Geometric Possibility, Ideological Parsimony, and Monistic Substantivalism

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ABSTRACT

Monistic substantivalists believe that material objects and regions of spacetime are not two distinct kinds of fundamental of entities. For the monist, objects either are identical with regions or are somehow derivative from them. *Dualistic substantivalists* view regions and objects as distinct kinds of fundamental entities. One virtue monists claim to have is that their view is more *ideologically parsimonious* than dualism because monists can do without a primitive notion of location. In this paper I provide an argument that undercuts some of the theoretical edge that monists claim over dualists. The assumption that the monist can provide a reduction of location unique to her position rests on a false assumption about the possible structures spacetime can have. If it is metaphysically possible for two distinct regions to coincide with respect to all their significant spatiotemporal properties and relations (call these 'coincident regions'), then analyses of location unique to monism will turn out to be inadequate.

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General Audience Abstract

You, I, a rock on the ground, electrons, and galaxies all have something in common: we are all *material objects*. Material objects are often defined as the things that have locations within spacetime. But what is it to have a location within spacetime? Some authors, monists, believe that to have a location in spacetime is to be no more than a bit of the spatiotemporal manifold. Others, dualists, think of spacetime like a box that objects get placed into. For them having a location is to "take up" part of the room in this box. As the debate currently stands, many philosophical considerations look to point in favor of monism over dualism. In this paper I discuss a novel argument that this assessment does not stand up to scrutiny. The argument makes use of contemporary theories in physics and advanced geometry to argue that distinct parts of spacetime can be located at one another. This is shown to undermine many of the considerations which are thought to favor monism over dualism.

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1. Region and Object: Monism, Dualism, and Ideological Parsimony

A popular view in current discussions within the metaphysics of physics is spacetime substantivalism: the view that regions of spacetime belong in our fundamental ontology. Given that spacetime regions are one type of fundamental entity, how should we think of material objects? Are material objects distinct fundamental entities from regions? *Monistic substantivalists* answer in the negative while *dualistic substantivalists* answer in the affirmative.

Monistic theories are of two sorts. *Non-reductive monists* deny that material objects are fundamental. They claim that material objects depend on, or are derived from regions in some way.[1] *Supersubstantivalists* or *reductive monists* deny that material objects are distinct entities from regions, and they identify objects with regions.[2]

Dualism has often been thought of as the default position since, arguably, it is the more conservative of the two theories; it requires fewer revisions of our pre-theoretic beliefs. However, monists claim their theories enjoy a number of theoretical virtues over dualism. For example, monism is claimed to be more ontologically parsimonious than dualism because monism posits fewer kinds of fundamental entities,[3] enjoys greater theoretical economy because monism can explain away brute necessities that vex dualist theories, and so on. These virtues collectively form an attractive case for favoring monistic theories over dualistic ones.

[1] Gilmore (2014) explores and provides some examples of various non-reductive monisms. Lehmkuhl (2015) discusses different arguments from physics in favor of a non-reductive monism.

[2] Skow (2005), and Schaffer (2009) explicitly endorse reductive monism while Lewis (1986, p. 76 n.55) states a preference for monism based on considerations of economy and Sider (2001 p. 110) claims that the "identification of objects with regions is just crying out to be made."

[3] I assume throughout the paper that what determines the degree of ontological parsimony of a theory is a function how many kinds of fundamental entities there are. For a defense of the idea that only fundamental entities are relevant to ontological parsimony see Schaffer (2015).

One important virtue monists claim over dualists is that monistic theories are more *ideologically parsimonious* than dualist theories.[4] This is because monism, unlike dualism, supposedly can do without a primitive notion of location.[5] Call this claim '*I-PARSIMONY*':

I-PARSIMONY
Monism is more ideologically parsimonious than dualism because monism can do without a primitive notion of location.

The monist's claim to I-PARSIMONY rests on the idea that location is analyzable in terms of more fundamental notions such as identity, dependence, constitution, or grounding.

In this paper I provide an argument that undercuts some of the theoretical edge that monists claim over dualists. The assumption that the monist can provide a reduction of location unique to her position rests on a false assumption about the possible structures spacetime can have. If it is metaphysically possible for two distinct regions to coincide with respect to all their significant spatiotemporal properties and relations (call these 'coincident regions'), then the monistic analyses of location will turn out to be inadequate.

This paper can be seen as an exercise in weighing explanatory virtues. The goal is to weaken the case in favor of monism by undercutting the monists' claim that their view enjoys a greater balance of virtues than dualism. My main target is undermining I-PARSIMONY.[6] Also, we will see important explanations monists claim they can offer are undercut. In the end, the dualist regains a step against the monist.

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[4] Roughly, while considerations of ontological parsimony are concerned with minimizing the amount of irreducible/fundamental objects, considerations of ideological parsimony are concerned with minimizing the amount of primitive concepts. See Cowling (2013) for a discussion and defense of ideological parsimony as an epistemic virtue of theories.

[5] Perhaps dualist views allow for other ways of simplifying our ideology, and at the end of the day these views balance out with respect to ideological parsimony. For the sake of argument I assume this isn't the case.

[6] As will become clear later (§3.3) I-PARSIMONY is not a stand-alone virtue, it also plays an important role in explaining necessities that dualists supposedly cannot explain.

The paper has the following structure. I provide an overview of theories of location and examine different possible monistic analyses of location is provided in (§2). In (§3) I argue for the possibility of coincident regions and show how they undermine the aforementioned monistic analyses of location. In (§4) I show some further implications the possibility of coincident regions has for the debate between monists and dualists. Monistic replies are considered in (§5). In (§6) I conclude with where this leaves the debate.

2. Reducing Location: Monism and Ideological Parsimony

For I-PARSIMONY to be true, the monist needs to be able to provide an adequate analysis of location. To see why monists think they can achieve this will require some unpacking. It will be helpful to understand what is at stake in providing a theory of location and to have some condition of adequacy for such theories before examining specific monistic analyses.

2.1. Theories of Location and a Condition of Adequacy

There are important features about the nature of spatiotemporal things that theories of location are intended to capture. One starting point for substantivalist theories of location is the observation that objects are related to regions in important ways. The family dog, Rupert, is importantly related to the dog-shaped region he is in. However, Rupert is not specially related in the same way to the human-shaped region that I am in. He is 'connected' to the dog-shaped region in some way, and not to the region I am in. Further, Rupert takes a fixed place relative to other objects. This is matched by how the region Rupert is specially related to takes a fixed place relative to the regions those other objects are specially related to. There is mirroring between Rupert and the Rupert-region. Finally, there seem to be other interesting correlations between the region Rupert is specially related to and Rupert. The region and Rupert are the same size, and shape, and they seem to share mirroring mereological features − every subregion of the Rupert-region is matched by a part of Rupert.

Three questions can be abstracted from these sorts of observations. First, what connects or unifies objects with the regions that they are specially related to (*e.g.* Rupert and the Rupert-region)? Second, what determines the position of something within the spatiotemporal manifold (*i.e.* what fixes or determines where something is)? Finally, what is the status of the correlations between the properties of a region and the properties of the object that is in that region (geometric, mereological, etc.)? Theories of location can be seen as primarily aimed at answering these questions.

Theories of location vary on what locative notions they take to be the most fundamental. The locative notion I will be concerned with throughout the rest of the paper is the locative notion that picks out the special relation Rupert stands in to the Rupert-shaped region he is in. This has been called '*exact location*'. This is the relation that something bears to a region when it stands in complete correspondence with that region with respect to all its spatiotemporal properties and relations.[7,8] Importantly, because exact location is implied by complete spatiotemporal correspondence I suggest the following as a minimal condition on any analysis of location:[9]

LOCATION
If some *x* has exactly the same shape, size, and stands in all the same spatiotemporal relations to other entities as a region *R*, then *x* is located at *R*.

[7] However, something can have an exact location without standing in exact correspondence with respect to spatiotemporal properties and relations to a region. We might think that it is possible for a pointy object to be located within an extended gunky region of spacetime. In this situation the pointy object wouldn't stand in any exact correspondence with respect to spatiotemporal properties as any region. See Parsons (2007) for a discussion on this.
[8] See Cody Gilmore (2014) for an example of a gloss of the *is-exactly-located-at* relation which is very similar to LOCATION.
[9] Throughout the rest of the paper whenever I say 'location' I mean 'exact location'.

I take it as a necessary condition on any adequate analysis of location that it satisfies LOCATION. By this I mean if it is possible that there is something which is the same size, shape, and stands in all of the same spatiotemporal relations as a given region, but the analysis under consideration does not imply that that entity is located at that region, then the analysis of location provided is unsuccessful.[10] Given LOCATION, we can give a rough assessment of different monistic analyses of location. We will see that, under some plausible assumptions, it is possible for multiple regions to share all the same spatiotemporal properties and relations. And, if it is possible for multiple regions to share all the same spatiotemporal properties, then none of the previously given monistic analyses of location will turn out to be adequate.

2.2. Monist Accounts of Location

Each version of monistic substantivalism will provide a different analysis of location. For example, since reductive monists identify objects with regions, reductive monists will analyze location in terms of identity:

> **Location as Identity (LAI)**
> Necessarily for any x, x is located at a region R *if and only if* $x = R$.

Non-reductive monists will analyze location in terms of whatever ontological priority relation they take objects and regions to stand in to each other. But not only will the analysis for the non-reductive monist vary depending on her favored priority relation, but the analysis of location will also likely vary depending on what priority theses she holds. For example, a non-reductive monist who takes the entire manifold to be ontologically prior to any of its parts will likely analyze location differently than a non-reductive monist who takes the parts of the manifold to be prior to

[10] Other conditions may also be required for an adequate analysis of the concept of location, but only LOCATION is used in my later argument, so I will restrict my discussion to LOCATION merely as a minimal condition of adequacy.

it. These analyses will have the common feature that the region will stand in some priority relation or relations to that which is located at it. For the sake of space, let's allow the following analysis to stand in for the other more specific analyses non-reductive monists might give:

> **Location as Priority (LAP)**
> Necessarily for any *x*, *x* is located at a region *R if and only if R* stands in the right sort of priority relation to *x*.

Since ontological priority will be a common feature of all non-reductive monistic analyses of location, nothing in my argument will hang on the details of any more specific analysis. One last note on this family of analyses: certain views about location will take regions to be located at themselves.[11] So for the sake of neutrality I will take it that the priority relations in question are antisymmetric.[12]

It is important to note is that, in each of these cases, the monist analyzes location in terms of relations that simplify their ontology. If objects aren't distinct from regions, then location is analyzed in terms of identity. If objects aren't fundamental, then location is analyzed in terms of some ontological priority relation. For the dualist these options aren't available. According to their view material objects simply just don't stand in any of these relations to regions. This is what prevents these analyses from being available to the dualist and gives the monists supposed explanatory edge of having a simpler ideology.

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[11] Casati and Varzi (1999) call this 'conditional reflexivity'. See Parsons (2007) and Gilmore (2014) for further discussion on this principle.

[12] A relation *R* is anti-symmetric when *aRb* and *bRa* entail that *a* and *b* are identical, or, equivalently, when *aRb* and $a \neq b$, then ~*aRb*.

3. Coincident Regions and I-PARSIMONY

I-PARSIMONY relies on the adequacy of the analyses given in the previous section. Without the ability to provide an analysis of location, the monist is stuck with primitive location leaving their ideology no more parsimonious than the dualist's. If, however, it can be shown that it is metaphysically possible for multiple regions to be the same size, shape, and stand in all the same spatiotemporal relations, then the monistic analyses of location in terms of identity and ontological priority can be shown to be inadequate.

To see why this is the case, assume for *reductio* that location is identity with a region as LAI claims. Since multiple regions have the same size, shape, and stand in all the same spatiotemporal relations as each other, then, by LOCATION, each of the multiple regions is located at each other. By LAI each of the multiple regions are identical with each other. But multiple things can't be identical with one another, and so we have a contradiction. Thus, if multiple regions can have the same size, shape, and all the same spatiotemporal relations, LAI is false. Now suppose that location is to be understood in terms of some ontological priority relation, as LAP contends. We know by LOCATION that the multiple regions are located at each other. So, by LAP, each of the multiple regions is ontologically prior to each of the other regions. But this violates the antisymmetry of those priority relations. Once again we have a contradiction. Therefore, if multiple regions can have the same size, shape, and stand in all the same spatiotemporal relations, then LAP is false. If these possibilities render LAI and LAP false, then the monistic accounts of location are inadequate. And so the possibility of multiple regions sharing the same spatiotemporal properties and relations undermines I-PARSIMONY.

3.1. Plenitude and Geometric Possibility

The previous argument relies on the premise that coincident regions are metaphysically possible. To support this claim it would be helpful to have some way to determine if a given situation is metaphysically possible. One promising method for figuring out geometric possibility finds its roots in Bricker (1991) on principles of plenitude for possible structures.[13] The governing intuition behind Bricker's theory is that the realm of metaphysically possible worlds contains no gaps or arbitrary or unnatural boundaries. Bricker's proposal can be roughly summed up as follows: we take the natural class of structures *S* of any structure that has played an explanatory role in our theorizing about the world as possible. Then any structure belonging to any *natural generalization* of *S* is metaphysically possible.[14, 15]

When considering what structures represent possible spaces Bricker's principle of plenitude is too strong. Generalizing out from the most obvious spatial structures might leave us with structures that aren't spatial.[16] In order to guarantee the structure under consideration is a geometric possibility, we need some further criteria to guarantee that the structure has the right credentials to count as a space. Belot (2011) and Brighouse (2014) have modified the principle of plenitude for structures to this end. They suggest that we constrain our natural generalizations to particular axes of generalization determined by which features are central to the concept of

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[13] Roughly, possible structures can be thought of as representations of ways that some or all of the concrete inhabitants in some possible world are or could be. Bricker formulates his discussion in terms of what structures could be instantiated rather than in terms of which structures can represent ways concreta could be. The representation talk, however, conforms better with the language used by those writing on the possible structures that space could have.

[14] Bricker (1991 p. 617)

[15] Not much is said about the notion of natural generalization other than "a natural generalization of a class of structures is always a natural class; but a natural class need not be a natural generalization of all of its subclasses." Bricker (1991 p. 617). The notion of a natural class is left unanalyzed but we are given a guide for figuring out which classes are natural: any class of structures which serves as the principle object of study for some area in mathematics will count as a natural class (*ibid.* p. 611).

[16] For example, as Belot (2011 p. 11) notes, the class of all structures in which some objects are arranged by finitely many relations. This class is a generalization of Euclidean spatial structure, and so will include spatial structures amongst the members of this class, but will presumably include non-spatial structures as well.

space.[17] To find the class of possible spatial structures take a paradigm class of spatial structures then find all the natural generalizations of this class which hold the central features of space fixed.[18]

Belot and Brighouse suggest different features as central to the concept of space. These different constraints provide similar results. Belot takes having a metric distance function as central to space. [19] This leads Belot to conclude that the class of metric spaces represent the possible ways space could be. [20] Brighouse takes having determinate dimension properties as central. [21] This leads Brighouse to conclude that either the full class of metrizable spaces or the class of separable metrizable spaces represent geometric possibility. However, for coincident regions to be possible the possible geometries of space need to be represented by a slightly larger class. I believe there is good reason to think a larger class does represent geometric possibility. But first, it will prove useful to clear up some of the terminology and ideas at play in the discussion to come.

3.2. A Topological Aside

For there to be coincident regions certain topological facts need to hold. Specifying these help provide a more rigorous understanding of coincident regions and what's involved in determining their possibility.

A topological space is a set X where a topology $\mathscr{T}$ has been specified for X. The topology $\mathscr{T}$ is a set of subsets of X called the *open sets* of X which have the following properties: (i) $\emptyset$ and X

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[17] For example, see Brighouse (2014 pp. 33-5).
[18] Note that because we use the process of natural generalization to carve out the boundaries of geometric possibility and this process always results in a natural class, the class of geometric structures will be a natural class.
[19] Belot (2011 p. 13).
[20] *Ibid.* pp. 27-8.
[21] Brighouse (2014, pp. 32 & 35).

are in $\mathscr{T}$; (ii) the union of the elements of any subcollection of $\mathscr{T}$ is in $\mathscr{T}$; and (iii) the intersection of the elements in any finite subcollection of $\mathscr{T}$ is in $\mathscr{T}$.[22] Different topological spaces obey different axioms. Relevant to deciding whether or not coincident regions are possible relate to the *topological separation*, or *distinguishability*, of the points within a space, and the topology induced by different "distance" functions on that space.[23]

Axioms which govern how points are distinguished topologically are called the *separation axioms*. Three commonly satisfied by topological spaces are the following:

> (T_0) For every pair of distinct points there exists a neighborhood of one of them which does not contain the other.
> (T_1) For every pair of distinct points there exists a neighborhood of each of them which does not contain the other.
> (T_2) For every pair of distinct points x and y there exists a neighborhood of x and a neighborhood of y which are disjoint.[24]

Neighborhood relations between distinct points determine their position within a space.[25] Points that share all of the same neighborhoods are indiscernible topologically. Each of these axioms has implications for the topological distinguishability of distinct points. Axiom T_0 implies that distinct points can be distinguished by their neighborhood relations. Each further condition is stronger than, and implies, the one before it. So a space's satisfying T_2 entails that it is T_1 which entails the space meets condition T_0. Because coincident regions stand in all of the same spatiotemporal relations as each other, they would have to be indistinguishable with respect to their neighborhood relations. Thus, if coincident regions are possible, the class of possible geometries must include spaces which don't satisfy T_0.

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[22] Munkres (2000 p. 75), Pears (1975, p. 3).
[23] Much of my presentation here follows Pears (1975, p. 1-10)
[24] These formulations come from Pears (*ibid.* p. 8)
[25] A subset $\mathcal{N}$ of a topological space is a neighborhood of a point x of X if there is an open set U such that $x \in U \subset \mathcal{N}$. Further, $\mathcal{N}$ is a neighborhood of a subset A of X if there exists an open set U such that $A \subset U \subset \mathcal{N}$. (Pears p.3)

Pseudo-metrics and *metrics* are functions that determine measure of distance within a given space.[26] A *pseudo-metric* on a topological space X is a function d: $X \times X \rightarrow \Re$ such that the following conditions hold: (i) $d(x, x) = 0$; (ii) $d(x, y) \geq 0$; (iii) $d(x, y) = d(y, x)$; and (iv) $d(x, z) \leq d(x, y) + d(y, z)$.[27] The function d is a *metric* when it satisfies a fifth condition: (v) $d(x, y) = 0$ then $x = y$.[28] All metrics are pseudo-metrics but the converse doesn't hold. Let's call any pseudo-metric which is not a metric a *strict pseudo-metric*. A *pseudo-metric space* is a pair (X, d) such that X is a set and d is a function which is a pseudo-metric on the set X. When d is a metric on X, then the pair (X, d) is called a *metric space*. The function d specifies a topology for X. Different topologies are specified when d is a metric, or a strict pseudo-metric. This is the *topology induced* by d. When the topology of a space is induced by a pseudo-metric, the space is *pseudo-metrizable*. When induced by a metric, the space is *metrizable*.[29]

There are two facts worth noting which pertain to the possibility of coincident regions. First, strictly pseudo-metric spaces have distinct points at zero distance from one another. Second, pseudo-metrizable spaces are metrizable provided they satisfy axiom T_0.[30] It follows that strictly pseudo-metrizable spaces fail to be T_0*-spaces*. Thus, indistinguishable points have all of the same neighborhoods, and are zero distance from one another in these spaces. The concrete points represented by the indistinguishable points in strictly pseudo-metrizable spaces are prime examples of coincident regions.[31] So the case for the possibility of coincident regions can be made

[26] Perhaps my use of 'distance' is too liberal here. Some will insist that distances can only satisfy the axioms of a metric (see Maudlin 2012 p. 72). If so, I intend the term in a less technical way as a useful way to group together functions that act in a distancey-way. However, I must note I think it would be quite surprising to find that all of this structure was built into the pre-theoretical concept of distance so that relations other than metrics are undeserving of the name 'distance'.
[27] Pears does not list (ii) as one of the axioms that a pseudo-metric must satisfy, instead he lists (i), (iii), and (iv) and then notes that the values provided by d must be non-negative.
[28] Pears (1975 p. 9)
[29] *ibid.* p. 9-10
[30] *ibid.* p. 10
[31] Also the regions composed of these points would be prime candidates for counting as coincident regions.

by showing that some class or subclass of strictly pseudo-metrizable spaces represent possible ways concrete space could be.

3.3. Strictly Pseudo-Metric Spaces and Coincident Regions

Considerations based on holding distance as central to space has led Belot to take the class of metric spaces as representing the possible geometries of space. In contrast, holding dimension as central has led Brighouse to claim that it is either the separable metrizable spaces, or metrizable spaces, that represent geometric possibility. Either of these results would rule out coincident regions from the realm of possibility. However, there are a number of good reasons to think the conclusions Belot and Brighouse come to rest upon too conservative of generalizations because they stop in arbitrary places. Further considerations provide pressure to think that larger classes of structures, which include pseudo-metrizable spaces, represent what is geometrically possible. This case will be made by considering distance and then dimension in turn.

Belot restricts his considerations of distance to spaces which have functions that satisfy the previously listed properties (i) – (v) for a metric function. The sole property distinguishing metrics from other pseudo-metrics is (v), which tells us that any points that are zero distance from one another are identical. Call this property 'positivity'.[32] Restricting the class of spaces to metric spaces implies that the positivity of a distance measure is necessary for a structure passing as spatial. However, given that the spacetime of the actual world is itself a spatial structure, this would mean that it must have a distance measure defined over it must satisfy positivity. In the spacetimes posited by the special theory of relativity and the general theory of relativity the metric structure is given by the interval between spacetime points and not by a function which

[32] Often conditions (i), (ii) and (v) for metric functions are listed just as (i*) $d(x, y) \geq 0$, with $d(x, y) = 0$ *iff* $x = y$ and referred to as 'positive definiteness' (See Maudlin 2012, p. 29).

satisfies positivity. Either we can understand spacetime interval as a distance measure of or not as a distance measure.[33] Say we do. Since the interval relation fails positivity, then we shouldn't constrain our theorizing about what the possible spatial structures are or aren't with distance functions that must satisfy positivity. On the other hand, if we don't take the interval relation to be a distance function, then considerations of distance shouldn't play a role in our theorizing about possible spaces because interval is what provides the geometric characterization between spacetime points in the actual world, not a distance function which obeys positivity. The interval relation captures the objective, frame of reference invariant relations between actual spacetime points whereas distance functions that obey positivity, like spatial interval, are not reference frame invariant.[34]

The upshot of this discussion is that since we are generalizing out along the axis of distance, the next natural class of spaces (with distance measures that don't satisfy positivity) is the full class of pseudo-metric spaces. We might be required to generalize out further, but, in the very least, the full class of pseudo-metric spaces will be representatives of geometric possibility. Amongst this class are those spaces where a strict pseudo-metric induces the topology – pseudo-metrizable spaces.

One might object that I have illicitly conflated the distinct notions of space and spacetime, and that to make judgments based on the possible spatial geometries from facts about the structure of spacetime is wrongheaded. Here are a few replies. First, the debate between the monist and the dualist is over the relationship between material objects and spacetime regions. So if there is a significant difference between regions of space and regions of spacetime, my discussion here should be read as restricted to structures that could represent possible spacetime

[33] I take it that the later two of these interpreatations of the interval relations in the more popular one. For example, see Sklar (1975 p. 62), Maudlin (2012 p. 77), Maudlin (2014 p. 287).
[34] Sklar (1975 pp. 55-64).

geometries. Second, I find it hard not to think of spacetime itself as a space. One of the take-aways from relativity is that space and time don't make for two differentiated aspects of the world, but, instead, are one undifferentiated one. Our best theory of space is a theory of spacetime.[35] Finally, consider a proposal like Skow's (2007) about the difference between timelike and spacelike directions. The difference between spacelike and timelike directions is not fundamental but is instead derived from the direction over which the laws of nature evolve.[36] If we find views like this attractive, then it seems that there is nothing fundamentally distinctive about spacetime that would warrant treating it as anything other than a spatial structure.

The other suggestion for determining geometric possibility is to take having determinate dimensions as central to space and restrict generalization along this axis. This has led Brighouse to think either the class of metrizable spaces or the class of separable metrizable spaces represent the possible spatial geometries. Brighouse's argument begins by introducing the three classical dimension functions: small inductive dimension (ind), large inductive dimension (Ind), and covering dimension (dim).[37] The functions, ind, Ind, and dim, all find their motivations in intuitive judgments about dimension.[38] Interestingly, the verdicts delivered by these functions exactly coincide in the class of separable metrizable spaces. Moreover, Ind and dim coincide in the full class of metrizable spaces. Brighouse takes this coincidence as a guide to determining which structures represent spatial possibilities. Brighouse's argument depends on the claim that the divergence of verdicts between dimension functions is indicative of pathological dimensional

[35] One reason for thinking this is that there is no unique way to divide up spacetime in general relativity into distinct three dimensional submanifolds which could be identified as instants of time. For more on this see Skow (2007 pp. 230-232).
[36] See Skow's (2007, pp 237-9) for a discussion of the position.
[37] Brighouse (pp. 38-42) provides a nice overview of each of these functions and what intuitions motivate each of them.
[38] For example, the judgment that the dimension of the surface of any object is one dimension less than the dimension of that object.

properties of that space.[39] This follows from Brighouse's claim that none of the dimension functions has a better claim to being correct than the others. This is why Brighouse concludes that we should either accept separable metrizable spaces or the full class of metrizable spaces as representing the class of possible spaces.

Whether or not we accept Brighouse's arguments for requiring coincidence as sound, there is no harm in agreeing with the verdict. This allows us to remain neutral on which, if any, dimension function is the correct one, and thus only strengthens the argument. As it happens, ind, Ind, and dim don't only coincide for the class of separable metrizable spaces, but they also coincide for the class of separable pseudo-metrizable spaces as well.[40] Similarly, just as Ind and dim coincide for the class of metrizable spaces, they coincide for the class of pseudo-metrizable spaces.[41] As such, the same considerations that led us to think either the separable metrizable spaces or the full class of metrizable spaces represent geometric possibility leads us to the same conclusion about the class of separable pseudo-metrizable spaces and the full class of pseudo-metrizable spaces. Therefore, even on Brighouse's most conservative criteria, some strictly pseudo-metrizable spaces will represent genuine possibilities for concrete space.

One might wonder why Brighouse doesn't suggest the larger classes of separable pseudo-metrizable and pseudo-metrizable spaces as candidates for representing geometric possibility. In fact, these spaces are never brought up during Brighouse's discussion. The reason seems to be that Brighouse assumes that possible spaces must satisfy the (T_2) separation axiom.[42] Since any (T_2)-*space* is a (T_0)-*space*, and pseudo-metrizable spaces which are (T_0)-*spaces* just are metrizable spaces, no (T_2)-*space* would be strictly pseudo-metrizable. So the assumption that space must be

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[39] *ibid.* p. 50
[40] Pears (1975 p. 186)
[41] *ibid.* p. 181
[42] Brighouse (2014 pp. 37-8)

(T_2) automatically rules out any strictly pseudo-metrizable space. But why think this? Brighouse offers little positive reason for stopping our generalizing here. She does say that (T_2) spaces represent a fair amount of spatial structure because they are, at least, locally just like Euclidean spaces.[43] And, this makes them prime candidates for a natural generalization of Euclidean structure.

But do we need to think that, for a structure to represent a possibility for space, it must be locally just like Euclidean space? Or must it merely be relevantly alike in some way? It is important to ask what, in the first place, led us to the belief that space at least satisfies (T_0). It doesn't seem like it was for empirical reasons. Any theory cast in terms where every point is topologically distinguishable from another, and thus at least satisfies (T_0), can be recast in terms of sets, pluralities, or fusions of indistinguishable points. Furthermore, it doesn't seem that it was settled that space is (T_0) for any purely conceptual reasons, either. Rather, it seems that we take space to be (T_0) because it provides us with a simpler metaphysic of space and a less messy system of representation to work with. These all seem like perfectly good reasons to assume that *our world* is this way, but absolutely the wrong sorts of reasons to assume that all possible spaces are this way. For these reasons, I find Brighouse's assumption that space must be (T_2) to be unwarranted.[44]

Thus both considerations from distance and dimension lead us to think that some strictly pseudo-metrizable spaces that represent possible ways space could be. These structures represent

[43] *ibid.* p. 38
[44] Compare this with a similar argument for a different conclusion found in Belot (2011, p. 27). Belot argues that simplicity considerations aren't the right kinds of considerations to decide whether mediated or unmediated distance relations are metaphysically possible. He allows that considerations of simplicity might give us reason to think that one type of distance relation is more likely to be actual than another, but not that one or the other is impossible. For this reason he takes both to be possible. Here my point is an analogous one about whether or not it is possible for there to be spatial structures which are not (T_2). So I take it that if these considerations work in the case offered by Belot, then they work in this case as well.

worlds in which there are coincident regions. So there is ample reason to think that coincident regions are metaphysically possible.

4. Further Virtues Undercut

Before discussing monistic responses to my argument, I would like to note some consequences that follow from the monist's losing the ability to reduce location. Several arguments given in favor of monism rely on the reduction of the location relation. For example, Schaffer (2009) uses LAI as a premise in his arguments that monist's are in a unique position to explain why the mereological features of objects must mirror the features of their locations, why that co-location is impossible, and why multi-location is impossible.[45] Each of these results is supposed to illustrate the explanatory economy of his specific monistic theory. However, without LAI, Schaffer's arguments fail. This shows how important the ability to do without with a primitive notion of location is to the monist's case against the dualist.

Further, the possibility of coincident regions can be used to show how even reductive monists lack an explanation for why these situations are impossible. Consider the situation where three points p_1, p_2, and p_3 are coincident.[46] So p_1, p_2, and p_3 have same size, shape, and stand in all the same spatiotemporal relations to everything as each other. By LOCATION, p_1 will be located at p_2 and p_3. Given that $p_1 \neq p_2$, p_1 will be located at more than one region, namely p_2 and itself. This is a violation of the ban on multi-location.

Similarly, by LOCATION p_1 and p_2 will both be located at p_3. Because p_1 has all of the same spatiotemporal properties and relations as p_3 this will entail that p_1 is located at p_3. p_2 also has all of the same spatiotemporal properties and relations as p_1 and so, by LOCATION, will count as

[45] Schaffer (2009, pp. 138-142)

[46] By 'point' here, I mean 'point-sized region'. Also note that points have no extent and as such can be thought of as having no size and shape and thus would vacuously fulfill two of the first three conditions of LOCATION.

being located at p_3 as well. Since $p_1 \neq p_2$, distinct things will count as having the same location. This contradicts the claim that co-located things are impossible.[47]

Finally, consider the fusion *R* of the three points. Presumably, *R* will be coincident with each of these points. So by LOCATION *R* will be located at each of the three points. But presumably each point is simple while *R* isn't. So we have a case of mereological mismatch between location and located. So not only do monists lose the edge they claim over the dualist with respect to ideological parsimony, they also lose a significant amount of the explanatory power they claim to have that the dualist is purported to lack. [48]

5. Regaining the Explanatory Edge?

How might the monist respond? Two ways suggest themselves. First, the monist might argue that, beyond the geometric constraints placed on determining the boundaries of geometric possibility, we need to consider certain metaphysical criteria as well. For example, the natures of regions need to be brought into consideration when determining which class of structures represents ways space could be. Second, the monist might try to reduce location in some way other than in terms of identity or priority to regain her claim to I-PARSIMONY.

Consider the first route, specifically the claim that points are individuated by their position within the structure of spatiotemporal relations, as moderate ontic structural realists

[47] Note too that this violates two weaker bans on co-location. Plausibly, points don't share any parts, so the case described above contradicts the claim that only things that share parts can be co-located. Also, since points are the same kind of thing, this contradicts the claim that no two things of the same kind can come to be co-located.

[48] This discussion above is cashed out entirely in terms of regions, but we might think that the monists interest is not in explaining bans on multi-location, co-location, and disharmony between regions, but between material objects and the regions they are located at. However, given plausible assumptions we can re-run each of these cases of material objects violating these bans. In the easiest case, assume with Schaffer that any region is identical with a material object, then all of the cases described above end up being straightforward violations of these bans in terms of how material objects can relate to regions. This sort of tactic generalizes to other monistic theories. So if we constrict our concern to just explaining certain necessities that hold about objects and regions, these arguments show that even the reductive monist doesn't fare much better than dualist in being able to offer brute-necessity free explanations.

contend.[49] Moderate ontic structural realism could then be provided as a purely metaphysical constraint for what structures represent possible geometries. Take the points p_1 and p_2. Say p_1 and p_2 stand in all the same spatiotemporal relations to everything else as one another. According to the structuralist, p_1 and p_2 would be identical. But for coincident regions to be possible distinct points would have to be indiscernible with respect to their spatiotemporal relations.

However, there are good reasons to think that this position has unfavorable consequences. Wüthrich (2009) offers one. Wüthrich's argument makes use of models of general relativity where the spacetimes are highly symmetric and exhibit very little spatial variation. He argues that, due to the lack of spatial variation in these models, moderate ontic structural realism entails that spacetime itself contains only one spatial point. This contradicts the fact that spacetime contains an uncountable number of points. Because moderate structural realism has this as a result, Wüthrich concludes that position within the structure of spatiotemporal relations is insufficient to ground the numerical diversity of points. But that is exactly what the moderate ontic structural realist's theory is supposed to be able to do.

Jantzen (2011) offers another compelling case against this view. Jantzen argues that relational structure alone can't ground there being a determinate number of objects. Since we are committed to there being a determinate number of points and our metaphysical theses shouldn't *a priori* rule out that there are a determinate number of points, we should reject the view that relational structure alone is responsible for the individuation of points. I find these objections compelling. For the structuralist to offer us a satisfactory metaphysic of spactime, her theory must be able to capture determinate numerical diversity amongst points.

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[49] Esfeld and Lam (2008) take this point of view. Schaffer (2009 pp. 132 & 136) uses this position to provide motivation for the view that the entirety of spacetime is prior to any of its subregions.

Perhaps the most interesting reply is given by Lam (2014). Lam suggests that primitive identity and primitive numerical diversity need to be distinguished. Lam then argues that the structuralist should supplement her theory of individuation with facts about primitive numerical diversity. On this view, primitive facts about identity wouldn't fix how many points there are. Instead, the facts about how many points there are will be primitive. There is a worry that such a maneuver is *ad hoc*, but, for the sake of argument, let's grant Lam this suggestion. Once we have primitive numerical diversity there doesn't seem to be any reason why structuralism would render coincident regions impossible. So if the structuralist follows Lam's suggestion, then she no longer has the resources to block the possibility of coincident regions.[50]

Another monist-friendly response is to accept that coincident regions are possible, but to provide a different analysis for location. The monist might analyze location in terms of having the same size, shape, and standing in all the same spatiotemporal relations as some region, or in terms of topological indistinguishability, or in terms of some other like notion. Assuming one of these analyses is adequate, the monist can do away with primitive location in spite of the possibility of coincident regions.

The problem with this reply is that it is unclear why analyses of these sorts aren't available to the dualist as well. So even if the monist can analyze location in this way, I-PARSIMONY is false. To see this, say the monist does analyze location in terms of being topologically indistinguishable from some region. To be located at a region is to be topologically indistinguishable from it.[51] This allows the monist to provide an account of location while allowing for the possibility of

[50] Perhaps there are other theories about the nature of regions that allows the monist to restrict geometric possibility in such a way as to rule out the possibility of coincident regions. I am unaware of any that give us any substantive reason for thinking this. In the very least, it seems that it is up to the monist to provide us with such a story.

[51] Strictly speaking topological equivalence is a relation between points or point-sized things. Where two points are topologically indistinguishable whenever they have exactly the same neighborhoods. This account of location would need to be extended to make sense out of the location of composites to something like the following: any x is located at a region R just in case any point-sized y that is a part of x is topologically indistinguishable from any point-sized region P that is a part of R.

coincident regions: coincident regions are topologically indistinguishable from one another, and, so, are located at each other. The issue is that the dualist can gladly accept the same account of location as part of their theory. For the dualist an object will be located at a region just in case it is topologically indistinguishable from that region. I'd imagine similar maneuvers would face exactly the same challenge. What allowed the monist to claim their view was more ideologically parsimonious than the dualist's was that monist's thought they could analyze location using resources and claims unique to their theory. Other attempts at analysis look like they will only make use of resources common to both the monist and the dualist. Because of this these analyses won't provide the monists anyway to gain a dialectical advantage over the dualist.

The previous analyses of location are what undergird other important explanatory advantages that the monist claims to have over the dualist as was noted in (§4). But these advantages could only be claimed because the previous analyses were *uniquely* available to the monist. However, if we analyze location any of the previously suggested ways, the analysis of location isn't uniquely available to the monist, but is available to the dualist, as well. Because of this, any explanatory work that the reduction of location is supposed to do for the monist can in turn be co-opted by the dualist. The result is that offering an analysis of this sort wouldn't allow the monist to regain any of the explanatory edge the monist lost over the dualist. Instead, the monist would have to be able to provide a different reduction of location that was unique to monist views. It is unclear how the monist would go about providing us with such an account. At best, the monist owes us a story of how she can provide a new reduction of location in a way unavailable to the dualist.

6. Conclusion

We've seen that the possibility of coincident regions renders standard monist analyses of location inadequate. Considerations from a principle of plenitude have led us to think that coincident regions are possible. We've also seen that monistic attempts to regain some explanatory edge over the dualist don't look promising. But how should we view this with respect to the dialectic between the monist and dualist as a whole? It depends on how we should weigh distinct virtues against one another. Various ways of answering these questions will deliver different results. In any case, it does seem like the possibility of coincident regions undercuts a significant argument monist's give in favor of their view over dualism. Not only is her claim to have a more parsimonious theory halved, but she also loses the important explanations that are purported to make her view more attractive than the dualist alternative.

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