

**Geometrically Nonlinear Analysis of Axially Symmetric, Composite Pressure Domes  
Using the Method of Multiple Shooting**

by

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**(ABSTRACT)**

An analysis is presented of the linear and geometrically nonlinear static response of “thin” doubly-curved shells of revolution, under internal pressure loading. The analysis is based upon direct numerical integration of the governing differential equations, written in first-order state vector form. It is assumed that the loading and response of the shell are both axially symmetric; the governing equations are thus ordinary differential equations. The geometry of the shell is limited in the analysis by the assumptions of axisymmetry and constant thickness. The shell is allowed to have general composite laminate construction, elastic supports at the edges and internal ring stiffeners. In addition, the analysis allows for the possibility of circumferential line loads at discrete locations along the dome meridian. The problem is a numerically unstable two-point boundary value problem; integrations are performed using the technique of multiple shooting. A development of the multiple shooting technique known as stabilized marching is given. Results achieved by use of the multiple shooting technique are verified by comparison to results of finite element analysis using the finite element analysis codes STAGS and ABAQUS. Parametric studies are performed for ellipsoidal domes constructed of symmetric, 8-ply laminates. The parametric studies examine the effects of dome geometry for a quasi-isotropic laminate first, then examine whether material properties may be adjusted to create a “better” design. Conclusions and recommendations for future work follow.