Multiscale Decision-Making for Multiple Decision Alternatives

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Thesis submitted to the faculty of the Virginia Polytechnic Institute and State University in partial fulfillment of the requirements for the degree of

Master of Science
in
Industrial and Systems Engineering

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12/10/2012
Blacksburg, Virginia

Keywords: Multiscale Decision-Making, Decision Analysis, Optimal Incentives, Hierarchical Agents, Health Care Decision Support

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ABSTRACT

In organizations with decision makers across multiple hierarchical levels, conflicting objectives are commonly observed. The decision maker, or agent, at the highest level usually makes decisions in the interest of the organization, while a subordinate agent may have a conflict of interest between taking a course of action that is best for the organization and the course of action that is best for itself.

The Multiscale Decision-Making (MSDM) model was established by Wernz (2008). The model has been developed to capture interactions in multi-agent systems, by integrating both the hierarchical and temporal scale of decisions made in organizations.

This thesis contributes towards expanding the results in the hierarchical interaction domain of MSDM by extending the model to incorporate *N* decision alternatives and outcomes instead of two, and studying its effect on the interaction between agents.

We consider decisions with uncertain outcomes, where the outcomes of the decisions made by agents lower in hierarchy affect the transition probabilities of the decisions made by agents above them in hierarchy. This leads to a game theoretic situation, where the lower-level agents need to be sufficiently incentivized in order to shift their best response strategy to one in the interest of their superior and the organization. Mathematical expressions for the optimal incentives at each hierarchical level are developed.

We analyze systems with agents interacting across two and three organizational levels. We then study the effect of introducing the cost of taking an action on the optimal incentives. We discuss a health care application of MSDM.

To my family...

ACKNOWLEDGMENTS

I would like to thank my advisor, Dr.Christian Wernz, for the opportunity to pursue research under his guidance. I thank him for the financial support and encouragement he provided me. The freedom he gave me helped me explore and get deeply involved with my area of study. His relentless effort in helping me shape my thoughts and research style has taught me some very valuable lessons. I would like to thank my committee members Dr.Ebru Bish and Dr.Michael Taaffe for their invaluable insights and perspectives that helped me improve my work. I also thank the Grado Department of Industrial and Systems Engineering and the Physics Department at Virginia Tech for giving me the resources that enabled me to pursue my education.

I thank my family and friends for their love, support and guidance throughout all my endeavors. I thank my brother, Dr.Raghuram Sudhaakar, for guiding me through graduate school and helping me learn from his experiences. I thank Rohit Kota for being a bouncing board for my thoughts and ideas. I thank Daniel Steeneck for his feedback and insights, and Dr.Urvashi Singh for her detailed account of hospital organizational environments, which impacted my thesis significantly. I can’t thank my parents enough for their hard work and dedication in ensuring I received quality education. They made it all worth it, and this wouldn’t have been possible without them.

Table of Contents

Page

List of Figures

Chapter 1

INTRODUCTION

1.1 Motivation

In organizations with decision makers across multiple hierarchical levels, conflicting objectives are commonly observed. The decision maker, or agent, at the highest level usually makes decisions in the interest of the organization, while a subordinate agent may have a conflict of interest between taking a course of action that is best for the organization and the course of action that is best for itself.

The Multiscale Decision-Making (MSDM) model was established by Wernz (2008) . The model describes a novel approach capturing interactions in multi-agent systems, by integrating both the hierarchical and temporal scales of decisions made in organizations. The model suggests a method to compute the optimal incentives that each agent should offer to its subordinate to enable cooperation, taking into consideration two decision alternatives which lead to one of two associated outcomes with uncertainty. The model and associated results for the optimal incentive have been developed for two-agent interactions and three-agent interactions, considering only two decision alternatives and outcomes. Existing work does not address the effect of having more than two decision alternatives and outcomes for each agent.

In order to take the MSDM model towards real-world applications, a general model is required, considering as many decision alternatives, outcomes and agents as the organization contains. This work has generalized the number of decision alternatives and outcomes available, contributing to the hierarchical section of MSDM, thereby making available general two-agent and three-agent interaction models, which can be applied in real-world applications discussed in the following chapters.

1.2 Overview

We begin with a review of the literature in Chapter 2, where an extensive review of existing work in economics and decision theory is presented. In Chapter 3, the model is motivated with an example in health care decision-making in a hierarchical hospital environment. We describe briefly the agents, interactions and the decisions made by each agent. Following the example, we introduce the MSDM model with two agents and two decision alternatives and outcomes and proceed to explain the general *N*-decision outcome model. We then discuss the two-decision outcome case as a special case of the general model, and discuss the optimal incentives in the different cases.

In Chapter 4, we introduce the three-agent model with *N* decision outcomes, and we study how the optimal incentives behave in this model. In Chapter 5, we present a variation of the model described, by introducing the cost of action for each decision, and we study the effect on the optimal incentive expressions. In Chapter 6, a discussion that gives insights to organizational designers on measuring organizational parameters relevant to the model is presented, and future research potential is discussed in conclusion.

Chapter 2

RELATED WORK

Since the aim of this work is to contribute to the hierarchical section of Multiscale Decision-Making, a survey of hierarchical decision-making literature is presented below.
A classification of hierarchical agent interactions is presented by Schneeweiss (2003) where constructional and organizational systems are defined as the primary classes of distributed decision- making. Constructional systems refer to those with only one decision maker at each level, where information symmetry prevails. Organizational systems, in contrast, are characterized by information asymmetry with decision makers in leadership roles, including time-based decision hierarchies such as operational and tactical decisions.

From an economics perspective, Williamson (1967) proposed a mathematical model to identify the optimal number of hierarchical levels in an organization. The input parameters include the number of employees a supervisor can handle effectively (span of control), fraction of work done by a subordinate that contributes to the objectives of the superior (compliance parameter), wage of employees at each level, among others, and the dependency of the optimal number of hierarchical levels on these parameters are discussed. While the author discussed structuring organizations, quantitative methods to improve degree of compliance among hierarchical levels are not dealt with. Organizational hierarchy is modeled by Prat (1997) and Qian (1994).

Harris, Kriebel et al. (1982) discussed intra-firm resource allocation among various division managers. The authors developed a model that makes a trade-off between information asymmetry at different levels, and potential conflict of objective between higher-level and lower-level decision makers, to suggest an optimal resource allocation among departments in an organization. The authors, however, did not discuss how objectives can be aligned, and incentives shared optimally within the departments, among hierarchies. Harris and Raviv (2002) also discussed the cost minimal organizational design, among matrix organizations, functional hierarchies, divisional hierarchies or flat hierarchies, based on the characteristics of activities and managerial costs of agents.

Vroom (2006) studied the interaction between remuneration structures in an organization and competitiveness of the organization, concluding that similar organizational structures promote increased rivalry among firms. Fershtman and Judd (1987), in their seminal paper on economic incentives in an owner-manager organization facing competition from another similar firm, discussed a two-stage game. In the first stage, the owners offered an incentive contract to the managers in the presence of demand uncertainty. In the second stage, the managers had full information about the realized demand as well as the incentive contracts of the managers of the other firm, and they played an oligopoly game to maximize their expected profit. The authors did not discuss a scenario when both the owner and manager have to act simultaneously under outcome uncertainty. The authors view the owner's role as only that of offering an incentive contract, and did not discuss the interaction between the owner and manager when the owner makes organizational decisions whose outcome is affected by the manager's actions. Knott (2001) compared hierarchy in organizations to market design and its impact on organizational performance.

Deng and Papadimitriou (1999) discussed hierarchical decision-making scenarios in organizations, where conflicting objectives exist between agents at different hierarchies. The authors suggested a mathematical programming approach to decision-making, where each rational agent intends to maximize its own objective function. The agents engage in game-theoretic reasoning process, where the top-most agent moves first, and the other agents follow, in choosing their strategies, which aligned with the most optimal Nash equilibrium of the game. The authors derived a set of conditions for which the firm is considered efficient. However, the authors did not consider stochastic decision outcomes, or attempt to provide incentives at each stage to enable cooperation among agents.

Principal-agent theory models a range of problems with two hierarchically interacting agents, with incomplete and asymmetric information. In Milgrom and Roberts (1992), an overview of the principal-agent theory is given. The principal is the superior decision maker who offers a contract to make the lower-level decision maker, known as the agent, which is designed to motivate the agent to perform work in interest of the principal. The agent involves in a game-theoretic reasoning process to select the option with the highest expected utility.

The MSDM model describes agent interaction that differs from the principal-agent problem in the following critical ways: The superior agent does not hire the lower-level agent as the organizational structure has been established by an external organizational designer. Additionally, the superior agent cannot impose a penalty on the subordinate agent.

Holmstrom and Milgrom (1991) discussed why employment is better than contracting using a linear principal-agent model developed in Holmstrom and Milgrom (1987). Grossman and Hart (1983) discussed a model where the agent needs to solve a convex linear program to find the optimal action. Rogerson (1985) compared the approaches of considering utility as a stationary point as opposed to the first order solution of utility maximization, where the author argues that the former is a better approach. Jewitt (1988), on the other hand, support the first order solution to be the better approach.

To motivate the healthcare application, existing performance measures for physicians in large hospitals were studied. Lanier, Roland et al. (2003) presented a comprehensive review of existing initiatives to measure physician performance in the US, the UK and the Netherlands.Werner and Bradlow (2006) discussed the relationship between the hospital compare performance measures used by the CMS (Centers for Medicare and Medicaid Services), and the risk adjusted mortality rates of hospitals. Chen, Yamauchi et al. (2006) discussed the use of BSC (Balanced Scorecards) to measure the performance of hospitals in Japan and China. The literature does not seem to address patient satisfaction as a performance metric for physicians.

Chapter 3

TWO-AGENT INTERACTION

3.1 An Example

With the Patient Protection and Affordable Care Act (PPACA) being passed in March 2010, the compensation structure of payment to hospitals from the Center for Medicare and Medicaid Services (CMS) has changed significantly. The PPACA has introduced the quality of care provided to the patients as additional criteria for reimbursement, instead of just the quantity of care. As a result, hospital profits are increasingly dependent on patient satisfaction.

Patient satisfaction is impacted by the overall experience in the hospital, and hence requires all the levels of employees in a hospital to work in unison towards ensuring that the patient is satisfied. Hence, studying the various levels in a hospital hierarchy, and the interactions between each level is required. Decision makers in hospitals are hierarchically structured as shown in Figure 1.

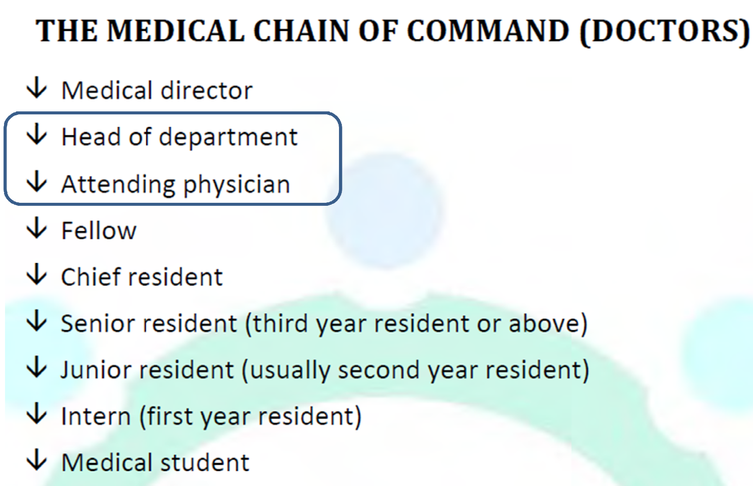

Figure 1: Hierarchy of Agents in a Hospital (Hallisy and Haskel 2009)

The medical director is interested in improving the patient satisfaction levels for the health check-up packages that the hospital offers, in order to increase the profitability of the program. Health check-up packages usually consist of a series of health screening tests, during which patients interact with the physicians and nurses to understand their current wellness, and measures to be taken to improve it. Each department head is allocated with the task of setting up training and incentive programs to ensure that the end goal of improving patient satisfaction is met. We consider the interaction between the head of the department and the attending physician to motivate our two-agent model.

In order to improve the patient satisfaction and associated profits, the head of the department decides to implement a nurse training program, with special focus on health check-up related patient care. During the health check-up program, the attending physician can decide to conduct an *in-depth physical examination*, spending more time per patient, explaining in detail the various aspects of the patient's health, or a *quick physical examination*, spending lesser time and providing a superficial overview of the patient's wellness. The patient's perception of satisfaction is affected by the experience with the physician.

The performance of the physicians employed by the hospital is mainly evaluated by their percent compliance to medical procedures, morbidity and mortality reviews and peer reviews (Lanier, Roland et al. 2003). These measures take into consideration response to inpatients which have higher medical and legal consequences, but do not cover their performance in health check-up programs as the consequences are not as critical. A physician is scheduled to conduct a fixed number of health check-ups in a day, irrespective of which type of service the physician offers, as the actual service offered can be measured only when the outcome of patient satisfaction levels has been realized. This eliminates the possibility of making increased profits by offering the quicker service and increasing the volume of patients per physician.

Physicians, who are not motivated purely by ensuring that their patients are satisfied, may prefer to conduct a quick physical examination, as it most likely involves spending lesser time with the patient. For the purpose of this example, we make the assumption that the physicians considered seek to spend minimal time with patients. The probability that the head of the department achieves his goal of improved patient satisfaction (profits) increases when the physician chooses to conduct an in-depth physical examination. This leads to a conflict of interest between the head of the department and the physician.

As a result, the head of the department is willing to share a portion of the additional profits received as a consequence of high patient satisfaction with the physician, to motivate the physician to switch to an initially less favorable action. One form of incentive offered could be in the form of medical liability insurance that the hospital offers the physician, as the existing rewards are not sufficient to motivate the physicians to behave in favor of the organization. The interaction is described in Figure 2.

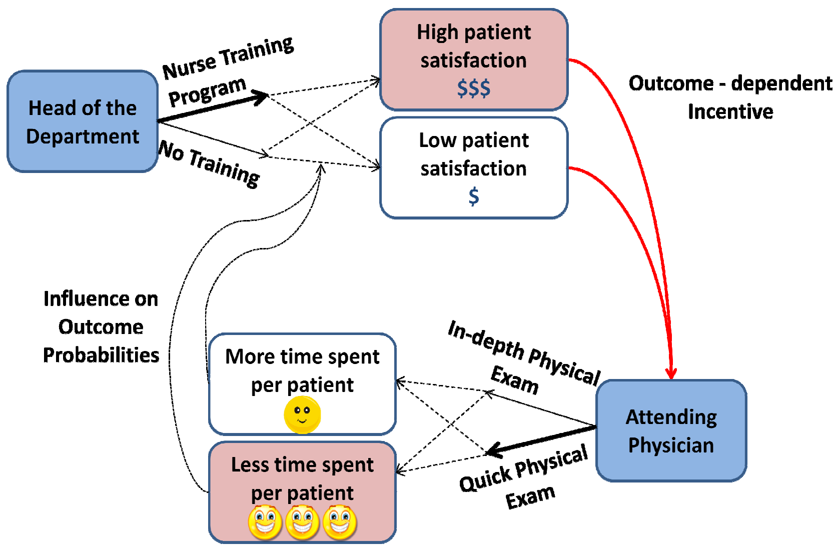

Figure 2: Agent interaction in the Health Check-up Scenario

The timeline of the decisions made by the agents is represented in Figure 3. The head of the department communicates to the attending physician the portion of outcome-dependent reward he/she is willing to share. The head of the department then chooses an action. Since we assume that the head always acts in favor of the organization, he/she always chooses to conduct the nurse training program, as the training program will ensure a higher chance of reaching the desired outcome of high patient satisfaction. The attending physician is aware of this information, and hence can take an action *simultaneously* or *sequentially* as shown in Figure 3. The outcome for the physician's action, which is the actual time spent with the patient, is realized just before the head's action outcome, which is high or low patient satisfaction. The patient satisfaction level is obtained using surveys conducted at the end of the health check-up program. Once the profit or reward corresponding to the patient satisfaction is obtained by the head, he/she shares the pre-determined portion with the attending physician.

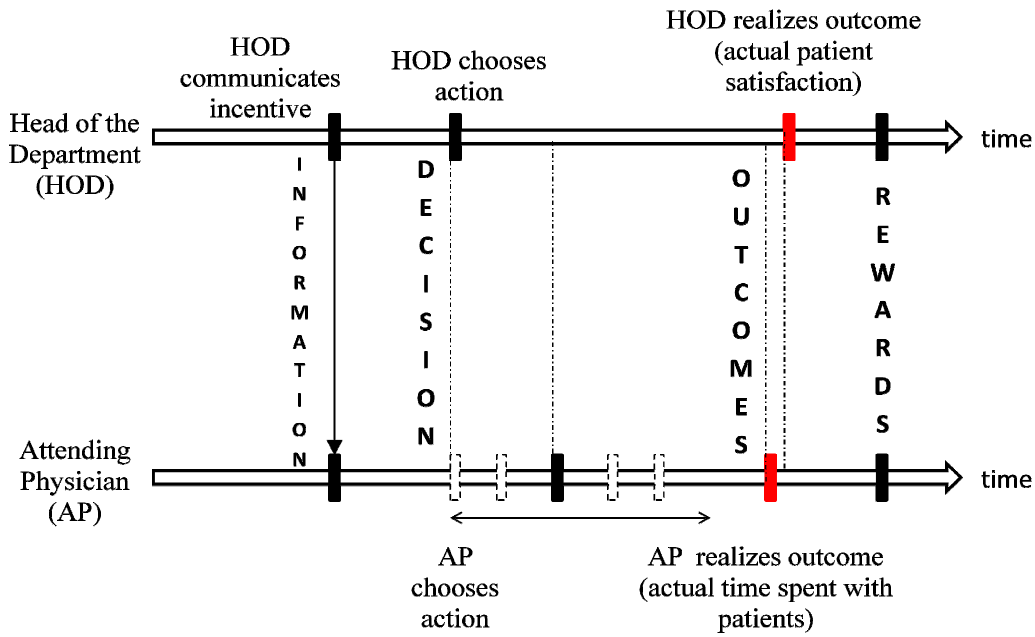

Figure 3: Timeline of Decisions made by Agents

The MSDM model establishes the optimal incentive that should be offered by the head of the department, so that the game-theoretic interaction is redefined, and the *Nash Equilibrium* of the *new* game is changed to ensure that both agents favor the decision which corresponds to high patient satisfaction, resolving the conflict of interest. If the incentive offered is insufficient, the lower agent, the attending physician in this example, will not shift decision strategies, as the Nash equilibrium of the new game is same as that of the old game.

The following sections discuss the above example mathematically to establish the optimal incentives to be offered when *N* decision alternatives and outcomes are considered. An application in a manufacturing firm is discussed in Wernz and Deshmukh (2007)

3.2 The Two-Agent, Two-Decision Case

In Wernz (2008), the interaction between two agents in a hierarchical setting has been analyzed. One agent is the supremal agent (SUP) and the other, the infimal agent (INF). Agent SUP shares a portion of its reward with agent INF, to shift INF's preferred strategy in favor of SUP, while agent INF influences the transition probability with which agent SUP reaches its desired state on taking an action. We further explain the interaction below. We consider a single decision epoch interaction, where agents are self-interested expected utility maximizing entities. In the example described above, the head of the department represents agent SUP, and the attending physician represents agent INF.

Agent SUP and INF have the following distinct action spaces:

$$A^{SUP} = \left\{ a_1^{SUP}, a_2^{SUP} \right\}, A^{INF} = \left\{ a_1^{INF}, a_2^{INF} \right\} \tag{1}$$

The corresponding state spaces are represented by the random variable S^{SUP} and S^{INF} which have the following observations:

$$S^{SUP} = \left\{ s_1^{SUP}, s_2^{SUP} \right\},\ S^{INF} = \left\{ s_1^{INF}, s_2^{INF} \right\} \tag{2}$$

When an action is chosen from the action spaces, the state space random variable is realized with an initial transition probability $p\left(s \mid a\right)$, with which an agent moves to the state corresponding to the action, without any influence from other agents. The transition probabilities for agents SUP and INF are given below:

$$\begin{gathered}
p^{SUP}\left(s_1^{SUP} \mid a_1^{SUP}\right) = \alpha_1^{SUP},\ p^{SUP}\left(s_2^{SUP} \mid a_1^{SUP}\right) = 1 - \alpha_1^{SUP} \\
p^{SUP}\left(s_1^{SUP} \mid a_2^{SUP}\right) = 1 - \alpha_2^{SUP},\ p^{SUP}\left(s_2^{SUP} \mid a_2^{SUP}\right) = \alpha_2^{SUP} \\
p^{INF}\left(s_1^{INF} \mid a_1^{INF}\right) = \alpha_1^{INF},\ p^{INF}\left(s_2^{INF} \mid a_1^{INF}\right) = 1 - \alpha_1^{INF} \\
p^{INF}\left(s_1^{INF} \mid a_2^{INF}\right) = 1 - \alpha_1^{INF},\ p^{INF}\left(s_2^{INF} \mid a_2^{INF}\right) = \alpha_2^{INF} \\
0 \le \alpha_i^{SUP}, \alpha_j^{INF} \le 1,\ \forall i, j = 1,2\ .
\end{gathered} \tag{3}$$

We can assume that $\alpha_i^{SUP}, \alpha_j^{INF} > \frac{1}{2}$ to link an action to its corresponding state. In other words, if an agent takes action a_i , the most likely final state is s_i .

On taking a decision, each agent receives a state-dependent reward, represented by:

$$r^{SUP}\left(s_i^{SUP}\right) = \rho_i^{SUP},\ r^{INF}\left(s_j^{INF}\right) = \rho_j^{INF},\ i, j = 1,2\ . \tag{4}$$

The reward matrix can be represented as:

$$R^{SUP} = \begin{pmatrix} \rho_1^{SUP} \\ \rho_2^{SUP} \end{pmatrix}, \qquad R^{INF} = \begin{pmatrix} \rho_1^{INF} \\ \rho_2^{INF} \end{pmatrix} \tag{5}$$

To represent the interaction mathematically, a *share coefficient* b is defined, to represent the portion of agent SUP's reward that it shares with infimal agent INF. The final reward received by each agent is represented as:

$$r_{final}^{SUP}\left(s_i^{SUP}\right)=\left(1-b\right)\rho_i^{SUP} \tag{6}$$

$$r_{final}^{INF}\left(s_i^{SUP},s_j^{INF}\right)=r^{INF}\left(s_j^{INF}\right)+b\cdot r^{SUP}\left(s_i^{SUP}\right)=\rho_j^{INF}+b\cdot\rho_i^{SUP}\,,\quad i,j=1,2\,. \tag{7}$$

Since the share coefficient represents the proportion shared, it follows that $0\le b\le 1$.

To model the influence that agent INF's final state has on agent SUP's transition probability, an additive influence function has been defined, represented as:

$$f\left(s_i^{SUP}\mid s_j^{INF},a_m^{SUP}\right)=\begin{cases}c, & i=j\\ -c, & i\ne j\end{cases},\; i,j,m=1,2 \tag{8}$$

where $c\ge 0$ is the *change coefficient* with which agent SUP's transition probability is influenced. The change coefficient has the following property:

$$0\le c\le \min\left\{\alpha_1^{SUP},1-\alpha_1^{SUP},1-\alpha_2^{SUP},\alpha_2^{SUP}\right\} \tag{9}$$

which ensure that the transition probabilities do not exceed one.

The additive influence function *increases* the probability of agent SUP reaching the state corresponding to its action by c when agent INF's final state index is the same as agent SUP's; while the function *decreases* that probability by c if agent INF's final state index is different from that of agent SUP's final state. The final transition probabilities, taking the influence function into consideration is represented as:

$$p_{final}^{SUP}\left(s_i^{SUP}\mid s_j^{INF},a_m^{SUP}\right)=p^{SUP}\left(s_i^{SUP}\mid a_m^{SUP}\right)+f\left(s_i^{SUP}\mid s_j^{INF},a_m^{SUP}\right),\; i,j,m=1,2 \tag{10}$$

The final transition probability of agent INF remains unchanged:

$$p_{final}^{INF}\left(s_i^{INF}\mid s_j^{SUP},a_n^{INF}\right)=p^{INF}\left(s_i^{INF}\mid a_n^{INF}\right),\; i,j,n=1,2\,. \tag{11}$$

We assume that full information about rewards, probabilities, share coefficient and change coefficient are available to all agents, which is enforced through organizational mechanisms. Once this information is available, each agent can take the decision which maximizes its own expected reward.

The final expected reward for agent INF is calculated as:

$$E\left(r_{final}^{INF}\mid a_m^{SUP},a_n^{INF}\right)=\sum_{i=1}^{2}\sum_{j=1}^{2}r_{final}^{INF}\left(s_i^{SUP},s_j^{INF}\right)\cdot p^{INF}\left(s_j^{INF}\mid a_n^{INF}\right)\cdot p_{final}^{SUP}\left(s_i^{SUP}\mid s_j^{INF},a_m^{SUP}\right). \tag{12}$$

The final expected reward for agent SUP is calculated as:

$$E\left(r_{final}^{SUP}\mid a_m^{SUP},a_n^{INF}\right)=\sum_{i=1}^{2}\sum_{j=1}^{2}r_{final}^{SUP}\left(s_i^{SUP}\right)\cdot p^{INF}\left(s_j^{INF}\mid a_n^{INF}\right)\cdot p_{final}^{SUP}\left(s_i^{SUP}\mid s_j^{INF},a_m^{SUP}\right). \tag{13}$$

As stated earlier, we assume that agent SUP always takes the decision that is in best interest of the organization. State s_1^{SUP} represents the state that is favored by the organization. Due to the influence function defined earlier, the probability that agent SUP reaches state s_1^{SUP} on taking action a_1^{SUP} is increased, when agent INF's final state is s_1^{INF} .

However, since the agents have conflicting objectives, $\rho_1^{SUP} > \rho_2^{SUP}$ while $\rho_1^{INF} < \rho_2^{INF}$, without reward sharing, agent INF's preferred strategy would be to take action a_2^{INF} , and is most likely to reach state s_2^{INF} . This is not a favorable outcome for agent SUP, as the influence function chosen reduces the probability that agent SUP will reach its desired state $\left(s_1^{SUP}\right)$ when agent INF reaches s_2^{INF} . In order to shift agent INF's preferred strategy from action a_2^{INF} to action a_1^{INF} , agent SUP offers a portion of its reward through share coefficient b to its subordinate agent.

Agent INF will switch its preferred strategy only when the expected reward when it takes action a_1^{INF} is greater than or at least equal to the expected reward when it takes action a_2^{INF} :

$$E\left[r_{final}^{INF} \mid a_1^{SUP}, a_1^{INF}\right] \geq E\left[r_{final}^{INF} \mid a_1^{SUP}, a_2^{INF}\right] \tag{14}$$

On solving for b , we get:

$$b \geq \frac{\rho_2^{INF} - \rho_1^{INF}}{2c\left(\rho_1^{SUP} - \rho_2^{SUP}\right)} \tag{15}$$

Here, we assume that if the expected rewards are equal for both decision strategies, the agent INF acts in favor of the organization.

The above expression represents the optimal share coefficient that would change the game such that the *new* game's Nash equilibrium corresponds to the decision strategies that favors the organization. In other words, the above expression represents the value of the share coefficient b that makes the expected reward received by agent INF on taking action a_1^{INF} greater than, or at least equal to, the expected reward for action a_2^{INF} .

In Wernz (2008), the above closed form expression has been obtained for the share coefficient in the two-agent, two-decision model. The share coefficient obtained is clearly independent of all transition probabilities.

On selecting numeric values for the rewards and transition probabilities in, the indifference curve which represents the share coefficient that makes agent INF indifferent to either action, for the corresponding change coefficient can be plotted as shown:

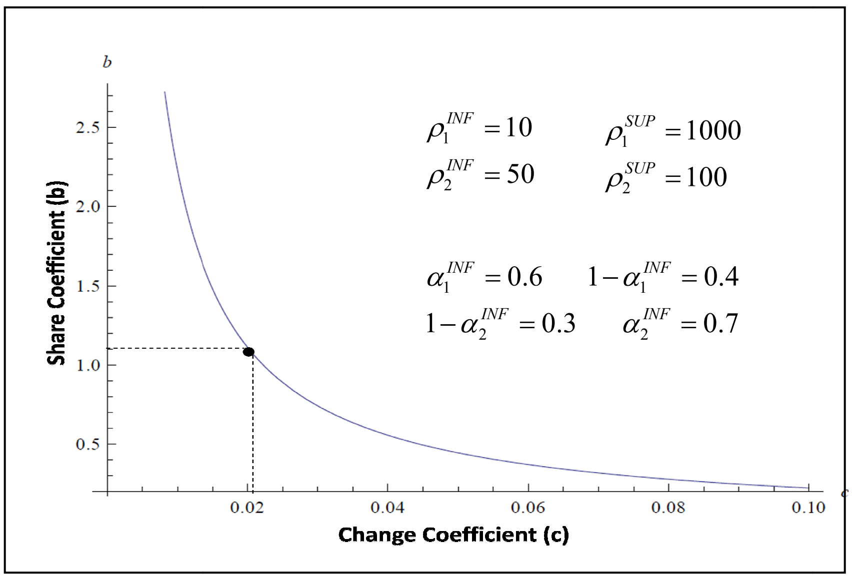

Figure 4: Indifference curve for Two-agent Interaction with two decision outcomes

3.3 The Two-Agent, *N*-Decision Case

We now consider two interacting agents, agent SUP and INF, in a hierarchical setting as described earlier, with *N* actions to choose from, and which have *N* corresponding states as the possible outcomes. The action spaces, state space random variables and the reward matrices for agents SUP and INF remain the same as represented in terms (1), (2) and (5), with the number of indices extended to *N*.

The initial transition probability matrices for agent SUP and agent INF can be represented as:

$$P^{SUP} = \begin{pmatrix} \alpha_{11}^{SUP} & \cdots & \alpha_{1N}^{SUP} \\ \vdots & \ddots & \vdots \\ \alpha_{N1}^{SUP} & \cdots & \alpha_{NN}^{SUP} \end{pmatrix}, \; P^{INF} = \begin{pmatrix} \alpha_{11}^{INF} & \cdots & \alpha_{1N}^{INF} \\ \vdots & \ddots & \vdots \\ \alpha_{N1}^{INF} & \cdots & \alpha_{NN}^{INF} \end{pmatrix} \tag{16}$$

In order to ensure that the final transition probabilities in (10) sum up to one, the general additive influence function is represented as follows:

$$f\left(s_i^{SUP} \mid s_j^{INF}, a_m^{SUP}\right) = \begin{cases} c, & i = j \\ \dfrac{-c}{n-1}, & i \neq j \end{cases}, \; i, j, m = 1, 2, \ldots N \tag{17}$$

with $c > 0$, where n represents the number of outcomes associated with the two decisions that the lower agent transitions between. When $n = 2$, the influence function represented in (8) is obtained. The final transition probabilities are similar to the expressions presented in (10) and (11), with the indices extending to *N*. Here, we make the simplifying assumption that change coefficients for all outcomes are equal.

In order to describe a meaningful problem, the following assumptions can be made without loss of generality:

- The agents have conflicting objectives; hence we can assume that the rewards have the property that $\rho_1^{SUP} > \rho_2^{SUP} > \ldots.\rho_n^{SUP}$ while $\rho_1^{INF} < \rho_2^{INF} < \ldots.\rho_n^{INF}$.
- Agent SUP being in state s_1^{SUP} is favorable to the organization.
- To define a link between an action and the particular state that the agent intends to reach when taking the action, we can assume that $\alpha_{mi}^{SUP} > \frac{1}{n}, \forall m = i$ and $\alpha_{nj}^{INF} > \frac{1}{n}, \forall n = j$, for all $i, j, m, n = 1, 2, \ldots.n$
- $0 \leq c \leq \min\left\{\alpha_{mi}^{SUP}\right\}, \forall m, i = 1, 2, \ldots.n$ is the assumption made to ensure the transition probabilities do not exceed one.

We derive the expression for the share coefficient that causes a shift in agent INF's preferred strategy from action a_n^{INF} to an initially less favorable action a_m^{INF} with agent SUP taking action a_1^{SUP}:

$$E\left[r_{final}^{INF} \mid a_1^{SUP}, a_m^{INF}\right] \geq E\left[r_{final}^{INF} \mid a_1^{SUP}, a_n^{INF}\right], \; m, n = 1, 2 \ldots.N$$

$$\Leftrightarrow \sum_{i=1}^{N}\sum_{j=1}^{N} r_{final}^{INF}\left(s_i^{SUP}, s_j^{INF}\right)\cdot p^{INF}\left(s_j^{INF} \mid a_m^{INF}\right)\cdot p_{final}^{SUP}\left(s_i^{SUP} \mid s_j^{INF}, a_1^{SUP}\right) \geq$$
$$\sum_{i=1}^{N}\sum_{j=1}^{N} r_{final}^{INF}\left(s_i^{SUP}, s_j^{INF}\right)\cdot p^{INF}\left(s_j^{INF} \mid a_n^{INF}\right)\cdot p_{final}^{SUP}\left(s_i^{SUP} \mid s_j^{INF}, a_1^{SUP}\right) \tag{18}$$

By solving the expressions for the share coefficient for different values of *N*, we get:

$$b \geq -\left(\frac{n-1}{nc}\right)\left(\frac{\sum_{i=1}^{N}\rho_i^{INF}\left(\alpha_{mi}^{INF}-\alpha_{ni}^{INF}\right)}{\sum_{i=1}^{N}\rho_i^{SUP}\left(\alpha_{mi}^{INF}-\alpha_{ni}^{INF}\right)}\right) \tag{19}$$

$$b \geq \left(\frac{n-1}{nc}\right)\left(\frac{\rho_n^{INF}\left(\alpha_{nn}^{INF}-\alpha_{mn}^{INF}\right)-\left(\sum_{i=1}^{n-1}\rho_i^{INF}\left(\alpha_{mi}^{INF}-\alpha_{ni}^{INF}\right)\right)}{\sum_{i=1}^{n}\rho_i^{SUP}\left(\alpha_{mi}^{INF}-\alpha_{ni}^{INF}\right)}\right) \tag{20}$$

The above expression represents the optimal share coefficient for a two-agent interaction with *N* decision alternatives and outcomes, when the agent shifts strategy from action a_n^{INF} to action a_m^{INF}. The share coefficient is dependent on the transition probabilities of agent INF, but independent of the transition probabilities of agent SUP, thereby the same property observed in the two-agent two-decision case is not observed here. In expression (20), the terms are rewritten such that they are positive, as $\alpha_{nn}^{INF} > \alpha_{mn}^{INF}$, with ρ_n^{INF} being the largest of all the rewards, due to the conflicting objectives. The first term in the numerator must be large enough to make the share coefficient positive, thereby making interaction feasible.

Since the expression can be used to move from the organization's least preferred action n to any action m, the cost of improving co-operation to the most preferred action, namely, when $m = 1$, can be evaluated, against the cost of moving to other intermediate actions. The organizational designer can use this information to determine the level of co-operation that is affordable.

An important observation in (19) is that the transition probabilities that appear in the expression are only those that describe agent INF's transition to the *outcomes* associated with the two decisions of interest – agent INF's originally preferred strategy and the organization's preferred strategy. Hence, it is only the number of outcomes available to these decision strategies that really affect the optimal share coefficient, and not the actual number of decision alternatives.

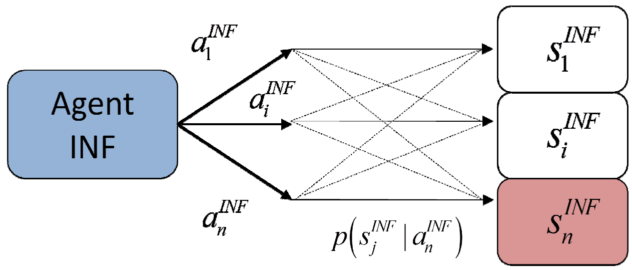

Figure 5: Graph that represents actions and associated outcomes

In Figure 4, the connectivity of the graph that represents the link between actions and outcomes determines which transition probabilities appear in expression (19). Hence we conclude that even when *N* decision alternatives are present, only the number of possible decision outcomes of the two decisions of interest determine the complexity of the optimal share coefficient expression.

3.4 The Two-Decision Case - A Special Case of the *N*-Decision Case

To understand why the two-decision, two-outcome case is independent of all transition probabilities, we substitute $n = 2$ in expression (20):

$$b \geq \left(\frac{2-1}{2c}\right)\left(\frac{\rho_2^{INF}\left(\alpha_{22}^{INF}-\alpha_{12}^{INF}\right)-\left(\rho_1^{INF}\left(\alpha_{11}^{INF}-\alpha_{21}^{INF}\right)\right)}{\rho_1^{SUP}\left(\alpha_{11}^{INF}-\alpha_{21}^{INF}\right)+\rho_2^{SUP}\left(\alpha_{12}^{INF}-\alpha_{22}^{INF}\right)}\right)$$

Using the notation used in Wernz (2008) for the two-agent two-decision model:

$$P^{INF} = \begin{pmatrix} \alpha_{11}^{INF} & \alpha_{12}^{INF} \\ \alpha_{21}^{INF} & \alpha_{22}^{INF} \end{pmatrix} = \begin{pmatrix} \alpha_1^{INF} & 1-\alpha_1^{INF} \\ 1-\alpha_2^{INF} & \alpha_2^{INF} \end{pmatrix}$$

Substituting, we get:

$$b \geq \frac{1}{2c}\left(\frac{\left(1-\alpha_1^{INF}-\alpha_2^{INF}\right)\left(\rho_2^{INF}-\rho_1^{INF}\right)}{\left(1-\alpha_1^{INF}-\alpha_2^{INF}\right)\left(\rho_1^{SUP}-\rho_2^{SUP}\right)}\right)$$

$$b \geq \frac{\left(\rho_2^{INF} - \rho_1^{INF}\right)}{2c\left(\rho_1^{SUP} - \rho_2^{SUP}\right)} \tag{21}$$

The above expression is the same as expression (15). It can be seen that the two-decision case with each decision having two possible is a special case of the *N* decision, *N* outcome case, where the transition probabilities factor out of the expression, thereby making the share coefficient independent of the probabilities. From the above result, we infer that organizational designers benefit from restricting the number of decision *outcomes* to two, for the two decisions of interest, whenever possible, due to the property of independence from transition probabilities.

It is important to note that even when there are *N* decision alternatives, if the two decisions of interest – agent INF's originally preferred strategy and the organization's preferred strategy result in only two outcomes, then the optimal share coefficient is independent of all transition probabilities.

Chapter 4

THREE-AGENT INTERACTION

4.1 The Three-Agent, Two-Decision Case

An MSDM model for three-agent interaction has been studied in Wernz and Henry (2009), with each agent having two decision alternatives to choose from, resulted in two associated outcomes. We extend the model to study the interaction among three agents with each agent having *N* decision alternatives to choose from, with *N* associated outcomes. The model presented in the above paper is explained below, to form the framework for our extension.

The three interacting agents are represented as $A1$, $A2$ and $A3$, highest to lowest in hierarchy, in the model. All three agents take a decision simultaneously, in a single decision epoch. In the model presented in Wernz and Henry (2009), each agent has two decision alternatives to choose from, their action spaces are represented as:

$$A^{Ax} = \left\{a_1^{Ax}, a_2^{Ax}\right\},\ x = 1,2,3 \tag{22}$$

The state space random variable has the following realizations:

$$S^{Ax} = \left\{s_1^{Ax}, s_2^{Ax}\right\},\ x = 1,2,3 \tag{23}$$

On taking a decision, $p\left(s \mid a\right)$ is the initial transition probability with which an agent moves to the state corresponding to the action, without considering the influence from other agents.

$$p^{Ax}\left(s_1^{Ax} \mid a_1^{Ax}\right) = \alpha_1^{Ax},\ p^{Ax}\left(s_2^{Ax} \mid a_1^{Ax}\right) = 1 - \alpha_1^{Ax}$$
$$p^{Ax}\left(s_1^{Ax} \mid a_2^{Ax}\right) = 1 - \alpha_2^{Ax},\ p^{Ax}\left(s_2^{Ax} \mid a_2^{Ax}\right) = \alpha_2^{Ax} \tag{24}$$

We can assume that $\alpha_i^{Ax} > \frac{1}{2}$ to link an action to its corresponding state (if an agent *Ax* takes action a_i^{Ax}, the most likely final state is s_i^{Ax}).

Each agent receives a reward depending on the final state to which the agent moves on receiving the reward.

$$r^{Ax}\left(s_i^{Ax}\right) = \rho_i^{Ax},\ \forall i = 1,2; \forall x = 1,2,3 \tag{25}$$

The reward matrices can be represented as:

$$R^{Ax} = \begin{pmatrix} \rho_1^{Ax} \\ \rho_2^{Ax} \end{pmatrix} \tag{26}$$

The model describes the interaction among agents through the influence that agents have on each others' rewards and transition probabilities. Each agent shares a portion of its reward with its immediate subordinate (the agent exactly one level below). As a result, the topmost agent indirectly shares a portion of its reward with the agent at the lowest level, through the middle agent. The final state each agent moves to influences the transition probabilities of its superior agents, and hence their expected reward when taking a particular action. The interaction has been represented using a dependency graph in Wernz and Henry (2009).

Mathematically, the portion of reward shared by an agent Ax with its immediate subordinate, represented as $b_x, x = 1,2$, is known as the *share coefficient* that agent Ax offers its immediate subordinate. Since agent $A1$ shares a portion b_1 of its reward with agent $A1$, the final reward it receives is:

$$r_{final}^{A1}\left(s_i^{A1}\right) = r^{A1}\left(s_i^{A1}\right)\left(1-b_1\right) \tag{27}$$

Agent $A1$ receives a portion of agent $A1$'s reward, but it also shares a portion of its net reward with agent $A3$, its immediate subordinate. The final reward agent $A2$ receives can be represented as:

$$r_{final}^{A2}\left(s_i^{A1}, s_j^{A2}\right) = \left(r^{A2}\left(s_j^{A2}\right) + b_1 \cdot r^{A1}\left(s_i^{A1}\right)\right) \cdot \left(1-b_1\right) \tag{28}$$

Agent $A3$ receives a share b_2 of agent $A2$'s reward. Its final reward is mathematically represented as:

$$r_{final}^{A3}\left(s_i^{A1}, s_j^{A2}, s_k^{A3}\right) = r^{A3}\left(s_k^{A3}\right) + b_2 \cdot r_{final}^{A2}\left(s_i^{A1}, s_j^{A2}\right) = r^{A3}\left(s_k^{A3}\right) + b_2 \cdot r^{A2}\left(s_j^{A2}\right) + b_1 \cdot b_2 \cdot r^{A1}\left(s_i^{A1}\right)$$

With $0 \le b_1, b_2 \le 1$ (29)

To model the influence exerted by an agent on its superior's transition probability, influence functions f_x, $x = 1,2$ for agents $A1$ and $A2$ are defined:

$$f_1\left(s_i^{A1}, s_j^{A2}\right) = \begin{cases} c_1, & i = j \\ -c_1, & i \ne j \end{cases}, \quad f_2\left(s_j^{A2}, s_k^{A3}\right) = \begin{cases} c_2, & j = k \\ -c_2, & j \ne k \end{cases} \tag{30}$$

c_x, $x = 1,2$ is known as the *change coefficient* with which agent Ax influences its superior agents' transition probabilities, and describes the extent of influence the subordinate agents have on its superiors.

Since $A3$ is the bottom most agent, its transition probabilities are not influenced by any agent. However, agent $A2$'s transition probability is influenced by agent $A3$'s final state, and is characterized through the influence function f_2 :

$$p_{final}^{A2}\left(s_j^{A2} \mid a_n^{A2}, s_k^{A3}\right) = p^{A2}\left(s_j^{A2} \mid a_n^{A2}\right) + f_2\left(s_j^{A2}, s_k^{A3}\right) \tag{31}$$

The final transition probability of agent $A1$, influenced by agent $A2$ directly and agent $A3$ indirectly, is represented as:

$$p_{final}^{A1}\left(s_i^{A1} \mid a_n^{A1}, s_j^{A2}, s_k^{A3}\right) = p^{A1}\left(s_i^{A1} \mid a_n^{A1}\right) + f_1\left(s_i^{A1}, s_j^{A2}\right) \cdot \left(1 + f_2\left(s_j^{A2}, s_k^{A3}\right)\right) \tag{32}$$

Since the transitions probabilities cannot exceed one,

$$0 < c_2 \leq \min\left\{\alpha_1^{A2}, \alpha_2^{A2}, 1-\alpha_1^{A2}, 1-\alpha_2^{A2}\right\},\ 0 < c_1\left(1+c_2\right) \leq \min\left\{\alpha_1^{A1}, \alpha_2^{A1}, 1-\alpha_1^{A1}, 1-\alpha_2^{A1}\right\}$$

As in the two-agent interaction discussed in the previous section, we assume that the topmost agent $A1$ always takes a decision to favor the organization, to reach state s_1^{A1}, which is the state the organization most prefers. We assume that without considering the incentives that are given by the superior agents, both agents $A2$ and $A3$ prefer states s_2^{A2} and s_2^{A3} respectively; and hence prefer corresponding actions a_2^{A2} and a_2^{A3} which are most likely to result in the outcome. In other words, the rewards have the properties that: $\rho_1^{A1} > \rho_2^{A1}$ while $\rho_1^{A2} < \rho_2^{A2}$, $\rho_1^{A3} < \rho_2^{A3}$

Once the share coefficients and change coefficients have been established, the decision process begins, with each self interested expected utility maximizing agent choosing that decision alternative which maximizes its expected reward. In order to switch the Nash equilibria of agents $A2$ and $A3$ to a decision choice that favors the organization, the share coefficients need to be sufficiently high.

To find the optimal share coefficient that agent $A1$ offers $A2$, we need to determine the value of b_1 that causes agent A2 to shift its best response strategy from a_2^{A2} to a_1^{A2}, which occurs when the expected rewards for the latter strategy is greater than that of the former. When the expected rewards for both strategies are equal, agents pick the strategy that favors the organization.

To find the optimal b_1 we solve:

$$E\left[r_{final}^{A2} \mid a_1^{A1}, a_1^{A2}, a_o^{A3}\right] \geq E\left[r_{final}^{A2} \mid a_1^{A1}, a_2^{A2}, a_o^{A3}\right]$$

$$\Leftrightarrow \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} r_{final}^{A2}\left(s_i^{A1}, s_j^{A2}\right) \cdot p_{final}^{A1}\left(s_i^{A1} \mid a_1^{A1}, s_j^{A2}, s_k^{A3}\right) \cdot p_{final}^{A2}\left(s_j^{A2} \mid a_1^{A2}, s_k^{A3}\right) \cdot p^{A3}\left(s_k^{A3} \mid a_o^{A3}\right) \geq$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} r_{final}^{A2}\left(s_i^{A1}, s_j^{A2}\right) \cdot p_{final}^{A1}\left(s_i^{A1} \mid a_1^{A1}, s_j^{A2}, s_k^{A3}\right) \cdot p_{final}^{A2}\left(s_j^{A2} \mid a_2^{A2}, s_k^{A3}\right) \cdot p^{A3}\left(s_k^{A3} \mid a_o^{A3}\right)$$

$$b_1 \geq \frac{\left(\rho_2^{A2} - \rho_1^{A2}\right)}{2c_1\left(\rho_1^{A1} - \rho_2^{A1}\right)} \tag{33}$$

Two important inferences are made from this result: Firstly, the share coefficient b_1 is independent of all transition probabilities. Secondly, b_1 is indifferent to the decision made by agent $A3$.

To find the optimal share coefficient that agent $A2$ offers $A3$, we need to determine the value of b_2 that causes agent $A3$ to shift its best response strategy from a_2^{A3} to a_1^{A3}. Here, agent $A2$ has already been offered the optimal b_1, and has shifted strategy to favors agent $A1$. To find the optimal b_2 we solve:

$$E\left[r_{final}^{A3} \mid a_1^{A1}, a_1^{A2}, a_1^{A3}\right] \geq E\left[r_{final}^{A3} \mid a_1^{A1}, a_1^{A2}, a_2^{A3}\right]$$

$$\Leftrightarrow \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} r_{final}^{A3}\left(s_i^{A1}, s_j^{A2}, s_k^{A3}\right) \cdot p_{final}^{A1}\left(s_i^{A1} \mid a_1^{A1}, s_j^{A2}, s_k^{A3}\right) \cdot p_{final}^{A2}\left(s_j^{A2} \mid a_1^{A2}, s_k^{A3}\right) \cdot p^{A3}\left(s_k^{A3} \mid a_1^{A3}\right) \geq \Leftrightarrow$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} r_{final}^{A3}\left(s_i^{A1}, s_j^{A2}, s_k^{A3}\right) \cdot p_{final}^{A1}\left(s_i^{A1} \mid a_1^{A1}, s_j^{A2}, s_k^{A3}\right) \cdot p_{final}^{A2}\left(s_j^{A2} \mid a_1^{A2}, s_k^{A3}\right) \cdot p^{A3}\left(s_k^{A3} \mid a_2^{A3}\right)$$

$$b_2 \geq \frac{\left(\rho_2^{A3} - \rho_1^{A3}\right)}{2c_2\left(\rho_2^{A2} - \rho_1^{A2} + 3b_1c_1\left(\rho_1^{A1} - \rho_2^{A1}\right)\right)} \tag{34}$$

The above result shows that the share coefficient b_2 is independent of all transition probabilities.

In the following section, we study the three-agent interaction with each agent having *N* decision alternatives to choose from instead of two. We characterize the results, and some critical differences from the two decision alternative case.

4.2 The Three-Agent, *N*-Decision Case

The three interacting agents in the order of their hierarchy from highest to lowest are $A1$, $A2$, $A3$. Each agent can choose from *N* different actions, the action spaces, state space random variable and the reward matrices are similar to those defined in (22), (23), and (26), with the indexes extending to *N*.

The transition probability with which an agent Ax reaches a particular state on choosing an action, without considering the influence of agents lower in hierarchy is given by:

$$p^{Ax}\left(s_j^{Ax} \mid a_i^{Ax}\right) = \alpha_{ij}^{Ax},\ i, j = 1, 2, ...N \tag{35}$$

The initial transition probability matrix of agent x is given by:

$$P^{Ax} = \begin{pmatrix} \alpha_{11}^{Ax} & \cdots & \alpha_{1n}^{Ax} \\ \vdots & \ddots & \vdots \\ \alpha_{n1}^{Ax} & \cdots & \alpha_{nn}^{Ax} \end{pmatrix} \tag{36}$$

The interaction between agents is similar to the structure described in the three-agent two-decision alternative system, with each agent sharing a portion of its state-dependent reward with its immediate subordinate. Each agent's final state affects the transition probability of its superiors, thereby impacting the probability with which they reach their desired state.

In order to ensure that the final transition probabilities sum up to one, we introduce the general influence function as:

$$f_1\left(s_i^{A1}, s_j^{A2}\right) = \begin{cases} c_1, & i = j \\ \dfrac{c_1}{n-1}, & i \neq j \end{cases}, \qquad f_2\left(s_j^{A2}, s_k^{A3}\right) = \begin{cases} c_2, & j = k \\ \dfrac{c_2}{n-1}, & j \neq k \end{cases} \tag{37}$$

where n represents the number of outcomes of the decisions that the agents switch between. The interaction is graphically represented in Figure 6.

The final rewards and final transition probabilities remain the same for the agents $A1$, $A2$, $A3$ as described in the three-agent, two-decision case. As before, the agents have conflicting objectives; hence we can assume that the rewards have the property that $\rho_1^{A1} > \rho_2^{A1} > ....\rho_n^{A1}$ while $\rho_1^{A2} < \rho_2^{A2} < ....\rho_n^{A2}$ and $\rho_1^{A3} < \rho_2^{A3} < ....\rho_n^{A3}$.

To find the share coefficient that agent $A1$ offers $A2$, we need to determine the value of b_1 that causes agent $A2$ to shift its best response strategy from a_n^{A2} to an initially less preferred strategy a_m^{A2}.

To find the optimal b_1 we solve:

$$E\left[r_{final}^{A2} \mid a_1^{A1}, a_m^{A2}, a_q^{A3}\right] \geq E\left[r_{final}^{A2} \mid a_1^{A1}, a_n^{A2}, a_q^{A3}\right]$$

$$\Leftrightarrow \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} r_{final}^{A2}\left(s_i^{A1}, s_j^{A2}\right)\cdot p_{final}^{A1}\left(s_i^{A1} \mid a_1^{A1}, s_j^{A2}, s_k^{A3}\right)\cdot p_{final}^{A2}\left(s_j^{A2} \mid a_m^{A2}, s_k^{A3}\right)\cdot p^{A3}\left(s_k^{A3} \mid a_q^{A3}\right) \geq$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} r_{final}^{A2}\left(s_i^{A1}, s_j^{A2}\right)\cdot p_{final}^{A1}\left(s_i^{A1} \mid a_1^{A1}, s_j^{A2}, s_k^{A3}\right)\cdot p_{final}^{A2}\left(s_j^{A2} \mid a_n^{A2}, s_k^{A3}\right)\cdot p^{A3}\left(s_k^{A3} \mid a_q^{A3}\right)$$

$m, n, q = 1, 2, \ldots. N$

$$b_1 = \frac{(n-1)^2 \sum_{i=1}^{n}\left(\alpha_{mi}^{A2} - \alpha_{ni}^{A2}\right)\rho_i^{A2}}{c_1 \sum_{i=1}^{n}\left(\alpha_{mi}^{A2} - \alpha_{ni}^{A2}\right)\left(\left(c_2\left(n\alpha_{mi}^{A3} - 1\right) + n - 1\right)\left(\sum_{j=1}^{n}\rho_j^{A1} - n\rho_j^{A1}\right)\right)} \tag{38}$$

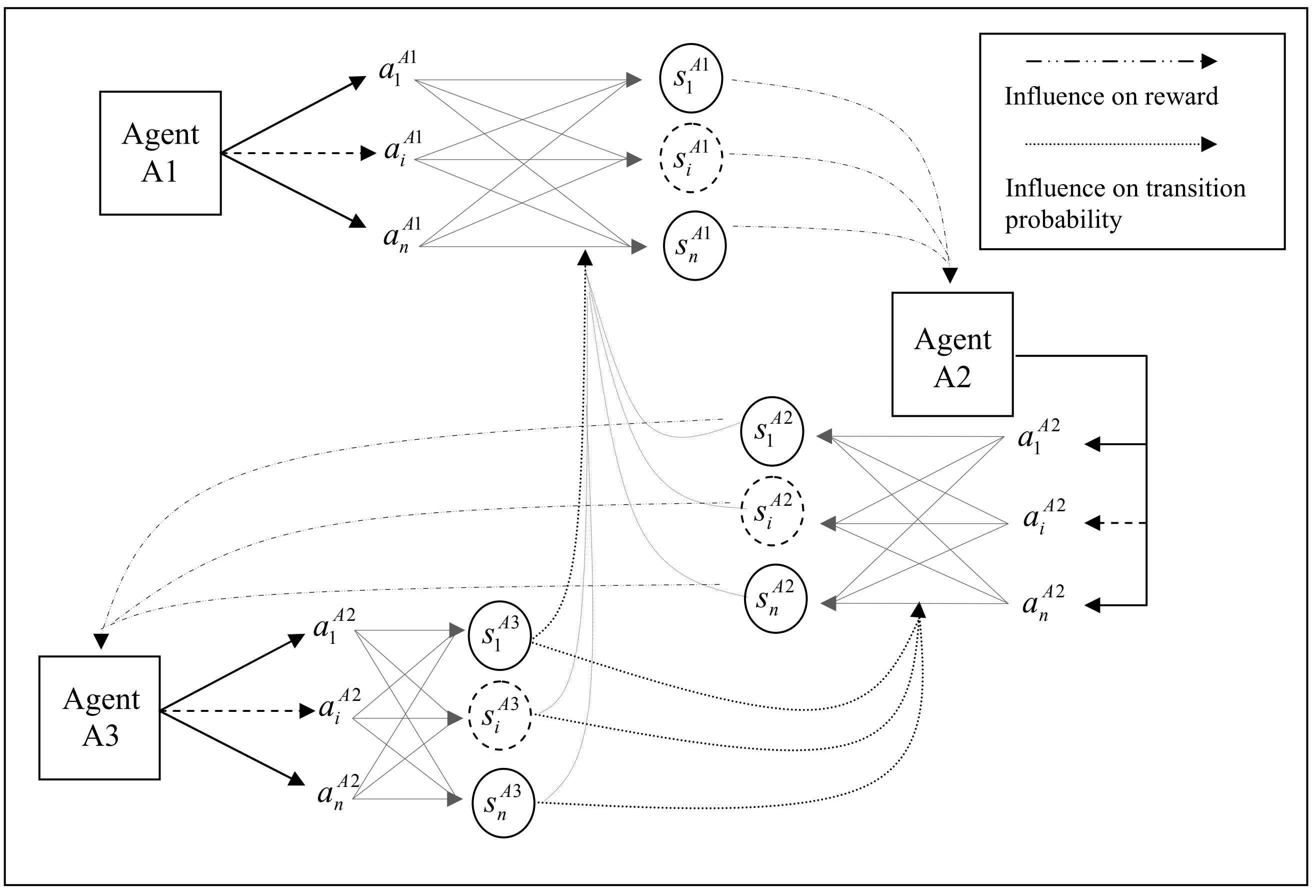

Figure 6: Graphical Representation of Three-agent Interaction

We can see that b_1 is dependent on the transition probabilities of agent $A2$ and agent $A3$. Hence, b_1 also is sensitive to the decision that agent $A3$ makes.

To find the share coefficient that agent $A2$ offers $A3$, we need to determine the value of b_2 that causes agent $A3$ to shift its best response strategy from a_n^{A3} to an initially less preferred strategy a_m^{A3}.

$$E\left[r_{final}^{A3} \mid a_1^{A1}, a_q^{A2}, a_m^{A3}\right] \geq E\left[r_{final}^{A3} \mid a_1^{A1}, a_q^{A2}, a_n^{A3}\right]$$

$$\begin{aligned}&\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} r_{final}^{A3}\left(s_i^{A1}, s_j^{A2}, s_k^{A3}\right)\cdot p_{final}^{A1}\left(s_i^{A1} \mid a_1^{A1}, s_j^{A2}, s_k^{A3}\right)\cdot p_{final}^{A2}\left(s_j^{A2} \mid a_q^{A2}, s_k^{A3}\right)\cdot p^{A3}\left(s_k^{A3} \mid a_m^{A3}\right) \geq \\ &\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n} r_{final}^{A3}\left(s_i^{A1}, s_j^{A2}, s_k^{A3}\right)\cdot p_{final}^{A1}\left(s_i^{A1} \mid a_1^{A1}, s_j^{A2}, s_k^{A3}\right)\cdot p_{final}^{A2}\left(s_j^{A2} \mid a_q^{A2}, s_k^{A3}\right)\cdot p^{A3}\left(s_k^{A3} \mid a_n^{A3}\right)\end{aligned} \tag{39}$$

Obtaining a closed form expression b_2 is cumbersome, as the number of terms contained in the expression is reasonably large. From the Mathematica code used to solve (39) for large values of n, we observed can see that the b_2 is dependent on the transition probabilities of agents $A2$ and $A3$.

As before, in order to describe a meaningful problem, the following assumptions are made without loss of generality:

- To define a link between an action and the particular state that the agent intends to reach when taking the action, we can assume that $\alpha_{ij}^{Ax} > \frac{1}{n}$ when $i = j$ for all $i, j = 1, 2, ...N$, $x = 1, 2, 3$
- $0 \leq c_1, c_2 \leq \min\left\{\alpha_{ij}^{Ax}\right\}, \forall i, j = 1, 2, ...N; x = 1, 2, 3$, to ensure the transition probabilities do not exceed one.

4.3 The Two-Decision Case - A Special Case of the *N*-Decision Case

To understand why the two decision alternative case is independent of all transition probabilities, while the n decision alternative case is not, we substitute $n = 2$ in the b_1 expression:

$$b_1 = \frac{(2-1)^2 \sum_{i=1}^{2}\left(\alpha_{1i}^{A2} - \alpha_{2i}^{A2}\right)\rho_i^{A2}}{c_1 \sum_{i=1}^{2}\left(\alpha_{1i}^{A2} - \alpha_{2i}^{A2}\right)\left(\left(c_2\left(2\alpha_{1i}^{A3} - 1\right) + 2 - 1\right)\left(\sum_{j=1}^{2}\rho_j^{A1} - 2\rho_j^{A1}\right)\right)} \tag{40}$$

Replacing the notation used in the two decision alternative model:

$$P^{Ax} = \begin{pmatrix} \alpha_{11}^{Ax} & \alpha_{12}^{Ax} \\ \alpha_{21}^{Ax} & \alpha_{22}^{Ax} \end{pmatrix} = \begin{pmatrix} \alpha_{1}^{Ax} & 1-\alpha_{1}^{Ax} \\ 1-\alpha_{2}^{Ax} & \alpha_{2}^{Ax} \end{pmatrix}$$

$$b_1 = \frac{\left(1-\alpha_1^{A2}-\alpha_2^{A2}\right)\left(\rho_2^{A2}-\rho_1^{A2}\right)}{2c_1\left(1-\alpha_1^{A2}-\alpha_2^{A2}\right)\left(\rho_1^{A1}-\rho_2^{A1}\right)}$$

$$b_1 = \frac{\left(\rho_2^{A2}-\rho_1^{A2}\right)}{2c_1\left(\rho_1^{A1}-\rho_2^{A1}\right)}$$

Similarly, by substituting $n = 2$ in the generic code developed to solve for b_2 using Mathematica, the following expression was obtained:

$$b_2 = \frac{\left(\rho_2^{A3}-\rho_1^{A3}\right)}{2c_2\left(\rho_2^{A2}-\rho_1^{A2}+3b_1c_1\left(\rho_1^{A1}-\rho_2^{A1}\right)\right)} \tag{41}$$

From the above equations, it is clear that due to the structure of the expectation functions, the probabilities factor out in the two decision alternative case. We can conclude that due to this special property, restricting the number of decision alternatives available to each agent to two reduced the complexity involved with the computation of the share coefficients.

Chapter 5

TWO-AGENT AND THREE-AGENT INTERACTION WITH COST OF ACTION

The *N* decision alternative and outcome model that was presented in the previous sections does not take into consideration the cost of taking an action. The cost associated with taking an action will affect the expected reward, thereby influencing the optimal share coefficient expression.

5.1 Cost of Action in the Two-Agent Interaction Model

The expected rewards represented for agent INF contains a term δ_i^{INF}, which represents the cost of taking action i. The share coefficient is now calculated as:

$$\left(\sum_{i=1}^{N}\sum_{j=1}^{N} r_{final}^{INF}\left(s_i^{SUP}, s_j^{INF}\right)\cdot p^{INF}\left(s_j^{INF} \mid a_m^{INF}\right)\cdot p_{final}^{SUP}\left(s_i^{SUP} \mid s_j^{INF}, a_1^{SUP}\right)\right) - \delta_m^{INF} \geq \left(\sum_{i=1}^{N}\sum_{j=1}^{N} r_{final}^{INF}\left(s_i^{SUP}, s_j^{INF}\right)\cdot p^{INF}\left(s_j^{INF} \mid a_n^{INF}\right)\cdot p_{final}^{SUP}\left(s_i^{SUP} \mid s_j^{INF}, a_1^{SUP}\right)\right) - \delta_n^{INF} \tag{42}$$

On solving, we get

$$b \geq \frac{\left(\delta_m^{INF} - \delta_n^{INF}\right) - \frac{(n-1)}{nc}\sum_{i=1}^{N}\rho_i^{INF}\left(\alpha_{mi}^{INF} - \alpha_{ni}^{INF}\right)}{\sum_{i=1}^{N}\rho_i^{SUP}\left(\alpha_{mi}^{INF} - \alpha_{ni}^{INF}\right)} \tag{43}$$

The above expression represents the optimal share coefficient when the cost of action is considered. When compared to equation (19), the term $\left(\delta_m^{INF} - \delta_n^{INF}\right)$, the cost difference between the two actions appears in the numerator. Hence if the scenario to be modeled involves significant cost of action, the organizational designer should include the cost in the evaluation of the optimal share coefficient.

5.2 Cost of Action in the Three-Agent Interaction Model

In the three-agent interaction model, when the cost of taking an action i for agent Ax, δ_i^{Ax} is considered in the evaluation of the expected reward, the optimal share coefficients are found to be:

$$b_1 = \frac{\left(\delta_n^{A2} - \delta_m^{A2}\right) + (n-1)^2\sum_{i=1}^{n}\left(\alpha_{mi}^{A2} - \alpha_{ni}^{A2}\right)\rho_i^{A2}}{c_1\sum_{i=1}^{n}\left(\alpha_{mi}^{A2} - \alpha_{ni}^{A2}\right)\left(\left(c_2\left(n\alpha_{mi}^{A3} - 1\right) + n - 1\right)\left(\sum_{j=1}^{n}\rho_j^{A1} - n\rho_j^{A1}\right)\right)} \tag{44}$$

In the above expression, the cost difference term appears in the numerator, similar to the two agent case analyzed in (43). For b_2 a similar behavior was observed, with the cost difference term, for the two actions between which the switch takes place, featuring in the numerator for large values of n. Again, obtaining a closed for expression for b_2 for a general case is cumbersome.

Chapter 6

DISCUSSION, CONCLUSION AND FUTURE RESEARCH

6.1 Discussion

We proceed to discuss some of the aspects of the Multiscale Decision-Making (MSDM) model to enable an organizational designer to effectively implement the model. The organizational designer must take care in measuring the organizational parameters, as the input to the model will affect the accuracy of the results. Some aspects of the MSDM model are discussed below.

6.1.1 On the Change Coefficient

The change coefficient defined in the MSDM model captures the influence that a subordinate has on its superiors' transition probabilities. The influence function considered in this work is an additive influence function. The organizational designer needs to investigate the nature of the interaction in the particular scenario being modeled, and may choose an influence function that best fits the application. Alternate influence functions have been discussed in (Wernz 2008). Since the model has been discussed in the function form, the expectation expressions can be solved to compute the optimal share coefficient. To capture the influence, the designer can use simulation techniques by studying historic data available on decisions and outcomes. In the absence of robust data, eliciting information from experts is a reasonable technique to use.

Scaling the data to ensure that the change coefficient is within its feasible limits is critical. The change coefficient should be smaller than the smallest of transition probability values, when additive influence functions are being considered, as shown in (9). In order to define a meaningful model, the final transition probabilities should always sum to one. When alternate influence functions are being considered, the range limits for the change coefficient should ensure that the final transition probability does not exceed one. Accuracy in fixing the change coefficient is critical to obtaining the optimal share coefficient that causes the Nash equilibrium of the new game to align with the organizational goals.

6.1.2 On the Share Coefficient

The share coefficient represents the share of its reward that an agent is willing to share with its subordinates, to obtain co-operation. When the optimal share coefficient is greater than one, the reward structure that exists is insufficient to cause the lower-level agents to shift strategy. Hence, the organizational designer receives insights about the rewards that the most superior agent should be offered, so that the sharing of reward is feasible, thereby ensuring co-operation.

When the share coefficient is negative, it indicates that for the given transition probabilities, interaction is not feasible. On altering the transition probabilities, interaction is feasible once again, for the same reward structure and agent hierarchy. This enables the organizational designer to understand the organization better, and work towards altering transition probabilities by changing the organizational environment, whenever possible.

6.2 Conclusion

The Multiscale Decision-Making (MSDM) model was extended to include N decision alternatives and associated outcomes. The optimal share coefficient expressions were developed for the general case, for two-agent interactions and partially for three-agent interactions.

In the two-agent interaction model, the optimal share coefficient was found to be dependent on the transition probabilities of the infimal agent, when the number of decision alternatives and associated outcomes were increased above two. On further investigation, we concluded that the number of possible decision outcomes for the two decisions of interest –

agent INF's originally preferred strategy and the organization's preferred strategy, affected the transition probabilities that appeared in the optimal share coefficient expression. This suggests to the organizational designer that the number of decision alternatives does not affect the optimal share coefficient but it is only the number of decision outcomes that affects it. Hence, there is benefit to restricting the number of decision outcomes available for the two decision of interest to two, as the optimal share coefficient is independent of transition probabilities.

For the three-agent interaction model, expressions were developed for the optimal share coefficients for agent $A2$, but the expression for the optimal share coefficient for agent $A3$ was cumbersome. It was found that as the number of decision alternatives and associated outcomes were increased beyond two, the optimal share coefficients at each level were dependent on transition probabilities of all the agents superior in hierarchy. As the expression was cumbersome, and useful properties were hard to elicit. Again, there is benefit to restricting the decision outcomes available for the two decisions of interest at *each* level, as the optimal share coefficients are independent of all transition probabilities. The cost of taking a decision was incorporated into the MSDM model, and it was found that the optimal share coefficient is affected by the difference in cost between the two decision alternatives of interest.

It is clearly seen that as the number of agents increase, closed form expressions get less tractable. When the number of hierarchies increases, organizational designers can resort to generating the indifference curve of solving the expectation to identify the optimal share coefficient graphically.

6.3 Future Research

The MSDM model integrates hierarchical and temporal scales of decision-making in an organization. This work contributes to the hierarchical portion of MSDM, with general models for the two-agent and three-agent hierarchical interactions being established. A general model capturing temporal scale interactions would possibly lead to some interesting observations, and help the organizational designer better understand how optimal share coefficients behave when the parameters change with time.

Two-agent and three-agent interactions in the hierarchical scale have been studied. Certain patterns seem to be emerging as the number of agents is increased. A vertical generalization, for an N agent interaction would provide further understanding about the behavior of the optimal share coefficient, and suggest to the designer if there is any benefit in increasing or restricting the number of hierarchical levels. Modeling the influence function as the number of agents grows beyond three is yet to be explored.

The influence function chosen to model interaction significantly impacts the optimal share coefficient and the organizational parameters it depends on. The current model assumes that all non-cooperative outcomes are equally bad. In the pursuit of taking the MSDM model closer to real-world applications, considering influence functions where the change coefficient c is not a constant may be useful to model a wider range of hierarchical interactions. Exploring various influence functions and identifying the properties of the optimal share coefficient would certainly expand the horizon of applications of MSDM.

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