THE CLOSED FORM ERROR EXPRESSION WITH PARSEVAL'S THEOREM

by

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I. INTRODUCTION

The closed form error expression of optimum Wiener's filter was first introduced in Yovit-Jackson's work, "Linear Filter Optimization with Game Theory Considerations," of 1955 [1]. The Y. W. Lee's minimum mean square error (MMSE) expression [2] of the optimum filter actually has the following disadvantages: (1) to factorize all power density spectra before the actual computation of the errors, and (2) to go through practically the whole procedure of finding optimum impulse response before one can evaluate the error.

However, Yovit-Jackson's formula for MMSE enables us to avoid the tedious factorization procedures in evaluating the error of the optimum filter system, and thereby gives us a potent analytic means for the study of optimum filters.

Since this closed form was introduced in 1955, a number of people, Viterbi and Cahn [3], Snyder [4] and Van Trees [5], have applied almost similar techniques to prove the error expression, but no one has succeeded to provide any extension of the error expression for the other cases than the zero delay filters.

The object of this paper is to confirm Yovit-Jackson's formula with Parseval's theorem and discuss a possible extension for some other cases such as delay and prediction filters.

II. THEORETICAL DEVELOPMENT

2.1 General Theory

The figure (1) shows the system under consideration for the closed form error expression.

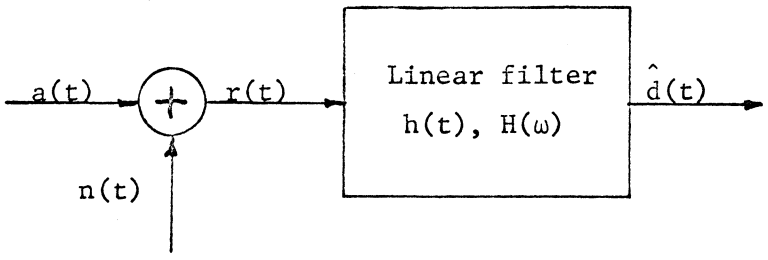

Fig. 1.

It is assumed that noise n(t) is a white Gaussian process with zero mean and whose power spectral height is $N_o/2$ Watts/Hz. It is also assumed that n(t) is orthogonal to the signal a(t).

Referring to Fig. 1, the estimated desired output $\hat{d}(t)$ is given by the convolution integral between the input signal r(t) and the filter impulse response h(t):

$$\hat{d}(t) = \int_{-\infty}^{\infty} h(\tau) \cdot r(t - \tau) \cdot d\tau$$

$$= \int_{-\infty}^{\infty} h(\tau) \cdot a(t - \tau) \cdot d\tau + \int_{-\infty}^{\infty} h(\tau) \cdot n(t - \tau) \cdot d\tau \qquad (1)$$

Using (1) the mean square error σ^2 can be obtained as:

$$\sigma^2 = E\{[\hat{d}(t) - d(t)]^2\}$$

$$= E\{[\int_{-\infty}^{\infty} h(\tau) \cdot r(t-\tau) \cdot d\tau - d(t)]^2\}$$

$$= [\phi_d(\tau) + \phi_{\hat{d}}(\tau) - \phi_{\hat{d}d}(\tau) - \phi_{d\hat{d}}(\tau)]_{\tau = 0}$$

$$= \int_{-\infty}^{\infty} \Phi_d(\omega)d\omega - \int_{-\infty}^{\infty} \Phi_{da}(\omega)H(\omega)d\omega - \int_{-\infty}^{\infty} \Phi_{ad}(\omega)H^*(\omega)d\omega +$$

$$+ \int_{-\infty}^{\infty} \Phi_a(\omega)H(\omega)H^*(\omega)d\omega + \int_{-\infty}^{\infty} \Phi_n(\omega)H(\omega)H^*(\omega)d\omega \qquad (2)$$

where ϕ's denote the auto- or crosscorrelation functions of respective subscripts and Φ's represent the auto- or crosspower spectral densities corresponding to ϕ's.

If we split the optimum filter into two parts as shown in Fig. 2 [5], the optimum filter function $H_o(\omega)$ can be expressed by

$$H_o(\omega) = W(\omega) \cdot \Psi_+(\omega) = \frac{1}{\Phi_r^+(\omega)} \cdot \left[\frac{\Phi_{rd}(\omega)}{\Phi_r^-(\omega)}\right]_+ \qquad (3)$$

where

$$W(\omega) = \frac{1}{\Phi_r^+(\omega)} \qquad (4)$$

$$\Psi_+(\omega) = \left[\frac{\Phi_{rd}(\omega)}{\Phi_r^-(\omega)}\right]_+ \qquad (5)$$

Lee's expression for error [2] is given by

$$\sigma^2 = \phi_d(0) - \frac{1}{2\pi} \int_o^{\infty} \psi^2(\tau)d\tau \qquad (6)$$

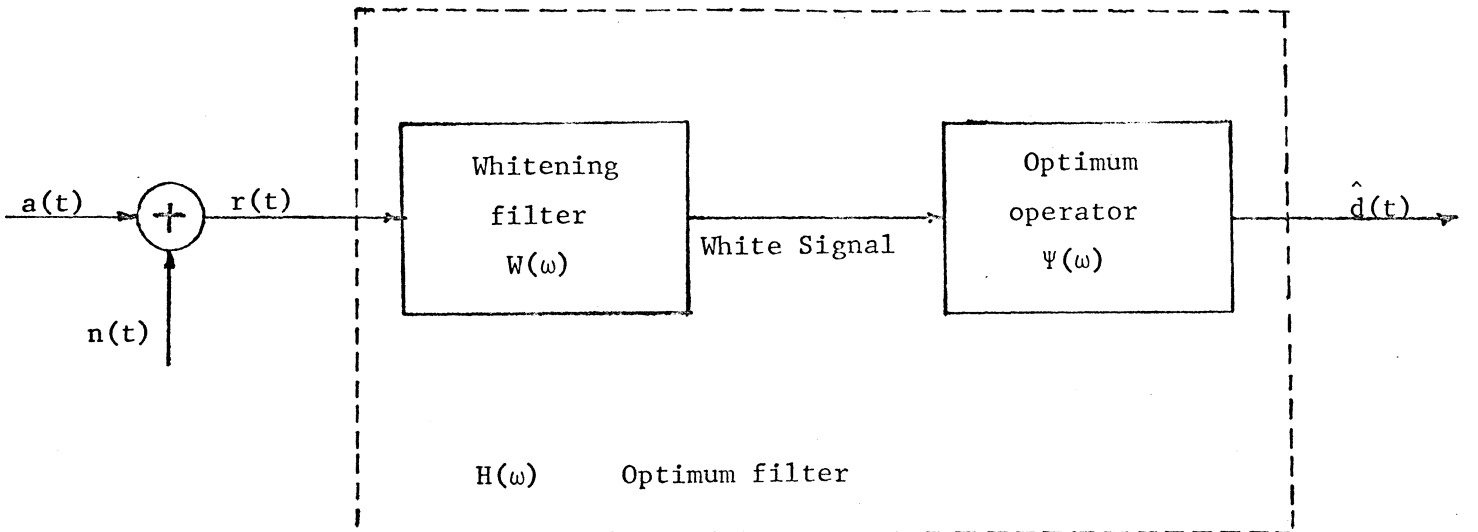
a(t)
r(t)
n(t)
Whitening
filter
W(ω)
White Signal
Optimum
operator
Ψ(ω)
d̂(t)
H(ω) Optimum filter

Figure 2

where

$$\psi(\tau) = F^{-1}[\Psi_{+}(\omega)] \tag{7}$$

Yovit-Jackson [1], Viterbi-Cahn [3], Snyder [4] and Van Trees [5] all started with the equation (2) in order to derive the closed form of the error expression. The equation (2) can be shown as the minimum mean square error (MMSE) by substituting the optimum transfer function $H_o(\omega)$ (3) into (2) for $H(\omega)$ and by applying the assumptions made at the beginning of this chapter.

As the matter of convenience for the direct comparison between previous methods done by several authors and a method to be presented in this paper, the following relations are introduced in the first place.

$$\Phi_n(\omega) = \frac{N_o}{2} \tag{8}$$

$$\sigma^2 = \int_{-\infty}^{\infty} \Phi_d(\omega)d\omega - \int_{-\infty}^{\infty} \Phi_{da}(\omega)H_o(\omega)d\omega - \int_{-\infty}^{\infty} \Phi_{ad}(\omega)H_o^*(\omega)d\omega$$

$$+ \int_{-\infty}^{\infty} \Phi_a(\omega)|H_o(\omega)|^2 d\omega + \int_{-\infty}^{\infty} \frac{N_o}{2}|H_o(\omega)|^2 d\omega \tag{9}$$

Considering $d(t) = a(t)$, equation (9) then becomes

$$\sigma^2 = \int_{-\infty}^{\infty} |1 - H_o(\omega)|^2 \Phi_a(\omega)d\omega + \frac{N_o}{2}\int_{-\infty}^{\infty} |H_o(\omega)|^2 d\omega \tag{10}$$

Now the optimum operator of zero delay filter can be written by the expression [5]

$$\psi_+(\omega) = \Phi_r^+(\omega) - \sqrt{\frac{N_o}{2}} \tag{11}$$

Using (3), the optimum transfer function becomes

$$H_o(\omega) = 1 - \frac{\sqrt{N_o/2}}{\Phi_r^+(\omega)} \tag{12}$$

Consequently, we have

$$|1 - H_o(\omega)|^2 = \frac{N_o/2}{\Phi_r(\omega)}, \tag{13}$$

and

$$|H_o(\omega)|^2 = -|1 - H_o(\omega)|^2 + 2|H_o(\omega)|\cos\phi(\omega) \tag{14}$$

in which

$$\phi(\omega) = \underline{/H_o(\omega)} \tag{15}$$

With the aid of realizability criteria and the relations of (13) and (14), the equation (10) can be shown to be

$$\sigma^2 = \frac{N_o}{2} \int_{-\infty}^{\infty} \ell n\left(\frac{\Phi_r(\omega)}{N_o/2}\right) d\omega, \tag{16}$$

Note that on the derivation of eq. (16), the following definite integral relation has been used:

$$\int_{-\infty}^{\infty} \ell n \frac{x^2 + b^2}{x^2 + a^2}\, dx = 2\pi(b - a) \tag{17}$$

2.2 New Method for Proof

If the signal power spectral density expresses as the rational function such that

$$\Phi_a(\omega) = \frac{p^2 \prod_{i=1}^{m} (\omega^2 + \gamma_i^2)}{\prod_{i=1}^{n} (\omega^2 + \alpha_i^2)} \quad (m < n), \tag{18}$$

where $R_e[\gamma_i] > 0$, $R_e[\alpha_i] > 0$, p = constant;

then we have

$$\Phi_a^+(\omega) = \frac{p \prod_{i=1}^{m} (j\omega + \gamma_i)}{\prod_{i=1}^{n} (j\omega + \alpha_i)} \quad (m < n) \tag{19}$$

$$\Phi_a^-(\omega) = \frac{p \prod_{i=1}^{m} (-j\omega + \gamma_i)}{\prod_{i=1}^{n} (-j\omega + \alpha_i)} \quad (m < n) \tag{20}$$

Hence, the input power density spectrum can also be shown by

$$\Phi_r(\omega) = \Phi_a(\omega) + \Phi_n(\omega)$$

$$= \frac{p^2 \prod_{i=1}^{m} (\omega^2 + \gamma_i^2)}{\prod_{i=1}^{n} (\omega^2 + \alpha_i^2)} + \frac{N_o}{2}$$

$$= \frac{\frac{N_o}{2} \prod_{i=1}^{n} (\omega^2 + \beta_i^2)}{\prod_{i=1}^{n} (\omega^2 + \alpha_i^2)} \qquad R_e[\beta_i] > 0 \tag{21}$$

where β_i are the roots of $\frac{2p^2}{N_o} \prod_{i=1}^{m} (\omega^2 + \gamma_i^2) + \prod_{i=1}^{n} (\omega^2 + \alpha_i^2)$.

Thus, we have

$$\Phi_r^+(\omega) = \frac{\sqrt{\frac{N_o}{2}} \prod_{i=1}^{n} (j\omega + \beta_i)}{\prod_{i=1}^{m} (j\omega + \alpha_i)} \tag{22}$$

and

$$\Phi_r^-(\omega) = \frac{\sqrt{\frac{N_o}{2}} \prod_{i=1}^{n} (-j\omega + \beta_i)}{\prod_{i=1}^{n} (-j\omega + \alpha_i)} \tag{23}$$

Bearing the above equations (18) through (23) in mind, we now return to equations (5) and (7) which define the transfer function of optimum operator (see Fig. 2).

Applying the Parserval's Theorem we get the following relation:

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} \psi^2(\tau) d\tau = \int_{-\infty}^{\infty} |\Psi_+(\omega)|^2 d\omega$$

$$= \int_{-\infty}^{\infty} [\Psi_+(\omega)][\Psi_+(\omega)]^* d\omega \tag{24}$$

Substituting Equation (11) into (24), it becomes

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\psi^2(\tau)d\tau = \int_{-\infty}^{\infty}[\Phi_r^+(\omega) - \sqrt{\frac{N_o}{2}}][\Phi_r^-(\omega) - \sqrt{\frac{N_o}{2}}]d\omega$$

$$= \int_{-\infty}^{\infty}\Phi_r(\omega)d\omega + \int_{-\infty}^{\infty}\frac{N_o}{2}\,d\omega - \sqrt{\frac{N_o}{2}}\int_{-\infty}^{\infty}[\Phi_r^+(\omega) - \Phi_r^-(\omega)]d\omega \qquad (25)$$

Using Equations (22) and (23), it can be further expanded as follows:

$$\frac{1}{2\pi}\int_{-\infty}^{\infty}\psi^2(\tau)d\tau = \int_{-\infty}^{\infty}\Phi_a(\omega)d\omega + \int_{-\infty}^{\infty}N_o d\omega - \frac{N_o}{2}\int_{-\infty}^{\infty}\prod_{i=1}^{n}\frac{(j\omega + \beta_i)}{(j\omega + \alpha_i)}\,d\omega$$

$$- \frac{N_o}{2}\int_{-\infty}^{\infty}\prod_{i=1}^{n}\frac{(-j\omega + \beta_i)}{(-j\omega + \alpha_i)}\,d\omega$$

$$= \int_{-\infty}^{\infty}\Phi_a(\omega)d\omega + \int_{-\infty}^{\infty}N_o d\omega - \frac{N_o}{2}\int_{-\infty}^{\infty}\prod_{i=1}^{n}(1 + \frac{\beta_i - \alpha_i}{j\omega + \alpha_i})d\omega$$

$$- \frac{N_o}{2}\int_{-\infty}^{\infty}\prod_{i=1}^{n}(1 + \frac{\beta_i - \alpha_i}{-j\omega + \alpha_i})d\omega$$

$$= \int_{-\infty}^{\infty}\Phi_\alpha(\omega)d\omega + \int_{-\infty}^{\infty}N_o d\omega - \int_{-\infty}^{\infty}\frac{N_o}{2}[1 + \sum_{i=1}^{n}\frac{\beta_i - \alpha_i}{j\omega + \alpha_i} +$$

$$\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\frac{(\beta_i - \alpha_i)(\beta_j - \alpha_j)}{(j\omega + \alpha_i)(j\omega + \alpha_j)} +$$

$$\sum_{i=1}^{n-2}\sum_{j=i+1}^{n-1}\sum_{k=j+1}^{n}\frac{(\beta_i - \alpha_i)(\beta_j - \alpha_j)(\beta_k - \alpha_k)}{(j\omega + \alpha_i)(j\omega + \alpha_j)(j\omega + \alpha_k)} + \ldots]d\omega$$

$$- \int_{-\infty}^{\infty} \frac{N_o}{2}\left[1 + \sum_{i=1}^{n} \frac{\beta_i - \alpha_i}{-j\omega + \alpha_i} + \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \frac{(\beta_i - \alpha_i)(\beta_j - \alpha_j)}{(-j\omega + \alpha_i)(-j\omega + \alpha_j)} + \ldots\right] d\omega \tag{25a}$$

Since the optimum filter is valid for the time greater than or equal to zero, the contour of these integrals encircles the left-half plane. Therefore, in Eq. (25) the last integral yields only the first term $\int_{-\infty}^{\infty} N_o/2 \, d\omega$ and the third integral will have the first two terms only (see Appendix I). That is,

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} \psi^2(\tau) d\tau = \int_{-\infty}^{\infty} \Phi_a(\omega) d\omega + \int_{-\infty}^{\infty} N_o d\omega - \int_{-\infty}^{\infty} \frac{N_o}{2} d\omega - \int_{-\infty}^{\infty} \frac{N_o}{2} d\omega$$

$$+ \frac{N_o}{2} \sum_{i=1}^{n} (\beta_i - \alpha_i) \cdot 2\pi$$

$$= \int_{-\infty}^{\infty} \Phi_a(\omega) d\omega + \frac{N_o}{2} \sum_{i=1}^{n} \int_{-\infty}^{\infty} \ell n\left(\frac{\omega^2 + \beta_i^2}{\omega^2 + \alpha_i^2}\right) d\omega$$

$$= \int_{-\infty}^{\infty} \Phi_a(\omega) d\omega + \frac{N_o}{2} \int_{-\infty}^{\infty} \ell n\left[\prod_{i=1}^{n} \frac{\omega^2 + \beta_i^2}{\omega^2 + \alpha_i^2}\right] d\omega \tag{25b}$$

Referring to Eq. (21), (25b) becomes immediately

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} \psi^2(\tau) d\tau = \int_{-\infty}^{\infty} \Phi_a(\omega) d\omega + \frac{N_o}{2} \int_{-\infty}^{\infty} \ell n\left[\frac{\Phi_r(\omega)}{N_o/2}\right] d\omega \tag{26}$$

Since

$$\psi(\tau) = F^{-1}[\Psi_+(\omega)], \tag{27}$$

and $$\psi(\tau) = 0 \quad \text{for} \quad \tau < 0$$

(26) becomes

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} \psi^2(\tau) d\tau = \frac{1}{2\pi} \int_{o}^{\infty} \psi^2(\tau) d\tau$$

$$= \int_{-\infty}^{\infty} \Phi_a(\omega) d\omega + \frac{N_o}{2} \int_{-\infty}^{\infty} \ell n[\frac{\Phi_r(\omega)}{N_o/2}] d\omega \qquad (28)$$

Substituting (28) into (6), we have the desired closed form of error expression. That is,

$$\sigma^2 = \frac{N_o}{2} \int_{-\infty}^{\infty} \ell n[\frac{\Phi_r(\omega)}{N_o/2}] d\omega \qquad (29)$$

III. ILLUSTRATIVE EXAMPLE

In this chapter, we shall compare the conventional Y. W. Lee's method with the closed form by an illustrative example to demonstrate the effectiveness of this form.

Let us assume that we have an aperiodic telegraphic signal whose power density spectrum is

$$\Phi_a(\omega) = \frac{p^2}{\omega^2 + \alpha^2} \tag{30}$$

The noise is again assumed to be white with spectral height $N_o/2$, i.e.,

$$\Phi_n(\omega) = \frac{N_o}{2}$$

Then the input power density spectrum becomes

$$\Phi_r(\omega) = \Phi_a(\omega) + \frac{N_o}{2}$$

$$= \frac{p^2}{\omega^2 + \alpha^2} + \frac{N_o}{2}$$

$$= \frac{\frac{N_o}{2}(\omega^2 + \beta^2)}{\omega^2 + \alpha^2} \tag{31}$$

where

$$\beta^2 = \alpha^2 + \frac{2p^2}{N_o} \quad \text{and} \quad \mathrm{Re}[\beta] > 0 \tag{32}$$

By factorizing Equation (31)

$$\Phi_r(\omega) = \sqrt{\frac{N_o}{2}}\,\frac{(j\omega+\beta)}{(j\omega+\alpha)} \cdot \sqrt{\frac{N_o}{2}}\,\frac{(-j\omega+\beta)}{(-j\omega+\alpha)}$$

$$= \Phi_r^{+}(\omega) \cdot \Phi_r^{-}(\omega) \tag{33}$$

Hence, from Equation (11),

$$\Psi_{+}(\omega) = \sqrt{\frac{N_o}{2}}\,\frac{(j\omega+\beta)}{(j\omega+\alpha)} - \sqrt{\frac{N_o}{2}}$$

$$= \sqrt{\frac{N_o}{2}} \cdot \frac{(\beta-\alpha)}{j\omega+\alpha} \tag{34}$$

By taking inverse transform of Eq. (34), we have

$$\psi(\tau) = \sqrt{\frac{N_o}{2}} \int_{-\infty}^{\infty} \frac{\beta-\alpha}{j\omega+\alpha}\, e^{j\omega\tau} d\omega = 2\pi \sqrt{\frac{N_o}{2}}\,(\beta-\alpha)e^{-\tau\alpha} \tag{35}$$

Since $\sigma^2 = \int_{-\infty}^{\infty} \Phi_a(\omega)d\omega - \frac{1}{2\pi}\int_o^{\infty} \psi^2(\tau)d\tau$

Let us compute the last term first using Eq. (35), that is

$$\frac{1}{2\pi}\int_o^{\infty} \psi^2(\tau)d\tau = 2\pi \int_o^{\infty} \frac{N_o}{2}\,(\beta-\alpha)^2 e^{-2\alpha\tau} d\tau$$

$$= \pi N_o(\beta-\alpha)^2/2\alpha \tag{36}$$

The first term becomes:

$$\int_{-\infty}^{\infty} \Phi_a(\omega)d\omega = \int_{-\infty}^{\infty} \frac{p^2}{\omega^2+\alpha^2}\, d\omega$$

$$= \frac{p^2}{\alpha} [\tan^{-1} \frac{\omega}{\alpha}]_{-\infty}^{\infty}$$

$$= \frac{\pi p^2}{\alpha} \tag{37}$$

Substituting Eq. (36) and (37), the MMSE becomes:

$$\sigma^2 = \frac{\pi p^2}{\alpha} - \frac{\pi N_o (\beta - \alpha)^2}{2\alpha} \tag{38}$$

Since from Eq. (32) we have

$$p^2 = \frac{N_o}{2} (\beta^2 - \alpha^2)$$

Equation (38) can be further reduced as

$$\sigma^2 = \frac{\pi N_o (\beta^2 - \alpha^2) - \pi N_o (\beta - \alpha)^2}{2\alpha}$$

$$= \pi N_o (\beta - \alpha) \tag{39}$$

Now, with the closed form method we can start from Eq. (31) as follows:

$$\sigma^2 = \frac{N_o}{2} \int_{-\infty}^{\infty} \ell n \left[\frac{\frac{N_o}{2}(\omega^2 + \beta^2)}{\frac{(\omega^2 + \alpha^2)}{N_o/2}} \right] d\omega$$

$$= \pi N_o (\beta - \alpha)$$

which gives us same answer as Eq. (39), with much less computations.

IV. CONCLUSION

It has been attempted by several authors to prove the Yovit-Jackson's error formula with individual efforts by different approach. However, Yovit-Jackson's error formula is strictly applicable only for the zero-delay optimum filtering. Due to the mathematical difficulties involved, no one has yet extended the closed-form error expression for some other cases such as delay or prediction filter and integration filter.

The main feature of the closed-form error expression lies in the fact that the MMSE can be obtained only once the signal and noise spectra are known. As stated above, Yovit-Jackson's closed form is good only for zero delay filtering. On the other hand, Lee's MMSE expression does require a lengthy calculation, but it is applicable to the analysis for any kind of filtering problem.

It is understood that the main reasons to fail in extension of Yovit-Jackson's original formula in the past can be summarized as:

(1) The starting point that most of other authors have chosen is for the special case that confines itself to the zero delay filtering system.

(2) The derivation procedure depends too much on cancellations among terms.

To compensate the above two points, the author presented an alternative method which made use of Lee's error expression [2]. As shown in this paper, the main problem to find the closed form expression of error is to find that of the transfer function of the optimum operator (see Fig. 2).

Thus, the new approach to the possible extension of closed-form error expression is now summarized as:

(1) To find a new method of derivation from generalized starting point such as Lee's error expression, which was the main object of this paper.

(2) To find a mathematical relations between the original spectrum and factorized component spectra.

(3) To find the closed form expression of generalized transfer function of optimum operator.

APPENDIX (I)

We wish to show that

$$\int_{-\infty}^{\infty} \frac{K d\omega}{\prod_{i=1}^{n} (j\omega + \alpha_i)} = 0 \quad \text{for} \quad n > 1 \tag{40}$$

where K and α_i's are constants and $\mathrm{Re}[\alpha_i] > 0$.

Let $Z = j\omega$ and $d\omega = -j dZ$, then

$$K\int_{-\infty}^{\infty} \frac{d\omega}{\prod_{i=1}^{n} (j\omega + \alpha_i)} = -jK \int_{-j\infty}^{j\infty} \frac{dZ}{\prod_{i=1}^{n} (Z + \alpha_i)}$$

If we let

$$F(Z) = \int \frac{dZ}{\prod_{i=1}^{n} (Z + \alpha_i)} \tag{41}$$

Then

$$\int_{-\infty}^{\infty} \frac{K d\omega}{\prod_{i=1}^{n} (j\omega + \alpha_i)} = -jK[F(Z)]_{-j\infty}^{j\infty}$$

$$= -jK[F(Z)]_{Z=j\infty} + jK[F(Z)]_{Z=-j\infty} \tag{42}$$

That is, the above integral entirely depends upon the boundary values (evaluated at $\pm j\infty$) of F(Z).

It shall also be noted that,

$$\text{as} \qquad Z \to \pm j\infty, \quad \frac{1}{\prod_{i=1}^{n} (Z + \alpha_i)} \to \frac{1}{Z^n} \tag{43}$$

$$\text{i.e.} \quad F(Z) = \int \frac{dZ}{\prod_{i=1}^{n} (Z + \alpha_i)} \to \int \frac{dZ}{Z^n} = -(n-1) \frac{1}{Z^{n-1}} \quad \text{for} \quad n > 1$$

which implies that as $Z \to \pm j\infty$

$$F(Z) \to -(n-1) \frac{1}{Z^{n-1}} \to 0 \quad \text{for} \quad n > 1$$

That comparing with Equation (42) completes the proof.

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ABSTRACT

The closed form error expression of optimum Wiener filter was first introduced by M. C. Yovit and J. L. Jackson in 1955. Since then, a number of people have proved the validity of this form using almost similar technique but no one has succeeded to extend the Yovit-Jackson's original formula for the other cases such as prediction and delay filters.

It is understood that the main reason to fail in the extension can be summarized as:

(1) The starting point that most of other authors have chosen is for the special case that confines itself to the zero delay filtering system.

(2) The derivation procedure depends too much on cancellation among terms.

To compensate the above two points, an alternative method utilizing Parseval's theorem was presented.

The major problems in the extension of Yovit-Jackson's form are summarized as:

(1) To find a new method of derivation from generalized starting point such as Y. W. Lee's error expression.

(2) To find mathematical relations between the original spectrum and factorized component spectra.

(3) To find the closed form expression of generalized transfer function of optimum operator.