

**A UNIFIED DECISION ANALYSIS FRAMEWORK  
FOR ROBUST SYSTEM DESIGN EVALUATION  
IN THE FACE OF UNCERTAINTY**

by

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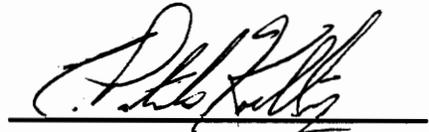
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(ABSTRACT)

Some engineered systems now in use are not adequately meeting the *needs* for which they were developed, nor are they very cost-effective in terms of consumer utilization. Many problems associated with unsatisfactory system performance and high life-cycle cost are the direct result of decisions made during early phases of system design.

To develop quality systems, both engineering and management need fundamental principles and methodologies to guide decision making during system design and advanced planning. In order to provide for the efficient resolution of complex system design decisions involving uncertainty, human judgments, and value trade-offs, an efficient and effective decision analysis framework is required.

Experience indicates that an effective approach to improving the quality of detail designs is through the application of Genichi Taguchi's philosophy of robust design. How to apply Taguchi's philosophy of robust design to system design evaluation at the preliminary design stage is an open question.

The goal of this research is to develop a unified decision analysis framework to support the need for developing better system designs in the face of various uncertainties. This goal is accomplished by adapting and integrating statistical decision theory, utility theory, elements of the systems engineering process, and Taguchi's philosophy of robust design. The result is a structured, systematic methodology for evaluating system design alternatives.

The decision analysis framework consists of two parts: (1) decision analysis foundations, and (2) an integrated approach. Part I (Chapters 2 through 5) covers the foundations for design decision analysis in the face of uncertainty. This research begins with an examination of the life cycle of engineered systems and identification of the elements of the decision process of system design and development. After investigating various types of uncertainty involved in the process of system design, the concept of robust design is defined from the perspective of system life-cycle engineering. Some common measures for assessing the robustness of candidate system designs are then identified and examined.

Then the problem of design evaluation in the face of uncertainty is studied within the context of decision theory. After classifying design decision problems into four categories, the structure of each type of problem in terms of sequence and causal relationships between various decisions and uncertain outcomes is represented by a decision tree. Based upon statistical decision theory, the foundations for choosing a best design in the face of uncertainty are identified. The assumptions underlying common objective functions in design optimization are also investigated. Some confusion and controversy which surround Taguchi's robust design criteria – loss functions and signal-to-noise ratios – are addressed and clarified.

Part II (Chapters 6 through 9) covers models and their application to design evaluation in the face of uncertainty. Based upon the decision analysis foundations, an integrated approach is developed and presented for resolving both discrete decisions, continuous decisions, and decisions involving both uncertainty and multiple attributes. Application of the approach is illustrated by two hypothetical examples: bridge design and repairable equipment population system design.

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# I. INTRODUCTION

- 
- 1.1 Problem Definition
  - 1.2 Problem Statement
  - 1.3 Research Objectives
  - 1.4 Uniqueness and Premise of this Research
  - 1.5 Organization of the Decision Framework
- 

## 1.1 Problem Definition

With the introduction of new technologies in design, engineered systems and products are becoming more complex. However, many of the systems in use are not meeting the needs for which they were developed, nor they are very cost-effective in terms of consumer utilization (Blanchard, 1991). Although various factors are contributing to such unacceptable situations, one of the major causes is that the system developed is not robust with respect to its operational environment. Many systems are not easily maintained and cannot be efficiently supported. Some systems are completely unavailable when needed and others are operating at less than full capacity in terms of desired output or at high operational cost.

Many problems associated with unsatisfactory system performance and high life-cycle cost are the direct result of decisions made during early phases of system design and advanced planning. Inefficient product design is viewed as one of the bottlenecks to improved product/system quality and time to market. Those early decisions pertaining to utilization of new technologies in design, the selection of component parts and materials, the selection of a manufacturing process, the identification of maintenance support policies, etc., have a major effect on both total quality and life-cycle cost.

In order to develop robust designs, only education pertaining to the importance and possible benefits alone is not sufficient. There is an urgent need for the development of new design methodologies and approaches for design engineers. Observations and research have shown that design theories and methodologies are essential to the development of sound pedagogical techniques (EiMaraghy, et al., 1989). To provide a theoretical basis for the development of tools to aid designers, the study of design theory and methodology is developing into a central field of research.

The overall goal of research in engineering design is to improve the performance and outcome of the design process. Due to various uncertainties, engineering designs are typically represented imprecisely at the early, conceptual and preliminary stages. Technical tools to aid this area of the design process are rare, largely because of the scarcity of techniques capable of handling imprecise data (Wood and Antonsson, 1989). Little research has been conducted on the development of design analysis and evaluation methodology for early system design activities. There is not a complete, cohesive structure for the determination of design criteria, their modeling in terms of system variables/parameters, the synthesis and screening of alternatives, and formal optimization. Most of these activities and decisions have been accomplished in an ad hoc or empirical manner. New tools are often built generally without consideration of the overall effects on the process, and without the use of any formal mathematical models of the process.

Taguchi's philosophy of robust design is very important to design decision analysis in the face of uncertainty. However, there has been relatively little research on the mathematical foundation, assumption, and techniques of Taguchi's approach. Furthermore, there has been little research to compare his techniques to other methods, either analytically or experimentally, except for comparisons with experimental design techniques from which Taguchi's approach is derived. In addition, as indicated by Otto and Antonsson (1991), there has been little research attempting to improve the approach itself.

How to apply Taguchi's philosophy of robust design for design evaluation at the preliminary system design stage is an open question. Taguchi's parameter design approach relies on direct experimentation. When a mathematical model or a computer model of the design exists, Box and Fung (1986) argued that a

more appropriate means of identifying a robust design is through nonlinear optimization techniques. More recently, a number of researchers have implemented Taguchi's philosophy using nonlinear programming, goal programming, and simulation approaches, including d'Entremont and Ragsdell (1988), Sandgren (1989), Sundaresan et al. (1989), Belegundu and Zhang (1989), Parkinson et al. (1990), and Ramakrishnan and Rao (1991). But the problem is to determine under what condition each approach should be used. What is needed is a unified framework for design evaluation which integrates various approaches based on solid mathematical foundation of robust design.

To obtain better system performance, both engineering and management needs fundamental principles and methodologies to guide decision making in design. In order to provide for the efficient resolution of complex system design decisions involving uncertainty, human judgments, and multiple attributes, an efficient and effective decision analysis framework is required. System optimization can be achieved only through a systematic approach to design evaluation.

## **1.2 Problem Statement**

System design and development requires that timely evaluations of design alternatives be made as the design concept evolves. In most instances, specified requirements can be satisfied by one or more design alternatives. The problem is to identify the best design alternatives through an iterative process of systems analysis using selected analytical methods. Design evaluation is invoked as a basis for choice in finalizing the design quickly.

Choice of the best design is a trade-off among design characteristics. The design selected should not only be feasible, but also optimal and robust with respect to various uncertainties over the system's life cycle. This research aims to improve the performance of engineering design processes through the development of a unified decision analysis framework for system design evaluation in the face of uncertainty.

### **1.3 Research Objectives**

The goal of this research is to develop a unified decision analysis framework to support the need and requirement for developing better system designs in the face of uncertainty. Specific objectives are to:

- Define and operationalize the concept of robust system design.
- Identify decision analysis foundations for design evaluation in the face of uncertainty through mathematically modeling the functional relationships between design decisions and the overall worth of a candidate design.
- Identify and integrate appropriate decision analysis approaches into a unified framework for system design evaluation in the face of uncertainty.
- Present examples to illustrate the application of the framework.

### **1.4 Uniqueness and Premise of this Research**

Developing explicit design evaluation procedures has been recognized as a crucial step toward development of a more formal theory and methodology of design (Chandrasekran, 1989; Finger and Dixon, 1989). It has been noted that a major research issue in design theory and methodology is the analysis and evaluation of designs in the preliminary stage of design (Finger and Dixon, 1989-II).

The uniqueness of this research is to integrate and adapt statistical decision theory, elements of the systems engineering process, and Taguchi's philosophy of robust design to meet the needs of system design and development. Instead of concentrating on performance variability alone, a structured approach is taken in this research to quantify uncertainties, risk attitudes, value trade-offs, and expected gains and losses during system life cycle. This approach is offensive in that it does not remove the uncertainty. The effect of uncertainty on the relative desirability of design alternatives is incorporated into the design evaluation process.

This approach is useful in the early stages of the design process. It can facilitate the integration of performance-related characteristics and logistic support requirements in system design. The approach will be applicable at the macro level for the evaluation of candidate systems, or at the micro level for

design iteration. Integration of the evaluation approach with CAE/CAD tools may increase design productivity, and provide technical capabilities needed to dramatically influence the decision process during system design evolution. Accordingly, this research is expected to impact the development and design of complex technological systems, both commercial and public sector, while also influencing some aspects of strategic planning.

The underlying premise of this research is that major decisions in the design process would be improved if the factors which influence the decisions are quantified and made visible. Such factors include uncertainty, hard and soft operational and technological considerations, human factors, and other judgmental elements. The need for visibility and quantification of uncertainty and judgmental factors arises not only from a desire for logical consistency in the treatment of decision elements, but also from the need for people to communicate, review, and discuss such factors as part of the total decision process.

In this research, the methodology and models will be developed to optimize the total problem-solving process rather than just the decision per se; provide for insufficiencies in data base as well as for uncertainties in cause-effect relationships. The idea is not to fully automate the design process, nor to automatically generate design alternatives. Rather, the goal is to make it easier for the designer to evaluate more alternatives in less time, and to provide more information on the performance of each alternative. Since most important (and costly) decisions in the design process are made in the early stages, the effect will be greater the earlier in the design process the information is made available. Thus, these developments form a semi-automated approach to design analysis and evaluation.

## **1.5 Organization of the Decision Framework**

A unified decision analysis framework is presented for system design evaluation in the face of uncertainty. This framework consists of two parts: (1) decision analysis foundations, and (2) an integrated approach. Figure 1.1 shows the basic organization.

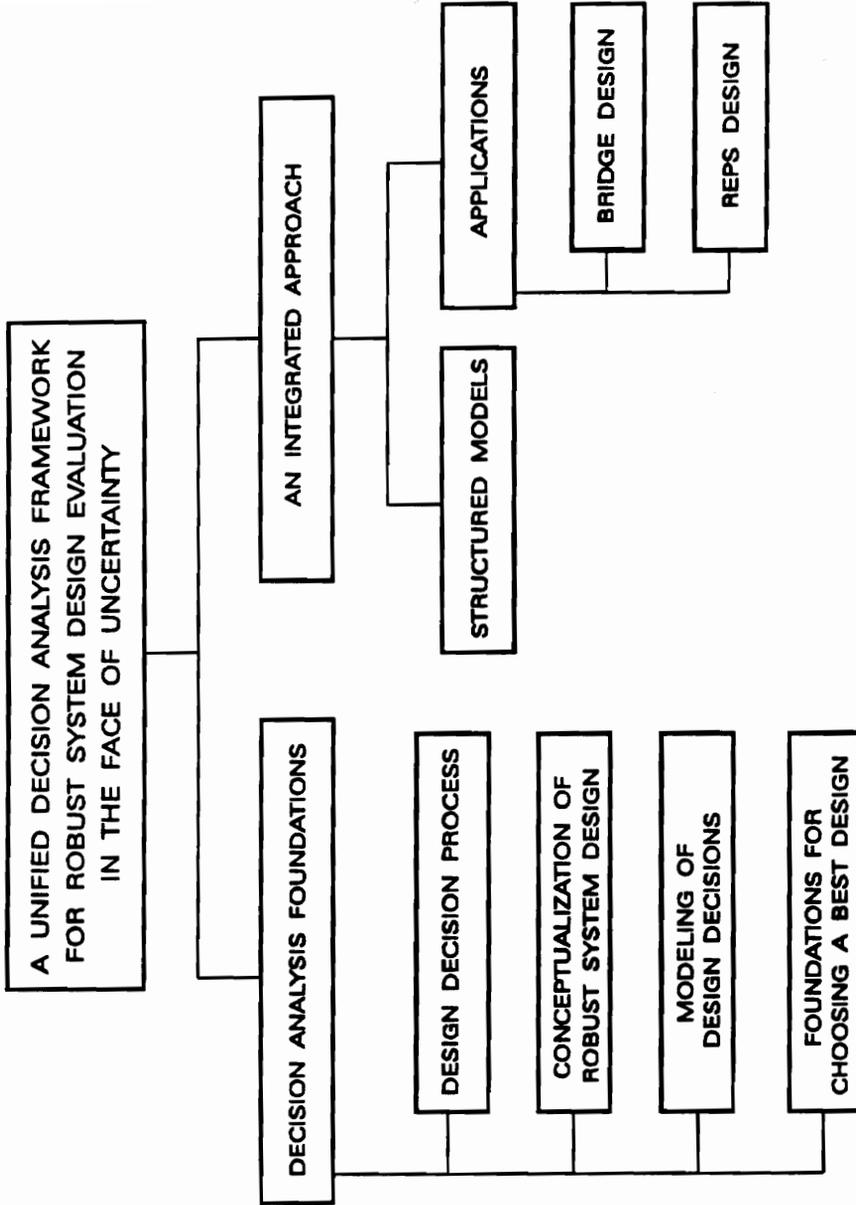


Figure 1.1. Organization of the decision analysis framework

Part I, consisting of Chapters 2 through 5, covers decision analysis foundations for design analysis and evaluation in the face of uncertainty. In Chapter 2, the decision process of system design and development is investigated from the perspective of concurrent life-cycle engineering. The elements of the decision process are identified. Then the focus of the research is defined.

Chapter 3 defines and operationalizes the concept of robust system design. After identifying various uncertainties involved in the process of engineered system design, the concept of robust design is defined from the perspective of system life-cycle engineering. Some common measures of the robustness of candidate designs are also examined.

Chapters 4 and 5 study the problem of design decisions in the face of uncertainty within the context of decision theory. The concept of best design is investigated and clarified. The focus of Chapter 4 is on the modeling of design decisions in the face of uncertainty. After classifying design decision problems into four categories, the structure of a decision problem in terms of the sequence and causal relationships between various decisions and uncertain outcomes are represented by decision trees.

Once the decision problem has been modeled, a choice must be made. Chapter 5 investigates the foundations for choosing a best design in the face of uncertainty. After summarizing the concepts of choices, preferences, and utility theory, three decision analysis approaches are identified for design evaluation in the face of uncertainty.

Part II, made up of Chapters 6 through 9, covers models and applications of design evaluation in the face of uncertainty. An integrated approach is developed and presented in Chapter 6 for conducting design analysis and evaluation for both discrete and continuous decisions. Chapter 7 presents a hypothetical bridge design example to explain the concepts underlying the decision analysis framework. In Chapter 8, the framework is extended to resolve design decision problems involving both uncertainties and multiple attributes. Chapter 9 presents an example of repairable equipment population system (REPS) design.

Chapter 10 summarizes the contribution of this research and discusses the possibilities for future research.

## **II. THE DECISION PROCESS FOR ENGINEERED SYSTEM DESIGN**

- 
- 2.1 Introduction
  - 2.2 The System Life Cycle
  - 2.3 System Design and Development
  - 2.4 Elements of the Design Decision Process
  - 2.5 Design Decisions in the Face of Uncertainty
  - 2.6 Decision Models for Design Analysis and Evaluation
- 

### **2.1 Introduction**

In this chapter, the decision process of system and development is examined from the perspective of concurrent life-cycle engineering. Elements of the decision process are identified. Then the focus of this research is defined. Two important decision models for design analysis and evaluation are also introduced.

### **2.2 The System Life Cycle**

In general, the life cycle of a system can be divided into two phases: the acquisition phase and the utilization phase. In the acquisition phase, decisions progress from identifying the need through conceptual design and preliminary design, detail design and development, and production/construction. The utilization phase includes activities of system deployment, use, phaseout, and disposal.

The concurrent life-cycle engineering design approach goes beyond consideration of the life-cycle of the product/system itself. This approach encompasses three concurrent life cycles as illustrated in Figure 2.1: product life cycle, manufacturing system life cycle, and support system life cycle (Fabrycky, 1991; Blanchard and Fabrycky, 1990; Midkiff and Fabrycky, 1991).

In this approach, conceptual design is initiated first to meet the need for the system. Then, during conceptual/preliminary design of the system, consideration is given simultaneously to its ease of manufacture. This gives rise to a parallel life cycle for bringing a manufacturability capability into being; that is, design for manufacture. Another life cycle is for the logistic activities needed to service the system during use and to support the manufacturing facility during its duty cycle. This approach indicates that logistics and maintenance requirements planning should begin during system conceptual design in a coordinated manner.

The knowledge acquired, life-cycle cost committed, and ease of design change for each stage in the system life-cycle process is illustrated in Figure 2.2. As indicated in this figure, a large portion of the total cost for a system is associated with its operation and support. The costs associated with different phases of the life cycle are interrelated. Commitment of these costs is based on the decisions made in the early stages of the system life cycle.

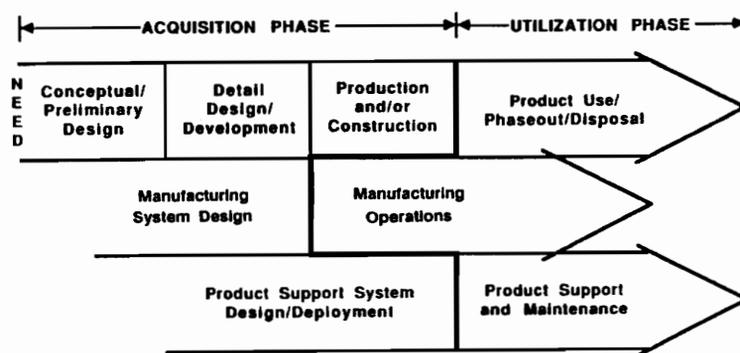


Figure 2.1. Product, process, and support life cycles (Blanchard and Fabrycky, 1990)

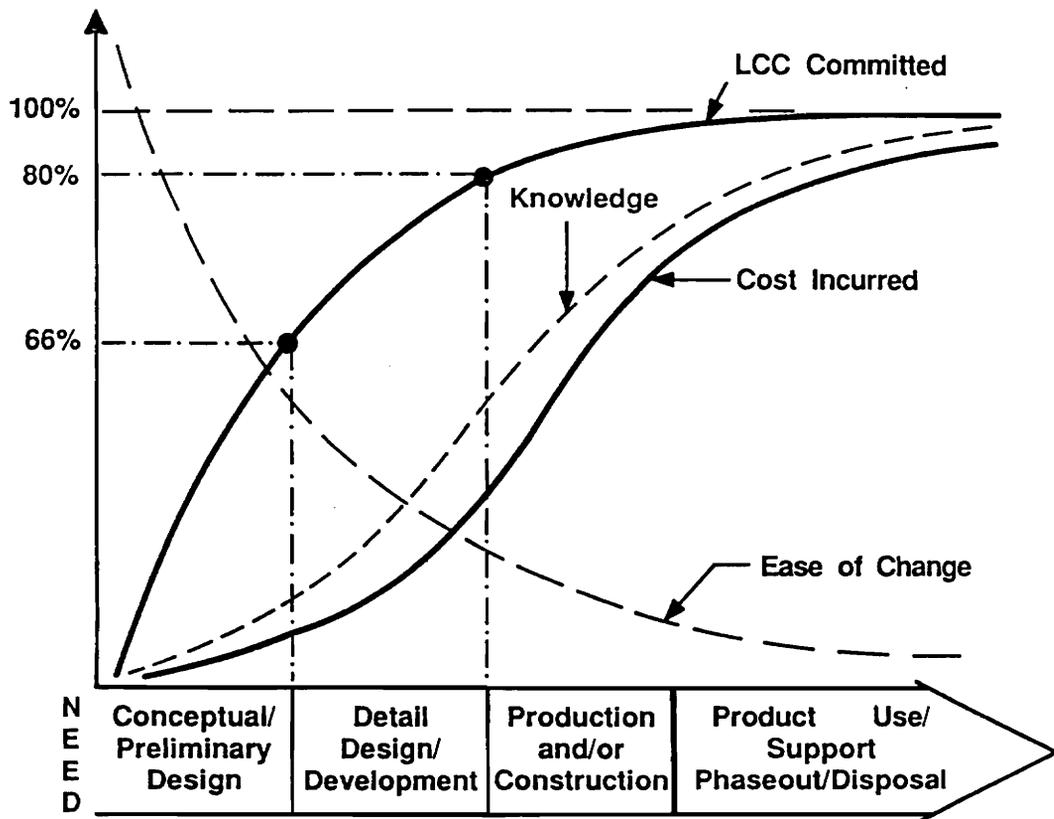


Figure 2.2. Commitment of resources, life-cycle cost committed, and cost incurred in a system's life cycle (Fabrycky and Blanchard, 1991)

## 2.3 System Design and Development

The system acquisition phase consists of two subphases: (1) design and development, and (2) production and/or construction. The design process follows from a set of stated requirements for a given system and evolves through three steps: (1) conceptual design, (2) preliminary design, and (3) detail design (Figure 2.3). This process generally begins with a visualization of what is required and extends through the development, test, and evaluation of an engineering or prototype model of the system. The output constitutes a configuration that can be directly produced or constructed from specifications, a set of drawings, and supporting documents.

Preliminary system design follows conceptual design and extends through the translation of established system-level requirements into detailed qualitative and quantitative design requirements (Blanchard and Fabrycky, 1990). As illustrated in Figure 2.3, preliminary design includes the process of functional analysis and requirement allocation, the accomplishment of trade-off studies and optimization, system synthesis, and configuration definition in the form of detailed specifications.

The emphasis of this research is on the process of system level trade-off studies and optimization. Various activities in this process can be grouped into four categories: design generation, design analysis, design evaluation, and design optimization. The relationships between these activities can be illustrated by the conceptual model in Figure 2.4.

***Design generation.*** Design generation is a process of identifying and describing candidate alternatives. Each alternative must be described in sufficient detail to permit subsequent estimates of outcomes. To identify the possible courses of action, as summarized by Ackoff (1962), two tasks need to be accomplished: (1) identifying the variables that significantly affect the outcome of the problem, and (2) determining which of these variables can be controlled directly or indirectly by the decision maker.

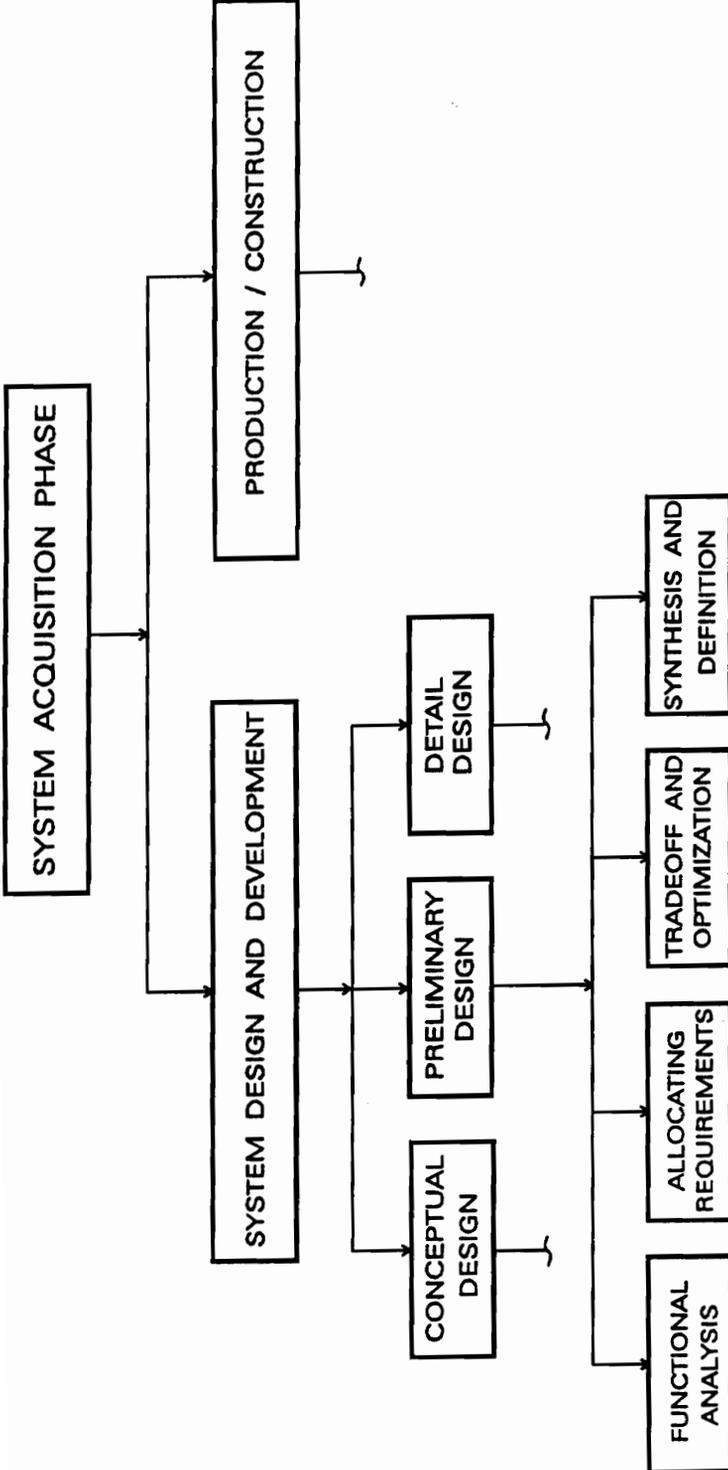


Figure 2.3. Activities of system design and development

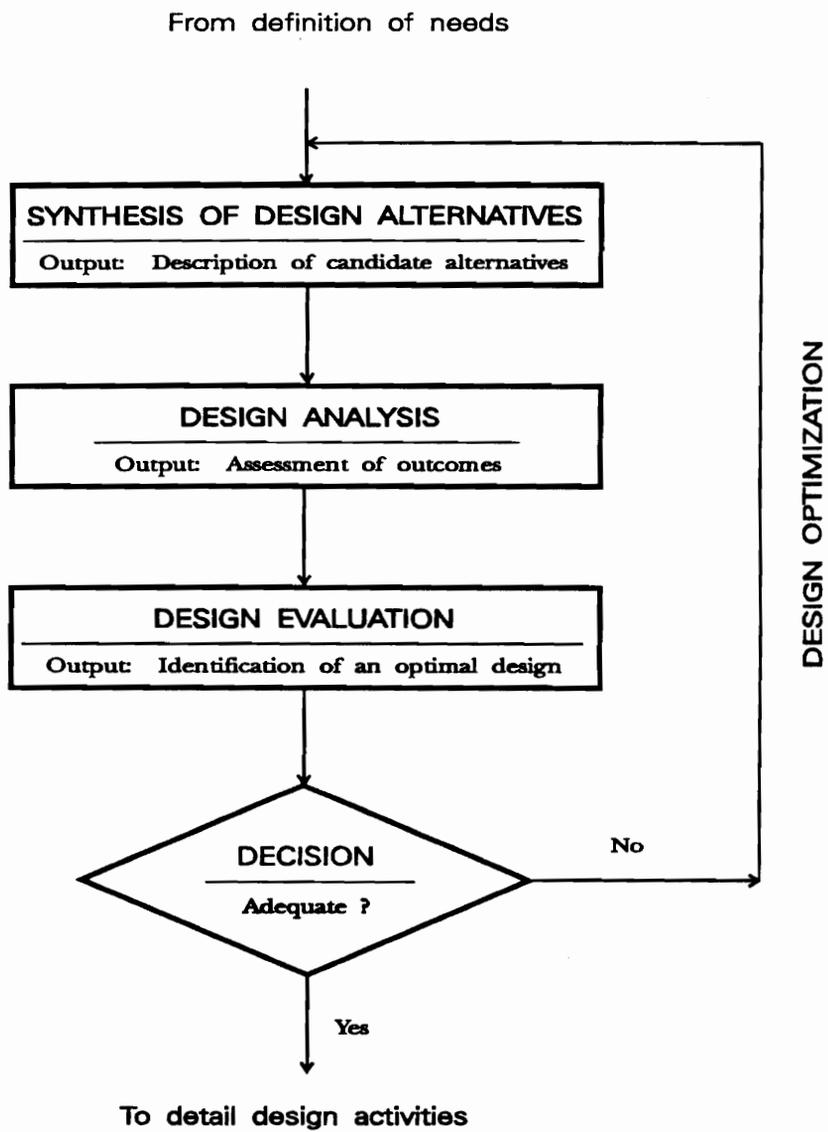


Figure 2.4. The process of system trade-off studies and optimization

**Design analysis.** Design analysis represents the activities to transform the description of each candidate alternative into estimates of outcomes. The results generated from design analysis will be used for design evaluation.

**Design evaluation.** Design evaluation denotes the activities to transform the estimates of outcomes into a utility estimate. All alternatives are compared equivalently under the same set of criteria. To make a logical decision, a common measure is required.

**Design optimization.** The information obtained by performing the foregoing computations can be used to obtain better candidate solutions. The design model is improved through iterative redesign. This decision process is continued until the utility of making another iteration is less than the utility of mapping to another level of abstraction. If the process is successful, the design evolves and the output of this decision process is mapped into a less abstract modeling schema, and the sequence repeats until a suitable system design is defined. Thus, design alternates between optimization and mapping (Bell, et al., 1991). Since only partial information is available at each stage, incorrect decisions are probable. The process must be iterative as well as concurrent.

## 2.4 Elements of the Design Decision Process

A typical design decision process consists of four elements: the decision maker, the candidate design alternatives, the states of nature, and the outcome. These are discussed below.

**Decision maker (DM):** An individual or groups of individuals who have the authority and responsibility to select the alternative to be implemented. Depending on the level of problem considered, the decision maker may be the designers or upper-level management.

**Candidate alternatives  $\{a_j\}$ :** A set of mutually exclusive courses of action which satisfy all functional design criteria and provide for the solution of a design

problem. Each alternative requires a description so that it can be identified and analyzed to determine the consequences of its selection. This description includes specifying characteristics which can be selected by the decision maker when a given decision is made. The set of these characteristics will be designated the control variables. The vector of control variables are specified by  $X$ .

**States of nature  $\{s_j\}$ :** A set of mutually exclusive and exhaustive states of nature. The states of nature represent those aspects of the problem environment which are not subject to the decision maker's control, but may affect the consequences of the choice of action.

**Outcome  $\{c_{ij}\}$ :** The consequences associated with implementing a candidate alternative given a state of nature. An outcome may consist of a single attribute or multiple attributes, or dimensions. Each dimension of an outcome which is significantly affected by the choice of an alternative, and which the decision maker considers to be important in making the decision, is designated an evaluation attribute or decision criterion.

Evaluation attributes are the variables used to rank or measure the desirability of possible outcomes. By the functions of the criteria, the set of decision criteria has three major subsets: effectiveness criteria, cost criteria, and schedule criteria (Lifson, 1972). Each subset represents an important area of concern. An effectiveness criterion is an attribute of a system which is directly related to the fulfillment of needs; a cost criterion reflects the resources required to implement a course of action; a schedule criterion is related to the time the system is needed. By the nature of the criteria, the set of criteria may also be partitioned into subsets of quantified criteria and nonquantified criteria. The focus of this research will be on quantified criteria.

## 2.5 Design Decisions in the Face of Uncertainty

The focus of this research is on resolving design decisions in the face of uncertainties. The relationships between the elements of the design decision process can be illustrated by a decision evaluation matrix as in Figure 2.5.

|       |          |          |   |   |   |          |   |   |   |          |
|-------|----------|----------|---|---|---|----------|---|---|---|----------|
|       | $s_1$    | $s_2$    | • | • | • | $s_j$    | • | • | • | $s_n$    |
| $a_1$ | $c_{11}$ | $c_{12}$ | • | • | • | $c_{1j}$ | • | • | • | $c_{1n}$ |
| $a_2$ | $c_{21}$ | $c_{22}$ | • | • | • | $c_{2j}$ | • | • | • | $c_{2n}$ |
| •     | •        | •        |   |   |   | •        |   |   |   | •        |
| •     | •        | •        |   |   |   | •        |   |   |   | •        |
| •     | •        | •        |   |   |   | •        |   |   |   | •        |
| $a_i$ | $c_{i1}$ | $c_{i2}$ | • | • | • | $c_{ij}$ | • | • | • | $c_{in}$ |
| •     | •        | •        |   |   |   | •        |   |   |   | •        |
| •     | •        | •        |   |   |   | •        |   |   |   | •        |
| •     | •        | •        |   |   |   | •        |   |   |   | •        |
| $a_m$ | $c_{m1}$ | $c_{m2}$ | • | • | • | $c_{mj}$ | • | • | • | $c_{mn}$ |
|       | $p_1$    | $p_2$    | • | • | • | $p_j$    | • | • | • | $p_n$    |

Where:

$a_i$  = design alternatives

$s_j$  = states of nature

$p_j$  = probability of  $s_j$

$c_{ij}$  = the outcome associated with  $a_i$  if  $s_j$  occurs

Figure 2.5. A decision evaluation matrix for making design decisions

The decision maker has identified each state of nature and the corresponding probability of its occurrence. The outcome associated with each alternative and each state of nature is also known. It can be assumed that the decisions are exclusive and exhaustive. That is, one of decisions has to be taken, and at most one of them can be taken. The choice of any one excludes the choice of any other. Now the problem is to select the alternative to maximize the expected worth of the system with respect to various uncertain states of nature. This best alternative is expected to satisfy recognized human needs and/or desires best according to some specified criterion of goodness.

Traditional decision theory classifies decisions into three categories (Luce and Raiffa, 1957): (1) decisions under certainty, (2) decisions under risk, and (3) decisions under uncertainty. Depending upon whether the probability of the state of nature is specified, "decisions under risk" and "decisions under uncertainty" are distinguished.

Now in both the communities of decision research and engineering design, this distinction between "decisions under risk" and "decisions under uncertainty" is not made strictly. According to Lindley (1985), there is only one logical way to make a decision in the presence of uncertainty. Three basic principles must be followed: (1) assigning probabilities to uncertain events, (2) assigning utilities to the possible outcomes, and (3) choosing that decision that maximizes expected utility. Thus, if the quantification of judgment in the form of probability and utility estimates can be made, then decisions under uncertainty can be converted into decisions under risk. Therefore, in this research, "decisions under uncertainty" and "decisions under risk" will not be distinguished. In keeping with the terminology of engineering design, the term "design evaluation in the face of uncertainty" will be used.

## **2.6 Decision Models for Design Analysis and Evaluation**

To study the design decision process quantitatively, decision models are very helpful. A model may be used as a representation of a system to be brought into being, or to analyze a system already in being. Two decision models which are particularly useful for design analysis and evaluation are

introduced below. One is the decision model presented by Churchman et al. (1957). The other is the Design Dependent Parameter Approach developed by Blanchard and Fabrycky (1990) and Fabrycky and Blanchard (1991).

### **2.6.1 Decision model of Churchman et al.**

For any system, its evaluation attribute is a function of various variables and parameters. Churchman et al. (1957) classified the variables and parameters which affect the outcome of a system into two groups: (1) the variables which are subject to control by the decision maker, and (2) the factors (variable or constant) which are not subject to the control by the decision maker within the scope of the problem as defined. The former are often called control variables, while the latter is called system parameters. The functional relationship between the evaluation attribute  $E$ , control variables  $X$ , and system parameters  $Y$ , in its unconstrained form, is expressed as

$$E = f(X, Y). \quad (2.1)$$

This decision model is useful for design optimization. The model enables the decision makers to determine what values of the controllable variables provide the best level of the evaluation attribute under the conditions described by the system parameters.

### **2.6.2 Design Dependent Parameter Approach (DDP)**

Blanchard and Fabrycky (1990) extended Churchman et al.'s decision model to design and operational decision situations involving multiple alternatives. This extension identifies and isolates design-dependent system parameters from design-independent parameters. The purpose of the design-dependent parameters is to define each alternative explicitly. In the process of design analysis and evaluation, the DDP approach uses a design evaluation function to express the relationship between the evaluation attribute(s), design variables, design-dependent parameters, and design-independent parameters. The design evaluation function has the following form:

$$E = f(X; Y_d, Y_i) \quad (2.2)$$

$$\text{subject to } g_j(X; Y_d, Y_i) \leq 0, \quad j = 1, \dots, k$$

where:  $E$  = a vector of evaluation attributes  
 $X$  = a vector of design variables  
 $Y_d$  = a vector of design-dependent parameters  
 $Y_i$  = a vector of design-independent parameters

The procedures to apply the DDP approach in design analysis and evaluation can be illustrated by Figure 2.6. According to the DDP approach, design decision analysis follows four steps:

**Step 1:** Identify possible levels of design-dependent parameters. Each set of design-dependent parameter values determines a unique design alternative.

**Step 2:** For each design alternative, determine the setting of design variable values which optimize the evaluation attribute. The optimum value of the evaluation attribute for each alternative is then obtained. This step provides optimization within an alternative.

**Step 3:** Compare the optimum values of the evaluation attribute for all alternatives and select the alternative which gives the best attribute value.

**Step 4:** Decide if the optimum attribute level obtained from the optimum alternative meets the design requirements. If yes, go to the next design phase. Otherwise, go back to step 1.

The design evaluation function provides a mathematical means to assess a system's response to changes in both controllable and uncontrollable factors. The importance of the DDP approach is in distinguishing choice-based design and optimization-based design (Fabrycky, 1992). By considering design variables and design-dependent parameters at different levels of the design

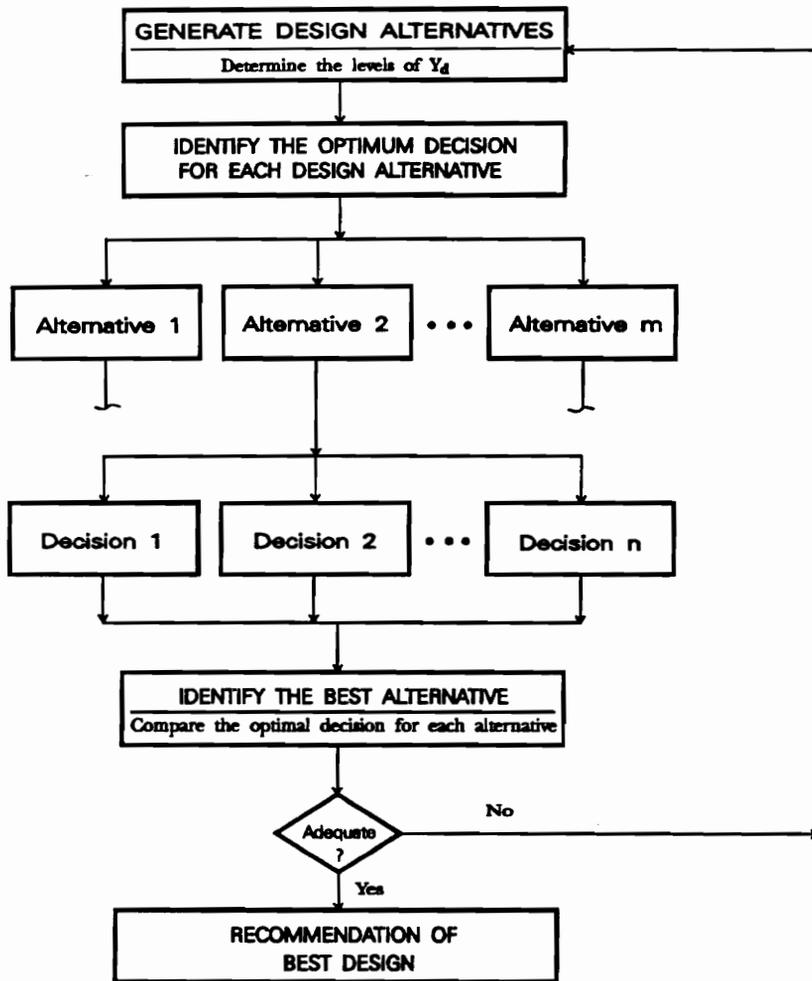


Figure 2.6. Procedures for application of the DDP approach

decision process, the implementation of system trade-off studies and optimization as illustrated in Figure 2.4 becomes more structured and systematic. When applied in the evaluation of system design, the evaluation function can be optimized in terms of life-cycle cost and/or the multiple system effectiveness measures, as shown in Figure 2.7.

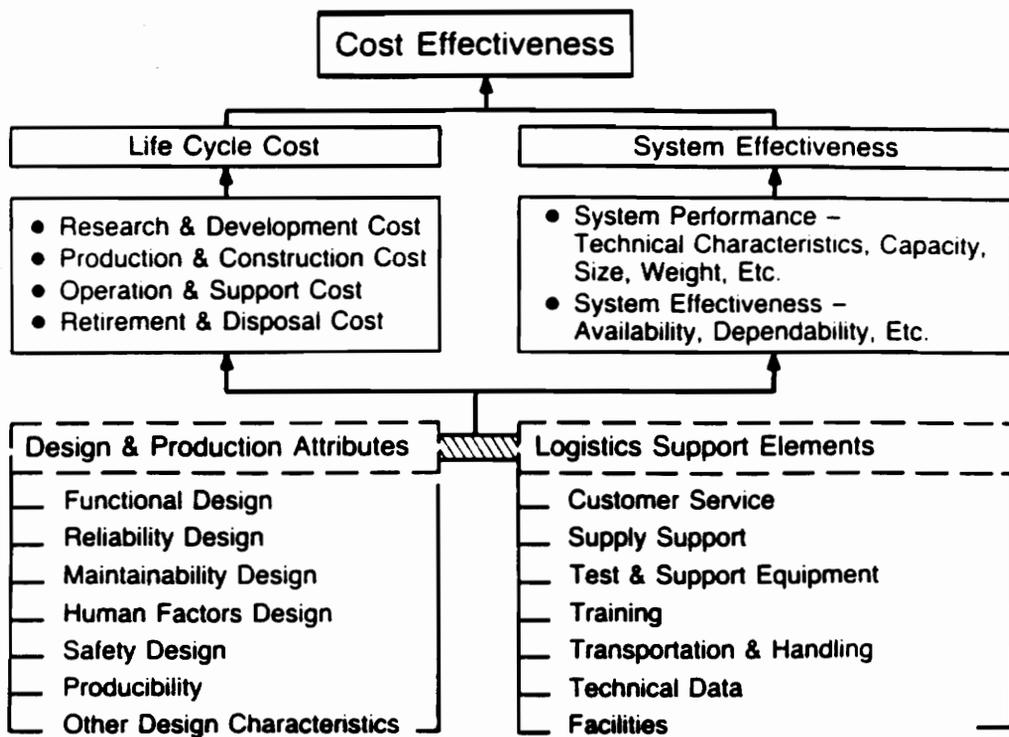


Figure 2.7. Multiple evaluation attributes used in design evaluation  
(Blanchard and Fabrycky, 1990)

### **III. CONCEPTUALIZATION OF ROBUST SYSTEM DESIGN**

- 
- 3.1 Introduction
  - 3.2 Sources of Uncertainty in Engineered Systems
  - 3.3 The Effect of Uncertainties
  - 3.4 The Concept of Robust System Design
  - 3.5 Measures of the Robustness of Candidate Systems
  - 3.6 Review of Some Existing Approaches
- 

#### **3.1 Introduction**

This chapter defines and operationalizes the concept of robust system design. After identifying various types of uncertainty involved in the process of system design, the concept of robust design is defined from the perspective of system life-cycle engineering. Some common measures for assessing the robustness of candidate designs are examined. Within the context of robust design, a brief review is then made on some existing approaches for design optimization.

#### **3.2 Sources of Uncertainty in Engineered Systems**

The English language has a number of words to describe the nature of various uncertainties: possible, odds, probable, plausible, chance, likely, and many others. The richness of the language reflects the ubiquity of the concept of uncertainty. In engineering design, the magnitude of a system evaluation attribute depends both on the state of nature and on the alternative selected. Since a state of nature is associated with some future date, the state which will occur cannot, in general, be determined with certainty at the time the decision is

made. The states of nature are inherently probabilistic. It is evident that, even given the state and the alternative, the magnitude of a given attribute cannot be known with certainty at the time the alternative must be selected. Whenever one can define possible states of nature it is possible to estimate the probability associated with the choice. This does not mean that we can always obtain estimates in which we have confidence.

### **3.2.1 Sources of uncertainty**

In the process of design decision making, there are various types of quantities to be considered. These include decision variables, empirical quantities, outcome criterion, defined constants, and others. For design evaluation in the face of uncertainty, the empirical quantities demand special attention because they represent measurable properties of the real-world systems being modeled.

There have been several attempts to create taxonomies of different kinds of uncertainty. Most of these have concentrated on uncertainty in empirical quantities which constitute the majority of quantities in models for design analysis and evaluation. Uncertainties in empirical quantities can arise from a variety of different kinds of sources. According to Morgan and Henrion (1990), various sources of uncertainty can be divided into seven categories:

- 1) Random error and statistical variation
- 2) Systematic error and subjective judgment
- 3) Linguistic imprecision
- 4) Variability
- 5) Inherent randomness
- 6) Disagreement
- 7) Approximation

In developing engineered systems, two major types of uncertainties are the uncertainty associated with the inherent variability of the physical process and the uncertainty associated with the imperfection in the modeling of the physical process (Ang and Tang, 1984).

***Uncertainty due to inherent variability.*** The randomness in a physical process contributes to uncertainty because it is inherently not possible to ascertain the realization of the process. From a practical standpoint, inherent variability is essentially a state of nature and the resulting uncertainty cannot be avoided. Even if the physical laws governing a system are well understood, its behavior may be unpredictable because of modeling and computational limitations. The issues of inherent randomness and the limits of predictability do not seem to pose practical difficulties for uncertainty in risk analysis and other quantitative policy analysis. In this context, the main objective is to distinguish uncertainty that might be reducible by further research or more detailed modeling from uncertainty that is unlikely to be reducible, whether because of "inherent randomness" or because of practical unpredictability.

***Uncertainty associated with prediction error.*** In most problem environments of engineering design, predictions and estimations of the states of nature are often performed under conditions of incomplete or inadequate information. The potential errors of an imperfect prediction model cannot be entirely corrected deterministically. Errors of prediction include estimation error (such as statistical sampling error) as well as the imperfection of the prediction model. Such prediction error may include a systematic component (bias) as well as a random component (random error). The systematic errors often arise from biases in the measuring apparatus and experimental procedure. The uncertainty associated with prediction or modeling error may be reduced through the use of more accurate models and/or the acquisition of additional data.

### **3.2.2 Variations over the system life cycle**

Uncertainties are involved in all phases of a system's life cycle. When the decision is made to begin concept formulation, uncertainties are great. In fact, there are few certainties. Needs may be known vaguely; cost of acquisition and use are essentially unknown; feasibility, both financial and technical, has not been established. As the life cycle progresses, uncertainty is reduced by gaining more information. Due to uncertainties involved in the system life cycle,

there exist three sources of variation when a system design is implemented (Taylor, 1991):

- 1) Manufacturing variation
- 2) Variation due to deterioration
- 3) Usage variation

Manufacturing variation is the variation in system performance resulting from such things as fluctuations in the process parameters and materials, wearing and changing in tooling, and changes in the methods, operators, and manufacturing environment. Statistical process control addresses only this source of variation. Formally, manufacturing variation should be defined as the variation up to the time the system is delivered to the customer.

In the customer's eyes, the last two types of variations are just as important as manufacturing variation. All three cause a system to deviate from the ideal. When reducing variation, these last two sources should not be overlooked.

The sources of performance variations are called noise factors in Taguchi's terminology. Taguchi (1986) classifies various noise factors in a system into three types: (1) internal noise errors inherent in the design, such as wear, storage deterioration of materials, etc., (2) variational noise errors due to variation in the supplied materials and manufacturing processes, and (3) external noise errors due to environmental fluctuations.

### **3.2.3 Uncertainty about models for design evaluation**

Design decision analysis depends on the decision models used to represent the system. The models are representation of states, objects, and events. The model form incorporates both the factual and value structure of the model being employed. They are idealized in the sense that they are less complicated than reality and hence easier to use for research purposes. The simplicity of models, compared with reality, lies in the fact that only the relevant properties of reality are represented. They are utilized to accumulate and relate the knowledge we have about different aspects of reality.

Uncertainty about the form of a model is generally harder to think about than uncertainty about the value of a quantity. In general, approximation uncertainty arises because the model is only a simplified version of the real-world system being modeled. There has been relatively little research into situations in which there is uncertainty or disagreement about what form of model to use, for either facts or values; and much remains to be done in developing methods of dealing with them. Ackoff (1962) pointed out that there are four ways in which a model could be in error:

- 1) The model may contain irrelevant variables which have no effect on the outcome.
- 2) The model may not include variables which are relevant.
- 3) The function which relates the controllable and uncontrollable variables to the outcome may be incorrect.
- 4) The numeric values assigned to the variables may be inaccurate.

### 3.3 The Effect of Uncertainties

Following the design decision model of Fabrycky and Blanchard (1991), a general design evaluation model has the form:

$$\begin{array}{ll} \text{Maximize} & E = f(X; Y_d, Y_i) \\ \text{subject to} & g_j(X; Y_d, Y_i) \leq 0, \quad j = 1, \dots, k \end{array}$$

where:

- $E$  = a vector of evaluation attributes
- $X$  = a vector of design variables
- $Y_d$  = a vector of design-dependent parameters
- $Y_i$  = a vector of design-independent parameters

In general, for a given set of nominal values for  $X$ ,  $Y_d$ , and  $Y_i$ , there can be fluctuations  $\delta X$ ,  $\delta Y_d$ , and  $\delta Y_i$  about these nominal values. We are interested in how these fluctuations are transmitted to the objective and constraint functions.

The fluctuations  $\delta X$  are variations from the derived values of the design variable that arise primarily when the design is implemented. In general, the design can specify a tolerance band for these fluctuations.  $\delta Y_d$  may be due to errors in estimating and/or predicting the values of the design-dependent parameters. The fluctuations  $\delta Y_i$  represent variations over which the designer has no control or very limited control. They are primarily because of uncertainty in the values of  $Y_i$ .

Because the amount of these fluctuations is unknown,  $X$ ,  $Y_d$ , and  $Y_i$  are actually random variables. Thus, the evaluation attribute  $E$  is optimized while a set of stochastic functional relationships constraint the vector of design variables. Since design-independent parameters are empirical quantities, the uncertainty associated with them can often be expressed by probability distributions. Design variables and design-dependent parameters are decision variables. As argued by Morgan and Henrion (1990), it is generally inappropriate to represent uncertainty about decision variables by probability distributions. Instead, a parametric sensitivity analysis should be conducted on these quantities, that is to examine the effect on the outcome of deterministic changes to the uncertain quantity.

The uncertainties about design variables, design-dependent parameters, and design-independent parameters may have significant effects on design decision making. Two of the problems they may cause are discussed below:

***Feasibility.*** In a constrained design space, the scope of the feasible region may be reduced due to variations in design-dependent and/or design-independent parameters. In many traditional design optimization formulations, an optimal solution is obtained by assuming a "best value" for each uncertain parameter. If some of the "best estimates" vary in practice, the optimal solution previously identified may not be feasible.

***Performance variations.*** The traditional approach to design optimization is to optimize an idealized model and then rely on a continuity principle: what is optimal at the model should be optimal nearby. Unfortunately, this reliance on continuity is confounded: the classical optimized procedures tend to be discontinuous in the statistically meaningful topologies (Huber, 1977). Because

of uncertainties, both the values of design variables and system parameters may deviate from the ideal conditions when a system is implemented. As a result, the variation of the evaluation attribute will increase. The uncertainty associated with estimates of outcomes is often regarded as a risk in system design.

### **3.4 The Concept of Robust System Design**

#### **3.4.1 Dealing with uncertainty in system design**

One of the principal aims of engineering design is the assurance of system performance within the constraint of economy. Indeed, the assurance of performance is primarily (if not solely) the responsibility of designers. The achievement of this objective, however, is generally not a simple problem, particularly for large systems. Risk is generally implicit in all engineered systems.

There are three ways to approach this uncertainty in the engineering design decision process:

- 1) Obtaining better estimates of uncertainties in design-independent parameters. If the decision maker must act and cannot delay the problem, then the estimate of the probability should be made in such a way as to take into account of the serious outcomes.
- 2) Controlling the variations in controllables. The variations in the settings of design variables can be reduced by enforcing tighter control. However, reducing the tolerance band will increase manufacturing costs.
- 3) Controlling the transmitted variation by minimizing sensitivities of constraints and objective function to various variations. Developing a design which is less sensitive to the uncertain factors is called robust design in Taguchi's terminology. For some key inputs which are outside of the manufacturer's control, e.g., usage conditions, only robust design will work. It is not possible to tighten up on usage conditions without reducing the functionality of the

system. Instead, interaction between usage conditions and other key inputs can be used to make the system insensitive to the variations in usage conditions.

### **3.4.2 The philosophy of robust design**

The logic of robust design can be illustrated with an example from the military. It is clear that the outcome of a battle often depends on what an enemy does. But what the enemy does cannot be accurately predicted. The strategy should be to attempt to develop equipment and tactics which are less sensitive to whatever the enemy does.

As applied in engineered system design, the concept of robust design is very important. When a candidate design is selected and realized, the system's response depends both on the values for design variables and uncontrollable system parameters (or noise factors). In many instances, the optimum values for the controllable system design variables are obtained to optimize the evaluation function with respect to its target value. The variation of the evaluation attribute with respect to uncontrollable parameters is often ignored in this process. Since the values of the uncontrollable factors are uncertain in the process of system design and planning, the robustness of the proposed design is essential in implementing a solution on a real system. By requiring the design to be insensitive to the uncertainty in the value of system parameters, an additional criterion is available to distinguish between designs which are approximately equivalent in meeting other design criteria.

### **3.4.3 Definitions of robustness**

As used in engineered product and process design, robustness is a vague construct or concept devised for measuring the desirability of a design. To study the concept, we need to operationalize and define it. The most common way to operationalize a concept is to select measurable variables to represent the concept. However, one must keep in mind that these variables only give an incomplete representation.

Before selecting specific variables to study, we need to review the literature to determine how other researchers operationalize the concept of robustness. The literature must be examined critically and problems with operational definitions of concepts should be noted.

According to the American Heritage Dictionary (Houghton Mifflin, 1985), the word "robust" has five meanings: (1) vigorous, (2) powerfully built, (3) requiring or suited to physical strength or endurance, (4) rough, and (5) marked by richness and fullness. In the scientific research community, the word "robust" is loaded with many – sometimes inconsistent – connotations. To study the robustness of statistical methods and models, "robust statistics" has been developed into an important branch of statistics. In the sense of statistical analysis, "robustness" means the insensitivity of the decision to uncertain assumptions in the analysis (Huber, 1977). It signifies insensitivity of the decision against small deviations from the assumptions.

Thanks to the recent success in applying Taguchi's philosophy of product and process design, "robustness" has become a popular term in the engineering design community. The original idea of Taguchi's robust design is to use statistically planned experiments to identify process control parameter settings that reduce the process's sensitivity to manufacturing variation (Kackar and Shoemaker, 1986).

The word "robustness" now means different things to different people. The connotation depends on the purposes of the study and the environment wherever the concept is used. For some researchers, robust design means minimizing the variations in system performance with respect to various settings of design variables (Sundaresan et al., 1991; d'Entremont and Ragsdell, 1988). Optimal tolerance design is considered as a part of robust design by Parkinson et al. (1990). Parameter sensitivity analysis is another term for robust design (Eggert and Mayne, 1990; Beltracchi and Gabriele, 1988). Among others, the term "robust design" is used as a buzzword to label any design optimization techniques. There is no general and formal definition given in the literature.

As indicated before, uncertainties are involved in all phases of a system's life cycle. Thus, the robustness of a system should be studied from the perspective of life-cycle engineering. Various definitions of robust design used in developing engineered systems can be summarized into a general definition:

In system design, robustness expresses the insensitivity of the system's performance to uncertainties in both the system acquisition phase and the system utilization phase.

In the preliminary design stage, design analysis and evaluation depends upon the decision model used to represent the system. As identified in Section 3.2, uncertainties are associated with decision variables and design-independent parameters. The uncertainty about design-independent parameters is not controllable. The variations of decision variables are controllable. With the help of a design evaluation function, the robustness of a candidate system can be estimated by assessing the variations of the evaluation attribute due to uncertainties in decision variables and design-independent parameters. Thus, by incorporating the general definition of robust design into a design decision model, two operational definitions are obtained, each representing the decision maker's concern to each type of uncertainty:

- 1) Robustness represents the insensitivity of the system's evaluation attribute to the uncertainty in uncontrollable (design-independent) parameters.
- 2) Robustness represents the insensitivity of the system's evaluation attribute to uncertainties in design-independent parameters as well as variations in design variables and design-dependent parameters.

### **3.5 Measures of the Robustness of Candidate Systems**

Due to various uncertainties, the evaluation attribute of a system is a random variable. The robustness of a candidate system can be expressed and estimated by studying the variations of the evaluation attribute due to various uncertainties. Some of the common measures of the variations in the evaluation attributes are identified next.

#### **3.5.1 Probability distribution**

As discussed in Section 3.2, there is a functional dependency between the evaluation attribute  $E$  and design variables  $X$ , design-dependent parameters  $Y_d$ ,

and design-independent parameters  $Y_i$ . Since  $E$  is a function of random variables, it is a random variable itself and cannot be described deterministically.

The randomness in a physical process and unknown states of nature contribute to uncertainty. The conceivable or possible realizations of system response can be represented with a probability mass function (PMF) or a probability distribution function.

If an evaluation attribute  $E$  is continuous, its cumulative distribution function (CDF) is

$$F_E(e) = P(E \leq e) \text{ for all } e \quad (3.1)$$

If  $F_E(e)$  has a first derivative, the probability density function (PDF) of  $E$  is

$$f_E(e) = \frac{dF_E(e)}{de}. \quad (3.2)$$

The probabilistic characteristics of an evaluation attribute would be described completely if the form of the distribution function (or PMF) and the associated parameters are specified. A probability distribution, in terms of the evaluation attribute, contains all the information. In practice, the form of the distribution function may not be known; consequently, approximate description of a random variable is often necessary. The probabilistic characteristics of the evaluation attribute may be described approximately in terms of certain main descriptors of the random variable. The most important of these quantities are the central value of the evaluation attribute, and a measure of dispersion of its values.

Moreover, even when the distribution function is known, the main descriptors remain useful. In practice, it usually is hard to look at probability distributions and internalize the risk and opportunities of various design alternatives. Rather than try to assimilate the entire probability distribution for the evaluation attribute, comparisons can be made on the basis of some summary measures.

### 3.5.2 Mean, variance, and standard deviation

One of the most important summary measures for a random variable is the expected value of  $E$ . For a discrete evaluation attribute  $E$  with probability mass function  $p_E(e_i)$ , its expected value, denoted by  $\mu_E$ , is

$$\mu_E = \sum_i e_i p_E(e_i). \quad (3.3)$$

Similarly, for a continuous evaluation attribute  $E$  with PDF  $f_E(e)$ , the mean value is

$$\mu_E = \int_{-\infty}^{\infty} e f_E(e) de. \quad (3.4)$$

Using the mean of the evaluation attribute is not enough to describe its probabilistic characteristics. The variation of the evaluation attribute around the mean results in a risk in system design. To measure the risk, we need to determine the variability or dispersion in the evaluation attribute.

If the deviations are taken with respect to the mean value, a suitable average measure of dispersion is the variance. For a discrete evaluation attribute  $E$  with probability mass function  $p_E(e_i)$ , the variance of  $E$ , denoted by  $\sigma_E^2$  is

$$\sigma_E^2 = \sum_i (e_i - \mu_E)^2 p_E(e_i). \quad (3.5)$$

If  $E$  is continuous with PDF  $f_E(e)$ , the variance is

$$\sigma_E^2 = \int_{-\infty}^{\infty} (e - \mu_E)^2 f_E(e) de. \quad (3.6)$$

Dimensionally, a more convenient measure of dispersion is the square root of the variance, or the standard deviation  $\sigma$ . That is:

$$\sigma_E = \sqrt{\sigma_E^2}. \quad (3.7)$$

It is hard to say, solely on the basis of the variance or standard deviation, whether the dispersion is large or small. For this purpose, the measure of dispersion relative to the mean is more useful. Thus, the coefficient of variation (COV),

$$\delta_E = \frac{\sigma_E}{\mu_E} \quad (3.8)$$

is often a preferred and convenient nondimensional measure of variability.

The use of variance as a measure of robustness of a system implies that deviations below the expected value are regarded in the same way as deviations above the expected value (Figure 3.1). Even though this measure has been criticized as too conservative, since it regards all extreme values as undesirable, variance is still a popular measure of risk because of its familiarity and ease of computation (Mantell, 1972).

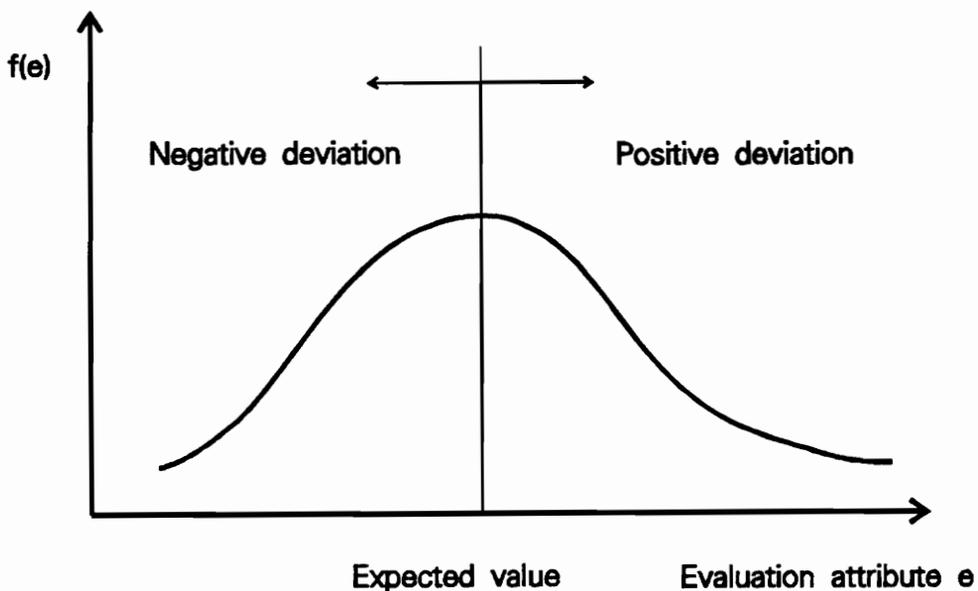


Figure 3.1. Variance as a measure of an evaluation attribute's variability

### 3.5.3 Semivariance

Variance is an even function of the deviations. Whether a deviation is above or below the mean value is of no significance. In some cases, however, we are concerned with the variability only on the undesirable side of the expected value. The semivariance of the evaluation attribute is a measure focusing on such variability (Figure 3.2).

For a continuous evaluation attribute  $E$  with PDF  $f_E(e)$ , the semivariance,  $S_h$ , is

$$S_h = \int_{-\infty}^h (h - e)^2 f_E(e) de. \quad (3.9)$$

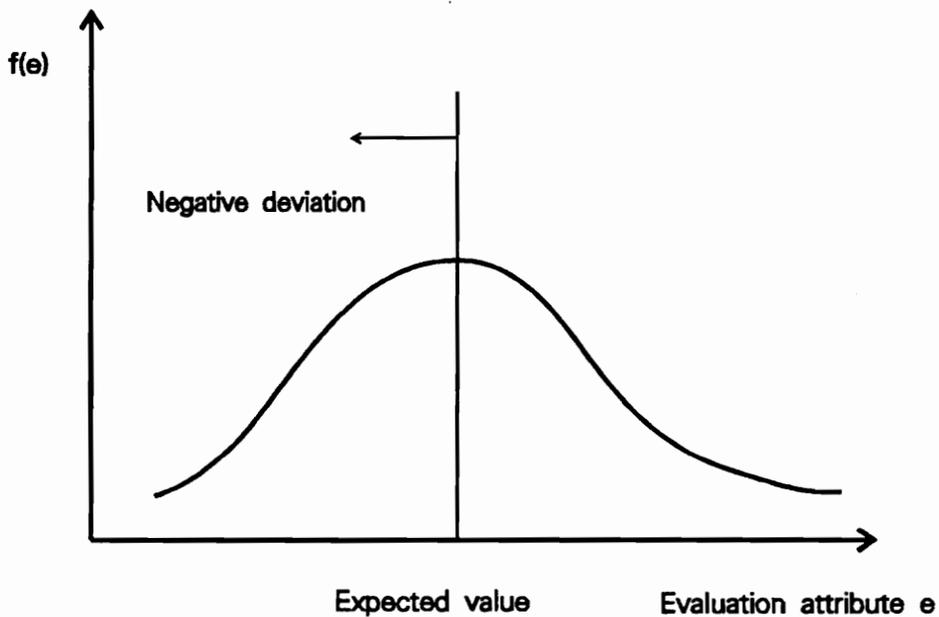


Figure 3.2. Semivariance as a measure of an evaluation attribute's variability

### 3.5.4 Probability of loss

Another measure of robustness for a system is the probability of loss criterion (Bonni, 1975). This measure, along with some variants of it, has become known as the reliability criterion or the safety-first rule in the community of engineering design, particularly in civil engineering. The measure treats only the values of the evaluation attribute below a certain value as unfavorable (more is preferable) (Figure 3.3). The critical level is called aspiration level, which is widely used in project evaluation. For example, if the evaluation attribute of concern is the reliability of a system, the probability of loss measure considers only the possibilities of reliability being below a critical level, say 0.8.

For an evaluation attribute  $E$  with PDF  $f_E(e)$ , if its aspiration level is  $e_h$ , the probability of loss is

$$P_L = P(E < e_h) = \int_{-\infty}^{e_h} f_E(e) de, \quad (3.10)$$

The probability-of-loss calculation obscures the magnitude of the variability of the evaluation attribute. Thus, this measure provides less information than the probability distribution itself.

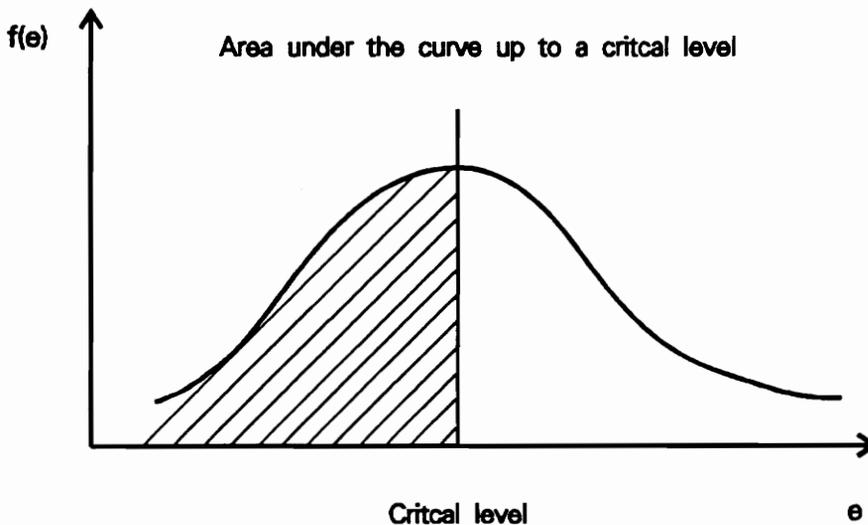


Figure 3.3. Probability of loss as a measure of a system's robustness

### 3.5.5 Taguchi's loss function

Taguchi (1986) recommends the use of a squared-error loss function to measure the loss in value due to the deviation of the evaluation attribute from its target value. For an evaluation attribute  $E$ , the loss function takes the following form:

$$L = k(E - e_T)^2 \quad (3.11)$$

where  $e_T$  is the target value of the evaluation attribute  $E$ , and  $k$  is a constant.

The function (3.11) can be expressed as

$$\begin{aligned} L &= k(E - e_T)^2 \\ &= k(E - \mu_E + \mu_E - e_T)^2 \\ &= k[(E - \mu_E)^2 + 2(E - \mu_E)(\mu_E - e_T) + (\mu_E - e_T)^2] \end{aligned}$$

Taking expectation of the loss function, we obtain the expected loss

$$E(L) = k[(\mu_E - e_T)^2 + \sigma_E^2]. \quad (3.12)$$

The first term within the brackets represents the bias. Thus, the expected loss is a function of both bias and variance.

### 3.5.6 Taguchi's signal-to-noise ratios

To evaluate the robustness of various candidate designs, Taguchi (1986) defined a series of statistics. These statistics are called signal-to-noise ratios. Taguchi classifies various design decision problems into three categories: smaller the better, larger the better, nominal the better. The signal-to-noise ratio is defined for each category below.

**Smaller the better (STB).** The overriding concern is to get the value of an evaluation attribute as close as to zero as possible. To obtain as many of the values as low as possible requires concentrating on both reducing the average and on reducing the variation around this average. Taguchi recommends the following performance measure (Taguchi, 1987):

$$PM = -10 \log[\mu_E^2]. \quad (3.13)$$

Let  $e_1, e_2, \dots, e_n$  approximate a random sample from the distribution of  $E$  for a given level of design-independent parameters. Taguchi presents the following signal-to-noise ratio to approximate the performance measure:

$$S/N = -10 \log \left[ \sum_{i=1}^n \left( \frac{e_i}{n} \right)^2 \right]. \quad (3.14)$$

**Larger the better (LTB).** In this type of problem the overriding concern is getting some characteristic as high as possible. Lower values must be guarded against. To get as many of the values as high as possible requires concentrating primarily on driving the average higher. However, variation cannot be ignored. No matter how high an average is obtained, excessive variation can still cause some units to fall below the lower specification limit. The performance measure and signal-to-noise ratio for this case are (Taguchi, 1987)

$$PM = -10 \log \left[ \frac{1}{\mu_E^2} \right], \quad (3.15)$$

$$S/N = -10 \log \left[ \sum_{i=1}^n \left( \frac{1}{ne_i^2} \right) \right]. \quad (3.16)$$

**Nominal the better (NTB).** The third category is characterized by the existence of an ideal value called target value. Every unit should be as close to this target value as possible. Both excessively high and excessively low values must be guarded against. This requires the average be as close to the target as

possible and minimizing the variation around this target. The performance measure and signal-to-noise ratio recommended by Taguchi are (Taguchi, 1987):

$$PM = 10 \log \left[ \frac{\mu_E^2}{\sigma_E^2} \right], \quad (3.17)$$

$$S/N = 10 \log \left[ \sum_{i=1}^n \left( \frac{e_i^2}{s^2} \right) \right], \quad (3.18)$$

where  $s$  is the standard deviation of the sample.

### 3.5.7 Sensitivity ratio

To use semivariance and the probability of loss as a measure of robustness in design evaluation, full knowledge of the probability distribution of each alternative's evaluation attribute is required. As an alternative, when the design evaluation function

$$E = f(X; Y_d, Y_i)$$

is defined for the design decision problem, the variation of the evaluation attribute with respect to the design-independent parameters may be estimated directly by using the theory of sensitivity analysis of linear systems.

Let  $Y_i^*$  = the estimated value of  $Y_i$ . If the design evaluation function is differentiable, the sensitivity of the evaluation attribute with respect to  $Y_i$  is

$$\left| \frac{df(X; Y_d, Y_i)}{dY_i} \right| \text{ for } Y_i = Y_i^*. \quad (3.19)$$

The objective of robust design is to find  $X$  so that the sensitivity ratio (3.19) becomes minimum. For specified values of  $X, Y_d$ , and  $Y_i$ , the change of  $E$  due to the variation  $\delta Y_i$  in  $Y_i$  is then given by

$$\delta E = \left| \frac{df(X^*; Y_d^*, Y_i^*)}{dY_i} \right| \delta Y_i. \quad (3.20)$$

### **3.6 Review of Some Existing Approaches**

There are a number of approaches developed for design analysis and evaluation. In this section, after discussing the limitations of some traditional techniques, the advantages and limitations of Taguchi's approach are reviewed. This review indicates some areas for further research.

#### **3.6.1 Limitations of the traditional approaches**

The difficulty with most existing tools for design analysis and evaluation is not that they solve the problem incorrectly, but they are being applied to solve the wrong problem (Sandgren, 1989). Little research has been conducted on the development of decision analysis methodology for early system design activities. There is not a complete, cohesive structure for the determination of design criteria, their modeling in terms of system variables and parameters, the synthesis and screening of alternatives, and formal optimization. Most of these activities and decisions have been accomplished in an ad hoc manner or, at best, in separate activities without close coordination with other segments of the process. The selection of design criteria is often subjective and influenced by factors such as design application, judgment of the designer, timing, etc.

Deterministic optimization techniques have been employed to solve a wide variety of engineering design problems. Nonlinear programming has shown some promise as a general design tool. But the rigid structure imposed by the problem formulation has made it difficult to include many important design issues. There is no convenient way to bring knowledge of the design trade-offs into the optimization. Deterministic models do not portray the nature and impact of random variations that occur in actual manufacturing processes or operating conditions (Eggert, 1991). In an optimally designed system based on deterministic considerations, the designs may be sensitive to variations in design variables or system parameters. Such variations may lead to unexpected constraint violations and an unsatisfactory design (Eggert, 1991). Consequently, the optimized system tends to be more sensitive to fabrication defects and improper definition of the environment (Ang and Tang, 1984).

### **3.6.2 Taguchi's approach**

To improve product quality, a new approach to engineering design has been developed by Taguchi. Different from traditional optimization methods, Taguchi's approach employs statistically designed experiments for product and process design. Taguchi (1986) divided the design process into three steps: system design, parameter design, and tolerance design. At the heart of Taguchi's philosophy is the concept of the quality loss function, which is used as a criterion to be optimized in parameter design. Quality loss is defined as the loss incurred by society from the time a product is released for shipment (Taguchi et al., 1989).

Taguchi's work is closely aligned with statistical experimental design and addresses the uncertainty issue as a normal part of the design process. The philosophy is to identify settings of controllable factors that minimize performance variations, while keeping performance as close as possible to its target value. Parameter design is usually accomplished by using an orthogonal experimental design approach.

Experiences indicated that Taguchi's parameter design approach worked well in manufacturing after the system design has been completed. But it is difficult to apply at the conceptual design level. Since the model development and interpretation of the approach relies on direct experimentation, it is difficult to apply to designs which do not yet exist. However, as Sandgren (1989) indicated, the time to consider the sensitivity of a design change should be during the initial design phase. If design sensitivities are considered early on in the process, it may well reduce the number of local minima present.

Taguchi's philosophy of reducing variation in performance through reducing the sensitivity of an engineering design to sources of variation rather than controlling the sources is very important to the development of quality design. However, this concept is often used without considering other concerns such as costs of experimentation and manufacturing. Otto and Antonsson (1991) argued that applying this concept in preliminary system design would generally lead to overly expensive products. In the preliminary design stage, the design-dependent parameters are selected. According to Taguchi's concept, there would be an illusion that we should always pick the parameters which minimize

performance variations, even if this means greatly increased expense to the designer, manufacturer, or company. This is unacceptable, as Taguchi readily admits (1986). In this sense, Taguchi's loss function is not complete. System life-cycle cost may be a more appropriate measure.

As with any new techniques, there are many criticisms and controversies regarding Taguchi's approach. Much of the controversies are focused on technical issues that pertain to certain pieces of the overall scheme. As stated by Box (1985), it is very important to separate Taguchi's quality engineering ideas from the statistical techniques he used to put these ideas in practice.

## **IV. DESIGN EVALUATION BY DECISION THEORY: CLASSIFICATION AND MODELING OF DESIGN DECISIONS**

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- 4.1 Introduction**
  - 4.2 Design Evaluation by Decision Theory**
  - 4.3 Concepts of Decision Modeling**
  - 4.4 Classification of System Design Decision Problems**
  - 4.5 Discrete Decisions and Discrete Events**
  - 4.6 Discrete Decisions and Continuous Events**
  - 4.7 Continuous Decisions and Discrete Events**
  - 4.8 Continuous Decisions and Continuous Events**
- 

### **4.1 Introduction**

The objective of design evaluation is to identify a best design. In this chapter and Chapter V, problems of design evaluation in the face of uncertainty are studied based upon statistical decision theory. The focus of this chapter is on the structuring and modeling of design decisions in the face of uncertainty. Chapter V will discuss the concepts and approaches for choosing a best design.

### **4.2 Design Evaluation by Decision Theory**

#### **4.2.1 Robust design vs. "best design"**

To solve a design problem is to make the best choice from among the available courses of action. In order to maximize the chance of attaining or approximating the best solution to a design problem, one must understand what

the "best" solution to the problem is. However, as pointed out by Ackoff (1962), it is not at all obvious what is meant by the "best" solution to a problem. A final definition of "best" in this context has not yet been attained, and it is not likely that it ever will be.

In resolving system design decisions in the face of uncertainty, does a robust design as defined in Chapter III represent the best design for the overall decision problem? To answer this question, three aspects of the design decision problem must be considered. One is the variations in the value of the system's evaluation attribute(s) due to various uncertainties. Experience indicates that use of mean as the decision criterion for design evaluation in the face of uncertainty may result in a poor design. Attempts to minimize the variation of the evaluation attribute have led to the philosophy of robust design. Actually, the concept of minimization of variation is often incorrectly interpreted as Taguchi method. This misconception leads some to believe that variance minimization is an objective criterion for identifying a best design.

A design which generates a minimum variance for the evaluation attribute is not necessarily the best design. Variations represent the risks involved in the process of design evaluation. In comparing various candidate designs, one must keep in mind that different decision makers may not have identical risk attitudes. This subjective nature of the DM must be considered in order to select a best design. Thus, the second aspect of the design decision problem concerns the risk attitude of the DM toward various levels of the evaluation attribute.

Taguchi's robust design approach is often used by considering only a single attribute. However, a design which is optimal for individual attributes of a system may not be best overall. Taguchi's approach does not address the third aspect of the design decision problem; that is, the value trade-offs among multiple attributes or objectives.

In many cases, to identify the best design, designers have to consider more than one attribute. Some attributes may be more important than others and some may be hard while others may be soft criteria. Specific requirements for these attributes may be associated with any level of the design process. Examples of the requirements include factors such as how well the design specifications are met as well as cost, reliability, and maintainability. To obtain

the best design possible with the resources available, the DM must weigh value judgments that involve various factors. Thus, to make the optimal decision is to choose the alternative from among those available that will give the best performance, considering all factors, including robustness. This best performance represents the optimum compromise of all the factors considered.

Thus, to resolve design decisions problems under uncertainty, one must consider performance variations, risk attitudes, and value trade-offs jointly. We would like to select the alternative which is expected to result in the greatest degree of achievement of our objectives. A best design is not only robust for an individual attribute, but also provides an optimal trade-off among various attributes of concern. In this sense, the "best design" is subjective. It depends upon the value preferences and risk attitudes of the decision maker.

#### **4.2.2 Design evaluation by decision theory**

As defined in Section 3.4.3, "robustness" means the insensitivity of a design to uncertainties in both the system acquisition stage and the system utilization stage. Since the design of engineered systems is often accomplished without complete information, the assurance of performance can seldom be perfect. Moreover, many decisions that are required during the process of planning and design are invariably made under conditions of uncertainty. Therefore, there is invariably some chance of nonperformance or failure and of its associated adverse consequences; hence, risk is often unavoidable. Under such conditions, it is not feasible (practically or economically) to assure absolute performance of engineered systems. Thus, instead of talking about robust design in a narrow sense, the purpose of design optimization should be to develop the "best design" by considering three aspects of the design decision problem discussed above.

In determining what a design decision is best, one is concerned with the choices a decision maker *should* make, not necessarily with those the DM normally makes. In order to apply the concept of "best decision" to design evaluation under uncertainty, it is necessary to evaluate the losses (and gains) from falsely (or correctly) rejecting or accepting an alternative. It is also necessary to evaluate the losses due to error in estimating the value of a

parameter, when this estimate may be used for many purposes of which the research cannot be aware. Thus, design evaluation in the face of uncertainty is actually a problem of decision making under uncertainty. As indicated by Singpurwalla (1991), the term "design by decision theory" would more accurately encapsulate the totality of Taguchi's ideas. Such an approach would raise the level of awareness in engineering design by shifting emphasis from the narrow aspect of experimental design to the more encompassing one of decision making under uncertainty.

The decision analysis approach can integrate the key steps of engineering design. Instead of focusing on certain parts of the design decision process, such as experimental design, signal to noise ratios, etc., the decision analysis approach concentrates on the overall design decision process. Statistical theory underlying this theme is well developed, and like the statistical theory of the design of experiments, design engineers should learn to apply the results of this theory to design practice.

### **4.3 Modeling of Design Decision Problems**

#### **4.3.1 Decision tree**

Design decision problems in the face of uncertainty are made up of decisions and uncertain events. The structure of a decision problem in terms of the sequence and causal relationships between various decisions and uncertain outcomes can be effectively represented by a decision tree.

Conceptually, the symbolic logic of the decision tree representation is closely akin to that of network analysis. Decision trees are built up as a connection of essentially two fundamental units, namely decision nodes and chance nodes. Decision nodes are conventionally represented by a square box (Figure 4.1) and indicate that subsequent nodes connected to this box can be reached according to deterministic choice on the part of the decision maker at this point. The set of subsequent nodes attached to the box will thus represent the DM's set of feasible decisions, and these nodes can be future decision nodes, terminal payoffs or, more usually, chance nodes.

Chance nodes are conventionally represented by circles (Figure 4.1) and indicate that the set of subsequent nodes connected to this circle will be reached according to some probabilistic process over which the DM has no control (although the DM will typically have some beliefs upon which are more or less likely than others). Thus this set of subsequent nodes will represent the set of possible outcomes and will be either future decision or chance nodes, or a final terminal payoff.

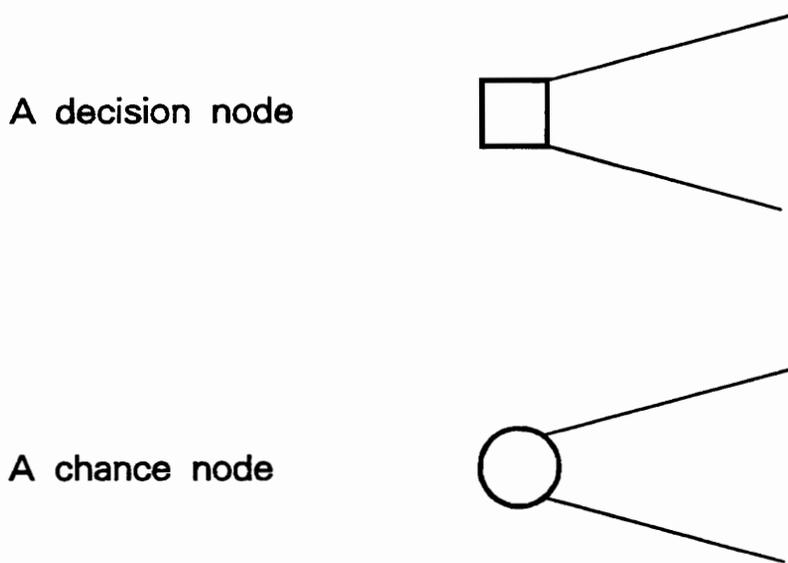


Figure 4.1. A decision node and a chance node

Decision trees link together these two types of nodes to represent possible outcomes. For example, suppose a particular decision, say,  $a_i$ , is selected and the uncertain event,  $s_j$ , occurs. The occurrence of the event will remove all uncertainty from the problem and the action,  $a_i$ , will produce a definite result which can be foreseen with certainty. In other words, the combination of  $a_i$  with  $s_j$  will result in a foreseeable consequence. This consequence can be written  $c_{ij}$ . An example of decision trees is given in Figure 4.2.

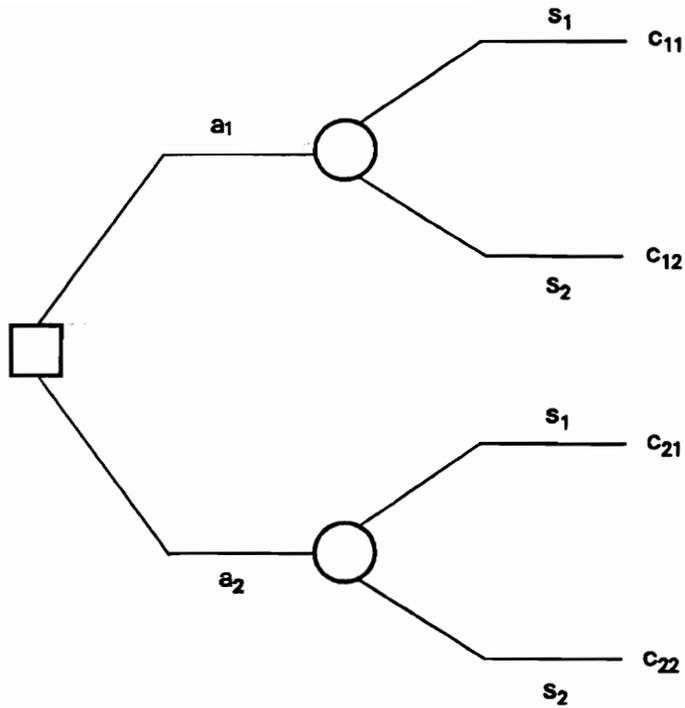


Figure 4.2. An example of decision trees

### 4.3.2 Two types of outcome spaces

The starting point for the modeling of decision making under uncertainty is to specify the outcome space. The end points in a decision tree represent the outcome space for the model of the decision problem. The levels of the evaluation attribute assigned to the end points are specific values of a random variable.

Basically, the outcome space for design decision problems can be modeled either discretely or continuously. The type of models used for describing an outcome space depends on the characteristics of both design alternatives and design-independent parameters. If the outcomes of a design decision problem are either continuous or consist of a large number of possible outcomes, a continuous model should be used. For example, in some cases, the outcome

space is actually continuous, such as the gas mileage of a car (neglecting limitations due to measurement accuracy). In other cases, the outcome space is defined essentially by continuous variables. For example, the number of new cars sold in a certain geographical area during a month might cover the range of whole numbers between 1,000 to 10,000. From a practical point of view, it may be impossible to separately assess probabilities for each of the 9001 points required by such a model. A model based on a continuous set of outcomes may be the best approximation available for this essentially continuous outcome space.

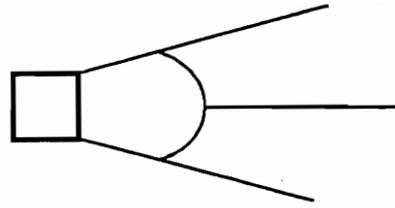
Decision diagrams represent continuous random variables by fans and a single representative outcome. They do not show individual branches. Instead, event fans and alternative fans are used. Figure 4.3 shows a hypothetical alternative fan and a hypothetical event fan.

### **4.3.3 Conversion of continuous probability models to discrete models**

Continuous probability models are often used to represent uncertain events with continuous outcomes or a large number of outcomes in order to obtain a good model. If a continuous probability distribution can be approximated by a discrete distribution, computations can be facilitated. The ability to generate discrete approximations for continuous distributions allows all definitions and manipulations for discrete random variables to be used for continuous random variables.

In principle, discrete approximations to continuous probability distributions can be made as accurate as desired. The limiting factor is the number of intervals used in the approximation. With the availability of computers, it is feasible to use a large number of discrete points similar to discrete approximations used in numerical integration. On the other hand, a small number of intervals usually provides an adequate approximation. There are several ways of making this approximation. The essential problem is to capture the important characteristics of a distribution with a few discrete points.

A continuous alternative



A continuous event

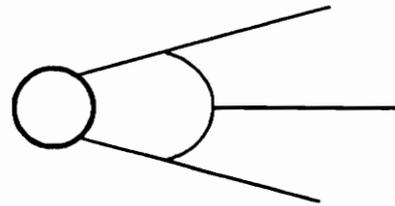


Figure 4.3. A continuous alternative and a continuous event

#### 4.4 Classification of System Design Decision Problems

The outcome space of a system design decision problem is determined by three factors: the set of design alternatives, the set of design variables, and the set of design-independent parameters. In preliminary system design, the most common situation is that there exists a list  $a_1, a_2, \dots, a_l$  of  $l$  exclusive and exhaustive design alternatives. The design alternatives are usually identified before design evaluation is started. For a design alternative, the design variables form the set of decision variables for the problem. The various combinations of the settings of the design variables determine a set of possible decisions available within the design alternative,  $\{d\}$ . Due to uncertainties in design-independent parameters, there is a set of uncertain events,  $\{s\}$ . The events are determined by the settings of the design-independent parameters. The decision problem is to select a single alternative and a single decision within the alternative, not knowing which member of the event set will be true.

Depending upon whether the decisions and the uncertain events can be represented by discrete models or by continuous models, various design decision problems can be classified into four categories (Table 4.1). Each category is discussed below.

Table 4.1. Classification of Design Decision Problems

| Decisions  | Uncertain Events |            |
|------------|------------------|------------|
|            | Discrete         | Continuous |
| Discrete   | Category 1       | Category 2 |
| Continuous | Category 3       | Category 4 |

#### 4.5 Discrete decisions and discrete events

In preliminary system design, a common situation is that for each design alternative, there exist a list  $d_1, d_2, \dots, d_m$  of  $m$  exclusive and exhaustive design decisions to be selected and there is a second list  $s_1, s_2, \dots, s_n$  of  $n$  exclusive and exhaustive uncertain events. Thus, both the decisions and the events can be represented by discrete models. The structure of this type of design decision problem can be represented by a multi-stage decision diagram as in Figure 4.4. Since both decisions and events are discrete, the outcome space of the problem is also discrete.

In constructing the decision tree, design alternatives are identified at the leftmost decision node. This corresponds to the first step of the Design Dependent Parameter approach. For each alternative, various decisions are identified at the decision nodes in the middle column by varying the settings of the design variables. The chance nodes reflect the uncertainties in the design-independent parameters. At each chance node, all of the possible uncertain

Design alternatives    Design decisions    Uncertain events

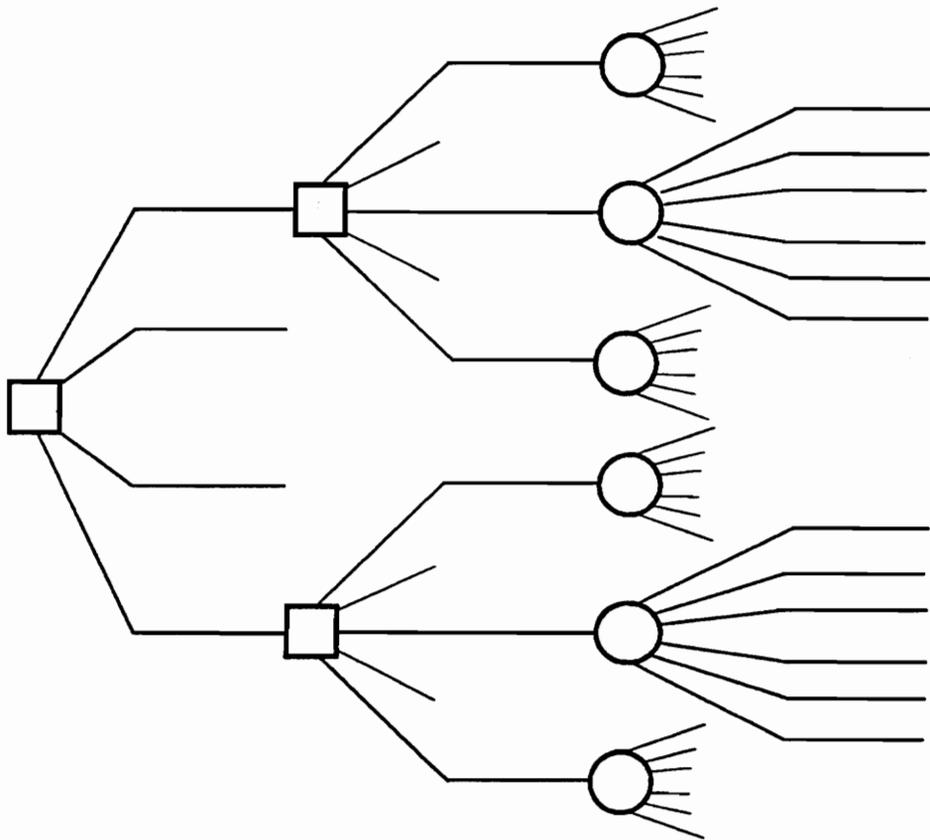


Figure 4.4. Discrete decisions and discrete events

events are identified by varying the possible settings of the design-independent parameters. Thus, the outcome for each decision within each design alternative depends upon the uncertain event taken.

The decision diagram provides a structured model for the decision process. Design evaluation begins with the right side of the tree and works backwards. For each uncertain event, the outcome of each decision is estimated based upon the design evaluation function. The outcomes from each decision within the same alternative are then compared to choose an optimal decision for each alternative. This step is called optimization within an alternative. Finally the optimal decisions from each alternative are compared to obtain the best decision for the overall problem.

#### **4.6 Discrete decisions and continuous events**

In this type of problem, for each design alternative identified, there are a finite number of exhaustive and mutually exclusive decisions,  $d_1, d_2, \dots, d_m$ , but there are a large or infinite number of uncertain events. The uncertain events must be represented by a continuous model. Such a decision problem can be represented by Figure 4.5. Since the events are continuous, the outcome space of this type of problem is also continuous. If the probability distribution of the events can be approximated by a discrete distribution, this category of problem is reduced to Category 1.

#### **4.7 Continuous decisions and discrete events**

When there are a large or infinite number of decisions for each alternative, decisions must be represented by a continuous model. Depending upon whether the events due to the uncertainty in design-independent parameters are discrete or continuous, the problems of continuous decisions can be divided into two categories: (1) the decisions are continuous, but the events are discrete, and (2) both decisions and events are continuous. The outcome space for both types of problems is continuous. The problems of continuous decisions and discrete events can be represented by Figure 4.6.

Design alternatives      Design decisions      Uncertain events

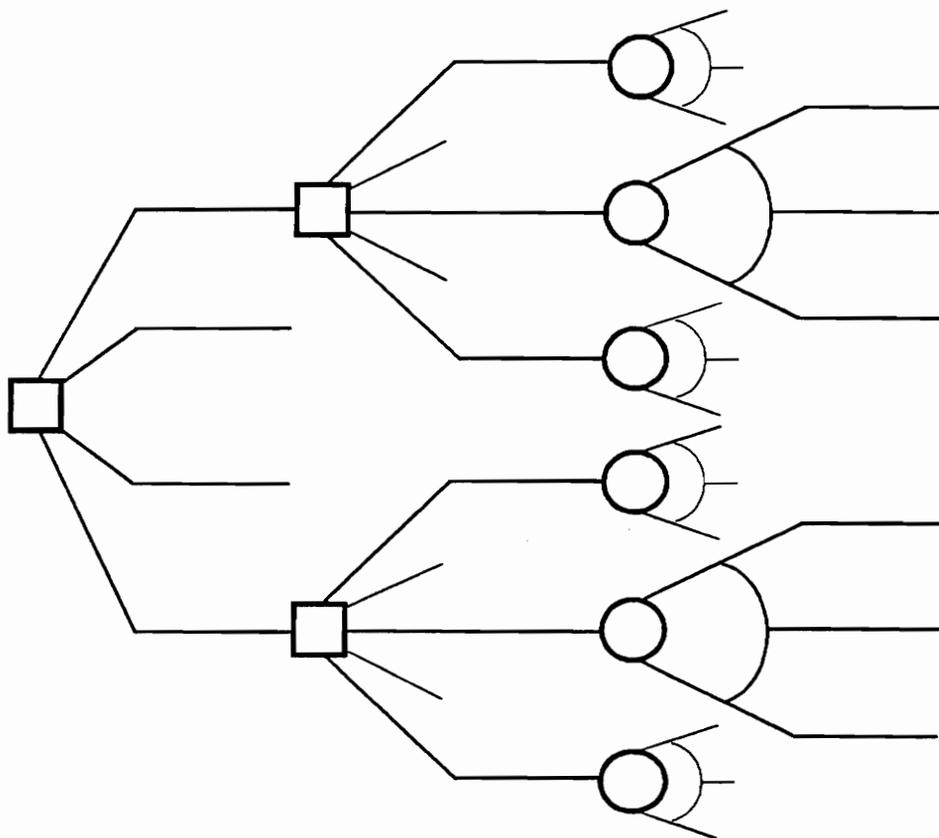


Figure 4.5. Discrete decisions and continuous events

Design alternatives    Design decisions    Uncertain events

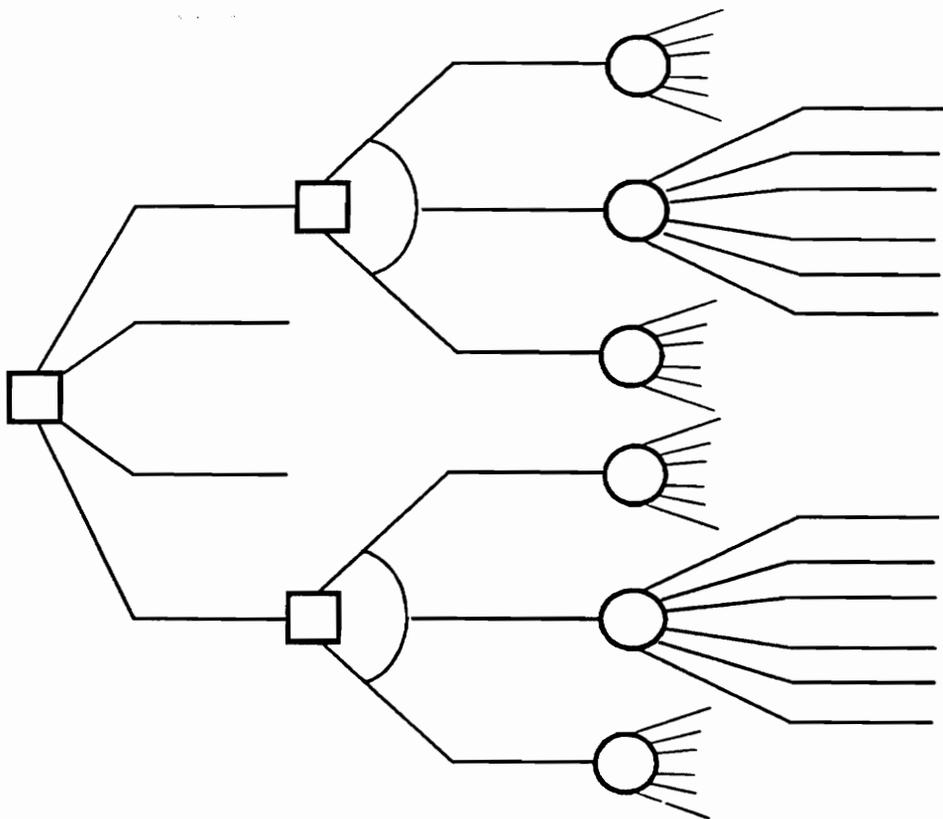


Figure 4.6. Continuous decisions and discrete events

#### **4.8 Continuous decisions and continuous events**

If both decisions and uncertain events are continuous, continuous models must be used. The decision tree for this category is given in Figure 4.7. If the probability distribution of the events can be approximated by a discrete distribution, this category is reduced to Category 3.

Design alternatives    Design decisions    Uncertain events

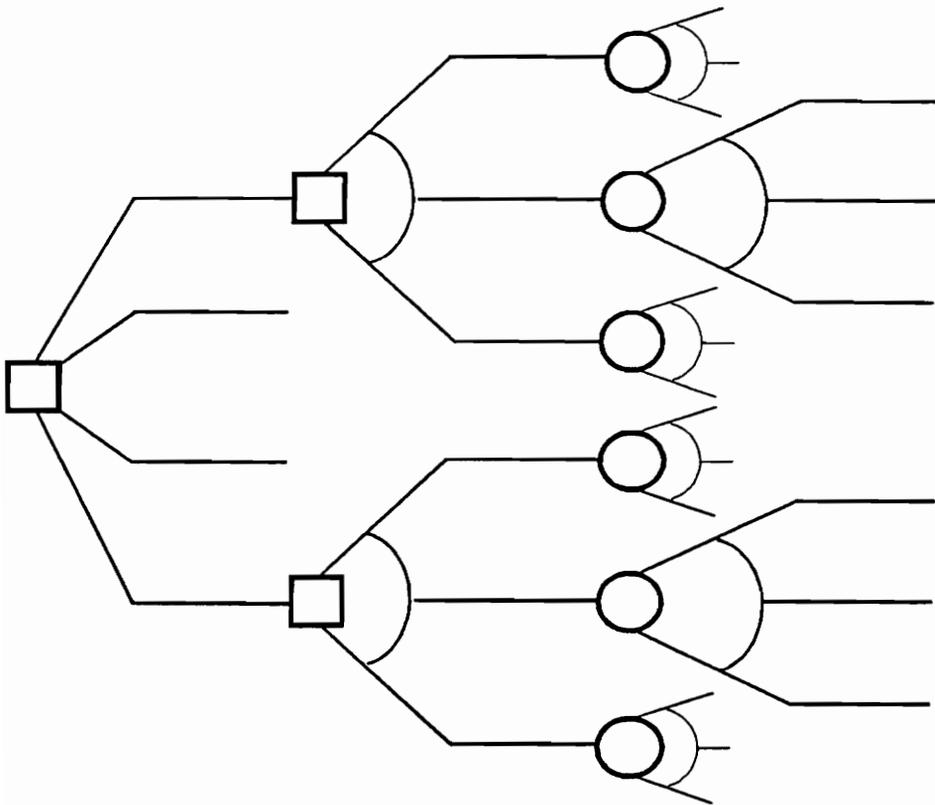


Figure 4.7. Continuous decisions and continuous events

## **V. DESIGN EVALUATION BY DECISION THEORY: FOUNDATIONS FOR CHOOSING A BEST DESIGN**

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- 5.1 Introduction
  - 5.2 Need for Decision Rules for Choosing a Best Design
  - 5.3 Existence of a Numerical Scale to Measure the Desirability of Designs
  - 5.4 The Concepts of Choices, Preferences, and Utility
  - 5.5 Sequential Decision Analysis Using the Maximum Expected Utility Principle
  - 5.6 Output Dominance and Stochastic Dominance
  - 5.7 Mean-Variance Analysis
  - 5.8 Assumptions Underlying Common Objective Functions
- 

### **5.1 Introduction**

Decision trees are useful for modeling and structuring the process of design evaluation in the face of uncertainty. To make a selection from various design alternatives, a general investigation of the desirability of system designs is needed. In this chapter, the concepts of preferences and choices are discussed under the context of design evaluation. By adapting statistical decision theory to the needs of design decision making, three decision analysis approaches are identified for design evaluation in the face of uncertainty. Then the assumptions underlying some objective functions which are commonly used in design optimization are also investigated.

### **5.2 Need for Decision Rules for Choosing a Best Design**

The objective of design evaluation is to identify a best design. But it is a long way between naming the objective and obtaining suitable decision rules for

representing the objective. Due to uncertainties with design-independent parameters, the evaluation attribute(s) of a candidate design is a random variable. The most complete way to describe the characteristics of a random variable is to use a probability distribution. In design evaluation, however, it is difficult to directly compare the probability distributions of the evaluation attribute from various design alternatives. Thus, decision rules are needed for the comparison.

Since the choice of the best design is a trade-off among different design characteristics, the evaluation of candidate designs depends on the decision rules applied. These decision rules form the basis under which alternative designs may be compared. They also provide the basis for formulating the design optimization problem.

For identifying appropriate rules and approaches for selecting a best design, some considerations are made in the research. First, the decision maker is assumed to be rational. Rationality means logical consistency in processing the information on which decisions are based. Consistency, in turn, requires that the information is stated explicitly and quantitatively. Rationality also implies validity in the models used for representing real-world systems. A model should accurately describe some set of system characteristics. However, the more accurately a real-world problem is described, the more complicated the description becomes. The models for implementing the decision process must, therefore, be manageable with the resources available to the decision maker. We would like to select the alternative which is expected to result in the greatest degree of achievement of our objectives. Furthermore, we would like our decision methodology to be generally applicable; to be applicable to any stage of the system life cycle.

Second, a quantitatively defined outcome is assumed for the design decision problem. A quantitative outcome is one which is (or is not) obtained in various degrees. A single evaluation attribute is used to represent the outcomes of the candidate designs. This attribute must be meaningful in the sense that it is adequate for the DM to choose among alternatives. To simplify the presentation, we assume that the DM would like to maximize the value of the evaluation attribute. This is true if the evaluation attribute is a measure of the design's performance effectiveness. However, the methods, theorems, and

rules developed for the larger-the-better case can be easily applied to the case of the smaller the better. The extension of the approaches to multiattribute design decision problems will be discussed in Chapter VIII.

### **5.3 Existence of a Numerical Scale to Measure the Desirability of Designs**

In order to develop the best design, a measure of the desirability of a design should be identified. To be consistent, the measure must be quantitative in character. It should be able to capture both the random nature of the evaluation attribute and the DM's attitudes towards it. To identify such a measure, we need to investigate the characteristics of the outcomes of design decisions.

The outcomes of design decisions in the face of uncertainty have two characteristics. One is the nonlinearity between usual evaluation measures, such as manufacturing cost, and their relative worth. In design evaluation, faced with similar sets of candidate designs with the same mean, two decision makers may not select the same alternative. This indicates that the mean of the evaluation attribute as the decision criterion may not reflect the DM's actual preferences for the attribute and his attitude toward risks.

Another characteristic of design decisions is that the outcomes, in general, are multidimensional. There is a fundamental difficulty involved in considering multidimensional outcomes. Evaluation and optimization of alternatives can be accomplished only with respect to a single criterion. Since all members of the set of criteria significantly influence the decision, no individual criterion can rationally be used as the only basis for the decision. These two characteristics indicate the existence of a preference scale, which measures relative contribution to success of the design.

The easiest and most useful way to order things is by means of numbers. It is natural to use this device in design evaluation. Our aim is to describe the desirability of design numerically, for numbers are the essence of the scientific method and it is by measuring things we know them. Specifically, what we want to do is to attach to any design alternative a number that describes the degrees of its desirability.

The needed scalar measure of relative contribution of design to success has been referred in the literature by various names: system worth, figure of merit, cost effectiveness, cost benefit, and utility. In fact, robustness is a new term for the same purpose. It describes the system's degree of fulfillment of needs and objectives under the influence of uncertain noises. However, robustness is not sufficient to represent the two characteristics of the outcomes of design decisions. A new measure is needed.

In this research, based upon statistical decision theory, a scalar measure, utility, will be used to represent the relative contribution of a design to success. This measure reflects the design's degree of fulfillment of the DM's needs and objectives. Although there are some unfortunate historical connotations to the term utility, there are advantages associated with the continued use of a term whose historical development can be traced and with which a considerable body of theory has been developed. Utility theory is introduced below under the context of design evaluation.

## **5.4 The Concepts of Choices, Preferences, and Utility Theory**

Decision trees can be used to model and structure the process of design evaluation. To make a selection from various design alternatives, a general investigation of the desirability of system designs is needed. Underlying all comparison methods of alternatives are assumptions regarding the DM's preferences and risk attitudes.

### **5.4.1 Basic concepts**

**Preferences.** The term "preference" is based on relationships among design alternatives. If a DM prefers one alternative to another, it is attributed to his "preferences." The focus of this research is on preferences for alternatives involving uncertain events. The uncertainty introduces an element called risk. An individual's risk preference reflects an underlying attitude toward uncertain outcomes.

**Utility.** Utility is defined on a numerical scale that represents the DM's preferences for a set of consequences. The higher the utility, the more desirable the consequence. Though any positive and negative numbers can be used to measure utility, it is convenient to measure utility on a probability scale so that the laws of probability can be used. The usual meaning of utility in economics is not the same as the preference scale here.

Utility is used to measure preferences for design alternatives with uncertain outcomes. It is sometimes difficult to attach a number to a consequence because the relevant features may not be naturally quantifiable. However, as indicated before, the scope of this research is limited to design problems in which a quantitative outcome is defined. If the outcome is denoted by  $e$ , its utility is represented by  $u(e)$ .

**Reference gamble.** A reference gamble can be established for any set of uncertain events. It is simply a two-outcome gamble (Figure 5.1). One outcome has a payoff greater than or equal to the maximum payoff for any outcome in the events considered. The other outcome has a payoff equal to or less than the minimum payoff for any outcome. In defining a reference gamble for design evaluation, two usable outcomes are the best possible outcome and the worst outcome possible.

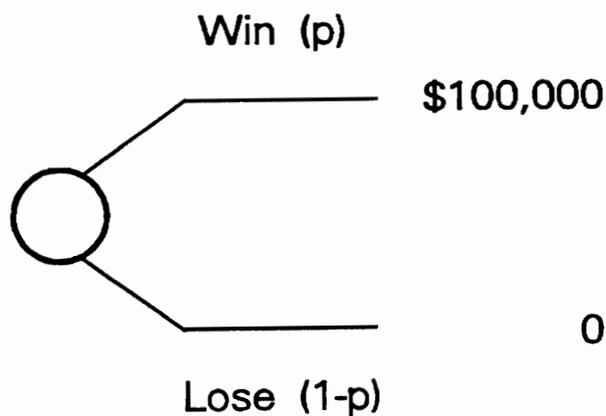


Figure 5.1. An example of reference gambles

***Certainty equivalent (CE).*** Certainty equivalents establish an equivalence between uncertain events and a certain value. For an uncertain event, its certainty equivalent is that certain value of an evaluation attribute which a DM is just willing to accept in lieu of the gamble represented by the uncertain event. A certainty equivalent is a decision, not an estimate. It is a value the DM decides to just accept in lieu of facing the uncertain event. It is not in any sense an estimate of what the DM thinks he will receive. Assessing certainty equivalents requires the DM to process information of two types simultaneously: (1) information on the probability that a set of outcomes will occur, and (2) information on the consequences of the outcomes as measured by the evaluation attribute.

#### **5.4.2 Attitudes toward risk**

Three basic attitudes toward risk can be identified: risk aversion, risk neutrality, and risk seeking. The choice process in design evaluation is affected by the type of risk attitude that the DM possesses.

##### **1) Risk aversion**

For an evaluation attribute of the greater the better, if the DM's certainty equivalent for an uncertain event is less than the expected value of the evaluation measure, the DM is called risk-averse. The difference between the expected value of the evaluation attribute and the certainty equivalent is called risk premium. Two special cases of risk aversion are deserve more attention:

***Decreasing risk aversion.*** One special case of risk aversion is decreasing risk aversion. This condition implies that the degree of risk aversion decreases as the value of the evaluation attribute increases. To be more precise, the risk premium decreases for gambles that are identical except for adding the same constant to each value of the evaluation attribute.

***Constant risk aversion.*** This condition implies that the risk premium is the same for gambles that are identical except for adding the same constant to

each level of the evaluation attribute. Constant risk aversion corresponds to an exponential preference function of the form

$$u(e) = a - b \exp(-\lambda e),$$

where  $\lambda$  is a constant that determines the degree of risk aversion, and  $a$  and  $b$  are scaling constants. These scaling constants can be used to make the preference function lie between 0 and 1 over the range of interest.

## 2) Risk neutrality

Risk neutrality corresponds to a zero risk premium. The preference curve is a straight line. Expected values are certainty equivalents for the special case of risk neutrality. Therefore, if the DM is risk neutral, choices can be made by comparing the expected values of the evaluation attribute for different design alternatives.

## 3) Risk seeking

Risk seeking behavior is the opposite of risk-averse behavior in that the certainty equivalent for a gamble is greater than the expected value of the evaluation attribute. Thus, the risk premium for risk seeking decision makers is negative.

### 5.4.3 Empirical evidence on risk-taking behavior

Empirical evidence indicates that individuals are risk neutral when the "stakes" are low. The most usual reaction when the "stakes" are high is risk aversion, although in special cases, including gambles with negative expected values, some individuals display risk-seeking characteristics. In general, it appears that decision makers in large companies are quite risk-averse (Holloway, 1979). In making design decisions, many designers have shown risk-averse behavior. As shown later in the chapter, Taguchi's philosophy of robust design by reducing performance variations is actually based upon the assumption that less risk is preferred in product design.

#### 5.4.4 Utility assessment using 50-50 gambles

The methods for obtaining utility curves can be divided into two categories. The first uses the basic reference gamble directly. The second method uses a variety of 50-50 gambles. The choice among the procedures should be based on ease of use by the DM. Presumably a procedure that is easier to think about will result in assessments that are more consistent and in which the DM will have more confidence. Since probabilities are difficult to conceptualize, particularly when small differences or small probabilities are being considered, the 50-50 method is recommended. The argument is that a 50-50 gamble is the simplest of all settings that include uncertainty and therefore is the best setting to use for assessing preferences. The utility assessment procedures using 50-50 gambles are summarized below (Holloway, 1979):

- 1) Establish the payoffs for a reference gamble for the decision problem.
- 2) Determine certainty equivalents  $CE_1$ ,  $CE_2$ , and  $CE_3$  for the reference gamble with

$$p = 1, p = 0, \text{ and } p = 0.5,$$

respectively. Record them on a plot with  $p$  on the vertical axis and the certainty equivalent on the horizontal axis. This establishes

$$u(CE_1) = 1.0, u(CE_2) = 0, \text{ and } u(CE_3) = 0.5$$

as the utilities for these certainty equivalents.

- 3) Create a sequence of new gambles, each with a probability of winning of  $p = 0.5$ . The payoffs  $CE_i$  and  $CE_j$  are varied and restricted to values of certainty equivalents previously specified (Figure 5.2).
- 4) Determine certainty equivalents,  $CE_k$ , for each gamble.
- 5) For each gamble calculate the expected utility of Alternatives A and B.  
For Alternative A,

$$E(U_a) = (0.5)u(CE_i) + (0.5)u(CE_j).$$

For Alternative B,

$$E(U_b) = (1)u(CE_k).$$

Indifference between A and B means that

$$E(U_a) = E(u_b) \text{ or}$$

$$u(CE_k) = (0.5)u(CE_i) + (0.5)u(CE_j).$$

- 6) Plot each  $[CE_k, u(CE_k)]$  pair with  $CE_k$  on the horizontal axis and  $u(CE_k)$  on the vertical axis.
- 7) Repeat steps 3, 4, 5 and 6 until the plot is well defined.
- 8) Draw a curve through the plotted points.

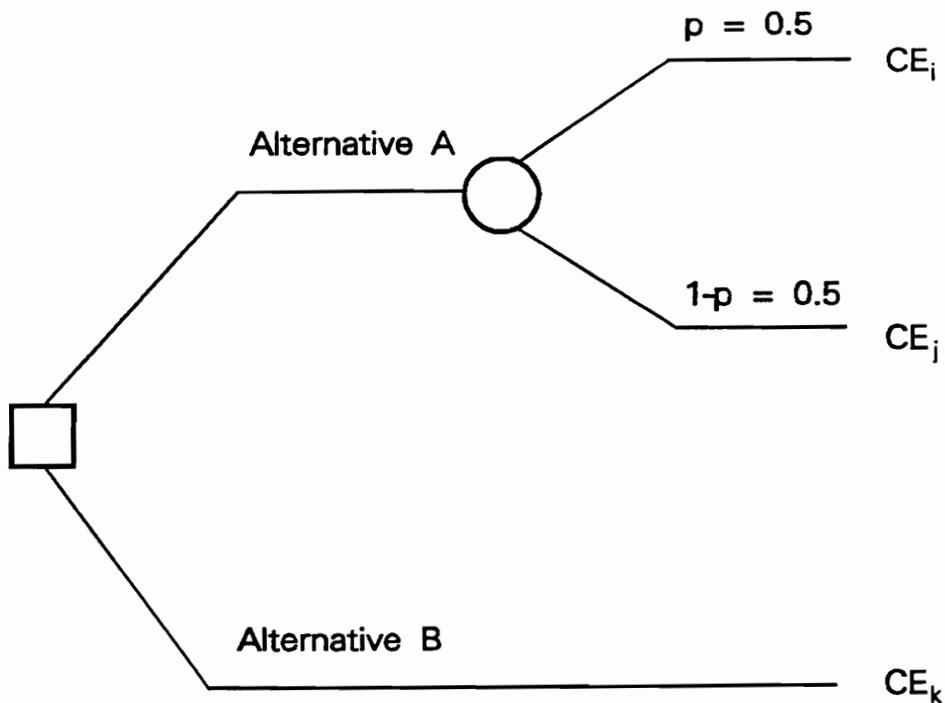


Figure 5.2. Utility assessment using 50-50 gambles

### 5.4.5 Utility functions for special risk attitudes

The shape of the utility curve depends on an individual's attitude toward risk. Three general categories of attitudes have been identified: risk averse, risk neutral, and risk seeking. Figure 5.3 shows examples of utility curves for each category. The shape of the risk-averse curve is concave. The risk-seeking curve is convex. The risk-neutral curve is a straight line.

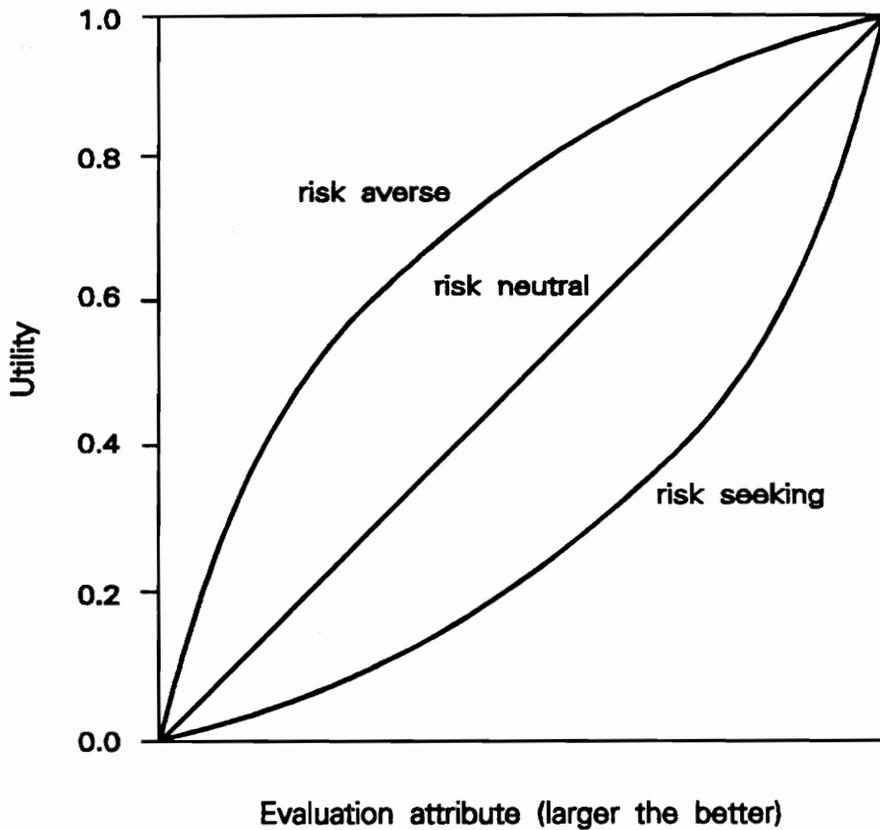


Figure 5.3. Three forms of utility functions

## 1) Risk neutrality – linear utility function

The utility curve for risk neutral decision makers is a straight line. The second derivative of the utility function

$$u''(e) = 0.$$

Since certainty equivalents are equal to expected values, the curve is not required for evaluating design alternatives.

## 2) Risk aversion – convex utility function

If the DM is risk-averse over the entire range of interest, the utility curve must be relatively smooth. Risk-averse individuals have a positive risk premium. The size of the risk premium depends upon (1) the degree of risk aversion, (2) the values taken on by the evaluation attribute, and (3) the probability distribution for the evaluation attribute. For a utility function with first and second derivatives, the risk aversion function is defined as

$$r(e) = \frac{-u''(e)}{u'(e)} \quad (5.1)$$

If the DM is decreasingly risk averse, the first derivative of the risk aversion function,  $r'(e)$ , is less than zero. If the DM is constantly risk averse,  $r'(e) = 0$ . For a DM of increasing risk aversion,  $r'(e) > 0$ .

Knowing that a positive risk premium exists restricts the shape of the utility curve. Since a risk averse individual has a positive risk premium, the utility curve will always lie to the left or above the risk-neutral curve. Thus, the utility curve is always concave. These requirements mean that a few assessments rapidly restrict the shape of the utility curve.

A special case of risk aversion is the constant risk aversion. Constant risk aversion implies a utility function of the form

$$u(e) = a - b \exp(-\lambda e), \quad (5.2)$$

where  $E$  is the evaluation measure. If we require

$$u(e_{min}) = 0, u(e_{max}) = 1.0,$$

where  $e_{min}$  is the lowest value for the preference scale and  $e_{max}$  is the highest, only one more equation is needed to find the values for parameters  $a$ ,  $b$ , and  $\lambda$ . This means that a single certainty equivalent assessment is all that is required to completely specify the utility function. By using the 50-50 gamble shown in Figure 5.4,

$$u(CE) = 0.5u(e_{max}) + 0.5u(e_{min}),$$

$$u(e_{max}) = 1.0,$$

$$u(e_{min}) = 0.$$

Thus,

$$a = \frac{\exp(-\lambda e_{min})}{\exp(-\lambda e_{min}) - \exp(-\lambda e_{max})}$$

$$b = \frac{1}{\exp(-\lambda e_{min}) - \exp(-\lambda e_{max})}$$

$$\exp(-\lambda CE) = 0.5[\exp(-\lambda e_{min}) + \exp(-\lambda e_{max})]$$

For any values of  $CE$ ,  $e_{max}$ ,  $e_{min}$ ,  $a$  and  $b$  can be evaluated directly.  $\lambda$  can be found by trial and error or by means of a complicated search procedure on a computer.

### 3) Risk seeking – convex utility function

The second derivative of the utility function  $u''(e) > 0$ .

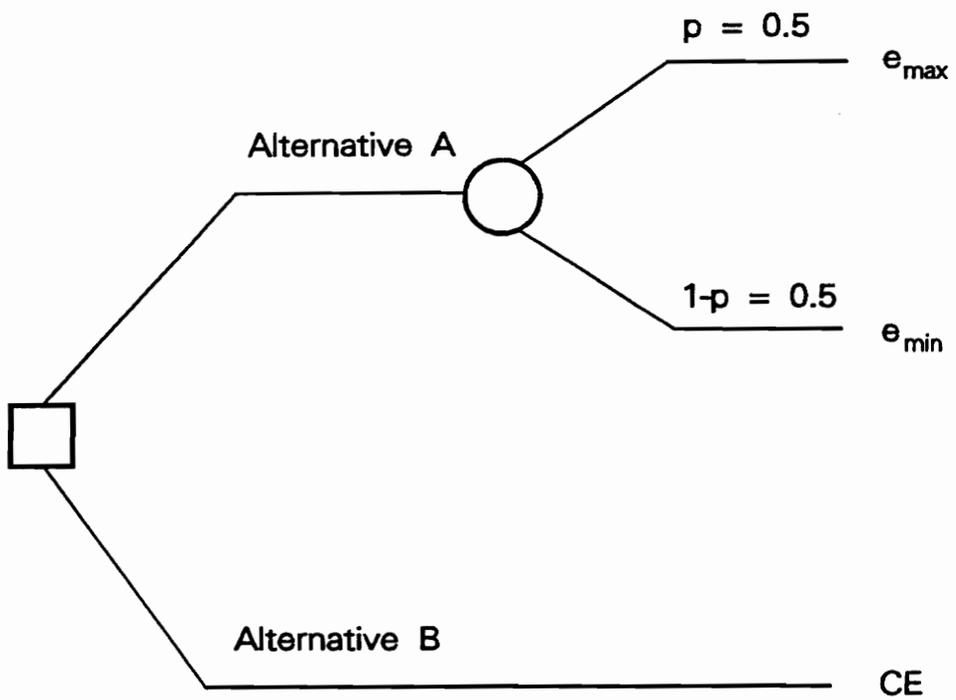


Figure 5.4. Estimating the utility function for constant risk aversion

#### 5.4.6 Procedures for assessing utility functions

In practice, according to Keeney (1977), the assessment of a utility function follows three phases: (1) ask some questions to determine the general shape of the utility function, (2) ask some specific questions to quantify a specific utility function, and (3) check consistency and make modifications. Once the attribute is specified, the assessment process can be broken into five parts (Keeney, 1977; Keeney and Raiffa, 1976):

**Step 1: Preparing for the assessment.** The decision maker is familiarized with the terminology and procedures used in the assessment. The analyst is familiarized with the design decision problem and the meaning of the attribute.

**Step 2: Identifying the relevant qualitative characteristics.** These characteristics can be determined by investigating three questions: (1) is the utility function monotonic, (2) is the decision maker risk averse, risk neutral, or risk prone, and (3) if the decision maker is risk averse, is his utility function increasingly, decreasingly, or constantly risk averse?

**Step 3: Specifying quantitative estimation.** The utilities of a few particular points on the utility function are determined. This usually involves determining the certainty equivalents for a few 50-50 gambles. If the decision maker is risk averse, his certainty equivalents (*CE*) must be larger than the expected consequences for monotonically decreasing utility functions. For increasing utility function, the *CE*'s must be less than the expected consequences.

Before determining the *CE*'s, the end points for the attribute *E*, i.e., the best value  $e^*$  and the worst value  $e^o$ , should be determined. These could be the best and worst conceivable values for the attribute; or they could be numbers bounding the alternatives to be considered by the analysis; or they could have some other convenient interpretation (Watson and Buede, 1987). In this research,  $u(e^*)$  is defined to be equal 1.0 and  $u(e^o)$  is equal to 0.0.

**Step 4: Choose a utility function.** In utility assessment, the question is whether or not a utility function exists that simultaneously satisfies all of the information obtained from the assessment. We would like to find a parametric family of utility functions that possesses the relevant characteristics (such as risk aversion). Then by using the certainty equivalents, a specific member of that family which is appropriate for the DM is identified. The *CE*'s are used to specify values for the parameters of the original family of utility functions.

**Step 5: Checking for consistency.** The consistency of above assessments must be examined. If some of the assessments are not consistent, more assessments should be conducted.

#### **5.4.7 Axioms for choices**

The utility analysis method introduced above is valid if certain behavioral assumptions are satisfied. These assumptions are: (1) Comparability, (2) Transitivity, (3) Reduction of compound uncertain events, (4) Continuity, (5) Substitutability, and (6) Monotonicity. If these assumptions are satisfied, there exist a utility so that the DM's preferences for various design alternatives can be determined by calculating expected preferences. These six axioms are explained in detail in Appendix A.1.

### **5.5 Sequential Decision Analysis Using the Maximum Expected Utility Principle**

Combining the decision models presented in Chapter IV with utility theory results in a logical decision process for design evaluation under uncertainty. This approach is called sequential decision analysis using the Maximum Expected Utility (MEU) principle. The decision criterion used in the approach is maximization of expected utility.

### 5.5.1 The Maximum Expected Utility principle

In the process of design evaluation, the selection of a design alternative is influenced by various uncertainties. Due to these uncertainties, the selection of a design decision may result in many possible consequences. Based upon utility theory, the DM's preferences for the various consequences of a design decision can be described in terms of utilities. Thus, the design alternatives can be compared based upon their expected utilities.

**Principle of Maximum Expected Utility (MEU):** There are a set of mutually exclusive design alternatives,  $a_1, a_2, \dots, a_m$ . The set of states of nature for the problem are identified as  $s_1, s_2, \dots, s_n$ . If the probability that  $s_k$  occurs,  $p_s(s_k)$ , is known, the expected utility of Alternative  $a_i$  is given by

$$\bar{u}_i = \sum_{k=1}^n p_s(s_k) \times u(a_i, s_k). \quad (5.3)$$

The alternative with the maximum expected utility is preferred.

If a single attribute,  $E$ , is sufficient to represent the consequences of the design alternatives, the expected utility for Alternative  $a_i$  is equal to

$$\bar{u}_i = \sum_{k=1}^n p_E(e_{ik}) \times u(e_{ik}), \quad (5.4)$$

where  $p_E(e_{ik})$  = the probability mass function of  $E$   
 $e_{ik}$  = the value of  $E$  for Alternative  $i$  when  $s_k$  occurs

### 5.5.2 Procedures of sequential decision analysis

As discussed in Section 4.2, design decision problems in the face of uncertainty are made up of decisions and uncertain events. The problem is to select an alternative from a set of mutually exclusive alternatives, not knowing which member of the uncertain event set will be true. Such a problem can be resolved by using the sequential analysis approach along with the MEU principle.

The paradigm of sequential design decision analysis includes structural analysis, uncertainty analysis, and utility analysis. According to Lindley (1985), three basic principles should be followed in the decision process: assigning probabilities to uncertain events; assigning utilities to possible consequences; and choosing the decision that maximizes expected utility. For evaluating design alternatives, the approach proceeds in seven steps below:

**Step 1.** Identify all design alternatives ( $a_1, a_2, \dots, a_m$ ).

**Step 2.** List the uncertain events ( $s_1, s_2, \dots, s_n$ ).

**Step 3.** Construct a decision tree to link the decision nodes and the chance nodes. The decision tree is written out in chronological order, the decisions and uncertain events being described by branches in the order in which they occur.

**Step 4.** Assign probabilities to the uncertain events. Probabilities are attached to the branches emanating from random nodes in any coherent and consistent way.

**Step 5.** Determine the value of the evaluation attribute for each alternative  $i$  under each possible uncertain event  $j$ , that is,  $e_{ij}$ .

**Step 6.** Assign utilities to the values of the evaluation attribute. Utility  $u(e_{ij})$  is attached to  $e_{ij}$  for each possible outcome.

**Step 7.** Choose that alternative of maximum expected utility. Proceeding back from the terminals to the base, (1) at a random node, take an expectation of the utilities; (2) at a decision node, choose among alternatives at this node which has a maximum expected utility; and (3) eliminate the decision node by crossing out all but the preferred alternative. Keep moving backward by taking expectations at random nodes and maximization at decision nodes. The best decisions and their expected utilities are then determined.

### **5.5.3 Justification of the use of the MEU principle**

By adapting decision theory and utility theory for design decision making, the expected utility of the evaluation attribute is recommended as the objective function for design evaluation. For each evaluation attribute  $E$ , three measures are incorporated into the function: the value of  $E$ , the probability distribution of  $E$ , and the utility of the various levels of  $E$ . The uncertainty associated with an attribute is characterized by its probability distribution. The relative worth of a design alternative is reflected by the utilities. By converting the values of the evaluation attribute into utilities, the contribution of different levels of the evaluation attribute to the desirability of a design is determined. Embedded in the utility function are the designer's value judgments and attitudes toward risk.

Probabilistic utility analysis is employed to determine the effect of uncertainty in the level of the evaluation attribute on the ultimate desirability and ordinary ranking of alternatives. The resulting expected utility over a range of possible ultimate levels of the evaluation attribute reflects the negative impact that uncertainty has on the desirability of a design alternative. The magnitude of the impact is determined by the degree of risk aversion exhibited by the DM and the extent of the uncertainty in the attribute level. The uncertainty of the outcome with respect to various states of nature and preferences for various levels of the evaluation attribute are processed in the computation of expected utility. Since the outcomes are the realization of random quantities, mathematical expectation is taken for the utilities.

Thus, expected utility combines information concerning the utility of outcomes and the probability of outcomes into an estimate of expected utility. The overall objective is to maximize the expected utility for the evaluation attribute by choosing elements of the design variable vector. This principle is in keeping with Taguchi's dictum that good quality is that which minimizes the total loss to society. However, the utility function is more general and complete than represented by Taguchi's loss function. It is capable of considering various attributes of concern, including system life-cycle costs and the cost of selecting a particular alternative.

## 5.6 Outcome Dominance and Stochastic Dominance

With the help of utility theory, stochastic dominance rules can be developed for design evaluation under uncertainty. These rules incorporate the DM's preferences and the probability distributions of the evaluation attribute.

### 5.6.1 Outcome dominance

The most basic method of choosing two design alternatives is to compare them directly and, using some intuitive process, select one over the other. In some cases designers may find this method adequate. However, as the complexity of a problem increases, it is hard to resolve the problem directly.

There are two ways that outcome dominance can arise. The first is when the worst outcome for Alternative  $A$  is at least as good as the best outcome for Alternative  $B$ . In this case,  $A$  dominates  $B$ . Another type of outcome dominance may exist when two alternatives are followed by the same uncertain event, say the same set of design-independent parameters. That is, the alternatives differ only in the consequences associated with the outcomes. In this case, the following rule applies:

**Outcome Dominance Rule:** If Alternative  $A$  is at least as preferred as Alternative  $B$  for each outcome, and if  $A$  is strictly preferred to  $B$  for one outcome, then  $A$  dominates  $B$ .

An example of outcome dominance is given in Table 5.1. There are two design alternatives. The life-cycle costs for each alternative are given with respect to different states of nature. According to the Outcome Dominance Rule, Alternative  $B$  dominates Alternative  $A$ .

Table 5.1. An Example of Outcome Dominance

| State of Nature | Outcome (life-cycle cost) |               |
|-----------------|---------------------------|---------------|
|                 | Alternative A             | Alternative B |
| 1               | \$2,500,000               | \$2,500,000   |
| 2               | \$2,800,000               | \$2,700,000   |
| 3               | \$3,000,000               | \$2,900,000   |

### 5.6.2 Stochastic dominance

If the outcome dominance rule is not sufficient to resolve a design decision problem, stochastic dominance can be applied. The most general form of stochastic dominance makes no assumption about the form of the probability distribution of the evaluation attribute. Furthermore, the user does not have to assume the specific form of the DM's utility functions. There are three progressively stronger assumptions about the DM's behavior that are used in stochastic dominance literature (Elton and Gruber, 1981). They lead to first-, second-, and third-order stochastic dominance.

**Definition of Stochastic Dominance** (Hanoch and Levy, 1969): Given two random variables  $X$  and  $Y$  with cumulative probability distribution function  $F(X)$  and  $G(Y)$ , we say that  $X$  dominates  $Y$  if

$$E[u(x)] \geq E[u(y)]$$

for every utility function in the class of functions, and if the inequality holds strictly for at least one function in the class.

**First-Order Stochastic Dominance Theorem** (Hanoch and Levy, 1969): Let  $F(X)$  and  $G(Y)$  be cumulative distributions for random variables  $X$  and  $Y$ . Let  $u$  be any nondecreasing function with finite values for any finite  $x$ . A necessary and sufficient condition for  $X$  to dominate  $Y$  is that

$$F(x) \leq G(x) \text{ for every } x$$

and  $F(x_0) < G(x_0)$  for some  $x_0$ .

The theorem states that the cumulative distribution function (CDF) of  $X$  must lie below that of  $Y$  for at least one value and must lie nowhere above it. This theorem is equally valid for continuous and discrete probability distributions. Applying the theorem to evaluate two design alternatives, we have the following decision rule:

**First-Order Stochastic Dominance Rule:** If the designer prefers more of the evaluation attribute to less, and if the cumulative probability of the evaluation attribute for Alternative  $A$  is never greater than the cumulative probability for Alternative  $B$  and sometimes less, then  $A$  is preferred to  $B$ .

Obviously, outcome dominance is contained in the First-Order Stochastic rule. If Alternative  $A$  dominates Alternative  $B$  by outcome dominance,  $A$  dominates  $B$  by first-order stochastic dominance. However, the reverse may not be true. That is, if  $A$  dominates  $B$  by first-order stochastic dominance,  $A$  may not dominate  $B$  by outcome dominance. A stochastically dominated alternative can have an actual outcome that is better than the actual outcome from the alternative dominated it. But the dominating alternative has higher chance of obtaining a favorable outcome.

**Second-Order Stochastic Dominance Theorem** (Hanoch and Levy, 1969): Let  $F(X)$  and  $G(Y)$  be cumulative distributions for random variables  $X$  and  $Y$ . Let  $u$  be any nondecreasing, concave utility function. A necessary and sufficient condition for  $X$  to dominate  $Y$  is that

$$\int_{-\infty}^x F(t)dt \leq \int_{-\infty}^x G(t)dt \quad \text{for every } x$$

and strict inequality holds at some  $x_0$ .

This theorem states that the integral of the CDF of  $X$  must lie below that of  $Y$  for at least one value and must lie nowhere above it. By interpreting the theorem in non-mathematical terms, the decision rule is:

**Second-Order Stochastic Dominance Rule:** If (1) the decision maker prefers more of the evaluation attribute to less, and (2) the decision maker is *risk-averse*, and (3) the sum of the cumulative probabilities for the evaluation attribute is never more with  $A$  than  $B$  and sometimes less, then  $A$  dominates  $B$  by second-order stochastic dominance.

**Third-Order Stochastic Dominance Theorem (Whitmore, 1970):** Let  $F(X)$  and  $G(Y)$  be cumulative distributions for random variables  $X$  and  $Y$ . Let  $u$  be any nondecreasing, concave utility function with nonnegative third derivative. A necessary and sufficient condition for  $X$  to dominate  $Y$  is that

$$\int_{-\infty}^x \int_{-\infty}^t F(t)dt dw \leq \int_{-\infty}^x \int_{-\infty}^t G(t)dt dw \quad \text{for every } x$$

and strict inequality holds at some  $x_0$  and  $E(X) > E(Y)$ .

Thus, another stochastic dominance rule for design evaluation is:

**Third-Order Stochastic Dominance Rule:** Alternative  $A$  dominates Alternative  $B$  if: (1) the DM prefers more of the evaluation attribute to less of the attribute, and (2) the DM is risk averse with decreasing absolute risk aversion, and (3) the mean of the evaluation attribute for  $A$  is greater than that for  $B$ , and (4) the sum of the sum of the cumulative probability distribution for all values of the evaluation attribute are never more with  $A$  than  $B$  and sometimes less.

Since the assumptions underlying the three rules are progressively stronger, if Alternative  $A$  dominates Alternative  $B$  by first-order stochastic dominance,  $A$  also dominates  $B$  by second- and third-order stochastic dominance. Similarly, if  $A$  dominates  $B$  by second-order dominance, it is certain that  $A$  dominates  $B$  by third-order dominance. Thus, if a design decision problem can be resolved using a lower order rule, a higher order rule does not need to be used.

In order to implement any of the three stochastic dominance tests, we need detailed information about the probability distributions of the performance measure. The analysis may become complicated and tedious for a large number of alternatives.

## **5.7 Mean-Variance Analysis**

According to the philosophy of Taguchi's robust design, two objectives need to be achieved in order to develop a best design: (1) make the mean as close to the target, and (2) make the variance as small as possible. If the DM's preferences for gains and losses can be fully represented by the mean and variance of the system's evaluation attribute, a natural way to evaluate design alternatives is to compare their means and variances. This approach is named Mean-Variance analysis. The term originated from Markowitz's outstanding book on portfolio selection (1959).

### **5.7.1 Mean-Variance (E-V) rules**

If a DM is risk averse, less risk is preferred to more risk. The notion of risk involves both uncertainty and the magnitude of the evaluation attribute. But there is not a precise definition of risk that can be used to calculate a value of risk. An often-used surrogate for risk is the variance (or standard deviation) of the probability distribution for the evaluation attribute. Since the variance is a measure of dispersion, it can be thought of as describing the amount of uncertainty, and consequently, it captures an important part of the notion of risk.

Following this line of reasoning, a risk-averse DM would want to minimize the variance, everything else being equal. Thus, if a DM prefers more of the

evaluation attribute and is risk averse, the following rules of thumb result for choosing between two alternatives:

**E-V Rule 1:** Alternative 1 is preferred to Alternative 2 if

$$\mu_e^{(1)} \geq \mu_e^{(2)} \text{ and } [\sigma_e^2]^{(1)} < [\sigma_e^2]^{(2)}.$$

**E-V Rule 2:** Alternative 1 is preferred to Alternative 2 if

$$\sigma_e^2 \leq \sigma_e^2 \text{ and } E(e_1) > E(e_2),$$

where  $\mu_e^{(1)}$  and  $[\sigma_e^2]^{(1)}$  denote the mean and variance of  $E$  for Alternative 1, and

$\mu_e^{(2)}$  and  $[\sigma_e^2]^{(2)}$  represent the mean and variance of  $E$  for Alternative 2.

In other words, the DM prefers to maximize the expected value and minimize the variance. Thus, for two alternatives with "well-behaved" symmetrical probability distributions for the evaluation measure, a risk-averse designer will prefer:

- 1) The alternative with the lower variance if the expected values are equal, or if the alternative with the lower variance has a higher expected value.
- 2) The alternative with the higher expected value if the variances are equal, or if the alternative with the higher expected value has a lower variance.

To compare various alternatives, we plot each pair  $(\mu_e, \sigma_e^2)$  on an E-V chart. The horizontal axis of the E-V chart represents the variance of the evaluation attribute, while the vertical axis denotes the mean of the attribute. Figure 5.5 is an example of the E-V chart.

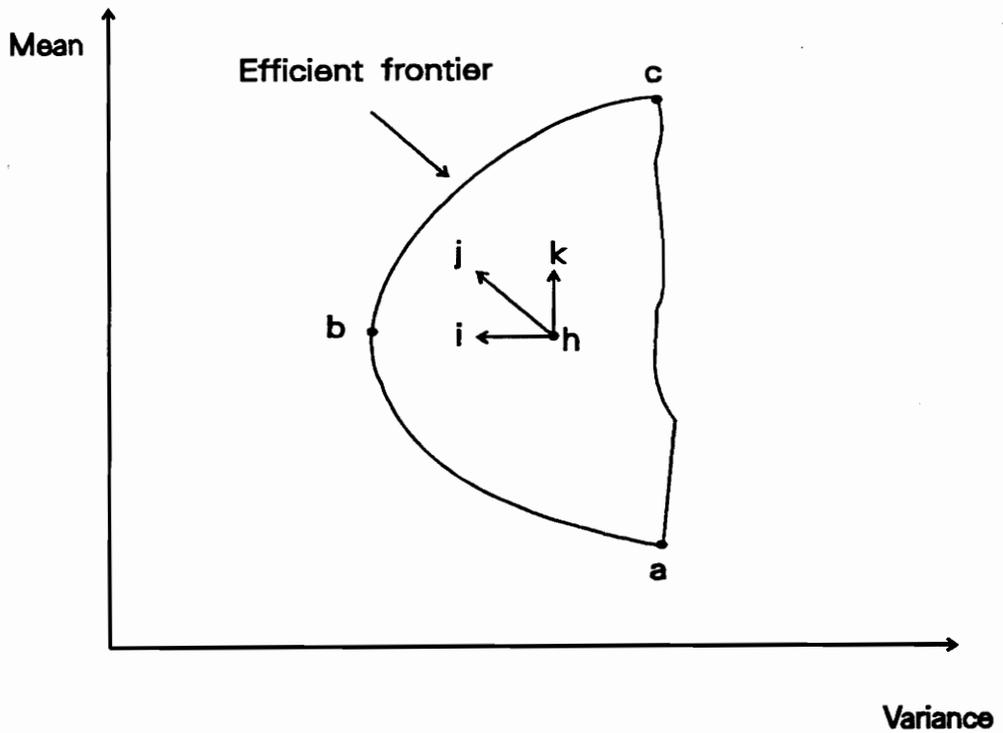


Figure 5.5. Efficient frontier

On an E-V chart, a possible point  $(\mu_e^*, \sigma_e^{*2})$  is called efficient if no other possible point  $(\mu_e, \sigma_e^2)$  has

$$\mu_e \geq \mu_e^* \text{ and } \sigma_e^2 \leq \sigma_e^{*2}.$$

The efficient points form the highest left boundary of the set of possible points. The boundary is called efficient frontier and the set of efficient points is called

efficient set. In Figure 5.5, the efficient frontier  $bc$  is a curve drawn through the points representing alternatives that are not dominated by some other alternatives. Any point below and to the right of the efficient frontier represents an alternative dominated by one on the frontier. For any obtainable E-V combination except on arc  $bc$  it is possible to find a feasible combination with at least as much mean and less variance; or to find one with less variance and no less mean, or both. Any such combination is considered inefficient. For example, from point  $h$ , we can move to  $i$  obtaining less variance and no less mean; we can move to  $k$  to obtain more mean and no more variance; or we can move diagonally from  $h$  to  $j$  obtaining both more mean and less variance. These points are considered inefficient. We cannot move upward from an E-V combination on the arc  $bc$ , except for  $b$ .

The application of the E-V rules can be illustrated with an example. Suppose we want to evaluate five design alternatives with the evaluation attribute for this case being reliability. The mean and variance of the reliability for each alternative are given in Table 5.2. Plotting each pair of mean and variance on Figure 5.6, we find that Alternative 1 is not efficient, since Alternative 2 has an equal mean and less variance. Similarly, Alternatives 3 and 4 are not efficient. The efficient set consist of Alternatives 2 and 5.

Table 5.2. Means and Variances for Five Alternatives

|        | ALT 1 | ALT 2 | ALT 3 | ALT 4 | ALT 5 |
|--------|-------|-------|-------|-------|-------|
| E(R)   | 0.90  | 0.90  | 0.85  | 0.80  | 0.80  |
| Var(R) | 5.00  | 3.00  | 3.50  | 4.00  | 2.50  |

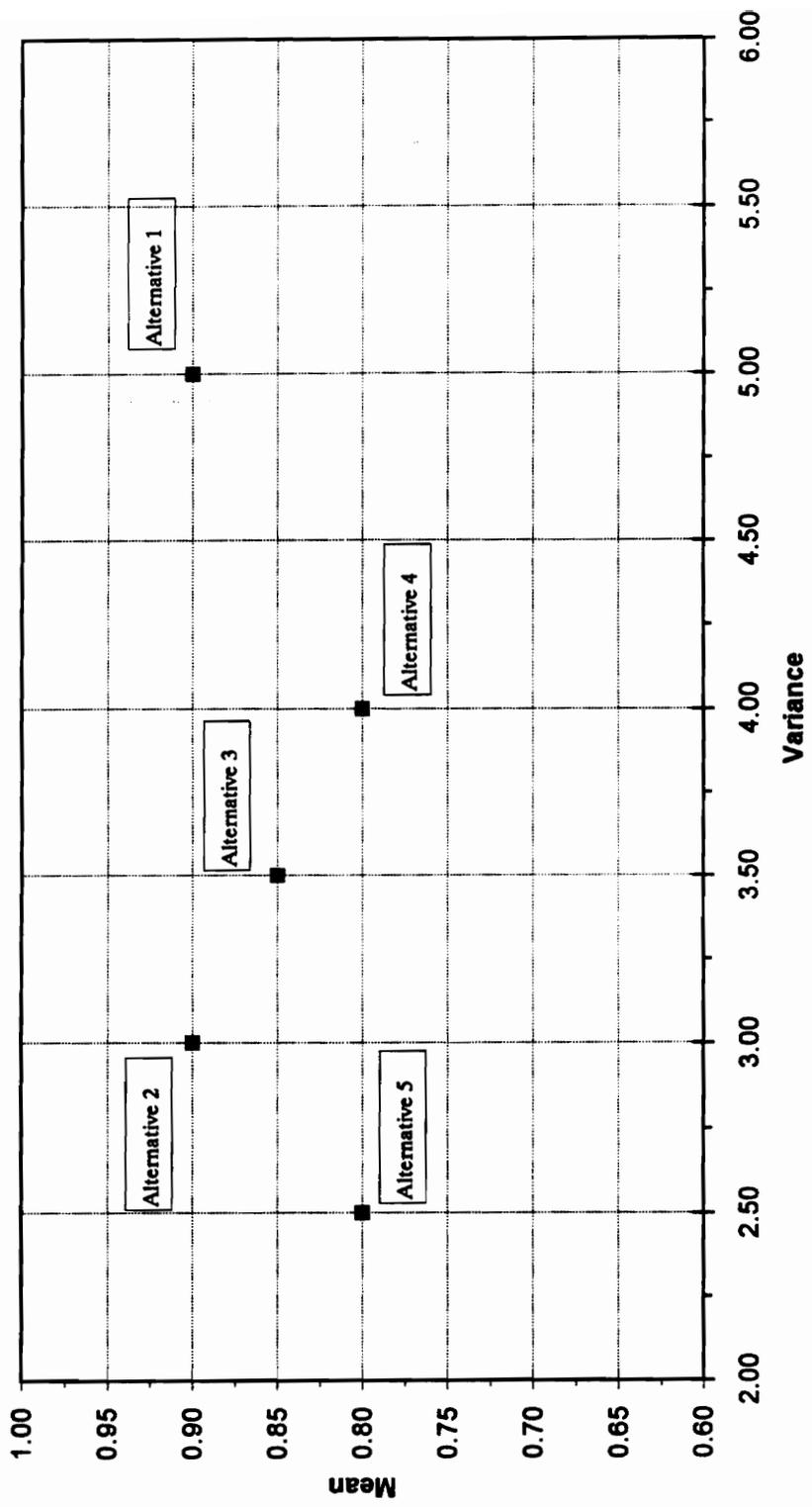


Figure 5.6. An example of the E-V chart

## 5.7.2 Utility indifference curves

As discussed above, to develop a best design, we must make the evaluation attribute meet a target value. The target value can be a specific number, or a value as large as possible, or a value as small as possible. If there is more than one alternative which meets the target value, the decision is easy. That is, according to the E-V rules, pick the alternative which has the minimum variance.

However, the problem of design evaluation in the face of uncertainty is often more complicated in practice. In some cases, the mean and variance of the evaluation attribute for a design may be dependent. As a result, more than one efficient alternative can be identified in the E-V analysis. This makes it impossible to achieve the optimum for both the mean and variance jointly.

Since the E-V rules cannot help us choose between the alternatives in the efficient set, we must resort other information to make a choice. The ultimate choice between the elements in the efficient set depends upon the DM's trade-off between the mean and variance, that is, the trade-off between the bias and variance. Thus, the DM must make a trade-off between attainment of a target value and the variability at the target value. Utility theory can be used to represent the DM's willingness to make such trade-offs.

For a rational decision maker, he can be assumed to be indifferent to sets of  $(\mu_e, \sigma_e^2)$ . That is, an indifference curve of mean and variance exists. The DM is indifferent to any point on the indifference curve. So, for the same level of utility, such an indifference curve can be defined:

$$U = u(\mu_e, \sigma_e^2). \quad (5.5)$$

Operationally, the utility indifference curve that relates the mean and variance of the evaluation attribute provides a foundation for Mean-Variance analysis. Thus, the decision problem can be solved if we can determine which point of  $(\mu_e, \sigma_e^2)$  on the E-V chart is on a higher utility curve (Figure 5.7).

Thus, to compare design alternatives in an efficient set, we must define a utility function to accurately reflect the DM's preference and willingness to make trade-offs over the mean and variance. In general, the utility function of mean and variance has the following characteristics (see Figure 5.7):

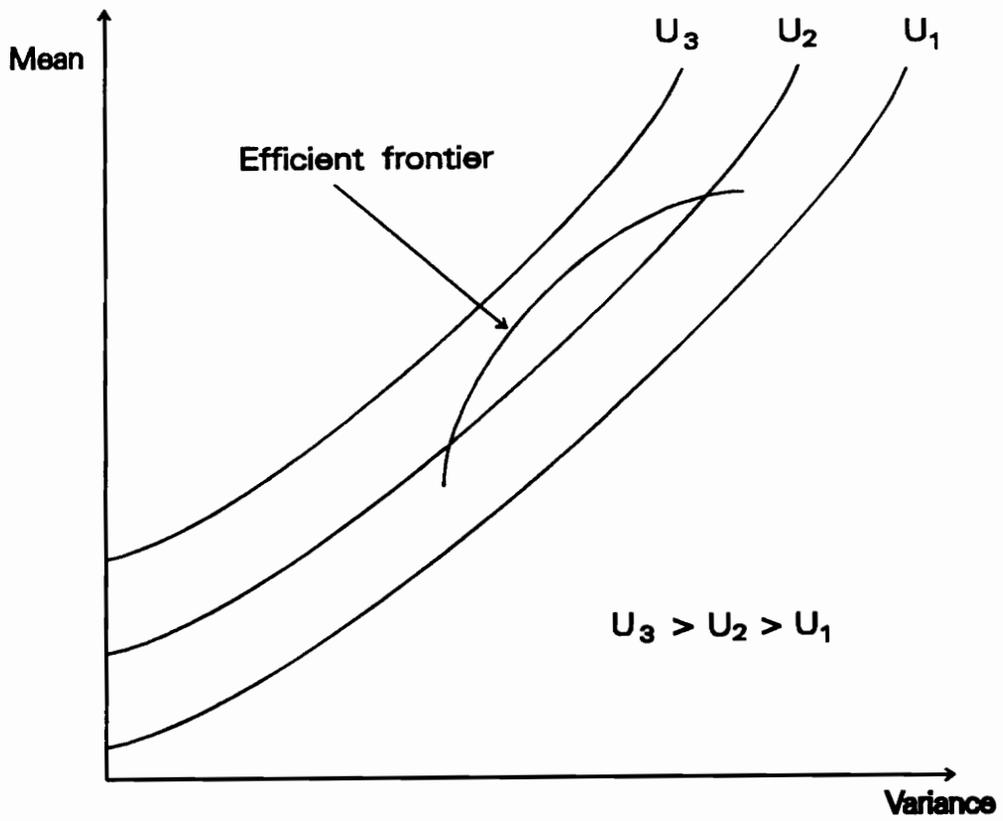


Figure 5.7. Utility indifference curves

- 1) It is an increasing function of the evaluation attribute (for the larger the better case).
- 2) It is a decreasing function of variance. The curve is convex on the E-V chart.
- 3) The intersection point of a curve with the vertical axis ( $\mu_e$ ) represents the certainty equivalent for all points on that curve. Since variance is zero at this point, the outcome is certain.

There are no easy guidelines currently available for assessing such a general multiattribute utility function. However, in many circumstances, certain conditions can be satisfied in order to decompose the joint utility function (5.5) into a function of single attribute utility functions, that is,

$$u(\mu_e, \sigma_e^2) = f[u_1(\mu_e), u_2(\sigma_e^2)]. \quad (5.6)$$

According to the multiattribute utility theory (Keeney and Raiffa, 1976), if the mean and variance are mutually utility independent, the utility function can be decomposed into a multiplicative model:

$$u(\mu_e, \sigma_e^2) = k_1[u_1(\mu_e)] + k_2[u_2(\sigma_e^2)] + (1 - k_1 - k_2)[u_1(\mu_e)][u_2(\sigma_e^2)]. \quad (5.7)$$

where  $k_1$  = scaling constant for the mean  
 $k_2$  = scaling constant for the variance  
 $u_1(\mu_e)$  = utility function for the mean  
 $u_2(\sigma_e^2)$  = utility function for the variance

If the mean and the variance are additively independent, utility function (5.6) is then reduced to the additive form:

$$u(\mu_e, \sigma_e^2) = k_1[u_1(\mu_e)] + k_2[u_2(\sigma_e^2)]. \quad (5.8)$$

Since the random nature of the decision problem is reflected by the variance, no expectation is needed. Thus, the utility function is simply a value model to help decision makers make a trade-off between mean and variance.

### 5.7.3 Relationships between E-V analysis and the MEU principle

As presented above, E-V analysis is based on utility theory. The relationship between the E-V analysis and the MEU principle was discussed by Markowitz (1987). Allen (1953) provided a simple mathematical derivation to illustrate the relationship:

For evaluation attribute  $E$ , take a Taylor series expansion of its utility function with respect to a constant  $c$ ,

$$u(E) = u(c) + (E - c) \frac{du(c)}{dE} + \frac{1}{2}(E - c)^2 \frac{d^2u(c)}{dE^2} + \frac{1}{3!}(E - c)^3 \frac{d^3u(c)}{dE^3} + \frac{1}{4!}(E - c)^4 \frac{d^4u(c)}{dE^4} + \dots$$

Letting  $c = \mu_E$ , the expected value of the evaluation attribute, then

$$u(E) = u(\mu_E) + (E - \mu_E) \frac{du(\mu_E)}{dE} + \frac{1}{2}(E - \mu_E)^2 \frac{d^2u(\mu_E)}{dE^2} + \frac{1}{3!}(E - \mu_E)^3 \frac{d^3u(\mu_E)}{dE^3} + \frac{1}{4!}(E - \mu_E)^4 \frac{d^4u(\mu_E)}{dE^4} + \dots$$

Taking mathematical expectation of each side of the equation, we obtain the expected utility of selecting a design alternative:

$$\bar{u}(E) = u(\mu_E) + \frac{1}{2}\sigma_E^2 \frac{d^2u(\mu_E)}{dE^2} + \frac{1}{3!}s \frac{d^3u(\mu_E)}{dE^3} + \frac{1}{4!}k \frac{d^4u(\mu_E)}{dE^4} + \dots \quad (5.9)$$

where  $s = E(E - \mu_E)^3$ , the skewness of the probability distribution of  $E$

$k = E(E - \mu_E)^4$ , the kurtosis of the probability distribution of  $E$

Equation (5.9) represents the expected utility of  $E$  in terms of (1) the moments of the PDF of  $E$ , that is, the mean, variance, skewness, and kurtosis; and (2) the first four derivatives of the utility function of  $E$ . Thus, the number of terms used to calculate the expected utility for a design alternative depends upon: (1) the number of moments that describe the distribution of  $E$ , and (2) the number of derivatives that can be taken from the utility function.

In order to represent the expected utility as a function of only the first two moments, mean and variance, either of the conditions must be satisfied: (1) the third and higher derivatives of the utility function are equal to zero, or (2) the probability distribution of  $E$  has only the first two moments.

Thus, the MEU principle is a general decision rule. E-V analysis is just a special case of the MEU principle. Under either of the following two conditions, using E-V analysis and the MEU principle will generate the same optimum:

- 1) If the utility function is a quadratic equation, then only the first two derivatives are non-zero, and the expected utility is derived using only the first two terms.
- 2) If the probability distribution of  $E$  is normal, which has only two moments, the mean and variance, the expected utility becomes a function of the first two moments.

Therefore, the E-V analysis approach holds exactly when the decision maker is an expected utility maximizer, prefers more to less, is risk averse, and either (1) the values of the evaluation attribute are normally distributed, or (2) the utility function of the evaluation attribute is quadratic. Furthermore, the analysis is robust in that, as Markowitz (1976) has shown, it frequently holds approximately even when assumptions (1) or (2) are violated. For example, quadratic approximations are almost always good local approximations to nonquadratic utility functions (Elton and Gruber, 1981).

In design evaluation, using E-V analysis has some advantages. First, the concept is straightforward and easy to understand. In the community of engineering design, mean and variance are more familiar terms than utility. In practice the mean and variance of a system's evaluation attribute are easy to estimate. Another advantage of working with the mean and variance is that a utility function is not needed in some cases. The problem may be solved by just using the E-V rules. However, the user must keep in mind the assumption underlying E-V analysis. The underlying assumption is that the DM's preferences for a design can be represented by a function of the mean and variance of the evaluation attribute.

## 5.8 Assumptions Underlying Common Objective Functions

There are a variety of objective functions (or decision rules) used to resolve problems of design evaluation under uncertainty. In this section, five commonly used functions are identified (Table 5.3). Based upon decision theory and utility theory introduced above, the decision analysis foundations underlying these rules are examined. Compared with the MEU principle, their limitations and advantages are obvious. Underlying all these rules are assumptions regarding the decision maker's preference or utility function.

Table 5.3. Some Decision Rules for Design Evaluation  
in the Face of Uncertainty

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Maximizing expected value  
Probability of loss  
Linear function of mean and variance  
Taguchi's loss function  
Taguchi's Signal-to-Noise ratios

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### 5.8.1 Maximizing expected value

In design optimization, a commonly used objective function is maximization (or minimization) of the mean of an evaluation attribute  $E$ ,  $\mu_E$ . The underlying assumption for the use of this objective function can be identified based on utility theory.

For an evaluation measure  $E$ , suppose its probability density function is  $f(e)$  and its utility function is  $u(e)$ . The expected utility of  $E$  is given by

$$E[u(e)] = \int_{-\infty}^{\infty} u(e)f(e)de.$$

If  $u(e) = be + c$ , where  $b$  and  $c$  are constants, then we have

$$E[u(e)] = \int_{-\infty}^{\infty} (be + c)f(e)de = b\mu_E + c.$$

Since  $b$  and  $c$  are constants, maximizing the expected utility is equivalent to maximizing  $\mu_E$ , the mean of  $E$ . Thus, if the objective function is to maximize the mean of an evaluation attribute, a linear utility function is implied. According to utility theory, a linear utility function implies that the DM is risk neutral. Since the expected value of an evaluation attribute is the certainty equivalent for the case of risk neutrality, choices of design alternatives can be made by comparing the expected values directly.

### 5.8.2 Probability of loss

The probability of loss criterion has been used as a measure of the desirability of a system design. The measure treats only the values of the evaluation attribute below a certain value as unfavorable (more is preferable) as in Figure 3.3. In applying the probability of loss rule for design evaluation, three decision rules can be employed (Elton and Gruber, 1981):

**Rule 1:** Minimize the probability of the evaluation measure  $E$  below a critical level  $e_h$ , that is,

$$\text{Minimize } P(E < e_h).$$

**Rule 2:** Maximize the aspiration level  $e_h$  subject to the constraint that the probability of the evaluation attribute  $E$  less than, or equal to, the aspiration level is not greater than some predetermined value  $\alpha$ . In symbols, the decision model is:

$$\text{Minimize } e_h,$$

$$\text{subject to } P(E < e_h) \leq \alpha.$$

**Rule 3:** Maximize the expected value of the evaluation attribute, subject to the constraint that the probability of the evaluation attribute less than, or equal to, the aspiration level is not greater than some predetermined number. The decision model is formulated as:

$$\begin{aligned} & \text{Maximize } \mu_E, \\ & \text{subject to } P(E \leq e_h) \leq \alpha. \end{aligned}$$

Among these rules, the first is the most widely used. These models have been used as an objective function for design optimization. To apply the rules, however, we must keep in mind that a strong assumption about the DM's behavior is underlain in these rules.

In applying the probability of loss criterion, the concern is on the probability of the evaluation attribute below a critical level. This rule is often used without specifying the DM's behavior. However, since probability and utility are both elements of decision making, treating only probability explicitly in the objective function does not cause utility to disappear. An objective function which contains only probabilities implies strong value judgments. The assumptions about utility implied in the use of the rule are discussed below.

For an evaluation attribute  $E$  with PDF  $f_E(e)$ , the reliability of the system, denoted by  $R$ , is

$$R = P(E < e_h) = \int_{-\infty}^{e_h} f_E(e) de.$$

If the utility of  $E$  is  $u(e)$ , the expected utility of  $E$  is given by

$$\begin{aligned} E[u(e)] &= \int_{-\infty}^{\infty} u(e) f_E(e) de \\ &= \int_{-\infty}^{e_h} u(e) f_E(e) de + \int_{e_h}^{\infty} u(e) f_E(e) de. \end{aligned}$$

Since the values of  $E$  below  $e_h$  are unfavorable, and amounts over  $e_h$  are treated as favorable and give the same contribution to the success of the

system, the utility function of  $E$  must be as shown in Figure 5.8. The expected utility of  $E$  is equal to

$$E[u(e)] = \int_{-\infty}^{e_h} (0) f_E(e) de + \int_{e_h}^{\infty} (1) f_E(e) de$$

$$= \int_{e_h}^{\infty} f_E(e) de = R.$$

Thus, when the probability of loss criterion is used in design evaluation, a utility function such as given in Figure 5.8 is implied. The underlying assumption is that there is an aspiration level that is important to the DM. Amounts below the aspiration level are of little or no importance. Amounts above the level give the same contribution.

### 5.8.3 Linear function of mean and variance

A linear function of the mean and variance for an evaluation attribute has been used as the objective function for design optimization in some literature. For evaluation attribute  $E$ , the objective function used is

$$\text{Maximize } w_1 \mu_E + w_2 \sigma_E^2 \quad (5.10)$$

where  $w_1$  and  $w_2$  are constants, representing the weights given to the mean and variance of  $E$ , respectively. If the objective is to maximize the evaluation attribute  $E$ ,  $w_1$  should be positive, while  $w_2$  must be negative.

The underlying foundation for model (5.10) can be identified by comparing this model with the additive utility model given by Equation (5.8). There are three assumptions underlying the model:

- 1) The DM's preferences for a design can be measured by the mean and variance of the evaluation attribute for the design.
- 2) The utility functions for both the mean and the variance are linear.
- 3) The mean and the variance are additively independent.

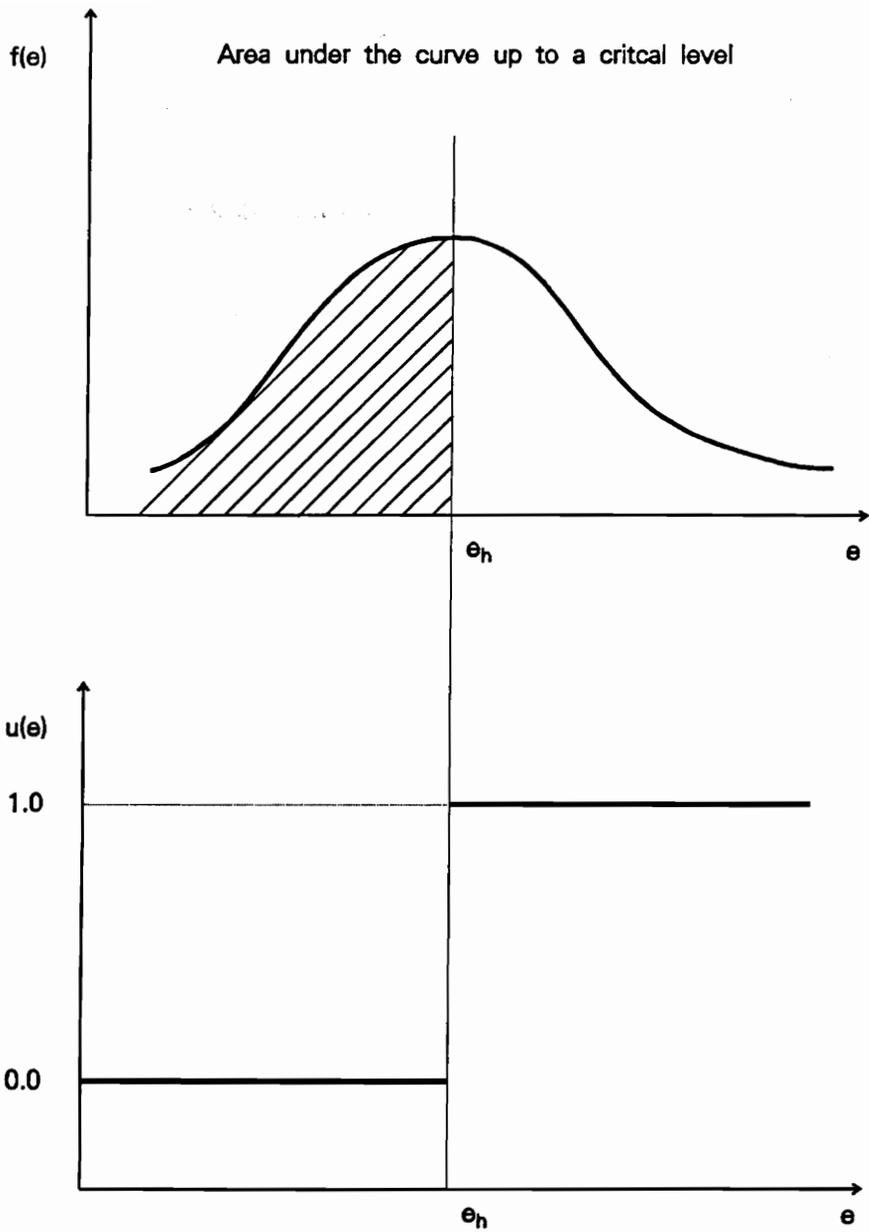


Figure 5.8. Utility function for the probability of loss criterion

#### 5.8.4 Taguchi's loss function

As indicated in Section 4.2, in the process of design evaluation by decision theory, there is the need to know the possible consequences of any design decision. Often the knowledge can be quantified by determining the gain or the loss that would be incurred for each possible decision for the various states of nature. So far the problem of design evaluation have been discussed in terms of gains – utility. In Taguchi's approach, this problem is discussed from the perspective of loss. Since a loss is just a negative of a gain, the loss function can be defined from the utility function.

If the state of nature can be represented by a vector  $s$ ,  $a$  represents an action to select a design alternative, the loss function is defined for all possible  $(s,a)$ , that is,  $L(s,a)$ . In making a decision, the loss function should, ideally, be developed according to the utility function (Berg, 1985), that is

$$L(s,a) = -u(s,a).$$

Since decisions are made in the presence of uncertainty, the incurred actual loss,  $L(s,a)$ , will never be known with certainty at the time of decision making. A natural method of proceeding in the face of this uncertainty is to consider the "expected loss" of selecting an alternative and then choose an "optimal" alternative with respect to this expected loss. In this way, instead of trying to estimate the actual loss, we can measure the amount "lost" by not having the most favorable possibility occur; that is, measure the regret we have for not using the best action. This measure is called *regret loss* in statistical decision theory.

One of the standard loss functions used in statistical decision analysis is the squared-error loss. As given in Section 3.5.5, Taguchi's loss function takes this form:

$$L = k(E - e_T)^2$$

where  $e_T$  is the target value of the evaluation attribute  $E$ , and  $k$  is a constant. The function is used to measure the loss in value as a part deviates from its target value.

It is understandable that some researchers have questioned the automatic use of a squared-error loss function. Squared-error loss functions are just one of the standard loss functions used in decision analysis. There are some criticisms of the function. For example, the function is not bounded. Depending on the DM's attitudes toward risk and preferences for the evaluation attribute, other forms of loss function may be used.

### **5.8.5 Taguchi's signal-to-noise ratios**

In the evaluation of various design alternatives, Taguchi does not use the loss function directly as the decision criterion. Instead, he defines a number of simple decision rules to operationalize the concept of loss. These decision rules are called signal-to-noise ratios. Taguchi classifies various design decision problems into three categories. The signal-to-noise ratio for each category is given in Section 3.5.6.

The automatic use of the S/N ratios has generated a lot of controversy among statisticians (Box et. al, 1988; Box, 1985; Easterling, 1985; Freund, 1985; Fung, 1986; Hunter, 1985; Leon et. al, 1987; Kackar, 1985). There is also some confusion regarding the relationship between Taguchi's squared loss function and S/N ratios (Leon et al., 1987). Some have incorrectly interpreted that Taguchi may be steering away from his notion of the squared error loss when he advocates the use of the S/N ratios.

The foundation of Taguchi's S/N ratios lies in utility theory. If these S/N ratios are considered under the context of decision theory, the performance measures for the smaller-the-better case and the larger-the-better case are just applications of the maximum expected utility principle. Since, these measures are directly derived from Taguchi's squared-error loss function, minimizing expected loss means maximizing expected utility.

The performance measure for the case of Nominal the Better can be regarded as an application of E-V analysis. The S/N ratio can be considered a multiattribute utility function representing the DM's preferences for mean and variance. Thus, the use of both the squared-error loss function and the S/N ratio is not contradictory if one understands the relationship between the MEU principle and E-V analysis (see Section 5.7.3). The foundation of the NTB S/N

ratio is in E-V analysis, while the foundation for the loss function is the principle of minimizing expected loss. For a quadratic utility function, the expected utility can be represented as a function of the mean and variance.

Observations have indicated that using S/N ratios cannot guarantee generation of a best design. Wilde (1991) gives a counter-example to the use of the NTB S/N ratio as a criterion for design optimization. This situation is easy to explain if we consider the NTB S/N ratio as a *special* utility function for the mean and variance. Under some situations, this function may not accurately represent the DM's preferences and risk attitude. Thus, using it as a *general* utility function does not guarantee generation of best design. Depending on the nature of the design decision problem and the DM's preferences and risk attitude, other forms of utility function may be more appropriate. Different engineering designs can lead to different functions. In fact, Taguchi has defined more than 60 different signal-to-noise ratios in conducting his parameter designs (Kackar, 1985). Kackar also indicates that the function, in general, is unknown and must be estimated. This point of view agrees with utility theory.

It should be noted that the NTB S/N ratio does not directly optimize the evaluation attribute. In practice Taguchi employs a two-phase approach to derive a robust design. At the first phase, the settings of design variables are identified to maximize the S/N ratio, that is, minimize the coefficient of variation  $\sigma_E / \mu_E$ . At the second step, the mean is moved toward a target value by changing the setting of some adjusting parameters. The adjustment parameters are special design variables that have a large effect on the mean, but almost no effect on the variance. However, in many cases, such a variable may not be available. In this case, design evaluation must be carried out by using the decision analysis approaches presented in Sections 5.5, 5.6, and 5.7.

## **VI. AN INTEGRATED APPROACH FOR DESIGN EVALUATION IN THE FACE OF UNCERTAINTY**

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- 6.1 Introduction
  - 6.2 A Conceptual Model for Preliminary System Design
  - 6.3 A Structured Model for Design Analysis
  - 6.4 Design Evaluation for Discrete Decisions
  - 6.5 Design Evaluation for Continuous Decisions
- 

### **6.1 Introduction**

Decision analysis foundations have been identified in Chapters IV and V for design evaluation in the face of uncertainty. The decision rules, decision diagrams, and decision analysis approaches form the basis under which alternative designs can be compared. In this chapter, these approaches are integrated into a structured, systematic approach for resolving different design decision problems.

### **6.2. A Conceptual Model for Preliminary System Design**

A conceptual model was presented in Chapter II for representing the design decision process in the preliminary design stage (Figure 6.1). This model divides the design decision process into four basic steps: synthesis of design alternatives, design analysis, design evaluation, and design optimization.

At the first step in Figure 6.1, based upon the information available, such as the need, user's requirements, and designer's experiences, various design alternatives are generated and synthesized. This task is accomplished by selecting, estimating, or predicting the levels of design-dependent parameters. Each set of design-dependent parameter values determines a unique design

alternative. The next step is design analysis. Design analysis is conducted to structure the decision problem and assess outcomes for each decision. After the design analysis is completed, design evaluation starts. The objective of design evaluation is to identify the best design strategy. When an optimal strategy is identified, the design is examined with respect to the user's needs and requirements defined. If the design is adequate, it is recommended for detail design. If the design is unsatisfactory, an iterative resign process is conducted. The information obtained is used to identify better designs.

### **6.3 A structured model for design analysis**

The objective of design analysis is to structure the decision problem and assess various possible outcomes for each design strategy. The process of design analysis is illustrated by Figure 6.2. This process is divided into four steps: preanalysis, structural analysis, outcome analysis, and uncertainty analysis. These steps are interrelated and concurrent.

***Preanalysis.*** The decision problem is identified and defined. This includes identification of all mutually exclusive decisions for each design alternative, identification of all possible uncertain events, and specification of objectives. Appropriate evaluation attributes are identified to represent the objectives. Also, general information about the DM's preferences and risk attitudes toward the evaluation attributes should be collected and processed.

***Structural analysis.*** The qualitative anatomy of the decision problem is structured by decision trees. Depending upon the characteristics of the decisions and events, the problem is modeled by one of the four models in Table 4.1. The construction of the decision tree follows the procedures of sequential decision analysis. At the leftmost decision node, each design alternative generated is represented by a branch. A decision node is attached for each alternative. Various decisions for each alternative are then attached to the decision node. Uncertain events are identified and represented by chance nodes.

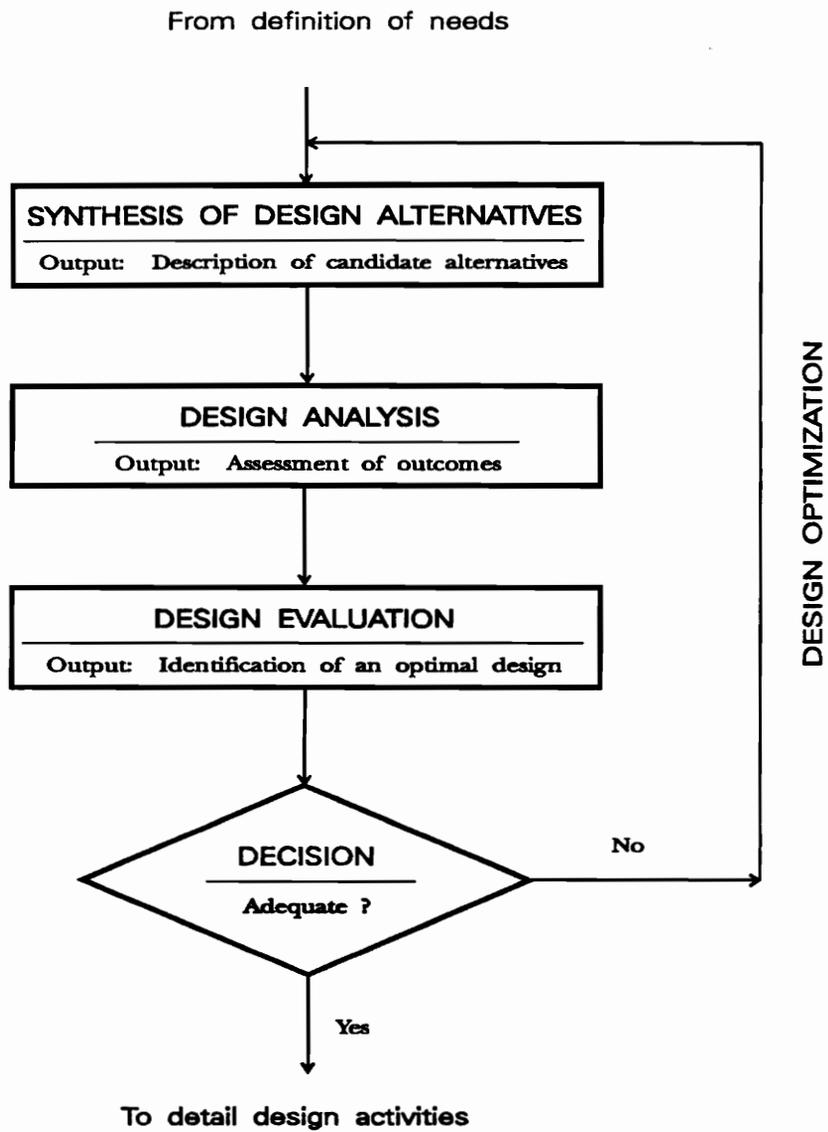


Figure 6.1. A conceptual model for preliminary system design

***Uncertainty analysis.*** The DM assigns probabilities to the branches emanating from the chance nodes. According to Keeney and Raiffa (1976), these assignments are made by actually mixing various techniques and procedures based on past empirical data, on assumptions fed into and results taken from various stochastic, dynamic models, on expert testimony, and on the subjective judgment of the DM. The assignments should be checked for internal inconsistencies.

***Outcome analysis.*** Design evaluation functions are determined to quantitatively describe the relationships between the evaluation attributes, design variables, design-dependent parameters, and design-independent parameters. Based on the evaluation functions, all outcomes for each decision within each design alternative are determined with respect to each uncertain event. Then a probability distribution for each outcome – the probability distribution over the set of evaluation attributes for each decision – is estimated.

Evaluation of the probability distributions of the evaluation attributes depends on the assumptions made, the amount of statistical data available, the complexity of the functional relationship, and the complexity of the analytical functions chosen to represent the probability distributions. When the probability distributions of the design-independent parameters are known, four techniques can be used (Sidall, 1982; Hahn and Shapiro, 1967): transformation of variables, independent cell method, moment transfer, and simulation.

After the four-step design analysis process is completed, the outputs are evaluated against the needs of design evaluation. If the outputs are adequate, go to the next stage in Figure 6.1; that is, design evaluation. Otherwise, the analysis process is repeated.

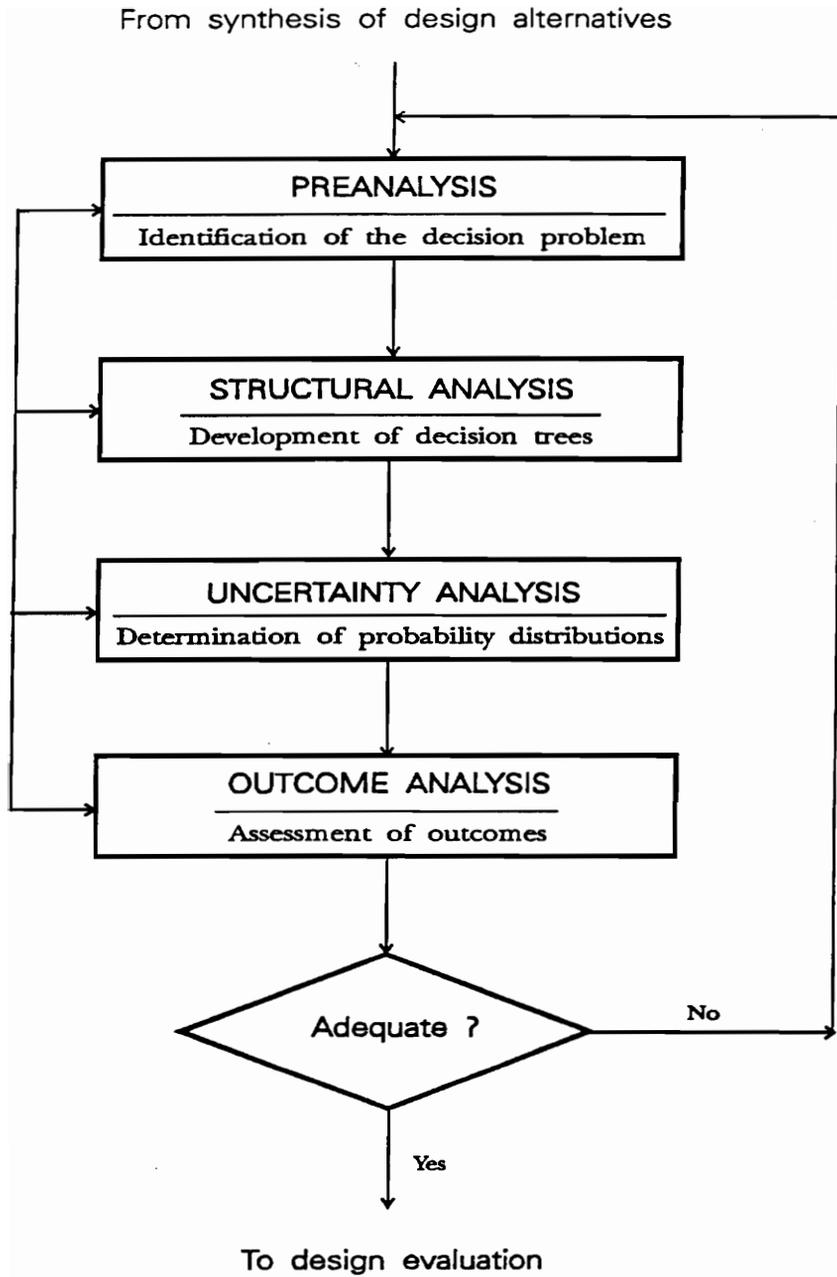


Figure 6.2. A structured model for design analysis

## **6.4 Design Evaluation for Discrete Decisions**

The objective of design evaluation is to select a decision strategy which gives the maximum expected utility for the DM. In practice, the approach used to identify such a strategy depends upon the outcome space of the decision problem. Some of the decision approaches identified in Chapter IV can be applied to both the problems of continuous outcome space and the problems of discrete outcome space. However, in general, the problem of discrete decisions and the problem of continuous decisions need to be resolved through different evaluation approaches. In this section, the decision analysis approaches which apply to the category of discrete decisions are identified. These approaches are then integrated into a structured design evaluation framework.

### **6.4.1 A structured model for design evaluation**

For a problem of discrete decisions, the outcome space is discrete if the events are discrete. For this category of problems (Category 1), the dominance rules, the Mean-Variance rules, and the Maximum Expected Utility principle presented in Chapter V can be directly applied for design evaluation. A structured framework is presented to help apply these approaches (Figure 6.3).

The framework is developed based on a consideration that the approaches which require fewer assumptions about the DM's behavior should be used first in evaluating design decisions. Four decision approaches are identified in the framework. The order of applying the approaches are determined according to their underlying assumptions. Outcome dominance is used first since it is an objective criterion. If all outcomes of Decision A dominate the corresponding outcomes of Decision B, Decision A is preferable. The only assumption made is that the DM prefers more (or less) of an evaluation attribute. If the decision problem is resolved by the application of the Outcome Dominance rule, other approaches are not needed. Then an optimal design strategy is recommended for examination by the decision makers.

The outcome dominance rule is helpful for eliminating some of the poor decisions. If it is not adequate to resolve the decision problem, a stronger approach should be used. If the DM is risk neutral, the decision problem can

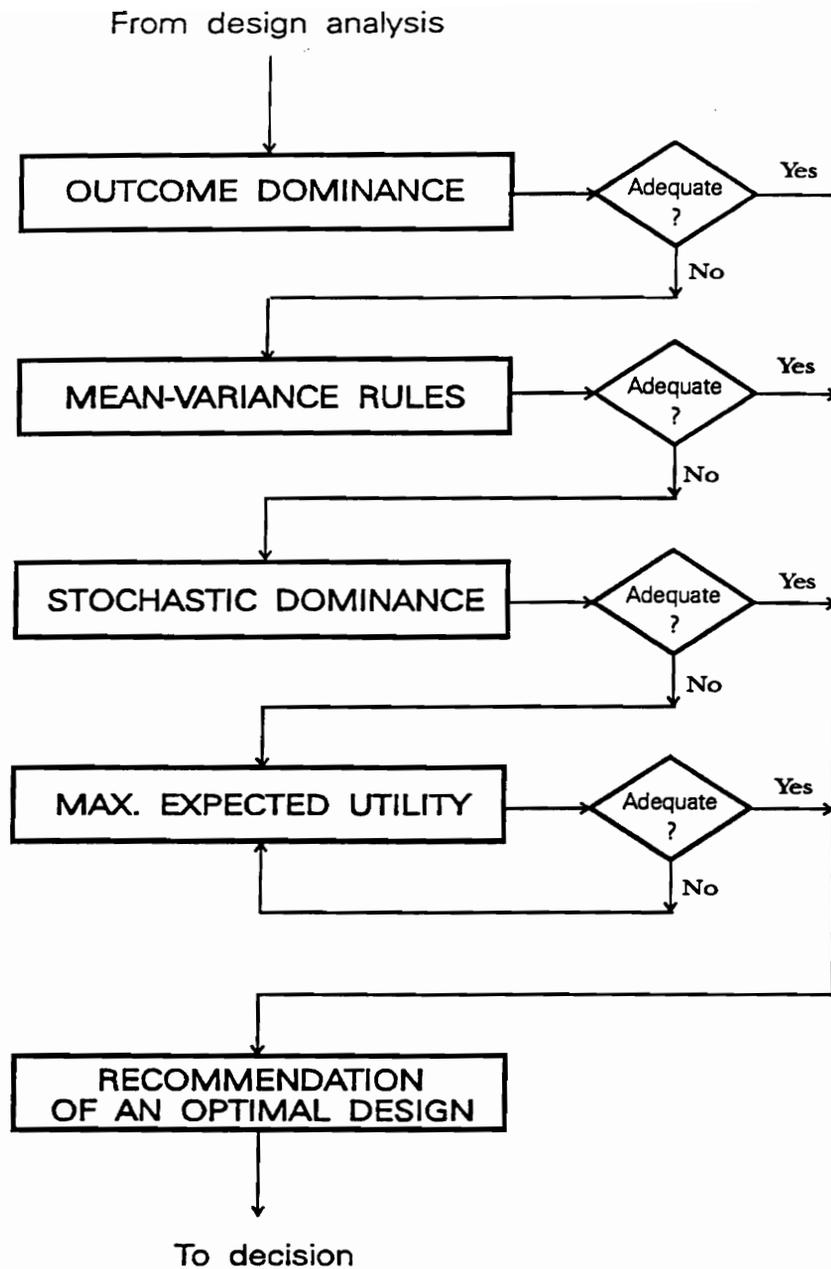


Figure 6.3. A structured evaluation model for problems of discrete decisions

be resolved by comparing the expected outcome for each decision. If the DM prefers more to less of the evaluation attribute, the optimal strategy is the one which has the maximum expected value.

If the DM is not risk neutral, design evaluation cannot be conducted by simply comparing the expected outcomes. Variance provides another dimension for comparison. Thus, the mean variance rules (E-V rules) may be used. If the DM prefers more (or less) of the evaluation attribute and also wants to minimize the attribute's variance, E-V diagrams can be developed to identify the efficient decisions. The inefficient decisions will be eliminated. If there is only one efficient decision, the problem is resolved. Thus, the efficient decision is the optimal decision.

If there exists more than one efficient decision, stochastic dominance may be used to compare the members of the efficient set. There are three stochastic dominance rules, which are based on progressively stronger assumptions. First, the First-Order Stochastic Dominance rule is used. If one decision is found to dominate other decisions, this decision is the optimal decision. The dominated decisions are eliminated. If some of the decisions do not dominate each other, the Second-Order Stochastic Dominance rule may be applied. A stronger assumption is made in the application of Second-Order Stochastic dominance; that is, the DM is risk averse. If the problem cannot be resolved by Second-Order Stochastic Dominance, Third-Order Stochastic Dominance may be used when the DM is risk averse with decreasing absolute risk aversion. In applying these rules, their underlying assumptions must be examined carefully. If any assumption is not true for a dominance rule, the rule itself and the higher-order rule cannot be used. Instead, the next approach in the framework, the MEU principle should be employed.

The MEU principle can be used as a general criterion for evaluating design decisions. The behavior assumptions underlying the principle are the six axioms for choice. The DM can be risk averse, risk neutral, or risk seeking. The application of the MEU principle follows the sequential decision analysis procedures in Section 5.5. After the utility function is assessed, the strategy which has the maximum expected utility is optimal. The consistency of utility analysis should be checked. If the assessment is not consistent, the process

should start again. After design evaluation, the optimal design is recommended to the DM for review.

If the events must be represented by continuous models, the structured evaluation framework presented above can still be used. There are no technical difficulties for application of the approaches. However, if the continuous probability distribution of the events can be approximated by a discrete distribution, the computation will be simplified. All the calculation procedures developed for discrete random variables can be used.

#### 6.4.2 Utility analysis of problems of continuous events

When the events are continuous, utility analysis for continuous outcome should be used. The application of utility analysis for problems of continuous events is explained below.

Since the outcome space is continuous due to continuous events, the expected utility for each decision must be defined by using a continuous model. The procedures for obtaining the expected utility for a given decision are illustrated in Figure 6.4.

In Figure 6.4a, the probability distribution of the events,  $g(s)$ , is identified. For each decision, the values of the evaluation attribute,  $E$ , are determined as a function of the event variable. Figure 6.4b shows the evaluation attribute of a selected decision for any event represented by a continuous variable,  $S$ . The values of the evaluation attribute are bounded by  $A$  and  $B$ . The probability distribution of the evaluation attribute for each decision,  $f_i(e)$ , is obtained based upon a design evaluation function (Figure 6.4c). In this figure, the expected value of the evaluation attribute is given by  $\mu_E$ . For each value of  $E$ , its corresponding utility,  $u_i(e)$ , is obtained according to the DM's utility function (Figure 6.4d). The probability distribution of the utilities is illustrated in Figure 6.4e. The expected utility for each decision is given by

$$E[u_i(e)] = \int_{-\infty}^{\infty} u(e) f_i(e) de. \quad (6.1)$$

where  $f_i(e)$  is the probability density function of  $E$  for alternative  $i$ .

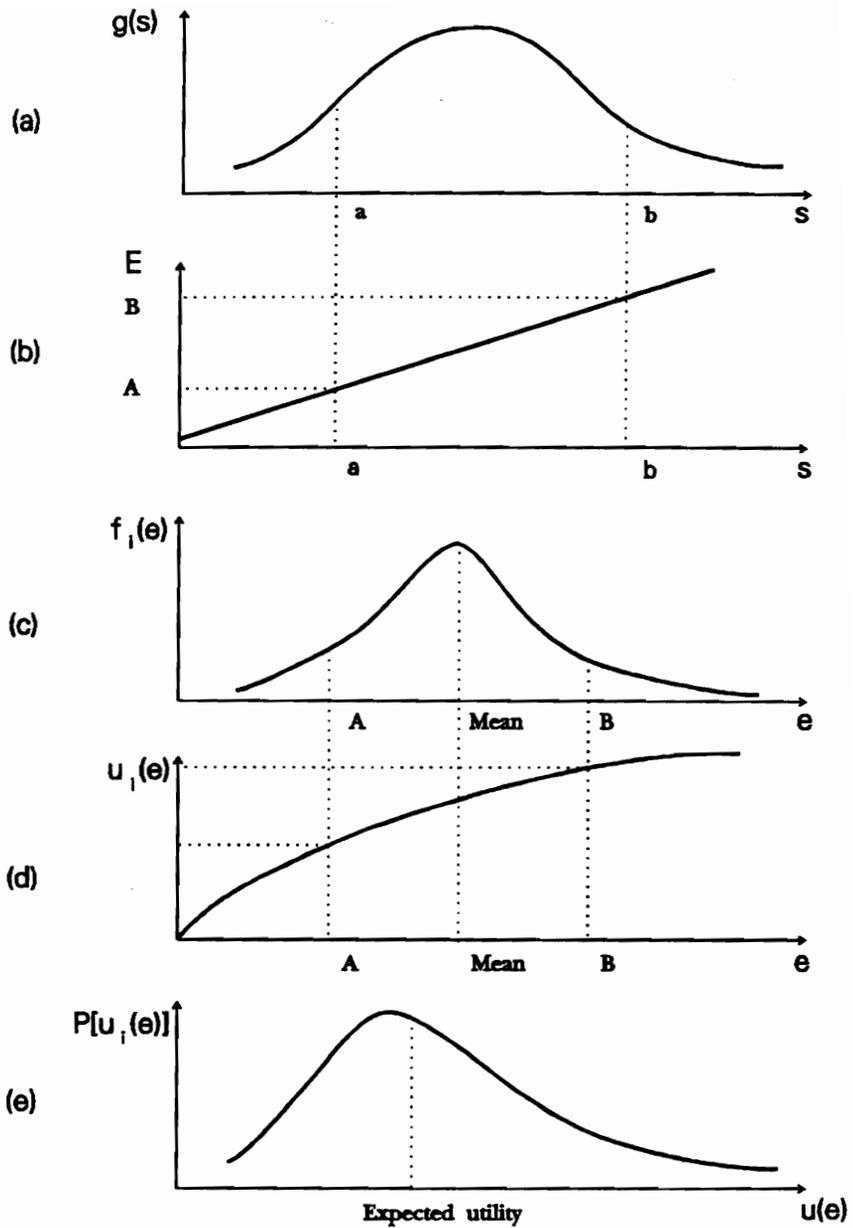


Figure 6.4. Utility analysis of problems of continuous events

## 6.5 Design Evaluation for Continuous Decisions

In the case of continuous decisions, it is impossible to compare individual decisions. For each design alternative, the evaluation attribute is a function of both the design variables and design-independent parameters. The objective is to determine a set of values for the design variables so that the system's expected utility is maximized. An evaluation process is presented for this type of problem (Figure 6.5).

The first step is to determine an objective function for design optimization. The function is actually the expected utility for the evaluation attribute. For a given utility function  $u(e)$ , the expected utility is

$$E[u(e)] = \int_{-\infty}^{\infty} u(e)f(e)de \quad (6.2)$$

where  $f(e)$  is the probability density function of  $E$ . For each alternative, the decision problem becomes one of determining the design variable settings which

$$\text{maximize } E[u(e)] = \int_{-\infty}^{\infty} u(e)f(e)de.$$

After an objective function is determined, the next step is to select an approach to solve this problem. An effective approach to the solution of the problem is to use optimization techniques. In practice, the problem may be formulated as a nonlinear optimization problem or as a stochastic optimization problem. If analytical models cannot be formulated, the problem may be formulated as a Monte-Carlo simulation model. Taguchi's parameter design may be used to convert a continuous problem into a discrete problem.

The optimization problem is to determine optimal settings of the design variables for each alternative. Then the expected utilities achieved by the optimal decision for each alternative are compared. The design which has the maximum expected utility is the optimal design.

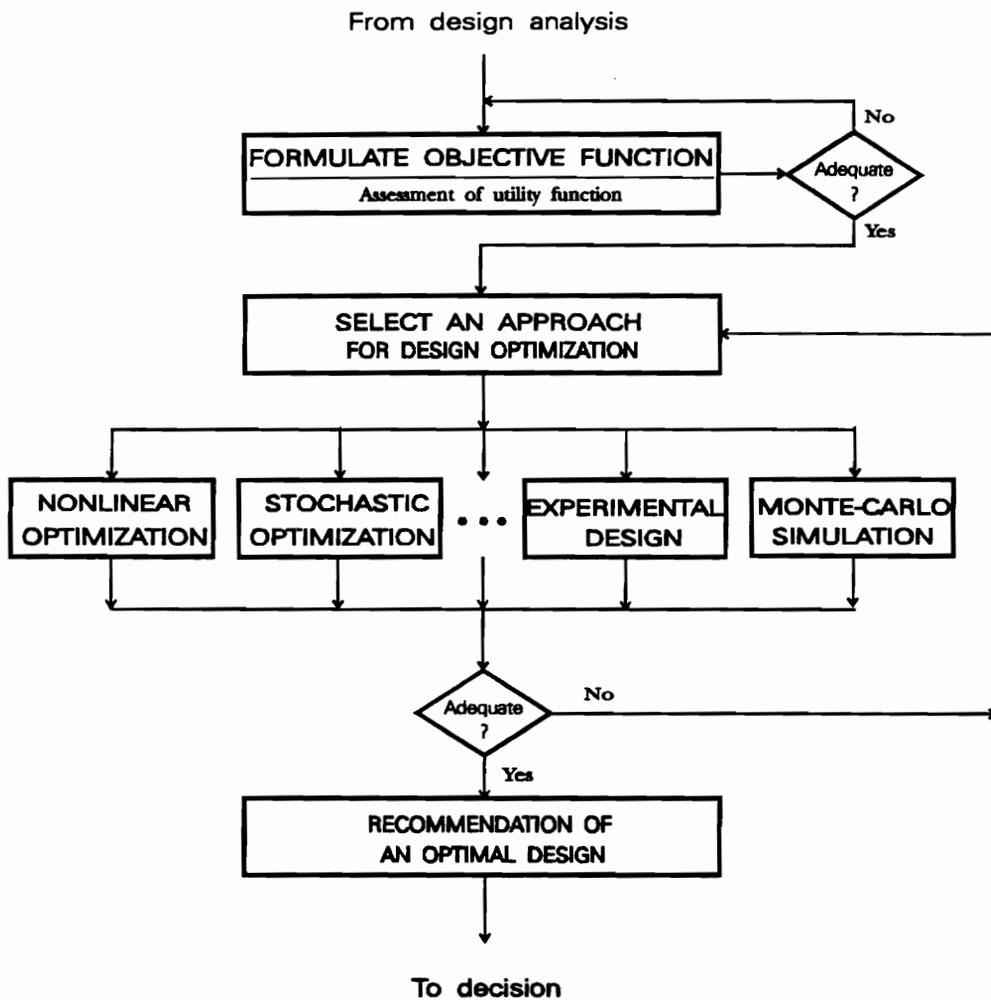


Figure 6.5. A structured evaluation model for problems of continuous decisions

In applying this approach, determination of the objective function should follow the procedures for the assessment of utility functions given in Section 5.4.6. In the literature on design optimization, a special function of mean and/or variance was often used as the objective function without giving the underlying decision analysis foundations. For example, a common problem formulation is a nonlinear optimization problem in which the variance of the evaluation attribute is minimized while the evaluation attribute is constrained to a target value (Ramakrishnan and Rao, 1991; d'Entremont and Ragsdell, 1988). In the formulation of stochastic optimization problems, a linear function of mean and variance was often assumed as the objective function (Eggert and Mayne, 1990; Rao and Reddy, 1979; Sundaresan et al., 1991). Though these formulations may be more appropriate to consider risks in design than the traditional problem formulation in which a mean is optimized, one must keep in mind their underlying assumptions as discussed in Section 5.8. The assumptions for each objective function should be examined to enable the objective function to accurately reflect the DM's preferences and risk attitudes. As indicated in Section 5.5.3, the objective function should combine information concerning the utility of outcomes and the probability of outcomes into an estimate of expected utility.

## **VII. AN EXAMPLE OF BRIDGE DESIGN**

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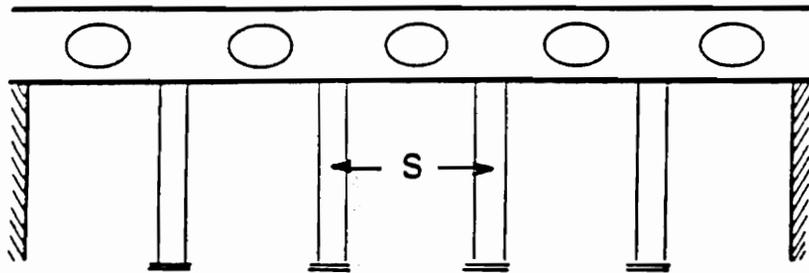
- 7.1 Introduction
  - 7.2 Description of the Problem
  - 7.3 Determination of Design Evaluation Function
  - 7.4 Problem Solution under Certainty
  - 7.5 The Uncertainty Problem
  - 7.6 Design Evaluation in the Face of Uncertainty
- 

### **7.1 Introduction**

A hypothetical example is presented in this chapter to illustrate the application of the framework presented in Chapter VI. The example originates from the bridge design evaluation model given in Chapter 10 of Fabrycky and Blanchard's *Life-Cycle Cost and Economic Analysis* (1991).

### **7.2 Description of the Problem**

The classical situation of bridge design was studied by Fabrycky and Blanchard (1991) to illustrate design optimization. An evaluation model was developed to help decision makers optimally allocate the anticipated capital investment to superstructure and to piers in the preliminary design stage. It assumes that there exists an inverse relationship between the cost of the superstructure and the number of piers. As the number of piers increases, the cost of the superstructure decreases. Conversely, the cost of the superstructure increases as the number of piers decreases. Pier cost is directly related to the number specified. Two bridge superstructure design alternatives were compared in their study (Figure 7.1). The objective is to select an alternative with the minimum total first cost.



$v/s$

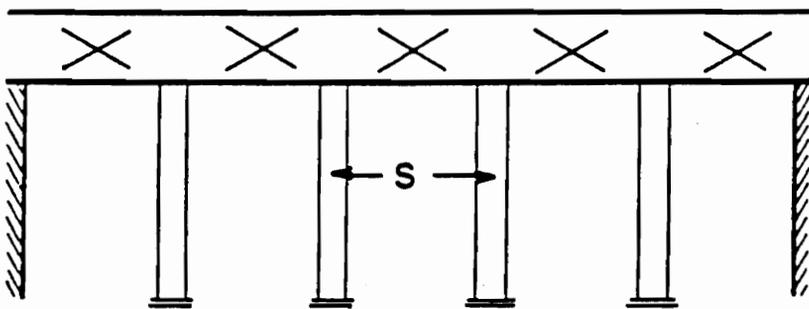


Figure 7.1. Two bridge superstructure design alternatives (Fabrycky and Blanchard, 1991)

### 7.3 Determination of Design Evaluation Function

All variables, parameters, and performance measures in the problem are classified into four groups: design variables, design-dependent parameters, design-independent parameters, and evaluation attributes (Table 7.1). The notation used in the example is:

$L$  = bridge length (feet)

$W$  = superstructure weight (pounds per foot)

$S$  = span between piers (feet)

$C_s$  = erected cost of superstructure (dollars per pound)

$C_p$  = installed cost of piers (dollars per pier)

$TFC$  = total first cost

Table 7.1. List of Design Variables, Parameters, and Evaluation Attribute

|                               |               |
|-------------------------------|---------------|
| Design Variable               | $S$           |
| Design-Dependent Parameters   | $A, B$        |
| Design-Independent Parameters | $C_s, C_p, L$ |
| Evaluation Attributes         | $TFC$         |

Assume that the weight of the superstructure is a linear function of the span between piers. That is,

$$W = AS + B,$$

where  $A$  and  $B$  are constants established by statistical estimation for the design alternative under consideration.

The total first cost of the bridge,  $TFC$ , is the sum of the superstructure cost,  $SC$ , and the total cost of piers,  $PC$ .  $TFC$  can be expressed as

$$TFC = SC + PC. \tag{7.1}$$

The superstructure cost,  $SC$ , is given by

$$\begin{aligned} SC &= W \times L \times C_s \\ &= (AS + B)L \times C_s \end{aligned} \quad (7.2)$$

The total cost of piers,  $PC$ , is the product of the number of piers and the installed cost for each pier. If the two abutments are considered as piers, the total cost of piers is equal to

$$PC = \left( \frac{L}{S} + 1 \right) C_p. \quad (7.3)$$

Thus, the total first cost is expressed as

$$TFC = (AS + B)L \times C_s + \left( \frac{L}{S} + 1 \right) C_p. \quad (7.4)$$

This is the design evaluation function for the bridge design problem, which represents the evaluation attribute,  $TFC$ , as a function of design variables, design-dependent parameters, and design-independent parameters. The decision problem is to select an alternative which has the minimum  $TFC$ .

#### 7.4 Problem Solution under Certainty

Two design alternatives are compared in this example. The basic evaluation process will follow the steps of the Design Dependent Parameter Approach. First, the decisions within each alternative, i.e., selecting span between piers, are compared separately to obtain an optimal solution. Then the optimal decision for Alternative 1 is compared with that of Alternative 2. The alternative with the lowest total first cost is selected as the best solution for the overall problem.

To find the optimum span between piers for each alternative, differentiate Equation (7.4) with respect to  $S$  and equate the result to zero, giving

$$\frac{d}{dS}(TFC) = A \times L \times C_s - \frac{L \times C_p}{S^2} = 0,$$

$$S^* = \sqrt{\frac{C_p}{A \times C_s}}. \quad (7.5)$$

Since

$$\frac{d^2}{dS^2}(TFC) = \frac{2L \times C_p}{S^3} > 0,$$

the minimum *TFC* is obtained by selecting  $S = S^*$ , giving

$$TFC^* = 2\sqrt{A \times C_p \times L^2 \times C_s} + B \times L \times C_s + C_p. \quad (7.6)$$

Thus, if a superstructure design is selected and the settings of all of the design-dependent parameters and design-independent parameters are determined, the optimal pier spacing can be found by using Equation (7.5).

#### 7.4.1 Data inputs

The values of various input parameters are given in Table 7.2, which are the same as given by Fabrycky and Blanchard (1991). In the following study, the unit pier cost will also be used as the estimated cost for each abutment.

Table 7.2. Data Inputs for the Bridge Design Example

|                                       |                                  |
|---------------------------------------|----------------------------------|
| Bridge length, $L$                    | 1,000 feet                       |
| Erected cost of superstructure, $C_s$ | \$0.65 per pound                 |
| Installed cost of piers, $C_p$        | \$80,000 per pier                |
| Weight of superstructure, $W$         |                                  |
| Alternative 1                         | $W = 16 S + 600$ pounds per foot |
| Alternative 2                         | $W = 22 S + 0$ pounds per foot   |

### 7.4.2 Alternative 1

For Alternative 1, based on Equation (7.5), the optimum span between piers is equal to

$$S^* = \sqrt{\frac{80000}{16 \times 0.65}} = 87.7 \text{ (feet).}$$

The number of piers is then given by

$$\frac{L}{S^*} + 1 = \frac{1000}{87.7} + 1 = 12.4.$$

This result must be adjusted to obtain an integer number of piers. The number can be found by calculating the *TFC* with respect to various numbers of piers around 12.4.

The total first costs for different number of piers are calculated by using the design evaluation function (7.4) and plotted along with the pier cost and the superstructure cost in Figure 7.2. The decision of 12 piers has the minimum *TFC*, which is equal to \$2,295,455.

### 7.4.3 Alternative 2

Similarly, the pier cost, superstructure cost, and total cost for various numbers of piers are calculated and plotted in Figure 7.3. The optimal span between piers,  $S^*$ , is found to be

$$S^* = \sqrt{\frac{80000}{22 + 0.65}} = 74.8 \text{ (feet).}$$

The lowest-cost integer number of piers is 14 piers, whose *TFC* is \$2,220,000.

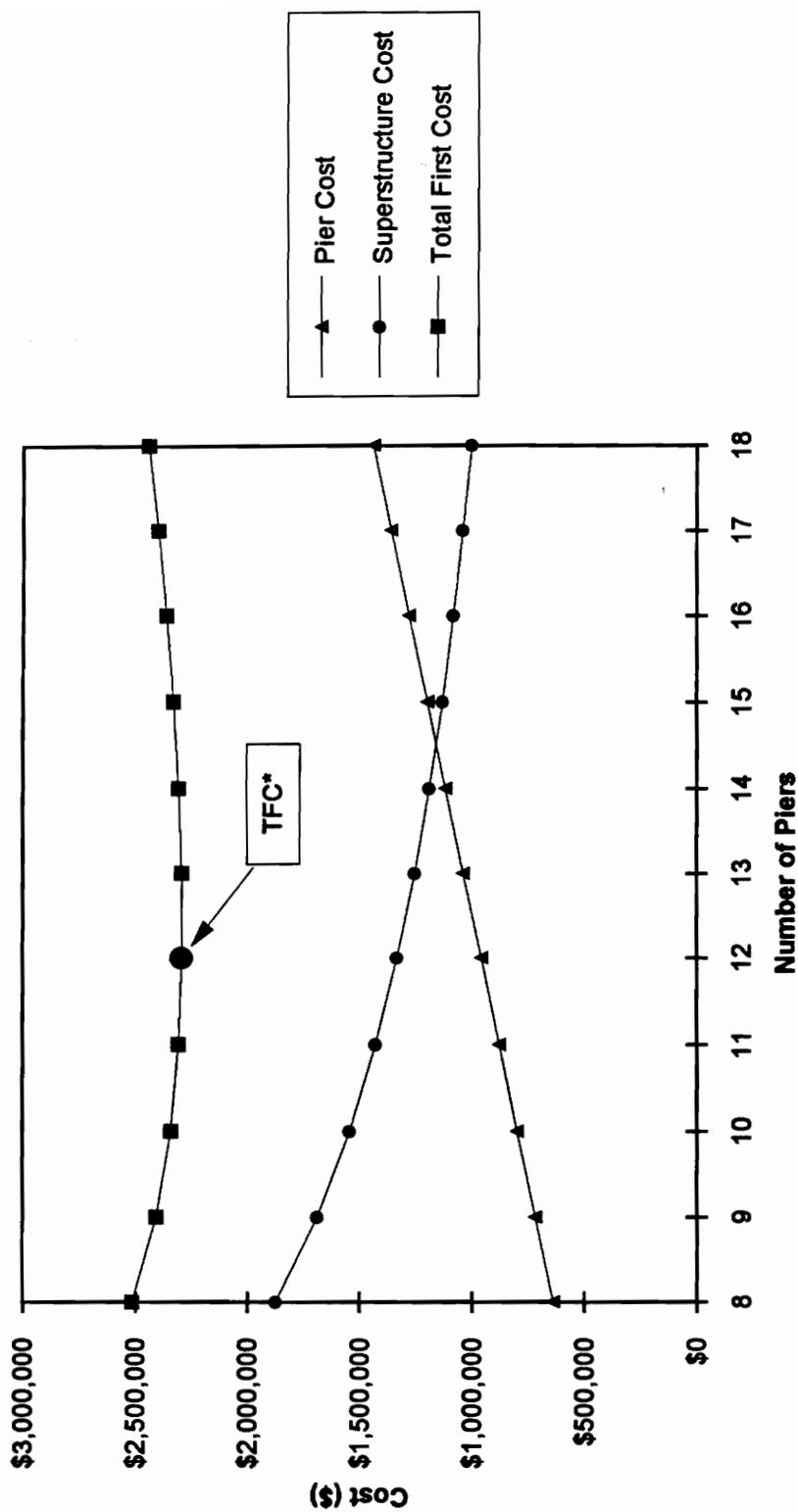


Figure 7.2. Pier cost, superstructure cost, and total first cost (Alternative 1)

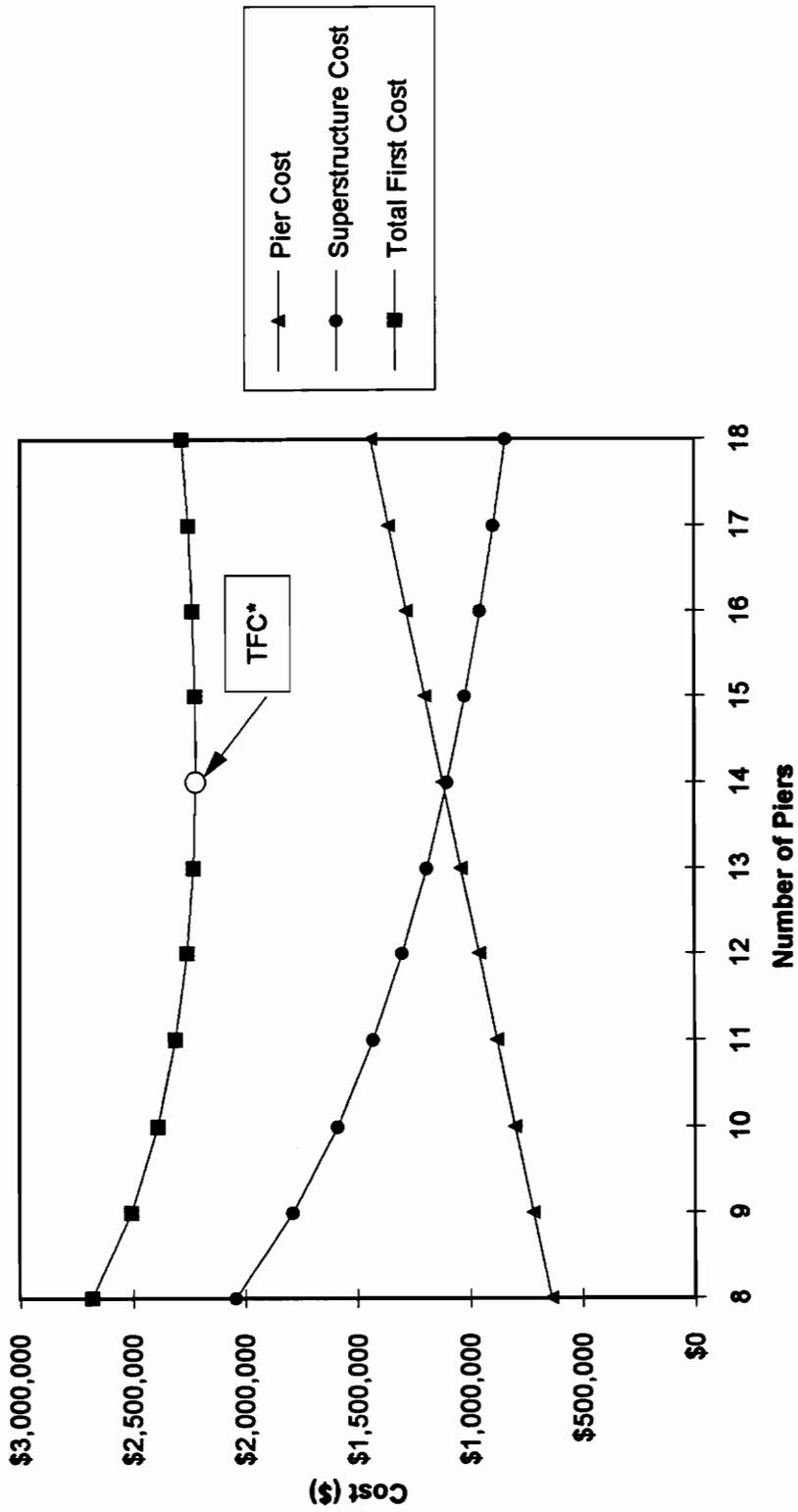


Figure 7.3. Pier cost, superstructure cost, and total first cost (Alternative 2)

#### 7.4.4 Comparison

The characteristics of the optimal solutions for Alternatives 1 and 2 are listed in Table 7.3. Alternative 2 has a lower total first cost. Thus, if the data inputs are as given in Table 7.2, Alternative 2 is preferable. The optimal number of piers is 14, with a total first cost of \$2,220,000.

Table 7.3. Optimal Solutions for Alternatives 1 and 2

| Alternative | Optimal Pier Number | Pier Cost (\$) | Superstructure Cost (\$) | Total First Cost (\$) |
|-------------|---------------------|----------------|--------------------------|-----------------------|
| 1           | 12                  | 960,000        | 1,300,000                | 2,260,000             |
| 2           | 14                  | 1,120,000      | 1,100,000                | 2,220,000             |

### 7.5 The Uncertainty Decision Problem

In the last section, the problem was solved by assuming that all design-independent parameter values in Table 7.2 are known constants. However, in practice, the settings of those parameters are not subjected to the designer's control. Their values may be difficult to estimate and are not known with certainty during the preliminary system design stage. This type of uncertainty causes difficulties in the evaluation of the design alternatives.

#### 7.5.1 Functional dependence

There are three design-independent parameters in the example problem:  $C_s$ ,  $C_p$ , and  $L$ . Among these parameters, bridge length  $L$  is the easiest to determine and is usually known in the preliminary design stage. When a superstructure configuration is selected, its corresponding design-dependent parameters  $A$  and

$B$  are determined. Thus, for each candidate design, the total first cost becomes a function of  $C_s$ ,  $C_p$ , and design variable  $S$ .

For a specific number of piers, according to Equation (7.4),  $TFC$  can be represented as a linear function of  $C_s$  and  $C_p$ . That is,

$$TFC = k_1 C_s + k_2 C_p \quad (7.7)$$

where  $k_1 = (AS + B)L$  and  $k_2 = (L/S + 1)$ . This equation represents the functional dependence of  $TFC$  on two design-independent parameters. Since  $C_s$  and  $C_p$  are not subject to the DM's control, they can be treated as random variables. Thus,  $TFC$  is a function of two random variables. If the probability distributions of  $C_s$  and  $C_p$  are known, the probability distribution of  $TFC$  can be determined by Equation (7.7).

Suppose that the mean and the variance of  $C_s$  and  $C_p$  are given as:

$$E(C_s) = \mu_1, \quad Var(C_s) = \sigma_1^2;$$

$$E(C_p) = \mu_2, \quad Var(C_p) = \sigma_2^2.$$

Taking expectation of both sides of Equation (7.7), we obtain the mean and the variance of  $TFC$ :

$$E(TFC) = k_1 \mu_1 + k_2 \mu_2,$$

$$Var(TFC) = k_1^2 \sigma_1^2 + k_2^2 \sigma_2^2 + 2 \text{cov}(C_s, C_p).$$

If  $C_s$  and  $C_p$  are not correlated, then

$$Var(TFC) = k_1^2 \sigma_1^2 + k_2^2 \sigma_2^2.$$

### 7.5.2 Probability distribution of $C_s$

In this hypothetical example, the original problem of bridge design is modified to illustrate the approach for design evaluation under uncertainty. We assume that  $C_s$  is a random variable with its probability distribution given in Figure 7.4, and let other parameters be the same as given in Table 7.2. As given in Figure 7.4,  $C_s$  has a mean of 0.65, which is equal to the constant setting of  $C_s$  given in the original problem (see Table 7.2). However, here  $C_s$  is not a constant, rather a random variable with a variance of 0.0039. To reduce the amount of computation, we let  $C_p$  be a constant. But the approach can be easily extended to consideration of  $C_s$  and  $C_p$  simultaneously.

### 7.5.3 The effect of uncertainties on $TFC$

As a random variable,  $C_s$  can take any of the six settings in Figure 7.4. What is the effect of the uncertainty with  $C_s$  on the evaluation of the design alternatives? For each setting of  $C_s$ , the optimal span and the corresponding  $TFC$  are calculated using Equations (7.5) and (7.6). The results for Alternative 1 are plotted in Figure 7.5.

As indicated in Figure 7.5, both the optimal span ( $S^*$ ) and the minimum  $TFC$  ( $TFC^*$ ) vary greatly with the settings of  $C_s$ . With the increase of  $C_s$ , the optimal span between piers decreases, while the  $TFC$  goes up. Not any single specific number of piers can achieve a minimum total first cost for all settings of  $C_s$ . These trends are also true for Alternative 2 (Figure 7.6).

In this example,  $TFC$  is a function of two independent variables,  $C_s$  and the number of piers. Since  $C_s$  is a random variable,  $TFC$  is also a random variable. To measure a random variable, one needs to specify its probability distribution. Based upon the design evaluation function (7.4), the probability distributions, mean, and variance of the  $TFC$  for each decision are obtained in Table 7.4 and Table 7.5. The variations of  $TFC$  with respect to  $C_s$  are illustrated in Figures 7.7 and 7.8 for each alternative.

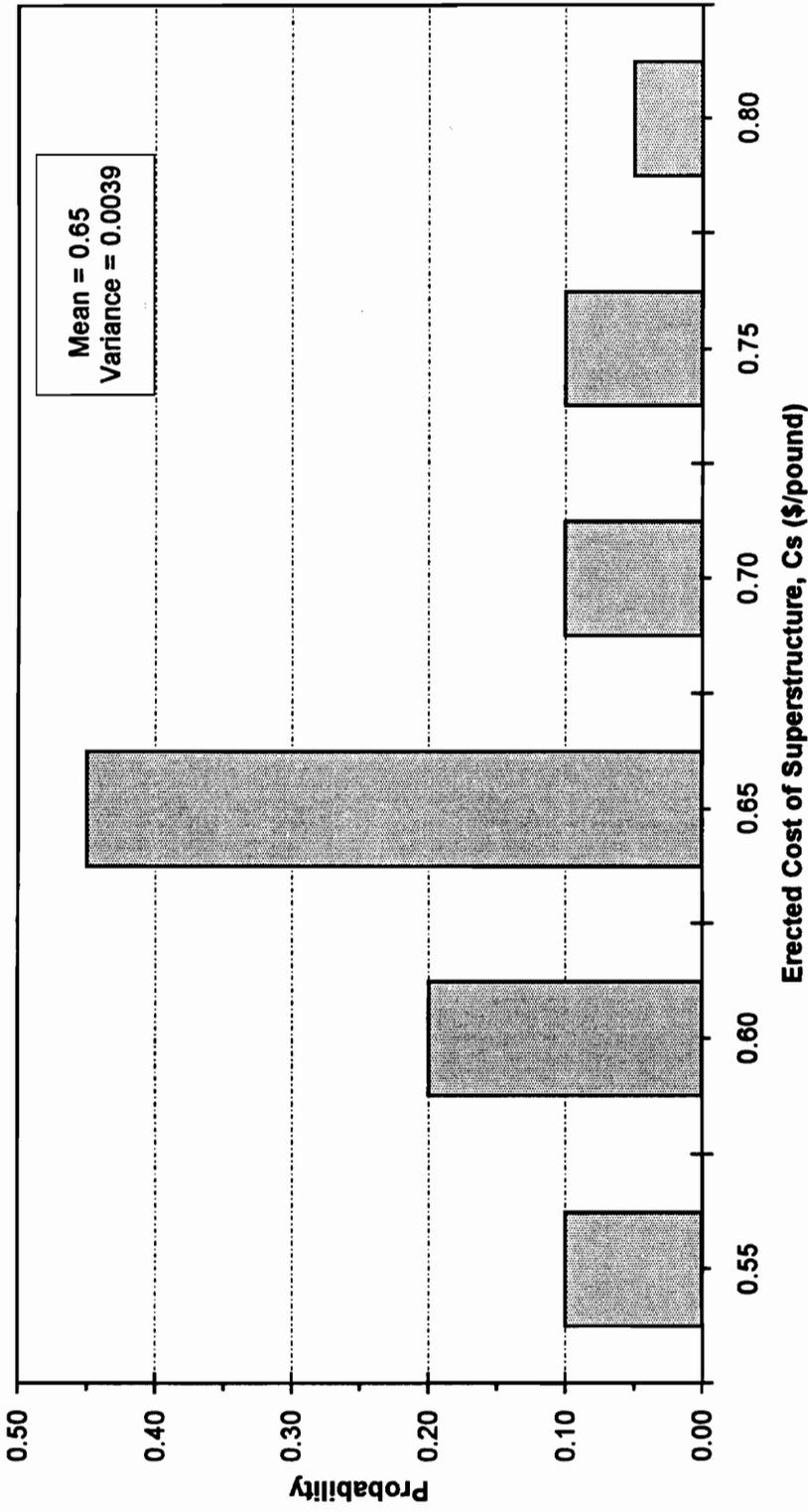


Figure 7.4. Probability distribution of the erected cost of superstructure

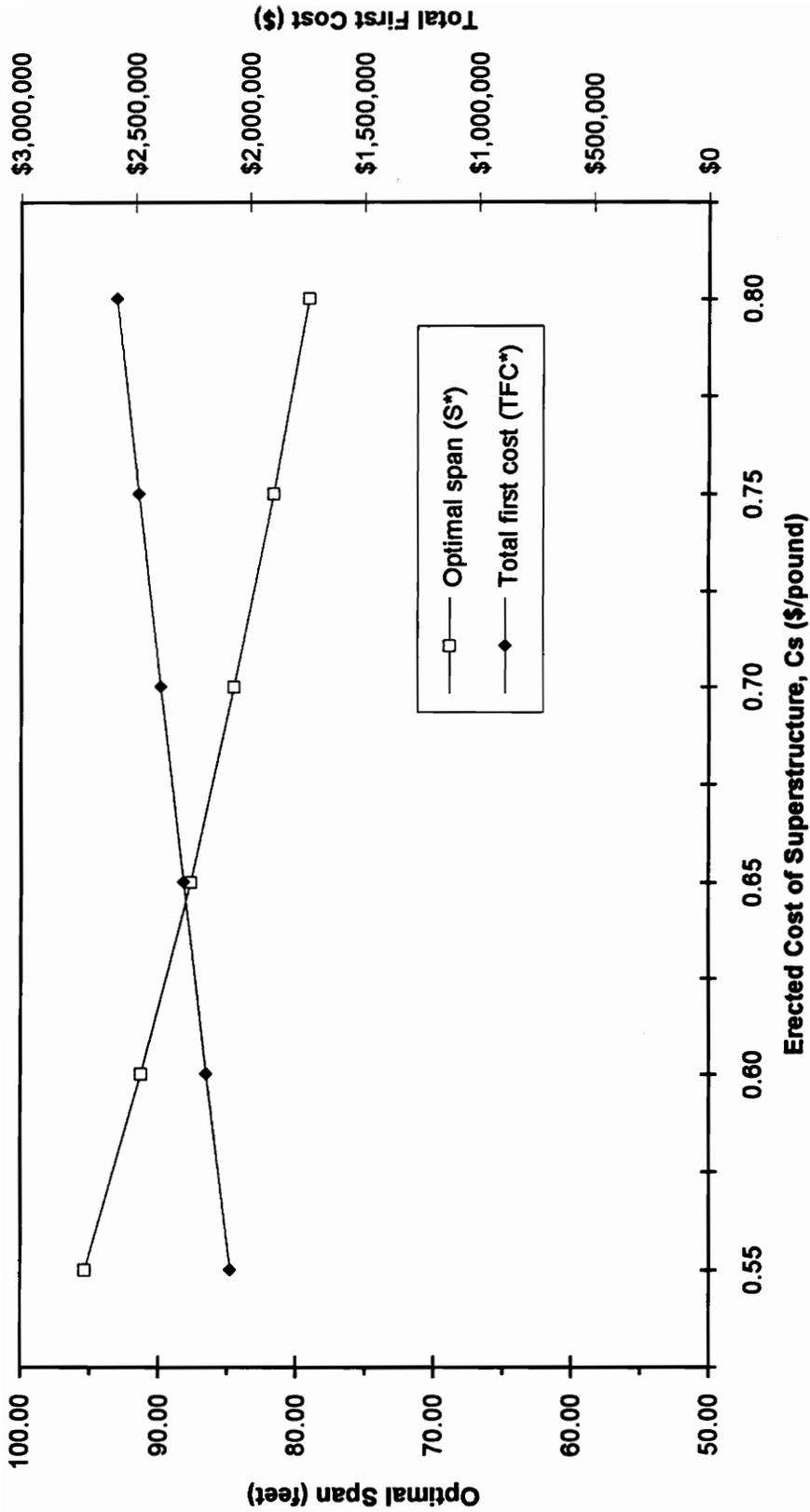


Figure 7.5. Optimal span and minimum  $TFC$  for various  $C_s$  (Alternative 1)

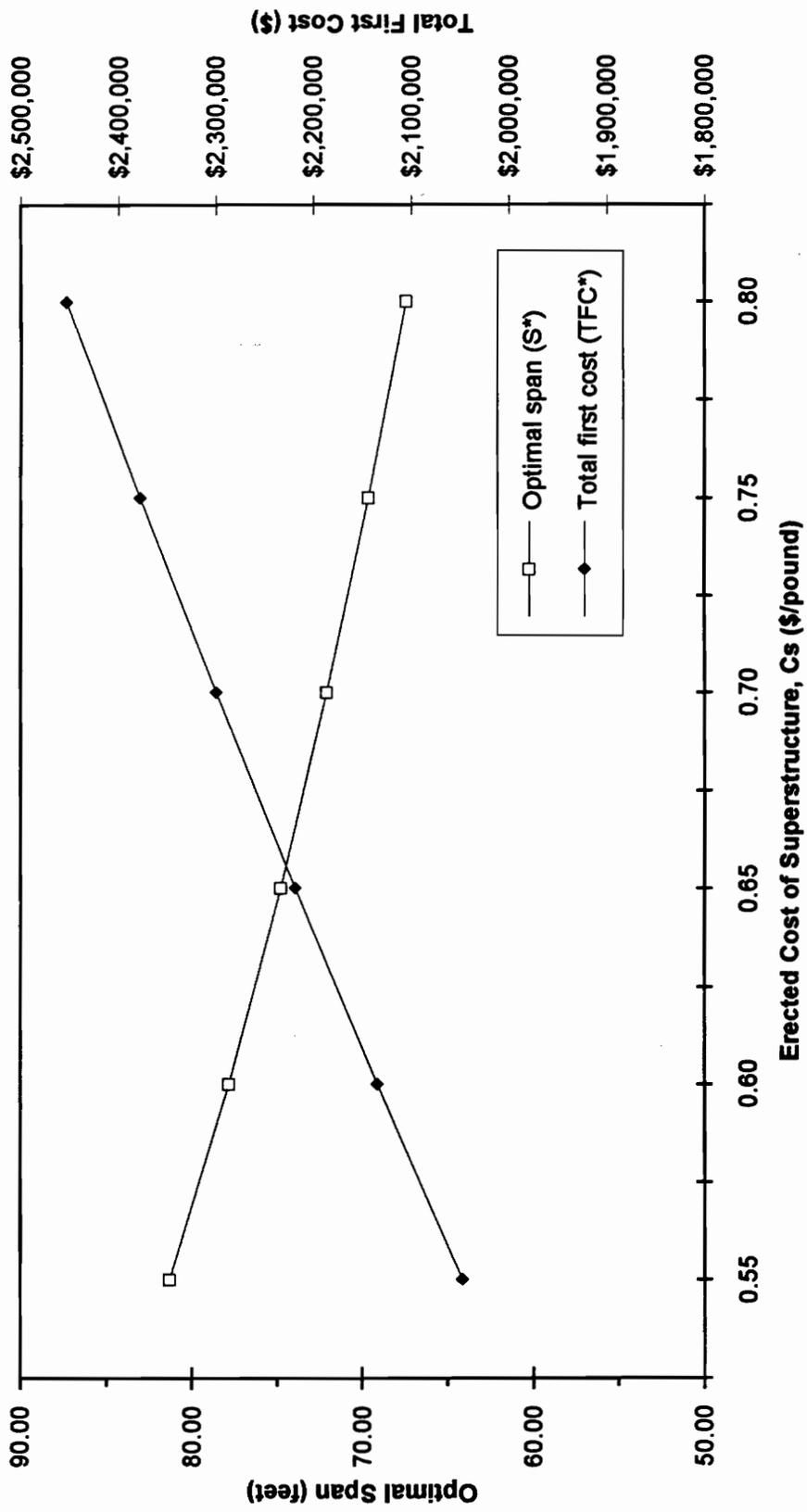


Figure 7.6. Optimal span and minimum  $TFC$  for various  $C_s$  (Alternative 2)



**Table 7.5. Total First Cost for Different Number of Piers and Cs (Alternative 2)**

| No. of Piers | Span (feet) | Erected Cost of Superstructure, Cs (\$/pound) |           |           |           |           |           |           |           |           |           | Mean      | Variance  |           |          |
|--------------|-------------|---|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|----------|
|              |             | 0.55  | 0.60      | 0.65      | 0.70      | 0.75      | 0.80      | 0.85      | 0.90      | 0.95      | 1.00      |           |           |           |          |
| 8            | 142.86      | 2,368,571                                     | 2,525,714 | 2,682,857 | 2,840,000 | 2,997,143 | 3,154,286 | 2,690,714 | 2,690,714 | 2,690,714 | 2,690,714 | 2,690,714 | 2,690,714 | 2,690,714 | 3.87E-03 |
| 9            | 125.00      | 2,232,500                                     | 2,370,000 | 2,507,500 | 2,645,000 | 2,782,500 | 2,920,000 | 2,514,375 | 2,514,375 | 2,514,375 | 2,514,375 | 2,514,375 | 2,514,375 | 2,514,375 | 2.93E+10 |
| 10           | 111.11      | 2,144,444                                     | 2,266,667 | 2,388,889 | 2,511,111 | 2,633,333 | 2,755,556 | 2,395,000 | 2,395,000 | 2,395,000 | 2,395,000 | 2,395,000 | 2,395,000 | 2,395,000 | 2.31E+10 |
| 11           | 100.00      | 2,090,000                                     | 2,200,000 | 2,310,000 | 2,420,000 | 2,530,000 | 2,640,000 | 2,315,500 | 2,315,500 | 2,315,500 | 2,315,500 | 2,315,500 | 2,315,500 | 2,315,500 | 1.87E+10 |
| 12           | 90.91       | 2,060,000                                     | 2,160,000 | 2,260,000 | 2,360,000 | 2,460,000 | 2,560,000 | 2,265,000 | 2,265,000 | 2,265,000 | 2,265,000 | 2,265,000 | 2,265,000 | 2,265,000 | 1.55E+10 |
| 13           | 83.33       | 2,048,333                                     | 2,140,000 | 2,231,667 | 2,323,333 | 2,415,000 | 2,506,667 | 2,236,250 | 2,236,250 | 2,236,250 | 2,236,250 | 2,236,250 | 2,236,250 | 2,236,250 | 1.30E+10 |
| 14           | 76.92       | 2,050,769                                     | 2,135,385 | 2,220,000 | 2,304,615 | 2,389,231 | 2,473,846 | 2,224,231 | 2,224,231 | 2,224,231 | 2,224,231 | 2,224,231 | 2,224,231 | 2,224,231 | 1.11E+10 |
| 15           | 71.43       | 2,064,286                                     | 2,142,857 | 2,221,429 | 2,300,000 | 2,378,571 | 2,457,143 | 2,225,357 | 2,225,357 | 2,225,357 | 2,225,357 | 2,225,357 | 2,225,357 | 2,225,357 | 9.55E+09 |
| 16           | 66.67       | 2,086,667                                     | 2,160,000 | 2,233,333 | 2,306,667 | 2,380,000 | 2,453,333 | 2,237,000 | 2,237,000 | 2,237,000 | 2,237,000 | 2,237,000 | 2,237,000 | 2,237,000 | 8.32E+09 |
| 17           | 62.50       | 2,116,250                                     | 2,185,000 | 2,253,750 | 2,322,500 | 2,391,250 | 2,460,000 | 2,257,188 | 2,257,188 | 2,257,188 | 2,257,188 | 2,257,188 | 2,257,188 | 2,257,188 | 7.31E+09 |
| 18           | 58.82       | 2,151,765                                     | 2,216,471 | 2,281,176 | 2,345,882 | 2,410,588 | 2,475,294 | 2,284,412 | 2,284,412 | 2,284,412 | 2,284,412 | 2,284,412 | 2,284,412 | 2,284,412 | 6.48E+09 |

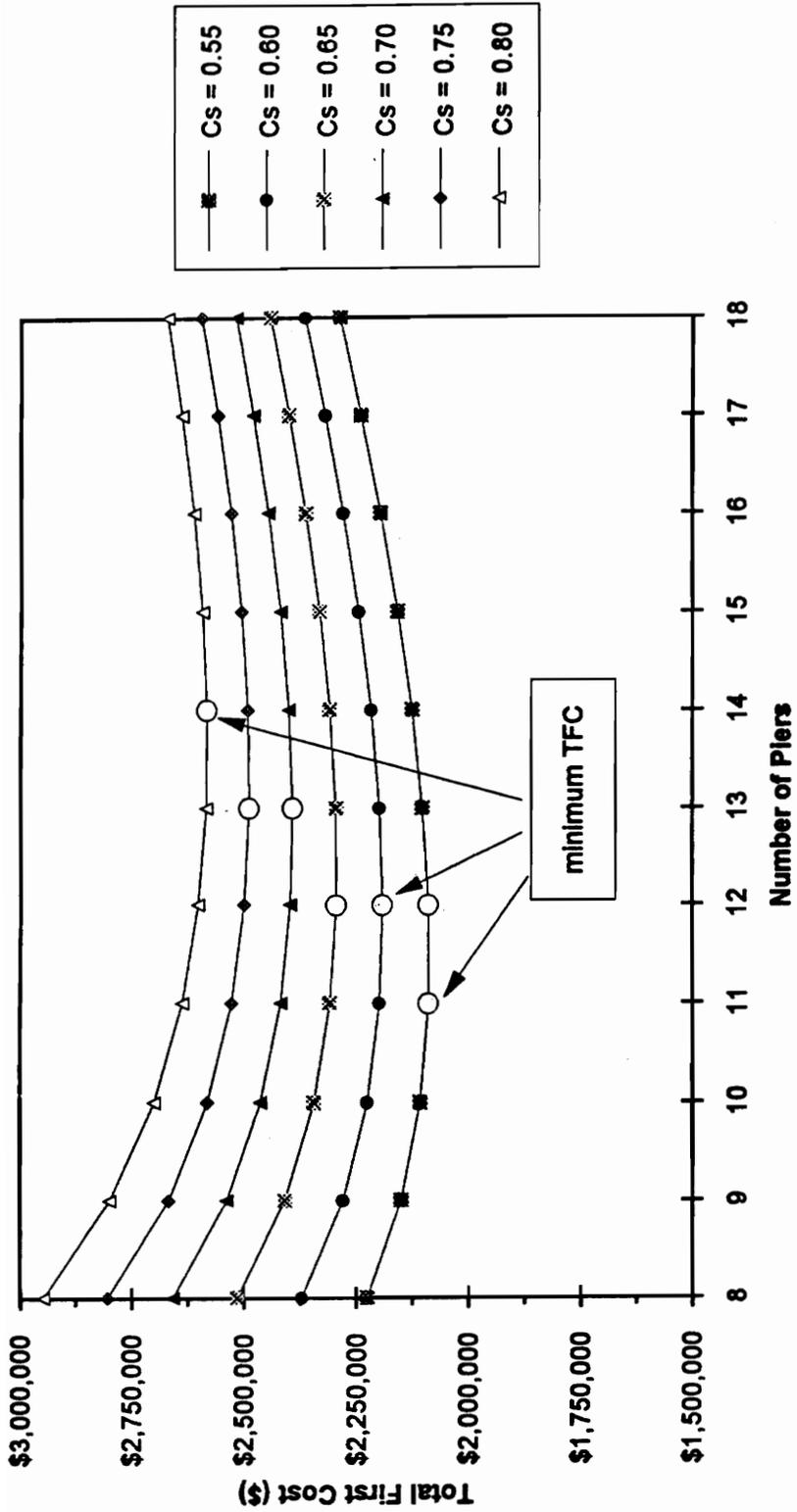


Figure 7.7. Total first cost as a function of  $C_s$  and the number of piers (Alternative 1)

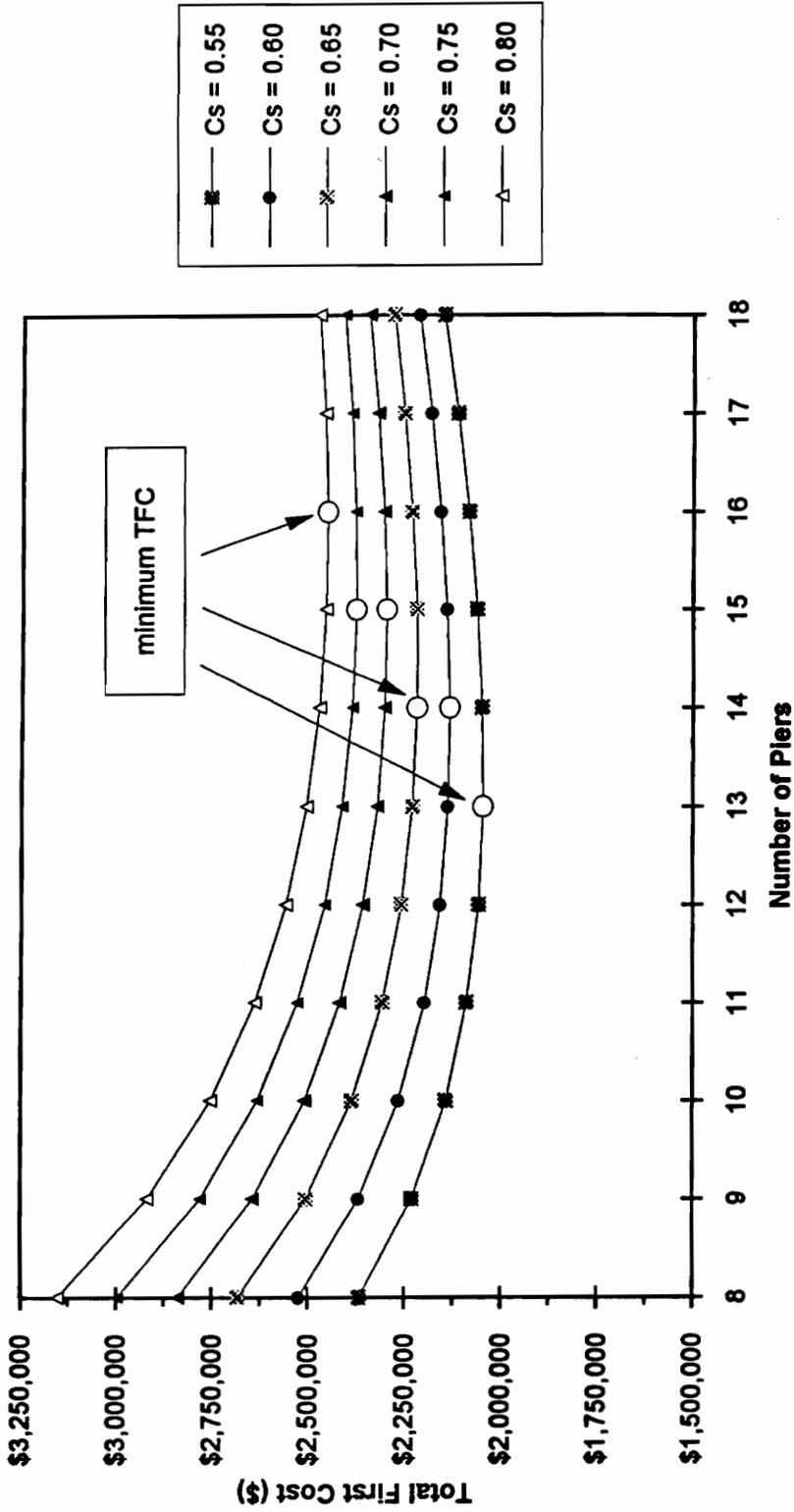


Figure 7.8. Total first cost as a function of  $C_s$  and the number of piers (Alternative 2)

## 7.6 Design Evaluation in the Face of Uncertainty

### 7.6.1 The evaluation approach

In the certainty case, the values obtained for  $TFC^*$  are considered to be a constant. We compared the values of the  $TFC$  for various number of piers and selected the decision that has the lowest  $TFC$ . However,  $TFC$  is a random variable under the uncertainty case. A selection cannot be made by simply comparing any particular values of  $TFC$ , e.g., the mean, since the setting which  $C_s$  will take is unknown. Because of uncertainties involved in  $C_s$ , there exists a problem of decision making under uncertainty. The problem can be represented by a decision tree (Figure 7.9).

In this design evaluation problem involving uncertainties, the different settings of  $C_s$  actually represent different states of nature. Though  $C_s$  can take any of the six settings in Figure 7.4, these settings are mutually exclusive. That is, only one particular setting of  $C_s$  will occur in practice. However, which setting  $C_s$  will actually take is unknown in the preliminary system design stage. Since the selections of the number of piers are mutually exclusive, the decision problem is which number of piers to select in the face of uncertainties in  $C_s$ .

As illustrated in the decision tree, a sequential decision analysis approach will be used to evaluate the various design decisions. Decision analyses begin with the right side of the tree and work backwards. For each decision, the evaluation attribute,  $TFC$ , is estimated with respect to the different settings of  $C_s$ . Then the values of  $TFC$  from each decision are compared to identify an optimal decision for each alternative. Finally, the optimal decision from Alternative 1 is compared with that of Alternative 2 to identify the best decision for the overall problem. Within each step of the decision tree, the evaluation approach for discrete decisions presented in Chapter V is used to make a selection.

### 7.6.2 Regret loss -- the cost of making a selection

As indicated in Figure 7.7, the optimal number of piers for Alternative 1 varies from 11 to 14, depending upon the setting of  $C_s$  taken. Since the actual setting which  $C_s$  will take is unknown during the preliminary design stage, making

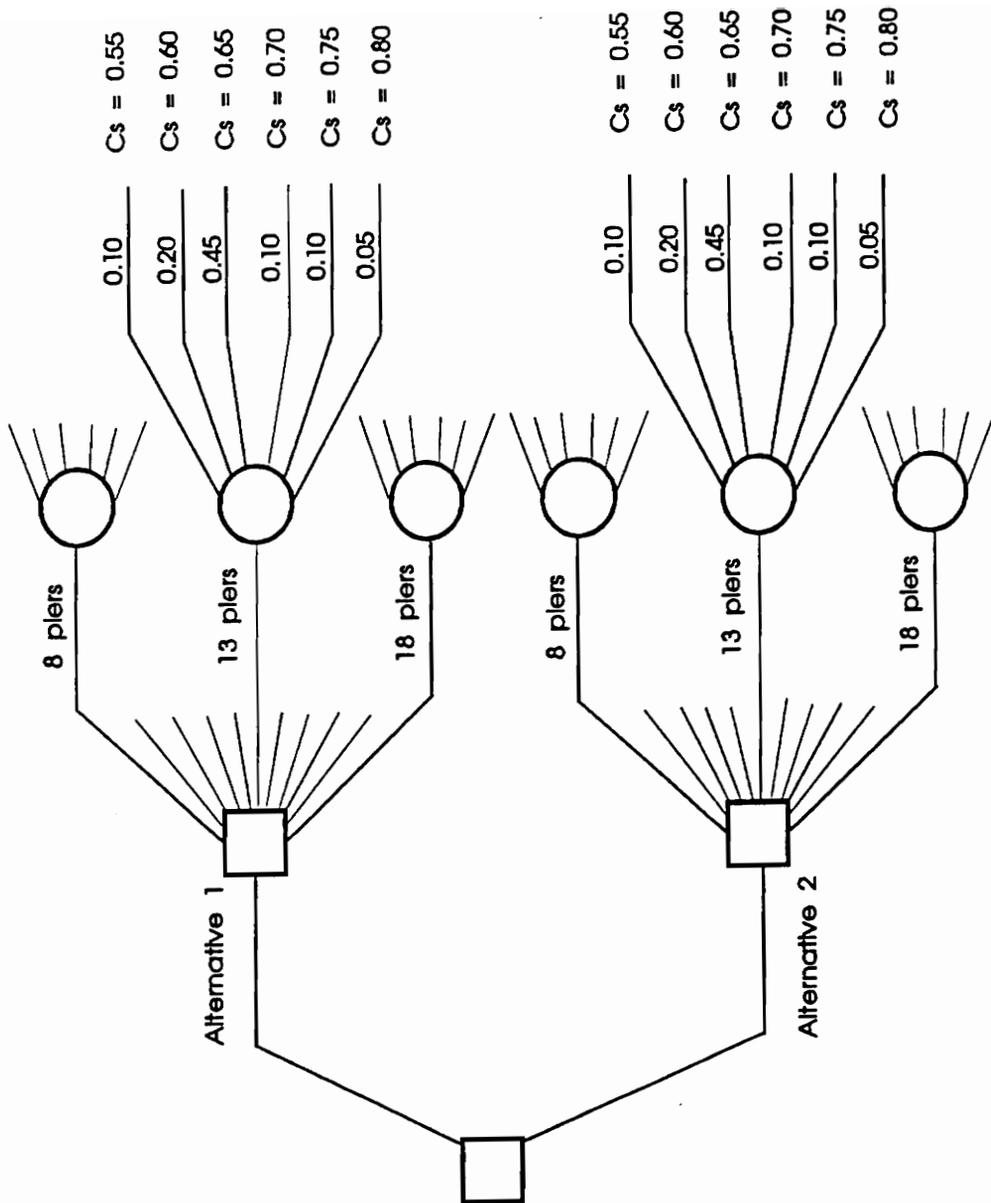


Figure 7.9. A decision tree for the bridge design problem

a selection from any of the decisions may result in some opportunity cost. In the term of statistical decision theory, this cost is called regret loss. The regret loss represents the difference between the actual cost of the decision selected and the minimum cost that could be achieved with perfect knowledge. The regret losses for various decisions are calculated below with respect to each possible setting of  $C_s$ .

For Alternative 1, regret losses are calculated in the following way: subtract the minimum *TFC* in each column of Table 7.4 from each value of *TFC* in the same column. The results for selecting 11 through 14 piers are given in Table 7.6. Several conclusions can be drawn from the calculations. The regret loss for each decision varies significantly with the level of  $C_s$ . For each level of  $C_s$ , if the decision is optimal, zero regret loss is incurred. However, not any single decision has zero regret loss for all levels of  $C_s$ . It seems that the decision of 12 piers is preferable because it has the lowest expected regret loss (\$2,527). However, if the minimax regret criterion is used, 13 piers should be selected.

Table 7.6. Regret Losses (RL) for Alternative 1 (dollars)

| No. of Piers | $C_s$  |        |        |        |        |        | Mean of RL | Max. RL |
|--------------|--------|--------|--------|--------|--------|--------|------------|---------|
|              | 0.55   | 0.60   | 0.65   | 0.70   | 0.75   | 0.80   |            |         |
| 11           | 0      | 7,273  | 14,545 | 26,667 | 40,000 | 55,385 | 17,436     | 55,385  |
| 12           | 0      | 0      | 0      | 4,848  | 10,909 | 19,021 | 2,527      | 19,021  |
| 13           | 13,333 | 7,273  | 1,212  | 0      | 0      | 2,051  | 3,436      | 13,333  |
| 14           | 36,923 | 25,734 | 14,545 | 8,205  | 3,077  | 0      | 16,513     | 36,923  |

Similarly, the regret losses for four decisions from Alternative 2 are calculated in Table 7.7. The decision of 14 piers has the lowest expected regret loss, while the decision of 15 piers has the smallest maximum regret loss.

Table 7.7. Regret Losses (*RL*) for Alternative 2 (dollars)

| No. of Piers | $C_s$  |        |        |        |        |        | Mean of RL | Max. RL |
|--------------|--------|--------|--------|--------|--------|--------|------------|---------|
|              | 0.55   | 0.60   | 0.65   | 0.70   | 0.75   | 0.80   |            |         |
| 13           | 0      | 4,615  | 11,667 | 23,333 | 36,429 | 53,333 | 14,816     | 53,333  |
| 14           | 2,436  | 0      | 0      | 4,615  | 10,659 | 20,513 | 2,797      | 20,513  |
| 15           | 15,952 | 7,473  | 1,429  | 0      | 0      | 3,810  | 3,923      | 15,952  |
| 16           | 38,333 | 24,615 | 13,333 | 6,667  | 1,429  | 0      | 15,566     | 38,333  |

### 7.6.3 Assumptions about the DM's preferences and risk attitude

The existence of non-zero regret losses for each decision represents the risk involved in the decision process. In order to identify the best alternative, we need to determine the decision maker's preferences for both gains and losses.

The preferences of a decision maker can be described by a utility function. However, the selection of utility functions is subjective. There is not a single utility function which can fit all types of decision makers. The type of the utility function depends on the decision maker's behavior.

In this example, two assumptions are made about the DM's preferences and risk attitudes: (1) the DM prefers less *TFC*, and (2) the DM is risk averse. The first assumption means that the utility function is a decreasing function of *TFC*. The second assumption implies that the utility function is a decreasing function of the variance of *TFC*.

### 7.6.4 Mean-Variance analysis

In evaluating various decisions in the problem, the first thought is to use the mean of the *TFC* as the decision criterion and select the decision which has the minimum expected *TFC*. However, as shown in Figures 7.10 and 7.11, each mean is associated with a variance. The variance measures the variability of the *TFC* for each decision. It reflects the risk involved in the decision process. Thus, as indicated in Chapter IV, if the DM is not risk neutral, the criterion of minimizing the expected *TFC* is not valid.

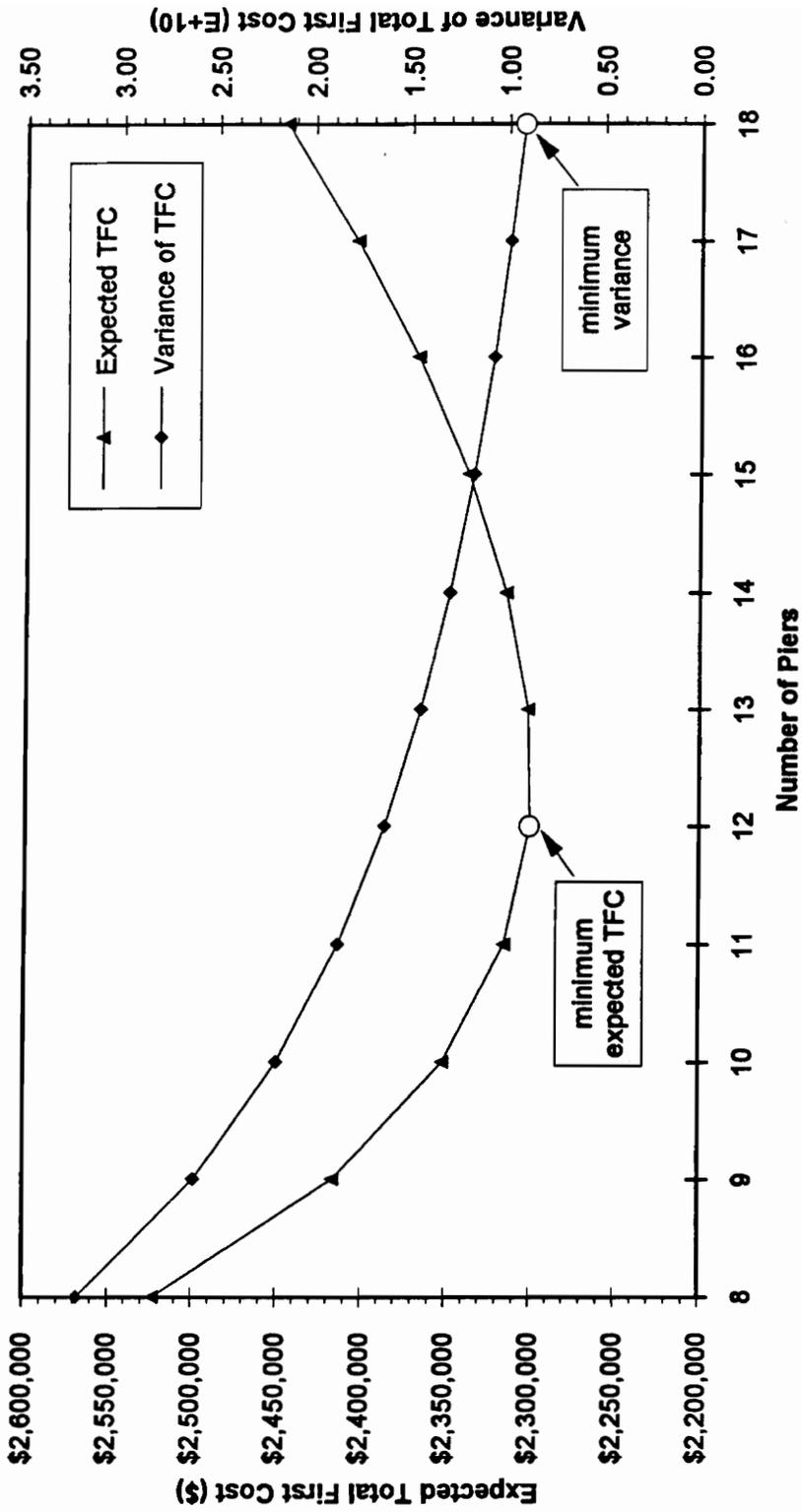


Figure 7.10. Expected value and variance of TFC (Alternative 1)

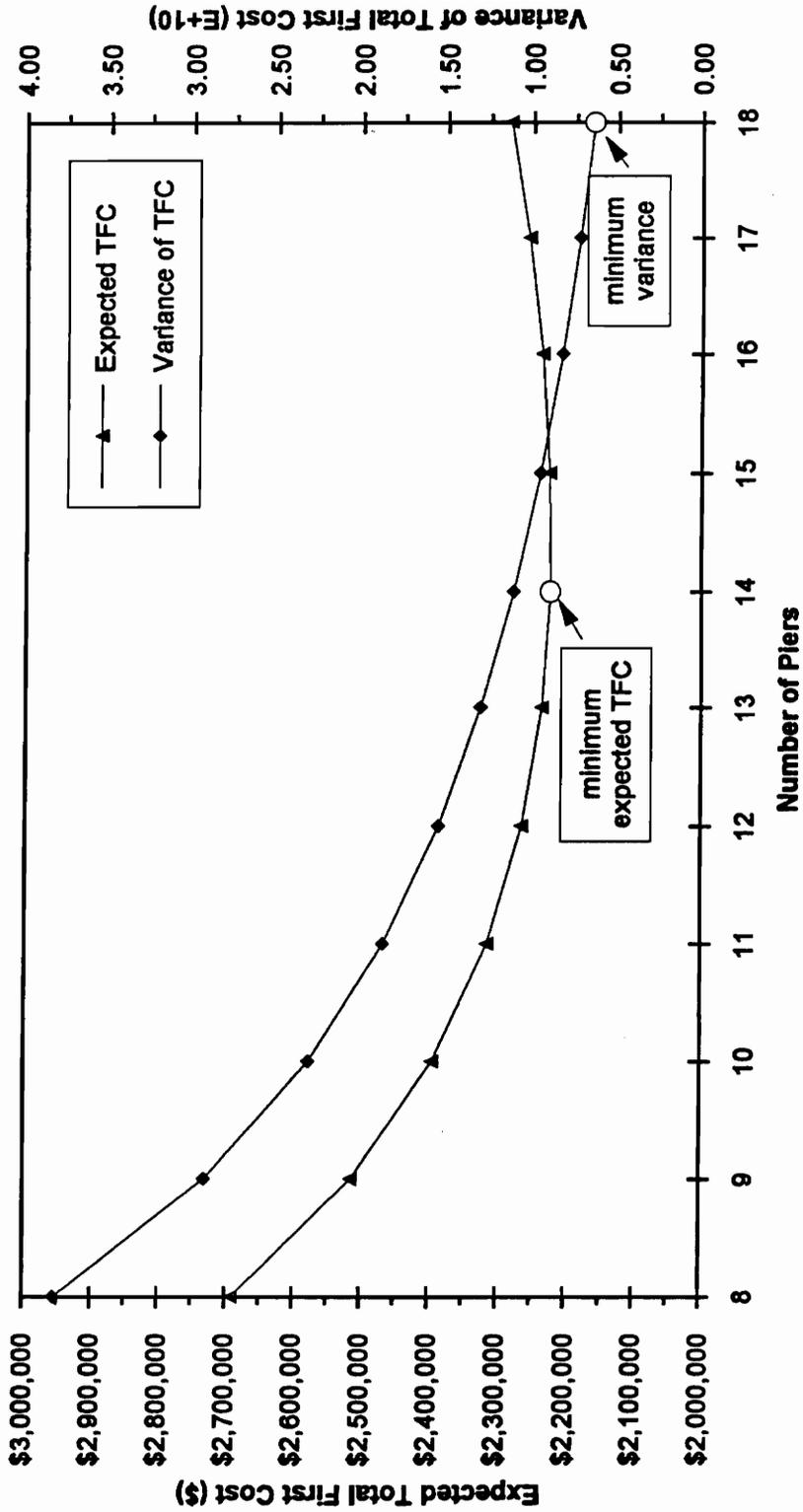


Figure 7.11. Expected value and variance of TFC (Alternative 2)

One way to evaluate the decisions involving uncertainty is to apply Mean-Variance analysis. Instead of comparing only the means of the decisions, we can compare the means and variances of the *TFC* together. As shown in Figures 7.10 and 7.11, both the mean and variance of *TFC* vary significantly with the number of piers. For Alternative 1, the decision of 12 piers has the minimum expected *TFC*, while the decision of 18 piers has the minimum variance for *TFC*. To find an optimal decision, a trade-off must be made between the mean and variance of the *TFC*. Given the two assumptions about the decision maker's behavior, the Mean-Variance analysis is now applied to resolve the trade-off problem.

In order to compare the decisions for each alternative, each pair of mean and variance of *TFC* is plotted on an E-V chart (Figures 7.12 and 7.13). The horizontal axis of the E-V charts represents the variance of the *TFC*, while the vertical axis denotes the mean of the *TFC*. Each point in the chart represents a decision, that is, selecting a specific number of piers. Thus, for each design alternative, there are a total of 11 points on its E-V chart, representing the number of piers from 8 through 18.

At the beginning of the section, it was assumed that the DM always prefers less value of *TFC* and is risk averse. Thus, on an E-V chart, a possible point  $(\sigma_{TFC}^*, \mu_{TFC}^*)$  is called efficient if no other possible point  $(\sigma_{TFC}^2, \mu_{TFC})$  has

$$\mu_{TFC} \leq \mu_{TFC}^*, \text{ and } \sigma_{TFC}^2 \leq \sigma_{TFC}^{*2}.$$

The efficient points form the lowest left boundary of the set of possible points on the E-V chart. The boundary is called the efficient frontier and the set of efficient points is called an efficient set. The efficient frontier is drawn through the points representing the decisions that are not dominated by some other decisions. Any point above and to the right of the efficient frontier represents a decision dominated by one on the frontier. For any obtainable E-V combination, except in the efficient set, it is possible to find a feasible combination with less mean and no more variance, or to find one with less variance and no more mean, or both. Any such combination is considered inefficient.

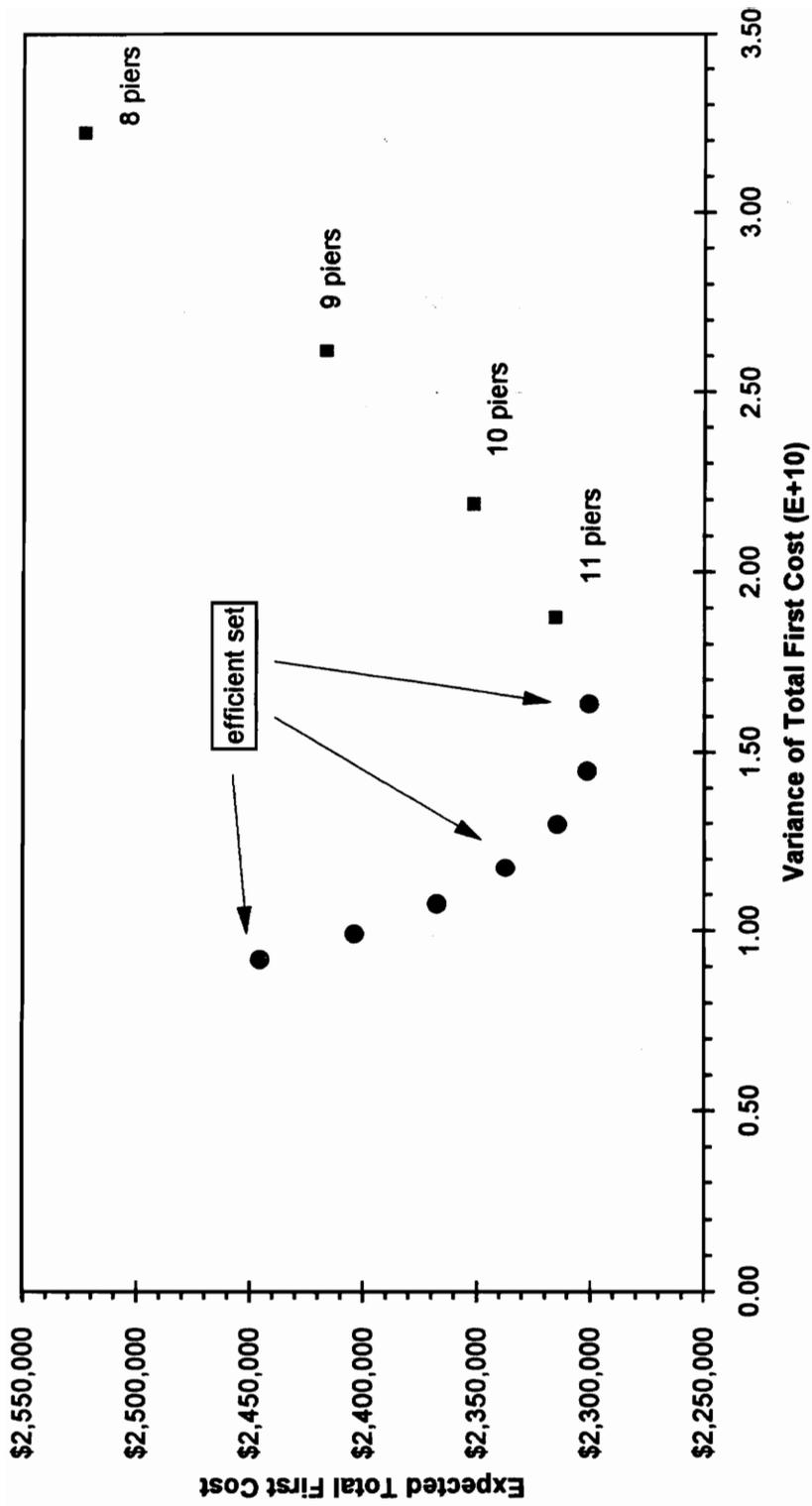


Figure 7.12. E-V chart for Alternative 1

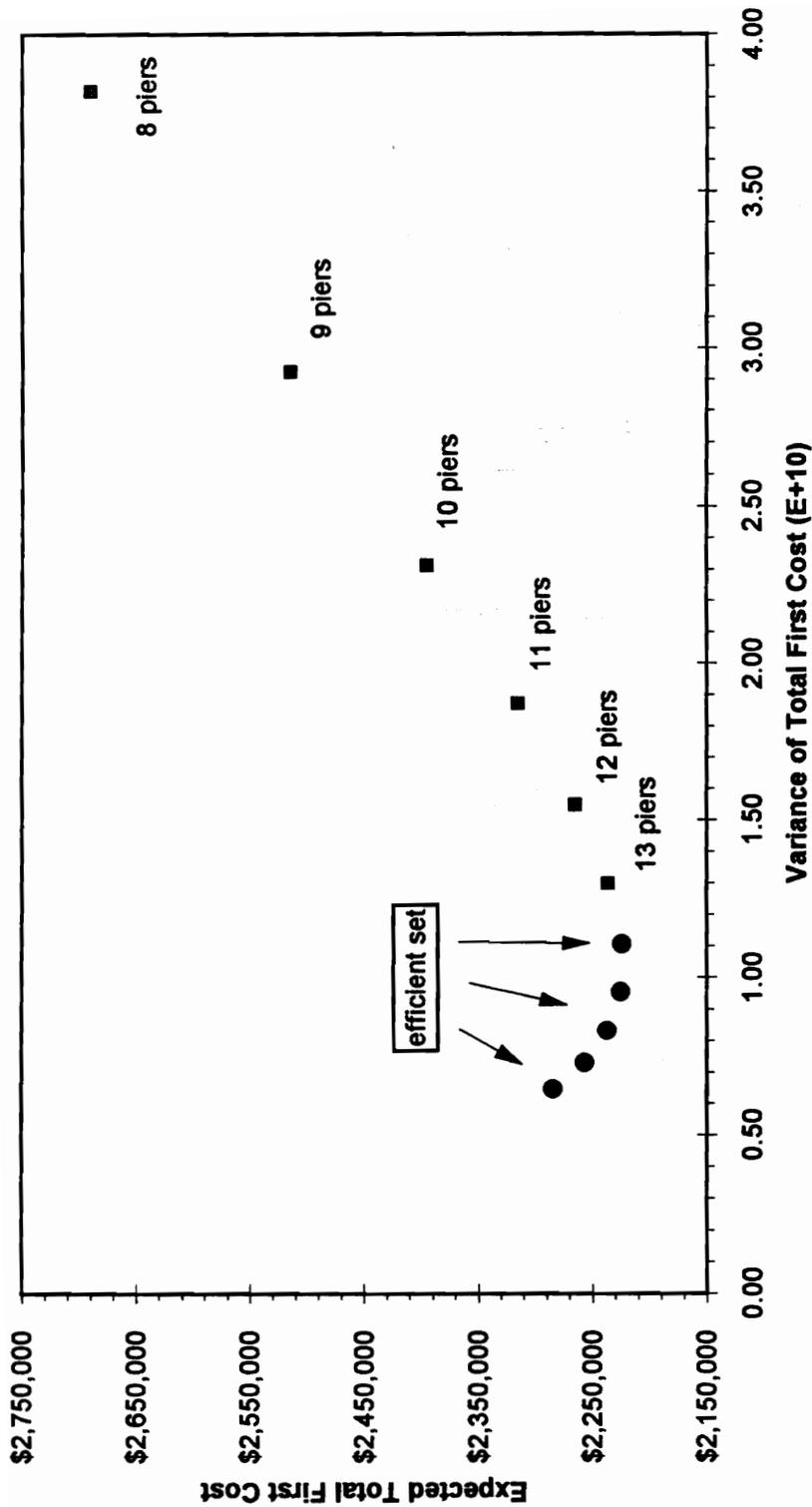


Figure 7.13. E-V chart for Alternative 2

The efficient set for Alternative 1 consists of decisions of selecting 12, 13, 14, 15, 16, 17, and 18 piers. None of the decisions dominates each other. The decisions of 8, 9, 10, and 11 piers are found to be inefficient, since we can always find a member from the efficient set which dominates them. Similarly, for Alternative 2, its efficient set consists of 14, 15, 16, 17, and 18 piers. The decisions of 8 through 13 piers are found to be inefficient. To resolve the decision problem, the E-V dominance rules introduced in Chapter IV are applied:

**E-V Rule 1:** If Decision *A* has a mean of *TFC* the same as or lower than that of Decision *B*, and has a lower variance of *TFC* than *B*, then Decision *A* is preferred.

**E-V Rule 2:** If Decision *A* has a variance of *TFC* the same as or lower than that of *B*, and has a lower mean of *TFC* than *B*, then Decision *A* is preferred.

By applying the E-V rules, those inefficient decisions are eliminated (Table 7.8). For Alternative 1, four decisions are dropped and seven decisions remain. For Alternative 2, six decisions are eliminated and five decisions remain. Thus, the decision problem is reduced to the evaluation of the remaining decisions. To compare the members in an efficient set using the E-V analysis, we need to develop a two-attribute utility function which quantifies the DM's preferences for the mean and variance of *TFC*. By plotting utility indifference curves on the E-V charts, the point on the efficient frontier which gives highest utility provides the optimal decision.

Table 7.8. List of the Decisions Eliminated and the Decisions Remaining

| Alternative | Decisions Eliminated | Decisions Remaining        |
|-------------|----------------------|----------------------------|
| 1           | 8, 9, 10, 11         | 12, 13, 14, 15, 16, 17, 18 |
| 2           | 8, 9, 10, 11, 12, 13 | 14, 15, 16, 17, 18         |

### 7.6.5 Stochastic dominance

As discussed in Chapter V, one consideration in applying the framework for design evaluation in the face of uncertainty is to use utility analysis as the last resort. It is recommended that the use of utility functions be deferred if other approaches which require less strict assumptions are valid. Stochastic dominance is applied below to evaluate the members of the efficient set identified by Mean-Variance analysis. As presented in Section 4.5, the stochastic dominance rules are based on several progressively stronger assumptions about the decision maker's behavior.

**First-Order Stochastic Dominance Rule:** If the DM prefers less *TFC* to more *TFC*, and if the cumulative probability of Decision *A* is never greater than the cumulative probability of Decision *B* and sometimes less, then *B* is preferred to *A*.

The cumulative probabilities of *TFC* for the members in the efficient set of Alternative 1 are plotted in Figure 7.14. In this figure, the horizontal axis and the vertical axis are exchanged so that all decisions can be compared simultaneously. For each level of cumulative probability, the value of *TFC* at which the cumulative probability is achieved is reflected by the height of the bar chart. Thus, if the bar height for Decision *A* is always lower than that for Decision *B* under the same level of the cumulative probability, the cumulative probability of Decision *A* is greater than that of Decision *B*. For example, the decision of 18 piers has a higher value of *TFC* than the decision of 17 piers for each level of cumulative probability. Thus, the decision of 17 piers dominates the decision of 18 piers by first-order stochastic dominance. Inspecting Figure 7.14, we find that the decisions of 13 and 14 piers dominate the decisions of 15, 16, 17, and 18 piers by first-order stochastic dominance. But for 12, 13, and 14 piers, none of them dominates each other. The conclusions are obvious in Table 7.4. Under any level of  $C_r$ , the decisions of 15 through 18 piers always have higher values of *TFC* than decisions of 13 piers and 14 piers. Similarly, the cumulative probabilities for various decisions in Alternative 2 are compared in Figures 7.15. The results are summarized in Table 7.9.

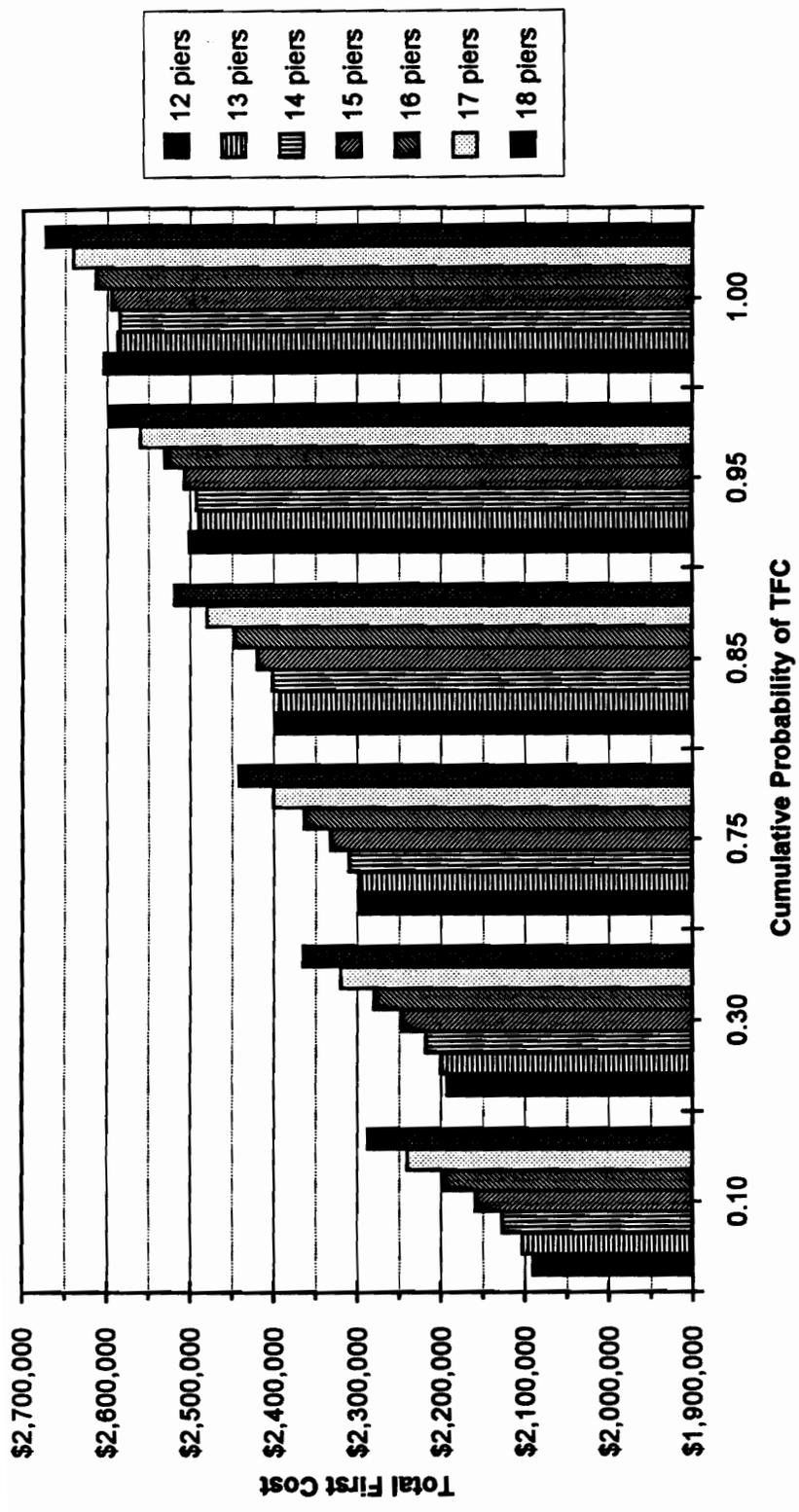


Figure 7.14. Comparison of cumulative probabilities of the TFC for the members of the efficient set (Alternative 1)

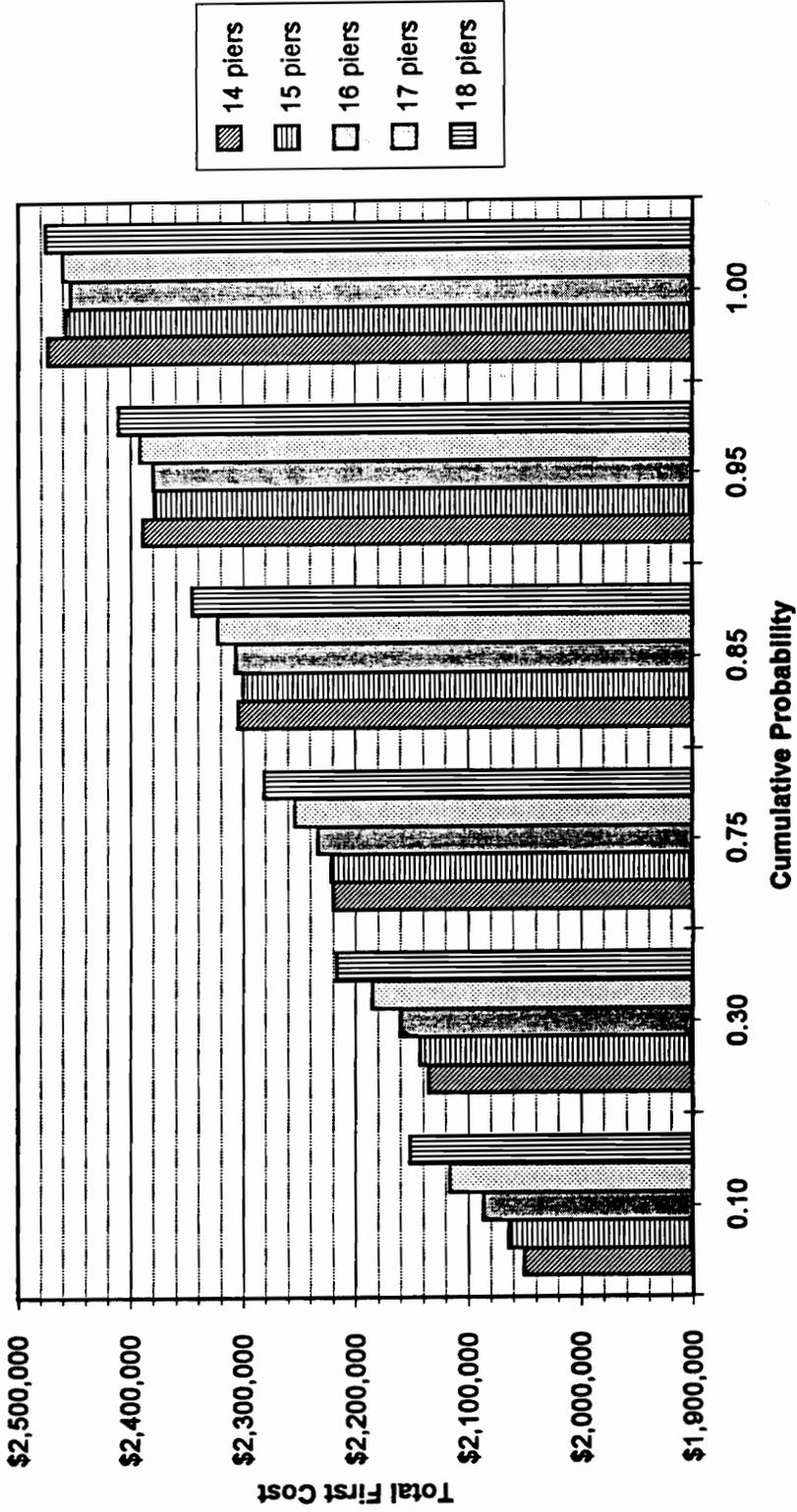


Figure 7.15. Comparison of cumulative probabilities of the TFC for the members of the efficient set (Alternative 2)

Table 7.9. List of Decisions Dominated and Decisions Remaining

| Alternative | Decisions Eliminated | Decisions Remaining |
|-------------|----------------------|---------------------|
| 1           | 15, 16, 17, 18       | 12, 13, 14          |
| 2           | 17, 18               | 14, 15, 16          |

In applying the First-Order Stochastic Dominance Rule, the only assumption made about the DM's behavior is that he prefers less *TFC* to more *TFC*. A more strict rule is needed in order to compare the decisions remaining in Table 7.9. Thus the Second-Order Stochastic Dominance Rule presented in Section 4.5 is applied to this problem.

**Second-Order Stochastic Dominance Rule:** If (1) the DM prefers less *TFC* to more *TFC*, and (2) the DM is *risk-averse*, and (3) the sum of the cumulative probabilities for all *TFC*'s are never more with *A* than *B* and sometime less, then *B* dominates *A* by second-order stochastic dominance.

Now this rule is applied to compare the remaining decisions for each alternative in Table 7.9 one by one. First, this rule is used to compare the decisions of 12 piers and 13 piers of Alternative 1. The sums of cumulative probabilities for both decisions are plotted in Figure 7.16. For each possible level of *TFC*, the sum of cumulative probabilities for 12 piers is always higher than that for 13 piers. This indicates that the decision of 12 piers dominates the decision of 13 piers by second-order stochastic dominance. This conclusion is further confirmed by inspecting the cumulative probabilities of both decisions in Figure 7.16. For the cumulative probability of *TFC* less than or equal to 0.75, the decision of 12 piers has less *TFC* than the decision of 13 piers. That is, the probability that selecting 12 piers will result in higher *TFC* than selecting 13 piers is no more than 0.25. Thus, if the decision maker is risk averse, he will select 12 piers.

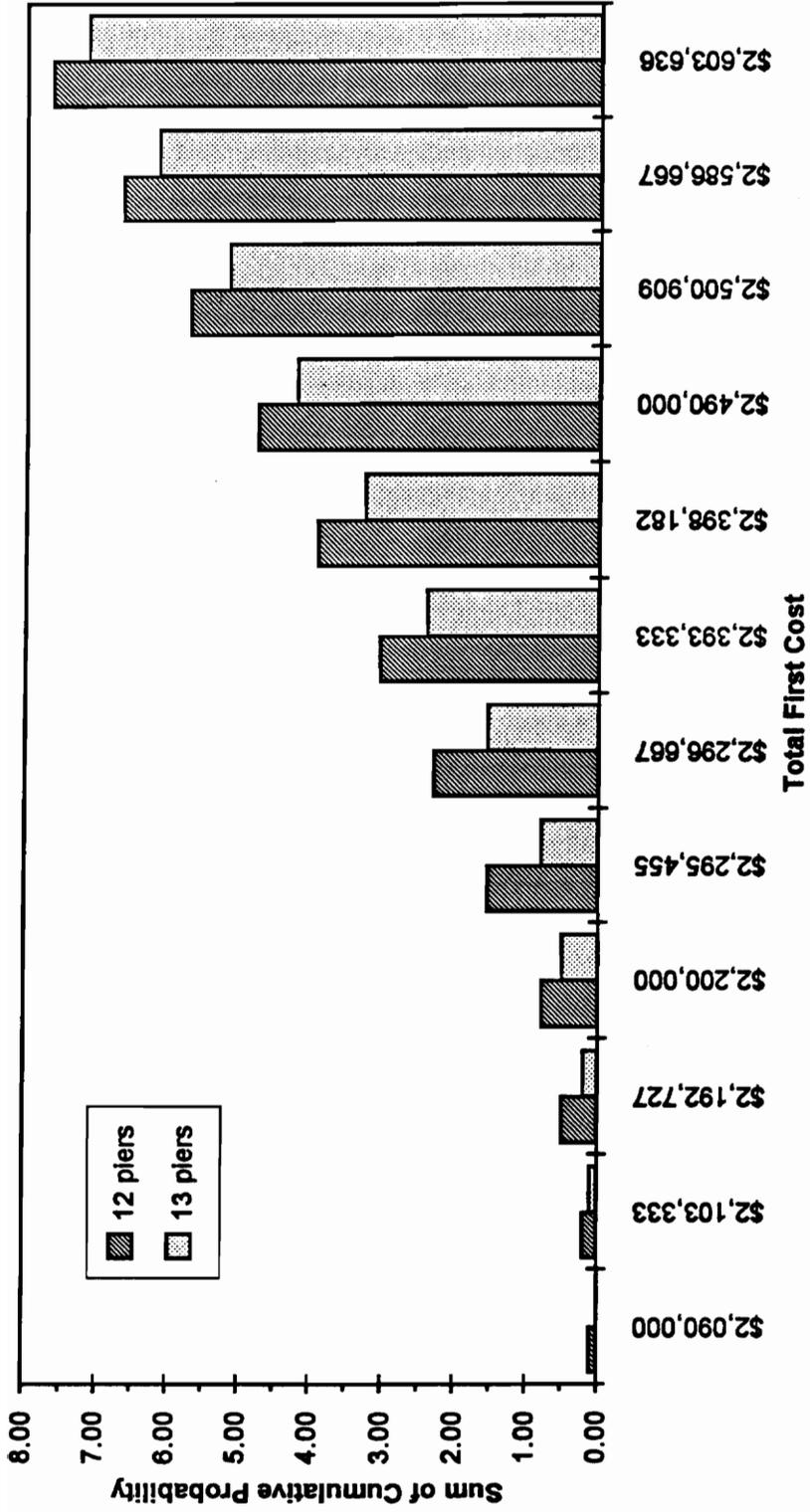


Figure 7.16. Comparison of the sums of cumulative probabilities (Alternative 1: 12 piers and 13 piers)

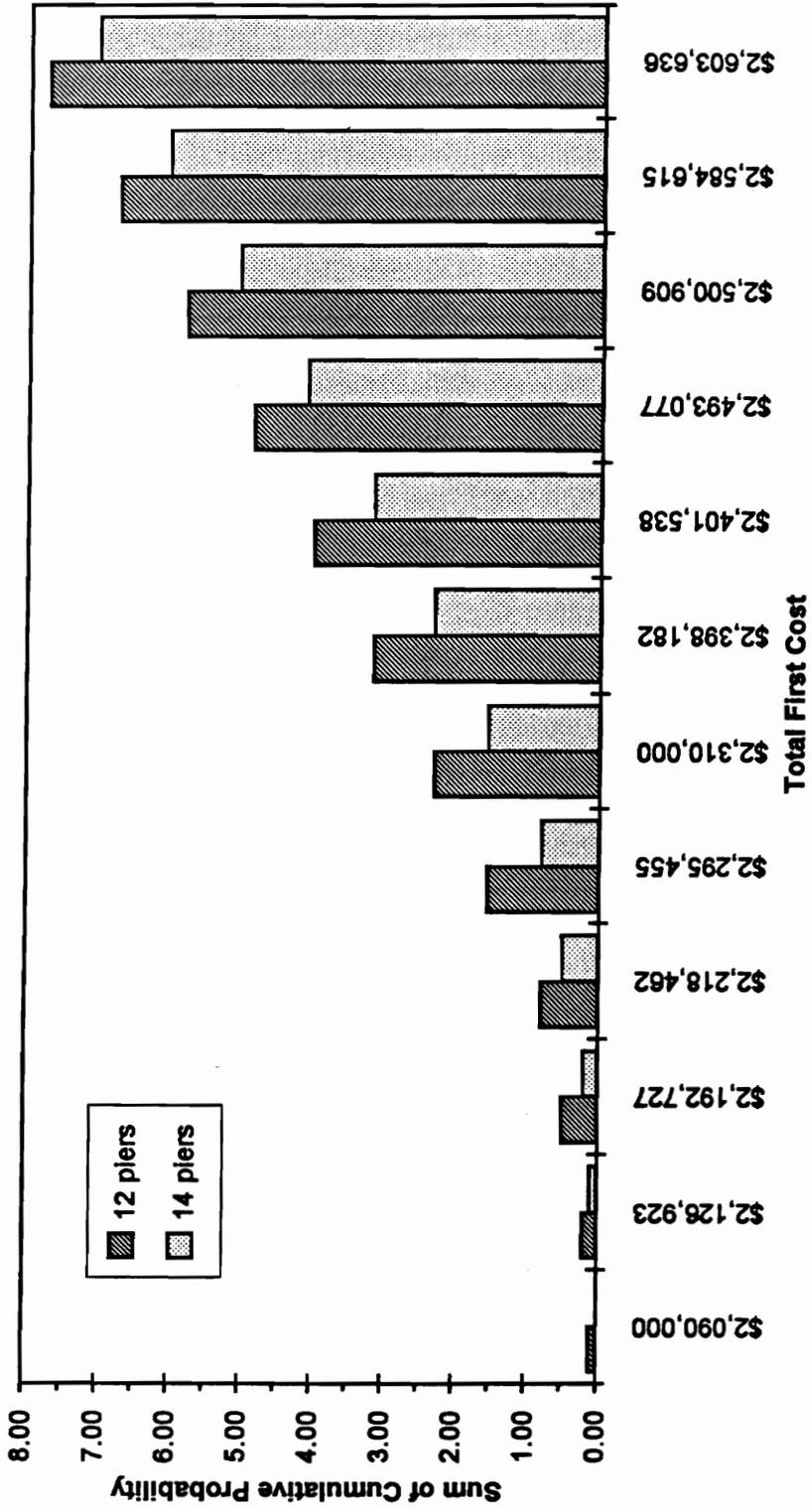


Figure 7.17. Comparison of the sums of cumulative probabilities (Alternative 1: 12 piers and 14 piers)

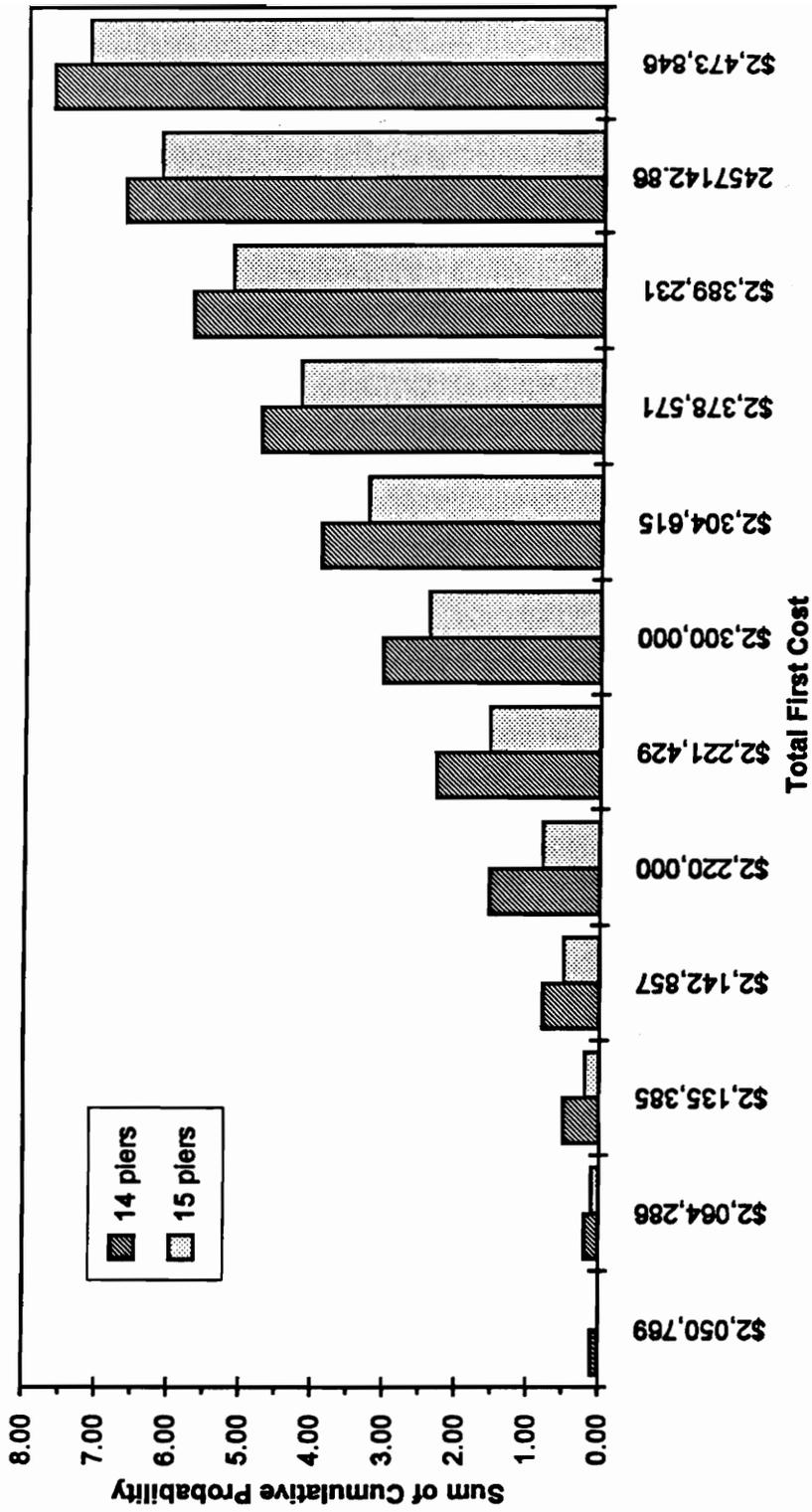


Figure 7.18. Comparison of the sums of cumulative probabilities (Alternative 2: 14 piers and 15 piers)

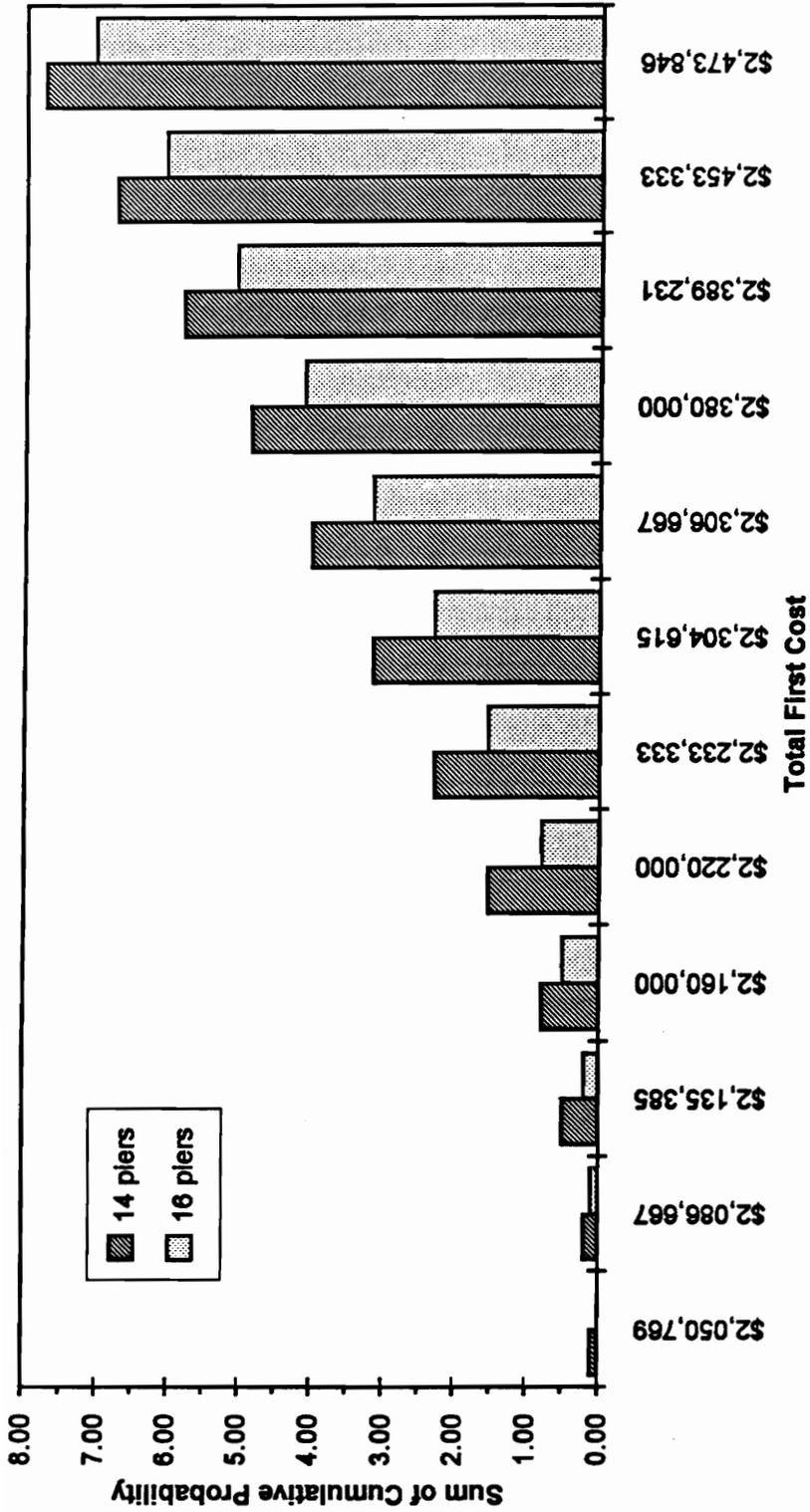


Figure 7.19. Comparison of the sums of cumulative probabilities (Alternative 2: 14 piers and 16 piers)

Then the decisions of 12 piers and 14 piers of Alternative 1 are compared. The sums of their cumulative probabilities are plotted in Figure 7.17. Since the sum of cumulative probabilities of 12 piers is higher for each level of *TFC*, the decision of 12 piers also dominates the decision of 14 piers by second-order stochastic dominance. Thus, the optimal decision for Alternative 1 is 12 piers.

Similarly, for Alternative 2, the decision of 14 piers is found to dominate the decisions of 15 piers and 16 piers by second-order dominance (see Figures 7.18 and 7.19). Thus, the optimal decision for Alternative 2 is 14 piers.

### 7.6.6 Comparison of Alternative 1 and Alternative 2

In the previous section, the optimal decision for each alternative has been identified. According to the decision tree in Figure 7.9, the next step is to compare the optimal decision from each alternative and select the better one. The probability distributions of *TFC* for the optimal decision from each alternative are given in Table 7.10. Both the Mean-Variance rules and the Stochastic Dominance rules are now used below to make a comparison.

**Mean-Variance analysis.** The means and variances for both alternatives are plotted in Figure 7.20. Alternative 1 is inefficient since it has both a higher mean and a greater variance than Alternative 2. According to the E-V rules, Alternative 2 is preferable.

**Stochastic dominance.** To further assure that Alternative 2 is preferred, we compare the optimal decisions for each alternative by employing the First-Order Stochastic Dominance rule. The cumulative probabilities of *TFC* for both alternatives are plotted in Figure 7.21. For each level of *TFC*, the cumulative probability of Alternative 2 is no more than that of Alternative 1. Thus, Alternative 2 dominates Alternative 1 by first-order stochastic dominance. This conclusion is obvious in Figure 7.22, where Alternative 2 always has less *TFC* for any level of  $C_r$ . Thus, the regret loss for selecting Alternative 2 is always zero.

**Table 7.10. Total First Costs for Optimal Decisions (dollars)**

| Cs     | 0.55      | 0.60      | 0.65      | 0.70      | 0.75      | 0.80      | Mean      | Variance |
|--------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|----------|
| Prob.  | 0.10      | 0.20      | 0.45      | 0.10      | 0.10      | 0.05      |           |          |
| Alt. 1 | 2,090,000 | 2,192,727 | 2,295,455 | 2,398,182 | 2,500,909 | 2,603,636 | 2,300,591 | 1.63E+10 |
| Alt. 2 | 2,050,769 | 2,135,385 | 2,220,000 | 2,304,615 | 2,389,231 | 2,473,846 | 2,224,231 | 1.11E+10 |

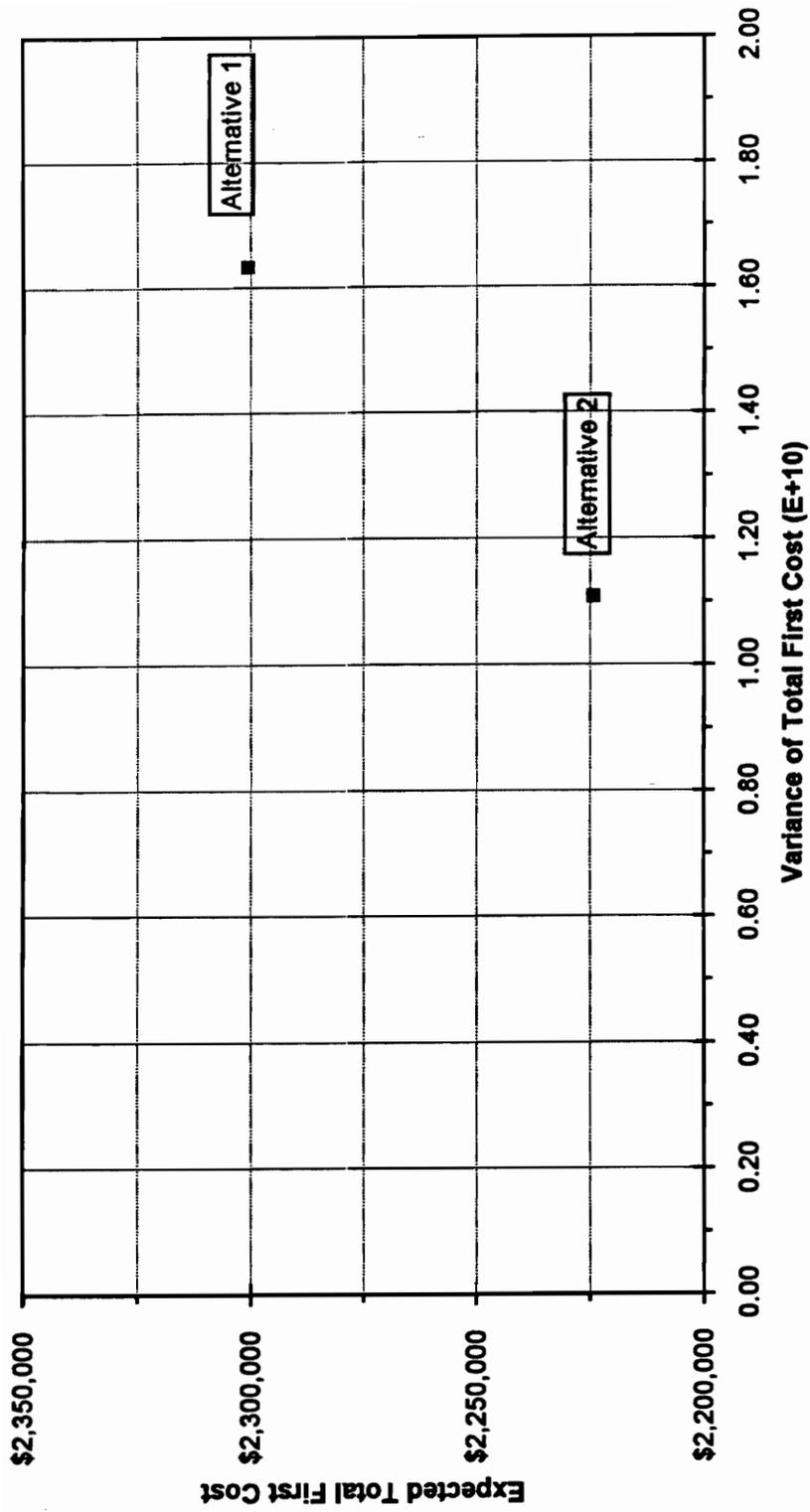


Figure 7.20. Comparison of the means and variances of the TFC for the optimal decisions of Alternative 1 and Alternative 2

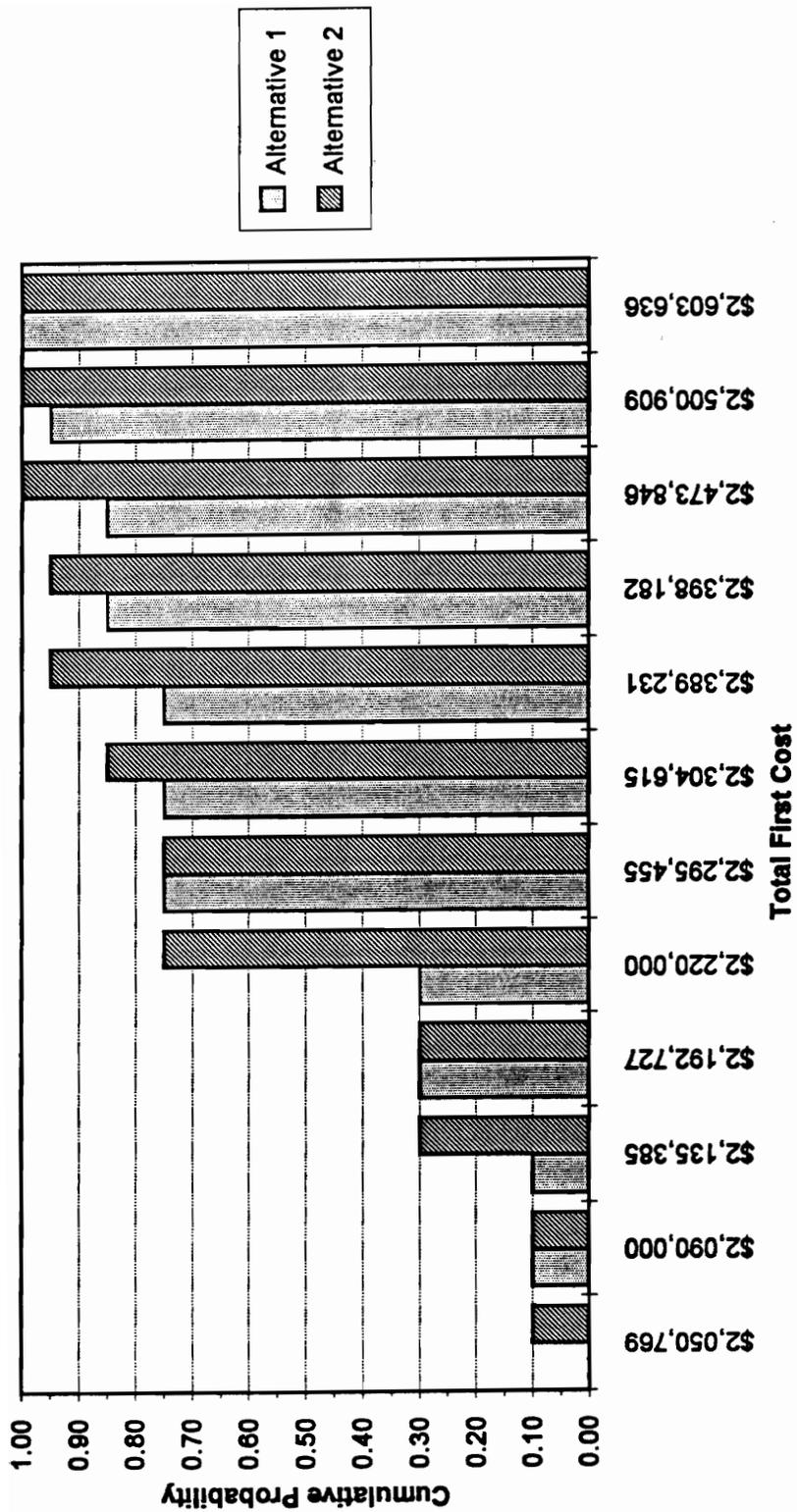


Figure 7.21. Comparison of the cumulative probabilities of the TFC for the optimal decisions of Alternative 1 and Alternative 2

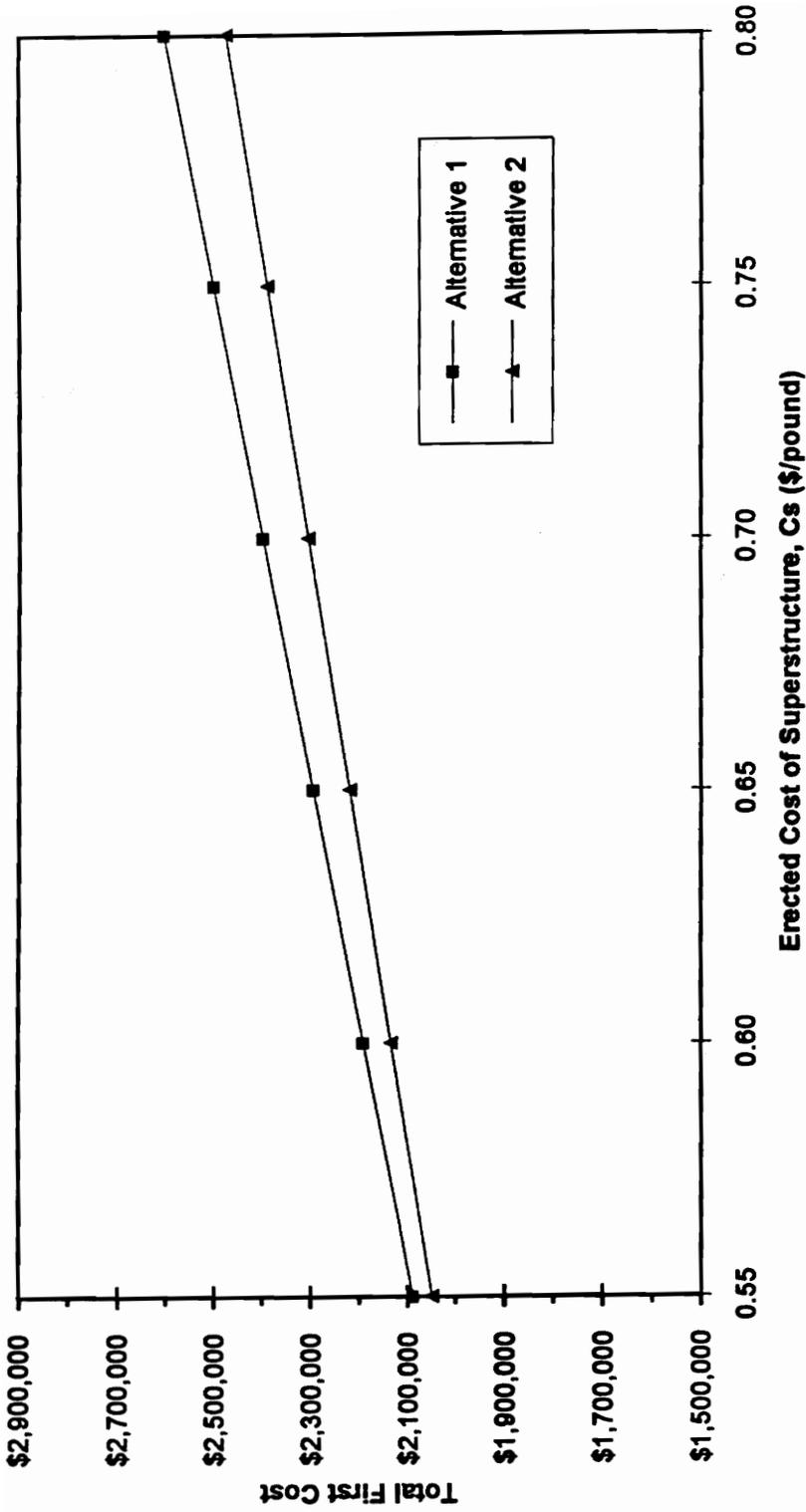


Figure 7.22. Comparison of the *TFC* for Alternative 1 and Alternative 2 with respect to different settings of  $C_s$

### 7.6.7 Characteristics of the optimal decision

The best decision for the bridge design problem involving uncertainty is Alternative 2 with 14 piers. The probability distribution of its *TFC* is given in Figure 7.23. The mean and variance of the *TFC* are \$2,224,231 and  $1.11 \times 10^{10}$ , respectively. Since the decision problem has been resolved by second-order stochastic dominance, there is no need to assess the DM's utility function quantitatively.

Though the best decision is the same as that identified in the case of evaluation under certainty, this coincidence is an exception rather than a general conclusion. For this problem, the coincidence in the solutions is mainly due to the type of probability distribution specified for the erected cost of the superstructure ( $C_s$ ). In the certainty case,  $C_s$  is a constant and equal to \$0.65 per pound. In the uncertainty case,  $C_s$  is a random variable. As given in Figure 7.4, the probability that  $C_s$  is equal to 0.65 is 0.45. Compared to other settings of  $C_s$ , this particular setting has a very large probability. As a result, it has a large effect on the final solution.

It must be noted that the nature of the decision problem in the face of uncertainty is different from that under certainty. Under the certainty case, the evaluation attribute is a constant. Thus, the best decision identified dominates other decisions deterministically. However, under the uncertainty case, the evaluation attribute is a random variable. The best decision was identified based on some assumptions about the decision maker's preferences and risk attitudes. Depending upon the actual setting of  $C_s$ , this best decision may not always dominate other decisions.

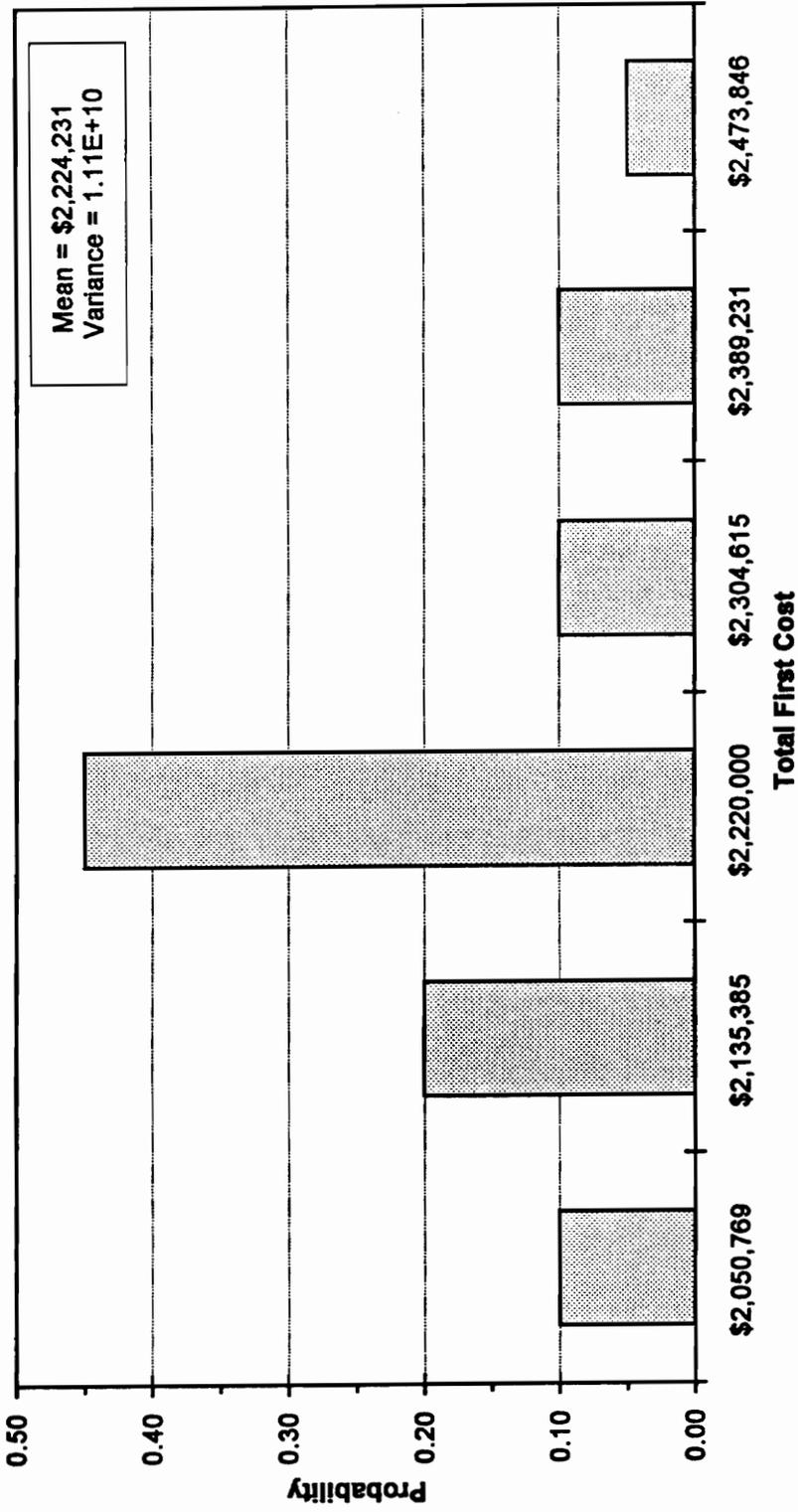


Figure 7.23. Probability distribution of the TFC of the optimal decision for the overall problem

## **VIII. MULTIATTRIBUTE DESIGN EVALUATION IN THE FACE OF UNCERTAINTY**

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- 8.1 Introduction**
  - 8.2 Descriptive Procedures for Choices with Multiple Attributes**
  - 8.3 The Need for a Systematic Approach to Deal with Multiple Attributes**
  - 8.4 Multiattribute Utility Theory (MAUT)**
  - 8.5 A Structured Approach for Multiattribute Design Evaluation  
in the Face of Uncertainty**
- 

### **8.1 Introduction**

The problem of design evaluation in the face of uncertainty was analyzed and modeled in Chapters V and VI by considering only one evaluation attribute. If a single evaluation attribute is not adequate to describe the outcome of a decision, a multiple attribute problem exists. The set of attributes might include reliability, maintainability, manufacturing cost, life-cycle cost, weight, speed, capacity, etc. Because multiple attributes are involved, the outcomes for the design decisions are multidimensional. Thus, this type of decision problem becomes a problem of multiattribute design evaluation under uncertainty.

Finding an optimal decision for a multiattribute design evaluation problem in the face of uncertainty is very difficult. Two major factors contributing to this difficulty are (1) the large uncertainties about what the impact of any alternative will eventually be and the difficulty in separating this from one's preferences concerning "possible" consequences, and (2) the multidimensional outcomes of the problem and the necessity to make value trade-offs among various levels of different attributes.

In Chapter VI, an integrated approach was developed to deal with uncertainty in design evaluation involving a single attribute. In this chapter, the approach is extended to resolve design decision problems involving both uncertainties and multiple attributes.

## 8.2 Descriptive Procedures for Choices with Multiple Attributes

Holloway (1979) summarized four descriptive procedures for dealing with multiattribute decision problems. These procedures are simple and straightforward. They are:

***Dominance:*** Alternatives are compared attribute by attribute. If Alternative A is at least preferred as Alternative B on all attributes and strictly preferred on at least one attribute, A dominates B.

***Satisficing:*** Satisfactory levels are set for each separate attribute. Any alternative that meets the satisficing levels for every attribute is kept. Others are discarded.

***Lexicographic procedure:*** Attributes are ranked in order of importance. Then alternatives are compared one attribute at a time, starting with the highest-ranked attribute. Lower-ranked attributes are used until they are exhausted or until a unique choice is made.

***Combination procedure:*** The dominance, satisficing, and lexicographic procedures are used in combination. First, dominance is used to eliminate any dominated alternatives. Next, satisficing is used to eliminate alternatives that are not adequate on one or more of the attributes. Those alternatives that survive both the dominance and satisficing procedures are subjected to the lexicographic procedure.

The dominance procedure works only in special cases. The satisficing procedure and the lexicographic procedure rely on strong assumptions

concerning the independence of the attributes. This is required because the attributes are treated separately. Holloway (1979) suggested that these procedures be used in combination. However, if decisions are complicated by both multiple attributes and uncertainty, the descriptive procedures are difficult to use, if possible, since the levels of the attributes are not known with certainty.

### **8.3 The Need for a Systematic Approach to Deal with Multiple Attributes**

Multiattribute design evaluation in the face of uncertainty is complicated because uncertainties and multidimensional outcomes must be considered together. As indicated in Section 8.1, there is a fundamental difficulty involved in considering multidimensional outcomes. Evaluation and optimization of alternatives can be accomplished only with respect to a single attribute; since all members of the set of attributes, by definition, significantly influence the decision, no criterion for any single attribute can rationally be used as the only basis for the decision.

During the early stages of system design, it is important to identify the most desirable combination of various attribute levels. Design analysis and evaluation must be based on a rigorously determined multiattribute objective function. The function must be defined to accurately reflect the DM's preferences and willingness to make trade-offs over multiple attributes. To be consistent, the multidimensional outcome must be transformed into a single figure of merit. A scale which measures relative contribution to success of the candidate design must be identified, and a means for measuring the multidimensional outcome on this scale must be formulated so that evaluation and optimization of alternatives can be accomplished. As identified in Chapter IV, the scale should be utility. Thus, the goal should not simply be to optimize any single attribute, such as performance variations claimed in Taguchi's philosophy. Instead, the goal should be to maximize the DM's utility.

The transformation of the multidimensional outcome into utility is not always accomplished explicitly. It may be done subjectively, intuitively and implicitly – but it must be done (Lindley, 1984; Lifson, 1972). The identification of the criteria on which the decision is really based and recognition of the relationships

between such criteria and human values constitute the value problem present in all decision situations. In the next section, multiattribute utility theory is introduced to show how such a transformation can be accomplished.

## 8.4 Multiattribute Utility Theory (MAUT)

Suppose we have two attributes  $E_1$  and  $E_2$ . The consequence space is  $E \equiv E_1 \times E_2$ . A specific consequence is designated by  $e$  or  $(e_1, e_2)$ . We are interested in assessing a utility function over  $E$ , denoted by  $u(e)$  or  $u(e_1, e_2)$ . The preference structure and all of the trade-offs between attributes are specified once  $u$  is known.

In order to determine how the worth of a design as a function of the multiple attributes is calculated, the conditions under which various forms of the utility function,  $u(e)$ , are appropriate should be determined. The utility function defined should accurately reflect the DM's preferences for each attribute and his risk attitude for various levels of the individual attribute. Three independence conditions which help in minimizing the level of effort required to determine such a utility function are described below.

### 8.4.1 Three independence conditions

**Preferential Independence (PI)** (Keeney and Raiffa, 1976): Attribute  $E_1$  is preferentially independent of its complement  $\bar{E}_1$  if the preference order of consequences involving only changes in the levels in  $E_1$  does not depend on the levels at which attributes in  $\bar{E}_1$  are held fixed.

Preferential independence implies that the conditional indifference curves over  $E_1$  do not depend on attributes in  $\bar{E}_1$ . The concept concerns the DM's preferences for consequences where no uncertainty is involved. Symbolically,  $E_1$  is PI if and only if for any consequences  $e'_1, e''_1, \bar{e}_1^+, \bar{e}_1$ ,

$$(e'_1, \bar{e}_1^+) \succ (e''_1, \bar{e}_1^+) \Rightarrow (e'_1, \bar{e}_1) \succ (e''_1, \bar{e}_1) \text{ for all } \bar{e}_1.$$

In design evaluation involving multiple attributes, PI means that the DM always prefers more of an attribute to less (or less to more, depending upon the attribute) regardless of the level of other attributes. It should be noted that PI does not refer to independence between different attributes, but rather to the worth a designer places on individual attribute levels. For example, the life-cycle cost of a system is related to the reliability of the system. But the relative worth to the designer over the range of acceptable levels of the life-cycle cost alone is independent of the level of reliability.

**Utility Independence (UI) (Keeney and Raiffa, 1976):** Attribute  $E_1$  is utility independent of its complement  $\bar{E}_1$  if the conditional preference order for lotteries involving only changes in the levels of attributes in  $E_1$  does not depend on the levels at which the attributes in  $\bar{E}_1$  are held fixed.

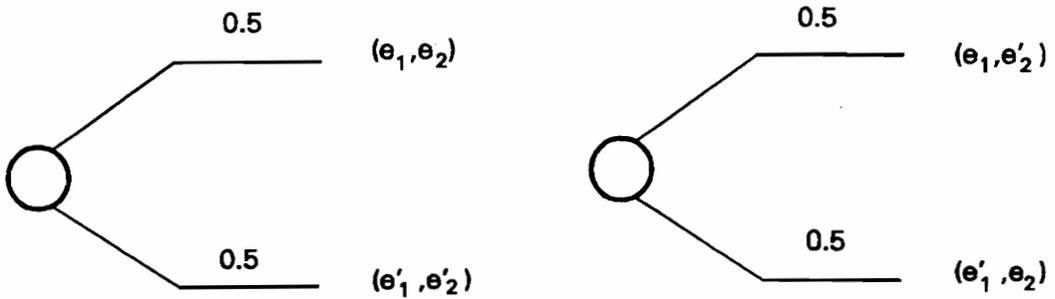
Utility independence concerns preferences for lotteries that involve uncertainty. For any lotteries  $\tilde{e}'_1, \tilde{e}''_1$ , and consequence  $\bar{e}_1^+$ ,  $E_1$  is UI if and only if

$$(\tilde{e}'_1, \bar{e}_1^+) \succ (\tilde{e}''_1, \bar{e}_1^+) \Rightarrow (\tilde{e}'_1, \bar{e}_1) \succ (\tilde{e}''_1, \bar{e}_1) \text{ for all } \bar{e}_1.$$

UI means that the general shape or degree of nonlinearity of the value function is not altered by changes in levels of the other attributes. By definition, it follows that if  $E_1$  is UI, then  $E_1$  is PI. But the converse is not necessarily true. The preferential independence can be stated in terms of the preference order for degenerate lotteries, those involving no uncertainty. Thus, UI is the stronger condition.

**Additive Independence (Keeney and Raiffa, 1976; Fishburn, 1988):** Attributes are additive independent if preferences over lotteries depend only on their *marginal* probability distributions and not on their *joint* probability distribution.

In two dimensions, an equivalent condition for two attributes,  $E_1$  and  $E_2$ , to be additive independent is that lotteries



must be equally preferable for all  $(e_1, e_2)$  given an arbitrarily chosen  $e'_1$  and  $e'_2$ . Note that in each of these two lotteries, there is a one-half probability of getting either  $e_1$  or  $e'_1$  and a one-half probability of getting either  $e_2$  or  $e'_2$ . The only difference is how the levels of  $E_1$  and  $E_2$  are combined.

Additive independence is a stronger condition than utility independence. If two attributes are additively independent, they must be mutually utility independent. But the converse is not true. Mutual utility independence does not imply that the attributes are additively independent.

### 8.4.2 Utility models

If PI and UI are satisfied, a general multiattribute utility function,  $u(e_1, e_2, \dots, e_n)$ , can be simplified and expressed as a function of single-attribute utility functions. The function can take either a multiplicative or an additive form.

**Multiplicative utility model.** Let  $E \equiv E_1 \times E_2 \times \dots \times E_n$ . If any attribute  $E_i$  is preferentially independent and utility independent of its complement  $\bar{E}_i$ , the utility  $u(e_1, e_2, \dots, e_n)$  can be represented as

$$u(e_1, e_2, \dots, e_n) = \frac{1}{K} \left[ \prod_{i=1}^n (K k_i u_i(e_i) + 1) - 1 \right], \quad (8.1)$$

where  $k_i$  = scaling constant for attribute  $e_i$   
 $u_i(e_i)$  = utility function for attribute  $e_i$   
 $K$  = a scaling constant for normalizing utility function  $u(e_1, e_2, \dots, e_n)$ .  
 $K$  can be obtained by solving

$$1 + K = \prod_{i=1}^n (1 + Kk_i).$$

**Additive utility model.** If the more restrictive additive independence condition is satisfied, the utility  $u(e_1, e_2, \dots, e_n)$  can be represented as

$$u(e_1, e_2, \dots, e_n) = \sum_{i=1}^n k_i u_i(e_i), \quad (8.2)$$

where  $\sum_{i=1}^n k_i = 1$ .

### 8.4.3 Assessing multiattribute utility functions

According to Keeney (1977), the assessment of multiattribute utility functions usually follows seven steps:

- Step 1. Verification of Preferential Independence
- Step 2. Verification of Utility Independence
- Step 3. Ordering of the scaling constants
- Step 4. Assessing the scaling constants
- Step 5. Selecting an additive or multiplicative utility function
- Step 6. Assessing single-attribute utility functions

## 8.5 A Structured Approach for Multiattribute Design Evaluation in the Face of Uncertainty

Based upon multiattribute utility theory, a structured approach can be presented to quantify uncertainties and value trade-offs in design evaluation.

This approach mathematically models the functional relationships between design decisions and the overall worth of a candidate design. It focuses on assessing and modeling uncertainties, not only on developing a value model to study multiple attributes. The approach is useful in the early stages of the design process. It can be used to (1) determine an objective function to be used in place of the arbitrary "loss function", and (2) determine the best combination of attribute levels available in the set of feasible design alternatives.

The approach centers around the development of a multiattribute utility function which represents the DM's preferences for the attributes and his risk attitude for the levels of each attribute. With such a utility function, the evaluation framework presented in Chapter V can be extended to multiattribute design evaluation under uncertainty. The decision criterion is the maximization of expected utility.

For a discrete outcome space, the procedures for multiattribute design evaluation are illustrated in Figure 8.1. After a design analysis is conducted to assess all possible outcomes and their probability distributions, an outcome dominance examination is made. Decisions or alternatives are compared attribute by attribute. The dominated decisions are eliminated. If there is only one decision remaining, the decision is optimal and is recommended. If there is more than one decision remaining after the outcome dominance test, a multiattribute utility function is defined and the utility for each possible outcome of each decision is estimated. The decision which has the highest expected utility is the optimal decision. If the decisions are continuous, a procedure similar to that given in Figure 6.5 should be used. However, instead of using a single-attribute utility function as the objective function, a multiattribute utility function needs to be employed.

In applying the approach to resolving problems of multiattribute design evaluation, because of uncertainties, expectation of the utilities should be taken. Once a multiattribute utility function  $u(e)$  or  $u(e_1, e_2, \dots, e_n)$  is determined, the expected utility  $E(u)$  can be obtained as follows:

If the utility function is additive,

$$u(e_1, e_2, \dots, e_n) = \sum_{i=1}^n k_i u_i(e_i),$$

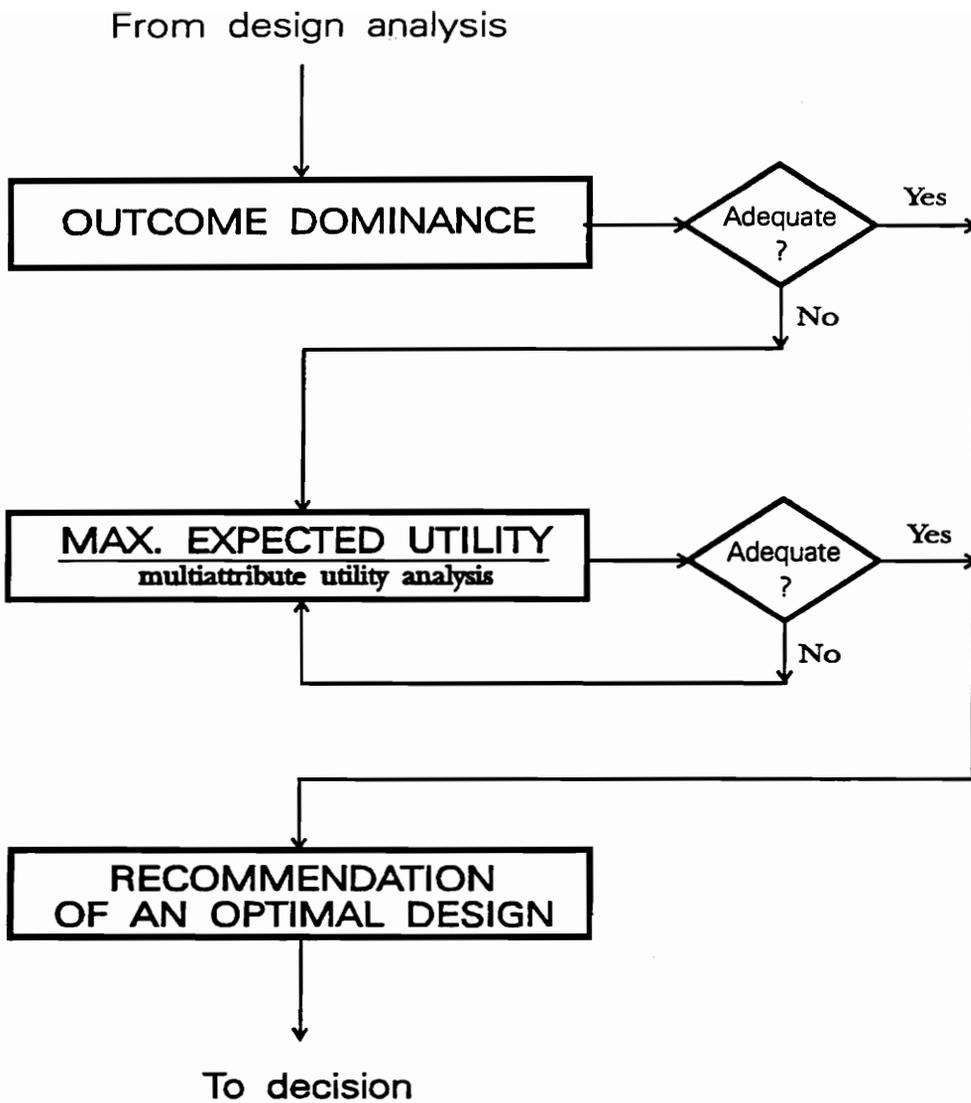


Figure 8.1. A structured approach for multiattribute design evaluation in the face of uncertainty

the expected utility is equal to

$$E[u(e_1, e_2, \dots, e_n)] = \sum_{i=1}^n k_i E[u_i(e_i)]. \quad (8.3)$$

If the utility function takes the multiplicative form,

$$u(e_1, e_2, \dots, e_n) = \frac{1}{K} \left[ \prod_{i=1}^n (Kk_i u_i(e_i) + 1) - 1 \right],$$

the expected utility becomes

$$E[u(e_1, e_2, \dots, e_n)] = \frac{1}{K} \left[ \prod_{i=1}^n (Kk_i E[u_i(e_i)] + 1) - 1 \right]. \quad (8.4)$$

In Equations (8.3) and (8.4),  $E[u_i(e_i)]$  represents the expected value of the single attribute utility function for Attribute  $e_i$ . If the Attribute  $e_i$  is a discrete variable, then

$$E[u_i(e_i)] = \sum_{j=1}^m u_i(e_{ij}) p_j. \quad (8.5)$$

If Attribute  $e_i$  is a continuous variable having probability distribution function  $f_i(e_i)$ , then

$$E[u_i(e_i)] = \int_{e_{i\min}}^{e_{i\max}} u_i(e_i) f_i(e_i) de_i. \quad (8.6)$$

## **IX. REPAIRABLE EQUIPMENT POPULATION SYSTEM DESIGN**

- 
- 9.1 Introduction
  - 9.2 Problem Description
  - 9.3 Determination of Design Evaluation Functions
  - 9.4 Decision Tree
  - 9.5 Experimentation
  - 9.6 Preliminary Findings
  - 9.7 Assessment of Utility Function
  - 9.8 Design Evaluation Using the Maximum Expected Utility Criterion
- 

### **9.1 Introduction**

In this chapter, an example is given to illustrate the framework presented in Chapter VII for multiattribute design evaluation in the face of uncertainty. This example is a modified version of the Repairable Equipment Population System (REPS) model, which is found in Chapter 13 of Fabrycky and Blanchard's Life-Cycle Cost and Economic Analysis (1991).

### **9.2 Problem Description**

A finite population of repairable equipment is to be procured and maintained in operation to meet a demand. As repairable equipment units fail, they will be repaired and returned to service. As they age, the older units will be removed from the system and replaced with new units. The system design problem is to determine the population size, the replacement age of units, and the number of repair channels so that design requirements will be met at a minimum life-cycle cost. This repairable equipment population system is illustrated in Figure 9.1.

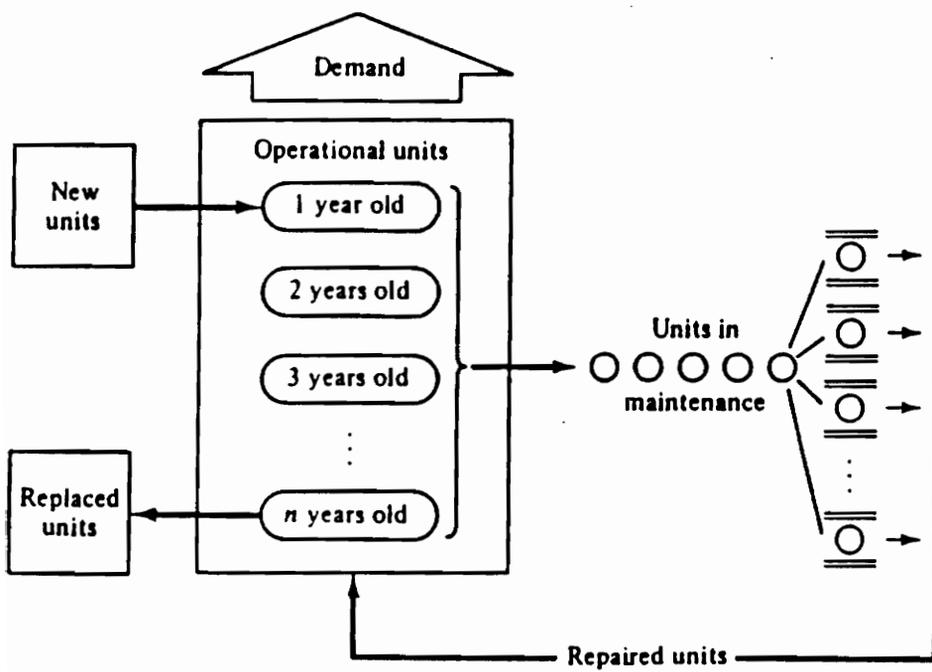


Figure 9.1. Repairable equipment population system  
(Fabrycky and Blanchard, 1991)

The REPS model may be used to represent the operation of numerous systems. For example, both the airlines and the military operate and maintain aircraft with these system characteristics. In ground transit, vehicles such as rental automobiles, taxis, and commercial trucks constitute repairable equipment populations. Production equipment such as weaving looms, drill presses, and autoclaves are populations of equipment which fit the repairable classification. The repairable unit may also be an inventory of components for the larger entities mentioned. For example, aircraft hydraulic pumps, automobile starters and alternators, truck engines, and electric motors also constitute repairable equipment population systems.

### **9.3 Determination of Design Evaluation Functions**

#### **9.3.1 Design variables, parameters, and attributes**

Various factors which influence decision making for the design of the REPS are identified in Table 9.1. They are categorized into four groups: system design variables, design-independent parameters, design-dependent parameters, and evaluation attributes. There are three system design variables. These controllable variables are the number of units to deploy or population size, the number of repair channels, and the replacement age of units. Design-independent parameters are those parameters not subject to the decision maker's control. They include the interest rate, shortage penalty cost, the cost of providing repair facilities, and demand. Design-dependent parameters include *MTBF*, *MTTR*, design life, unit acquisition cost and operating cost, and the estimated salvage value of a unit.

Two attributes are identified to measure the effectiveness of the REPS. One is the system's annual equivalent life-cycle cost. This attribute measures the cost-effectiveness of the REPS. Another attribute is the probability of no units short. This measure reflects the availability of the overall system in meeting the demand. It measures the performance effectiveness of the REPS. For a commercial system, a shortage of units results in loss of revenue and goodwill. For military systems, a shortage of units may be critical or even catastrophic.

Table 9.1. List of Design Variables, Parameters, and Attributes

| Category                              | Notations | Measures                  |
|---------------------------------------|-----------|---------------------------|
| <b>System design variables:</b>       |           |                           |
| population size                       | $N$       | units                     |
| number of repair channels             | $M$       | channels                  |
| replacement age                       | $n$       | years                     |
| <b>Design-independent parameters:</b> |           |                           |
| interest rate                         | $ir$      | %                         |
| shortage penalty cost                 | $C_s$     | \$ / unit short / year    |
| cost of repair channels               | $C_r$     | dollars per channel       |
| demand                                | $D$       | units                     |
| <b>Design-dependent parameters:</b>   |           |                           |
| mean time between failures            | $MTBF$    | years                     |
| mean time to repair                   | $MTTR$    | years                     |
| design life                           | $L$       | years                     |
| first or acquisition cost             | $P$       | dollars                   |
| unit operating cost                   | $C_o$     | dollars per unit per year |
| salvage value                         | $F$       | dollars per unit          |
| <b>Attributes:</b>                    |           |                           |
| annual equivalent life-cycle cost     | $AELCC$   | dollars                   |
| probability of no units short         | $PNUS$    |                           |

### 9.3.2 Functional relationships

In Fabrycky and Blanchard's example, the following assumptions were adopted in the development of the mathematical model for the REPS:

- 1) The interarrival times are exponentially distributed.
- 2) The repair times are exponentially distributed.
- 3) The interarrival times are statistically independent of the repair times.
- 4) The number of units in the population is small such that finite population queueing models must be used.
- 5) The repair channels are parallel and each is capable of identical performance.
- 6) The population size is always larger than or at least equal to the number of repair channels.
- 7) Each repair channel performs service on one unit at a time.
- 8) *MTBF* and *MTTR* values vary for each age group and represent the expected value of these variables for that age group.
- 9) Repaired units return to operation with the same operational characteristics as their age group.
- 10) Only steady-state modes of operation are considered in the formulation of the REPS model.

Given these assumptions, design evaluation functions are developed below for each of the two attributes: (1) the annual equivalent life-cycle cost, and (2) the probability of no units short.

#### ***Annual equivalent life-cycle cost***

The annual equivalent life-cycle cost for REPS, *AELCC*, is composed of four types of costs and can be expressed as

$$AELCC = PC + OC + RC + SC \quad (9.1)$$

where  $PC$  = annual equivalent population cost (dollars)

$OC$  = annual operating cost (dollars)  
 $RC$  = annual repair facility cost (dollars)  
 $SC$  = annual shortage penalty cost (dollars)

*Annual equivalent population cost (PC):*

The annual equivalent population cost of a deployed population of  $N$  units is

$$PC = C_i N$$

where

$$C_i = (P - B)(A^{iP,ir,n}) + B \times ir$$

$P$  = acquisition cost of a unit

$$B = P - n \frac{P - F}{L}$$

$F$  = estimated salvage value of a unit

$L$  = design life

$n$  = replacement age

$ir$  = annual interest rate

*Annual operating cost (OC):*

The annual cost of operating  $N$  units is given by

$$OC = C_o N$$

where

$C_o$  = annual cost of energy, labor, and preventive maintenance, and other operating costs

*Annual repair facility cost (RC):*

The annual cost of providing a repair facility to repair failed units is:

$$RC = C_r M$$

where  $C_r$  = annual fixed and variable repair cost per repair channel  
 $M$  = number of repair channels

*Annual shortage penalty cost (SC):*

The annual shortage penalty cost is the product of the shortage cost per unit short per year and the expected number of units short. It can be expressed as

$$SC = C_s E(S)$$

where  $C_s$  = shortage cost per unit short per year  
 $E(S)$  = expected number of units short

In a finite population repairable equipment system, if the number of units in operation is less than the demand due to random failures, a shortage is incurred. The expected number of units short,  $E(S)$ , can be found by using finite population queueing theory. The results are summarized below:

For a population of  $N$  units and  $M$  repair channels, the probability that  $k$  units fail is

$$P_k = a_k P_0$$

where

$$P_0 = \left( \sum_{k=1}^N a_k \right)^{-1}$$

if  $k = 0, 1, 2, \dots, M,$

$$a_k = \frac{N!}{(N-k)!k!} \left( \frac{\lambda}{\mu} \right)^k$$

if  $k = M+1, M+2, \dots, N,$

$$a_k = \frac{N!}{(N-k)!M!M^{k-M}} \left( \frac{\lambda}{\mu} \right)^k$$

where  $\lambda$  = failure rate of a unit,  $1/MTBF$   
 $\mu$  = repair rate of a unit,  $1/MTTR$

For a population size of  $N$  and a demand of  $D$ , if the number of failed units is greater than  $(N - D)$ , a shortage of units is incurred. The expected number of units short is then given by

$$E(S) = \sum_{j=1}^D j \times P_{(N-D+j)}$$

### **Probability of no units short**

For a repairable equipment population system with a population size of  $N$  and a demand of  $D$ , the probability of no units short is equal to

$$PNUS = \sum_{k=0}^{N-D} P_k \tag{9.2}$$

## **9.4 Decision Tree**

Design evaluation functions are determined above to represent two attributes, *AELCC* and *PNUS*, as functions of various design variables, design-dependent parameters, and design-independent parameters. The next step in design evaluation is to develop a decision tree which models the REPS decision problem as a sequential decision process.

In developing a sequential decision model, several simplifications are made to the original REPS model of Fabrycky and Blanchard (1991) in order to reduce the amount of calculation. First, only two of the three system design variables are selected as the decision variables: the population size  $N$  and the number of repair channels  $M$ . The replacement age of each unit is held fixed at its design life, that is,  $n = L$ . Thus,  $n$  becomes a design-dependent parameter. For each alternative, the *MTBF* and the *MTTR* represent the expected values of units within their design life.

Second, two of the design-independent parameters are considered as random variables. They are the annual interest rate and the shortage penalty cost. A discrete probability distribution will be specified for each of the variables. The values of other design-independent parameters are assumed to be constant and known.

Given these two assumptions, the REPS decision problem can be represented by a decision-tree diagram in Figure 9.2. At the left side decision node, an alternative is selected. For each alternative, a population size is selected at the next decision node in the middle. Then the number of repair channels is determined at the right side decision nodes. Two types of chance nodes in the figure represent various settings of two design-independent parameters, the interest rate and the shortage penalty cost.

This decision tree in Figure 9.2 can be decomposed into the diagram given in Figure 9.3. The decomposed decision tree combines the two types of chance nodes for the interest rate and the shortage penalty cost. Thus, each combination of the settings of the interest rate and the shortage penalty cost represents a state of nature for the problem. In Figure 9.3, the decision nodes for the population size and the decision nodes for the number of repair channels are also combined. Thus, for each alternative, each decision in the diagram represents a possible combination of the settings of two decision variables,  $N$  and  $M$ .

The evaluation process begins with the right side of the tree in Figure 9.3 and works backwards. The outcomes of each decision are estimated by calculating the values of two attributes for various possible states of nature. Then the utility for each decision is assessed. The decision which has the highest expected utility is identified for each alternative. Finally, the optimal decision for the problem is obtained by comparing the expected utility of the best decision for each alternative.

## 9.5 Experimentation

A computer model is developed to facilitate the computation of *AELCC* and *PNUS*. The flowchart of the model is illustrated in Figure 9.4. The model is programmed using C language (see Appendix A.2 for the program listing).

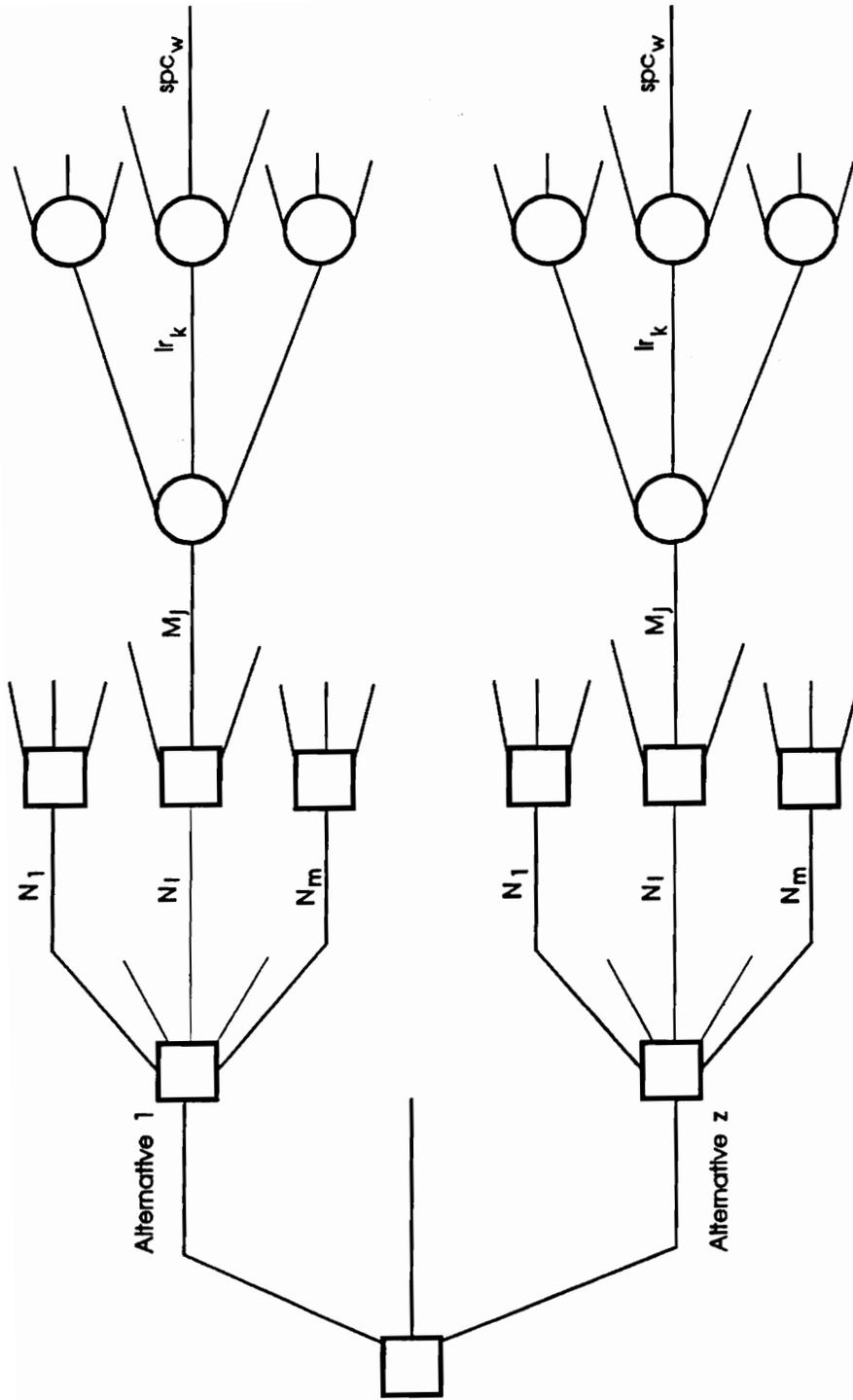


Figure 9.2. A decision-tree representation of the REPS problem

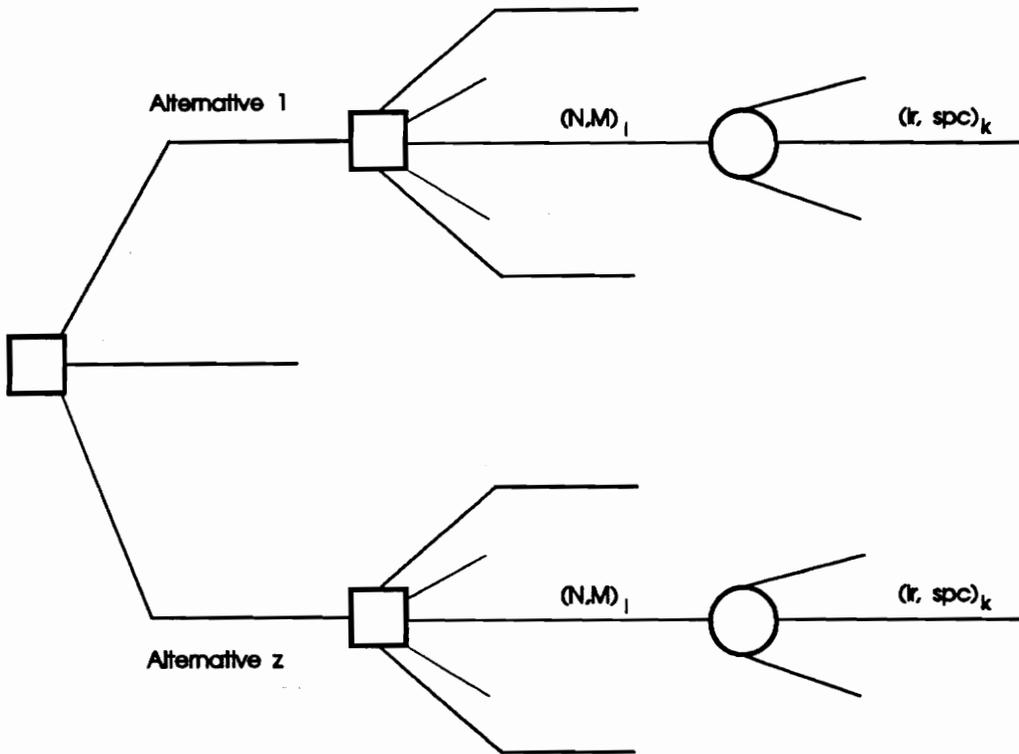


Figure 9.3. A simplified decision-tree representation of the REPS problem

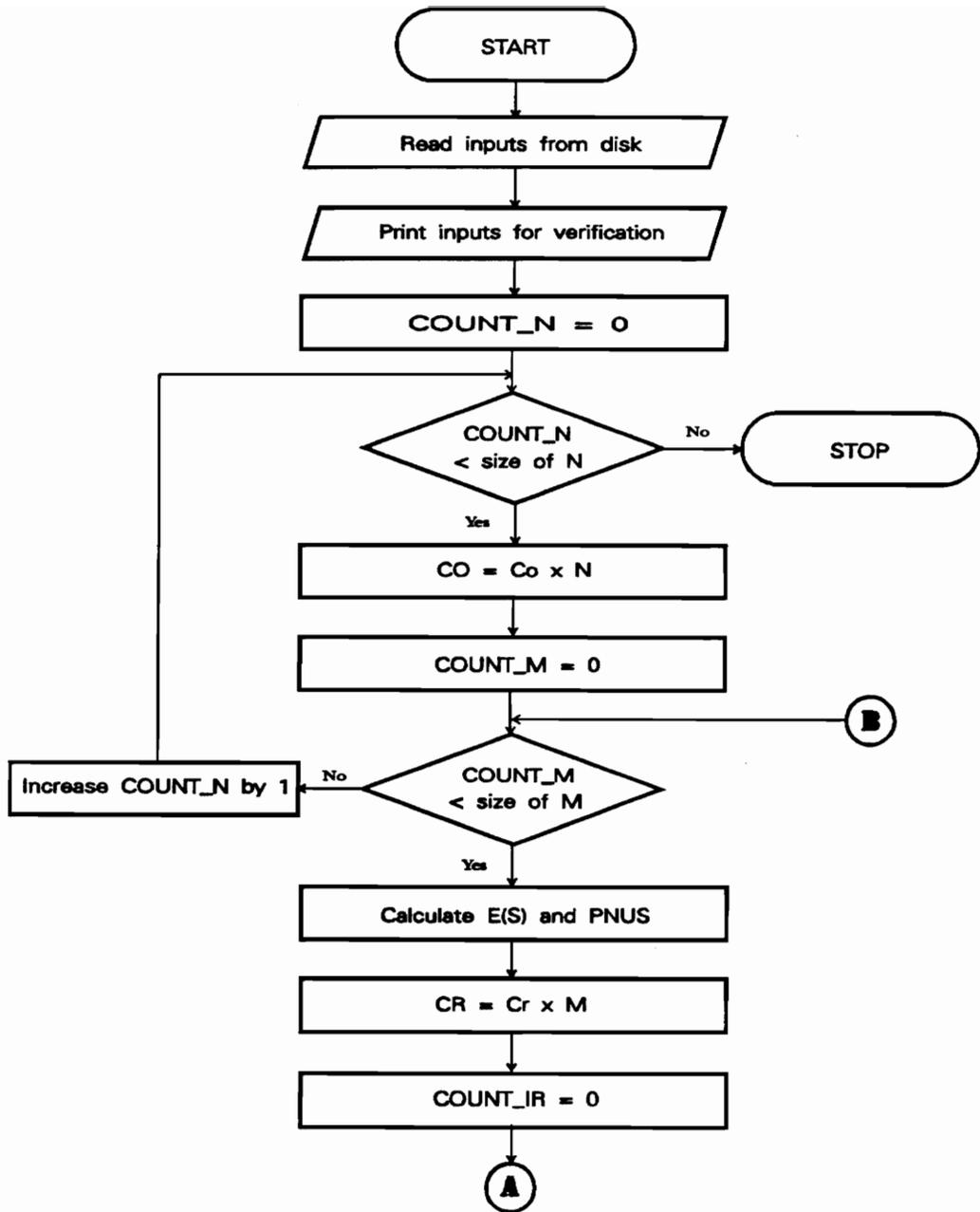


Figure 9.4. Flowcharts for the REPS model

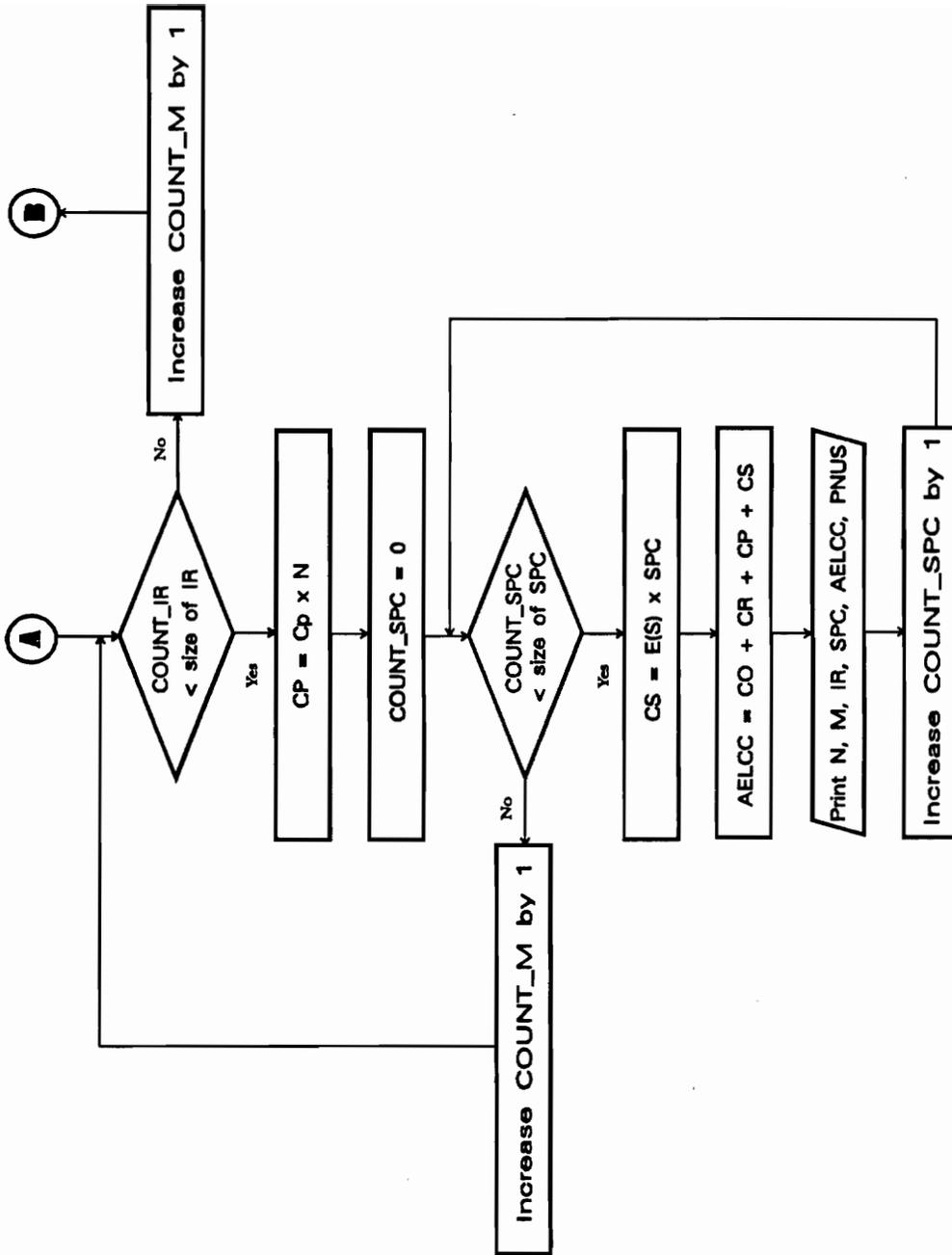


Figure 9.4. Flowchart for the REPS model (cont.)

The experiments are conducted by using the input values from the examples given by Blanchard and Fabrycky (1990). Two candidate systems are identified in Table 9.2. The design life and salvage values are adjusted for the simplified model formulation. The demand is assumed to be 15 units. The repair channel cost is \$45,000 per channel per year. The probability distributions of the interest rate and the shortage penalty cost are given in Tables 9.3a and 9.3b, respectively.

Table 9.2. Design-Dependent Parameters for Candidate Systems

| Parameter                  | Candidate System 1 | Candidate System 2 |
|----------------------------|--------------------|--------------------|
| Unit acquisition cost, $P$ | \$52,000           | \$43,000           |
| Unit design life, $L$      | 4 years            | 4 years            |
| Unit salvage value, $F$    | \$22,000           | \$17,667           |
| Unit operating cost, $C_o$ | \$1,750            | \$2,300            |
| $MTBF$                     | 0.2550             | 0.2225             |
| $MTTR$                     | 0.0425             | 0.0450             |

Table 9.3a. Probability Distribution of Annual Interest Rate

|                     |      |      |      |
|---------------------|------|------|------|
| Interest rate, $ir$ | 9%   | 10%  | 11%  |
| Probability         | 0.30 | 0.40 | 0.30 |

Table 9.3b. Probability Distribution of Shortage Penalty Cost

|   |        |        |        |
|---|--------|--------|--------|
| Shortage penalty cost, $C_s$ (\$/unit short/year) | 68,000 | 73,000 | 78,000 |
| Probability                                       | 0.25   | 0.50   | 0.25   |

The possible states of nature are determined by the combinations of the interest rate and the shortage penalty cost. There are a total of 9 (3x3) combinations, representing nine states of nature for the REPS problem. Since the interest rate and the shortage penalty cost are independent, the probability that a state occurs is equal to the product of the probabilities of the corresponding interest rate level and the shortage penalty cost level. The probabilities for the states of nature are plotted in Figure 9.5.

Experiments are conducted by letting  $N = 18, 19, 20, 21, 22$  and  $M = 2, 3, 4, 5$ . For each alternative, there are 20 decisions (5x4). The results for each alternative are listed in Table 9.4 and Table 9.5, respectively.

## 9.6 Preliminary Findings

### 9.6.1 Probability of no units short

Tables 9.6a and 9.6b summarize the probabilities of no units short for Alternative 1 and Alternative 2 respectively. The value of  $PNUS$  depends upon both the settings of  $N$  and  $M$ . Based upon the experimentation results, the following conclusions can be drawn:

- For a constant population size, the  $PNUS$  increases with the number of repair channels.
- For a constant number of repair channels, the  $PNUS$  increases with population size.
- The decision of 22 units and 5 repair channels has the highest  $PNUS$  for both alternatives.

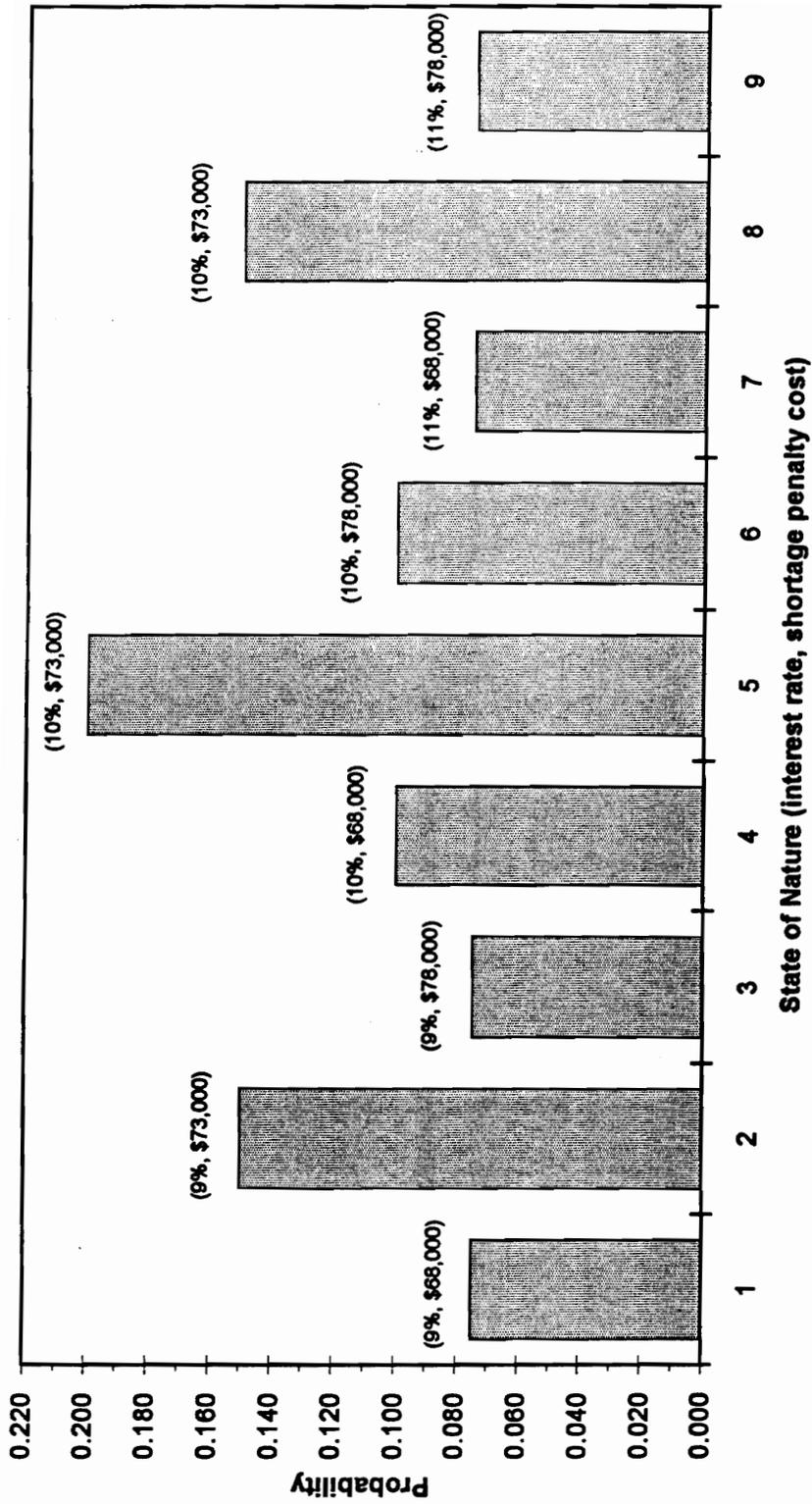


Figure 9.5. Probability distribution of the states of nature

**Table 9.4. AELCC and PNUS for the Decisions of Alternative 1**

|          |    | 9       | 9               | 9       | 10      | 10      | 10      | 10      | 11      | 11      | 11      |         |        |
|----------|----|---------|-----------------|---------|---------|---------|---------|---------|---------|---------|---------|---------|--------|
|          |    | ir (%)  | 68,000          | 73,000  | 78,000  | 88,000  | 93,000  | 103,000 | 113,000 | 123,000 | 133,000 |         |        |
|          |    | Cs (\$) | 68,000          | 73,000  | 78,000  | 88,000  | 93,000  | 103,000 | 113,000 | 123,000 | 133,000 | 78,000  |        |
|          |    | Prob.   | 0.075           | 0.150   | 0.075   | 0.100   | 0.200   | 0.100   | 0.075   | 0.150   | 0.075   | 0.075   |        |
| Decision | N  | M       | AELCC (dollars) |         |         |         |         |         |         |         |         |         | PNUS   |
| 1        | 18 | 2       | 567,378         | 585,286 | 603,195 | 575,011 | 592,919 | 610,828 | 582,673 | 600,581 | 618,490 | 618,490 | 0.1872 |
| 2        | 18 | 3       | 451,089         | 457,138 | 463,188 | 458,723 | 464,772 | 470,821 | 466,385 | 472,434 | 478,483 | 478,483 | 0.5459 |
| 3        | 18 | 4       | 455,089         | 458,124 | 461,158 | 462,723 | 465,757 | 468,791 | 470,385 | 473,419 | 476,454 | 476,454 | 0.7007 |
| 4        | 18 | 5       | 489,431         | 491,682 | 493,933 | 497,064 | 499,315 | 501,566 | 504,726 | 506,977 | 509,228 | 509,228 | 0.7396 |
| 5        | 19 | 2       | 575,138         | 592,662 | 610,186 | 583,195 | 600,719 | 618,243 | 591,283 | 608,807 | 626,331 | 626,331 | 0.2046 |
| 6        | 19 | 3       | 450,262         | 455,295 | 460,328 | 458,319 | 463,352 | 468,385 | 466,407 | 471,440 | 476,473 | 476,473 | 0.6222 |
| 7        | 19 | 4       | 454,221         | 456,236 | 458,251 | 462,278 | 464,293 | 466,309 | 470,366 | 472,381 | 474,396 | 474,396 | 0.8012 |
| 8        | 19 | 5       | 488,338         | 489,554 | 490,769 | 496,396 | 497,611 | 498,826 | 504,483 | 505,699 | 506,914 | 506,914 | 0.8594 |
| 9        | 20 | 2       | 585,000         | 602,294 | 619,588 | 593,481 | 610,775 | 628,069 | 601,994 | 619,288 | 636,582 | 636,582 | 0.2151 |
| 10       | 20 | 3       | 453,790         | 458,127 | 462,465 | 462,271 | 466,609 | 470,946 | 470,785 | 475,122 | 479,459 | 479,459 | 0.6744 |
| 11       | 20 | 4       | 458,870         | 460,272 | 461,674 | 467,351 | 468,753 | 470,155 | 475,865 | 477,267 | 478,669 | 478,669 | 0.8617 |
| 12       | 20 | 5       | 494,180         | 494,870 | 495,560 | 502,662 | 503,351 | 504,041 | 511,175 | 511,865 | 512,554 | 512,554 | 0.9202 |
| 13       | 21 | 2       | 596,185         | 613,346 | 630,507 | 605,090 | 622,251 | 639,412 | 614,029 | 631,190 | 648,351 | 648,351 | 0.2211 |
| 14       | 21 | 3       | 460,201         | 464,055 | 467,908 | 469,106 | 472,960 | 476,814 | 478,045 | 481,899 | 485,753 | 485,753 | 0.7107 |
| 15       | 21 | 4       | 466,641         | 467,660 | 468,678 | 475,547 | 476,565 | 477,583 | 484,486 | 485,504 | 486,522 | 486,522 | 0.8995 |
| 16       | 21 | 5       | 503,370         | 503,780 | 504,190 | 512,275 | 512,685 | 513,096 | 521,214 | 521,624 | 522,035 | 522,035 | 0.9525 |
| 17       | 22 | 2       | 608,172         | 625,260 | 642,347 | 617,502 | 634,589 | 651,677 | 626,866 | 643,954 | 661,041 | 661,041 | 0.2244 |
| 18       | 22 | 3       | 468,577         | 472,091 | 475,606 | 477,906 | 481,421 | 484,935 | 487,271 | 490,786 | 494,300 | 494,300 | 0.7362 |
| 19       | 22 | 4       | 476,250         | 477,020 | 477,790 | 485,580 | 486,350 | 487,119 | 494,945 | 495,714 | 496,484 | 496,484 | 0.9241 |
| 20       | 22 | 5       | 514,250         | 514,505 | 514,760 | 523,579 | 523,834 | 524,089 | 532,944 | 533,199 | 533,454 | 533,454 | 0.9705 |

**Table 9.5. AELCC and PNUS for the Decisions of Alternative 2**

| Decision | N  | M       | AELCC (dollars) |         |         |         |         |         |         |         |         |        |        | PNUS   |        |        |  |
|----------|----|---------|-----------------|---------|---------|---------|---------|---------|---------|---------|---------|--------|--------|--------|--------|--------|--|
|          |    |         | 9               | 9       | 9       | 10      | 10      | 10      | 10      | 10      | 11      | 11     | 11     |        |        |        |  |
|          |    | ir (%)  | 68,000          | 73,000  | 78,000  | 88,000  | 68,000  | 73,000  | 78,000  | 88,000  | 68,000  | 73,000 | 78,000 | 88,000 | 73,000 | 78,000 |  |
|          |    | Cs (\$) | 68,000          | 73,000  | 78,000  | 88,000  | 68,000  | 73,000  | 78,000  | 88,000  | 68,000  | 73,000 | 78,000 | 88,000 | 73,000 | 78,000 |  |
|          |    | Prob.   | 0.075           | 0.150   | 0.075   | 0.100   | 0.100   | 0.200   | 0.100   | 0.100   | 0.075   | 0.150  | 0.075  | 0.150  | 0.075  |        |  |
|          |    |         |                 |         |         |         |         |         |         |         |         |        |        |        |        |        |  |
| 1        | 18 | 2       | 658,212         | 684,494 | 710,777 | 664,494 | 690,776 | 717,059 | 670,800 | 697,082 | 723,365 | 0.0688 |        |        |        |        |  |
| 2        | 18 | 3       | 492,175         | 502,940 | 513,705 | 498,457 | 509,222 | 519,987 | 504,763 | 515,528 | 526,293 | 0.3566 |        |        |        |        |  |
| 3        | 18 | 4       | 463,628         | 468,985 | 474,343 | 469,910 | 475,267 | 480,624 | 476,216 | 481,573 | 486,931 | 0.5563 |        |        |        |        |  |
| 4        | 18 | 5       | 486,932         | 490,694 | 494,456 | 493,214 | 498,976 | 500,737 | 499,520 | 503,282 | 507,044 | 0.6200 |        |        |        |        |  |
| 5        | 19 | 2       | 668,152         | 694,304 | 720,457 | 674,783 | 700,935 | 727,087 | 681,439 | 707,592 | 733,744 | 0.0734 |        |        |        |        |  |
| 6        | 19 | 3       | 491,433         | 501,282 | 511,132 | 498,064 | 507,913 | 517,763 | 504,720 | 514,570 | 524,419 | 0.4113 |        |        |        |        |  |
| 7        | 19 | 4       | 458,188         | 462,284 | 466,381 | 464,819 | 468,915 | 473,011 | 471,476 | 475,572 | 479,668 | 0.6607 |        |        |        |        |  |
| 8        | 19 | 5       | 479,781         | 482,156 | 484,531 | 486,412 | 488,787 | 491,162 | 493,068 | 495,443 | 497,818 | 0.7601 |        |        |        |        |  |
| 9        | 20 | 2       | 678,963         | 705,049 | 731,136 | 685,943 | 712,029 | 738,115 | 692,950 | 719,036 | 745,122 | 0.0757 |        |        |        |        |  |
| 10       | 20 | 3       | 494,697         | 503,925 | 513,153 | 501,676 | 510,905 | 520,133 | 508,683 | 517,912 | 527,140 | 0.4484 |        |        |        |        |  |
| 11       | 20 | 4       | 458,581         | 461,845 | 465,109 | 465,560 | 468,824 | 472,088 | 472,567 | 475,831 | 479,095 | 0.7286 |        |        |        |        |  |
| 12       | 20 | 5       | 480,583         | 482,156 | 483,729 | 487,563 | 489,136 | 490,709 | 494,570 | 496,143 | 497,716 | 0.8411 |        |        |        |        |  |
| 13       | 21 | 2       | 690,237         | 716,291 | 742,346 | 697,566 | 723,620 | 749,674 | 704,923 | 730,977 | 757,032 | 0.0769 |        |        |        |        |  |
| 14       | 21 | 3       | 500,724         | 509,534 | 518,345 | 508,053 | 516,863 | 525,674 | 515,410 | 524,220 | 533,031 | 0.4734 |        |        |        |        |  |
| 15       | 21 | 4       | 462,614         | 465,313 | 468,013 | 469,943 | 472,642 | 475,342 | 477,300 | 479,999 | 482,699 | 0.7764 |        |        |        |        |  |
| 16       | 21 | 5       | 485,717         | 486,807 | 487,896 | 493,046 | 494,135 | 495,225 | 500,403 | 501,493 | 502,582 | 0.8900 |        |        |        |        |  |
| 17       | 22 | 2       | 701,745         | 727,784 | 753,824 | 709,423 | 735,462 | 761,501 | 717,130 | 743,170 | 769,209 | 0.0774 |        |        |        |        |  |
| 18       | 22 | 3       | 508,668         | 517,201 | 525,735 | 516,345 | 524,879 | 533,413 | 524,053 | 532,587 | 541,120 | 0.4899 |        |        |        |        |  |
| 19       | 22 | 4       | 469,002         | 471,310 | 473,618 | 476,680 | 478,988 | 481,296 | 484,387 | 486,695 | 489,004 | 0.8088 |        |        |        |        |  |
| 20       | 22 | 5       | 493,310         | 494,096 | 494,883 | 500,987 | 501,774 | 502,561 | 508,695 | 509,482 | 510,269 | 0.9205 |        |        |        |        |  |

**Table 9.6a. Probability of No Units Short (Alternative 1)**

| <i>N</i> | <i>M</i> |        |        |        |
|----------|----------|--------|--------|--------|
|          | 2        | 3      | 4      | 5      |
| 18       | 0.1872   | 0.5459 | 0.7007 | 0.7396 |
| 19       | 0.2046   | 0.6222 | 0.8012 | 0.8594 |
| 20       | 0.2151   | 0.6744 | 0.8617 | 0.9202 |
| 21       | 0.2211   | 0.7107 | 0.8995 | 0.9525 |
| 22       | 0.2244   | 0.7362 | 0.9241 | 0.9705 |

**Table 9.6b. Probability of No Units Short (Alternative 2)**

| <i>N</i> | <i>M</i> |        |        |        |
|----------|----------|--------|--------|--------|
|          | 2        | 3      | 4      | 5      |
| 18       | 0.0688   | 0.3566 | 0.5563 | 0.6200 |
| 19       | 0.0734   | 0.4113 | 0.6607 | 0.7601 |
| 20       | 0.0757   | 0.4484 | 0.7296 | 0.8411 |
| 21       | 0.0769   | 0.4734 | 0.7764 | 0.8900 |
| 22       | 0.0774   | 0.4899 | 0.8088 | 0.9205 |

As indicated in the experimentation, for a specified combination of *N* and *M*, the probability of no units short, *PNUS*, does not depend on the interest rate and the shortage penalty cost. If the population size and the number of repair channels are determined, the probability of no units short becomes a certainty. Thus, in comparing several decisions, if the values of *AEELCC* are the same, the decisions which provides the highest *PNUS* is preferred. Based upon the three conclusions above, those inefficient decisions can be eliminated by comparing the *AEELCC*'s for the decisions.

### 9.6.2 Annual equivalent life-cycle cost

As indicated in Section 9.3.2, the *AELCC* is a function of  $N$ ,  $M$ ,  $ir$ , and  $C_s$ . For a specified combination of  $N$  and  $M$ , the value of the *AELCC* depends upon the settings of the interest rate and the shortage penalty cost. Since it is not clear which state of nature will occur, uncertainties will result for the value of *AELCC*. To compare the various decisions for each alternative, we apply an outcome dominance test first.

For each decision, since the *PNUS* is constant and known, outcome dominance can be carried out by applying the following rule:

If Decision A's *PNUS* is equal to or higher than that of Decision B and A's *AELCC* is less than that of B under each state of nature, then A is preferred.

By applying the outcome dominance test, a number of decisions are eliminated for each alternative (Table 9.7). The expected *AELCC* and the *PNUS* for the remaining decisions of each alternative are shown in Figures 9.6 and 9.7. In order to identify the best decision for each alternative, a multiattribute utility analysis is required.

**Table 9.7. List of Decisions Dominated and Decisions Remaining**

| Alternative | Decision Dominated                         | Decision Remaining           |
|-------------|--|------------------------------|
| 1           | 1, 2, 3, 4, 5, 8, 9, 12, 13, 14, 17, 18    | 6, 7, 10, 11, 15, 16, 19, 20 |
| 2           | 1, 2, 3, 4, 5, 6, 8, 9, 10, 13, 14, 17, 18 | 7, 11, 12, 15, 16, 19, 20    |

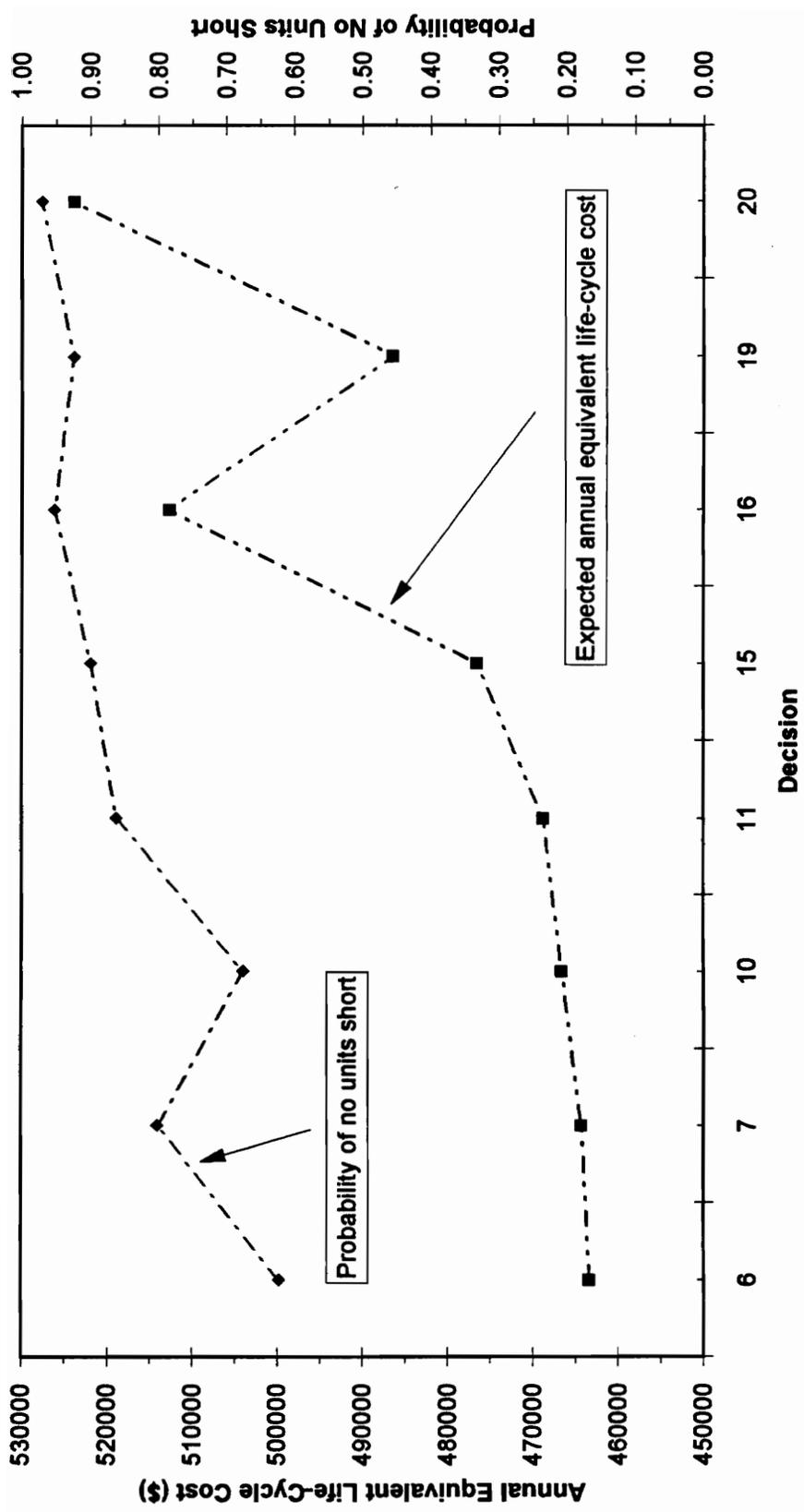


Figure 9.6. Expected AELCC and PNUS for the decisions of Alternative 1

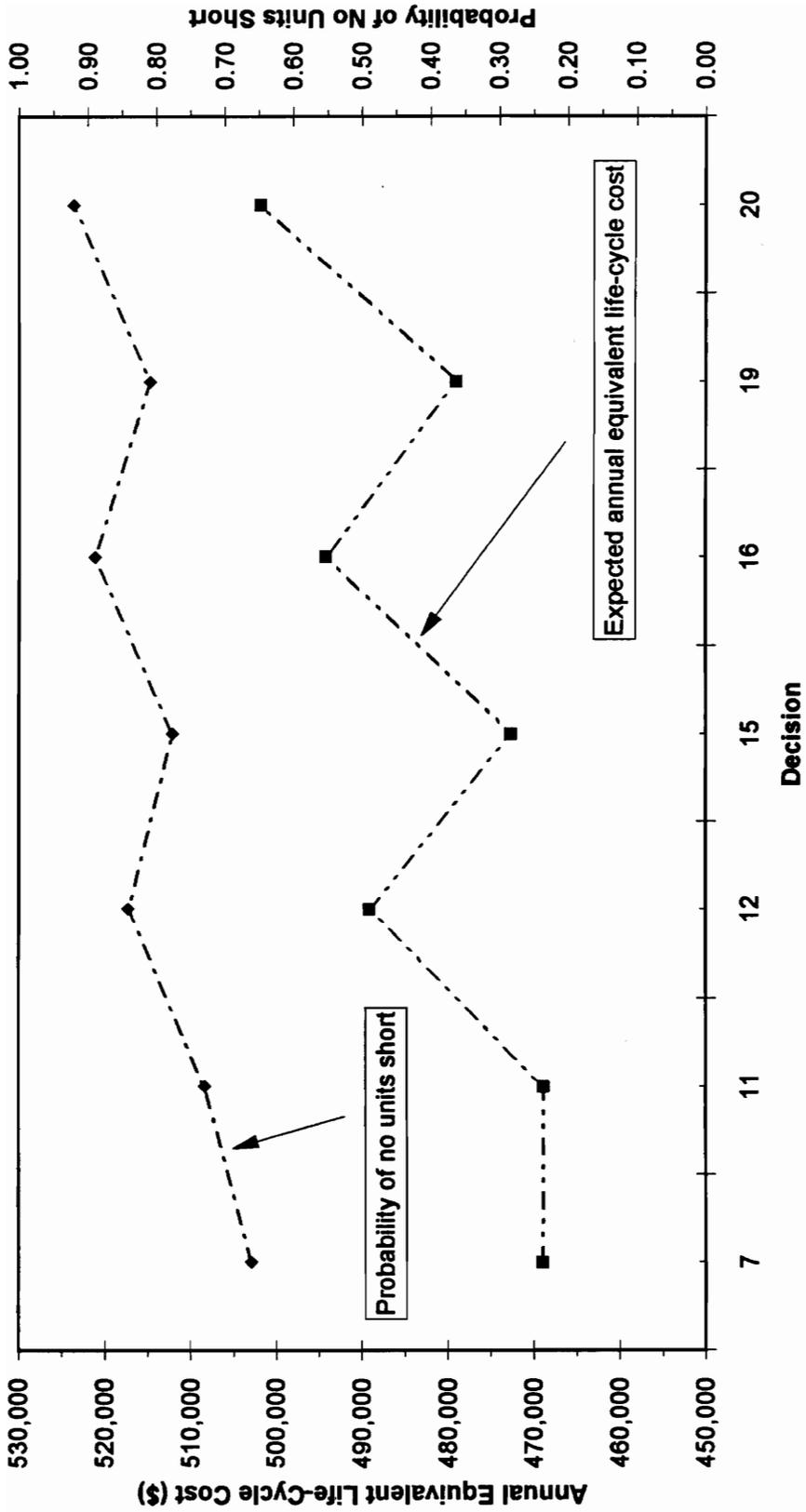


Figure 9.7. Expected AELCC and PNUS for the decisions of Alternative 2

## 9.7 Assessment of Utility Function

The REPS decision problem is complicated because of two factors. One is the multiple objective aspect of the problem and the necessity to make value trade-offs among various levels of different attributes. The other factor is the uncertainties about what the impact of any alternative will eventually be and the difficulty in separating this from one's preferences concerning "possible" consequences. As indicated by Keeney (1977), no decision procedure can circumvent the fact that preferences are a critical aspect in such problems and further, that preferences are inherently subjective. A multiattribute utility function is needed in order to identify the best decision for each alternative. A utility function of this type is assessed below.

### 9.7.1 Determination of the best value and the worst value for each attribute

From Tables 9.3 and 9.4, the maximum and minimum values of *AELCC* and *PNUS* each alternative will possibly take are identified and listed in Table 9.8a. If everything else is the same, the DM will always prefer a lower value of *AELCC* and a higher value of *PNUS*. According to these extreme values for each attribute, a best value and a worst value are determined for each attribute for the assessment of utility functions (Table 9.8b). The levels in the table are adjusted slightly to facilitate the assessment. Then a survey was conducted to assess the DM's preferences and risk attitudes.

Table 9.8a. The Maximum and Minimum Values of *AELCC* and *PNUS*

| Alternative | <i>AELCC</i> |           | <i>PNUS</i> |         |
|-------------|--------------|-----------|-------------|---------|
|             | Maximum      | Minimum   | Maximum     | Minimum |
| 1           | \$661,041    | \$450,262 | 0.9705      | 0.1872  |
| 2           | \$769,209    | \$458,188 | 0.9205      | 0.0688  |

Table 9.8b. Best Level and Worst Level of *AELCC* and *PNUS*

| Attribute                   | Best Level | Worst Level |
|-----------------------------|------------|-------------|
| <i>AELCC</i> (1000 dollars) | 400        | 800         |
| <i>PNUS</i>                 | 1.00       | 0.05        |

### 9.7.2 Verification of preferential independence

In order to identify the form of the utility function for the REPS problem, preferential independence assumptions need to be examined. The PI assumption is verified through the survey below, where DA is the decision analyst and DM denotes the decision maker.

DA: If everything else is the same, do you always prefer less *AELCC* to more *AELCC*?

DM: Yes.

DA: This means that you have a decreasing utility function for *AELCC*. Similarly, if everything else is the same, do you always prefer higher level of *PNUS* to lower level of *PNUS*?

DM: Yes.

DA: This means that your utility function for *PNUS* is increasing. Now we consider two attributes at once. Given the following consequences (*AELCC* = \$500,000, *PNUS* = 0.7) and (*AELCC* = \$550,000, *PNUS* = 0.7), which consequence do you prefer?

DM: The first one.

DA: O.K. If we change *PNUS* from 0.7 to 0.8, that is, we have these two consequences,

( $AELCC = \$500,000$ ,  $PNUS = 0.8$ ) and ( $AELCC = \$550,000$ ,  $PNUS = 0.8$ ), do you still prefer the first one?

DM: Yes.

DA: This indicates that if the setting of  $PNUS$  is the same, you always prefer lower level of  $AELCC$  regardless what value  $PNUS$  is held fixed at, is that true?

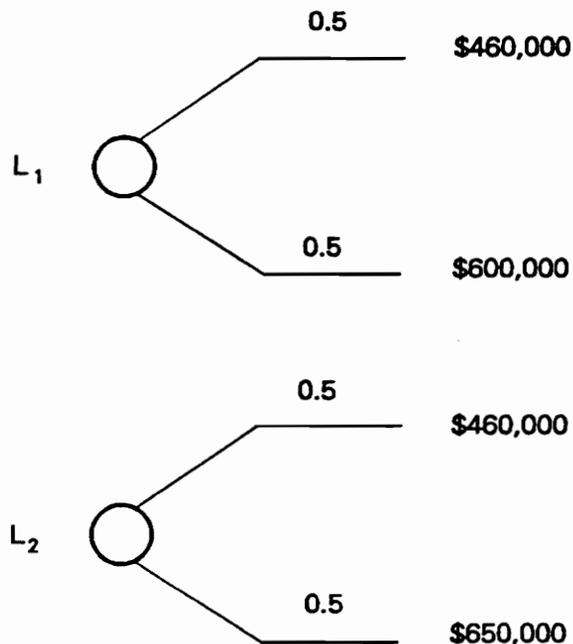
DM: Yes.

DA: This implies that  $AELCC$  is preferentially independent of  $PNUS$ .

### 9.7.3 Verification of utility independence

The utility independence condition is examined below:

DA: Suppose we have two lotteries for  $AELCC$ ,



For both lotteries,  $PNUS$  is held fixed at the same level. Which lottery do you prefer?

DM: The first one.

DA: O.K. Do you always prefer the first lottery if the *PNUS* is held fixed at another level?

DM: Yes.

DA: This implies that *AELCC* is utility independent of *PNUS*.

According to the MAUT theory introduced in Section 7.4, the preferential independence and utility independence assumptions imply that the DM's utility function must be either additive or multiplicative. That is, the utility function will take either of the forms:

$$u(AELCC, PNUS) = k_1 u_1(AELCC) + k_2 u_2(PNUS), \text{ or}$$

$$u(AELCC, PNUS) = k_1 u_1(AELCC) + k_2 u_2(PNUS) + (k_1 + k_2 - 1) u_1(AELCC) u_2(PNUS)$$

where

- $k_1$  = scaling constant for *AELCC*
- $k_2$  = scaling constant for *PNUS*
- $u_1(AELCC)$  = utility function of *AELCC*
- $u_2(PNUS)$  = utility function of *PNUS*

### 9.7.4 Assessing the scaling constants

The scaling constants measure the relative importance of attributes as they progress from their worst to best states. The constants are assessed through a survey below:

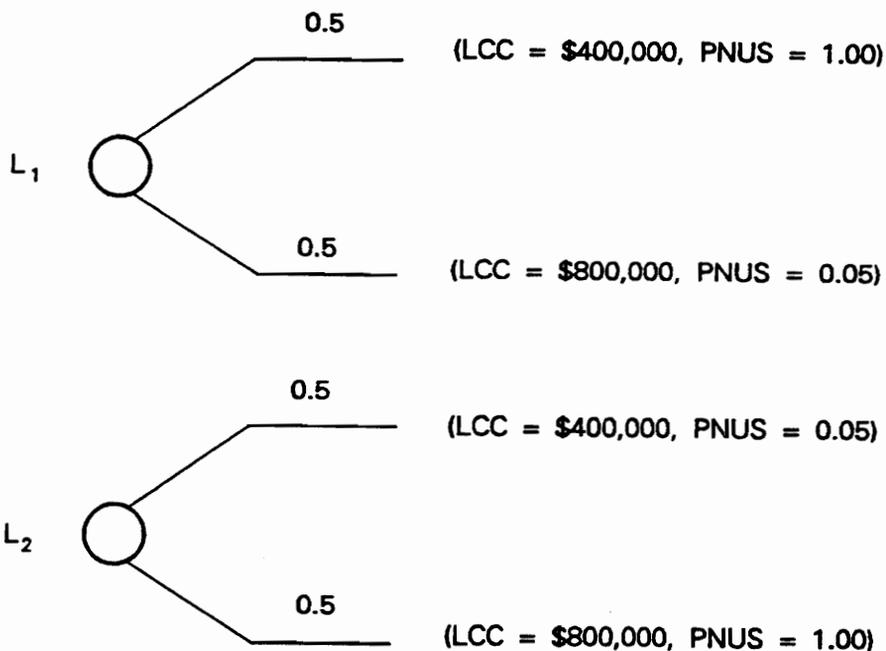
DA: Assuming that *AELCC* and *PNUS* are at their worst levels in Table 9.8b. If you have a choice to push one attribute at a time from its worst level to its best level, which attribute will you push first?

DM: I'll push *PNUS* first.

DA: This implies that  $k_2 > k_1$ . Now we compare a lottery to a certain consequence in order to assess the scaling constants.

Then a series of questions were asked to assess the scaling constants (Figures 9.8 and 9.9). For the *AELCC*, the probability at which the DM is indifferent between the lottery and the sure thing is around 0.45. Thus,  $k_1 = 0.45$ . For the *PNUS*, the probability  $p$  at which the DM is indifferent between the lottery and the sure thing is close to 0.55. This implies that  $k_2 = 0.55$ .

Since  $k_1 + k_2 = 1.0$ , the utility function is additive. The additive independence assumption was further confirmed by asking the DM to compare two lotteries:



Since the DM is indifferent to these two lotteries, the utility function must take the form

$$u(AELCC, PNUS) = k_1 u_1(AELCC) + k_2 u_2(PNUS).$$

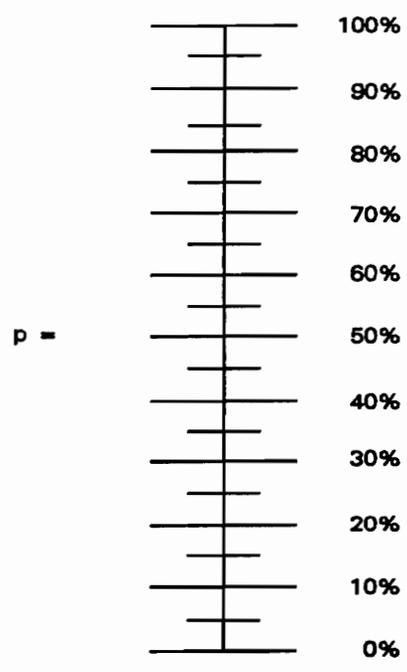
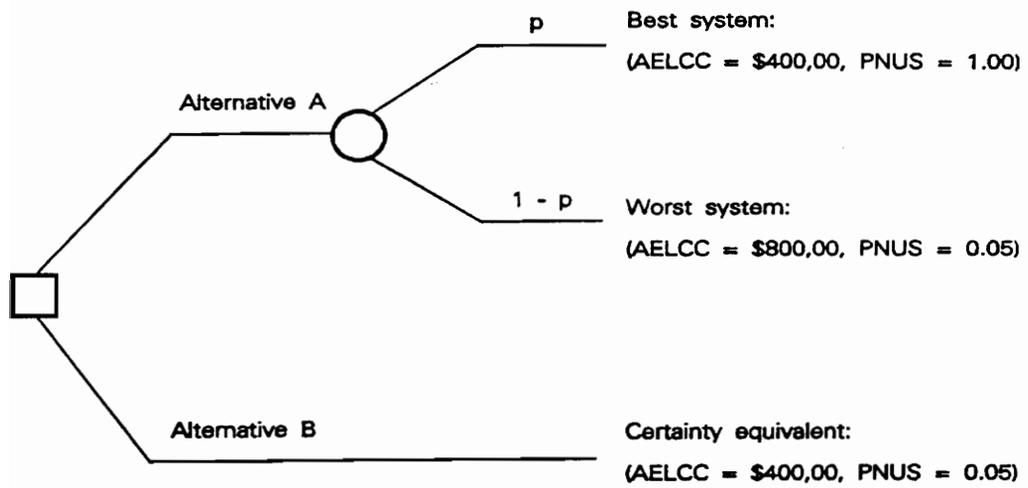


Figure 9.8. Assessment of scaling constant  $k_1$

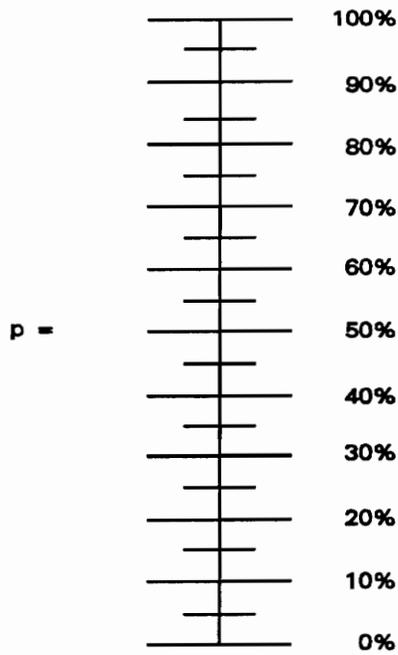
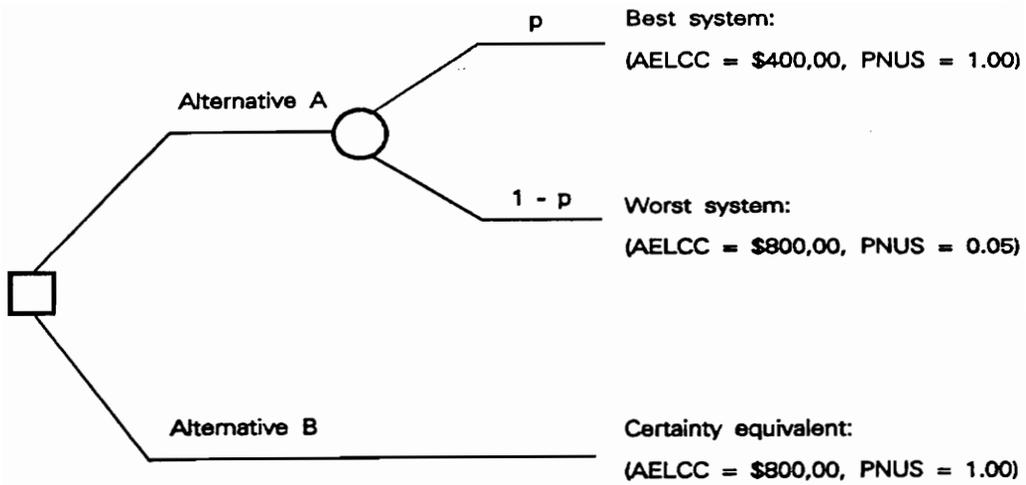


Figure 9.9. Assessment of scaling constant  $k_2$

### 9.7.5 Assessing single-attribute utility functions

The last step is to assess the utility function for *AELCC* and the utility function for *PNUS*. The 50-50 lottery approach introduced in Section 5.4.4 is used for this assessment. The certainty equivalent for the *AELCC* is found to be \$675,000 (Figure 9.10). The certainty equivalent for the *PNUS* is found to be 0.45 (Figure 9.11).

Exponential functions are fit to the data.  $u_1(AELCC)$  and  $u_2(PNUS)$  are plotted in Figure 9.12 and Figure 9.13, respectively.

### 9.7.6 Summary of assessments

The results of utility function assessment are summarized in Table 9.9. The overall utility function is

$$u(AELCC, PNUS) = 0.45 [1.2342 - 0.0444 \exp(0.0042 AELCC)] + 0.55 [2.1100 - 2.1826 \exp(-0.6761 PNUS)]. \quad (9.3)$$

Table 9.9. Utility Function and Scaling Constant for *AELCC* and *PNUS*

| Attribute    | Utility Function                     | Scaling Constant |
|--------------|--------------------------------------|------------------|
| <i>AELCC</i> | $1.2342 - 0.0444 \exp(0.0042 AELCC)$ | 0.45             |
| <i>PNUS</i>  | $2.1100 - 2.1826 \exp(-0.6761 PNUS)$ | 0.55             |

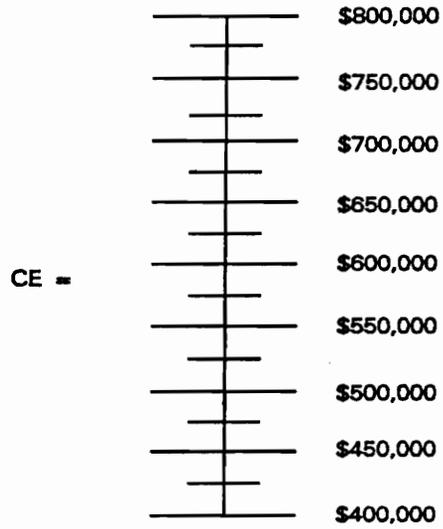
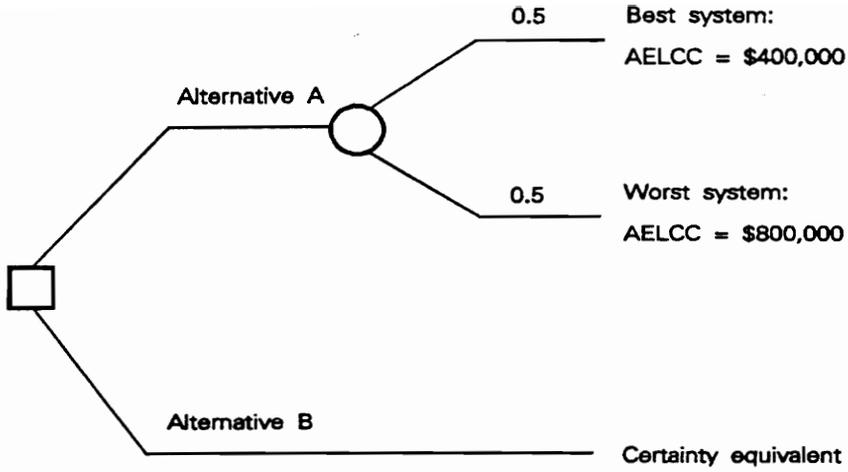


Figure 9.10. Assessment of the utility function for AELCC

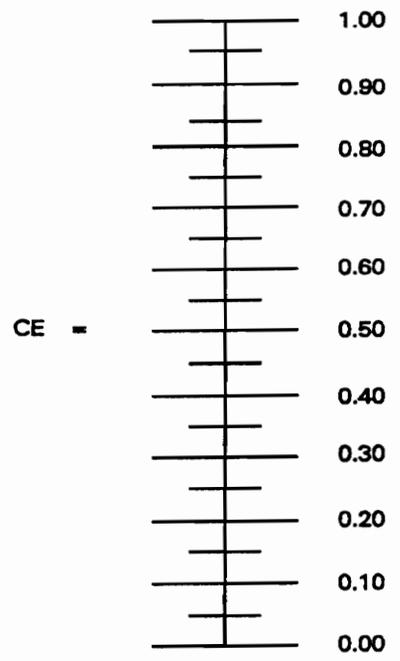
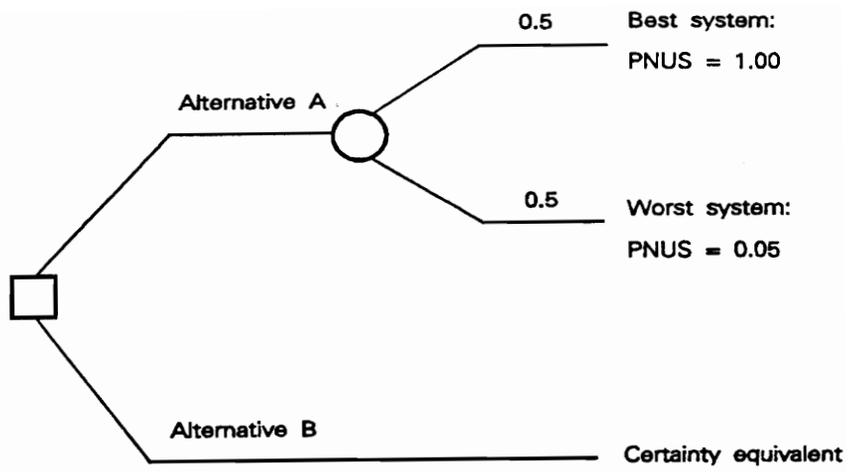


Figure 9.11. Assessment of utility function for PNUS

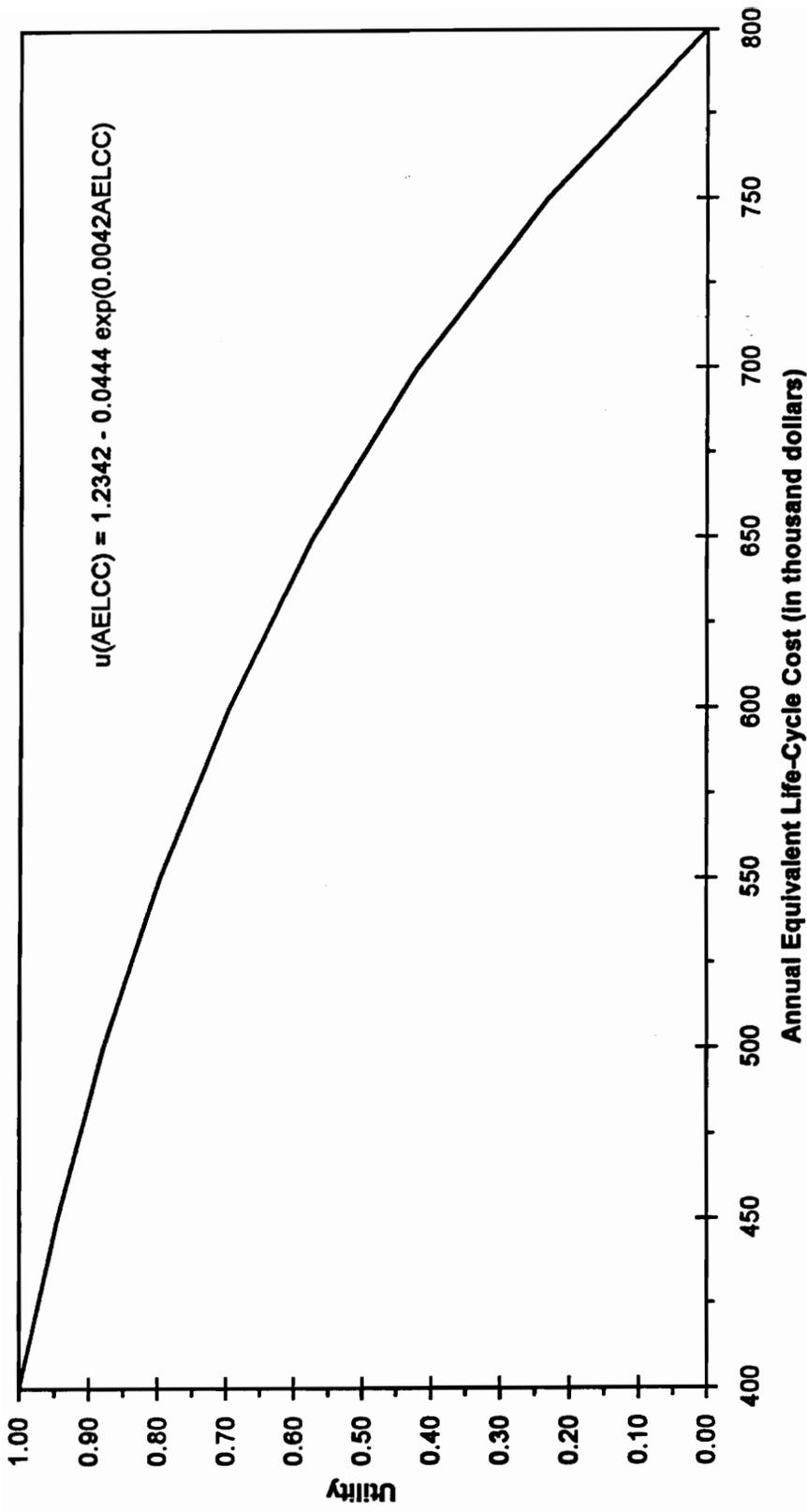


Figure 9.12. Utility function for annual equivalent life-cycle cost

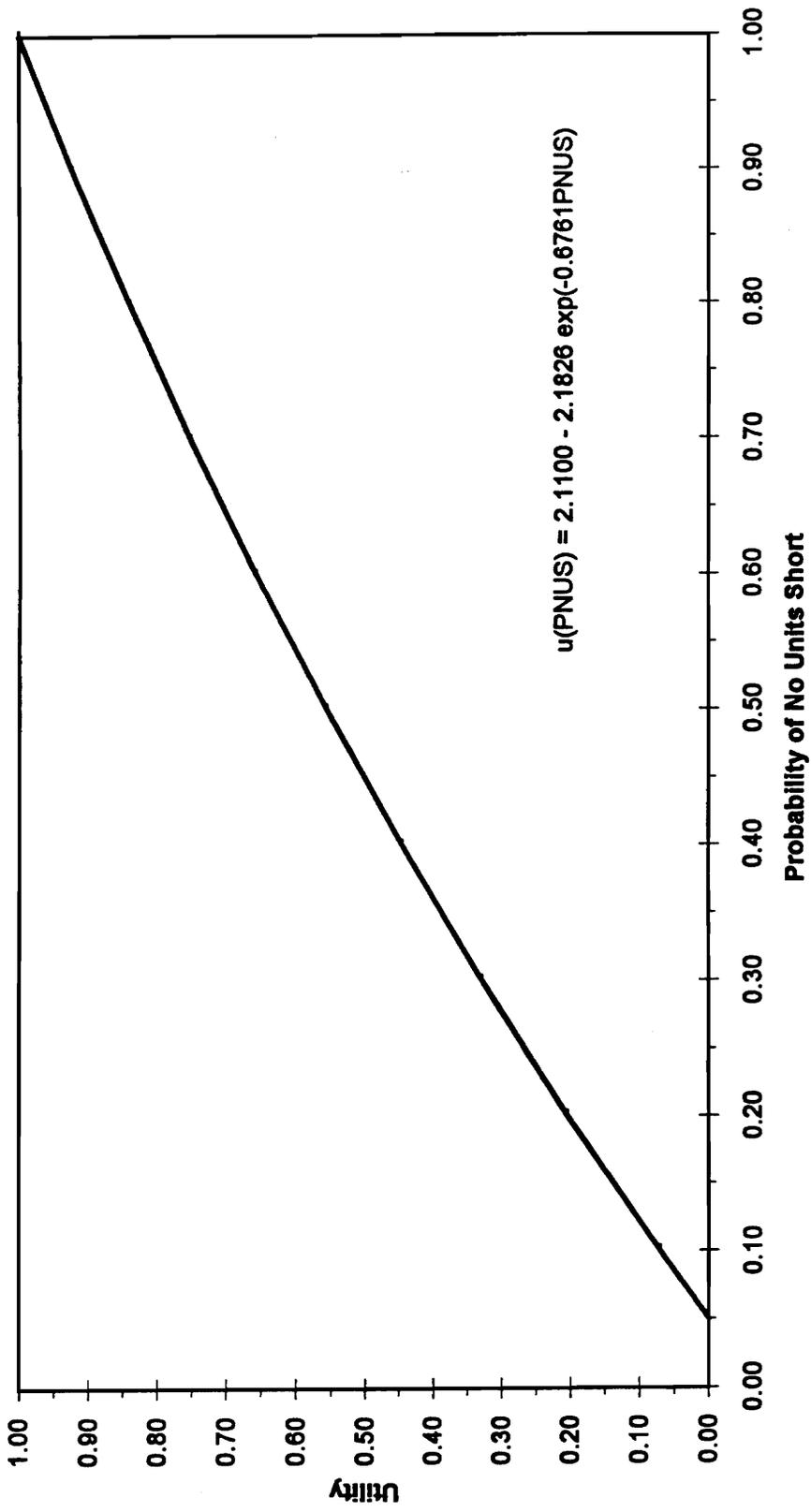


Figure 9.13. Utility function for probability of no units short

## 9.8 Design Evaluation Using the Maximum Expected Utility Criterion

The Maximum Expected Utility (MEU) criterion is applied to compare the remaining decisions for each alternative listed in Table 9.7. By using the single-attribute utility functions given in Table 9.9, the utilities for various levels of *AELCC* and *PNUS* of the decisions for each alternative are calculated in Tables 9.10 and 9.11.

Since *AELCC* is utility independent of *PNUS*, the expected overall utility for a decision is

$$E[u(AELCC, PNUS)] = k_1 E[u_1(AELCC)] + k_2 E[u_2(PNUS)]. \quad (9.4)$$

For each decision, *PNUS* is degenerated to a constant, thus,

$$E[u(AELCC, PNUS)] = k_1 E[u_1(AELCC)] + k_2 [u_2(PNUS)]. \quad (9.5)$$

The expected overall utility for Alternatives 1 and 2 are calculated and plotted in Figures 9.14 and 9.15 respectively. According to the MEU criterion, the decision which has maximum utility is preferred. For Alternative 1, Decision 19 has the maximum expected utility (0.9223). For Alternative 2, Decision 20 provides the maximum expected utility (0.9108).

After the best decision for each alternative is identified, Alternatives 1 and 2 can be evaluated by comparing their best decisions. Since the utility of the best decision from Alternative 1 is higher than that of the utility of the best decision from Alternative 2, the optimal decision for the overall problem is Decision 19 of Alternative 1. The characteristics of the optimal decision are summarized in Table 9.12. The probability distribution of *AELCC* for the optimal decision is plotted in Figure 9.16.

Table 9.10. Utilities of AELCC and PNUS for the Remaining Decisions of Alternative 1

| Decision | N  | M | #(AELCC) |        |        |        |        |        |        |        |        |        |        | #(PNUS) |  |
|----------|----|---|----------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|---------|--|
|          |    |   | 9        | 9      | 9      | 10     | 10     | 10     | 10     | 10     | 11     | 11     | 11     |         |  |
|          |    |   | ir (%)   | 9      | 9      | 9      | 10     | 10     | 10     | 10     | 10     | 11     | 11     | 11      |  |
|          |    |   | Cs (\$)  | 68,000 | 73,000 | 78,000 | 68,000 | 73,000 | 78,000 | 78,000 | 68,000 | 73,000 | 78,000 | 78,000  |  |
|          |    |   | Prob.    | 0.075  | 0.150  | 0.075  | 0.100  | 0.200  | 0.100  | 0.100  | 0.075  | 0.150  | 0.075  | 0.075   |  |
|          |    |   |          |        |        |        |        |        |        |        |        |        |        |         |  |
| 6        | 19 | 3 |          | 0.9456 | 0.9395 | 0.9333 | 0.9358 | 0.9295 | 0.9230 | 0.9230 | 0.9256 | 0.9191 | 0.9124 | 0.6770  |  |
| 7        | 19 | 4 |          | 0.9408 | 0.9384 | 0.9359 | 0.9308 | 0.9283 | 0.9257 | 0.9257 | 0.9205 | 0.9178 | 0.9152 | 0.8403  |  |
| 10       | 20 | 3 |          | 0.9414 | 0.9360 | 0.9306 | 0.9308 | 0.9253 | 0.9197 | 0.9197 | 0.9199 | 0.9142 | 0.9084 | 0.7266  |  |
| 11       | 20 | 4 |          | 0.9351 | 0.9334 | 0.9316 | 0.9244 | 0.9226 | 0.9207 | 0.9207 | 0.9132 | 0.9113 | 0.9095 | 0.8912  |  |
| 15       | 21 | 4 |          | 0.9253 | 0.9240 | 0.9227 | 0.9136 | 0.9123 | 0.9109 | 0.9109 | 0.9015 | 0.9001 | 0.8987 | 0.9220  |  |
| 16       | 21 | 5 |          | 0.8744 | 0.8737 | 0.8731 | 0.8608 | 0.8602 | 0.8595 | 0.8595 | 0.8467 | 0.8460 | 0.8453 | 0.9638  |  |
| 19       | 22 | 4 |          | 0.9127 | 0.9117 | 0.9106 | 0.9000 | 0.8989 | 0.8979 | 0.8979 | 0.8867 | 0.8856 | 0.8845 | 0.9415  |  |
| 20       | 22 | 5 |          | 0.8577 | 0.8573 | 0.8569 | 0.8428 | 0.8424 | 0.8420 | 0.8420 | 0.8273 | 0.8269 | 0.8265 | 0.9776  |  |

Table 9.11. Utilities of AELCC and PNUS for the Remaining Decisions of Alternative 2

| Decision | N  | M | u(AELCC) |        |        |        |        | u(PNUS) |        |        |        |        |        |  |
|----------|----|---|----------|--------|--------|--------|--------|---------|--------|--------|--------|--------|--------|--|
|          |    |   | 9        | 9      | 10     | 10     | 11     | 11      | 11     | 11     | 11     |        |        |  |
|          |    |   | ir (%)   | 9      | 9      | 10     | 10     | 10      | 10     | 11     | 11     | 11     | 11     |  |
|          |    |   | Cs (\$)  | 68,000 | 73,000 | 78,000 | 68,000 | 73,000  | 78,000 | 68,000 | 73,000 | 78,000 | 78,000 |  |
|          |    |   | Prob.    | 0.075  | 0.150  | 0.075  | 0.100  | 0.200   | 0.100  | 0.075  | 0.150  | 0.075  | 0.075  |  |
|          |    |   |          |        |        |        |        |         |        |        |        |        |        |  |
| 7        | 19 | 4 |          | 0.9360 | 0.9308 | 0.9256 | 0.9276 | 0.9224  | 0.9170 | 0.9190 | 0.9136 | 0.9081 | 0.7138 |  |
| 11       | 20 | 4 |          | 0.9355 | 0.9314 | 0.9272 | 0.9267 | 0.9225  | 0.9182 | 0.9176 | 0.9133 | 0.9089 | 0.7773 |  |
| 12       | 20 | 5 |          | 0.9069 | 0.9047 | 0.9026 | 0.8972 | 0.8950  | 0.8928 | 0.8873 | 0.8850 | 0.8827 | 0.8741 |  |
| 15       | 21 | 4 |          | 0.9304 | 0.9270 | 0.9235 | 0.9210 | 0.9175  | 0.9139 | 0.9113 | 0.9077 | 0.9040 | 0.8188 |  |
| 16       | 21 | 5 |          | 0.8998 | 0.8983 | 0.8968 | 0.8895 | 0.8879  | 0.8863 | 0.8788 | 0.8772 | 0.8755 | 0.9143 |  |
| 19       | 22 | 4 |          | 0.9222 | 0.9192 | 0.9162 | 0.9121 | 0.9090  | 0.9059 | 0.9017 | 0.8984 | 0.8952 | 0.8468 |  |
| 20       | 22 | 5 |          | 0.8891 | 0.8880 | 0.8868 | 0.8779 | 0.8767  | 0.8756 | 0.8663 | 0.8651 | 0.8639 | 0.9387 |  |

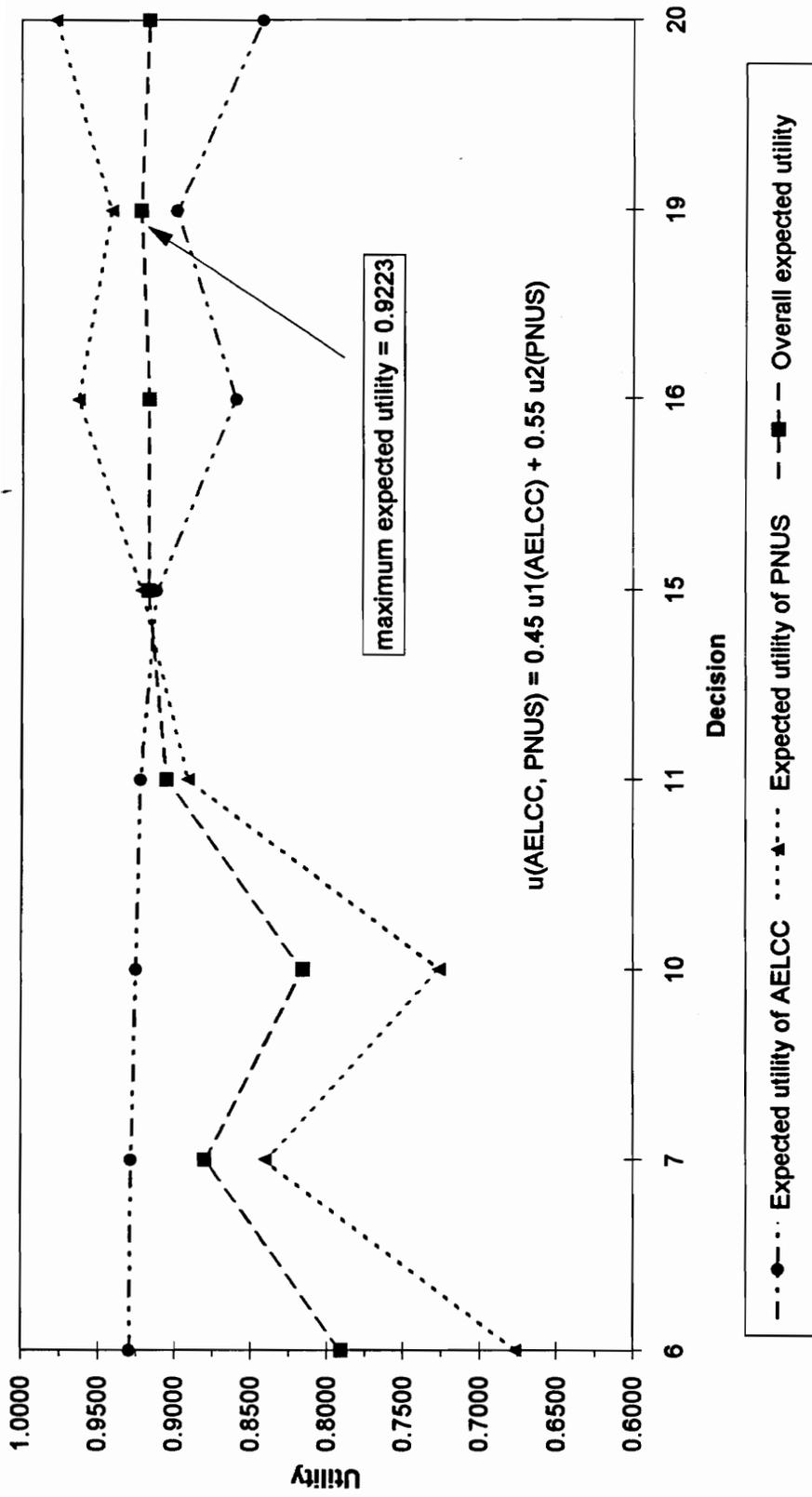


Figure 9.14. Expected utility of AELCC and PNUS, and overall expected utility (Alternative 1)

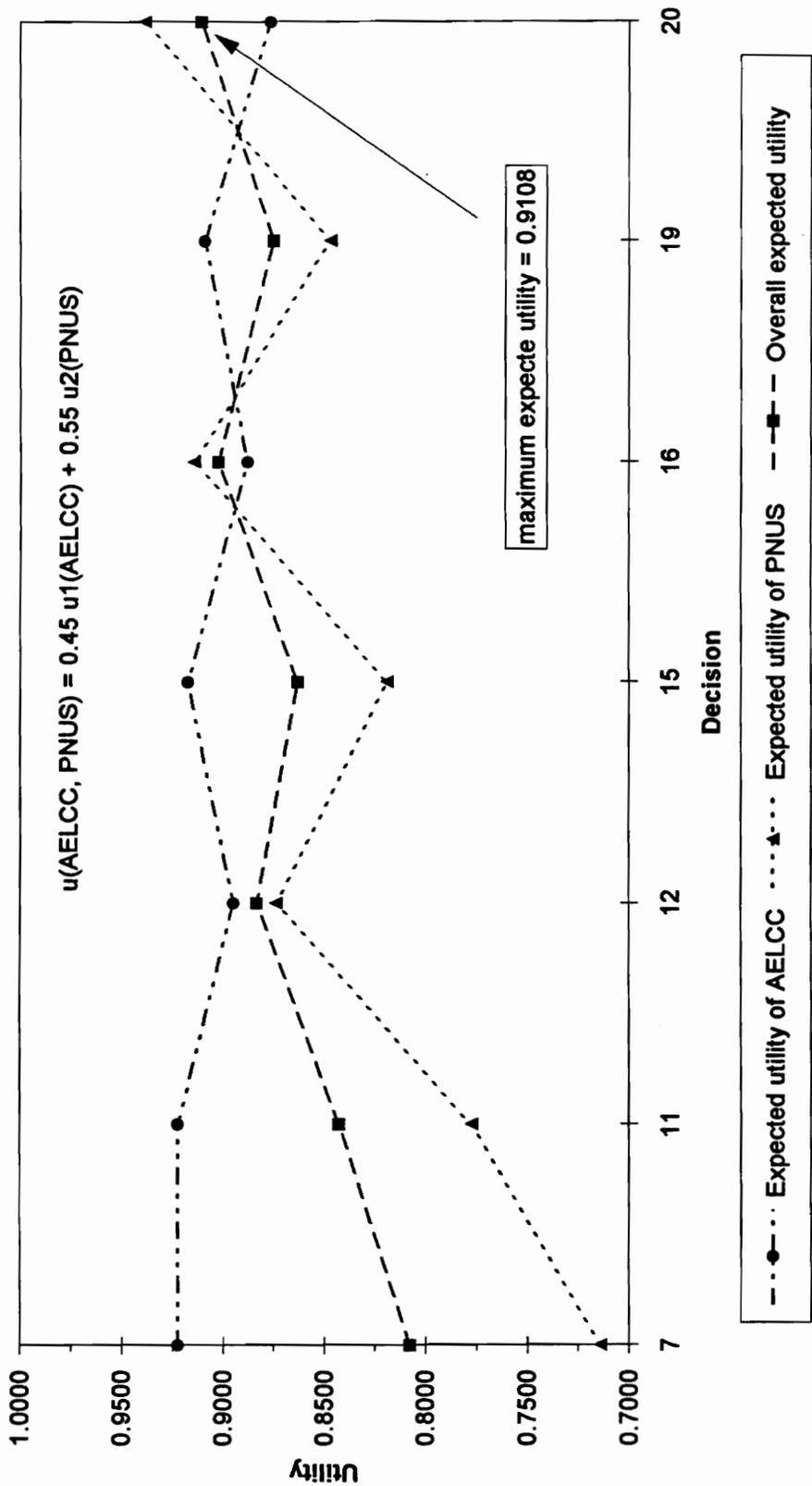


Figure 9.15. Expected utility of AELCC and PNUS, and overall expected utility (Alternative 2)

**Table 9.12. Characteristics of the Optimal Decision (Decision 19, Alt. 1)**

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|                                      |                 |
|--------------------------------------|-----------------|
| <b>Population size</b>               | <b>22</b>       |
| <b>Number of repair channels</b>     | <b>4</b>        |
| <b>Unit acquisition cost</b>         | <b>\$52,000</b> |
| <b>Unit design life</b>              | <b>4 years</b>  |
| <b>Unit salvage value</b>            | <b>\$22,000</b> |
| <b>Unit operating cost</b>           | <b>\$1,750</b>  |
| <i>MTBF</i>                          | 0.2550          |
| <i>MTTR</i>                          | 0.0425          |
| <b>Probability of no units short</b> | <b>0.9241</b>   |

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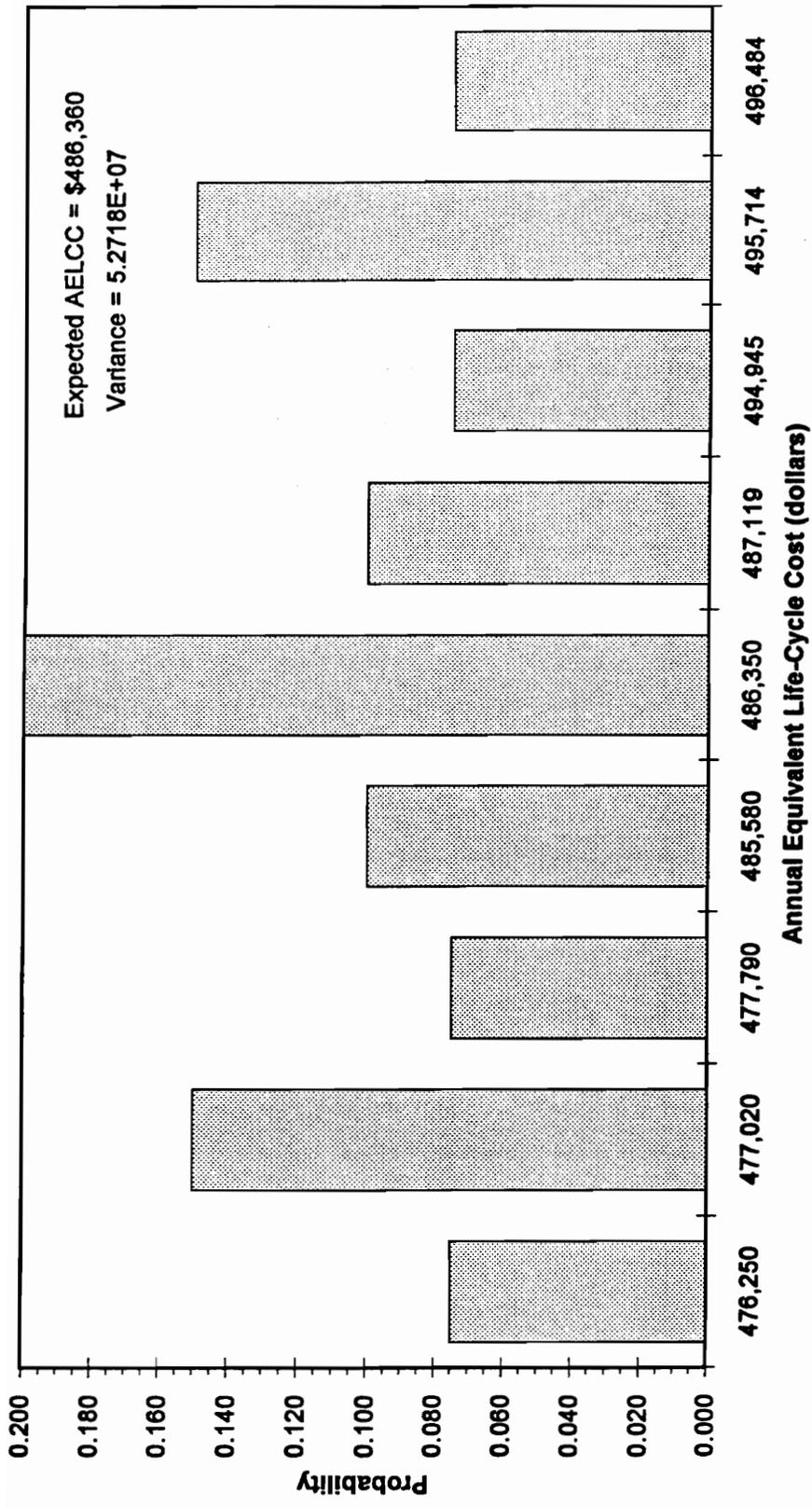


Figure 9.16. Probability distribution of AELCC for the optimal decision (Alternative 1: population size = 20, number of repair channels = 4)

## **X. SUMMARY, CONTRIBUTIONS, AND EXTENSIONS**

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|      |               |
|------|---------------|
| 10.1 | Summary       |
| 10.2 | Contributions |
| 10.3 | Extensions    |

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### **10.1 Summary**

Many problems associated with unsatisfactory system performance and excessive life-cycle cost are the direct result of decisions made during the early phases of system design and advanced planning. To develop quality systems, both engineering and management require fundamental principles and methodologies to guide design decision making and advanced planning. In order to provide for the efficient resolution of complex design decisions involving uncertainty, human judgments, and multiple attributes, a systematic decision analysis framework is needed.

The goal of this research is to develop a unified decision analysis framework to support the need and requirement for developing better system designs in the face of uncertainty. To accomplish this goal, the research is divided into seven parts:

- 1) The process of system design and development is examined from the perspective of concurrent life-cycle engineering. Elements of the decision process are identified. The Design Dependent Parameter Approach, an important paradigm for design analysis and evaluation, is invoked.

- 2) Types of uncertainty involved in the process of engineered system design are identified. The concept of robust system design is then defined from the perspective of life-cycle engineering. Two operational definitions of robustness are given based on the Design Dependent Parameter Approach. Some common measures for assessing the robustness of candidate system designs are identified. After a brief review of the existing approaches to design analysis and evaluation, the focus of this research is defined.
- 3) The problem of design evaluation in the face of uncertainty is studied within the context of decision theory. After classifying design decision problems into four categories, these problems are structured and modeled by decision trees. Then the concept of choices, preferences, and utility theory are discussed from the perspective of engineered system design. Based upon statistical decision theory, three decision analysis approaches are identified for design evaluation in the face of uncertainty. They are: (1) sequential decision analysis using the maximum expected utility principle, (2) stochastic dominance, and (3) mean-variance analysis. Under the context of statistical decision theory, the assumptions underlying some objective functions commonly used in design optimization are also clarified.
- 4) The decision analysis approaches identified and other effective approaches are integrated into a structured, systematic approach for resolving design decision problems under uncertainty. Structured models are developed for design analysis and design evaluation.
- 5) A hypothetical bridge design example is presented to demonstrate the concepts underlying the decision analysis framework. This example illustrates the application of the framework for a single evaluation attribute case for a simple static system.
- 6) The problem of multiattribute design evaluation in the face of uncertainty is investigated. Descriptive approaches and Multiattribute utility analysis are integrated to resolve design decision problems involving both uncertainties and multiple attributes.

- 7) An example from repairable equipment population system design is presented to demonstrate the application of the framework for multiattribute design evaluation in the face of uncertainty.

## **10.2 Contributions**

The major contribution of this research is the adaptation and integration of statistical decision theory, elements of the systems engineering process, and Taguchi's philosophy of robust design for design decision analysis in the face of uncertainty. As a result, a structured, systematic methodology is developed and presented for evaluating system design alternatives.

The following findings were obtained from this research:

- 1) By investigating the concept of robust design from the perspective of system life-cycle engineering, a general definition is presented for the robustness of system designs:

In system design, robustness expresses the insensitivity of the system's performance to uncertainties in both the system acquisition phase and the system utilization phase.

To facilitate the application of the concept of robust system design in design analysis and evaluation, two operational definitions are presented: (1) Robustness represents the insensitivity of the system's evaluation attribute(s) to the uncertainty in uncontrollable (design-independent) parameters, and (2) robustness represents the insensitivity of the system's evaluation attribute(s) to uncertainties in design-independent parameters as well as variations in design variables and design-dependent parameters.

- 2) The foundations for design evaluation in the face of uncertainty are studied within the context of statistical decision theory. This research indicates that design evaluation in the face of uncertainty is actually a problem of decision making under uncertainty. This concept more accurately encompasses the

totality of Taguchi's ideas. Instead of focusing on certain parts of the design decision process, such as experimental design, the decision analysis approach emphasizes the overall design decision process.

- 3) There are a variety of decision rules (or objective functions) used to resolve problems associated with design evaluation in the face of uncertainty, including the probability of loss criterion, maximization of expected value, Taguchi's loss functions and signal-to-noise ratios, etc. Before applying these rules in design evaluation, the assumptions underlying each rule must be examined carefully. From the perspectives of statistical decision theory, the foundations and assumptions are identified in this research for these commonly used decision criteria in design evaluation. Some confusion and controversy which surround Taguchi's loss function and signal-to-noise ratios are clarified. The results indicate that each of these rules rely on some strong assumptions about the decision maker's preferences and risk attitudes.
- 4) To identify a best system design in the face of various uncertainties, one must understand what the "best" solution is. Results of the research indicate that three factors of system design decision problems need to be considered in design evaluation: (1) performance variations, (2) risk attitudes, and (3) value trade-offs.

Experience indicates that, when uncertainty exists, use of the mean as the decision criterion for design evaluation may result in a poor design. Attempts to minimize the variation of the evaluation attribute have led to the philosophy of robust design. Taguchi's approaches focus on variance minimization. However, a design which generates a minimum variance for the evaluation attribute is not necessarily the best design. Variations represent the risks involved in the process of design evaluation. In comparing various candidate designs, different decision makers may not have identical risk attitudes. Instead of concentrating on performance variability alone, this subjective nature of the decision maker must be considered in order to select a best design.

Taguchi's robust design approach is often used by considering only a single attribute. In many cases, to identify the best design, designers have to consider more than one attribute. A design which is optimal for individual attributes of a system may not be best overall. Thus, if there exists more than one evaluation attribute, value trade-offs among these attributes must be considered.

To resolve system design decision problems under uncertainty, one must consider performance variations, risk attitudes, and value trade-offs jointly. A best design is not only robust for an individual attribute, but also provides an optimal trade-off among various attributes of concern. In this sense, the "best" design is subjective. It depends upon the value preferences and risk attitudes of the decision maker.

- 5) The research approach used herein placed emphasis on the need for visibility and quantification of uncertainty and the judgmental factors involved in major decisions within the design process. Following the concurrent life-cycle engineering design philosophy, a structured approach was taken to quantify uncertainties, value trade-offs, and expected gains and losses during the system life cycle. By making these factors visible and quantitative, design decisions can be improved, since this not only results in logical consistency in the treatment of decision elements, but also facilitates the communication and review of such factors as part of the total design decision process. Such a systematic approach to design analysis and evaluation can help the designer evaluate more alternatives in less time, and also provides more information about the performance of each of those alternatives.
- 6) Based upon utility theory, this research indicates that a numerical scale exists to measure the desirability of system designs. Instead of concentrating on performance variability alone, the overall objective of design evaluation should be that of maximizing expected utility rather than just minimizing variation. Expected utility provides for three measures of each evaluation attribute: (1) the values of the evaluation attribute, (2) the

probability distribution of the evaluation attribute, and (3) the utility of the evaluation attribute. Thus, information concerning uncertainty (made explicit by probability measures) and relative worth (made explicit by utility functions) is combined into a rational and theoretically sound decision rule for design evaluation – maximizing expected utility.

A utility function represents an objective function resulting from adapting decision and utility theory to the needs of decision making for system design. A utility function which incorporates value trade-offs and designer's preferences is more general and complete than Taguchi's loss function. Taguchi's loss function and signal-to-noise ratios are just special cases of utility functions. Utility functions may apply to any single evaluation attribute or set of evaluation attributes. During the early stages of system design, it is important to identify the decision rules for evaluating various design alternatives. With the help of an accurately defined utility function, the decision maker is able to consider system life-cycle costs and the cost of selecting a particular alternative. Optimization methods and Taguchi's parameter design approach can only be used after the utility function has been defined.

In design evaluation, utility is simply a reflection of the resulting costs and rewards from each candidate design. In the process of design decision making, the importance for undertaking utility analysis is due to that the complexity arising from uncertainty associated with the alternatives being so great that the decision maker feels unsure of which choice to make. That is, the decision maker realizes that the choice revolves around his preferences as they relate to taking risks, that his feelings toward risk are not entirely clear in his own mind, and that he cannot informally apply his feelings using direct choice. Thus, utility analysis can help the decision maker clarify the difficulty. The potential benefit is that a decision can be made which is consistent with his attitude toward risk.

- 7) A unified decision analysis framework is developed for making design decisions in the face of uncertainty. This framework integrates sequential

decision analysis, utility theory, elements of the system engineering process, and Taguchi's philosophy of robust design. Three effective approaches are identified in the framework: (1) maximization of expected utility, (2) stochastic dominance, and (3) mean-variance analysis.

Design decision problems in the face of uncertainty are made up of decisions and uncertain events. The structure of a decision problem in terms of the sequence and causal relationships between various decisions and uncertain outcomes can be effectively represented by a decision tree. By integrating sequential decision analysis with utility theory and the Design Dependent Parameter Approach, the implementation of design analysis and evaluation becomes more structured and systematic.

The decision analysis framework presented herein is useful for making design decisions during the early stages of system design and development. It is more systematic and complete than Taguchi's parameter design approach, since it is capable of dealing with design decision problems involving both uncertainty and multiple attributes. It applies to both discrete decisions and continuous decisions. The approach facilitates the integration of performance-related characteristics and logistic support requirements in system design. It may be applied at the macro level for the evaluation of candidate systems, or at the micro level for design iteration.

This approach is offensive in that it does not remove uncertainty. The effect that uncertainty has on the relative desirability of design alternatives is incorporated into the design evaluation process. Among other benefits of this approach are increased objectivity, less risk of overlooking significant factors, and perhaps most importantly, the ability to reconstruct the selection process rather than invoking intuition in explaining the alternative selected. Because the results are quantitative, evaluators can conduct sensitivity and "what if" analyses at an early stage in system design to determine the robustness of the results and to identify key factors that can affect the results.

In summary, this research (1) lays down theoretical foundations for employing and developing more efficient techniques for system design evaluation, (2) integrates Taguchi's philosophy of robust design and traditional design approaches, (3) streamlines system design evaluation efforts and resolves much confusion and controversy surrounding Taguchi's approaches, and (4) helps develop strategies for dealing with a broader range of decision problems pertaining to system design and development.

### **10.3 Extensions**

There are several opportunities for further research related to system design analysis and evaluation in the face of uncertainty. Additional work may be done by extending the work presented in this dissertation. Areas identified for additional study include the following:

- 1) As indicated in this research, uncertainties are associated with both design-independent parameters and decision variables. The focus of the research is on the uncertainty involved in the design-independent parameters. The variations in the value of decision variables are not investigated. The effect of variations in the decision variables on system design are usually evaluated by sensitivity analysis. Further research needs to be done on how to integrate the methodology presented herein with sensitivity analysis approaches.
- 2) To facilitate the application of the approach for design evaluation under uncertainty, a series of computer programs could be developed. Some examples include: (1) programs for assessing utility functions of various evaluation attributes of engineered systems, (2) programs for quantifying various uncertainties and assessing joint probability distributions of an evaluation attribute, (3) programs for automating the process of conducting stochastic dominance and mean-variance analysis for engineered systems, and (4) programs for documenting and presenting the results of analysis and evaluation involving various uncertainties. Integration of the evaluation

approach with CAE/CAD tools may increase design productivity, and provide technical capabilities needed to dramatically influence the decision process during system design evolution.

- 3) If the utilities and/or the probabilities of the evaluation attribute(s) are known only approximately, they can be represented as fuzzy numbers (Whalen and Bronn, 1982). Fuzzy expected utilities can be calculated by the extension principle of fuzzy mathematics; this process reduces to ordinary arithmetic when the operands are crisp. Further research is needed to integrate the approaches presented herein with fuzzy utility analysis.
- 4) In identifying and developing approaches for design evaluation in the face of uncertainty, the emphasis herein has been on discrete decisions. More research is needed for resolving problems of continuous decisions, including developing systematic procedures for the definition of objective functions, problem formulation, problem solution, and multiattribute sensitivity analysis.
- 5) As indicated herein, there are many approaches available for design analysis and evaluation. In order to efficiently evaluate design alternatives in the face of uncertainty, there is a need to develop an expert system for selecting appropriate approaches for different types of design decision problems. Such an expert system may help designers find the most efficient and effective ways to resolve decision problems.
- 6) Most decision models and approaches presented herein assume that a single decision maker exists. Further research is needed to adapt the approaches and models for group decision making situations.

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## APPENDICES

### A.1. Axioms of Utility Theory

**Comparability.** A DM can order (establish preference or indifference) any two outcomes. That is, either  $A_i \succ A_j, A_j \succ A_i$ , or  $A_i \sim A_j$ .

**Transitivity.** The ordering of outcomes is transitive. That is, if

$$A_i \succ A_j, A_j \succ A_k, \text{ then } A_i \succ A_k.$$

**Reduction of compound uncertain events.** The DM is indifferent between a compound uncertain event and the simple uncertain event determined by reduction according to the rules of the probability calculus.

**Continuity.** For each outcome  $A$ , the DM is indifferent between the outcome and some uncertain event (lottery) involving only two basic outcomes –  $A_1$ , which is better than  $A$ , and  $A_2$ , which is worse than  $A$ .

This assumption suggests that the DM can always find  $p$ , the probability of obtaining  $A_1$  in Figure A.1 such that he is indifferent between  $a_1$  and  $a_2$ .

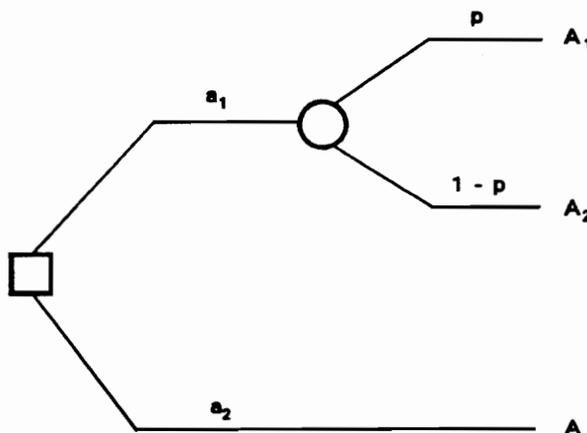


Figure A.1 Illustration of the Continuity Axiom

**Substitutability.** The DM is indifferent between any original uncertain event and one formed by substituting, for some outcome  $A$ , an uncertain event that the DM has judged to be equivalent to the outcome  $A$ .

**Monotonicity.** For the two uncertain events given in Figure A.2, event  $E_1$  is preferred to  $E_2$  if and only if  $p_1 > p_2$ , where  $A_1 \succ A_2$ .

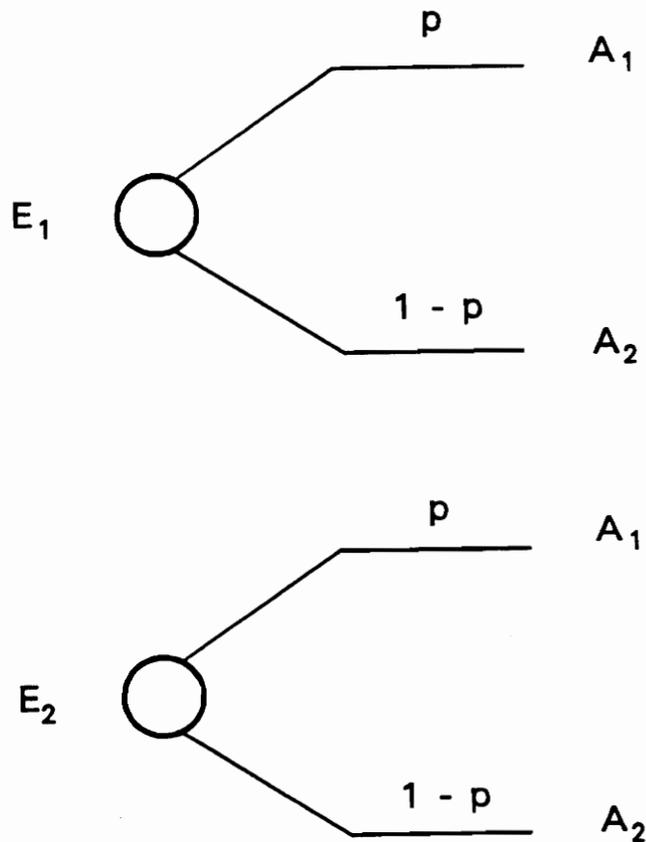


Figure A.2. Illustration of the Monotonicity Axiom

## A.2. Program Listing of repsmod.c

```
/* reps.c */
/* Definitions

    Arrays:

        ir[]           interest rate
        spc[]         shortage penalty cost per unit short per year
        p[]           probability of n failed units
        Cn[]          coefficient Cn

    Variables:

        AELCC         annual equivalent life-cycle cost
        PNUS          probability of no units short
        CP            annual equivalent population cost
        CO            annual operating cost
        CR            annual repair facility cost
        CS            annual shortage cost
        FC            first cost of a unit (P)
        kcp           annual equivalent cost per unit (Ci)
        kco           annual operating cost per unit
        kcr           annual repair cost per channel (Cr)
        life         life of the unit (life = replacement age, L, n)
        salv         salvage value
        D            demand (D)
        N            population size
        M            number of repair channels
        ES           expected number of units short (E(S))
        NIR          number of elements of IR[]
        NCS          number of elements of kcs[]
        LM           ratio of lambda and mu
*/
#include <stdio.h>
#define SIZE_ir 3
#define SIZE_spc 3
#define N_min 18
#define N_max 22
#define M_min 2
#define M_max 5

/* get inputs */
void getinput(int *, float *, int *, float *, float *, float *, float *, float *, float *);
/* print inputs */
void prtinput(int, float, int, float, float, float, float, float *, float *);
float getkcp(float, int, float, float); /* calculate kcp */
/* calculate prob and E(S) */
void getprob(int, int, int, float, float *, float *);
FILE *fptr;
```

```

main()
{
    float ir[SIZE_ir], spc[SIZE_spc];
    int N, M, D, life, count, j;
    float LM, CP, CO, CR, CS, kcp, kco, kcr, ES, FC, salv, AELCC, PNUS;

    printf("\x1B[2J");      /* clear screen */
    printf("\x1B[10;10f");
    printf("REPSMOD is running, be patient...");

    /* input */
    getinput(&D, &FC, &life, &salv, &kco, &kcr, &LM, ir, spc); /* get inputs from disk */

    fptro=fopen("output.dat","w"); /* open output file */
    prtinput(D, FC, life, salv, kco, kcr, LM, ir, spc); /* print inputs to disk*/
    fprintf(fptro, "SUMMARY OF RESULTS:\n\n");
    fprintf(fptro, " N  M  ir  spc  AELCC  PNUS  CP  CO  CR  CS
    ES\n\n");

    /* calculation process */
    for (N=N_min; N<=N_max; N++)
    {
        CO = kco*N;          /* annual operation cost */

        for (M=M_min; M<=M_max; M++)
        {
            CR = kcr*M;      /* annual repair facility cost */

            for (count=0; count<SIZE_ir; count++)
            {
                /* find annual equiv. cost per unit */
                kcp = getkcp(FC, life, salv, ir[count]);
                CP = kcp*N;    /* annual population cost */

                for (j=0; j<SIZE_spc; j++)
                {
                    /* find E(S) and Prob of no units short */
                    getprob(N, M, D, LM, &ES, &PNUS);
                    CS = spc[j]*ES; /* annual shortage cost */
                    AELCC = CO + CP + CR +CS; /* calculate total cost */
                    /* print outputs to the disk */
                    fprintf(fptro, "%5d %5d %7.0f %8.0f %10.0f %8.4f %8.0f
                    %8.0f %8.0f %8.0f %8.4f\n", N, M, ir[count], spc[j],
                    AELCC, PNUS, CP, CO, CR, CS, ES);
                }
            }
        }
    }
    fprintf(fptro, "\n ----- END OF RESULTS ----- \n\n");
    fclose (fptro);
    printf("\x1B[12;10f");
}

```

```

printf("The modeling process has been completed successfully.");
printf("\x1B[14;10r");
printf("Check file 'output.dat' for results.\n\n\n");

}

/* getinput() */
/* get inputs from the disk */
void getinput(int *pD, float *pFC, int *plife, float *psalv, float *pkco, float *pkcr,
float *pLM, float *ptr1, float *ptr2)
{
FILE *fptr;
int size;
if ( (fptr=fopen("input.dat","r"))==NULL)
{printf("Can't open file input.dat."); exit();}
fscanf(fptr, "%d %f %d %f %f %f %f", pD, pFC, plife, psalv, pkco, pkcr, pLM);
for (size=0; size<SIZE_ir; size++)
fscanf(fptr, "%f", ptr1++);
for (size=0; size<SIZE_spc; size++)
fscanf(fptr, "%f", ptr2++);
fclose(fptr);
}

/* prtinput() */
/* print inputs for verification */

void prtinput(int D, float FC, int life, float salv, float kco,
float kcr, float LM, float *ptr1, float *ptr2)
{
int size;

fprintf(fptr, "\nLIST OF INPUTS:\n\n");
fprintf(fptr, "Demand = %d\nFirst cost = %0.2f\nDesign life = %d\n", D, FC, life);
fprintf(fptr, "salvage value = %0.2f\nkco = %0.2f\nkcr = %0.2f\nLM = %0.7f\n",
salv, kco, kcr, LM);
for (size=0; size<SIZE_ir; size++)
fprintf(fptr, "ir(%d) = %8.1f", size, *(ptr1+size));
fprintf(fptr, "\n");
for (size=0; size<SIZE_spc; size++)
fprintf(fptr, "spc(%d) = %8.1f", size, *(ptr2+size));
fprintf(fptr, "\n-----\n\n");
}

/* getkcp() */
/* calculate annual equivalent cost per unit */
float getkcp(float FC, int life, float salv, float irr)
{
int n;
float ip1, AP;
ip1 = 1;
irr = irr/100.0;

```

```

    for (n=1; n<=life; n++)
        ip1 = ip1 * (1 + irr);
    AP = irr * ip1 / (ip1 - 1);    /* calculate A/P factor */
    return ( (FC-salv)*AP + salv*irr );
}

/* getprob() */
/* find expected number of units short and prob of no units short */
void getprob(int N, int M, int D, float LM, float *ptrn, float *ptrp)
{
    float C[30], p[30], sum;
    int count;
    C[0] = 1.0;

    for (count=1; count<=M; count++)    /* calculate Cn for n <= M */
        C[count] = C[count-1] * LM * (N+1-count)/count;

    /* calculate Cn for n > M */
    for (count=M+1; count<=N; count++)
        C[count] = C[count-1] * LM * (N+1-count)/M;

    /* calculate p0 */
    sum = 0.0;
    for (count=0; count<=N; count++)
        sum = sum + C[count];
    p[0] = 1.0/sum;

    for (count=1; count<=N; count++)    /* calculate pn */
        p[count] = p[0] * C[count];

    /* find prob of no units short */
    *ptrp = 0;
    for (count=0; count<=N-D; count++)
        *ptrp = *ptrp + p[count];

    /* find expected number of units short */
    *ptrn = 0;
    for (count=1; count <=D; count++)
        *ptrn = *ptrn + count * p[N - D + count];
}

```

## VITA

Chunming Duan was born on April 7, 1962 in Hunan Province, China. He received his B.S. and M.S. degrees in Mining Engineering from Central-South University of Technology (CSUT), Changsha, China in 1982 and 1985. Then he worked as a design engineer/research associate within the Materials Handling Research Center at CSUT.

In 1986, he was admitted for doctorate graduate study in the Department of Mining and Minerals Engineering at Virginia Polytechnic Institute and State University. After completing his Ph.D. in Mining Engineering in 1990, he decided to further enhance his interest and capability in systems engineering by becoming a candidate for the Ph.D. in Industrial and Systems Engineering.

During the last six years, Duan has been an investigator on three sponsored projects. He published ten research articles in the areas of systems analysis, robust design, and logistic support.