalent orifice area for use in the pressure-dependent leakage equation.

The equivalent orifice area is found using the following equation:

$$A_{Lij} = N * \left(\frac{p d_{Lij}^{2}}{4} \right) \tag{8.3}$$

Where, A_{Lij} = Equivalent orifice area

N = Number of leaks along the pipe length (integer value)

d_{Lij} = Diameter of the orifice opening for pipe ij = z/10.

The diameter of the orifice opening is difficult to estimate due to the fact that corrosion holes due to pitting are not uniform in shape. However, one may assume that the orifice opening will be approximately 1/10 of the wall thickness.

Table 8.8 summarizes all of the corrosion data that is used in the corrosion equations. Table 8.9 gives a summary of the results of the corrosion calculations.

Table 8.8: Summary of corrosion data for the Mt. Tabor subnetwork

Pipe Data						Corrosion Data										
Pipe No.	Head-node	Tail-node	Pipe Length	Pipe Diameter	Pipe Radius	Aeration Constant	Aeration Constant	Average pH value	Soil Resistivity	Wall Thickness	Relative Pit Depth	Constant	Total Surface Area	Vulnerability Index	Susceptible Surface Area	Time of Exposure
			l_{ij}	d_{ij}	R_{ij}	n	K_n	pH	rho	z	K_a	a	A_T	V	A_S	t
			(ft.)	(in.)	(in.)				(ohm-cm)	(mils)			(ft^2)	(%)	(ft^2)	(years)
4240	4200	4210	1363	10	5	0.3333	222	5.10	4000	460	1.40	0.22	3568.33	20	713.67	35
4250	4210	4215	991	10	5	0.3333	222	4.95	4000	460	1.40	0.22	2594.43	20	518.89	35
4255	4215	4220	841	10	5	0.3333	222	5.10	4000	460	1.40	0.22	2201.73	20	440.35	35
4260	4220	4230	888	10	5	0.3333	222	5.10	4000	460	1.40	0.22	2324.78	20	464.96	35
7180	4200	6990	194	8	4	0.3333	222	5.10	4000	430	1.40	0.22	406.31	20	81.26	35
7190	6990	7000	1098	8	4	0.3333	222	4.80	4000	430	1.40	0.22	2299.65	20	459.93	35
7200	7000	7010	578	8	4	0.3333	222	4.50	4000	430	1.40	0.22	1210.56	20	242.11	35
7210	7010	7020	95	8	4	0.3333	222	4.50	4000	430	1.40	0.22	198.97	20	39.79	35
7220	7030	7020	419	8	4	0.3333	222	4.80	4000	430	1.40	0.22	877.55	20	175.51	35
7230	4210	7030	151	8	4	0.3333	222	5.10	4000	430	1.40	0.22	316.25	20	63.25	35
7250	7070	7060	823	8	4	0.3333	222	4.50	4000	430	1.40	0.22	1723.69	20	344.74	35
7260	7060	4220	155	8	4	0.3333	222	5.10	4000	430	1.40	0.22	324.63	20	64.93	35
7300	7110	7060	303	8	4	0.3333	222	5.10	4000	430	1.40	0.22	634.60	20	126.92	35
7320	7130	7110	499	8	4	0.3333	222	4.80	4000	430	1.40	0.22	1045.10	20	209.02	35
7340	4230	7130	189	8	4	0.3333	222	5.10	4000	430	1.40	0.22	395.84	20	79.17	35
10135	7040	7070	59	6	3	0.3333	222	4.50	4000	250	1.40	0.22	92.68	20	18.54	35
10140	9780	7040	271	6	3	0.3333	222	4.50	4000	250	1.40	0.22	425.69	20	85.14	35
10145	9795	9780	730	6	3	0.3333	222	4.80	4000	250	1.40	0.22	1146.68	20	229.34	35
10150	7020	9795	208	6	3	0.3333	222	4.50	4000	250	1.40	0.22	326.73	20	65.35	35

Table 8.9: Results of the corrosion equations for the Mt. Tabor subnetwork

Pipe No.	Time to First Leak, t_L (years)	Number of Leaks, N	Number of Leaks, N (integer value)	Equivalent Orifice Area, A_{Lij} (ft^2)
4240	34.62	1.02	1	0.00166
4250	41.46	0.77	0	0
4255	47.62	0.63	0	0
4260	45.94	0.66	0	0
7180	118.66	0.16	0	0
7190	35.62	0.97	0	0
7200	51.43	0.56	0	0
7210	169.35	0.09	0	0
7220	67.26	0.37	0	0
7230	140.00	0.12	0	0
7250	40.73	0.79	0	0
7260	137.60	0.13	0	0
7300	88.41	0.25	0	0
7320	59.94	0.44	0	0
7340	120.72	0.15	0	0
10135	55.11	0.50	0	0
10140	20.15	2.31	2	0.00098
10145	11.08	5.71	5	0.00245
10150	23.99	1.77	1	0.00049

8.7 Application to the mathematical formulation

The key findings from the corrosion equations are knowledge of which pipes are leaking and values for the orifice area for the leaking pipes. Two situations will be analyzed using the Mt. Tabor subnetwork.

1. Increased nodal demands based on the percent leakage as determined by a water audit, with minimal use of corrosion equations. (Illustrative example of Method #3 from Chapter 5).
2. Pressure-dependent leakage based on the orifice equation for leaking pipes, based entirely on the corrosion equations. (Illustrative example of Method #2 from Chapter 5).

8.7.1 Percent increase in nodal demands

Assume that a water audit has been done on the entire Blacksburg water distribution system. During the audit it was found that the Mt. Tabor subnetwork loses approximately 10% of the inflow to leakage. This value of 10% will be added to each nodal demand to account for these losses with the assumption that leakage could occur in any pipe in the network. In this example we assume that corrosion potential is not a major factor, but rather the water audit is the driving factor for the working knowledge of the system. This is a fairly simple way to quickly assess the distribution of leakage in the network. (Please refer to Method #3 in Section 5.2.2.3 of Chapter 5).

However, one may use knowledge of leaking pipes in the objective function, which contains only the pipes that were found to be leaking due to corrosion pitting. The pipes identified as having corrosion holes are used in the objective function since these pipes have a higher probability of leaking. However, the assumption is made that all of the pipes could be leaking and does not limit leakage to the corroded pipes. This is a more conservative approach, but may oversimplify the problem. The following formulation beginning on the next page was input into LINGO. Please refer to Appendix A for an introduction to the LINGO software package. Appendix C gives the actual LINGO input for this run.

Objective Function

Maximize:
$(Qp4240 - Q4240)^2 + (Qp10140 - Q10140)^2 + (Qp10145 - Q10145)^2 + (Qp10150 - Q10150)^2$

Set 1: Continuity for demand written at each node using Equation (5.2)

1) $-Q4240 - Q7180 = D4200 - S4200$ (Node 4200)
2) $Q4240 - Q4250 - Q7230 = D4210$ (Node 4210)
3) $Q4250 - Q4255 = D4215$ (Node 4215)
4) $Q4255 + Q7260 - Q4260 = D4220$ (Node 4220)
5) $Q4260 - Q7340 = D4230$ (Node 4230)
6) $Q7180 - Q7190 = D6990$ (Node 6990)
7) $Q7190 - Q7200 = D7000$ (Node 7000)
8) $Q7200 - Q7210 = D7010$ (Node 7010)
9) $Q7210 + Q7220 - Q10150 = D7020$ (Node 7020)
10) $Q7230 - Q7220 = D7030$ (Node 7030)
11) $Q10140 - Q10135 = D7040$ (Node 7040)
12) $Q7250 + Q7300 - Q7260 = D7060$ (Node 7060)
13) $Q10135 - Q7250 = D7070$ (Node 7070)
14) $Q7320 - Q7300 = D7110$ (Node 7110)
15) $Q7340 - Q7320 = D7130$ (Node 7130)
16) $Q10145 - Q10140 = D9780$ (Node 9780)
17) $Q10150 - Q10145 = D9795$ (Node 9795)

Set 2: Continuity equation for combined demand and leakage using Equation (5.6)

1) $-Qp4240 - Qp7180 = (1.1*D4200) - Sp4200$ (Node 4200)
2) $Qp4240 - Qp4250 - Qp7230 = 1.1*D4210$ (Node 4210)
3) $Qp4250 - Qp4255 = 1.1*D4215$ (Node 4215)
4) $Qp4255 + Qp7260 - Qp4260 = 1.1*D4220$ (Node 4220)
5) $Qp4260 - Qp7340 = 1.1*D4230$ (Node 4230)
6) $Qp7180 - Qp7190 = 1.1*D6990$ (Node 6990)
7) $Qp7190 - Qp7200 = 1.1*D7000$ (Node 7000)
8) $Qp7200 - Qp7210 = 1.1*D7010$ (Node 7010)
9) $Qp7210 + Qp7220 - Qp10150 = 1.1*D7020$ (Node 7020)
10) $Qp7230 - Qp7220 = 1.1*D7030$ (Node 7030)
11) $Qp10140 - Qp10135 = 1.1*D7040$ (Node 7040)
12) $Qp7250 + Qp7300 - Qp7260 = 1.1*D7060$ (Node 7060)
13) $Qp10135 - Qp7250 = 1.1*D7070$ (Node 7070)
14) $Qp7320 - Qp7300 = 1.1*D7110$ (Node 7110)
15) $Qp7340 - Qp7320 = 1.1*D7130$ (Node 7130)
16) $Qp10145 - Qp10140 = 1.1*D9780$ (Node 9780)
17) $Qp10150 - Qp10145 = 1.1*D9795$ (Node 9795)

Set 3: Head Loss equations (including leakage flows) using Equation (5.14)

1) $(H4200 + 2163) - (H4210 + 2141) = r4240*Qp4240*|Qp4240|^{0.852}$ (Pipe 4240)
2) $(H4210 + 2141) - (H4215 + 2110) = r4250*Qp4250*|Qp4250|^{0.852}$ (Pipe 4250)
3) $(H4215 + 2110) - (H4220 + 2132) = r4255*Qp4255*|Qp4255|^{0.852}$ (Pipe 4255)
4) $(H4220 + 2132) - (H4230 + 2121) = r4260*Qp4260*|Qp4260|^{0.852}$ (Pipe 4260)
5) $(H4200 + 2163) - (H6990 + 2153.5) = r7180*Qp7180*|Qp7180|^{0.852}$ (Pipe 7180)
6) $(H6990 + 2153.5) - (H7000 + 2141.5) = r7190*Qp7190*|Qp7190|^{0.852}$ (Pipe 7190)
7) $(H7000 + 2141.5) - (H7010 + 2129) = r7200*Qp7200*|Qp7200|^{0.852}$ (Pipe 7200)
8) $(H7010 + 2129) - (H7020 + 2127) = r7210*Qp7210*|Qp7210|^{0.852}$ (Pipe 7210)
9) $(H7030 + 2127) - (H7020 + 2127) = r7220*Qp7220*|Qp7220|^{0.852}$ (Pipe 7220)
10) $(H4210 + 2141) - (H7030 + 2127) = r7230*Qp7230*|Qp7230|^{0.852}$ (Pipe 7230)
11) $(H7070 + 2110) - (H7060 + 2139) = r7250*Qp7250*|Qp7250|^{0.852}$ (Pipe 7250)
12) $(H7060 + 2139) - (H4220 + 2132) = r7260*Qp7260*|Qp7260|^{0.852}$ (Pipe 7260)
13) $(H7110 + 2144.5) - (H7060 + 2139) = r7300*Qp7300*|Qp7300|^{0.852}$ (Pipe 7300)
14) $(H7130 + 2116) - (H7110 + 2144.5) = r7320*Qp7320*|Qp7320|^{0.852}$ (Pipe 7320)
15) $(H4230 + 2121) - (H7130 + 2116) = r7340*Qp7340*|Qp7340|^{0.852}$ (Pipe 7340)
16) $(H7040 + 2109.5) - (H7070 + 2110) = r10135*Qp10135*|Qp10135|^{0.852}$ (Pipe 10135)
17) $(H9780 + 2099.5) - (H7040 + 2109.5) = r10140*Qp10140*|Qp10140|^{0.852}$ (Pipe 10140)
18) $(H9795 + 2120) - (H9780 + 2099.5) = r10145*Qp10145*|Qp10145|^{0.852}$ (Pipe 10145)
19) $(H7020 + 2127) - (H9795 + 2120) = r10150*Qp10150*|Qp10150|^{0.852}$ (Pipe 10150)

Set 4: Constraint on leakage flows using Equation (5.16)

1) $|Sp4200| \geq |S4200|$ (Supply)
2) $|Qp4240| \geq |Q4240|$ (Pipe 4240)
3) $|Qp4250| \geq |Q4250|$ (Pipe 4250)
4) $|Qp4255| \geq |Q4255|$ (Pipe 4255)
5) $|Qp4260| \geq |Q4260|$ (Pipe 4260)
6) $|Qp7180| \geq |Q7180|$ (Pipe 7180)
7) $|Qp7190| \geq |Q7190|$ (Pipe 7190)
8) $|Qp7200| \geq |Q7200|$ (Pipe 7200)
9) $|Qp7210| \geq |Q7210|$ (Pipe 7210)
10) $|Qp7220| \geq |Q7220|$ (Pipe 7220)
11) $|Qp7230| \geq |Q7230|$ (Pipe 7230)
12) $|Qp7250| \geq |Q7250|$ (Pipe 7250)
13) $|Qp7260| \geq |Q7260|$ (Pipe 7260)
14) $|Qp7300| \geq |Q7300|$ (Pipe 7300)
15) $|Qp7320| \geq |Q7320|$ (Pipe 7320)
16) $|Qp7340| \geq |Q7340|$ (Pipe 7340)
17) $|Qp10135| \geq |Q10135|$ (Pipe 10135)
18) $|Qp10140| \geq |Q10140|$ (Pipe 10140)
19) $|Qp10145| \geq |Q10145|$ (Pipe 10145)
20) $|Qp10150| \geq |Q10150|$ (Pipe 10150)

Set 5: Set flow direction for leakage and normal flows using Equation (5.17)

1) Sp4200*S4200 - |Sp4200|*|S4200| = 0 (Supply)
2) Qp4240*Q4240 - |Qp4240|*|Q4240| = 0 (Pipe 4240)
3) Qp4250*Q4250 - |Qp4250|*|Q4250| = 0 (Pipe 4250)
4) Qp4255*Q4255 - |Qp4255|*|Q4255| = 0 (Pipe 4255)
5) Qp4260*Q4260 - |Qp4260|*|Q4260| = 0 (Pipe 4260)
6) Qp7180*Q7180 - |Qp7180|*|Q7180| = 0 (Pipe 7180)
7) Qp7190*Q7190 - |Qp7190|*|Q7190| = 0 (Pipe 7190)
8) Qp7200*Q7200 - |Qp7200|*|Q7200| = 0 (Pipe 7200)
9) Qp7210*Q7210 - |Qp7210|*|Q7210| = 0 (Pipe 7210)
10) Qp7220*Q7220 - |Qp7220|*|Q7220| = 0 (Pipe 7220)
11) Qp7230*Q7230 - |Qp7230|*|Q7230| = 0 (Pipe 7230)
12) Qp7250*Q7250 - |Qp7250|*|Q7250| = 0 (Pipe 7250)
13) Qp7260*Q7260 - |Qp7260|*|Q7260| = 0 (Pipe 7260)
14) Qp7300*Q7300 - |Qp7300|*|Q7300| = 0 (Pipe 7300)
15) Qp7320*Q7320 - |Qp7320|*|Q7320| = 0 (Pipe 7320)
16) Qp7340*Q7340 - |Qp7340|*|Q7340| = 0 (Pipe 7340)
17) Qp10135*Q10135 - |Qp10135|*|Q10135| = 0 (Pipe 10135)
18) Qp10140*Q10140 - |Qp10140|*|Q10140| = 0 (Pipe 10140)
19) Qp10145*Q10145 - |Qp10145|*|Q10145| = 0 (Pipe 10145)
20) Qp10150*Q10150 - |Qp10150|*|Q10150| = 0 (Pipe 10150)

The following demands values, in units of cfs, are given as estimates by the Town of Blacksburg.

1) D4200 = 0.00129 (Node 4200)
2) D4210 = 0.00470 (Node 4210)
3) D4215 = 0 (Node 4215)
4) D4220 = 0.01201 (Node 4220)
5) D4230 = 0.02012 (Node 4230)
6) D6990 = 0.00856 (Node 6990)
7) D7000 = 0.01328 (Node 7000)
8) D7010 = 0.00385 (Node 7010)
9) D7020 = 0.00129 (Node 7020)
10) D7030 = 0.00602 (Node 7030)
11) D7040 = 0.00985 (Node 7040)
12) D7060 = 0.00172 (Node 7060)
13) D7070 = 0.00470 (Node 7070)
14) D7110 = 0.00813 (Node 7110)
15) D7130 = 0.00085 (Node 7130)
16) D9780 = 0.00900 (Node 9780)
17) D9795 = 0.00428 (Node 9795)

The following are known values given by the Town of Blacksburg:
S4200 = 0.10965 (Initial supply into the subnetwork at Node 4200)
Sp4200 = 1.1*S4200 (Supply necessary to satisfy leakage in the subnetwork)
H4200 = 113.65 (Fixed head at Node 4200)

8.7.1.1 Results of LINGO runs

The input and corresponding output for this LINGO run is given in the first half of Appendix C. Tables 8.10 and 8.11 summarize the results of the LINGO runs for the case of percent leakage for increased demands.

Table 8.10: Results of pipe flows and pipe leakage for the case of percent leakage

Pipe No.	Head Node	Tail Node	Qp_{ij} (cfs)	Q_{ij} (cfs)	Qp_{ij} - Q_{ij} (cfs)	L_{ij} (cfs)
4240	4200	4210	0.07275	0.06448	0.00827	0.00047
4250	4210	4215	0.05412	0.04937	0.00475	0
4255	4215	4220	0.05412	0.04937	0.00475	0.00120
4260	4220	4230	0.02010	0.02010	0	0
7180	4200	6990	0.04645	0.04388	0.00257	0.00086
7190	6990	7000	0.03703	0.03532	0.00171	0.00133
7200	7000	7010	0.02242	0.02204	0.00038	0.00039
7210	7010	7020	0.01819	0.01819	0	0
7220	7030	7020	0.00683	0.00439	0.00244	0.00013
7230	4210	7030	0.01345	0.01041	0.00304	0.00060
7250	7070	7060	-0.00701	-0.00654	-0.00047	0.00047
7260	7060	4220	-0.02082	-0.01727	-0.00355	0.00017
7300	7110	7060	-0.01191	-0.00900	-0.00291	0.00081
7320	7130	7110	-0.00297	-0.00087	-0.00210	0.00008
7340	4230	7130	-0.002036	-0.00002	-0.00201	0.00201
10135	7040	7070	-0.00184	-0.00184	0	0
10140	9780	7040	0.00899	0.00801	0.00098	0.00098
10145	9795	9780	0.01889	0.01701	0.00188	0.00090
10150	7020	9795	0.02360	0.02129	0.00231	0.00043

Table 8.11: Results of nodal heads and outflows

Node No.	Demand D_i (cfs)	% Leakage (%)	Total outflow (cfs)	Nodal leakage L_i (cfs)	Head H_i (ft)
4200	0.00129	10	0.00142	0.00013	113.65
4210	0.00470	10	0.00517	0.00047	135.63
4215	0	10	0	0	166.63
4220	0.01201	10	0.01321	0.00120	144.62
4230	0.02012	10	0.02213	0.00201	155.62
6990	0.00856	10	0.00941	0.00086	123.15
7000	0.01328	10	0.01461	0.00133	135.14
7010	0.00385	10	0.00424	0.00039	147.63
7020	0.00129	10	0.00142	0.00013	149.63
7030	0.00602	10	0.00662	0.00060	149.63
7040	0.00985	10	0.01083	0.00098	167.12
7060	0.00172	10	0.00189	0.00017	137.62
7070	0.00470	10	0.00517	0.00047	166.62
7110	0.00813	10	0.00895	0.00081	132.12
7130	0.00085	10	0.00093	0.00008	160.62
9780	0.00900	10	0.00990	0.00090	177.12
9795	0.00428	10	0.00471	0.00043	156.63

8.7.2 Pressure-dependent leakage for corrosion holes

This second case relies on the results of the corrosion equations to come up with equivalent orifice areas to be used in the orifice equation. (Please refer to Method #2 in Section 5.2.2.2 of Chapter 5). The leakage leaving the corroded pipes is found by using the average nodal pressure in the orifice equation. In this method, the leakage losses are found directly through solving the formulation, there is no second step necessary to allocate the leakage among pipes. Leakage allocation among pipes is built-in to the formulation of Method #2. Once again, the objective function in this case only contains the pipes that are known to be leaking based on the corrosion equations. The following equations were input into LINGO and solved. The input for this case is shown in Appendix C.

Objective Function

Maximize:
$(Qp4240 - Q4240)^2 + (Qp10140 - Q10140)^2 + (Qp10145 - Q10145)^2 + (Qp10150 - Q10150)^2$

Set 1: Continuity for demand written at each node using Equation (5.2)

1) -Q4240 - Q7180 = D4200 - S4200 (Node 4200)
2) Q4240 - Q4250 - Q7230 = D4210 (Node 4210)
3) Q4250 - Q4255 = D4215 (Node 4215)
4) Q4255 + Q7260 - Q4260 = D4220 (Node 4220)
5) Q4260 - Q7340 = D4230 (Node 4230)
6) Q7180 - Q7190 = D6990 (Node 6990)
7) Q7190 - Q7200 = D7000 (Node 7000)
8) Q7200 - Q7210 = D7010 (Node 7010)
9) Q7210 + Q7220 - Q10150 = D7020 (Node 7020)
10) Q7230 - Q7220 = D7030 (Node 7030)
11) Q10140 - Q10135 = D7040 (Node 7040)
12) Q7250 + Q7300 - Q7260 = D7060 (Node 7060)
13) Q10135 - Q7250 = D7070 (Node 7070)
14) Q7320 - Q7300 = D7110 (Node 7110)
15) Q7340 - Q7320 = D7130 (Node 7130)
16) Q10145 - Q10140 = D9780 (Node 9780)
17) Q10150 - Q10145 = D9795 (Node 9795)

Set 2: Continuity equation for combined demand and leakage using Equation (5.4)

1) -Qp4240 - Qp7180 = D4200 + (L4240/2) - Sp4200 (Node 4200)
2) Qp4240 - Qp4250 - Qp7230 = D4210 + (L4240/2) (Node 4210)
3) Qp4250 - Qp4255 = D4215 (Node 4215)
4) Qp4255 + Qp7260 - Qp4260 = D4220 (Node 4220)
5) Qp4260 - Qp7340 = D4230 (Node 4230)
6) Qp7180 - Qp7190 = D6990 (Node 6990)
7) Qp7190 - Qp7200 = D7000 (Node 7000)
8) Qp7200 - Qp7210 = D7010 (Node 7010)
9) Qp7210 + Qp7220 - Qp10150 = D7020 + (L10150/2) (Node 7020)
10) Qp7230 - Qp7220 = D7030 (Node 7030)
11) Qp10140 - Qp10135 = D7040 + (L10140/2) (Node 7040)
12) Qp7250 + Qp7300 - Qp7260 = D7060 (Node 7060)
13) Qp10135 - Qp7250 = D7070 (Node 7070)
14) Qp7320 - Qp7300 = D7110 (Node 7110)
15) Qp7340 - Qp7320 = D7130 (Node 7130)
16) Qp10145 - Qp10140 = D9780 + (L10140/2) + (L10145/2) (Node 9780)
17) Qp10150 - Qp10145 = D9795 + (L10145/2) + (L10150/2) (Node 9795)
18) Sp4200 = S4200 + L4240 + L10140 + L10145 + L10150 (Overall balance)

Set 3: Head Loss equations (including leakage flows) using Equation (5.13)

1) $(H4200 + 2163) - (H4210 + 2141) = r4240*Qp4240*|Qp4240|^{0.852}$ (Pipe 4240)
2) $(H4210 + 2141) - (H4215 + 2110) = r4250*Qp4250*|Qp4250|^{0.852}$ (Pipe 4250)
3) $(H4215 + 2110) - (H4220 + 2132) = r4255*Qp4255*|Qp4255|^{0.852}$ (Pipe 4255)
4) $(H4220 + 2132) - (H4230 + 2121) = r4260*Qp4260*|Qp4260|^{0.852}$ (Pipe 4260)
5) $(H4200 + 2163) - (H6990 + 2153.5) = r7180*Qp7180*|Qp7180|^{0.852}$ (Pipe 7180)
6) $(H6990 + 2153.5) - (H7000 + 2141.5) = r7190*Qp7190*|Qp7190|^{0.852}$ (Pipe 7190)
7) $(H7000 + 2141.5) - (H7010 + 2129) = r7200*Qp7200*|Qp7200|^{0.852}$ (Pipe 7200)
8) $(H7010 + 2129) - (H7020 + 2127) = r7210*Qp7210*|Qp7210|^{0.852}$ (Pipe 7210)
9) $(H7030 + 2127) - (H7020 + 2127) = r7220*Qp7220*|Qp7220|^{0.852}$ (Pipe 7220)
10) $(H4210 + 2141) - (H7030 + 2127) = r7230*Qp7230*|Qp7230|^{0.852}$ (Pipe 7230)
11) $(H7070 + 2110) - (H7060 + 2139) = r7250*Qp7250*|Qp7250|^{0.852}$ (Pipe 7250)
12) $(H7060 + 2139) - (H4220 + 2132) = r7260*Qp7260*|Qp7260|^{0.852}$ (Pipe 7260)
13) $(H7110 + 2144.5) - (H7060 + 2139) = r7300*Qp7300*|Qp7300|^{0.852}$ (Pipe 7300)
14) $(H7130 + 2116) - (H7110 + 2144.5) = r7320*Qp7320*|Qp7320|^{0.852}$ (Pipe 7320)
15) $(H4230 + 2121) - (H7130 + 2116) = r7340*Qp7340*|Qp7340|^{0.852}$ (Pipe 7340)
16) $(H7040 + 2109.5) - (H7070 + 2110) = r10135*Qp10135*|Qp10135|^{0.852}$ (Pipe 10135)
17) $(H9780 + 2099.5) - (H7040 + 2109.5) = r10140*Qp10140*|Qp10140|^{0.852}$ (Pipe 10140)
18) $(H9795 + 2120) - (H9780 + 2099.5) = r10145*Qp10145*|Qp10145|^{0.852}$ (Pipe 10145)
19) $(H7020 + 2127) - (H9795 + 2120) = r10150*Qp10150*|Qp10150|^{0.852}$ (Pipe 10150)

Set 4: Pressure-Dependent Leakage equations using Equation (5.8)

1) $L4240 = c_d*A4240*(2*g*((H4200+H4210)/2))^{0.5}$ (Pipe 4240)
2) $L10140 = c_d*A10140*(2*g*((H9780+H7040)/2))^{0.5}$ (Pipe 10140)
3) $L10145 = c_d*A10145*(2*g*((H9795+H9780)/2))^{0.5}$ (Pipe 10145)
4) $L10150 = c_d*A10150*(2*g*((H7020+H9795)/2))^{0.5}$ (Pipe 10150)

Set 5: Constraint on leakage flows using Equation (5.15)

1) $|Sp4200| \geq |S4200|$ (Supply)
2) $|Qp4240| \geq |Q4240|$ (Pipe 4240)
3) $|Qp4250| \geq |Q4250|$ (Pipe 4250)
4) $|Qp4255| \geq |Q4255|$ (Pipe 4255)
5) $|Qp4260| \geq |Q4260|$ (Pipe 4260)
6) $|Qp7180| \geq |Q7180|$ (Pipe 7180)
7) $|Qp7190| \geq |Q7190|$ (Pipe 7190)
8) $|Qp7200| \geq |Q7200|$ (Pipe 7200)
9) $|Qp7210| \geq |Q7210|$ (Pipe 7210)
10) $|Qp7220| \geq |Q7220|$ (Pipe 7220)
11) $|Qp7230| \geq |Q7230|$ (Pipe 7230)
12) $|Qp7250| \geq |Q7250|$ (Pipe 7250)
13) $|Qp7260| \geq |Q7260|$ (Pipe 7260)
14) $|Qp7300| \geq |Q7300|$ (Pipe 7300)
15) $|Qp7320| \geq |Q7320|$ (Pipe 7320)
16) $|Qp7340| \geq |Q7340|$ (Pipe 7340)
17) $|Qp10135| \geq |Q10135|$ (Pipe 10135)
18) $|Qp10140| \geq |Q10140|$ (Pipe 10140)
19) $|Qp10145| \geq |Q10145|$ (Pipe 10145)
20) $|Qp10150| \geq |Q10150|$ (Pipe 10150)

Set 6: Set flow direction for leakage and normal flows using Equation (5.16)

1) $Sp4200*S4200 - |Sp4200|*|S4200| = 0$ (Supply)
2) $Qp4240*Q4240 - |Qp4240|*|Q4240| = 0$ (Pipe 4240)
3) $Qp4250*Q4250 - |Qp4250|*|Q4250| = 0$ (Pipe 4250)
4) $Qp4255*Q4255 - |Qp4255|*|Q4255| = 0$ (Pipe 4255)
5) $Qp4260*Q4260 - |Qp4260|*|Q4260| = 0$ (Pipe 4260)
6) $Qp7180*Q7180 - |Qp7180|*|Q7180| = 0$ (Pipe 7180)
7) $Qp7190*Q7190 - |Qp7190|*|Q7190| = 0$ (Pipe 7190)
8) $Qp7200*Q7200 - |Qp7200|*|Q7200| = 0$ (Pipe 7200)
9) $Qp7210*Q7210 - |Qp7210|*|Q7210| = 0$ (Pipe 7210)
10) $Qp7220*Q7220 - |Qp7220|*|Q7220| = 0$ (Pipe 7220)
11) $Qp7230*Q7230 - |Qp7230|*|Q7230| = 0$ (Pipe 7230)
12) $Qp7250*Q7250 - |Qp7250|*|Q7250| = 0$ (Pipe 7250)
13) $Qp7260*Q7260 - |Qp7260|*|Q7260| = 0$ (Pipe 7260)
14) $Qp7300*Q7300 - |Qp7300|*|Q7300| = 0$ (Pipe 7300)
15) $Qp7320*Q7320 - |Qp7320|*|Q7320| = 0$ (Pipe 7320)
16) $Qp7340*Q7340 - |Qp7340|*|Q7340| = 0$ (Pipe 7340)
17) $Qp10135*Q10135 - |Qp10135|*|Q10135| = 0$ (Pipe 10135)
18) $Qp10140*Q10140 - |Qp10140|*|Q10140| = 0$ (Pipe 10140)
19) $Qp10145*Q10145 - |Qp10145|*|Q10145| = 0$ (Pipe 10145)
20) $Qp10150*Q10150 - |Qp10150|*|Q10150| = 0$ (Pipe 10150)

<u>The following demands values, in units of cfs, are given as estimates by the Town of Blacksburg</u>.

1) D4200 = 0.00129 (Node 4200)
2) D4210 = 0.00470 (Node 4210)
3) D4215 = 0 (Node 4215)
4) D4220 = 0.01201 (Node 4220)
5) D4230 = 0.02012 (Node 4230)
6) D6990 = 0.00856 (Node 6990)
7) D7000 = 0.01328 (Node 7000)
8) D7010 = 0.00385 (Node 7010)
9) D7020 = 0.00129 (Node 7020)
10) D7030 = 0.00602 (Node 7030)
11) D7040 = 0.00985 (Node 7040)
12) D7060 = 0.00172 (Node 7060)
13) D7070 = 0.00470 (Node 7070)
14) D7110 = 0.00813 (Node 7110)
15) D7130 = 0.00085 (Node 7130)
16) D9780 = 0.00900 (Node 9780)
17) D9795 = 0.00428 (Node 9795)

The following are known values from the Town of Blacksburg. The orifice areas were computed previously from the corrosion equations.

S4200 = 0.10965 (Initial supply into the subnetwork at Node 4200)
H4200 = 113.65 (Fixed head at Node 4200)
C = 0.61 (Orifice coefficient)
g = 32.2 (Acceleration due to gravity)
A4240 = 0.0415 (Equivalent orifice area for leakage in Pipe 4240)
A10140 = 0.0245 (Equivalent orifice area for leakage in Pipe 10140)
A10145 = 0.0614 (Equivalent orifice area for leakage in Pipe 10145)
A10150 = 0.0123 (Equivalent orifice area for leakage in Pipe 10150)

8.7.2.1 Results of LINGO runs

The input and corresponding output for this LINGO run is given in the second half of Appendix C. Table 8.11 and 8.12 summarize the results of this case of orifice leakage through corrosion holes. Note that a direct solution for pipe leakage losses, L_{ij}, was made in this run as shown in Table 8.12. This method is the most sophisticated and challenging of the three methods. However, it does provide the most direct and accurate results for determining actual leakage losses from specific pipes in water distribution system.

Table 8.12: Results of pipe flows and leakage for the case of orifice leakage

Pipe No.	Head Node	Tail Node	Qp_{ij} (cfs)	Q_{ij} (cfs)	Qp_{ij} - Q_{ij} (cfs)	L_{ij} (cfs)
4240	4200	4210	0.26355	0.06810	0.19545	0.09069
4250	4210	4215	0.15032	0.05738	0.09294	0
4255	4215	4220	0.15032	0.05738	0.09294	0
4260	4220	4230	0.04576	0.04537	0.00039	0
7180	4200	6990	0.13746	0.04026	0.09720	0
7190	6990	7000	0.12890	0.03170	0.09720	0
7200	7000	7010	0.11562	0.01842	0.09720	0
7210	7010	7020	0.11177	0.01457	0.09720	0
7220	7030	7020	0.05717	0	0.05717	0
7230	4210	7030	0.06319	0.00602	0.05717	0
7250	7070	7060	-0.10749	-0.01455	-0.09294	0
7260	7060	4220	-0.09255	0	-0.09255	0
7300	7110	7060	0.01666	0.01627	0.00039	0
7320	7130	7110	0.02479	0.02440	0.00039	0
7340	4230	7130	0.02564	0.02525	0.00039	0
10135	7040	7070	-0.10279	-0.00985	-0.09294	0
10140	9780	7040	-0.06150	0	-0.06150	0.06287
10145	9795	9780	0.05632	0.00900	0.04732	0.15477
10150	7020	9795	0.15281	0.01328	0.13953	0.02966

Table 8.13: Nodal demands and resultant pressure heads

Node #	Demand D_i (cfs)	Head H_i (ft)
4200	0.00129	113.65
4210	0.00470	135.46
4215	0	166.42
4220	0.01201	144.37
4230	0.02012	155.37
6990	0.00856	123.13
7000	0.01328	135.01
7010	0.00385	147.46
7020	0.00129	149.45
7030	0.00602	149.46
7040	0.00985	166.78
7060	0.00172	137.37
7070	0.00470	166.30
7110	0.00813	131.87
7130	0.00085	160.37
9780	0.00900	176.75
9795	0.00428	156.32

Chapter 9

Thesis Summary

9.1 Review of the thesis

This thesis has provided a comprehensive overview of the problem of leakage in water distribution systems. This thesis has also provided a model by which leakage flows can be determined for a pipe network. The first three chapters give a comprehensive overview of leakage and leak location techniques. The emphasis is placed on the operational problem, mainly that the outflow from a pipe network is pressure-dependent as discussed in Chapter 4. The mathematical formulation presented in Chapter 5 includes all of the hydraulic equations and rules necessary to determine pipe flows, nodal heads, demands, and leakage from a network. In Chapter 6 corrosion is identified as the main culprit in leakage. The formation of corrosion perforations is utilized to enhance and solidify the usefulness of the mathematical formation. The examples given in Chapters 7 and 8 clearly show how the formulation can be used to solve pipe network problems.

9.2 Contributions of this work

This thesis has provided several contributions to the area of water distribution system leakage analysis. This thesis has provided a comprehensive formulation for leakage assessment in water distribution systems. The research has formed a methodology by which leakage may be allocated at nodes based on the findings of a well-conducted water audit. In addition, the work done on this thesis has contributed a new formulation that can account for demand and leakage flows separately based on the use of Qp_{ij} and Q_{ij} terms in the continuity equation, thereby permitting the allocation of nodal leakage among converging pipes using a weighted average method. The final contribution is that corrosion has been identified as a major contributor to leakage. The introduction of a vulnerability index to identify the pipe surface area that is susceptible to corrosion has been presented. Corrosion hole information is used to solve directly for the leakage loss flowrate from corroded pipes.

9.3 Further research in this area

There is much potential for research in this area of leakage in water distribution systems. The main impetus for research needs to be in the area of corrosion potential. One research project could focus on a better method for determining the diameter of corrosion holes due to pitting. By getting a better handle on the size and shape of corrosion holes a better estimate of the equivalent orifice area can be obtained. Another research interest could also focus on locating points where corrosion potential is the greatest for a given pipe. By understanding the corrosion potential, the vulnerability index of the pipe can be more closely estimated to determine what percentage of the surface area to input as being exposed.

Another area of research could focus on the financial aspects involved in the rehabilitation of pipe networks. Finding ways to use the formulation presented in this thesis to determine whole-life costing of pipe replacement or rehabilitation schemes will be an important contribution. In order to maximize the benefits, an innovative cost analysis will relate the cost of the pipe servicing customers to the amount paid by those customers. Whole-life costing of a pipe could potentially allow for older pipes to be replaced before significant failures occur in that pipe. By continually putting money back into the system, replacement costs will supersede costly maintenance of the system. Theoretically, a given pipe should pay for itself over its life through water costs paid by the consumers who have service connections on that given water main.

Chapter 10

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Appendix A

LINGO Primer

LINGO Primer

LINGO overview and capabilities

LINGO is a software program that allows the user to either solve a system of equations or optimize a linear or nonlinear mathematical program. Mathematical programs involve three main components. The following three components are necessary to formulate a linear or nonlinear program:

1. Variables: The first step in problem formulation is to identify all the unknown variables involved in the problem. After choosing the variables, they must be represented by algebraic symbols.
2. Constraints: In a model there are always limitations or physical laws that restrict the values of variables. These limitation formulas may be written in the form of an equality or inequality. (In LINGO, inequalities may be expressed with either >= or just >, the equal sign is not necessary.) Constraints are used to constrain the values that variables can take.
3. Objective Function: The purpose of the objective function is to optimize a certain formula that is based on the decision variables. The objective function may be either maximized or minimized depending on the desired outcome.

Entering the model into LINGO

The first step in entering the model is to type in “MODEL:” at the top of the screen. Every model must begin with the word MODEL followed by a colon. Next, the objective function should be typed in the form “MAX = …” or “MIN = …” with a semi-colon at the end of the line. The next step is to type in all of the constraints. Each constraint must include either >, <, or = to separate the left and right hand sides of the equation. The ">" sign constrains the left hand side of the equation to be greater than or equal to the right hand side. The "<" sign constrains the left hand side of the equation to be less than or equal to the right hand side. The "=" sign constrains the left hand side of the equation to be equal to the right hand side. Each constraint must be followed by a semi-colon. After typing in all of the constraints, the word "END" must be typed in at the end of the model. The following example shows how the model is to be input.

```
!Jonathan's Model;
MODEL:
MAX = 100 * X + 150 * Y;
X < 100;
Y < 120;
X + 2 * Y < 160;
END
```

The following bulleted items give some helpful hints for writing a LINGO model:

- An initial guess for variables can be made by using a subroutine within the model. Many times LINGO will not be able to converge on a solution unless initial values are given, this is especially true for nonlinear programs. The following is an example of what the INIT section should look like within a model.

  ```
  INIT:
  X=50;
  Y=100;
  ENDINIT
  ```

- The symbol "@abs" is used to take the absolute value of a variable or term.
- The symbol "@free" is used to allow a variable to take a negative value. (LINGO assumes nonnegativity unless otherwise specified using this symbol.)
- Commentary can be made within the model by beginning the comment with an exclamation point, !, and ending the comment with a semi-colon.

Appendix B

LINGO Runs for the Pudar-Liggett Formulation

```
!This model is the same formulation given by Pudar and Liggett;
!Even-determined problem;
!Zero leaks condition;

Model:

!Continuity Equation for Demand and Leakage;
  -Qp12-Qp17=-Sp1;                  !Node 1;
  Qp12+Qp72-Qp24-Qp23=0;            !Node 2;
  Qp23+Qp43=D3;                     !Node 3;
  Qp54+Qp64+Qp74+Qp24-Qp43=0;       !Node 4;
  Qp65-Qp54=-S5;                    !Node 5;
  -Qp64-Qp65-Qp67=-S6;              !Node 6;
  Qp17+Qp67-Qp72-Qp74=0;            !Node 7;

!Energy equation with leakage flows;
  H1-H2=r12*Qp12*((@abs(Qp12))^0.852);    !Pipe 1;
  H1-H7=r17*Qp17*((@abs(Qp17))^0.852);    !Pipe 2;
  H7-H2=r72*Qp72*((@abs(Qp72))^0.852);    !Pipe 3;
  H2-H4=r24*Qp24*((@abs(Qp24))^0.852);    !Pipe 4;
  H2-H3=r23*Qp23*((@abs(Qp23))^0.852);    !Pipe 5;
  H4-H3=r43*Qp43*((@abs(Qp43))^0.852);    !Pipe 6;
  H7-H4=r74*Qp74*((@abs(Qp74))^0.852);    !Pipe 7;
  H6-H4=r64*Qp64*((@abs(Qp64))^0.852);    !Pipe 8;
  H5-H4=r54*Qp54*((@abs(Qp54))^0.852);    !Pipe 9;
  H6-H5=r65*Qp65*((@abs(Qp65))^0.852);    !Pipe 10;
  H6-H7=r67*Qp67*((@abs(Qp67))^0.852);    !Pipe 11;

!Resistance Coefficients based on the Hazen-Williams equation;
  r12=392.9136;
  r17=276.9719;
  r72=821.1570;
  r24=1164.8971;
  r23=276.9719;
  r43=276.9719;
  r74=276.9719;
  r64=392.9136;
  r54=821.1570;
  r65=3333.7099;
  r67=3333.7099;

!Pressure-dependent outflow (orifice equation);
  D3=C*A3*((2*g*H3)^0.5);

!Allow for flow reversal in each pipe;
  @free(Qp12);
  @free(Qp17);
  @free(Qp72);
  @free(Qp24);
  @free(Qp23);
  @free(Qp43);
  @free(Qp74);
  @free(Qp64);
  @free(Qp54);
  @free(Qp65);
  @free(Qp67);
```

```
!Given values in the Pudar-Liggett paper;
  H1=30.48;
  S5=0.01893;
  S6=0.01262;
  A3=0.0054154736;
  C=1;
  g=9.81;
End
```

```
Rows=     20 Vars=     19 No. integer vars=      0
 Nonlinear rows=     12 Nonlinear vars=     12 Nonlinear constraints=     12
 Nonzeros=     61 Constraint nonz=    57 Density=0.153
 No. < :   0 No. =:  19 No. > :   0, Obj=MIN Single cols=    1

 Feasible solution found at step:       50


                                                       Variable           Value
                                                           QP12       0.4676561E-01
                                                           QP17       0.4743015E-01
                                                            SP1       0.9419577E-01
                                                           QP72       0.1568544E-01
                                                           QP24      -0.4049197E-03
                                                           QP23       0.6285597E-01
                                                           QP43       0.6288979E-01
                                                             D3       0.1257458
                                                           QP54       0.1547429E-01
                                                           QP64       0.1968513E-01
                                                           QP74       0.2813529E-01
                                                           QP65      -0.3455705E-02
                                                             S5       0.1893000E-01
                                                           QP67      -0.3609422E-02
                                                             S6       0.1262000E-01
                                                             H1        30.48000
                                                             H2        29.12811
                                                            R12        392.9136
                                                             H7        29.50155
                                                            R17        276.9719
                                                            R72        821.1570
                                                             H4        29.12941
                                                            R24        1164.897
                                                             H3        27.47989
                                                            R23        276.9719
                                                            R43        276.9719
                                                            R74        276.9719
                                                             H6        29.40170
                                                            R64        392.9136
                                                             H5        29.49381
                                                            R54        821.1570
                                                            R65        3333.710
                                                            R67        3333.710
                                                              C        1.000000
                                                             A3       0.5415474E-02
                                                              G        9.810000

                                                            Row    Slack or Surplus
                                                              1       0.0000000
                                                              2       0.0000000
                                                              3       0.0000000
                                                              4       0.0000000
                                                              5       0.0000000
                                                              6       0.0000000
                                                              7       0.0000000
                                                              8      -0.1900189E-03
                                                              9      -0.1130802E-03
                                                             10      -0.2326881E-03
```

```
11      -0.6864792E-03
12      -0.1648987E-03
13      -0.1842216E-03
14      -0.2223710E-03
15      -0.2444417E-06
16      -0.5170276E-06
17      -0.2754639E-05
18      -0.7060599E-05
19       0.0000000
20       0.0000000
21       0.0000000
22       0.0000000
23       0.0000000
24       0.0000000
25       0.0000000
26       0.0000000
27       0.0000000
28       0.0000000
29       0.0000000
30      -0.1292936E-07
31       0.0000000
32       0.0000000
33       0.0000000
34       0.0000000
35       0.0000000
36       0.0000000
```

```
!This model is the same formulation given by Pudar and Liggett;
!Even-determined problem;

!One leak condition (Leak at Node 2);

Model:

!Continuity Equation for Demand and Leakage;
  -Qp12-Qp17=-Sp1;                   !Node 1;
  Qp12+Qp72-Qp24-Qp23=L2;            !Node 2;
  Qp23+Qp43=D3;                      !Node 3;
  Qp54+Qp64+Qp74+Qp24-Qp43=0;        !Node 4;
  Qp65-Qp54=-S5;                     !Node 5;
  -Qp64-Qp65-Qp67=-S6;               !Node 6;
  Qp17+Qp67-Qp72-Qp74=0;             !Node 7;

!Energy equation with leakage flows;
  H1-H2=r12*Qp12*((@abs(Qp12))^0.852);    !Pipe 1;
  H1-H7=r17*Qp17*((@abs(Qp17))^0.852);    !Pipe 2;
  H7-H2=r72*Qp72*((@abs(Qp72))^0.852);    !Pipe 3;
  H2-H4=r24*Qp24*((@abs(Qp24))^0.852);    !Pipe 4;
  H2-H3=r23*Qp23*((@abs(Qp23))^0.852);    !Pipe 5;
  H4-H3=r43*Qp43*((@abs(Qp43))^0.852);    !Pipe 6;
  H7-H4=r74*Qp74*((@abs(Qp74))^0.852);    !Pipe 7;
  H6-H4=r64*Qp64*((@abs(Qp64))^0.852);    !Pipe 8;
  H5-H4=r54*Qp54*((@abs(Qp54))^0.852);    !Pipe 9;
  H6-H5=r65*Qp65*((@abs(Qp65))^0.852);    !Pipe 10;
  H6-H7=r67*Qp67*((@abs(Qp67))^0.852);    !Pipe 11;

!Resistance Coefficients based on the Hazen-Williams equation;
  r12=392.9136;
  r17=276.9719;
  r72=821.1570;
  r24=1164.8971;
  r23=276.9719;
  r43=276.9719;
  r74=276.9719;
  r64=392.9136;
  r54=821.1570;
  r65=3333.7099;
  r67=3333.7099;

!Pressure-dependent outflow (orifice equation);
  D3=C*A3*((2*g*H3)^0.5);
  L2=C*A2*((2*g*H2)^0.5);

!Allow for flow reversal in each pipe;
  @free(Qp12);
  @free(Qp17);
  @free(Qp72);
  @free(Qp24);
  @free(Qp23);
  @free(Qp43);
  @free(Qp74);
  @free(Qp64);
  @free(Qp54);
  @free(Qp65);
```

```
  @free(Qp67);

!Given values in the Pudar-Liggett paper;
  H1=30.48;
  S5=0.01893;
  S6=0.01262;
  A2=0.00232;
  A3=0.0054154736;
  C=1;
  g=9.81;

End
```

```
Rows=     21 Vars=     20 No. integer vars=      0
 Nonlinear rows=     13 Nonlinear vars=     13 Nonlinear constraints=     13
 Nonzeros=     64 Constraint nonz=    60 Density=0.145
 No. < :   0 No. =:  20 No. > :   0, Obj=MIN Single cols=    1

 Feasible solution found at step:       19


                                                       Variable           Value
                                                           QP12       0.7295987E-01
                                                           QP17       0.7157494E-01
                                                            SP1       0.1445348
                                                           QP72       0.2647295E-01
                                                           QP24      -0.1206994E-01
                                                           QP23       0.5771227E-01
                                                             L2       0.5379050E-01
                                                           QP43       0.6458204E-01
                                                             D3       0.1222943
                                                           QP54       0.1620982E-01
                                                           QP64       0.2212254E-01
                                                           QP74       0.3831962E-01
                                                           QP65      -0.2720175E-02
                                                             S5       0.1893000E-01
                                                           QP67      -0.6782361E-02
                                                             S6       0.1262000E-01
                                                             H1        30.48000
                                                             H2        27.39911
                                                            R12        392.9136
                                                             H7        28.38395
                                                            R17        276.9719
                                                            R72        821.1570
                                                             H4        27.72492
                                                            R24        1164.897
                                                             H3        25.99206
                                                            R23        276.9719
                                                            R43        276.9719
                                                            R74        276.9719
                                                             H6        28.06293
                                                            R64        392.9136
                                                             H5        28.12206
                                                            R54        821.1570
                                                            R65        3333.710
                                                            R67        3333.710
                                                              C        1.000000
                                                             A3       0.5415474E-02
                                                              G        9.810000
                                                             A2       0.2320000E-02

                                                            Row    Slack or Surplus
                                                              1       0.0000000
                                                              2       0.0000000
                                                              3       0.0000000
                                                              4       0.0000000
                                                              5       0.0000000
                                                              6       0.0000000
                                                              7       0.0000000
                                                              8      -0.3713310E-03
```

```
 9      -0.2522351E-03
10      -0.1996757E-03
11      -0.4761582E-03
12      -0.1181113E-04
13      -0.3304709E-05
14      -0.4022328E-04
15      -0.5239059E-05
16      -0.5847977E-06
17      -0.3105951E-05
18      -0.7953134E-04
19       0.0000000
20       0.0000000
21       0.0000000
22       0.0000000
23       0.0000000
24       0.0000000
25       0.0000000
26       0.0000000
27       0.0000000
28       0.0000000
29       0.0000000
30      -0.2188991E-07
31      -0.1671276E-07
32       0.0000000
33       0.0000000
34       0.0000000
35       0.0000000
36       0.0000000
37       0.0000000
38       0.0000000
```

```
!This model is the same formulation given by Pudar and Liggett;
!Even-determined problem;

!Two leaks condition (Leaks at Nodes 2 and 4);

Model:

!Continuity Equation for Demand and Leakage;
  Qp12+Qp17=Sp1;                    !Node 1;
  Qp12+Qp72-Qp24-Qp23=L2;           !Node 2;
  Qp23+Qp43=D3;                     !Node 3;
  Qp54+Qp64+Qp74+Qp24-Qp43=L4;      !Node 4;
  Qp65-Qp54=-S5;                    !Node 5;
  Qp64+Qp65+Qp67=S6;                !Node 6;
  Qp17+Qp67-Qp72-Qp74=0;            !Node 7;

!Energy equation with leakage flows;
  H1-H2=r12*Qp12*((@abs(Qp12))^0.852);    !Pipe 1;
  H1-H7=r17*Qp17*((@abs(Qp17))^0.852);    !Pipe 2;
  H7-H2=r72*Qp72*((@abs(Qp72))^0.852);    !Pipe 3;
  H2-H4=r24*Qp24*((@abs(Qp24))^0.852);    !Pipe 4;
  H2-H3=r23*Qp23*((@abs(Qp23))^0.852);    !Pipe 5;
  H4-H3=r43*Qp43*((@abs(Qp43))^0.852);    !Pipe 6;
  H7-H4=r74*Qp74*((@abs(Qp74))^0.852);    !Pipe 7;
  H6-H4=r64*Qp64*((@abs(Qp64))^0.852);    !Pipe 8;
  H5-H4=r54*Qp54*((@abs(Qp54))^0.852);    !Pipe 9;
  H6-H5=r65*Qp65*((@abs(Qp65))^0.852);    !Pipe 10;
  H6-H7=r67*Qp67*((@abs(Qp67))^0.852);    !Pipe 11;

!Resistance Coefficients based on the Hazen-Williams equation;
  r12=392.9136;
  r17=276.9719;
  r72=821.1570;
  r24=1164.8971;
  r23=276.9719;
  r43=276.9719;
  r74=276.9719;
  r64=392.9136;
  r54=821.1570;
  r65=3333.7099;
  r67=3333.7099;

!Pressure-dependent outflow (orifice equation);
  D3=C*A3*((2*g*H3)^0.5);
  L2=C*A2*((2*g*H2)^0.5);
  L4=C*A4*((2*g*H4)^0.5);

!Allow for flow reversal in each pipe;
  @free(Qp12);
  @free(Qp17);
  @free(Qp72);
  @free(Qp24);
  @free(Qp23);
  @free(Qp43);
  @free(Qp74);
  @free(Qp64);
  @free(Qp54);
```

```
  @free(Qp65);
  @free(Qp67);

!Given values in the Pudar-Liggett paper;
  H1=30.48;
  S5=0.01893;
  S6=0.01262;
  A2=0.00232;
  A3=0.0054154736;
  A4=0.00232;
  C=1;
  g=9.81;

End
```

```
Rows=     22 Vars=     21 No. integer vars=      0
 Nonlinear rows=     14 Nonlinear vars=     14 Nonlinear constraints=     14
 Nonzeros=     67 Constraint nonz=    63 Density=0.138
 No. < :   0 No. =:  21 No. > :   0, Obj=MIN Single cols=    1

 Feasible solution found at step:       43


                                           Variable           Value
                                               QP12       0.9306903E-01
                                               QP17       0.9699652E-01
                                                SP1       0.1900655
                                               QP72       0.2885790E-01
                                               QP24       0.8966650E-02
                                               QP23       0.6092163E-01
                                                 L2       0.5203864E-01
                                               QP43       0.5680852E-01
                                                 D3       0.1177302
                                               QP54       0.1761377E-01
                                               QP64       0.2574944E-01
                                               QP74       0.5632541E-01
                                                 L4       0.5184674E-01
                                               QP65      -0.1316234E-02
                                                 S5       0.1893000E-01
                                               QP67      -0.1181320E-01
                                                 S6       0.1262000E-01
                                                 H1        30.48000
                                                 H2        25.64349
                                                R12        392.9136
                                                 H7        26.79956
                                                R17        276.9719
                                                R72        821.1570
                                                 H4        25.45462
                                                R24        1164.897
                                                 H3        24.08809
                                                R23        276.9719
                                                R43        276.9719
                                                R74        276.9719
                                                 H6        25.90237
                                                R64        392.9136
                                                 H5        25.91779
                                                R54        821.1570
                                                R65        3333.710
                                                R67        3333.710
                                                  C        1.000000
                                                 A3       0.5415474E-02
                                                  G        9.810000
                                                 A2       0.2320000E-02
                                                 A4       0.2320000E-02

                                                Row    Slack or Surplus
                                                  1       0.0000000
                                                  2       0.0000000
                                                  3       0.0000000
                                                  4       0.0000000
                                                  5       0.0000000
                                                  6       0.0000000
```

```
  7       0.0000000
  8      -0.8612917E-04
  9      -0.6784902E-04
 10      -0.3888568E-03
 11      -0.6988585E-03
 12      -0.3919538E-04
 13      -0.3305282E-04
 14      -0.1172287E-03
 15      -0.6664097E-05
 16      -0.3883983E-05
 17      -0.9108582E-05
 18      -0.1394207E-03
 19       0.0000000
 20       0.0000000
 21       0.0000000
 22       0.0000000
 23       0.0000000
 24       0.0000000
 25       0.0000000
 26       0.0000000
 27       0.0000000
 28       0.0000000
 29       0.0000000
 30      -0.1724420E-06
 31      -0.1333254E-07
 32      -0.7944073E-07
 33       0.0000000
 34       0.0000000
 35       0.0000000
 36       0.0000000
 37       0.0000000
 38       0.0000000
 39       0.0000000
 40       0.0000000
```

```
!This model is the same formulation given by Pudar and Liggett;
!Even-determined problem;

!Three leaks condition (Leaks at Nodes 2, 4, and 7);

Model:

!Continuity Equation for Demand and Leakage;
  -Qp12-Qp17=-Sp1;                   !Node 1;
  Qp12+Qp72-Qp24-Qp23=L2;            !Node 2;
  Qp23+Qp43=D3;                      !Node 3;
  Qp54+Qp64+Qp74+Qp24-Qp43=L4;       !Node 4;
  Qp65-Qp54=-S5;                     !Node 5;
  -Qp64-Qp65-Qp67=-S6;               !Node 6;
  Qp17+Qp67-Qp72-Qp74=L7;            !Node 7;

!Energy equation with leakage flows;
  H1-H2=r12*Qp12*((@abs(Qp12))^0.852);    !Pipe 1;
  H1-H7=r17*Qp17*((@abs(Qp17))^0.852);    !Pipe 2;
  H7-H2=r72*Qp72*((@abs(Qp72))^0.852);    !Pipe 3;
  H2-H4=r24*Qp24*((@abs(Qp24))^0.852);    !Pipe 4;
  H2-H3=r23*Qp23*((@abs(Qp23))^0.852);    !Pipe 5;
  H4-H3=r43*Qp43*((@abs(Qp43))^0.852);    !Pipe 6;
  H7-H4=r74*Qp74*((@abs(Qp74))^0.852);    !Pipe 7;
  H6-H4=r64*Qp64*((@abs(Qp64))^0.852);    !Pipe 8;
  H5-H4=r54*Qp54*((@abs(Qp54))^0.852);    !Pipe 9;
  H6-H5=r65*Qp65*((@abs(Qp65))^0.852);    !Pipe 10;
  H6-H7=r67*Qp67*((@abs(Qp67))^0.852);    !Pipe 11;

!Resistance Coefficients based on the Hazen-Williams equation;
  r12=392.9136;
  r17=276.9719;
  r72=821.1570;
  r24=1164.8971;
  r23=276.9719;
  r43=276.9719;
  r74=276.9719;
  r64=392.9136;
  r54=821.1570;
  r65=3333.7099;
  r67=3333.7099;

!Pressure-dependent outflow (orifice equation);
  D3=C*A3*((2*g*H3)^0.5);
  L2=C*A2*((2*g*H2)^0.5);
  L4=C*A4*((2*g*H4)^0.5);
  L7=C*A7*((2*g*H7)^0.5);

!Allow for flow reversal in each pipe;
  @free(Qp12);
  @free(Qp17);
  @free(Qp72);
  @free(Qp24);
  @free(Qp23);
  @free(Qp43);
  @free(Qp74);
  @free(Qp64);
```

```
  @free(Qp54);
  @free(Qp65);
  @free(Qp67);

!Given values in the Pudar-Liggett paper;
  H1=30.48;
  S5=0.01893;
  S6=0.01262;
  A2=0.00232;
  A3=0.0054154736;
  A4=0.00232;
  A7=0.0000929;
  C=1;
  g=9.81;

End
```

```
Rows=     23 Vars=     22 No. integer vars=      0
 Nonlinear rows=     15 Nonlinear vars=     15 Nonlinear constraints=     15
 Nonzeros=     70 Constraint nonz=    66 Density=0.132
 No. < :   0 No. =:  22 No. > :   0, Obj=MIN Single cols=    1

 ** WARNING ** Problem is poorly scaled.  The units
 of the rows and variables should be changed so the
 coefficients cover a much smaller range.

 Feasible solution found at step:       33


                                           Variable           Value
                                               QP12       0.9368283E-01
                                               QP17       0.9823022E-01
                                                SP1       0.1919130
                                               QP72       0.2849130E-01
                                               QP24       0.9236021E-02
                                               QP23       0.6095937E-01
                                                 L2       0.5197873E-01
                                               QP43       0.5662192E-01
                                                 D3       0.1175813
                                               QP54       0.1757842E-01
                                               QP64       0.2567111E-01
                                               QP74       0.5591260E-01
                                                 L4       0.5177624E-01
                                               QP65      -0.1351578E-02
                                                 S5       0.1893000E-01
                                               QP67      -0.1169953E-01
                                                 S6       0.1262000E-01
                                                 L7       0.2126778E-02
                                                 H1        30.48000
                                                 H2        25.58443
                                                R12        392.9136
                                                 H7        26.71246
                                                R17        276.9719
                                                R72        821.1570
                                                 H4        25.38552
                                                R24        1164.897
                                                 H3        24.02726
                                                R23        276.9719
                                                R43        276.9719
                                                R74        276.9719
                                                 H6        25.83077
                                                R64        392.9136
                                                 H5        25.84698
                                                R54        821.1570
                                                R65        3333.710
                                                R67        3333.710
                                                  C        1.000000
                                                 A3       0.5415474E-02
                                                  G        9.810000
                                                 A2       0.2320000E-02
                                                 A4       0.2320000E-02
                                                 A7       0.9290000E-04

                                                Row    Slack or Surplus
```

```
  1       0.0000000
  2       0.0000000
  3       0.0000000
  4       0.0000000
  5       0.0000000
  6       0.0000000
  7       0.0000000
  8      -0.9480279E-04
  9      -0.1378454E-03
 10      -0.6067232E-03
 11      -0.1379716E-03
 12      -0.1836610E-04
 13      -0.5608177E-04
 14      -0.8659031E-04
 15      -0.1781055E-04
 16      -0.2688335E-05
 17      -0.1514717E-04
 18      -0.2841325E-03
 19       0.0000000
 20       0.0000000
 21       0.0000000
 22       0.0000000
 23       0.0000000
 24       0.0000000
 25       0.0000000
 26       0.0000000
 27       0.0000000
 28       0.0000000
 29       0.0000000
 30      -0.3048766E-07
 31      -0.3778888E-07
 32       0.0000000
 33       0.0000000
 34       0.0000000
 35       0.0000000
 36       0.0000000
 37       0.0000000
 38       0.0000000
 39       0.0000000
 40       0.0000000
 41       0.0000000
 42       0.0000000
```

```
!This model is the same formulation given by Pudar and Liggett;
!Under-determined problem;

!One head measurement taken at Node 3;
!One leak condition (Possible leaks at Nodes 2, 4, and 7);

Model:

Min=(A4^2)+(A7^2);

!Continuity Equation for Demand and Leakage;
  -Qp12-Qp17=-Sp1;                  !Node 1;
  Qp12+Qp72-Qp24-Qp23=L2;           !Node 2;
  Qp23+Qp43=D3;                     !Node 3;
  Qp54+Qp64+Qp74+Qp24-Qp43=L4;      !Node 4;
  Qp65-Qp54=-S5;                    !Node 5;
  -Qp64-Qp65-Qp67=-S6;              !Node 6;
  Qp17+Qp67-Qp72-Qp74=L7;           !Node 7;

!Overall balance of inflows and outflows;
  Sp1+S5+S6=L2+L4+L7+D3;

!Energy equation with leakage flows;
  H1-H2=r12*Qp12*((@abs(Qp12))^0.852);    !Pipe 1;
  H1-H7=r17*Qp17*((@abs(Qp17))^0.852);    !Pipe 2;
  H7-H2=r72*Qp72*((@abs(Qp72))^0.852);    !Pipe 3;
  H2-H4=r24*Qp24*((@abs(Qp24))^0.852);    !Pipe 4;
  H2-H3=r23*Qp23*((@abs(Qp23))^0.852);    !Pipe 5;
  H4-H3=r43*Qp43*((@abs(Qp43))^0.852);    !Pipe 6;
  H7-H4=r74*Qp74*((@abs(Qp74))^0.852);    !Pipe 7;
  H6-H4=r64*Qp64*((@abs(Qp64))^0.852);    !Pipe 8;
  H5-H4=r54*Qp54*((@abs(Qp54))^0.852);    !Pipe 9;
  H6-H5=r65*Qp65*((@abs(Qp65))^0.852);    !Pipe 10;
  H6-H7=r67*Qp67*((@abs(Qp67))^0.852);    !Pipe 11;

!Resistance Coefficients based on the Hazen-Williams equation;
  r12=392.9136;
  r17=276.9719;
  r72=821.1570;
  r24=1164.8971;
  r23=276.9719;
  r43=276.9719;
  r74=276.9719;
  r64=392.9136;
  r54=821.1570;
  r65=3333.7099;
  r67=3333.7099;

!Pressure-dependent outflow (orifice equation);
  D3=C*A3*((2*g*H3)^0.5);
  L2=C*A2*((2*g*H2)^0.5);
  L4=C*A4*((2*g*H4)^0.5);
  L7=C*A7*((2*g*H7)^0.5);

!Allow for flow reversal in each pipe;
  @free(Qp12);
  @free(Qp17);
```

```
  @free(Qp72);
  @free(Qp24);
  @free(Qp23);
  @free(Qp43);
  @free(Qp74);
  @free(Qp64);
  @free(Qp54);
  @free(Qp65);
  @free(Qp67);

!Given values in the Pudar-Liggett paper;
  H1=30.48;
  H3=25.99206;
  S5=0.01893;
  S6=0.01262;
  A3=0.0054154736;
  C=1;
  g=9.81;

End
```

```
Rows=     23 Vars=     23 No. integer vars=      0
 Nonlinear rows=     15 Nonlinear vars=     17 Nonlinear constraints=     14
 Nonzeros=     78 Constraint nonz=    68 Density=0.141
 No. < :   0 No. =:  22 No. > :   0, Obj=MIN Single cols=    1

 Optimal solution found at step:        45
 Objective value:                0.0000000E+00


                              Variable           Value        Reduced Cost
                                    A4       0.0000000           0.1000000E-07
                                    A7       0.0000000           0.1000000E-07
                                  QP12       0.7296822E-01       0.0000000
                                  QP17       0.7156973E-01       0.0000000
                                   SP1       0.1445380           0.0000000
                                  QP72       0.2647189E-01       0.0000000
                                  QP24      -0.1206400E-01       0.0000000
                                  QP23       0.5771048E-01       0.0000000
                                    L2       0.5379363E-01       0.0000000
                                  QP43       0.6458384E-01       0.0000000
                                    D3       0.1222943           0.0000000
                                  QP54       0.1620958E-01       0.0000000
                                  QP64       0.2212153E-01       0.0000000
                                  QP74       0.3831672E-01       0.0000000
                                    L4       0.0000000           0.0000000
                                  QP65      -0.2720415E-02       0.0000000
                                    S5       0.1893000E-01       0.0000000
                                  QP67      -0.6781121E-02       0.0000000
                                    S6       0.1262000E-01       0.0000000
                                    L7       0.0000000           0.0000000
                                    H1        30.48000           0.0000000
                                    H2        27.39901           0.0000000
                                   R12        392.9136           0.0000000
                                    H7        28.38398           0.0000000
                                   R17        276.9719           0.0000000
                                   R72        821.1570           0.0000000
                                    H4        27.72501           0.0000000
                                   R24        1164.897           0.0000000
                                    H3        25.99206           0.0000000
                                   R23        276.9719           0.0000000
                                   R43        276.9719           0.0000000
                                   R74        276.9719           0.0000000
                                    H6        28.06299           0.0000000
                                   R64        392.9136           0.0000000
                                    H5        28.12213           0.0000000
                                   R54        821.1570           0.0000000
                                   R65        3333.710           0.0000000
                                   R67        3333.710           0.0000000
                                     C        1.000000           0.0000000
                                    A3       0.5415474E-02       0.0000000
                                     G        9.810000           0.0000000
                                    A2       0.2320138E-02       0.0000000

                                   Row    Slack or Surplus      Dual Price
                                     1       0.0000000           -1.000000
                                     2       0.0000000           0.0000000
                                     3       0.0000000           0.0000000
```

```
 4       0.0000000           0.0000000
 5       0.0000000           0.0000000
 6       0.0000000           0.0000000
 7       0.0000000           0.0000000
 8       0.0000000           0.0000000
 9       0.0000000           0.0000000
10      -0.9285315E-03       0.0000000
11       0.0000000           0.0000000
12       0.0000000           0.0000000
13       0.0000000           0.0000000
14       0.0000000           0.0000000
15       0.0000000           0.0000000
16       0.0000000           0.0000000
17       0.0000000           0.0000000
18       0.0000000           0.0000000
19       0.0000000           0.0000000
20       0.0000000           0.0000000
21       0.0000000           0.0000000
22       0.0000000           0.0000000
23       0.0000000           0.0000000
24       0.0000000           0.0000000
25       0.0000000           0.0000000
26       0.0000000           0.0000000
27       0.0000000           0.0000000
28       0.0000000           0.0000000
29       0.0000000           0.0000000
30       0.0000000           0.0000000
31       0.0000000           0.0000000
32       0.0000000           0.0000000
33       0.0000000           0.0000000
34       0.0000000           0.0000000
35       0.0000000           0.0000000
36       0.0000000           0.0000000
37       0.0000000           0.0000000
38       0.0000000           0.0000000
39       0.0000000           0.0000000
40       0.0000000           0.0000000
41       0.0000000           0.0000000
42       0.0000000           0.0000000
```

```
!This model is the same formulation given by Pudar and Liggett;
!Under-determined problem;

!One head measurement taken at Node 5;
!One leak condition (Possible leaks at Nodes 2, 4, and 7);

Model:

Min=(A4^2)+(A7^2);

!Continuity Equation for Demand and Leakage;
  -Qp12-Qp17=-Sp1;                  !Node 1;
  Qp12+Qp72-Qp24-Qp23=L2;           !Node 2;
  Qp23+Qp43=D3;                     !Node 3;
  Qp54+Qp64+Qp74+Qp24-Qp43=L4;      !Node 4;
  Qp65-Qp54=-S5;                    !Node 5;
  -Qp64-Qp65-Qp67=-S6;              !Node 6;
  Qp17+Qp67-Qp72-Qp74=L7;           !Node 7;

!Overall balance of inflows and outflows;
  Sp1+S5+S6=L2+L4+L7+D3;

!Energy equation with leakage flows;
  H1-H2=r12*Qp12*((@abs(Qp12))^0.852);    !Pipe 1;
  H1-H7=r17*Qp17*((@abs(Qp17))^0.852);    !Pipe 2;
  H7-H2=r72*Qp72*((@abs(Qp72))^0.852);    !Pipe 3;
  H2-H4=r24*Qp24*((@abs(Qp24))^0.852);    !Pipe 4;
  H2-H3=r23*Qp23*((@abs(Qp23))^0.852);    !Pipe 5;
  H4-H3=r43*Qp43*((@abs(Qp43))^0.852);    !Pipe 6;
  H7-H4=r74*Qp74*((@abs(Qp74))^0.852);    !Pipe 7;
  H6-H4=r64*Qp64*((@abs(Qp64))^0.852);    !Pipe 8;
  H5-H4=r54*Qp54*((@abs(Qp54))^0.852);    !Pipe 9;
  H6-H5=r65*Qp65*((@abs(Qp65))^0.852);    !Pipe 10;
  H6-H7=r67*Qp67*((@abs(Qp67))^0.852);    !Pipe 11;

!Resistance Coefficients based on the Hazen-Williams equation;
  r12=392.9136;
  r17=276.9719;
  r72=821.1570;
  r24=1164.8971;
  r23=276.9719;
  r43=276.9719;
  r74=276.9719;
  r64=392.9136;
  r54=821.1570;
  r65=3333.7099;
  r67=3333.7099;

!Pressure-dependent outflow (orifice equation);
  D3=C*A3*((2*g*H3)^0.5);
  L2=C*A2*((2*g*H2)^0.5);
  L4=C*A4*((2*g*H4)^0.5);
  L7=C*A7*((2*g*H7)^0.5);

!Allow for flow reversal in each pipe;
  @free(Qp12);
  @free(Qp17);
```

```
  @free(Qp72);
  @free(Qp24);
  @free(Qp23);
  @free(Qp43);
  @free(Qp74);
  @free(Qp64);
  @free(Qp54);
  @free(Qp65);
  @free(Qp67);

!Given values in the Pudar-Liggett paper;
  H1=30.48;
  H5=28.12206;
  S5=0.01893;
  S6=0.01262;
  A3=0.0054154736;
  C=1;
  g=9.81;

End
```

```
Rows=     24 Vars=     24 No. integer vars=      0
 Nonlinear rows=     16 Nonlinear vars=     18 Nonlinear constraints=     15
 Nonzeros=     81 Constraint nonz=    72 Density=0.135
 No. < :   0 No. =:  23 No. > :   0, Obj=MIN Single cols=    1

 Optimal solution found at step:       59
 Objective value:                0.0000000E+00


                       Variable           Value        Reduced Cost
                             A4       0.0000000           0.0000000
                             A7       0.0000000           0.0000000
                           QP12       0.7295591E-01        78.21059
                           QP17       0.7156927E-01       0.0000000
                            SP1       0.1445252           0.0000000
                           QP72       0.2647174E-01        81.20888
                           QP24      -0.1206382E-01       0.0000000
                           QP23       0.5771072E-01       0.0000000
                             L2       0.5378075E-01       0.0000000
                           QP43       0.6458371E-01       0.0000000
                             D3       0.1222944           0.0000000
                           QP54       0.1620791E-01       0.0000000
                           QP64       0.2212289E-01       0.0000000
                           QP74       0.3831673E-01       0.0000000
                             L4       0.0000000           -4.435036
                           QP65      -0.2722093E-02       0.0000000
                             S5       0.1893000E-01       0.0000000
                           QP67      -0.6780794E-02       0.0000000
                             S6       0.1262000E-01       0.0000000
                             L7       0.0000000            12.29956
                             H1        30.48000           0.0000000
                             H2        27.39906            2.132918
                            R12        392.9136           0.0000000
                             H7        28.38402           0.0000000
                            R17        276.9719           0.0000000
                            R72        821.1570           0.0000000
                             H4        27.72504           0.0000000
                            R24        1164.897           0.0000000
                             H3        25.99210           0.0000000
                            R23        276.9719           0.0000000
                            R43        276.9719           0.0000000
                            R74        276.9719           0.0000000
                             H6        28.06306           0.0000000
                            R64        392.9136           0.0000000
                             H5        28.12206           0.0000000
                            R54        821.1570           0.0000000
                            R65        3333.710           0.0000000
                            R67        3333.710           0.0000000
                              C        1.000000           0.0000000
                             A3       0.5415474E-02       0.0000000
                              G        9.810000           0.0000000
                             A2       0.2319575E-02       0.0000000

                            Row    Slack or Surplus      Dual Price
                              1       0.0000000           -1.000000
                              2       0.0000000           -1.000000
                              3       0.0000000           -1.000000
```

```
  4        0.0000000            1.000000
  5        0.0000000            3.435036
  6        0.0000000            25.51357
  7        0.0000000            8.425984
  8        0.0000000           -13.29956
  9        0.0000000            1.000000
 10       -0.1719500E-04       -1.000000
 11       -0.9932381E-05      -0.2267700
 12       -0.7457239E-05       -1.000000
 13       -0.7399611E-05       0.8862243E-01
 14       -0.1221764E-05       0.4429582E-01
 15       -0.2569791E-06      -0.4900088E-01
 16       -0.1328195E-05       0.5254016
 17       -0.8359379E-05      -0.1763761
 18       -0.2772095E-04      -0.4866488
 19       -0.2013393E-03       0.4242045
 20       -0.2087735E-05      -0.2478284
 21        0.0000000          -0.7841298E-02
 22        0.0000000          -0.1716087E-02
 23        0.0000000          -0.1199481E-02
 24        0.0000000          -0.2480011E-04
 25        0.0000000           0.2250141E-03
 26        0.0000000          -0.3065855E-03
 27        0.0000000           0.1250048E-02
 28        0.0000000          -0.1517354E-03
 29        0.0000000          -0.2353043E-03
 30        0.0000000          -0.7533827E-05
 31        0.0000000           0.2386067E-04
 32        0.0000000            2.000000
 33       -0.1302591E-06       0.0000000
 34        0.0000000           0.0000000
 35        0.0000000           0.0000000
 36        0.0000000            1.226770
 37        0.0000000           0.9108533
 38        0.0000000           -26.51357
 39        0.0000000           -9.425984
 40        0.0000000            45.16481
 41        0.0000000           0.2445888
 42        0.0000000           0.1246622E-01
```

```
!This model is the same formulation given by Pudar and Liggett;
!Under-determined problem;

!One head measurement taken at Node 6;
!One leak condition (Possible leaks at Nodes 2, 4, and 7);

Model:

Min=(A4^2)+(A7^2);

!Continuity Equation for Demand and Leakage;
  -Qp12-Qp17=-Sp1;                  !Node 1;
  Qp12+Qp72-Qp24-Qp23=L2;           !Node 2;
  Qp23+Qp43=D3;                     !Node 3;
  Qp54+Qp64+Qp74+Qp24-Qp43=L4;      !Node 4;
  Qp65-Qp54=-S5;                    !Node 5;
  -Qp64-Qp65-Qp67=-S6;              !Node 6;
  Qp17+Qp67-Qp72-Qp74=L7;           !Node 7;

!Overall balance of inflows and outflows;
  Sp1+S5+S6=L2+L4+L7+D3;

!Energy equation with leakage flows;
  H1-H2=r12*Qp12*((@abs(Qp12))^0.852);    !Pipe 1;
  H1-H7=r17*Qp17*((@abs(Qp17))^0.852);    !Pipe 2;
  H7-H2=r72*Qp72*((@abs(Qp72))^0.852);    !Pipe 3;
  H2-H4=r24*Qp24*((@abs(Qp24))^0.852);    !Pipe 4;
  H2-H3=r23*Qp23*((@abs(Qp23))^0.852);    !Pipe 5;
  H4-H3=r43*Qp43*((@abs(Qp43))^0.852);    !Pipe 6;
  H7-H4=r74*Qp74*((@abs(Qp74))^0.852);    !Pipe 7;
  H6-H4=r64*Qp64*((@abs(Qp64))^0.852);    !Pipe 8;
  H5-H4=r54*Qp54*((@abs(Qp54))^0.852);    !Pipe 9;
  H6-H5=r65*Qp65*((@abs(Qp65))^0.852);    !Pipe 10;
  H6-H7=r67*Qp67*((@abs(Qp67))^0.852);    !Pipe 11;

!Resistance Coefficients based on the Hazen-Williams equation;
  r12=392.9136;
  r17=276.9719;
  r72=821.1570;
  r24=1164.8971;
  r23=276.9719;
  r43=276.9719;
  r74=276.9719;
  r64=392.9136;
  r54=821.1570;
  r65=3333.7099;
  r67=3333.7099;

!Pressure-dependent outflow (orifice equation);
  D3=C*A3*((2*g*H3)^0.5);
  L2=C*A2*((2*g*H2)^0.5);
  L4=C*A4*((2*g*H4)^0.5);
  L7=C*A7*((2*g*H7)^0.5);

!Allow for flow reversal in each pipe;
  @free(Qp12);
  @free(Qp17);
```

```
  @free(Qp72);
  @free(Qp24);
  @free(Qp23);
  @free(Qp43);
  @free(Qp74);
  @free(Qp64);
  @free(Qp54);
  @free(Qp65);
  @free(Qp67);

!Given values in the Pudar-Liggett paper;
  H1=30.48;
  H6=28.06293;
  S5=0.01893;
  S6=0.01262;
  A3=0.0054154736;
  C=1;
  g=9.81;

End
```

```
Rows=     24 Vars=     24 No. integer vars=      0
 Nonlinear rows=     16 Nonlinear vars=     18 Nonlinear constraints=     15
 Nonzeros=     81 Constraint nonz=    71 Density=0.135
 No. < :   0 No. =:  23 No. > :   0, Obj=MIN Single cols=    1

 Optimal solution found at step:        51
 Objective value:                0.0000000E+00


                       Variable           Value        Reduced Cost
                             A4       0.0000000            103.4390
                             A7       0.0000000           0.0000000
                           QP12       0.7295734E-01       0.0000000
                           QP17       0.7157062E-01       0.0000000
                            SP1       0.1445280           0.0000000
                           QP72       0.2647231E-01        71.10902
                           QP24      -0.1206436E-01       0.0000000
                           QP23       0.5771021E-01       0.0000000
                             L2       0.5378379E-01       0.0000000
                           QP43       0.6458395E-01       0.0000000
                             D3       0.1222942           0.0000000
                           QP54       0.1620966E-01       0.0000000
                           QP64       0.2212157E-01       0.0000000
                           QP74       0.3831708E-01       0.0000000
                             L4       0.0000000           0.0000000
                           QP65      -0.2720339E-02       0.0000000
                             S5       0.1893000E-01       0.0000000
                           QP67      -0.6781234E-02       0.0000000
                             S6       0.1262000E-01       0.0000000
                             L7       0.0000000            15.46046
                             H1        30.48000           0.0000000
                             H2        27.39893           0.6746317
                            R12        392.9136           0.0000000
                             H7        28.38394           0.0000000
                            R17        276.9719           0.0000000
                            R72        821.1570           0.0000000
                             H4        27.72495           0.0000000
                            R24        1164.897           0.0000000
                             H3        25.99199           0.0000000
                            R23        276.9719           0.0000000
                            R43        276.9719           0.0000000
                            R74        276.9719           0.0000000
                             H6        28.06293           0.0000000
                            R64        392.9136           0.0000000
                             H5        28.12207           0.0000000
                            R54        821.1570           0.0000000
                            R65        3333.710           0.0000000
                            R67        3333.710           0.0000000
                              C        1.000000           0.0000000
                             A3       0.5415474E-02       0.0000000
                              G        9.810000           0.0000000
                             A2       0.2319717E-02       0.0000000

                            Row    Slack or Surplus      Dual Price
                              1       0.0000000           -1.000000
                              2       0.0000000            1.000000
                              3       0.0000000            1.000000
```

```
 4       0.0000000           -1.000000
 5       0.0000000           -3.435061
 6       0.0000000           -1.085115
 7       0.0000000            1.000000
 8       0.0000000           -14.46046
 9       0.0000000           -1.000000
10      -0.2888861E-06       0.0000000
11      -0.1375050E-06      -0.2850438
12      -0.1587172E-06      -0.8075475
13      -0.7055530E-07      -0.8861958E-01
14       0.0000000          -0.4429615E-01
15       0.0000000           0.4900122E-01
16      -0.2845986E-07       0.3461523
17      -0.7579763E-06      -0.1567394
18      -0.1481122E-05      -0.5179206E-01
19      -0.4731716E-05      -0.5179206E-01
20      -0.1132344E-05      -0.1763514
21       0.0000000           0.0000000
22       0.0000000          -0.2157150E-02
23       0.0000000          -0.9686759E-03
24       0.0000000           0.2480135E-04
25       0.0000000          -0.2250121E-03
26       0.0000000           0.3065898E-03
27       0.0000000           0.8235877E-03
28       0.0000000          -0.1348272E-03
29       0.0000000          -0.2504750E-04
30       0.0000000           0.9187236E-06
31       0.0000000           0.1698097E-04
32       0.0000000           -2.000000
33       0.0000000           0.0000000
34       0.0000000           -4.435061
35       0.0000000           0.0000000
36       0.0000000           0.2850438
37       0.0000000           0.3848828
38       0.0000000            2.085115
39       0.0000000           0.0000000
40       0.0000000           -45.16472
41       0.0000000          -0.2445883
42       0.0000000          -0.1246620E-01
```

```
!This model is the same formulation given by Pudar and Liggett;
!Under-determined problem;

!One head measurement taken at Node 3;
!Two leaks condition (Possible leaks at Nodes 2, 4, and 7);

Model:

Min=(A7^2);

!Continuity Equation for Demand and Leakage;
  -Qp12-Qp17=-Sp1;                  !Node 1;
  Qp12+Qp72-Qp24-Qp23=L2;           !Node 2;
  Qp23+Qp43=D3;                     !Node 3;
  Qp54+Qp64+Qp74+Qp24-Qp43=L4;      !Node 4;
  Qp65-Qp54=-S5;                    !Node 5;
  -Qp64-Qp65-Qp67=-S6;              !Node 6;
  Qp17+Qp67-Qp72-Qp74=L7;           !Node 7;

!Overall balance of inflows and outflows;
  Sp1+S5+S6=L2+L4+L7+D3;

!Energy equation with leakage flows;
  H1-H2=r12*Qp12*((@abs(Qp12))^0.852);    !Pipe 1;
  H1-H7=r17*Qp17*((@abs(Qp17))^0.852);    !Pipe 2;
  H7-H2=r72*Qp72*((@abs(Qp72))^0.852);    !Pipe 3;
  H2-H4=r24*Qp24*((@abs(Qp24))^0.852);    !Pipe 4;
  H2-H3=r23*Qp23*((@abs(Qp23))^0.852);    !Pipe 5;
  H4-H3=r43*Qp43*((@abs(Qp43))^0.852);    !Pipe 6;
  H7-H4=r74*Qp74*((@abs(Qp74))^0.852);    !Pipe 7;
  H6-H4=r64*Qp64*((@abs(Qp64))^0.852);    !Pipe 8;
  H5-H4=r54*Qp54*((@abs(Qp54))^0.852);    !Pipe 9;
  H6-H5=r65*Qp65*((@abs(Qp65))^0.852);    !Pipe 10;
  H6-H7=r67*Qp67*((@abs(Qp67))^0.852);    !Pipe 11;

!Resistance Coefficients based on the Hazen-Williams equation;
  r12=392.9136;
  r17=276.9719;
  r72=821.1570;
  r24=1164.8971;
  r23=276.9719;
  r43=276.9719;
  r74=276.9719;
  r64=392.9136;
  r54=821.1570;
  r65=3333.7099;
  r67=3333.7099;

!Pressure-dependent outflow (orifice equation);
  D3=C*A3*((2*g*H3)^0.5);
  L2=C*A2*((2*g*H2)^0.5);
  L4=C*A4*((2*g*H4)^0.5);
  L7=C*A7*((2*g*H7)^0.5);

!Allow for flow reversal in each pipe;
  @free(Qp12);
  @free(Qp17);
```

```
  @free(Qp72);
  @free(Qp24);
  @free(Qp23);
  @free(Qp43);
  @free(Qp74);
  @free(Qp64);
  @free(Qp54);
  @free(Qp65);
  @free(Qp67);

!Given values in the Pudar-Liggett paper;
  H1=30.48;
  H3=24.08809;
  S5=0.01893;
  S6=0.01262;
  A3=0.0054154736;
  C=1;
  g=9.81;

End
```

```
Rows=     23 Vars=     23 No. integer vars=      0
 Nonlinear rows=     15 Nonlinear vars=     17 Nonlinear constraints=     14
 Nonzeros=     77 Constraint nonz=    68 Density=0.139
 No. < :   0 No. =:  22 No. > :   0, Obj=MIN Single cols=    2

 Optimal solution found at step:        33
 Objective value:                0.1998789E-06


                       Variable           Value        Reduced Cost
                             A7       0.4470781E-03       0.0000000
                           QP12       0.8925796E-01       0.0000000
                           QP17       0.1002004            72.64294
                            SP1       0.1894584           0.0000000
                           QP72       0.1965300E-01       0.0000000
                           QP24       0.2030472E-01       0.0000000
                           QP23       0.6817954E-01       0.0000000
                             L2       0.2042671E-01       0.0000000
                           QP43       0.4955046E-01       0.0000000
                             D3       0.1177300           0.0000000
                           QP54       0.1776231E-01       0.0000000
                           QP64       0.2607015E-01       0.0000000
                           QP74       0.5805557E-01       0.0000000
                             L4       0.7264230E-01       0.0000000
                           QP65      -0.1167684E-02       0.0000000
                             S5       0.1893000E-01       0.0000000
                           QP67      -0.1228247E-01       0.0000000
                             S6       0.1262000E-01       0.0000000
                             L7       0.1020938E-01       0.0000000
                             H1        30.48000           0.0000000
                             H2        26.00395           0.0000000
                            R12        392.9136           0.0000000
                             H7        26.57130            1.000000
                            R17        276.9719           0.0000000
                            R72        821.1570           0.0000000
                             H4        25.14896           0.0000000
                            R24        1164.897           0.0000000
                             H3        24.08809           0.0000000
                            R23        276.9719           0.0000000
                            R43        276.9719           0.0000000
                            R74        276.9719           0.0000000
                             H6        25.60709           0.0000000
                            R64        392.9136           0.0000000
                             H5        25.61939           0.0000000
                            R54        821.1570           0.0000000
                            R65        3333.710           0.0000000
                            R67        3333.710           0.0000000
                              C        1.000000           0.0000000
                             A3       0.5415474E-02       0.0000000
                              G        9.810000           0.0000000
                             A2       0.9043347E-03       0.0000000
                             A4       0.3270243E-02       0.0000000

                            Row    Slack or Surplus      Dual Price
                              1       0.1998789E-06       -1.000000
                              2       0.0000000           0.0000000
                              3       0.0000000           0.0000000
```

```
  4       0.0000000           0.0000000
  5       0.0000000           0.0000000
  6       0.0000000           0.0000000
  7       0.0000000           0.0000000
  8       0.0000000           0.0000000
  9       0.0000000           0.0000000
 10       0.0000000           0.0000000
 11      -0.1278838E-03       -1.000000
 12       0.0000000           0.0000000
 13       0.0000000           0.0000000
 14       0.0000000           0.0000000
 15       0.0000000           0.0000000
 16      -0.2335660E-03       0.0000000
 17      -0.1035531E-04       0.0000000
 18      -0.7962225E-05       0.0000000
 19      -0.4801297E-04       0.0000000
 20      -0.2485138E-03       0.0000000
 21       0.0000000           0.0000000
 22       0.0000000          -0.1411271E-01
 23       0.0000000           0.0000000
 24       0.0000000           0.0000000
 25       0.0000000           0.0000000
 26       0.0000000           0.0000000
 27       0.0000000           0.0000000
 28       0.0000000           0.0000000
 29       0.0000000           0.0000000
 30       0.0000000           0.0000000
 31       0.0000000           0.0000000
 32       0.0000000           0.0000000
 33       0.0000000           0.0000000
 34       0.0000000           0.0000000
 35      -0.1412211E-05       0.0000000
 36       0.0000000            1.000000
 37       0.0000000           0.0000000
 38       0.0000000           0.0000000
 39       0.0000000           0.0000000
 40       0.0000000           0.0000000
 41       0.0000000           0.0000000
 42       0.0000000           0.0000000
```

```
!This model is the same formulation given by Pudar and Liggett;
!Under-determined problem;

!One head measurement taken at Node 5;
!Two leaks condition (Possible leaks at Nodes 2, 4, and 7);

Model:

Min=(A7^2);

!Continuity Equation for Demand and Leakage;
  -Qp12-Qp17=-Sp1;                    !Node 1;
  Qp12+Qp72-Qp24-Qp23=L2;             !Node 2;
  Qp23+Qp43=D3;                       !Node 3;
  Qp54+Qp64+Qp74+Qp24-Qp43=L4;        !Node 4;
  Qp65-Qp54=-S5;                      !Node 5;
  -Qp64-Qp65-Qp67=-S6;                !Node 6;
  Qp17+Qp67-Qp72-Qp74=L7;             !Node 7;

!Overall balance of inflows and outflows;
  Sp1+S5+S6=L2+L4+L7+D3;

!Energy equation with leakage flows;
  H1-H2=r12*Qp12*((@abs(Qp12))^0.852);    !Pipe 1;
  H1-H7=r17*Qp17*((@abs(Qp17))^0.852);    !Pipe 2;
  H7-H2=r72*Qp72*((@abs(Qp72))^0.852);    !Pipe 3;
  H2-H4=r24*Qp24*((@abs(Qp24))^0.852);    !Pipe 4;
  H2-H3=r23*Qp23*((@abs(Qp23))^0.852);    !Pipe 5;
  H4-H3=r43*Qp43*((@abs(Qp43))^0.852);    !Pipe 6;
  H7-H4=r74*Qp74*((@abs(Qp74))^0.852);    !Pipe 7;
  H6-H4=r64*Qp64*((@abs(Qp64))^0.852);    !Pipe 8;
  H5-H4=r54*Qp54*((@abs(Qp54))^0.852);    !Pipe 9;
  H6-H5=r65*Qp65*((@abs(Qp65))^0.852);    !Pipe 10;
  H6-H7=r67*Qp67*((@abs(Qp67))^0.852);    !Pipe 11;

!Resistance Coefficients based on the Hazen-Williams equation;
  r12=392.9136;
  r17=276.9719;
  r72=821.1570;
  r24=1164.8971;
  r23=276.9719;
  r43=276.9719;
  r74=276.9719;
  r64=392.9136;
  r54=821.1570;
  r65=3333.7099;
  r67=3333.7099;

!Pressure-dependent outflow (orifice equation);
  D3=C*A3*((2*g*H3)^0.5);
  L2=C*A2*((2*g*H2)^0.5);
  L4=C*A4*((2*g*H4)^0.5);
  L7=C*A7*((2*g*H7)^0.5);

!Allow for flow reversal in each pipe;
  @free(Qp12);
  @free(Qp17);
```

```
  @free(Qp72);
  @free(Qp24);
  @free(Qp23);
  @free(Qp43);
  @free(Qp74);
  @free(Qp64);
  @free(Qp54);
  @free(Qp65);
  @free(Qp67);

!Given values in the Pudar-Liggett paper;
  H1=30.48;
  H5=25.91779;
  S5=0.01893;
  S6=0.01262;
  A3=0.0054154736;
  C=1;
  g=9.81;

End
```

```
Rows=     24 Vars=     24 No. integer vars=      0
 Nonlinear rows=     16 Nonlinear vars=     18 Nonlinear constraints=     15
 Nonzeros=     80 Constraint nonz=    72 Density=0.133
 No. < :   0 No. =:  23 No. > :   0, Obj=MIN Single cols=    2

 Optimal solution found at step:        44
 Objective value:                0.0000000E+00


                              Variable           Value        Reduced Cost
                                    A7       0.0000000           0.0000000
                                  QP12       0.1048202           0.0000000
                                  QP17       0.1002851           0.0000000
                                   SP1       0.2051053           0.0000000
                                  QP72       0.3997187E-01        114.7004
                                  QP24      -0.2241593E-01       0.0000000
                                  QP23       0.4682845E-01        38.67855
                                    L2       0.1203795           0.0000000
                                  QP43       0.6944733E-01        50.86488
                                    D3       0.1162758           0.0000000
                                  QP54       0.1710655E-01       0.0000000
                                  QP64       0.2457589E-01       0.0000000
                                  QP74       0.5018083E-01       0.0000000
                                    L4       0.0000000           0.0000000
                                  QP65      -0.1823452E-02       0.0000000
                                    S5       0.1893000E-01       0.0000000
                                  QP67      -0.1013244E-01       0.0000000
                                    S6       0.1262000E-01       0.0000000
                                    L7       0.0000000            16.80507
                                    H1        30.48000           0.0000000
                                    H2        24.45216           0.0000000
                                   R12        392.9136           0.0000000
                                    H7        26.56505           0.0000000
                                   R17        276.9719           0.0000000
                                   R72        821.1570           0.0000000
                                    H4        25.47904           0.0000000
                                   R24        1164.897           0.0000000
                                    H3        23.49669            2.023574
                                   R23        276.9719           0.0000000
                                   R43        276.9719           0.0000000
                                   R74        276.9719           0.0000000
                                    H6        25.88973           0.0000000
                                   R64        392.9136           0.0000000
                                    H5        25.91779           0.0000000
                                   R54        821.1570           0.0000000
                                   R65        3333.710           0.0000000
                                   R67        3333.710           0.0000000
                                     C        1.000000           0.0000000
                                    A3       0.5415474E-02       0.0000000
                                     G        9.810000           0.0000000
                                    A2       0.5495975E-02       0.0000000
                                    A4       0.0000000           -44.71684

                                   Row    Slack or Surplus      Dual Price
                                     1       0.0000000           -1.000000
                                     2       0.0000000           -1.000000
                                     3       0.0000000           -1.000000
```

```
 4       0.0000000           -1.000000
 5       0.0000000            1.000000
 6       0.0000000            44.24403
 7       0.0000000            19.02530
 8       0.0000000           -17.80507
 9       0.0000000            1.000000
10      -0.6373368E-05       0.0000000
11      -0.3815525E-05      -0.2324396
12      -0.2559606E-05       -1.000000
13      -0.2403915E-05       0.2357367E-01
14      -0.8157110E-06       -1.023574
15      -0.1685302E-06       -1.000000
16      -0.3967188E-06       0.4691806
17      -0.9883355E-05      -0.5824124
18      -0.3097546E-04      -0.9103419
19      -0.1313077E-03       0.8807921
20      -0.9847244E-06      -0.2983797
21       0.0000000           0.0000000
22       0.0000000          -0.3285491E-02
23       0.0000000          -0.2573056E-02
24       0.0000000          -0.2078061E-04
25       0.0000000          -0.3531044E-02
26       0.0000000          -0.7157249E-02
27       0.0000000           0.1839654E-02
28       0.0000000          -0.6087706E-03
29       0.0000000          -0.4864310E-03
30       0.0000000          -0.7448172E-05
31       0.0000000           0.6044375E-04
32       0.0000000           0.0000000
33      -0.7682344E-07       0.0000000
34       0.0000000            2.000000
35       0.0000000           0.0000000
36       0.0000000           0.2324396
37       0.0000000            1.791134
38       0.0000000           -45.24403
39       0.0000000           -20.02530
40       0.0000000           0.0000000
41       0.0000000           0.0000000
42       0.0000000           0.0000000
```

```
!This model is the same formulation given by Pudar and Liggett;
!Under-determined problem;

!One head measurement taken at Node 6;
!Two leaks condition (Possible leaks at Nodes 2, 4, and 7);

Model:

Min=(A7^2);

!Continuity Equation for Demand and Leakage;
  -Qp12-Qp17=-Sp1;                  !Node 1;
  Qp12+Qp72-Qp24-Qp23=L2;           !Node 2;
  Qp23+Qp43=D3;                     !Node 3;
  Qp54+Qp64+Qp74+Qp24-Qp43=L4;      !Node 4;
  Qp65-Qp54=-S5;                    !Node 5;
  -Qp64-Qp65-Qp67=-S6;              !Node 6;
  Qp17+Qp67-Qp72-Qp74=L7;           !Node 7;

!Overall balance of inflows and outflows;
  Sp1+S5+S6=L2+L4+L7+D3;

!Energy equation with leakage flows;
  H1-H2=r12*Qp12*((@abs(Qp12))^0.852);    !Pipe 1;
  H1-H7=r17*Qp17*((@abs(Qp17))^0.852);    !Pipe 2;
  H7-H2=r72*Qp72*((@abs(Qp72))^0.852);    !Pipe 3;
  H2-H4=r24*Qp24*((@abs(Qp24))^0.852);    !Pipe 4;
  H2-H3=r23*Qp23*((@abs(Qp23))^0.852);    !Pipe 5;
  H4-H3=r43*Qp43*((@abs(Qp43))^0.852);    !Pipe 6;
  H7-H4=r74*Qp74*((@abs(Qp74))^0.852);    !Pipe 7;
  H6-H4=r64*Qp64*((@abs(Qp64))^0.852);    !Pipe 8;
  H5-H4=r54*Qp54*((@abs(Qp54))^0.852);    !Pipe 9;
  H6-H5=r65*Qp65*((@abs(Qp65))^0.852);    !Pipe 10;
  H6-H7=r67*Qp67*((@abs(Qp67))^0.852);    !Pipe 11;

!Resistance Coefficients based on the Hazen-Williams equation;
  r12=392.9136;
  r17=276.9719;
  r72=821.1570;
  r24=1164.8971;
  r23=276.9719;
  r43=276.9719;
  r74=276.9719;
  r64=392.9136;
  r54=821.1570;
  r65=3333.7099;
  r67=3333.7099;

!Pressure-dependent outflow (orifice equation);
  D3=C*A3*((2*g*H3)^0.5);
  L2=C*A2*((2*g*H2)^0.5);
  L4=C*A4*((2*g*H4)^0.5);
  L7=C*A7*((2*g*H7)^0.5);

!Allow for flow reversal in each pipe;
  @free(Qp12);
  @free(Qp17);
```

```
  @free(Qp72);
  @free(Qp24);
  @free(Qp23);
  @free(Qp43);
  @free(Qp74);
  @free(Qp64);
  @free(Qp54);
  @free(Qp65);
  @free(Qp67);

!Given values in the Pudar-Liggett paper;
  H1=30.48;
  H6=25.90237;
  S5=0.01893;
  S6=0.01262;
  A3=0.0054154736;
  C=1;
  g=9.81;

End
```

```
Rows=     24 Vars=     24 No. integer vars=      0
 Nonlinear rows=     16 Nonlinear vars=     18 Nonlinear constraints=     15
 Nonzeros=     80 Constraint nonz=    71 Density=0.133
 No. < :   0 No. =:  23 No. > :   0, Obj=MIN Single cols=    2

 Optimal solution found at step:        46
 Objective value:                0.2344281E-08


                       Variable           Value        Reduced Cost
                             A7       0.4841777E-04       0.0000000
                           QP12       0.9581256E-01       0.0000000
                           QP17       0.9805964E-01        70.92943
                            SP1       0.1938722           0.0000000
                           QP72       0.3136313E-01        79.61896
                           QP24      -0.6038320E-02       0.0000000
                           QP23       0.5773099E-01       0.0000000
                             L2       0.7548302E-01       0.0000000
                           QP43       0.5970602E-01       0.0000000
                             D3       0.1174370           0.0000000
                           QP54       0.1744424E-01       0.0000000
                           QP64       0.2537028E-01       0.0000000
                           QP74       0.5432306E-01       0.0000000
                             L4       0.3139323E-01       0.0000000
                           QP65      -0.1485761E-02       0.0000000
                             S5       0.1893000E-01       0.0000000
                           QP67      -0.1126451E-01        135.0914
                             S6       0.1262000E-01       0.0000000
                             L7       0.1108941E-02       0.0000000
                             H1        30.48000           0.0000000
                             H2        25.37623            1.000000
                            R12        392.9136           0.0000000
                             H7        26.72444            1.000000
                            R17        276.9719           0.0000000
                            R72        821.1570           0.0000000
                             H4        25.46673           0.0000000
                            R24        1164.897           0.0000000
                             H3        23.96835           0.0000000
                            R23        276.9719           0.0000000
                            R43        276.9719           0.0000000
                            R74        276.9719           0.0000000
                             H6        25.90237           0.0000000
                            R64        392.9136           0.0000000
                             H5        25.92168           0.0000000
                            R54        821.1570           0.0000000
                            R65        3333.710           0.0000000
                            R67        3333.710           0.0000000
                              C        1.000000           0.0000000
                             A3       0.5415474E-02       0.0000000
                              G        9.810000           0.0000000
                             A2       0.3382877E-02       0.0000000
                             A4       0.1404433E-02       0.0000000

                            Row   Slack or Surplus      Dual Price
                              1       0.0000000           -1.000000
                              2       0.0000000           0.0000000
                              3       0.0000000           0.0000000
```

```
 4       0.0000000           0.0000000
 5       0.0000000           0.0000000
 6       0.0000000           0.0000000
 7       0.0000000           0.0000000
 8       0.0000000           0.0000000
 9       0.0000000           0.0000000
10       0.0000000           0.0000000
11      -0.9197211E-05       -1.000000
12      -0.1115000E-03       -1.000000
13      -0.2124130E-04       0.0000000
14      -0.9656257E-06       0.0000000
15      -0.1173822E-05       0.0000000
16      -0.1284344E-03       0.0000000
17      -0.2496282E-04       0.0000000
18      -0.3507856E-05       0.0000000
19      -0.1920127E-04       0.0000000
20      -0.3966834E-03       -1.000000
21       0.0000000           0.0000000
22       0.0000000          -0.1355939E-01
23       0.0000000          -0.1641983E-02
24       0.0000000           0.0000000
25       0.0000000           0.0000000
26       0.0000000           0.0000000
27       0.0000000           0.0000000
28       0.0000000           0.0000000
29       0.0000000           0.0000000
30       0.0000000           0.0000000
31       0.0000000           0.2464742E-03
32       0.0000000           0.0000000
33       0.0000000           0.0000000
34      -0.6641537E-07       0.0000000
35      -0.2551091E-06       0.0000000
36       0.0000000            1.000000
37       0.0000000            1.000000
38       0.0000000           0.0000000
39       0.0000000           0.0000000
40       0.0000000           0.0000000
41       0.0000000           0.0000000
42       0.0000000           0.0000000
```

```
!This model is the same formulation given by Pudar and Liggett;
!Under-determined problem;

!Two head measurements taken at Nodes 3 and 5;
!One leak condition (Possible leaks at Nodes 2, 4, and 7);

Model:

Min=(A4^2)+(A7^2);

!Continuity Equation for Demand and Leakage;
  -Qp12-Qp17=-Sp1;                  !Node 1;
  Qp12+Qp72-Qp24-Qp23=L2;           !Node 2;
  Qp23+Qp43=D3;                     !Node 3;
  Qp54+Qp64+Qp74+Qp24-Qp43=L4;      !Node 4;
  Qp65-Qp54=-S5;                    !Node 5;
  -Qp64-Qp65-Qp67=-S6;              !Node 6;
  Qp17+Qp67-Qp72-Qp74=L7;           !Node 7;

!Overall balance of inflows and outflows;
  Sp1+S5+S6=L2+L4+L7+D3;

!Energy equation with leakage flows;
  H1-H2=r12*Qp12*((@abs(Qp12))^0.852);    !Pipe 1;
  H1-H7=r17*Qp17*((@abs(Qp17))^0.852);    !Pipe 2;
  H7-H2=r72*Qp72*((@abs(Qp72))^0.852);    !Pipe 3;
  H2-H4=r24*Qp24*((@abs(Qp24))^0.852);    !Pipe 4;
  H2-H3=r23*Qp23*((@abs(Qp23))^0.852);    !Pipe 5;
  H4-H3=r43*Qp43*((@abs(Qp43))^0.852);    !Pipe 6;
  H7-H4=r74*Qp74*((@abs(Qp74))^0.852);    !Pipe 7;
  H6-H4=r64*Qp64*((@abs(Qp64))^0.852);    !Pipe 8;
  H5-H4=r54*Qp54*((@abs(Qp54))^0.852);    !Pipe 9;
  H6-H5=r65*Qp65*((@abs(Qp65))^0.852);    !Pipe 10;
  H6-H7=r67*Qp67*((@abs(Qp67))^0.852);    !Pipe 11;

!Resistance Coefficients based on the Hazen-Williams equation;
  r12=392.9136;
  r17=276.9719;
  r72=821.1570;
  r24=1164.8971;
  r23=276.9719;
  r43=276.9719;
  r74=276.9719;
  r64=392.9136;
  r54=821.1570;
  r65=3333.7099;
  r67=3333.7099;

!Pressure-dependent outflow (orifice equation);
  D3=C*A3*((2*g*H3)^0.5);
  L2=C*A2*((2*g*H2)^0.5);
  L4=C*A4*((2*g*H4)^0.5);
  L7=C*A7*((2*g*H7)^0.5);

!Allow for flow reversal in each pipe;
  @free(Qp12);
  @free(Qp17);
```

```
  @free(Qp72);
  @free(Qp24);
  @free(Qp23);
  @free(Qp43);
  @free(Qp74);
  @free(Qp64);
  @free(Qp54);
  @free(Qp65);
  @free(Qp67);

!Given values in the Pudar-Liggett paper;
  H1=30.48;
  H3=25.99206;
  H5=28.12206;
  S5=0.01893;
  S6=0.01262;
  A3=0.0054154736;
  C=1;
  g=9.81;

End
```

```
Rows=     23 Vars=     22 No. integer vars=      0
 Nonlinear rows=     15 Nonlinear vars=     17 Nonlinear constraints=     14
 Nonzeros=     78 Constraint nonz=    66 Density=0.147
 No. < :   0 No. =:  22 No. > :   0, Obj=MIN Single cols=    1

 Optimal solution found at step:        24
 Objective value:                0.1479587E-12


                        Variable          Value         Reduced Cost
                              A4      0.0000000            0.0000000
                              A7      0.3846540E-06        0.0000000
                            QP12      0.7295760E-01        0.0000000
                            QP17      0.7157073E-01        0.0000000
                             SP1      0.1445283            0.0000000
                            QP72      0.2647347E-01        0.0000000
                            QP24     -0.1205282E-01        0.0000000
                            QP23      0.5770886E-01        0.0000000
                              L2      0.5377503E-01        0.0000000
                            QP43      0.6458546E-01        0.0000000
                              D3      0.1222943            0.0000000
                            QP54      0.1620741E-01        0.0000000
                            QP64      0.2212007E-01        0.0000000
                            QP74      0.3831079E-01        0.0000000
                              L4      0.0000000             2.215086
                            QP65     -0.2722584E-02        0.0000000
                              S5      0.1893000E-01        0.0000000
                            QP67     -0.6777488E-02         63.12762
                              S6      0.1262000E-01        0.0000000
                              L7      0.8985945E-05        0.0000000
                              H1       30.48000            0.0000000
                              H2       27.39892            0.0000000
                             R12       392.9136            0.0000000
                              H7       28.38394            0.5314291
                             R17       276.9719            0.0000000
                             R72       821.1570            0.0000000
                              H4       27.72507            0.0000000
                             R24       1164.897            0.0000000
                              H3       25.99206            0.0000000
                             R23       276.9719            0.0000000
                             R43       276.9719            0.0000000
                             R74       276.9719            0.0000000
                              H6       28.06301            0.0000000
                             R64       392.9136            0.0000000
                              H5       28.12206            0.0000000
                             R54       821.1570            0.0000000
                             R65       3333.710            0.0000000
                             R67       3333.710            0.0000000
                               C       1.000000            0.0000000
                              A3      0.5415474E-02        0.0000000
                               G       9.810000            0.0000000
                              A2      0.2319336E-02        0.0000000

                             Row   Slack or Surplus       Dual Price
                               1      0.0000000            -1.000000
                               2      0.0000000            -1.000000
                               3      0.0000000            -1.000000
```

```
 4       0.0000000            1.000000
 5       0.0000000           -3.215086
 6       0.0000000            1.000000
 7       0.0000000           -11.46511
 8       0.0000000           -1.000000
 9       0.0000000            1.000000
10      -0.8847191E-05       0.0000000
11      -0.5685167E-05       0.0000000
12      -0.6890117E-04       0.0000000
13      -0.7133044E-03      -0.4429704E-01
14      -0.1573118E-04       0.4429704E-01
15      -0.1554090E-04       0.8481933E-01
16      -0.7507673E-04      -0.6955433E-01
17      -0.4353202E-05       0.2915808
18      -0.3458932E-04      -0.9291012E-01
19      -0.1747062E-03       0.3094026
20      -0.2504789E-03      -0.6009834
21       0.0000000           0.0000000
22       0.0000000           0.0000000
23       0.0000000           0.0000000
24       0.0000000           0.1237516E-04
25       0.0000000           0.2250069E-03
26       0.0000000           0.5307187E-03
27       0.0000000          -0.1654379E-03
28       0.0000000           0.2507862E-03
29       0.0000000          -0.4492135E-04
30       0.0000000          -0.5496792E-05
31       0.0000000           0.5780984E-04
32       0.0000000            2.000000
33      -0.9639542E-07       0.0000000
34       0.0000000           0.0000000
35      -0.9134222E-07       0.0000000
36       0.0000000           0.0000000
37       0.0000000           0.1338214
38       0.0000000           0.4023127
39       0.0000000           -2.000000
40       0.0000000            10.46511
41       0.0000000            45.16477
42       0.0000000           0.2445886
43       0.0000000           0.1246621E-01
```

```
!This model is the same formulation given by Pudar and Liggett;
!Under-determined problem;

!Two head measurements taken at Nodes 5 and 6;
!One leak condition (Possible leaks at Nodes 2, 4, and 7);

Model:

Min=(A4^2)+(A7^2);

!Continuity Equation for Demand and Leakage;
  -Qp12-Qp17=-Sp1;                  !Node 1;
  Qp12+Qp72-Qp24-Qp23=L2;           !Node 2;
  Qp23+Qp43=D3;                     !Node 3;
  Qp54+Qp64+Qp74+Qp24-Qp43=L4;      !Node 4;
  Qp65-Qp54=-S5;                    !Node 5;
  -Qp64-Qp65-Qp67=-S6;              !Node 6;
  Qp17+Qp67-Qp72-Qp74=L7;           !Node 7;

!Overall balance of inflows and outflows;
  Sp1+S5+S6=L2+L4+L7+D3;

!Energy equation with leakage flows;
  H1-H2=r12*Qp12*((@abs(Qp12))^0.852);    !Pipe 1;
  H1-H7=r17*Qp17*((@abs(Qp17))^0.852);    !Pipe 2;
  H7-H2=r72*Qp72*((@abs(Qp72))^0.852);    !Pipe 3;
  H2-H4=r24*Qp24*((@abs(Qp24))^0.852);    !Pipe 4;
  H2-H3=r23*Qp23*((@abs(Qp23))^0.852);    !Pipe 5;
  H4-H3=r43*Qp43*((@abs(Qp43))^0.852);    !Pipe 6;
  H7-H4=r74*Qp74*((@abs(Qp74))^0.852);    !Pipe 7;
  H6-H4=r64*Qp64*((@abs(Qp64))^0.852);    !Pipe 8;
  H5-H4=r54*Qp54*((@abs(Qp54))^0.852);    !Pipe 9;
  H6-H5=r65*Qp65*((@abs(Qp65))^0.852);    !Pipe 10;
  H6-H7=r67*Qp67*((@abs(Qp67))^0.852);    !Pipe 11;

!Resistance Coefficients based on the Hazen-Williams equation;
  r12=392.9136;
  r17=276.9719;
  r72=821.1570;
  r24=1164.8971;
  r23=276.9719;
  r43=276.9719;
  r74=276.9719;
  r64=392.9136;
  r54=821.1570;
  r65=3333.7099;
  r67=3333.7099;

!Pressure-dependent outflow (orifice equation);
  D3=C*A3*((2*g*H3)^0.5);
  L2=C*A2*((2*g*H2)^0.5);
  L4=C*A4*((2*g*H4)^0.5);
  L7=C*A7*((2*g*H7)^0.5);

!Allow for flow reversal in each pipe;
  @free(Qp12);
  @free(Qp17);
```

```
  @free(Qp72);
  @free(Qp24);
  @free(Qp23);
  @free(Qp43);
  @free(Qp74);
  @free(Qp64);
  @free(Qp54);
  @free(Qp65);
  @free(Qp67);

!Given values in the Pudar-Liggett paper;
  H1=30.48;
  H5=28.12206;
  H6=28.06293;
  S5=0.01893;
  S6=0.01262;
  A3=0.0054154736;
  C=1;
  g=9.81;

End
```

```
Rows=     16 Vars=     15 No. integer vars=      0
 Nonlinear rows=      8 Nonlinear vars=     10 Nonlinear constraints=      7
 Nonzeros=     50 Constraint nonz=    40 Density=0.195
 No. < :   0 No. =:  15 No. > :   0, Obj=MIN Single cols=    1

 Optimal solution found at step:        16
 Objective value:                0.3185288E-08


                       Variable           Value        Reduced Cost
                             A4       0.5112617E-04       0.0000000
                             A7       0.2390404E-04       0.0000000
                           QP12       0.7245852E-01       0.0000000
                           QP17       0.7156941E-01       0.0000000
                            SP1       0.1440279           0.0000000
                           QP72       0.2590360E-01       0.0000000
                           QP24      -0.1126399E-01        52.07709
                           QP23       0.5814414E-01       0.0000000
                             L2       0.5148198E-01       0.0000000
                           QP43       0.6419530E-01       0.0000000
                             D3       0.1223394           0.0000000
                           QP54       0.1620980E-01       0.0000000
                           QP64       0.2212219E-01       0.0000000
                           QP74       0.3831971E-01       0.0000000
                             L4       0.1192417E-02       0.0000000
                           QP65      -0.2720198E-02       0.0000000
                             S5       0.1893000E-01       0.0000000
                           QP67      -0.6781996E-02       0.0000000
                             S6       0.1262000E-01       0.0000000
                             L7       0.5641020E-03       -56.38007
                             H1        30.48000           0.0000000
                             H2        27.43784           0.0000000
                            R12        392.9136           0.0000000
                             H7        28.38400           0.0000000
                            R17        276.9719           0.0000000
                            R72        821.1570           0.0000000
                             H4        27.72493           0.0000000
                            R24        1164.897           0.0000000
                             H3        26.01124           0.0000000
                            R23        276.9719           0.0000000
                            R43        276.9719           0.0000000
                            R74        276.9719           0.0000000
                             H6        28.06293           0.0000000
                            R64        392.9136           0.0000000
                             H5        28.12206           0.0000000
                            R54        821.1570           0.0000000
                            R65        3333.710           0.0000000
                            R67        3333.710           0.0000000
                              C        1.000000           0.0000000
                             A3       0.5415474E-02       0.0000000
                              G        9.810000           0.0000000
                             A2       0.2218885E-02        93.84790

                            Row    Slack or Surplus      Dual Price
                              1       0.0000000           -1.000000
                              2       0.0000000            1.000000
                              3       0.0000000           -3.045635
```

```
 4       0.0000000           -1.000000
 5       0.0000000            1.000000
 6       0.0000000           -725.8972
 7       0.0000000            376.0858
 8       0.0000000            57.38007
 9       0.0000000           -1.000000
10      -0.2014765E-06      -0.6367071E-01
11       0.0000000            1.037984
12      -0.3648169E-05      -0.8885223
13      -0.7952588E-06       -1.000000
14      -0.1025803E-05       0.4509920E-01
15      -0.8051145E-06      -0.4039538E-01
16       0.0000000           -1.765210
17       0.0000000           -13.19359
18       0.0000000            15.91841
19       0.0000000           -28.46660
20       0.0000000           -3.691716
21       0.0000000          -0.4929756E-03
22       0.0000000           0.7854993E-02
23       0.0000000          -0.1023791E-02
24       0.0000000           0.2464529E-03
25       0.0000000           0.2322918E-03
26       0.0000000          -0.2499354E-03
27       0.0000000          -0.4200436E-02
28       0.0000000          -0.1134971E-01
29       0.0000000           0.7698533E-02
30       0.0000000           0.5049120E-03
31       0.0000000           0.3555514E-03
32       0.0000000           -2.000000
33      -0.4792209E-06       -4.045635
34       0.0000000           0.0000000
35       0.0000000           0.0000000
36       0.0000000          -0.9743131
37       0.0000000           -44.38501
38       0.0000000            45.35191
39       0.0000000            726.8972
40       0.0000000           -375.0858
41       0.0000000           -45.18144
42       0.0000000          -0.4529581
43       0.0000000          -0.2308641E-01
```

```
!This model is the same formulation given by Pudar and Liggett;
!Under-determined problem;

!Two head measurements taken at Nodes 3 and 6;
!One leak condition (Possible leaks at Nodes 2, 4, and 7);

Model:

Min=(A4^2)+(A7^2);

!Continuity Equation for Demand and Leakage;
  -Qp12-Qp17=-Sp1;                  !Node 1;
  Qp12+Qp72-Qp24-Qp23=L2;           !Node 2;
  Qp23+Qp43=D3;                     !Node 3;
  Qp54+Qp64+Qp74+Qp24-Qp43=L4;      !Node 4;
  Qp65-Qp54=-S5;                    !Node 5;
  -Qp64-Qp65-Qp67=-S6;              !Node 6;
  Qp17+Qp67-Qp72-Qp74=L7;           !Node 7;

!Overall balance of inflows and outflows;
  Sp1+S5+S6=L2+L4+L7+D3;

!Energy equation with leakage flows;
  H1-H2=r12*Qp12*((@abs(Qp12))^0.852);    !Pipe 1;
  H1-H7=r17*Qp17*((@abs(Qp17))^0.852);    !Pipe 2;
  H7-H2=r72*Qp72*((@abs(Qp72))^0.852);    !Pipe 3;
  H2-H4=r24*Qp24*((@abs(Qp24))^0.852);    !Pipe 4;
  H2-H3=r23*Qp23*((@abs(Qp23))^0.852);    !Pipe 5;
  H4-H3=r43*Qp43*((@abs(Qp43))^0.852);    !Pipe 6;
  H7-H4=r74*Qp74*((@abs(Qp74))^0.852);    !Pipe 7;
  H6-H4=r64*Qp64*((@abs(Qp64))^0.852);    !Pipe 8;
  H5-H4=r54*Qp54*((@abs(Qp54))^0.852);    !Pipe 9;
  H6-H5=r65*Qp65*((@abs(Qp65))^0.852);    !Pipe 10;
  H6-H7=r67*Qp67*((@abs(Qp67))^0.852);    !Pipe 11;

!Resistance Coefficients based on the Hazen-Williams equation;
  r12=392.9136;
  r17=276.9719;
  r72=821.1570;
  r24=1164.8971;
  r23=276.9719;
  r43=276.9719;
  r74=276.9719;
  r64=392.9136;
  r54=821.1570;
  r65=3333.7099;
  r67=3333.7099;

!Pressure-dependent outflow (orifice equation);
  D3=C*A3*((2*g*H3)^0.5);
  L2=C*A2*((2*g*H2)^0.5);
  L4=C*A4*((2*g*H4)^0.5);
  L7=C*A7*((2*g*H7)^0.5);

!Allow for flow reversal in each pipe;
  @free(Qp12);
  @free(Qp17);
```

```
  @free(Qp72);
  @free(Qp24);
  @free(Qp23);
  @free(Qp43);
  @free(Qp74);
  @free(Qp64);
  @free(Qp54);
  @free(Qp65);
  @free(Qp67);

!Given values in the Pudar-Liggett paper;
  H1=30.48;
  H3=25.99206;
  H6=28.06293;
  S5=0.01893;
  S6=0.01262;
  A3=0.0054154736;
  C=1;
  g=9.81;

End
```

```
Rows=     23 Vars=     22 No. integer vars=      0
 Nonlinear rows=     15 Nonlinear vars=     17 Nonlinear constraints=     14
 Nonzeros=     78 Constraint nonz=    65 Density=0.147
 No. < :   0 No. =:  22 No. > :   0, Obj=MIN Single cols=    1

 Optimal solution found at step:        16
 Objective value:                0.0000000E+00


                       Variable           Value        Reduced Cost
                             A4       0.0000000            21.64551
                             A7       0.0000000            2.320110
                           QP12       0.7295628E-01       0.0000000
                           QP17       0.7157009E-01       0.0000000
                            SP1       0.1445264           0.0000000
                           QP72       0.2647157E-01       0.0000000
                           QP24      -0.1206490E-01       0.0000000
                           QP23       0.5771070E-01       0.0000000
                             L2       0.5378205E-01       0.0000000
                           QP43       0.6458362E-01        11.48376
                             D3       0.1222943           0.0000000
                           QP54       0.1620930E-01        1.423962
                           QP64       0.2212244E-01       0.0000000
                           QP74       0.3831679E-01       0.0000000
                             L4       0.0000000           0.0000000
                           QP65      -0.2720702E-02       0.0000000
                             S5       0.1893000E-01       0.0000000
                           QP67      -0.6781735E-02       0.0000000
                             S6       0.1262000E-01       0.0000000
                             L7       0.0000000           0.0000000
                             H1        30.48000           0.0000000
                             H2        27.39902           0.0000000
                            R12        392.9136           0.0000000
                             H7        28.38397           0.0000000
                            R17        276.9719           0.0000000
                            R72        821.1570           0.0000000
                             H4        27.72499           0.0000000
                            R24        1164.897           0.0000000
                             H3        25.99206           0.0000000
                            R23        276.9719           0.0000000
                            R43        276.9719           0.0000000
                            R74        276.9719           0.0000000
                             H6        28.06293           0.0000000
                            R64        392.9136           0.0000000
                             H5        28.12211           0.0000000
                            R54        821.1570           0.0000000
                            R65        3333.710           0.0000000
                            R67        3333.710           0.0000000
                              C        1.000000           0.0000000
                             A3       0.5415474E-02       0.0000000
                              G        9.810000           0.0000000
                             A2       0.2319638E-02       0.0000000

                            Row    Slack or Surplus      Dual Price
                              1       0.0000000           -1.000000
                              2       0.0000000          -0.9016844
                              3       0.0000000          -0.9016844
```

```
 4       0.0000000           0.0000000
 5       0.0000000           -1.829758
 6       0.0000000           -1.000000
 7       0.0000000            1.000000
 8       0.0000000           -1.000000
 9       0.0000000           0.9016844
10      -0.8571204E-06       0.0000000
11      -0.4103318E-06      -0.1812652E-02
12      -0.3797920E-05       0.1426747E-02
13      -0.6537011E-04      -0.1854368E-01
14      -0.3075378E-05       0.1997043E-01
15      -0.3016415E-05      -0.1942702
16      -0.7610506E-05      -0.2605117E-01
17      -0.7116084E-04      -0.1000031
18      -0.4744315E-05      -0.4967223E-01
19      -0.2567180E-04      -0.4967223E-01
20      -0.1362593E-04      -0.2281178E-01
21       0.0000000           0.0000000
22       0.0000000          -0.1371758E-04
23       0.0000000           0.1711335E-05
24       0.0000000           0.5190125E-05
25       0.0000000           0.1014458E-03
26       0.0000000          -0.1215494E-02
27       0.0000000          -0.6198177E-04
28       0.0000000          -0.8602886E-04
29       0.0000000          -0.2402132E-04
30       0.0000000           0.8813389E-06
31       0.0000000           0.2196860E-05
32       0.0000000           0.9016844
33       0.0000000           0.0000000
34       0.0000000          -0.9280739
35       0.0000000          -0.9831562E-01
36       0.0000000           0.1812652E-02
37       0.0000000          -0.1721785
38       0.0000000           0.1724871
39       0.0000000           0.9831560E-01
40       0.0000000           -1.901684
41       0.0000000            20.36219
42       0.0000000           0.1102709
43       0.0000000           0.5620295E-02
```

```
!This model is the same formulation given by Pudar and Liggett;
!Under-determined problem;

!Two head measurements taken at Nodes 3 and 5;
!Two leaks condition (Possible leaks at Nodes 2, 4, and 7);

Model:

Min=(A7^2);

!Continuity Equation for Demand and Leakage;
  -Qp12-Qp17=-Sp1;                  !Node 1;
  Qp12+Qp72-Qp24-Qp23=L2;           !Node 2;
  Qp23+Qp43=D3;                     !Node 3;
  Qp54+Qp64+Qp74+Qp24-Qp43=L4;      !Node 4;
  Qp65-Qp54=-S5;                    !Node 5;
  -Qp64-Qp65-Qp67=-S6;              !Node 6;
  Qp17+Qp67-Qp72-Qp74=L7;           !Node 7;

!Overall balance of inflows and outflows;
  Sp1+S5+S6=L2+L4+L7+D3;

!Energy equation with leakage flows;
  H1-H2=r12*Qp12*((@abs(Qp12))^0.852);    !Pipe 1;
  H1-H7=r17*Qp17*((@abs(Qp17))^0.852);    !Pipe 2;
  H7-H2=r72*Qp72*((@abs(Qp72))^0.852);    !Pipe 3;
  H2-H4=r24*Qp24*((@abs(Qp24))^0.852);    !Pipe 4;
  H2-H3=r23*Qp23*((@abs(Qp23))^0.852);    !Pipe 5;
  H4-H3=r43*Qp43*((@abs(Qp43))^0.852);    !Pipe 6;
  H7-H4=r74*Qp74*((@abs(Qp74))^0.852);    !Pipe 7;
  H6-H4=r64*Qp64*((@abs(Qp64))^0.852);    !Pipe 8;
  H5-H4=r54*Qp54*((@abs(Qp54))^0.852);    !Pipe 9;
  H6-H5=r65*Qp65*((@abs(Qp65))^0.852);    !Pipe 10;
  H6-H7=r67*Qp67*((@abs(Qp67))^0.852);    !Pipe 11;

!Resistance Coefficients based on the Hazen-Williams equation;
  r12=392.9136;
  r17=276.9719;
  r72=821.1570;
  r24=1164.8971;
  r23=276.9719;
  r43=276.9719;
  r74=276.9719;
  r64=392.9136;
  r54=821.1570;
  r65=3333.7099;
  r67=3333.7099;

!Pressure-dependent outflow (orifice equation);
  D3=C*A3*((2*g*H3)^0.5);
  L2=C*A2*((2*g*H2)^0.5);
  L4=C*A4*((2*g*H4)^0.5);
  L7=C*A7*((2*g*H7)^0.5);

!Allow for flow reversal in each pipe;
  @free(Qp12);
  @free(Qp17);
```

```
  @free(Qp72);
  @free(Qp24);
  @free(Qp23);
  @free(Qp43);
  @free(Qp74);
  @free(Qp64);
  @free(Qp54);
  @free(Qp65);
  @free(Qp67);

!Given values in the Pudar-Liggett paper;
  H1=30.48;
  H3=24.08809;
  H5=25.91779;
  S5=0.01893;
  S6=0.01262;
  A3=0.0054154736;
  C=1;
  g=9.81;

End
```

```
Rows=     23 Vars=     22 No. integer vars=      0
 Nonlinear rows=     15 Nonlinear vars=     17 Nonlinear constraints=     14
 Nonzeros=     77 Constraint nonz=    66 Density=0.146
 No. < :   0 No. =:  22 No. > :   0, Obj=MIN Single cols=    2

 Optimal solution found at step:        21
 Objective value:                0.1897848E-05


                       Variable           Value        Reduced Cost
                             A7       0.1377624E-02       0.0000000
                           QP12       0.9363703E-01       0.0000000
                           QP17       0.1038754            74.49817
                            SP1       0.1975124           0.0000000
                           QP72       0.2223138E-01       0.0000000
                           QP24       0.5754640E-02       0.0000000
                           QP23       0.5975556E-01       0.0000000
                             L2       0.5035821E-01       0.0000000
                           QP43       0.5797444E-01       0.0000000
                             D3       0.1177300           0.0000000
                           QP54       0.1650890E-01       0.0000000
                           QP64       0.2299680E-01       0.0000000
                           QP74       0.4239315E-01       0.0000000
                             L4       0.2967906E-01       0.0000000
                           QP65      -0.2421101E-02        8.370163
                             S5       0.1893000E-01       0.0000000
                           QP67      -0.7955701E-02       0.0000000
                             S6       0.1262000E-01       0.0000000
                             L7       0.3129519E-01       0.0000000
                             H1        30.48000           0.0000000
                             H2        25.58877           0.0000000
                            R12        392.9136           0.0000000
                             H7        26.30155           0.9800894
                            R17        276.9719           0.0000000
                            R72        821.1570           0.0000000
                             H4        25.50698           0.0000000
                            R24        1164.897           0.0000000
                             H3        24.08809           0.0000000
                            R23        276.9719           0.0000000
                            R43        276.9719           0.0000000
                            R74        276.9719           0.0000000
                             H6        25.87014           0.0000000
                            R64        392.9136           0.0000000
                             H5        25.91779           0.0000000
                            R54        821.1570           0.0000000
                            R65        3333.710           0.0000000
                            R67        3333.710           0.0000000
                              C        1.000000           0.0000000
                             A3       0.5415474E-02       0.0000000
                              G        9.810000           0.0000000
                             A2       0.2247483E-02       0.0000000
                             A4       0.1326695E-02       0.0000000

                            Row    Slack or Surplus      Dual Price
                              1       0.1897848E-05       -1.000000
                              2       0.0000000            1.000000
                              3       0.0000000            1.000000
```

```
 4       0.0000000            1.000000
 5       0.0000000            1.000000
 6       0.0000000            4.151478
 7       0.0000000           -1.000000
 8       0.0000000            1.000000
 9       0.0000000           -1.000000
10      -0.1957552E-06       0.0000000
11      -0.2386293E-04       -1.000000
12      -0.8360912E-04       0.0000000
13      -0.9763938E-03       0.0000000
14      -0.6223540E-06       0.0000000
15      -0.6248777E-06       0.0000000
16      -0.1010330E-03       0.0000000
17      -0.4560799E-05       0.6838348E-01
18      -0.1962967E-05      -0.6838348E-01
19      -0.1052323E-04      -0.8829404E-01
20      -0.9495530E-04       0.1991056E-01
21       0.0000000           0.0000000
22       0.0000000          -0.1508626E-01
23       0.0000000           0.0000000
24       0.0000000           0.0000000
25       0.0000000           0.0000000
26       0.0000000           0.0000000
27       0.0000000           0.0000000
28       0.0000000           0.6320615E-04
29       0.0000000          -0.3421095E-04
30       0.0000000           0.1262186E-05
31       0.0000000          -0.2577151E-05
32       0.0000000           0.0000000
33      -0.3660152E-07       0.0000000
34       0.0000000           0.0000000
35      -0.4769888E-06       0.0000000
36       0.0000000            1.000000
37       0.0000000           0.0000000
38       0.0000000          -0.1991056E-01
39       0.0000000           -3.151478
40       0.0000000            2.000000
41       0.0000000           0.0000000
42       0.0000000           0.0000000
43       0.0000000           0.0000000
```

```
!This model is the same formulation given by Pudar and Liggett;
!Under-determined problem;

!Two head measurements taken at Nodes 5 and 6;
!Two leaks condition (Possible leaks at Nodes 2, 4, and 7);

Model:

Min=(A7^2);

!Continuity Equation for Demand and Leakage;
  -Qp12-Qp17=-Sp1;                  !Node 1;
  Qp12+Qp72-Qp24-Qp23=L2;           !Node 2;
  Qp23+Qp43=D3;                     !Node 3;
  Qp54+Qp64+Qp74+Qp24-Qp43=L4;      !Node 4;
  Qp65-Qp54=-S5;                    !Node 5;
  -Qp64-Qp65-Qp67=-S6;              !Node 6;
  Qp17+Qp67-Qp72-Qp74=L7;           !Node 7;

!Overall balance of inflows and outflows;
  Sp1+S5+S6=L2+L4+L7+D3;

!Energy equation with leakage flows;
  H1-H2=r12*Qp12*((@abs(Qp12))^0.852);    !Pipe 1;
  H1-H7=r17*Qp17*((@abs(Qp17))^0.852);    !Pipe 2;
  H7-H2=r72*Qp72*((@abs(Qp72))^0.852);    !Pipe 3;
  H2-H4=r24*Qp24*((@abs(Qp24))^0.852);    !Pipe 4;
  H2-H3=r23*Qp23*((@abs(Qp23))^0.852);    !Pipe 5;
  H4-H3=r43*Qp43*((@abs(Qp43))^0.852);    !Pipe 6;
  H7-H4=r74*Qp74*((@abs(Qp74))^0.852);    !Pipe 7;
  H6-H4=r64*Qp64*((@abs(Qp64))^0.852);    !Pipe 8;
  H5-H4=r54*Qp54*((@abs(Qp54))^0.852);    !Pipe 9;
  H6-H5=r65*Qp65*((@abs(Qp65))^0.852);    !Pipe 10;
  H6-H7=r67*Qp67*((@abs(Qp67))^0.852);    !Pipe 11;

!Resistance Coefficients based on the Hazen-Williams equation;
  r12=392.9136;
  r17=276.9719;
  r72=821.1570;
  r24=1164.8971;
  r23=276.9719;
  r43=276.9719;
  r74=276.9719;
  r64=392.9136;
  r54=821.1570;
  r65=3333.7099;
  r67=3333.7099;

!Pressure-dependent outflow (orifice equation);
  D3=C*A3*((2*g*H3)^0.5);
  L2=C*A2*((2*g*H2)^0.5);
  L4=C*A4*((2*g*H4)^0.5);
  L7=C*A7*((2*g*H7)^0.5);

!Allow for flow reversal in each pipe;
  @free(Qp12);
  @free(Qp17);
```

```
  @free(Qp72);
  @free(Qp24);
  @free(Qp23);
  @free(Qp43);
  @free(Qp74);
  @free(Qp64);
  @free(Qp54);
  @free(Qp65);
  @free(Qp67);

!Given values in the Pudar-Liggett paper;
  H1=30.48;
  H5=25.91779;
  H6=25.90237;
  S5=0.01893;
  S6=0.01262;
  A3=0.0054154736;
  C=1;
  g=9.81;

End
```

```
Rows=     16 Vars=     15 No. integer vars=      0
 Nonlinear rows=      8 Nonlinear vars=      9 Nonlinear constraints=      7
 Nonzeros=     49 Constraint nonz=    40 Density=0.191
 No. < :   0 No. =:  15 No. > :   0, Obj=MIN Single cols=    2

 Optimal solution found at step:        25
 Objective value:                0.0000000E+00


                       Variable           Value        Reduced Cost
                             A7       0.0000000           0.1000000E-07
                           QP12       0.9306545E-01       0.0000000
                           QP17       0.9699469E-01       0.0000000
                            SP1       0.1900601           0.0000000
                           QP72       0.2885816E-01       0.0000000
                           QP24       0.8987354E-02       0.0000000
                           QP23       0.6092649E-01       0.0000000
                             L2       0.5200977E-01       0.0000000
                           QP43       0.5680408E-01       0.0000000
                             D3       0.1177306           0.0000000
                           QP54       0.1761352E-01       0.0000000
                           QP64       0.2574909E-01       0.0000000
                           QP74       0.5632393E-01       0.0000000
                             L4       0.5186980E-01       0.0000000
                           QP65      -0.1316482E-02       0.0000000
                             S5       0.1893000E-01       0.0000000
                           QP67      -0.1181261E-01       0.0000000
                             S6       0.1262000E-01       0.0000000
                             L7       0.0000000           0.0000000
                             H1        30.48000           0.0000000
                             H2        25.64392           0.0000000
                            R12        392.9136           0.0000000
                             H7        26.79962           0.0000000
                            R17        276.9719           0.0000000
                            R72        821.1570           0.0000000
                             H4        25.45463           0.0000000
                            R24        1164.897           0.0000000
                             H3        24.08833           0.0000000
                            R23        276.9719           0.0000000
                            R43        276.9719           0.0000000
                            R74        276.9719           0.0000000
                             H6        25.90237           0.0000000
                            R64        392.9136           0.0000000
                             H5        25.91779           0.0000000
                            R54        821.1570           0.0000000
                            R65        3333.710           0.0000000
                            R67        3333.710           0.0000000
                              C        1.000000           0.0000000
                             A3       0.5415474E-02       0.0000000
                              G        9.810000           0.0000000
                             A2       0.2318668E-02       0.0000000
                             A4       0.2321035E-02       0.0000000

                            Row    Slack or Surplus      Dual Price
                              1       0.0000000          -1.000000
                              2       0.0000000           0.0000000
                              3       0.0000000           0.0000000
```

```
 4       0.0000000           0.0000000
 5       0.0000000           0.0000000
 6       0.0000000           0.0000000
 7       0.0000000           0.0000000
 8       0.0000000           0.0000000
 9       0.0000000           0.0000000
10       0.0000000           0.0000000
11       0.0000000           0.0000000
12       0.0000000           0.0000000
13      -0.3145052E-03       0.0000000
14       0.0000000           0.0000000
15       0.0000000           0.0000000
16       0.0000000           0.0000000
17       0.0000000           0.0000000
18       0.0000000           0.0000000
19       0.0000000           0.0000000
20       0.0000000           0.0000000
21       0.0000000           0.0000000
22       0.0000000           0.0000000
23       0.0000000           0.0000000
24       0.0000000           0.0000000
25       0.0000000           0.0000000
26       0.0000000           0.0000000
27       0.0000000           0.0000000
28       0.0000000           0.0000000
29       0.0000000           0.0000000
30       0.0000000           0.0000000
31       0.0000000           0.0000000
32       0.0000000           0.0000000
33      -0.5542434E-06       0.0000000
34       0.0000000           0.0000000
35       0.0000000           0.0000000
36       0.0000000           0.0000000
37       0.0000000           0.0000000
38       0.0000000           0.0000000
39       0.0000000           0.0000000
40       0.0000000           0.0000000
41       0.0000000           0.0000000
42       0.0000000           0.0000000
43       0.0000000           0.0000000
```

```
!This model is the same formulation given by Pudar and Liggett;
!Under-determined problem;

!Two head measurements taken at Nodes 3 and 6;
!Two leaks condition (Possible leaks at Nodes 2, 4, and 7);

Model:

Min=(A7^2);

!Continuity Equation for Demand and Leakage;
  -Qp12-Qp17=-Sp1;                  !Node 1;
  Qp12+Qp72-Qp24-Qp23=L2;           !Node 2;
  Qp23+Qp43=D3;                     !Node 3;
  Qp54+Qp64+Qp74+Qp24-Qp43=L4;      !Node 4;
  Qp65-Qp54=-S5;                    !Node 5;
  -Qp64-Qp65-Qp67=-S6;              !Node 6;
  Qp17+Qp67-Qp72-Qp74=L7;           !Node 7;

!Overall balance of inflows and outflows;
  Sp1+S5+S6=L2+L4+L7+D3;

!Energy equation with leakage flows;
  H1-H2=r12*Qp12*((@abs(Qp12))^0.852);    !Pipe 1;
  H1-H7=r17*Qp17*((@abs(Qp17))^0.852);    !Pipe 2;
  H7-H2=r72*Qp72*((@abs(Qp72))^0.852);    !Pipe 3;
  H2-H4=r24*Qp24*((@abs(Qp24))^0.852);    !Pipe 4;
  H2-H3=r23*Qp23*((@abs(Qp23))^0.852);    !Pipe 5;
  H4-H3=r43*Qp43*((@abs(Qp43))^0.852);    !Pipe 6;
  H7-H4=r74*Qp74*((@abs(Qp74))^0.852);    !Pipe 7;
  H6-H4=r64*Qp64*((@abs(Qp64))^0.852);    !Pipe 8;
  H5-H4=r54*Qp54*((@abs(Qp54))^0.852);    !Pipe 9;
  H6-H5=r65*Qp65*((@abs(Qp65))^0.852);    !Pipe 10;
  H6-H7=r67*Qp67*((@abs(Qp67))^0.852);    !Pipe 11;

!Resistance Coefficients based on the Hazen-Williams equation;
  r12=392.9136;
  r17=276.9719;
  r72=821.1570;
  r24=1164.8971;
  r23=276.9719;
  r43=276.9719;
  r74=276.9719;
  r64=392.9136;
  r54=821.1570;
  r65=3333.7099;
  r67=3333.7099;

!Pressure-dependent outflow (orifice equation);
  D3=C*A3*((2*g*H3)^0.5);
  L2=C*A2*((2*g*H2)^0.5);
  L4=C*A4*((2*g*H4)^0.5);
  L7=C*A7*((2*g*H7)^0.5);

!Allow for flow reversal in each pipe;
  @free(Qp12);
  @free(Qp17);
```

```
  @free(Qp72);
  @free(Qp24);
  @free(Qp23);
  @free(Qp43);
  @free(Qp74);
  @free(Qp64);
  @free(Qp54);
  @free(Qp65);
  @free(Qp67);

!Given values in the Pudar-Liggett paper;
  H1=30.48;
  H3=24.08809;
  H6=25.90237;
  S5=0.01893;
  S6=0.01262;
  A3=0.0054154736;
  C=1;
  g=9.81;

End
```

```
Rows=     23 Vars=     22 No. integer vars=      0
 Nonlinear rows=     15 Nonlinear vars=     17 Nonlinear constraints=     14
 Nonzeros=     77 Constraint nonz=    65 Density=0.146
 No. < :   0 No. =:  22 No. > :   0, Obj=MIN Single cols=    2

 Optimal solution found at step:        30
 Objective value:                0.2934081E-05


                       Variable           Value        Reduced Cost
                             A7       0.1712916E-02       0.0000000
                           QP12       0.9426296E-01       0.0000000
                           QP17       0.1050755           0.0000000
                            SP1       0.1993385           0.0000000
                           QP72       0.2173437E-01       0.0000000
                           QP24      -0.3868206E-02       0.0000000
                           QP23       0.5843742E-01        4.353664
                             L2       0.6142811E-01       0.0000000
                           QP43       0.5929258E-01       0.0000000
                             D3       0.1177300           0.0000000
                           QP54       0.1617566E-01       0.0000000
                           QP64       0.2201732E-01       0.0000000
                           QP74       0.3785365E-01       0.0000000
                             L4       0.1288585E-01       0.0000000
                           QP65      -0.2754340E-02       0.0000000
                             S5       0.1893000E-01       0.0000000
                           QP67      -0.6642979E-02        88.14334
                             S6       0.1262000E-01       0.0000000
                             L7       0.3884455E-01       0.0000000
                             H1        30.48000           0.0000000
                             H2        25.52804           0.0000000
                            R12        392.9136           0.0000000
                             H7        26.21167            1.000000
                            R17        276.9719           0.0000000
                            R72        821.1570           0.0000000
                             H4        25.56731           0.0000000
                            R24        1164.897           0.0000000
                             H3        24.08809           0.0000000
                            R23        276.9719           0.0000000
                            R43        276.9719           0.0000000
                            R74        276.9719           0.0000000
                             H6        25.90237           0.0000000
                            R64        392.9136           0.0000000
                             H5        25.96290           0.0000000
                            R54        821.1570           0.0000000
                            R65        3333.710           0.0000000
                            R67        3333.710           0.0000000
                              C        1.000000           0.0000000
                             A3       0.5415474E-02       0.0000000
                              G        9.810000           0.0000000
                             A2       0.2744788E-02       0.0000000
                             A4       0.5753381E-03       0.0000000

                            Row    Slack or Surplus      Dual Price
                              1       0.2934081E-05       -1.000000
                              2       0.0000000            1.000000
                              3       0.0000000            1.000000
```

```
 4       0.0000000            5.353664
 5       0.0000000            1.000000
 6       0.0000000          -0.5354688E-01
 7       0.0000000           -1.000000
 8       0.0000000            1.000000
 9       0.0000000           -1.000000
10      -0.1173924E-05       0.0000000
11      -0.1136452E-04       0.0000000
12      -0.5446157E-05       0.0000000
13      -0.3920870E-03       0.0000000
14      -0.3422477E-05       0.0000000
15      -0.3407359E-05       0.9422843E-01
16      -0.5420038E-04       0.0000000
17      -0.1796483E-04       0.7096702E-01
18      -0.3386360E-05       0.2326141E-01
19      -0.1761107E-04       0.2326141E-01
20      -0.3110795E-03      -1.000000
21       0.0000000           0.0000000
22       0.0000000           0.0000000
23       0.0000000           0.0000000
24       0.0000000           0.0000000
25       0.0000000           0.0000000
26       0.0000000           0.5032440E-03
27       0.0000000           0.0000000
28       0.0000000           0.6051406E-04
29       0.0000000           0.1120595E-04
30       0.0000000          -0.4222295E-06
31       0.0000000           0.9268642E-04
32       0.0000000            4.353664
33       0.0000000           0.0000000
34      -0.6600668E-07       0.0000000
35      -0.2530848E-06       0.0000000
36       0.0000000           0.0000000
37       0.0000000           0.1048676
38       0.0000000           0.9057716
39       0.0000000            1.053547
40       0.0000000            2.000000
41       0.0000000            94.64673
42       0.0000000           0.5125569
43       0.0000000           0.2612404E-01
```

Appendix C

LINGO Runs for the S-L formulation

```
     !Improvement on the Pudar-Liggett Formulation for the Small Network;
!Possible leak locations are Nodes 2, 4, and 7;
!This model is based on the formulation in Chapter 5 using the orifice
equation for pressure-dependent demand and leakage terms;

!Zero leaks condition;

Model:

MAX=((Qp12-Q12)^2)+((Qp17-Q17)^2)+((Qp72-Q72)^2)+((Qp24-Q24)^2)+((Qp23-
Q23)^2)+((Qp43-Q43)^2)+((Qp74-Q74)^2)+((Qp64-Q64)^2)+((Qp54-Q54)^2)+((Qp65-
Q65)^2)+((Qp67-Q67)^2);

!Continuity Equation for Demand;
  -Q12-Q17=-S1;                     !Node 1;
  Q12+Q72-Q24-Q23=0;                !Node 2;
  Q23+Q43=D3;                       !Node 3;
  Q54+Q64+Q74+Q24-Q43=0;            !Node 4;
  Q65-Q54=-S5;                      !Node 5;
  -Q64-Q65-Q67=-S6;                 !Node 6;
  Q17+Q67-Q72-Q74=0;                !Node 7;

!Continuity Equation for Demand and Leakage;
  -Qp12-Qp17=-Sp1;                  !Node 1;
  Qp12+Qp72-Qp24-Qp23=L2;           !Node 2;
  Qp23+Qp43=D3;                     !Node 3;
  Qp54+Qp64+Qp74+Qp24-Qp43=L4;      !Node 4;
  Qp65-Qp54=-S5;                    !Node 5;
  -Qp64-Qp65-Qp67=-S6;              !Node 6;
  Qp17+Qp67-Qp72-Qp74=L7;           !Node 7;

!Overall balance of supply and leakage;
  Sp1=S1+L2+L4+L7;

!Energy equation with leakage flows;
  H1-H2=r12*Qp12*((@abs(Qp12))^0.852);    !Pipe 1;
  H1-H7=r17*Qp17*((@abs(Qp17))^0.852);    !Pipe 2;
  H7-H2=r72*Qp72*((@abs(Qp72))^0.852);    !Pipe 3;
  H2-H4=r24*Qp24*((@abs(Qp24))^0.852);    !Pipe 4;
  H2-H3=r23*Qp23*((@abs(Qp23))^0.852);    !Pipe 5;
  H4-H3=r43*Qp43*((@abs(Qp43))^0.852);    !Pipe 6;
  H7-H4=r74*Qp74*((@abs(Qp74))^0.852);    !Pipe 7;
  H6-H4=r64*Qp64*((@abs(Qp64))^0.852);    !Pipe 8;
  H5-H4=r54*Qp54*((@abs(Qp54))^0.852);    !Pipe 9;
  H6-H5=r65*Qp65*((@abs(Qp65))^0.852);    !Pipe 10;
  H6-H7=r67*Qp67*((@abs(Qp67))^0.852);    !Pipe 11;

!Resistance Coefficients based on the Hazen-Williams equation;
  r12=392.9136;
  r17=276.9719;
  r72=821.1570;
  r24=1164.8971;
  r23=276.9719;
  r43=276.9719;
  r74=276.9719;
  r64=392.9136;
  r54=821.1570;
```

```
  r65=3333.7099;
  r67=3333.7099;

!Pressure-dependent outflow (orifice equation);
  D3=C*A3*((2*g*H3)^0.5);
  L2=0;
  L4=0;
  L7=0;

!Flow magnitude constraint;
  @abs(Sp1)>@abs(S1);
  @abs(Qp12)>@abs(Q12);
  @abs(Qp17)>@abs(Q17);
  @abs(Qp72)>@abs(Q72);
  @abs(Qp24)>@abs(Q24);
  @abs(Qp23)>@abs(Q23);
  @abs(Qp43)>@abs(Q43);
  @abs(Qp74)>@abs(Q74);
  @abs(Qp64)>@abs(Q64);
  @abs(Qp54)>@abs(Q54);
  @abs(Qp65)>@abs(Q65);
  @abs(Qp67)>@abs(Q67);

!Flow direction constraint;
  Qp12*Q12-@abs(Qp12)*@abs(Q12)=0;
  Qp17*Q17-@abs(Qp17)*@abs(Q17)=0;
  Qp72*Q72-@abs(Qp72)*@abs(Q72)=0;
  Qp24*Q24-@abs(Qp24)*@abs(Q24)=0;
  Qp23*Q23-@abs(Qp23)*@abs(Q23)=0;
  Qp43*Q43-@abs(Qp43)*@abs(Q43)=0;
  Qp74*Q74-@abs(Qp74)*@abs(Q74)=0;
  Qp64*Q64-@abs(Qp64)*@abs(Q64)=0;
  Qp54*Q54-@abs(Qp54)*@abs(Q54)=0;
  Qp65*Q65-@abs(Qp65)*@abs(Q65)=0;
  Qp67*Q67-@abs(Qp67)*@abs(Q67)=0;

!Allow for flow reversal in each pipe;
  @free(Q12);
  @free(Q17);
  @free(Q72);
  @free(Q24);
  @free(Q23);
  @free(Q43);
  @free(Q74);
  @free(Q64);
  @free(Q54);
  @free(Q65);
  @free(Q67);
  @free(Qp12);
  @free(Qp17);
  @free(Qp72);
  @free(Qp24);
  @free(Qp23);
  @free(Qp43);
  @free(Qp74);
  @free(Qp64);
  @free(Qp54);
```

```
  @free(Qp65);
  @free(Qp67);

!Given values in the Pudar-Liggett paper;
  H1=30.48;
  S5=0.01893;
  S6=0.01262;
  A3=0.0054154736;
  C=1;
  g=9.81;

End
```

```
Rows=     51 Vars=     31 No. integer vars=      0
 Nonlinear rows=     36 Nonlinear vars=     25 Nonlinear constraints=     35
 Nonzeros=    157 Constraint nonz=   129 Density=0.096
 No. < :   0 No. =:  38 No. > :  12, Obj=MAX Single cols=    0

 Optimal solution found at step:        52
 Objective value:                 0.5115751E-34
```

(Zero Leaks Condition)

```
                       Variable           Value        Reduced Cost
                           QP12       0.4676858E-01       0.0000000
                            Q12       0.4676858E-01       0.0000000
                           QP17       0.4742725E-01       0.0000000
                            Q17       0.4742725E-01       0.0000000
                           QP72       0.1569094E-01        44.13267
                            Q72       0.1569094E-01       0.0000000
                           QP24      -0.3924415E-03       0.0000000
                            Q24      -0.3924415E-03       0.0000000
                           QP23       0.6285196E-01       0.0000000
                            Q23       0.6285196E-01       0.0000000
                           QP43       0.6289387E-01        107.0429
                            Q43       0.6289387E-01       0.0000000
                           QP74       0.2812730E-01       0.0000000
                            Q74       0.2812730E-01       0.0000000
                           QP64       0.1968485E-01       0.0000000
                            Q64       0.1968485E-01       0.0000000
                           QP54       0.1547416E-01       0.0000000
                            Q54       0.1547416E-01       0.0000000
                           QP65      -0.3455835E-02       0.0000000
                            Q65      -0.3455835E-02        122.1296
                           QP67      -0.3609013E-02       0.0000000
                            Q67      -0.3609013E-02        123.1296
                             S1       0.9419583E-01       0.0000000
                             D3       0.1257458           0.0000000
                             S5       0.1893000E-01       0.0000000
                             S6       0.1262000E-01       0.0000000
                            SP1       0.9419583E-01       0.0000000
                             L2       0.0000000           0.0000000
                             L4       0.0000000           0.0000000
                             L7       0.0000000           0.0000000
                             H1        30.48000           0.0000000
                             H2        29.12803           0.0000000
                            R12        392.9136           0.0000000
                             H7        29.50161           0.0000000
                            R17        276.9719           0.0000000
                            R72        821.1570           0.0000000
                             H4        29.12956           0.0000000
                            R24        1164.897           0.0000000
                             H3        27.47992          -0.2153438
                            R23        276.9719           0.0000000
                            R43        276.9719           0.0000000
                            R74        276.9719           0.0000000
                             H6        29.40184           0.0000000
                            R64        392.9136           0.0000000
                             H5        29.49396           0.0000000
                            R54        821.1570           0.0000000
                            R65        3333.710           0.0000000
```

```
R67        3333.710           0.0000000
  C        1.000000           0.0000000
 A3       0.5415474E-02       0.0000000
  G        9.810000           0.0000000

Row    Slack or Surplus      Dual Price
  1       0.0000000            1.000000
  2       0.0000000            1.000000
  3       0.0000000            1.000000
  4       0.0000000            1.000000
  5       0.0000000            1.000000
  6       0.0000000           0.0000000
  7       0.0000000           -122.1296
  8       0.0000000            1.000000
  9       0.0000000           -1.000000
 10       0.0000000            1.000000
 11       0.0000000            53.91201
 12       0.0000000           -4.547189
 13       0.0000000            29.31623
 14       0.0000000            66.51388
 15       0.0000000            1.000000
 16       0.0000000            1.000000
 17      -0.2611737E-03       0.3735003E-01
 18      -0.1616118E-03       0.5235732E-01
 19      -0.3254061E-03       -1.000000
 20      -0.9597427E-03       -2.052357
 21      -0.2419203E-03        1.089707
 22      -0.2614194E-03       -1.000000
 23      -0.3286778E-03      -0.2266413
 24      -0.4080358E-05        2.032528
 25      -0.2810368E-06      -0.7535293
 26      -0.1508385E-05      -0.7535293
 27      -0.5022686E-04       -1.278999
 28       0.0000000           0.1285423E-03
 29       0.0000000           0.1849189E-03
 30       0.0000000          -0.4553463E-03
 31       0.0000000           0.1009076E-05
 32       0.0000000           0.6483303E-02
 33       0.0000000          -0.5956932E-02
 34       0.0000000          -0.3041756E-03
 35       0.0000000           0.1408540E-02
 36       0.0000000          -0.3343895E-03
 37       0.0000000           0.2082093E-04
 38       0.0000000           0.3829600E-04
 39      -0.1831836E-07        54.91201
 40       0.0000000            2.000000
 41       0.0000000           -3.547189
 42       0.0000000            2.000000
 43       0.0000000           0.0000000
 44       0.0000000           0.0000000
 45       0.0000000           0.0000000
 46       0.0000000           0.0000000
 47       0.0000000           0.0000000
 48       0.0000000           0.0000000
 49       0.0000000           0.0000000
 50       0.0000000           0.0000000
 51       0.0000000            123.1296
```

```
52       0.0000000            1.000000
53       0.0000000           0.0000000
54       0.0000000           0.0000000
55       0.0000000           0.0000000
56       0.0000000           0.0000000
57       0.0000000           0.0000000
58       0.0000000           0.0000000
59       0.0000000           0.0000000
60       0.0000000           0.0000000
61       0.0000000           0.0000000
62       0.0000000           0.0000000
63       0.0000000           0.0000000
64       0.0000000           0.0000000
65       0.0000000           0.0000000
66       0.0000000          -0.8970736E-01
67       0.0000000           -29.31623
68       0.0000000            55.61570
69       0.0000000            1275.042
70       0.0000000            6.904957
71       0.0000000           0.3519324
```

```
!Improvement on the Pudar-Liggett Formulation for the Small Network;
!Possible leak locations are Nodes 2, 4, and 7;
!This model is based on the formulation in Chapter 5 using the orifice
equation for pressure-dependent demand and leakage terms;

!One leak condition (Leak at Node 2);

Model:

MAX=((Qp12-Q12)^2)+((Qp17-Q17)^2)+((Qp72-Q72)^2)+((Qp24-Q24)^2)+((Qp23-
Q23)^2)+((Qp43-Q43)^2)+((Qp74-Q74)^2)+((Qp64-Q64)^2)+((Qp54-Q54)^2)+((Qp65-
Q65)^2)+((Qp67-Q67)^2);

!Continuity Equation for Demand;
  -Q12-Q17=-S1;                     !Node 1;
  Q12+Q72-Q24-Q23=0;                !Node 2;
  Q23+Q43=D3;                       !Node 3;
  Q54+Q64+Q74+Q24-Q43=0;            !Node 4;
  Q65-Q54=-S5;                      !Node 5;
  -Q64-Q65-Q67=-S6;                 !Node 6;
  Q17+Q67-Q72-Q74=0;                !Node 7;

!Continuity Equation for Demand and Leakage;
  -Qp12-Qp17=-Sp1;                  !Node 1;
  Qp12+Qp72-Qp24-Qp23=L2;           !Node 2;
  Qp23+Qp43=D3;                     !Node 3;
  Qp54+Qp64+Qp74+Qp24-Qp43=L4;      !Node 4;
  Qp65-Qp54=-S5;                    !Node 5;
  -Qp64-Qp65-Qp67=-S6;              !Node 6;
  Qp17+Qp67-Qp72-Qp74=L7;           !Node 7;

!Overall balance of supply and leakage;
  Sp1=S1+L2+L4+L7;

!Energy equation with leakage flows;
  H1-H2=r12*Qp12*((@abs(Qp12))^0.852);    !Pipe 1;
  H1-H7=r17*Qp17*((@abs(Qp17))^0.852);    !Pipe 2;
  H7-H2=r72*Qp72*((@abs(Qp72))^0.852);    !Pipe 3;
  H2-H4=r24*Qp24*((@abs(Qp24))^0.852);    !Pipe 4;
  H2-H3=r23*Qp23*((@abs(Qp23))^0.852);    !Pipe 5;
  H4-H3=r43*Qp43*((@abs(Qp43))^0.852);    !Pipe 6;
  H7-H4=r74*Qp74*((@abs(Qp74))^0.852);    !Pipe 7;
  H6-H4=r64*Qp64*((@abs(Qp64))^0.852);    !Pipe 8;
  H5-H4=r54*Qp54*((@abs(Qp54))^0.852);    !Pipe 9;
  H6-H5=r65*Qp65*((@abs(Qp65))^0.852);    !Pipe 10;
  H6-H7=r67*Qp67*((@abs(Qp67))^0.852);    !Pipe 11;

!Resistance Coefficients based on the Hazen-Williams equation;
  r12=392.9136;
  r17=276.9719;
  r72=821.1570;
  r24=1164.8971;
  r23=276.9719;
  r43=276.9719;
  r74=276.9719;
  r64=392.9136;
  r54=821.1570;
```

```
  r65=3333.7099;
  r67=3333.7099;

!Pressure-dependent outflow (orifice equation);
  D3=C*A3*((2*g*H3)^0.5);
  L2=C*A2*((2*g*H2)^0.5);
  L4=0;
  L7=0;

!Flow magnitude constraint;
  @abs(Sp1)>@abs(S1);
  @abs(Qp12)>@abs(Q12);
  @abs(Qp17)>@abs(Q17);
  @abs(Qp72)>@abs(Q72);
  @abs(Qp24)>@abs(Q24);
  @abs(Qp23)>@abs(Q23);
  @abs(Qp43)>@abs(Q43);
  @abs(Qp74)>@abs(Q74);
  @abs(Qp64)>@abs(Q64);
  @abs(Qp54)>@abs(Q54);
  @abs(Qp65)>@abs(Q65);
  @abs(Qp67)>@abs(Q67);

!Flow direction constraint;
  Qp12*Q12-@abs(Qp12)*@abs(Q12)=0;
  Qp17*Q17-@abs(Qp17)*@abs(Q17)=0;
  Qp72*Q72-@abs(Qp72)*@abs(Q72)=0;
  Qp24*Q24-@abs(Qp24)*@abs(Q24)=0;
  Qp23*Q23-@abs(Qp23)*@abs(Q23)=0;
  Qp43*Q43-@abs(Qp43)*@abs(Q43)=0;
  Qp74*Q74-@abs(Qp74)*@abs(Q74)=0;
  Qp64*Q64-@abs(Qp64)*@abs(Q64)=0;
  Qp54*Q54-@abs(Qp54)*@abs(Q54)=0;
  Qp65*Q65-@abs(Qp65)*@abs(Q65)=0;
  Qp67*Q67-@abs(Qp67)*@abs(Q67)=0;

!Allow for flow reversal in each pipe;
  @free(Q12);
  @free(Q17);
  @free(Q72);
  @free(Q24);
  @free(Q23);
  @free(Q43);
  @free(Q74);
  @free(Q64);
  @free(Q54);
  @free(Q65);
  @free(Q67);
  @free(Qp12);
  @free(Qp17);
  @free(Qp72);
  @free(Qp24);
  @free(Qp23);
  @free(Qp43);
  @free(Qp74);
  @free(Qp64);
  @free(Qp54);
```

```
  @free(Qp65);
  @free(Qp67);

!Given values in the Pudar-Liggett paper;
  H1=30.48;
  S5=0.01893;
  S6=0.01262;
  A2=0.00232;
  A3=0.0054154736;
  C=1;
  g=9.81;

End
```

```
Rows=     52 Vars=     32 No. integer vars=      0
 Nonlinear rows=     37 Nonlinear vars=     26 Nonlinear constraints=     36
 Nonzeros=    161 Constraint nonz=   133 Density=0.094
 No. < :   0 No. =:  39 No. > :  12, Obj=MAX Single cols=    0

 Optimal solution found at step:        71
 Objective value:                0.2893377E-02
```

(One Leak Condition)

```
                       Variable           Value        Reduced Cost
                           QP12       0.7296048E-01       0.0000000
                            Q12       0.1917036E-01       0.0000000
                           QP17       0.7157337E-01       0.0000000
                            Q17       0.7157337E-01       0.0000000
                           QP72       0.2647375E-01       0.0000000
                            Q72       0.2647375E-01       0.0000000
                           QP24      -0.1206554E-01       0.0000000
                            Q24      -0.1206554E-01       0.0000000
                           QP23       0.5770965E-01        38.48348
                            Q23       0.5770965E-01        1.000000
                           QP43       0.6458408E-01       0.0000000
                            Q43       0.6458408E-01       0.0000000
                           QP74       0.3831757E-01       0.0000000
                            Q74       0.3831757E-01       0.0000000
                           QP64       0.2212226E-01       0.0000000
                            Q64       0.2212226E-01        79.06229
                           QP54       0.1620978E-01       0.0000000
                            Q54       0.1620978E-01        2.000000
                           QP65      -0.2720216E-02       0.0000000
                            Q65      -0.2720216E-02       0.0000000
                           QP67      -0.6782042E-02       0.0000000
                            Q67      -0.6782042E-02       0.0000000
                             S1       0.9074373E-01       0.0000000
                             D3       0.1222937           0.0000000
                             S5       0.1893000E-01       0.0000000
                             S6       0.1262000E-01       0.0000000
                            SP1       0.1445338           0.0000000
                             L2       0.5379012E-01       0.0000000
                             L4       0.0000000           0.0000000
                             L7       0.0000000           0.0000000
                             H1        30.48000           0.0000000
                             H2        27.39870           0.0000000
                            R12        392.9136           0.0000000
                             H7        28.38379           0.0000000
                            R17        276.9719           0.0000000
                            R72        821.1570           0.0000000
                             H4        27.72477           0.0000000
                            R24        1164.897           0.0000000
                             H3        25.99181           0.0000000
                            R23        276.9719           0.0000000
                            R43        276.9719           0.0000000
                            R74        276.9719           0.0000000
                             H6        28.06278           0.0000000
                            R64        392.9136           0.0000000
                             H5        28.12190           0.0000000
                            R54        821.1570           0.0000000
                            R65        3333.710           0.0000000
```

```
R67        3333.710           0.0000000
  C        1.000000           0.0000000
 A3       0.5415474E-02       0.0000000
  G        9.810000           0.0000000
 A2       0.2320000E-02       0.0000000

Row    Slack or Surplus      Dual Price
  1       0.2893377E-02        1.000000
  2       0.0000000           -1.000000
  3       0.0000000           -1.000000
  4       0.0000000           0.1152189E-10
  5       0.0000000           0.1152189E-10
  6       0.0000000           -1.000000
  7       0.0000000           -79.06229
  8       0.0000000           0.0000000
  9       0.0000000            1.000000
 10       0.0000000           -1.000000
 11       0.0000000            23.78052
 12       0.0000000            11.47867
 13       0.0000000           -15.36075
 14       0.0000000            39.77518
 15       0.0000000           -1.541023
 16       0.0000000           -1.000000
 17      -0.1239123E-04      -0.2557062E-01
 18      -0.7606124E-05      -0.2841084E-01
 19      -0.8166551E-05      -0.6660165E-02
 20      -0.1759337E-05       0.2693028
 21      -0.2431662E-04      -0.3034968
 22      -0.5356391E-05       0.2475521
 23      -0.1465376E-04       0.4087605
 24      -0.3237852E-05       -1.000000
 25      -0.8262430E-06       0.5694888
 26      -0.9797899E-06       0.5694888
 27      -0.5619423E-04       0.4305111
 28       0.0000000          -0.2005301E-03
 29       0.0000000          -0.2150224E-03
 30       0.0000000          -0.7989864E-05
 31       0.0000000          -0.7538159E-04
 32       0.0000000          -0.1541652E-02
 33       0.0000000           0.1548884E-02
 34       0.0000000           0.9725725E-03
 35       0.0000000          -0.8602489E-03
 36       0.0000000           0.2754182E-03
 37       0.0000000          -0.1010114E-04
 38       0.0000000          -0.4146332E-04
 39       0.0000000            23.78052
 40       0.0000000           -2.000000
 41       0.0000000            10.47867
 42       0.0000000           -2.541023
 43       0.5379012E-01       0.0000000
 44       0.5379012E-01       0.0000000
 45       0.0000000            1.000000
 46       0.0000000           -1.000000
 47       0.0000000           -1.000000
 48       0.0000000           0.0000000
 49       0.0000000           0.0000000
 50       0.0000000           0.1152189E-10
```

```
51       0.0000000           0.0000000
52       0.0000000          -1.000000
53       0.0000000          -78.06229
54       0.0000000          -79.06229
55       0.0000000           0.0000000
56       0.0000000           0.0000000
57       0.0000000           0.0000000
58       0.0000000           0.0000000
59       0.0000000           0.0000000
60       0.0000000           0.0000000
61       0.0000000           0.0000000
62       0.0000000           0.0000000
63       0.0000000           0.0000000
64       0.0000000           0.0000000
65       0.0000000           0.0000000
66       0.0000000           0.5398146E-01
67       0.0000000            16.36075
68       0.0000000            39.28712
69       0.0000000          -46.37079
70       0.0000000            537.0182
71       0.0000000            2.800628
72       0.0000000           0.1427426
```

```
!Improvement on the Pudar-Liggett Formulation for the Small Network;
!Possible leak locations are Nodes 2, 4, and 7;
!This model is based on the formulation in Chapter 5 using the orifice
equation for pressure-dependent demand and leakage terms;

!Two leaks condition (Leaks at Nodes 2 and 4);

Model:

MAX=((Qp12-Q12)^2)+((Qp17-Q17)^2)+((Qp72-Q72)^2)+((Qp24-Q24)^2)+((Qp23-
Q23)^2)+
((Qp43-Q43)^2)+((Qp74-Q74)^2)+((Qp64-Q64)^2)+((Qp54-Q54)^2)+((Qp65-
Q65)^2)+((Qp67-Q67)^2);

INIT:
Q12=0.04;
S1=0.08;
ENDINIT

!Continuity Equation for Demand;
  -Q12-Q17=-S1;                    !Node 1;
  Q12+Q72-Q24-Q23=0;               !Node 2;
  Q23+Q43=D3;                      !Node 3;
  Q54+Q64+Q74+Q24-Q43=0;           !Node 4;
  Q65-Q54=-S5;                     !Node 5;
  -Q64-Q65-Q67=-S6;                !Node 6;
  Q17+Q67-Q72-Q74=0;               !Node 7;

!Continuity Equation for Demand and Leakage;
  -Qp12-Qp17=-Sp1;                 !Node 1;
  Qp12+Qp72-Qp24-Qp23=L2;          !Node 2;
  Qp23+Qp43=D3;                    !Node 3;
  Qp54+Qp64+Qp74+Qp24-Qp43=L4;     !Node 4;
  Qp65-Qp54=-S5;                   !Node 5;
  -Qp64-Qp65-Qp67=-S6;             !Node 6;
  Qp17+Qp67-Qp72-Qp74=L7;          !Node 7;

!Overall balance of supply and leakage;
  Sp1=S1+L2+L4+L7;

!Energy equation with leakage flows;
  H1-H2=r12*Qp12*((@abs(Qp12))^0.852);    !Pipe 1;
  H1-H7=r17*Qp17*((@abs(Qp17))^0.852);    !Pipe 2;
  H7-H2=r72*Qp72*((@abs(Qp72))^0.852);    !Pipe 3;
  H2-H4=r24*Qp24*((@abs(Qp24))^0.852);    !Pipe 4;
  H2-H3=r23*Qp23*((@abs(Qp23))^0.852);    !Pipe 5;
  H4-H3=r43*Qp43*((@abs(Qp43))^0.852);    !Pipe 6;
  H7-H4=r74*Qp74*((@abs(Qp74))^0.852);    !Pipe 7;
  H6-H4=r64*Qp64*((@abs(Qp64))^0.852);    !Pipe 8;
  H5-H4=r54*Qp54*((@abs(Qp54))^0.852);    !Pipe 9;
  H6-H5=r65*Qp65*((@abs(Qp65))^0.852);    !Pipe 10;
  H6-H7=r67*Qp67*((@abs(Qp67))^0.852);    !Pipe 11;

!Resistance Coefficients based on the Hazen-Williams equation;
  r12=392.9136;
  r17=276.9719;
  r72=821.1570;
```

```
  r24=1164.8971;
  r23=276.9719;
  r43=276.9719;
  r74=276.9719;
  r64=392.9136;
  r54=821.1570;
  r65=3333.7099;
  r67=3333.7099;

!Pressure-dependent outflow (orifice equation);
  D3=C*A3*((2*g*H3)^0.5);
  L2=C*A2*((2*g*H2)^0.5);
  L4=C*A4*((2*g*H4)^0.5);
  L7=0;

!Flow magnitude constraint;
  @abs(Sp1)>@abs(S1);
  @abs(Qp12)>@abs(Q12);
  @abs(Qp17)>@abs(Q17);
  @abs(Qp72)>@abs(Q72);
  @abs(Qp24)>@abs(Q24);
  @abs(Qp23)>@abs(Q23);
  @abs(Qp43)>@abs(Q43);
  @abs(Qp74)>@abs(Q74);
  @abs(Qp64)>@abs(Q64);
  @abs(Qp54)>@abs(Q54);
  @abs(Qp65)>@abs(Q65);
  @abs(Qp67)>@abs(Q67);

!Flow direction constraint;
  Qp12*Q12-@abs(Qp12)*@abs(Q12)=0;
  Qp17*Q17-@abs(Qp17)*@abs(Q17)=0;
  Qp72*Q72-@abs(Qp72)*@abs(Q72)=0;
  Qp24*Q24-@abs(Qp24)*@abs(Q24)=0;
  Qp23*Q23-@abs(Qp23)*@abs(Q23)=0;
  Qp43*Q43-@abs(Qp43)*@abs(Q43)=0;
  Qp74*Q74-@abs(Qp74)*@abs(Q74)=0;
  Qp64*Q64-@abs(Qp64)*@abs(Q64)=0;
  Qp54*Q54-@abs(Qp54)*@abs(Q54)=0;
  Qp65*Q65-@abs(Qp65)*@abs(Q65)=0;
  Qp67*Q67-@abs(Qp67)*@abs(Q67)=0;

!Allow for flow reversal in each pipe;
  @free(Q12);
  @free(Q17);
  @free(Q72);
  @free(Q24);
  @free(Q23);
  @free(Q43);
  @free(Q74);
  @free(Q64);
  @free(Q54);
  @free(Q65);
  @free(Q67);
  @free(Qp12);
  @free(Qp17);
  @free(Qp72);
```

```
  @free(Qp24);
  @free(Qp23);
  @free(Qp43);
  @free(Qp74);
  @free(Qp64);
  @free(Qp54);
  @free(Qp65);
  @free(Qp67);

!Given values in the Pudar-Liggett paper;
  H1=30.48;
  S5=0.01893;
  S6=0.01262;
  A2=0.00232;
  A3=0.0054154736;
  A4=0.00232;
  C=1;
  g=9.81;

End
```

```
Rows=     53 Vars=     33 No. integer vars=      0
 Nonlinear rows=     38 Nonlinear vars=     27 Nonlinear constraints=     37
 Nonzeros=    165 Constraint nonz=   137 Density=0.092
 No. < :   0 No. =:  40 No. > :  12, Obj=MAX Single cols=    0

 Optimal solution found at step:        72
 Objective value:                0.8084208E-02
```

(Two Leaks Condition)

```
                       Variable           Value        Reduced Cost
                           QP12       0.9307065E-01       0.0000000
                            Q12       0.4103206E-01       0.0000000
                           QP17       0.9699503E-01       0.0000000
                            Q17       0.4514817E-01       0.0000000
                           QP72       0.2886457E-01       0.1163427
                            Q72       0.2886457E-01       0.0000000
                           QP24       0.8977793E-02       0.0000000
                            Q24       0.8977793E-02       0.0000000
                           QP23       0.6091884E-01       0.0000000
                            Q23       0.6091884E-01       0.0000000
                           QP43       0.5681139E-01       0.0000000
                            Q43       0.5681139E-01       0.0000000
                           QP74       0.5631915E-01       0.0000000
                            Q74       0.4472289E-02       0.1003104
                           QP64       0.2574820E-01       0.0000000
                            Q64       0.2574820E-01       0.0000000
                           QP54       0.1761311E-01       0.0000000
                            Q54       0.1761311E-01       0.0000000
                           QP65      -0.1316889E-02       0.0000000
                            Q65      -0.1316889E-02       0.0000000
                           QP67      -0.1181131E-01       0.0000000
                            Q67      -0.1181131E-01       0.2934701E-01
                             S1       0.8618023E-01       0.0000000
                             D3       0.1177302           0.0000000
                             S5       0.1893000E-01       0.0000000
                             S6       0.1262000E-01       0.0000000
                            SP1       0.1900657           0.0000000
                             L2       0.5203858E-01       0.0000000
                             L4       0.5184686E-01       0.0000000
                             L7       0.0000000           0.0000000
                             H1        30.48000           0.0000000
                             H2        25.64342           0.0000000
                            R12        392.9136           0.0000000
                             H7        26.79959           0.0000000
                            R17        276.9719           0.0000000
                            R72        821.1570           0.0000000
                             H4        25.45482           0.0000000
                            R24        1164.897           0.0000000
                             H3        24.08819           0.0000000
                            R23        276.9719           0.0000000
                            R43        276.9719           0.0000000
                            R74        276.9719           0.0000000
                             H6        25.90253           0.0000000
                            R64        392.9136           0.0000000
                             H5        25.91796           0.0000000
                            R54        821.1570           0.0000000
                            R65        3333.710           0.0000000
```

```
R67        3333.710           0.0000000
  C        1.000000           0.0000000
 A3      0.5415474E-02        0.0000000
  G        9.810000           0.0000000
 A2      0.2320000E-02        0.0000000
 A4      0.2320000E-02        0.0000000

Row    Slack or Surplus      Dual Price
  1      0.8084208E-02         1.000000
  2      0.0000000            0.1330407
  3      0.0000000            0.2896357E-01
  4      0.0000000            0.2596371E-01
  5      0.0000000            0.2596370E-01
  6      0.0000000            0.1006140E-07
  7      0.0000000            0.0000000
  8      0.0000000            0.2934702E-01
  9      0.0000000           -0.1330407
 10      0.0000000            0.0000000
 11      0.0000000            0.0000000
 12      0.0000000            0.0000000
 13      0.0000000            0.1657923E-01
 14      0.0000000            0.1239791E-01
 15      0.0000000           -0.7396130E-01
 16      0.0000000            0.1330407
 17      0.0000000            0.3009437E-03
 18      0.0000000           -0.6348728E-03
 19     -0.9313133E-07       -0.5764866E-03
 20     -0.3240980E-06       -0.7710390E-04
 21      0.0000000           -0.6344816E-04
 22      0.0000000           -0.2219471E-09
 23     -0.2426811E-07       -0.6723491E-03
 24      0.0000000            0.4212597E-03
 25      0.0000000            0.1927031E-03
 26      0.0000000            0.1927031E-03
 27     -0.3550004E-07       -0.6139628E-03
 28      0.0000000            0.3704473E-05
 29      0.0000000           -0.8436199E-05
 30      0.0000000           -0.8116811E-06
 31      0.0000000           -0.1248379E-07
 32      0.0000000           -0.3562699E-06
 33      0.0000000           -0.1095125E-11
 34      0.0000000           -0.3264449E-05
 35      0.0000000            0.4800131E-06
 36      0.0000000            0.1086869E-06
 37      0.0000000           -0.8918543E-09
 38      0.0000000            0.1652107E-06
 39      0.0000000            0.2596371E-01
 40      0.0000000            0.1330407
 41      0.0000000            0.1330407
 42      0.0000000            0.5907943E-01
 43      0.1038854            0.0000000
 44      0.5203858E-01        0.0000000
 45      0.5184686E-01        0.0000000
 46      0.0000000           -0.3834585E-03
 47      0.0000000           -0.2999882E-02
 48      0.0000000           -0.2999872E-02
 49      0.0000000            0.0000000
```

```
50       0.5184686E-01       0.0000000
51       0.0000000           0.2596369E-01
52       0.0000000           0.2596368E-01
53       0.0000000           0.0000000
54       0.0000000           0.0000000
55       0.0000000           0.0000000
56       0.0000000           0.0000000
57       0.0000000           0.0000000
58       0.0000000           0.0000000
59       0.0000000           0.0000000
60       0.0000000           0.0000000
61       0.0000000           0.0000000
62       0.0000000           0.0000000
63       0.0000000           0.0000000
64       0.0000000           0.0000000
65       0.0000000           0.0000000
66       0.0000000           0.3339291E-03
67       0.0000000          -0.1657924E-01
68       0.0000000          -0.1239791E-01
69       0.0000000            2.984160
70       0.0000000           0.5644406
71       0.0000000            2.973165
72       0.0000000           0.1687771E-01
73       0.0000000           0.8602244E-03
```

```
!Improvement on the Pudar-Liggett Formulation for the Small Network;
!Possible leak locations are Nodes 2, 4, and 7;
!This model is based on the formulation in Chapter 5 using the orifice
equation for pressure-dependent demand and leakage terms;
```

!Three leaks condition (Leaks at Nodes 2, 4, and 7);

```
Model:

MAX=((Qp12-Q12)^2)+((Qp17-Q17)^2)+((Qp72-Q72)^2)+((Qp24-Q24)^2)+((Qp23-
Q23)^2)+
((Qp43-Q43)^2)+((Qp74-Q74)^2)+((Qp64-Q64)^2)+((Qp54-Q54)^2)+((Qp65-
Q65)^2)+((Qp67-Q67)^2);

!Continuity Equation for Demand;
  -Q17-Q12=-S1;                      !Node 1;
  Q12+Q72-Q24-Q23=0;                 !Node 2;
  Q23+Q43=D3;                        !Node 3;
  Q54+Q64+Q74+Q24-Q43=0;             !Node 4;
  Q65-Q54=-S5;                       !Node 5;
  -Q64-Q65-Q67=-S6;                  !Node 6;
  Q17+Q67-Q72-Q74=0;                 !Node 7;

!Continuity Equation for Demand and Leakage;
  -Qp12-Qp17=-Sp1;                   !Node 1;
  Qp12+Qp72-Qp24-Qp23=L2;            !Node 2;
  Qp23+Qp43=D3;                      !Node 3;
  Qp54+Qp64+Qp74+Qp24-Qp43=L4;       !Node 4;
  Qp65-Qp54=-S5;                     !Node 5;
  -Qp64-Qp65-Qp67=-S6;               !Node 6;
  Qp17+Qp67-Qp72-Qp74=L7;            !Node 7;

!Overall balance of supply and leakage;
  Sp1=S1+L2+L4+L7;

!Energy equation with leakage flows;
  H1-H2=r12*Qp12*((@abs(Qp12))^0.852);    !Pipe 1;
  H1-H7=r17*Qp17*((@abs(Qp17))^0.852);    !Pipe 2;
  H7-H2=r72*Qp72*((@abs(Qp72))^0.852);    !Pipe 3;
  H2-H4=r24*Qp24*((@abs(Qp24))^0.852);    !Pipe 4;
  H2-H3=r23*Qp23*((@abs(Qp23))^0.852);    !Pipe 5;
  H4-H3=r43*Qp43*((@abs(Qp43))^0.852);    !Pipe 6;
  H7-H4=r74*Qp74*((@abs(Qp74))^0.852);    !Pipe 7;
  H6-H4=r64*Qp64*((@abs(Qp64))^0.852);    !Pipe 8;
  H5-H4=r54*Qp54*((@abs(Qp54))^0.852);    !Pipe 9;
  H6-H5=r65*Qp65*((@abs(Qp65))^0.852);    !Pipe 10;
  H6-H7=r67*Qp67*((@abs(Qp67))^0.852);    !Pipe 11;

!Resistance Coefficients based on the Hazen-Williams equation;
  r12=392.9136;
  r17=276.9719;
  r72=821.1570;
  r24=1164.8971;
  r23=276.9719;
  r43=276.9719;
  r74=276.9719;
  r64=392.9136;
```

```
  r54=821.1570;
  r65=3333.7099;
  r67=3333.7099;

!Pressure-dependent outflow (orifice equation);
  D3=C*A3*((2*g*H3)^0.5);
  L2=C*A2*((2*g*H2)^0.5);
  L4=C*A4*((2*g*H4)^0.5);
  L7=C*A7*((2*g*H7)^0.5);

!Flow magnitude constraint;
  @abs(Sp1)>@abs(S1);
  @abs(Qp12)>@abs(Q12);
  @abs(Qp17)>@abs(Q17);
  @abs(Qp72)>@abs(Q72);
  @abs(Qp24)>@abs(Q24);
  @abs(Qp23)>@abs(Q23);
  @abs(Qp43)>@abs(Q43);
  @abs(Qp74)>@abs(Q74);
  @abs(Qp64)>@abs(Q64);
  @abs(Qp54)>@abs(Q54);
  @abs(Qp65)>@abs(Q65);
  @abs(Qp67)>@abs(Q67);

!Flow direction constraint;
  Qp12*Q12-@abs(Qp12)*@abs(Q12)=0;
  Qp17*Q17-@abs(Qp17)*@abs(Q17)=0;
  Qp72*Q72-@abs(Qp72)*@abs(Q72)=0;
  Qp24*Q24-@abs(Qp24)*@abs(Q24)=0;
  Qp23*Q23-@abs(Qp23)*@abs(Q23)=0;
  Qp43*Q43-@abs(Qp43)*@abs(Q43)=0;
  Qp74*Q74-@abs(Qp74)*@abs(Q74)=0;
  Qp64*Q64-@abs(Qp64)*@abs(Q64)=0;
  Qp54*Q54-@abs(Qp54)*@abs(Q54)=0;
  Qp65*Q65-@abs(Qp65)*@abs(Q65)=0;
  Qp67*Q67-@abs(Qp67)*@abs(Q67)=0;

!Allow for flow reversal in each pipe;
  @free(Q12);
  @free(Q17);
  @free(Q72);
  @free(Q24);
  @free(Q23);
  @free(Q43);
  @free(Q74);
  @free(Q64);
  @free(Q54);
  @free(Q65);
  @free(Q67);
  @free(Qp12);
  @free(Qp17);
  @free(Qp72);
  @free(Qp24);
  @free(Qp23);
  @free(Qp43);
  @free(Qp74);
  @free(Qp64);
```

```
  @free(Qp54);
  @free(Qp65);
  @free(Qp67);

!Given values in the Pudar-Liggett paper;
  H1=30.48;
  S5=0.01893;
  S6=0.01262;
  A2=0.00232;
  A3=0.0054154736;
  A4=0.00232;
  A7=0.0000929;
  C=1;
  g=9.81;

End
```

```
Rows=     54 Vars=     34 No. integer vars=      0
 Nonlinear rows=     39 Nonlinear vars=     28 Nonlinear constraints=     38
 Nonzeros=    169 Constraint nonz=   141 Density=0.089
 No. < :   0 No. =:  41 No. > :  12, Obj=MAX Single cols=    0

 ** WARNING ** Problem is poorly scaled.  The units
 of the rows and variables should be changed so the
 coefficients cover a much smaller range.

 Optimal solution found at step:        61
 Objective value:                0.1083203E-01
```

(Three Leaks Condition)

```
                       Variable           Value        Reduced Cost
                           QP12       0.9368406E-01      0.0000000
                            Q12       0.7019131E-01      0.0000000
                           QP17       0.9822850E-01      0.0000000
                            Q17       0.1583978E-01      0.1177920
                           QP72       0.2848573E-01      0.0000000
                            Q72      -0.3498510E-09      0.5697143E-01
                           QP24       0.9234580E-02      0.0000000
                            Q24       0.9234580E-02      0.0000000
                           QP23       0.6095673E-01      0.0000000
                            Q23       0.6095673E-01      0.0000000
                           QP43       0.5662436E-01      0.0000000
                            Q43       0.5662436E-01      0.0000000
                           QP74       0.5591464E-01      0.0000000
                            Q74       0.4138415E-02      0.1035524
                           QP64       0.2567266E-01      0.0000000
                            Q64       0.2567266E-01      0.0000000
                           QP54       0.1757870E-01      0.0000000
                            Q54       0.1757870E-01      0.0000000
                           QP65      -0.1351302E-02      0.0000000
                            Q65      -0.1351302E-02      0.1006140E-07
                           QP67      -0.1170136E-01      0.0000000
                            Q67      -0.1170136E-01      0.0000000
                             S1       0.8603109E-01      0.0000000
                             D3       0.1175811          0.0000000
                             S5       0.1893000E-01      0.0000000
                             S6       0.1262000E-01      0.0000000
                            SP1       0.1919126          0.0000000
                             L2       0.5197848E-01      0.0000000
                             L4       0.5177622E-01      0.0000000
                             L7       0.2126777E-02      0.0000000
                             H1        30.48000          0.0000000
                             H2        25.58422          0.0000000
                            R12        392.9136          0.0000000
                             H7        26.71244          0.0000000
                            R17        276.9719          0.0000000
                            R72        821.1570          0.0000000
                             H4        25.38550          0.0000000
                            R24        1164.897          0.0000000
                             H3        24.02719          0.0000000
                            R23        276.9719          0.0000000
                            R43        276.9719          0.0000000
                            R74        276.9719          0.0000000
                             H6        25.83078          0.0000000
```

```
R64        392.9136           0.0000000
 H5        25.84697           0.0000000
R54        821.1570           0.0000000
R65        3333.710           0.0000000
R67        3333.710           0.0000000
  C        1.000000           0.0000000
 A3       0.5415474E-02       0.0000000
  G        9.810000           0.0000000
 A2       0.2320000E-02       0.0000000
 A4       0.2320000E-02       0.0000000
 A7       0.9290000E-04       0.0000000

Row    Slack or Surplus      Dual Price
  1       0.1083203E-01        1.000000
  2       0.0000000           0.4698547E-01
  3       0.0000000          -0.2012279E-07
  4       0.0000000           0.0000000
  5       0.0000000          -0.1006140E-07
  6       0.0000000           0.0000000
  7       0.0000000          -0.2012279E-07
  8       0.0000000           0.0000000
  9       0.0000000          -0.4698547E-01
 10       0.0000000           0.6981052E-01
 11       0.0000000           0.7455962E-01
 12       0.0000000           0.8712441E-01
 13       0.0000000           0.8225186E-01
 14       0.0000000           0.8002892E-01
 15       0.0000000           0.3522054E-01
 16       0.0000000           0.4698547E-01
 17       0.0000000           0.7213124E-03
 18       0.0000000          -0.1162430E-02
 19       0.0000000          -0.3051260E-03
 20       0.0000000           0.4344402E-03
 21       0.0000000           0.1003913E-03
 22       0.0000000          -0.2828263E-03
 23       0.0000000          -0.1175141E-02
 24       0.0000000           0.2208889E-03
 25       0.0000000           0.1002206E-03
 26       0.0000000           0.1002206E-03
 27       0.0000000          -0.3211095E-03
 28       0.0000000           0.8987695E-05
 29       0.0000000          -0.1581214E-04
 30       0.0000000          -0.4192267E-06
 31       0.0000000           0.7411098E-07
 32       0.0000000           0.5643599E-06
 33       0.0000000          -0.1387017E-05
 34       0.0000000          -0.5629985E-05
 35       0.0000000           0.2503307E-06
 36       0.0000000           0.5632127E-07
 37       0.0000000          -0.4865310E-09
 38       0.0000000           0.8492333E-07
 39       0.0000000           0.7455962E-01
 40       0.0000000           0.1167960
 41       0.0000000           0.1341099
 42       0.0000000           0.8220602E-01
 43       0.1058815           0.0000000
 44       0.2349275E-01       0.0000000
```

```
45       0.8238873E-01       0.0000000
46       0.2848573E-01       0.0000000
47       0.0000000           0.0000000
48       0.0000000           0.1006140E-07
49       0.0000000           0.0000000
50       0.5177622E-01       0.0000000
51       0.0000000           0.0000000
52       0.0000000          -0.2012279E-07
53       0.0000000           0.0000000
54       0.0000000          -0.1006140E-07
55       0.0000000           0.0000000
56       0.0000000           0.0000000
57       0.0000000           0.0000000
58       0.0000000           0.0000000
59       0.0000000           0.0000000
60       0.0000000           0.0000000
61       0.0000000           0.0000000
62       0.0000000           0.0000000
63       0.0000000           0.0000000
64       0.0000000           0.0000000
65       0.0000000           0.0000000
66       0.0000000           0.4411176E-03
67       0.0000000          -0.8225186E-01
68       0.0000000          -0.8002890E-01
69       0.0000000            2.616758
70       0.0000000            1.618843
71       0.0000000            2.992975
72       0.0000000            1.881958
73       0.0000000           0.2195622E-01
74       0.0000000           0.1119066E-02
```

Appendix D

LINGO Runs for the Mt. Tabor Subnetwork

```
!Mt. Tabor Subnetwork within the Blacksburg water distribution system;

!In this situation, it is assumed that 10% of the water supply is being lost
due to leakage based on the results of a water audit;

Model:

!Objective Function;
Max=(Qp4240-Q4240)^2+(Qp10140-Q10140)^2+(Qp10145-Q10145)^2+(Qp10150-
Q10150)^2;

!Continuity for Demand;
-Q4240-Q7180=D4200-S4200;           !Node 4200;
Q4240-Q4250-Q7230=D4210;            !Node 4210;
Q4250-Q4255=D4215;                  !Node 4215;
Q4255+Q7260-Q4260=D4220;            !Node 4220;
Q4260-Q7340=D4230;                  !Node 4230;
Q7180-Q7190=D6990;                  !Node 6990;
Q7190-Q7200=D7000;                  !Node 7000;
Q7200-Q7210=D7010;                  !Node 7010;
Q7210+Q7220-Q10150=D7020;           !Node 7020;
Q7230-Q7220=D7030;                  !Node 7030;
Q10140-Q10135=D7040;                !Node 7040;
Q7250+Q7300-Q7260=D7060;            !Node 7060;
Q10135-Q7250=D7070;                 !Node 7070;
Q7320-Q7300=D7110;                  !Node 7110;
Q7340-Q7320=D7130;                  !Node 7130;
Q10145-Q10140=D9780;                !Node 9780;
Q10150-Q10145=D9795;                !Node 9795;

!Continuity Equation for Demand and Leakage;
-Qp4240-Qp7180=(1.1*D4200)-Sp4200;              !Node 4200;
Qp4240-Qp4250-Qp7230=1.1*D4210;                 !Node 4210;
Qp4250-Qp4255=1.1*D4215;                        !Node 4215;
Qp4255+Qp7260-Qp4260=1.1*D4220;                 !Node 4220;
Qp4260-Qp7340=1.1*D4230;                        !Node 4230;
Qp7180-Qp7190=1.1*D6990;                        !Node 6990;
Qp7190-Qp7200=1.1*D7000;                        !Node 7000;
Qp7200-Qp7210=1.1*D7010;                        !Node 7010;
Qp7210+Qp7220-Qp10150=1.1*D7020;                !Node 7020;
Qp7230-Qp7220=1.1*D7030;                        !Node 7030;
Qp10140-Qp10135=1.1*D7040;                      !Node 7040;
Qp7250+Qp7300-Qp7260=1.1*D7060;                 !Node 7060;
Qp10135-Qp7250=1.1*D7070;                       !Node 7070;
Qp7320-Qp7300=1.1*D7110;                        !Node 7110;
Qp7340-Qp7320=1.1*D7130;                        !Node 7130;
Qp10145-Qp10140=1.1*D9780;                      !Node 9780;
Qp10150-Qp10145=1.1*D9795;                      !Node 9795;

!Energy equation with leakage flows;
((H4200)+2163)-((H4210)+2141)=r4240*Qp4240*((@abs(Qp4240))^0.852);!Pipe 4240;
(H4210+2141)-(H4215+2110)=r4250*Qp4250*((@abs(Qp4250))^0.852);    !Pipe 4250;
(H4215+2110)-(H4220+2132)=r4255*Qp4255*((@abs(Qp4255))^0.852);    !Pipe 4255;
(H4220+2132)-(H4230+2121)=r4260*Qp4260*((@abs(Qp4260))^0.852);    !Pipe 4260;
(H4200+2163)-(H6990+2153.5)=r7180*Qp7180*((@abs(Qp7180))^0.852);  !Pipe 7180;
(H6990+2153.5)-(H7000+2141.5)=r7190*Qp7190*((@abs(Qp7190))^0.852);!Pipe 7190;
(H7000+2141.5)-(H7010+2129)=r7200*Qp7200*((@abs(Qp7200))^0.852);  !Pipe 7200;
```

```
(H7010+2129)-(H7020+2127)=r7210*Qp7210*((@abs(Qp7210))^0.852);    !Pipe 7210;
(H7030+2127)-(H7020+2127)=r7220*Qp7220*((@abs(Qp7220))^0.852);    !Pipe 7220;
(H4210+2141)-(H7030+2127)=r7230*Qp7230*((@abs(Qp7230))^0.852);    !Pipe 7230;
(H7070+2110)-(H7060+2139)=r7250*Qp7250*((@abs(Qp7250))^0.852);    !Pipe 7250;
(H7060+2139)-(H4220+2132)=r7260*Qp7260*((@abs(Qp7260))^0.852);    !Pipe 7260;
(H7110+2144.5)-(H7060+2139)=r7300*Qp7300*((@abs(Qp7300))^0.852);  !Pipe 7300;
(H7130+2116)-(H7110+2144.5)=r7320*Qp7320*((@abs(Qp7320))^0.852);  !Pipe 7320;
(H4230+2121)-(H7130+2116)=r7340*Qp7340*((@abs(Qp7340))^0.852);    !Pipe 7340;
(H7040+2109.5)-(H7070+2110)=r10135*Qp10135*((@abs(Qp10135))^0.852);
!Pipe 10135;
(H9780+2099.5)-(H7040+2109.5)=r10140*Qp10140*((@abs(Qp10140))^0.852);
!Pipe 10140;
(H9795+2120)-(H9780+2099.5)=r10145*Qp10145*((@abs(Qp10145))^0.852);
!Pipe 10145;
(H7020+2127)-(H9795+2120)=r10150*Qp10150*((@abs(Qp10150))^0.852);!Pipe 10150;

!Resistance Coefficients based on the Hazen-Williams equation;
r4240=2.2084;
r4250=1.6057;
r4255=1.3626;
r4260=1.4388;
r7180=0.9319;
r7190=5.2745;
r7200=2.7765;
r7210=0.4564;
r7220=2.0128;
r7230=0.7254;
r7250=3.9535;
r7260=0.7446;
r7300=1.4555;
r7320=2.3971;
r7340=0.9079;
r10135=1.1506;
r10140=5.2850;
r10145=14.2364;
r10150=4.0564;

!Flow magnitude constraint;
@abs(Sp4200)>@abs(S4200);
@abs(Qp4240)>@abs(Q4240);
@abs(Qp4250)>@abs(Q4250);
@abs(Qp4255)>@abs(Q4255);
@abs(Qp4260)>@abs(Q4260);
@abs(Qp7180)>@abs(Q7180);
@abs(Qp7190)>@abs(Q7190);
@abs(Qp7200)>@abs(Q7200);
@abs(Qp7210)>@abs(Q7210);
@abs(Qp7220)>@abs(Q7220);
@abs(Qp7230)>@abs(Q7230);
@abs(Qp7250)>@abs(Q7250);
@abs(Qp7260)>@abs(Q7260);
@abs(Qp7300)>@abs(Q7300);
@abs(Qp7320)>@abs(Q7320);
@abs(Qp7340)>@abs(Q7340);
@abs(Qp10135)>@abs(Q10135);
@abs(Qp10140)>@abs(Q10140);
@abs(Qp10145)>@abs(Q10145);
```

```
@abs(Qp10150)>@abs(Q10150);

!Flow direction constraint;
Sp4200*S4200-@abs(Sp4200)*@abs(S4200)=0;
Qp4240*Q4240-@abs(Qp4240)*@abs(Q4240)=0;
Qp4250*Q4250-@abs(Qp4250)*@abs(Q4250)=0;
Qp4255*Q4255-@abs(Qp4255)*@abs(Q4255)=0;
Qp4260*Q4260-@abs(Qp4260)*@abs(Q4260)=0;
Qp7180*Q7180-@abs(Qp7180)*@abs(Q7180)=0;
Qp7190*Q7190-@abs(Qp7190)*@abs(Q7190)=0;
Qp7200*Q7200-@abs(Qp7200)*@abs(Q7200)=0;
Qp7210*Q7210-@abs(Qp7210)*@abs(Q7210)=0;
Qp7220*Q7220-@abs(Qp7220)*@abs(Q7220)=0;
Qp7230*Q7230-@abs(Qp7230)*@abs(Q7230)=0;
Qp7250*Q7250-@abs(Qp7250)*@abs(Q7250)=0;
Qp7260*Q7260-@abs(Qp7260)*@abs(Q7260)=0;
Qp7300*Q7300-@abs(Qp7300)*@abs(Q7300)=0;
Qp7320*Q7320-@abs(Qp7320)*@abs(Q7320)=0;
Qp7340*Q7340-@abs(Qp7340)*@abs(Q7340)=0;
Qp10135*Q10135-@abs(Qp10135)*@abs(Q10135)=0;
Qp10140*Q10140-@abs(Qp10140)*@abs(Q10140)=0;
Qp10145*Q10145-@abs(Qp10145)*@abs(Q10145)=0;
Qp10150*Q10150-@abs(Qp10150)*@abs(Q10150)=0;

!Allow for flow reversal in each pipe;
@free(Q4240);
@free(Q4250);
@free(Q4255);
@free(Q4260);
@free(Q7180);
@free(Q7190);
@free(Q7200);
@free(Q7210);
@free(Q7220);
@free(Q7230);
@free(Q7250);
@free(Q7260);
@free(Q7300);
@free(Q7320);
@free(Q7340);
@free(Q10135);
@free(Q10140);
@free(Q10145);
@free(Q10150);
@free(Qp4240);
@free(Qp4250);
@free(Qp4255);
@free(Qp4260);
@free(Qp7180);
@free(Qp7190);
@free(Qp7200);
@free(Qp7210);
@free(Qp7220);
@free(Qp7230);
@free(Qp7250);
@free(Qp7260);
@free(Qp7300);
```

```
@free(Qp7320);
@free(Qp7340);
@free(Qp10135);
@free(Qp10140);
@free(Qp10145);
@free(Qp10150);

!Estimated demand flows;
D4200=0.00129;
D4210=0.00470;
D4215=0;
D4220=0.01201;
D4230=0.02012;
D6990=0.00856;
D7000=0.01328;
D7010=0.00385;
D7020=0.00129;
D7030=0.00602;
D7040=0.00985;
D7060=0.00172;
D7070=0.00470;
D7110=0.00813;
D7130=0.00085;
D9780=0.00900;
D9795=0.00428;

!Given values;
S4200=0.10965;
Sp4200=1.1*S4200;
H4200=113.65;

End
```

```
Rows=     95 Vars=     55 No. integer vars=      0
 Nonlinear rows=     60 Nonlinear vars=     39 Nonlinear constraints=     59
 Nonzeros=    271 Constraint nonz=   211 Density=0.051
 No. < :   0 No. =:  74 No. > :  20, Obj=MAX Single cols=    0

 Optimal solution found at step:        53
 Objective value:                0.7821653E-04


                        Variable           Value        Reduced Cost
                          QP4240       0.7274776E-01       0.0000000
                           Q4240       0.6448077E-01       0.0000000
                         QP10140       0.8993099E-02        1.841996
                          Q10140       0.8008099E-02       0.0000000
                         QP10145       0.1889310E-01       0.0000000
                          Q10145       0.1700810E-01       0.0000000
                         QP10150       0.2360110E-01       0.0000000
                          Q10150       0.2128810E-01       0.0000000
                           Q7180       0.4387923E-01       0.0000000
                           D4200       0.1290000E-02       0.0000000
                           S4200       0.1096500           0.0000000
                           Q4250       0.4937190E-01       0.0000000
                           Q7230       0.1040887E-01       0.0000000
                           D4210       0.4700000E-02       0.0000000
                           Q4255       0.4937190E-01       0.0000000
                           D4215       0.0000000           0.0000000
                           Q7260      -0.1726542E-01       0.2989412
                           Q4260       0.2009648E-01       0.0000000
                           D4220       0.1201000E-01       0.0000000
                           Q7340      -0.2351634E-04       0.0000000
                           D4230       0.2012000E-01       0.0000000
                           Q7190       0.3531923E-01       0.0000000
                           D6990       0.8560000E-02       0.0000000
                           Q7200       0.2203923E-01       0.0000000
                           D7000       0.1328000E-01       0.0000000
                           Q7210       0.1818923E-01       0.0000000
                           D7010       0.3850000E-02       0.0000000
                           Q7220       0.4388865E-02       0.0000000
                           D7020       0.1290000E-02       0.0000000
                           D7030       0.6020000E-02       0.0000000
                          Q10135      -0.1841901E-02       0.0000000
                           D7040       0.9850000E-02       0.0000000
                           Q7250      -0.6541901E-02       0.0000000
                           Q7300      -0.9003516E-02       0.0000000
                           D7060       0.1720000E-02       0.0000000
                           D7070       0.4700000E-02       0.0000000
                           Q7320      -0.8735164E-03       0.0000000
                           D7110       0.8130000E-02       0.0000000
                           D7130       0.8500000E-03       0.0000000
                           D9780       0.9000000E-02       0.0000000
                           D9795       0.4280000E-02       0.0000000
                          QP7180       0.4644823E-01       0.0000000
                          SP4200       0.1206150           0.0000000
                          QP4250       0.5412489E-01       0.0000000
                          QP7230       0.1345287E-01       0.0000000
                          QP4255       0.5412489E-01       0.0000000
                          QP7260      -0.2081742E-01       0.0000000
```

```
 QP4260       0.2009648E-01       0.0000000
 QP7340      -0.2035516E-02       0.0000000
 QP7190       0.3703223E-01       0.0000000
 QP7200       0.2242423E-01       0.0000000
 QP7210       0.1818923E-01       0.0000000
 QP7220       0.6830865E-02       0.0000000
QP10135      -0.1841901E-02       0.0000000
 QP7250      -0.7011901E-02       0.0000000
 QP7300      -0.1191352E-01       0.0000000
 QP7320      -0.2970516E-02       0.0000000
  H4200        113.6500           0.0000000
  H4210        135.6328           0.0000000
  R4240        2.208400           0.0000000
  H4215        166.6255           0.0000000
  R4250        1.605700           0.0000000
  H4220        144.6194           0.0000000
  R4255        1.362600           0.0000000
  H4230        155.6183           0.0000000
  R4260        1.438800           0.0000000
  H6990        123.1468           0.0000000
  R7180       0.9319000           0.0000000
  H7000        135.1351           0.0000000
  R7190        5.274500           0.0000000
  H7010        147.6326           0.0000000
  R7200        2.776500           0.0000000
  H7020        149.6323           0.0000000
  R7210       0.4564000           0.0000000
  H7030        149.6325           0.0000000
  R7220        2.012800           0.0000000
  R7230       0.7254000           0.0000000
  H7070        166.6184           0.0000000
  H7060        137.6188           0.0000000
  R7250        3.953500           0.0000000
  R7260       0.7446000           0.0000000
  H7110        132.1184           0.0000000
  R7300        1.455500           0.0000000
  H7130        160.6183           0.0000000
  R7320        2.397100           0.0000000
  R7340       0.9079000           0.0000000
  H7040        167.1184           0.0000000
 R10135        1.150600           0.0000000
  H9780        177.1193           0.0000000
 R10140        5.285000           0.0000000
  H9795        156.6284           0.0000000
 R10145        14.23640           0.0000000
 R10150        4.056400           0.0000000

    Row    Slack or Surplus      Dual Price
      1       0.7821653E-04        1.000000
      2       0.0000000           0.0000000
      3       0.0000000           0.0000000
      4       0.0000000           0.0000000
      5       0.0000000           0.0000000
      6       0.0000000          -0.5551115E-16
      7       0.0000000           0.0000000
      8       0.0000000           0.0000000
      9       0.0000000           0.0000000
```

```
10       0.0000000           0.0000000
11       0.0000000           0.0000000
12       0.0000000           0.0000000
13       0.0000000          -0.2989412
14       0.0000000          -0.2989412
15       0.0000000          -0.5551115E-16
16       0.0000000          -0.5551115E-16
17       0.0000000           0.0000000
18       0.0000000           0.0000000
19       0.0000000           0.0000000
20       0.0000000           0.2606262E-01
21       0.0000000           0.2738959
22      -0.1022301E-07       0.4842077
23       0.0000000           0.3887305
24       0.0000000          -0.7504073E-02
25       0.0000000          -0.4252127E-01
26       0.0000000          -0.5454334E-01
27       0.0000000          -0.5619674E-01
28       0.0000000          -0.6100529E-02
29       0.0000000           0.4040410
30       0.0000000           0.5860419
31       0.0000000           0.6930150
32       0.0000000           0.3489649
33       0.0000000           0.3801667
34       0.0000000           -1.261181
35       0.0000000          -0.3648850
36       0.0000000           0.5943318E-01
37      -0.6906973E-07        1.000000
38      -0.5861196E-07        1.000000
39      -0.7390757E-05       -1.000000
40       0.0000000          -0.5943318E-01
41       0.0000000          -0.5943318E-01
42       0.0000000          -0.5943318E-01
43       0.0000000          -0.5943318E-01
44      -0.1013069E-06      -0.9405668
45      -0.3312288E-07      -0.9405668
46      -0.2314958E-06       -1.000000
47      -0.4850079E-05       -2.000000
48      -0.8677803E-05       -1.000000
49      -0.1353093E-05       -1.000000
50      -0.1196185E-05       -1.000000
51      -0.8399196E-07       -1.000000
52      -0.2947849E-06       -1.000000
53      -0.7132805E-06       -1.000000
54      -0.1967460E-06       -1.000000
55       0.0000000           0.4635738E-03
56       0.0000000           0.4510776E-02
57       0.0000000           0.4510776E-02
58       0.0000000          -0.7200761E-03
59       0.0000000          -0.2019556E-03
60       0.0000000          -0.1327514E-03
61       0.0000000          -0.5242736E-04
62       0.0000000          -0.3557994E-04
63       0.0000000          -0.9179916E-04
64       0.0000000          -0.3220733E-03
65       0.0000000           0.1024443E-03
66       0.0000000           0.1537292E-02
```

```
 67       0.0000000           0.2734173E-03
 68       0.0000000           0.2087780E-04
 69       0.0000000           0.1036729E-04
 70       0.0000000           0.8615355E-05
 71       0.0000000          -0.1624203E-03
 72       0.0000000          -0.6422639E-03
 73       0.0000000          -0.9697739E-03
 74       0.1096499E-01       0.0000000
 75       0.8266989E-02       0.0000000
 76       0.4752989E-02       0.0000000
 77       0.4752989E-02       0.0000000
 78       0.0000000           0.0000000
 79       0.2569001E-02       0.0000000
 80       0.1713001E-02       0.0000000
 81       0.3849999E-03       0.0000000
 82       0.0000000           0.0000000
 83       0.2442001E-02       0.0000000
 84       0.3044000E-02       0.0000000
 85       0.4699999E-03       0.0000000
 86       0.3551999E-02       0.0000000
 87       0.2909999E-02       0.2989412
 88       0.2096999E-02       0.0000000
 89       0.2011999E-02       0.0000000
 90       0.0000000           0.2989412
 91       0.9850003E-03       0.0000000
 92       0.1885001E-02       0.0000000
 93       0.2313001E-02       0.0000000
 94       0.0000000           0.0000000
 95       0.0000000           0.0000000
 96       0.0000000           0.0000000
 97       0.0000000           0.0000000
 98       0.0000000           0.0000000
 99       0.0000000           0.0000000
100       0.0000000           0.0000000
101       0.0000000           0.0000000
102       0.0000000           0.0000000
103       0.0000000           0.0000000
104       0.0000000           0.0000000
105       0.0000000           0.0000000
106       0.0000000           0.0000000
107       0.0000000           0.0000000
108       0.0000000           0.0000000
109       0.0000000           0.0000000
110       0.0000000           0.0000000
111       0.0000000           0.0000000
112       0.0000000           0.0000000
113       0.0000000           0.0000000
114       0.0000000           0.0000000
115       0.0000000           0.2866888E-01
116       0.0000000           0.3012855
117       0.0000000           0.5326285
118       0.0000000           0.4276036
119       0.0000000          -0.8254481E-02
120       0.0000000          -0.4677340E-01
121       0.0000000          -0.5999767E-01
122       0.0000000          -0.6181641E-01
123       0.0000000          -0.6710582E-02
```

```
124       0.0000000           0.4444451
125       0.0000000           0.3457049
126       0.0000000           0.4633753
127       0.0000000           0.3838614
128       0.0000000           0.4181833
129       0.0000000           -1.387299
130       0.0000000          -0.4013735
131       0.0000000           0.0000000
132       0.0000000           0.0000000
133       0.0000000           0.0000000
```

```
!Mt. Tabor Subnetwork within the Blacksburg water distribution system;

!In this situation, four of the pipes were found to contain leakage.  The
leakage from those four pipes is modeled using average nodal pressure in the
orifice equation;

Model:

!Objective Function;
MAX=(Qp4240-Q4240)^2+(Qp10140-Q10140)^2+(Qp10145-Q10145)^2+(Qp10150-
Q10150)^2;

INIT:
Sp4200=1;
ENDINIT

!Continuity Equation for Demand;
-Q4240-Q7180=D4200-S4200;           !Node 4200;
Q4240-Q4250-Q7230=D4210;            !Node 4210;
Q4250-Q4255=D4215;                  !Node 4215;
Q4255+Q7260-Q4260=D4220;            !Node 4220;
Q4260-Q7340=D4230;                  !Node 4230;
Q7180-Q7190=D6990;                  !Node 6990;
Q7190-Q7200=D7000;                  !Node 7000;
Q7200-Q7210=D7010;                  !Node 7010;
Q7210+Q7220-Q10150=D7020;           !Node 7020;
Q7230-Q7220=D7030;                  !Node 7030;
Q10140-Q10135=D7040;                !Node 7040;
Q7250+Q7300-Q7260=D7060;            !Node 7060;
Q10135-Q7250=D7070;                 !Node 7070;
Q7320-Q7300=D7110;                  !Node 7110;
Q7340-Q7320=D7130;                  !Node 7130;
Q10145-Q10140=D9780;                !Node 9780;
Q10150-Q10145=D9795;                !Node 9795;

!Continuity Equation for Demand and Leakage;
-Qp4240-Qp7180=D4200+(L4240/2)-Sp4200;          !Node 4200;
Qp4240-Qp4250-Qp7230=D4210+(L4240/2);           !Node 4210;
Qp4250-Qp4255=D4215;                            !Node 4215;
Qp4255+Qp7260-Qp4260=D4220;                     !Node 4220;
Qp4260-Qp7340=D4230;                            !Node 4230;
Qp7180-Qp7190=D6990;                            !Node 6990;
Qp7190-Qp7200=D7000;                            !Node 7000;
Qp7200-Qp7210=D7010;                            !Node 7010;
Qp7210+Qp7220-Qp10150=D7020+(L10150/2);         !Node 7020;
Qp7230-Qp7220=D7030;                            !Node 7030;
Qp10140-Qp10135=D7040+(L10140/2);               !Node 7040;
Qp7250+Qp7300-Qp7260=D7060;                     !Node 7060;
Qp10135-Qp7250=D7070;                           !Node 7070;
Qp7320-Qp7300=D7110;                            !Node 7110;
Qp7340-Qp7320=D7130;                            !Node 7130;
Qp10145-Qp10140=D9780+(L10140/2)+(L10145/2);    !Node 9780;
Qp10150-Qp10145=D9795+(L10145/2)+(L10150/2);    !Node 9795;

!Overall balance of supply and leakage;
Sp4200=S4200+L4240+L10140+L10145+L10150;
```

```
!Energy Equation with leakage flows;
((H4200)+2163)-((H4210)+2141)=r4240*Qp4240*((@abs(Qp4240))^0.852);!Pipe 4240;
(H4210+2141)-(H4215+2110)=r4250*Qp4250*((@abs(Qp4250))^0.852);    !Pipe 4250;
(H4215+2110)-(H4220+2132)=r4255*Qp4255*((@abs(Qp4255))^0.852);    !Pipe 4255;
(H4220+2132)-(H4230+2121)=r4260*Qp4260*((@abs(Qp4260))^0.852);    !Pipe 4260;
(H4200+2163)-(H6990+2153.5)=r7180*Qp7180*((@abs(Qp7180))^0.852);  !Pipe 7180;
(H6990+2153.5)-(H7000+2141.5)=r7190*Qp7190*((@abs(Qp7190))^0.852);!Pipe 7190;
(H7000+2141.5)-(H7010+2129)=r7200*Qp7200*((@abs(Qp7200))^0.852);  !Pipe 7200;
(H7010+2129)-(H7020+2127)=r7210*Qp7210*((@abs(Qp7210))^0.852);    !Pipe 7210;
(H7030+2127)-(H7020+2127)=r7220*Qp7220*((@abs(Qp7220))^0.852);    !Pipe 7220;
(H4210+2141)-(H7030+2127)=r7230*Qp7230*((@abs(Qp7230))^0.852);    !Pipe 7230;
(H7070+2110)-(H7060+2139)=r7250*Qp7250*((@abs(Qp7250))^0.852);    !Pipe 7250;
(H7060+2139)-(H4220+2132)=r7260*Qp7260*((@abs(Qp7260))^0.852);    !Pipe 7260;
(H7110+2144.5)-(H7060+2139)=r7300*Qp7300*((@abs(Qp7300))^0.852);  !Pipe 7300;
(H7130+2116)-(H7110+2144.5)=r7320*Qp7320*((@abs(Qp7320))^0.852);  !Pipe 7320;
(H4230+2121)-(H7130+2116)=r7340*Qp7340*((@abs(Qp7340))^0.852);    !Pipe 7340;
(H7040+2109.5)-(H7070+2110)=r10135*Qp10135*((@abs(Qp10135))^0.852);
!Pipe 10135;
(H9780+2099.5)-(H7040+2109.5)=r10140*Qp10140*((@abs(Qp10140))^0.852);
!Pipe 10140;
(H9795+2120)-(H9780+2099.5)=r10145*Qp10145*((@abs(Qp10145))^0.852);
!Pipe 10145;
(H7020+2127)-(H9795+2120)=r10150*Qp10150*((@abs(Qp10150))^0.852);!Pipe 10150;

!Resistance Coefficients based on the Hazen-Williams equation;
r4240=2.2084;
r4250=1.6057;
r4255=1.3626;
r4260=1.4388;
r7180=0.9319;
r7190=5.2745;
r7200=2.7765;
r7210=0.4564;
r7220=2.0128;
r7230=0.7254;
r7250=3.9535;
r7260=0.7446;
r7300=1.4555;
r7320=2.3971;
r7340=0.9079;
r10135=1.1506;
r10140=5.2850;
r10145=14.2364;
r10150=4.0564;

!Pressure-dependent outflow (orifice equation);
L4240=C*A4240*((g*(H4200+H4210))^0.5);
L10140=C*A10140*((g*(H9780+H7040))^0.5);
L10145=C*A10145*((g*(H9795+H9780))^0.5);
L10150=C*A10150*((g*(H7020+H9795))^0.5);

!Flow magnitude constraint;
@abs(Sp4200)>@abs(S4200);
@abs(Qp4240)>@abs(Q4240);
@abs(Qp4250)>@abs(Q4250);
@abs(Qp4255)>@abs(Q4255);
@abs(Qp4260)>@abs(Q4260);
```

```
@abs(Qp7180)>@abs(Q7180);
@abs(Qp7190)>@abs(Q7190);
@abs(Qp7200)>@abs(Q7200);
@abs(Qp7210)>@abs(Q7210);
@abs(Qp7220)>@abs(Q7220);
@abs(Qp7230)>@abs(Q7230);
@abs(Qp7250)>@abs(Q7250);
@abs(Qp7260)>@abs(Q7260);
@abs(Qp7300)>@abs(Q7300);
@abs(Qp7320)>@abs(Q7320);
@abs(Qp7340)>@abs(Q7340);
@abs(Qp10135)>@abs(Q10135);
@abs(Qp10140)>@abs(Q10140);
@abs(Qp10145)>@abs(Q10145);
@abs(Qp10150)>@abs(Q10150);

!Flow direction constraint;
Sp4200*S4200-@abs(Sp4200)*@abs(S4200)=0;
Qp4240*Q4240-@abs(Qp4240)*@abs(Q4240)=0;
Qp4250*Q4250-@abs(Qp4250)*@abs(Q4250)=0;
Qp4255*Q4255-@abs(Qp4255)*@abs(Q4255)=0;
Qp4260*Q4260-@abs(Qp4260)*@abs(Q4260)=0;
Qp7180*Q7180-@abs(Qp7180)*@abs(Q7180)=0;
Qp7190*Q7190-@abs(Qp7190)*@abs(Q7190)=0;
Qp7200*Q7200-@abs(Qp7200)*@abs(Q7200)=0;
Qp7210*Q7210-@abs(Qp7210)*@abs(Q7210)=0;
Qp7220*Q7220-@abs(Qp7220)*@abs(Q7220)=0;
Qp7230*Q7230-@abs(Qp7230)*@abs(Q7230)=0;
Qp7250*Q7250-@abs(Qp7250)*@abs(Q7250)=0;
Qp7260*Q7260-@abs(Qp7260)*@abs(Q7260)=0;
Qp7300*Q7300-@abs(Qp7300)*@abs(Q7300)=0;
Qp7320*Q7320-@abs(Qp7320)*@abs(Q7320)=0;
Qp7340*Q7340-@abs(Qp7340)*@abs(Q7340)=0;
Qp10135*Q10135-@abs(Qp10135)*@abs(Q10135)=0;
Qp10140*Q10140-@abs(Qp10140)*@abs(Q10140)=0;
Qp10145*Q10145-@abs(Qp10145)*@abs(Q10145)=0;
Qp10150*Q10150-@abs(Qp10150)*@abs(Q10150)=0;

!Allow for flow reversal in each pipe;
@free(Q4240);
@free(Q4250);
@free(Q4255);
@free(Q4260);
@free(Q7180);
@free(Q7190);
@free(Q7200);
@free(Q7210);
@free(Q7220);
@free(Q7230);
@free(Q7250);
@free(Q7260);
@free(Q7300);
@free(Q7320);
@free(Q7340);
@free(Q10135);
@free(Q10140);
@free(Q10145);
```

```
@free(Q10150);
@free(Qp4240);
@free(Qp4250);
@free(Qp4255);
@free(Qp4260);
@free(Qp7180);
@free(Qp7190);
@free(Qp7200);
@free(Qp7210);
@free(Qp7220);
@free(Qp7230);
@free(Qp7250);
@free(Qp7260);
@free(Qp7300);
@free(Qp7320);
@free(Qp7340);
@free(Qp10135);
@free(Qp10140);
@free(Qp10145);
@free(Qp10150);

!Estimated demand flows;
D4200=0.00129;
D4210=0.00470;
D4215=0;
D4220=0.01201;
D4230=0.02012;
D6990=0.00856;
D7000=0.01328;
D7010=0.00385;
D7020=0.00129;
D7030=0.00602;
D7040=0.00985;
D7060=0.00172;
D7070=0.00470;
D7110=0.00813;
D7130=0.00085;
D9780=0.00900;
D9795=0.00428;

!Given values;
S4200=0.10965;
H4200=113.65;
C=0.61;
g=32.2;
A4240=0.00166;
A10140=0.00098;
A10145=0.00245;
A10150=0.00049;

End
```

```
Rows=     99 Vars=     59 No. integer vars=      0
 Nonlinear rows=     64 Nonlinear vars=     44 Nonlinear constraints=     63
 Nonzeros=    295 Constraint nonz=   234 Density=0.050
 No. < :   0 No. =:  78 No. > :  20, Obj=MAX Single cols=    0

 Optimal solution found at step:       109
 Objective value:                0.6369365E-01


                       Variable           Value        Reduced Cost
                         QP4240       0.2635506          0.0000000
                          Q4240       0.6810000E-01      0.3909011
                        QP10140      -0.6150200E-01       3.820278
                         Q10140      -0.9228027E-20      0.0000000
                        QP10145       0.5632188E-01      0.0000000
                         Q10145       0.9000000E-02      0.0000000
                        QP10150       0.1528181          0.0000000
                         Q10150       0.1328000E-01      0.2507158
                         SP4200       0.4476475          0.0000000
                          Q7180       0.4026000E-01      0.0000000
                          D4200       0.1290000E-02      0.0000000
                          S4200       0.1096500          0.0000000
                          Q4250       0.5738000E-01      0.0000000
                          Q7230       0.6020000E-02      0.0000000
                          D4210       0.4700000E-02      0.0000000
                          Q4255       0.5738000E-01      0.0000000
                          D4215       0.0000000          0.0000000
                          Q7260      -0.1593356E-11      0.0000000
                          Q4260       0.4537000E-01      0.0000000
                          D4220       0.1201000E-01      0.0000000
                          Q7340       0.2525000E-01      0.0000000
                          D4230       0.2012000E-01      0.0000000
                          Q7190       0.3170000E-01      0.0000000
                          D6990       0.8560000E-02      0.0000000
                          Q7200       0.1842000E-01      0.0000000
                          D7000       0.1328000E-01      0.0000000
                          Q7210       0.1457000E-01      0.0000000
                          D7010       0.3850000E-02      0.0000000
                          Q7220      -0.1022234E-12      0.0000000
                          D7020       0.1290000E-02      0.0000000
                          D7030       0.6020000E-02      0.0000000
                         Q10135      -0.9850000E-02      0.0000000
                          D7040       0.9850000E-02      0.0000000
                          Q7250      -0.1455000E-01      0.0000000
                          Q7300       0.1627000E-01      0.0000000
                          D7060       0.1720000E-02      0.0000000
                          D7070       0.4700000E-02      0.0000000
                          Q7320       0.2440000E-01      0.0000000
                          D7110       0.8130000E-02      0.0000000
                          D7130       0.8500000E-03      0.0000000
                          D9780       0.9000000E-02      0.0000000
                          D9795       0.4280000E-02      0.0000000
                         QP7180       0.1374615          0.0000000
                          L4240       0.9069100E-01      0.0000000
                         QP4250       0.1503191          0.0000000
                         QP7230       0.6318599E-01      0.0000000
                         QP4255       0.1503191          0.0000000
```

```
         QP7260       -0.9254657E-01         3.574447
         QP4260        0.4576250E-01        0.0000000
         QP7340        0.2564250E-01        0.0000000
         QP7190        0.1289015            0.0000000
         QP7200        0.1156215            0.0000000
         QP7210        0.1117715            0.0000000
         QP7220        0.5716599E-01        0.0000000
         L10150        0.2965876E-01        0.0000000
        QP10135       -0.1027891            0.0000000
         L10140        0.6287413E-01        0.0000000
         QP7250       -0.1074891            0.0000000
         QP7300        0.1666250E-01        0.0000000
         QP7320        0.2479250E-01        0.0000000
         L10145        0.1547736            0.0000000
          H4200         113.6500            0.0000000
          H4210         135.4631            0.0000000
          R4240         2.208400            0.0000000
          H4215         166.4151            0.0000000
          R4250         1.605700            0.0000000
          H4220         144.3744            0.0000000
          R4255         1.362600            0.0000000
          H4230         155.3696            0.0000000
          R4260         1.438800            0.0000000
          H6990         123.1264            0.0000000
          R7180        0.9319000            0.0000000
          H7000         135.0077            0.0000000
          R7190         5.274500            0.0000000
          H7010         147.4566            0.0000000
          R7200         2.776500            0.0000000
          H7020         149.4487            0.0000000
          R7210        0.4564000            0.0000000
          H7030         149.4588            0.0000000
          R7220         2.012800            0.0000000
          R7230        0.7254000            0.0000000
          H7070         166.3017            0.0000000
          H7060         137.3653            0.0000000
          R7250         3.953500            0.0000000
          R7260        0.7446000            0.0000000
          H7110         131.8660            0.0000000
          R7300         1.455500            0.0000000
          H7130         160.3686            0.0000000
          R7320         2.397100            0.0000000
          R7340        0.9079000            0.0000000
          H7040         166.7847            0.0000000
         R10135         1.150600            0.0000000
          H9780         176.7545            0.0000000
         R10140         5.285000            0.0000000
          H9795         156.3236            0.0000000
         R10145         14.23640            0.0000000
         R10150         4.056400            0.0000000
              C        0.6100000            0.0000000
          A4240        0.1660000E-02        0.0000000
              G         32.20000            0.0000000
         A10140        0.9800000E-03        0.0000000
         A10145        0.2450000E-02        0.0000000
         A10150        0.4900000E-03        0.0000000
```

```
Row    Slack or Surplus      Dual Price
  1       0.6369365E-01        1.000000
  2       0.0000000           0.0000000
  3       0.0000000           0.0000000
  4       0.0000000           0.0000000
  5       0.0000000           0.0000000
  6       0.0000000           0.0000000
  7       0.0000000           0.0000000
  8       0.0000000           0.0000000
  9       0.0000000           0.0000000
 10       0.0000000           0.0000000
 11       0.0000000           0.0000000
 12       0.0000000           0.0000000
 13       0.0000000           0.0000000
 14       0.0000000           0.0000000
 15       0.0000000           0.0000000
 16       0.0000000           0.0000000
 17       0.0000000          -0.1230040
 18       0.0000000          -0.2836027E-01
 19       0.0000000           0.0000000
 20       0.0000000           0.3924723
 21       0.0000000           0.9056102
 22       0.0000000            1.341060
 23       0.0000000           0.4284532
 24       0.0000000           0.0000000
 25       0.0000000           0.0000000
 26       0.0000000           0.0000000
 27       0.0000000           0.0000000
 28       0.0000000           0.2818500
 29       0.0000000           0.0000000
 30       0.0000000           -1.215404
 31       0.0000000          -0.2659848
 32       0.0000000          -0.8250449
 33       0.0000000           0.7689179E-01
 34       0.0000000           -2.908658
 35       0.0000000           -1.033747
 36       0.0000000           0.0000000
 37       0.0000000           0.1196556E-02
 38       0.0000000           0.8671869
 39       0.0000000           0.8671869
 40       0.0000000           -4.741197
 41       0.0000000           0.0000000
 42       0.0000000           0.0000000
 43       0.0000000           0.0000000
 44       0.0000000           0.0000000
 45       0.0000000          -0.8659546
 46       0.0000000          -0.8659546
 47       0.0000000          -0.8671869
 48       0.0000000           -5.608384
 49       0.0000000           -4.741197
 50       0.0000000           -4.741197
 51       0.0000000           -4.741197
 52       0.0000000          -0.8671869
 53       0.0000000          -0.8670538
 54       0.0000000          -0.8664627
 55       0.0000000          -0.8659797
 56       0.0000000           0.1012448E-03
```

```
 57       0.0000000           0.2593838E-01
 58       0.0000000           0.2593838E-01
 59       0.0000000          -0.1567299E-01
 60       0.0000000           0.0000000
 61       0.0000000           0.0000000
 62       0.0000000           0.0000000
 63       0.0000000           0.0000000
 64       0.0000000          -0.4322293E-02
 65       0.0000000          -0.5202892E-02
 66       0.0000000           0.1393797E-01
 67       0.0000000           0.6831830E-01
 68       0.0000000          -0.2412963E-02
 69       0.0000000          -0.5036967E-02
 70       0.0000000          -0.5361452E-02
 71       0.0000000           0.1283035E-01
 72       0.0000000           0.4955288E-02
 73       0.0000000          -0.4207304E-02
 74       0.0000000          -0.2670541E-01
 75       0.0000000           0.1962362
 76       0.0000000           -1.454329
 77       0.0000000           -1.971203
 78       0.0000000          -0.5168737
 79       0.3379975           0.0000000
 80       0.1954506           0.0000000
 81       0.9293907E-01       0.0000000
 82       0.9293907E-01       0.0000000
 83       0.3924965E-03       0.0000000
 84       0.9720146E-01       0.0000000
 85       0.9720146E-01       0.0000000
 86       0.9720146E-01       0.0000000
 87       0.9720146E-01       0.0000000
 88       0.5716599E-01       0.0000000
 89       0.5716599E-01       0.0000000
 90       0.9293907E-01       0.0000000
 91       0.9254657E-01       0.0000000
 92       0.3924965E-03       0.0000000
 93       0.3924965E-03       0.0000000
 94       0.3924965E-03       0.0000000
 95       0.9293907E-01       0.0000000
 96       0.6150200E-01       0.0000000
 97       0.4732188E-01       0.0000000
 98       0.1395381           0.0000000
 99       0.0000000           0.0000000
100       0.0000000           0.0000000
101       0.0000000           0.0000000
102       0.0000000           0.0000000
103       0.0000000           0.0000000
104       0.0000000           0.0000000
105       0.0000000           0.0000000
106       0.0000000           0.0000000
107       0.0000000           0.0000000
108       0.0000000           0.0000000
109       0.0000000           0.0000000
110       0.0000000           0.0000000
111       0.0000000           0.0000000
112       0.0000000           0.0000000
113       0.0000000           0.0000000
```

```
114       0.0000000           0.0000000
115       0.0000000           0.0000000
116       0.0000000           0.0000000
117       0.0000000           0.0000000
118       0.0000000           0.0000000
119       0.0000000           0.0000000
120       0.0000000           0.3924723
121       0.0000000           0.9056102
122       0.0000000            1.341060
123       0.0000000           0.4284532
124       0.0000000           0.0000000
125       0.0000000           0.0000000
126       0.0000000           0.0000000
127       0.0000000           0.0000000
128       0.0000000           0.2818500
129       0.0000000           0.0000000
130       0.0000000           -1.215404
131       0.0000000          -0.2659848
132       0.0000000          -0.8250449
133       0.0000000           0.7689179E-01
134       0.0000000           -3.031662
135       0.0000000           -1.062108
136       0.0000000           0.0000000
137       0.0000000          -0.1160835E-02
138       0.0000000          -0.6460046
139       0.0000000          -0.6118976E-02
140       0.0000000            10.72100
141       0.0000000           -93.30579
142       0.0000000           -124.5266
143       0.0000000           -31.28537
```

Vita

Jonathan A. Stathis

Jonathan A. Stathis was born on November 12, 1972 in Washington, D.C. He graduated from Annandale High School in Annandale, Virginia in 1990. Upon graduation from high school, Jonathan entered the University of Virginia as an Engineering major. However, after one year he transferred to Brigham Young University in Provo, Utah.

Jonathan interrupted his studies to serve a full-time mission for The Church of Jesus Christ of Latter-day Saints. He served in the Canada Montreal Mission from January 1992 - February 1994. Upon finishing his mission, Jonathan returned to BYU and received a Bachelor of Science degree in Civil Engineering in April 1997.

In the fall of 1997, Jonathan entered Virginia Polytechnic Institute and State University as a Via Scholar in the Department of Civil Engineering. He has served as a hydraulics laboratory instructor and graduate research assistant while at Virginia Tech. Jonathan will receive his Master of Science degree in Civil Engineering in December 1998.

Jonathan is married to the former Kathryn Anne Painter of Price, Utah. They are the parents of one daughter, Emma Victoria.