Precise Geolocation for Drones, Metaverse Users, and Beyond: Exploring Ranging Techniques Spanning 40 KHz to 400 GHz

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ABSTRACT

This dissertation explores the realm of high-accuracy localization through the utilization of ranging-based techniques, encompassing a spectrum of signals ranging from low-frequency ultrasound acoustic signals to more intricate high-frequency signals like Wireless Fidelity (Wi-Fi) IEEE 802.11az, 5G New Radio (NR), and 6G. Moreover, another contribution is the conception of a novel timing mechanism and synchronization protocol grounded in tunable quantum photonic oscillators. In general, our primary focus is to facilitate precise indoor localization, where conventional GPS signals are notably absent. To showcase the significance of this innovation, we present two vital use cases at the forefront: drone localization and metaverse user positioning.

In the context of indoor drone localization, the spectrum of applications ranges from recreational enthusiasts to critical missions requiring pinpoint accuracy. At the hobbyist level, drones can autonomously navigate intricate indoor courses, enriching the recreational experience. As a finer illustration of a hobbyist application, consider the case of “follow me drones”. These specialized drones are tailored for indoor photography and videography, demanding an exceptionally accurate autonomous flight capability. This precision is essential to ensure the drone can consistently track and capture its designated subject, even as it moves within the confined indoor environment. Moving on from hobby use cases, the technology extends its profound impact to more crucial scenarios, such as search and rescue operations.
within confined spaces. The ability of drones to localize with high precision enhances their autonomy, allowing them to maneuver seamlessly, even in environments where human intervention proves challenging. Furthermore, the technology holds the potential to revolutionize the metaverse.

Within the metaverse, where augmented and virtual realities converge, the importance of high-accuracy localization is amplified. Immersive experiences like Augmented/Virtual/Mixed Reality (AR/VR/MR) gaming rely heavily on precise user positioning to create seamless interactions between digital and physical environments. In entertainment, this innovation sparks innovation in narrative design, enhancing user engagement by aligning virtual elements with real-world surroundings. Beyond entertainment, applications extend to areas like telemedicine, enabling remote medical procedures with virtual guidance that matches physical reality.

In light of all these examples, the imperative for an advanced high-accuracy localization system has become increasingly pronounced. The core objective of this dissertation is to address this pressing need by engineering systems endowed with exceptional precision in localization. Among the array of potential techniques suitable for GPS-absent scenarios, we have elected to focus on ranging-based methods. Specifically, our methodologies are built upon the fundamental principles of time of arrival, time difference of arrival, and time of flight measurements. In essence, each of our devised systems harnesses the capabilities of beacons such as ultrasound acoustic sensors, 5G femtocells, or Wi-Fi access points, which function as the pivotal positioning nodes. Through the application of trilateration techniques, based on the calculated distances between these positioning nodes and the integrated sensors on the drone or metaverse user side, we facilitate robust three-dimensional localization. This strategic approach empowers us to realize our ambition of creating localization systems that not only compensate for the absence of GPS signals but also deliver unparalleled accuracy
and reliability in complex and dynamic indoor environments.

A significant challenge that we confronted during our research pertained to the disparity in z-axis localization performance compared to that of the x-y plane. This nuanced yet pivotal concern often remains overlooked in much of the prevailing state-of-the-art literature, which predominantly emphasizes two-dimensional localization methodologies. Given the demanding context of our work, where drones and metaverse users navigate dynamically across all three dimensions, the imperative for three-dimensional localization became evident. To address this, we embarked on a comprehensive analysis, encompassing mathematical derivations of error bounds for our proposed localization systems. Our investigations unveiled that localization errors trace their origins to two distinct sources: errors induced by ranging-based factors and errors stemming from geometric considerations.

The former category is chiefly influenced by factors encompassing the quality of measurement devices, channel quality in which the signal communication between the sensor on the user and the positioning nodes takes place, environmental noise, multipath interference, and more. In contrast, the latter category, involving geometry-induced errors, arises primarily from the spatial configuration of the positioning nodes relative to the user. Throughout our journey, we dedicated efforts to mitigate both sources of error, ensuring the robustness of our system against diverse error origins. Our approach entails a two-fold strategy for each proposed localization system. Firstly, we introduce innovative techniques such as Frequency-Hopping Spread Spectrum (FHSS) and Frequency-Hopping Code Division Multiple Access (FH-CDMA) and incorporate devices such as Reconfigurable Intelligent Surfaces (RIS) and photonic oscillators to fortify the system against errors stemming from ranging-related factors. Secondly, we devised novel evolutionary-based optimization algorithms, adept at addressing the complex NP-Hard challenge of optimal positioning node placement. This strategic placement mitigates the impact of geometry-induced errors on localization
accuracy across the entire environmental space.

By meticulously addressing both these sources of error, our localization systems stand as a testament to comprehensive robustness and accuracy. Our methodologies not only extend the frontiers of three-dimensional localization but also equip the systems to navigate the intricacies of indoor environments with precision and reliability, effectively fulfilling the evolving demands of drone navigation and metaverse user interaction.
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GENERAL AUDIENCE ABSTRACT

In this dissertation, we first explore some promising substitutes for the Global Positioning System (GPS) for the autonomous navigation of drones and metaverse user positioning in indoor spaces. Then, we will make the scope of research more comprehensive and try to explore substitutes to GPS for autonomous navigation of drones in general, both in indoor environments and outdoors. For the first part, we make our small indoor GPS. Similar to GPS, in our system, a receiver onboard the drone or the metaverse user can receive signals from our small semi-satellites in the room, and with that, it can localize itself. The idea is very similar to how the well-known GPS works, with some modifications. Unlike the GPS, we are using acoustic ultrasound signals or some RF signal based on 5G or Wi-Fi for transmission. Also, we have more freedom compared to GPS because, in GPS, they have to transmit signals from far ahead distances, whereas, in our scenario, it is just a room in which we put all of our semi-satellite transmitters. Moreover, we can put them anywhere we want in the room. This is, in fact important, because the positions of these semi-satellites have a huge effect on the accuracy of our system. Also, we can decide how many of them we need to cover every point in the room and not have any blind spots. We propose our novel techniques for finding the optimal placement to improve localization accuracy. In GPS, they propose a technique that is suitable for the case of those satellites and their distance to the targets. Similarly, we offer our novel techniques to have a robust transmission against noise
and other factors and guarantee a localization scheme with high accuracy. All being said, our proposed system for indoor localization of drones and metaverse users in three dimensions has considered all the possible sources of error and proposed solutions to conquer them; hence a robust system with high accuracy in three-dimensional space.
Dedicated to my family.
As I sit down to write this section, a myriad of emotions courses through me, ranging from the initial trepidation to the eventual triumph. Yet, amid this whirlwind of feelings, one prevailing sentiment stands out – gratitude.

Reaching this point in my journey and completing my Ph.D. is not a solitary achievement; indeed, it owes everything to the important people in my life. First and foremost, my family, despite being thousands of miles away, has been a constant source of warmth, supporting me at every step. Moreover, my other families here in the USA have treated me as their own, providing care and encouragement. Moving forward, heartfelt thanks are due to my friends, colleagues, and, not least, my supportive advisors.

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Throughout these years, my interactions extended beyond colleagues to friends outside our group, professors, friends from previous schools, and so many more. While I can’t name everyone individually, I am grateful to each one of you.

Last but certainly not least, my family – mom, dad, sister, grandpa, grandma, uncles, and aunts – words cannot adequately express my gratitude. Your presence wasn’t just cheering; you were with me in every step of this process and carrying most of the load. I owe you the most, and I acknowledge that I can never fully repay this significant debt. The only thing I can do is be appreciative and grateful to all of you always and forever. Thank you so much.
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List of Abbreviations

2D  Two-Dimensional

3D  Three-Dimensional

3GPP  Third-Generation Partnership Project

6G  Sixth Generation

ADMM  Alternating Direction Method of Multipliers

AGV  Automated Guided Vehicle

AMF  Access and Mobility Management Function

AOA  Angle of Arrival

API  Application Programming Interface

AR  Augmented Reality

AV  Autonomous Vehicle

AWGN  Additive White Gaussian Noise

BPSK  Binary Phase Shift Keying

CBRS  Citizens Broadband Radio Service

CDF  Cumulative Distribution Function

CDMA  Code Division Multiple Access
<table>
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<tr>
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<th>Description</th>
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<tbody>
<tr>
<td>CFR</td>
<td>Channel Frequency Response</td>
</tr>
<tr>
<td>CID</td>
<td>Cell Identification</td>
</tr>
<tr>
<td>COTS</td>
<td>Commercial-Off-the-Shelf</td>
</tr>
<tr>
<td>CRB</td>
<td>Cramér-Rao Bound</td>
</tr>
<tr>
<td>CRLB</td>
<td>Cramér-Rao Lower Bound</td>
</tr>
<tr>
<td>CSI</td>
<td>Channel State Information</td>
</tr>
<tr>
<td>DL</td>
<td>Downlink</td>
</tr>
<tr>
<td>DLOS</td>
<td>Direct-Line-of-Sight</td>
</tr>
<tr>
<td>EA</td>
<td>Evolutionary Algorithm</td>
</tr>
<tr>
<td>emBB</td>
<td>Enhanced Mobile Broadband</td>
</tr>
<tr>
<td>FFH</td>
<td>Fast Frequency Hopping</td>
</tr>
<tr>
<td>FFT</td>
<td>Fast Fourier Transform</td>
</tr>
<tr>
<td>FH</td>
<td>Frequency Hopping</td>
</tr>
<tr>
<td>FH-CDMA</td>
<td>Frequency Hopping Code Division Multiple Access</td>
</tr>
<tr>
<td>FHSS</td>
<td>Frequency Hopping Spread Spectrum</td>
</tr>
<tr>
<td>FIM</td>
<td>Fisher Information Matrix</td>
</tr>
<tr>
<td>FTM</td>
<td>Fine-Time Measurement</td>
</tr>
<tr>
<td>GDOP</td>
<td>Geometric Dilution of Precision</td>
</tr>
<tr>
<td>GNSS</td>
<td>Global Navigation Satellite System</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
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</tr>
<tr>
<td>GPS</td>
<td>Global Positioning System</td>
</tr>
<tr>
<td>HDOP</td>
<td>Horizontal Dilution of Precision</td>
</tr>
<tr>
<td>HE</td>
<td>High-Efficiency</td>
</tr>
<tr>
<td>IMU</td>
<td>Inertial Measurement Unit</td>
</tr>
<tr>
<td>IRS</td>
<td>Intelligent Reflecting Surface</td>
</tr>
<tr>
<td>LBS</td>
<td>Location-Based Service</td>
</tr>
<tr>
<td>LIS</td>
<td>Large Intelligent Surface</td>
</tr>
<tr>
<td>LMF</td>
<td>Location Management Function</td>
</tr>
<tr>
<td>LOS</td>
<td>Line-of-Sight</td>
</tr>
<tr>
<td>LTE</td>
<td>Long-Term Evolution</td>
</tr>
<tr>
<td>LTF</td>
<td>Long Training Field</td>
</tr>
<tr>
<td>MIMO</td>
<td>Multiple-Input Multiple-Output</td>
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<tr>
<td>mMTC</td>
<td>Massive Machine Type Communications</td>
</tr>
<tr>
<td>mmWave</td>
<td>Millimeter-Wave</td>
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<tr>
<td>MUSIC</td>
<td>Multiple Signal Classification</td>
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<tr>
<td>N3IWF</td>
<td>Non-3GPP Interworking Function</td>
</tr>
<tr>
<td>NDP</td>
<td>Null Data Packet</td>
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<tr>
<td>NFV</td>
<td>Network Functions Virtualization</td>
</tr>
<tr>
<td>NGP</td>
<td>Next-Generation Positioning</td>
</tr>
</tbody>
</table>
NLOS  Non-Line-of-Sight

NP    Non-Polynomial

NR    New Radio

OFDM  Orthogonal Frequency-Division Multiplexing

OTDOA Observed Time Difference of Arrival

PDSCH Physical Downlink Shared Channel

PEB   Positioning Error Bound

PHY   Physical

PPDU  Physical Layer Protocol Data Unit

PRC   Phase Response Curve

PRS   Positioning Reference Signal

PTC   Phase Transition Curve

QoS   Quality of Service

RAN   Radio Access Network

RAT   Radio Access Technology

RF    Radio Frequency

RFID  Radio Frequency Identification

RIS   Reconfigurable Intelligent Surfaces

RSS   Received Signal Strength
RTT  Round Trip Time

SA   Standalone Access

SDN  Software-Defined Networking

SFH  Slow Frequency Hopping

SLAM Simultaneous Localization and Mapping

SNR  Signal-to-Noise Ratio

SoOp Signal of Opportunity

TB   Trigger-Based

TDOA Time Difference of Arrival

THz  Terahertz

TOA  Time of Arrival

TOF  Time of Flight

TRP  Transmission and Reception Point

UAV  Unmanned Aerial Vehicle

UE   User Equipment

UL   Uplink

URLLC Ultra-Reliable and Low Latency Communications

V2X  Vehicular to Everything

VDOP Vertical Dilution of Precision
VO  Visual Odometry

VR  Virtual Reality

Wi-Fi  Wireless Fidelity

WLAN  Wireless Local Area Network
Chapter 1

Introduction

In this dissertation, our primary focuses are directed toward self-localization for drones and high-accuracy positioning for metaverse users, particularly in contexts where GPS signals are absent or unreliable, such as indoor environments. The predominant emphasis is placed on ranging-based techniques. Our overarching objective is to propose a high-accuracy localization system that effectively addresses the challenges inherent to indoor environments. Our examination reveals that the localization errors within ranging-based techniques can be attributed to two primary sources: errors in ranging measurements and errors arising from the relative geometry between transmitters and receivers. The former is often linked to measurement equipment errors, noise, and multi-path fading. The latter, however, emanates from the spatial relationship between the drone and the reference beacons.

Our research endeavors encompass a multifaceted approach to mitigating these sources of error within ranging-based localization. By developing strategies that counteract error-inducing factors, we aim to provide robust, high-accuracy (centimeter-level) localization for drones navigating indoor spaces, and achieve an even higher level of accuracy (millimeter-level) for positioning users within the metaverse. Subsequently, we delve into addressing the error attributed to the relative geometry between transmitters and receivers. By introducing innovative solutions to minimize this form of error, we establish a comprehensive system for indoor drone localization.
1.1 Use Cases

1.1.1 Drone Applications

Unmanned Aerial Vehicles (UAVs), better known as drones, have embarked on an exceptional trajectory of growth, captivating enthusiasts and industry alike with their multifaceted applications. As they surge in popularity, these aerial agents have transitioned from being technological novelties to indispensable tools, spanning a spectrum of functions that range from vital missions to leisurely diversions. Drones have been called upon for missions as critical as surveillance at nuclear power plants and as groundbreaking as autonomous merchandise delivery for industry giants like Amazon, Google, and Facebook. These remarkable devices have extended their influence, supporting firefighters in hazardous rescues, serving as knowledgeable tour guides in museums, and bringing a new dimension to creative endeavors through captivating aerial photography and videography. Their versatility underscores the power of unmanned aerial systems across a multitude of scenarios.

In many of these versatile applications, the pursuit of autonomous flight emerges as a common thread, necessitating the capability for self-guided navigation. Autonomy allows drones to operate independently, a feature that proves invaluable across a range of contexts. The Global Positioning System (GPS) has proven effective in addressing self-localization and navigation needs in outdoor settings. However, this reliance on GPS becomes less tenable within confined indoor environments, characterized by a scarcity or absence of GPS signals. This predicament poses a formidable challenge to achieving seamless autonomous flight indoors.

The journey toward autonomous flight encompasses three pivotal steps: self-positioning, real-time trajectory tracking, and navigation command generation. While the latter phase of navigation holds relative simplicity, the initial two steps introduce intricate complex-
1.1. USE CASES

The task of achieving precise self-localization and positioning for a moving object in three-dimensional space, with the added demand for centimeter-level accuracy, presents a formidable challenge. This challenge is particularly pronounced in contexts where GPS signals are unreliable or unavailable, such as indoor environments.

1.1.2 Metaverse Applications

Of particular significance is the integration of localization advancements within the rapidly evolving metaverse. The metaverse, an emerging digital realm that intertwines virtual and physical realities, offers a diverse array of applications spanning leisure and serious pursuits alike. The metaverse’s immersive experiences encompass augmented and virtual reality (AR/VR) applications, reshaping industries as diverse as entertainment, education, healthcare, and real estate. This paradigm shift introduces a heightened demand for high-accuracy, real-time positioning, particularly as users seamlessly transition between physical and virtual realms. From augmented reality games that transform everyday spaces into interactive battlefields to virtual reality simulations that enable medical students to conduct intricate surgeries, the metaverse’s potential is vast. Central to these experiences is the capacity for precise localization, allowing virtual content to align seamlessly with the physical world.

Our research extends beyond the boundaries of drones, embarking on a dual mission to enhance both drone autonomy and metaverse experiences. By synergizing our advancements in ranging-based localization, we enable drones to navigate indoor environments with heightened accuracy. Simultaneously, we bolster metaverse applications by providing the high-accuracy real-time positioning essential for the seamless integration of virtual and physical spaces. Through the convergence of these endeavors, we pioneer a novel realm of possibilities, bridging the physical and virtual domains with unprecedented precision.
In the pages that follow, we embark on a comprehensive exploration of our contributions to the realms of drone autonomy and metaverse applications. Our work not only addresses the technological challenges of indoor self-localization but also lays the groundwork for transformative experiences within the metaverse. Through innovative solutions and cross-disciplinary synergy, we illuminate a path toward enhanced autonomy and integration in the ever-evolving landscape of technology.

1.2 Available Localization Solutions

The research landscape dedicated to addressing the aforementioned localization challenges is diverse and dynamic, encompassing a plethora of techniques and approaches. Vision-based methods offer promise in indoor self-navigation, leveraging visual sensors and computational power to determine drone positions in real time. Yet, the costs associated with high-quality visual equipment and computational resources impede their widespread adoption. Additionally, these methods often necessitate a pre-existing map of the environment and can be hampered by image blurring due to drone vibrations during flight. Although vision-based techniques stand as crucial solutions for metaverse localization, particularly in applications like hand-tracking for AR or VR applications, their performance is hindered in environments devoid of light. In other words, they are highly dependent on lighting conditions and are ineffective in visually impaired settings.

Another avenue of exploration lies in ranging-based localization techniques. These techniques establish a user’s position based on signal characteristics measured between known sensors and the user. Measurement metrics include time of arrival (TOA), angle of arrival (AOA), and received signal strength (RSS), among others. Techniques such as lateration, angulation, and fingerprinting are employed to determine the user’s position. The signals themselves can
span acoustic, radio frequency (RF), or other domains. For instance, Signals of Opportunity (SoOp), such as cellular signals, have been considered within this framework. Yet, challenges arise, as signals like these may be scarce within indoor spaces, and historically, the accuracy of this class of techniques has been limited, typically extending beyond tens of meters.

1.3 Contributions

The central objective of this dissertation is to establish comprehensive localization systems that operate effectively without reliance on GPS signals. This aim forms the cornerstone of our research. We have chosen to spotlight two prime exemplars: drones and metaverse users, as these applications currently enjoy significant prominence. Initially, our focus was directed towards drone navigation due to the fervent interest in drones at that time. Self-localization for drones was a particularly compelling subject. More recently, the metaverse has surged in popularity, prompting us to extend our efforts to deliver highly accurate localization for metaverse users. Notably, metaverse accuracy requirements proved even more stringent than those for drones, necessitating the use of distinct techniques.

It is essential to recognize that this dissertation primarily revolves around comprehensive ranging-based localization methodologies, which find utility across various applications. These systems are applicable wherever GPS signals are absent or inadequate, even when GPS signals are available but insufficiently accurate. Thus, our focus on drones and the metaverse is not intended to limit the applicability of our system. While tailored to address the complex challenges posed by drones and metaverse users’ constant three-dimensional movements, our approach remains adaptable for simpler scenarios with less demanding prerequisites.

Additionally, a significant objective of this study was to underscore the importance of the
second source of error. Existing state-of-the-art systems predominantly concentrate on refining localization by mitigating the first source of error through techniques like modulation improvements. Nonetheless, we have demonstrated that the second source of error holds substantial significance, highly impacting final three-dimensional localization accuracy, even when the primary source of error is effectively addressed and distance estimation is highly precise.

With this aim in mind, the five systems we introduce in this dissertation consist of two primary components. The initial facet of each system revolves around pioneering strategies to design systems that combat the first source of error. This results in reduced ranging error and the provision of high-precision distance estimation. The subsequent component of each system focuses on addressing the second source of error, aiming to minimize geometry-induced errors. This holistic approach ensures that our systems effectively tackle all error sources, resulting in exceptional accuracy.

Our systems encompass five distinct entities: PILOT, iDROP, OFDRA, Wi-Six, and IsoPos. The first two chapters revolve around acoustic ultrasound signals, designed to foster robustness against multipath effects. These chapters focus on indoor drone localization. PILOT relies on the Frequency Hopping Spread Spectrum (FHSS) technique for signal communication, while iDROP utilizes Frequency Hopping Code Division Multiple Access (FH-CDMA). Comparing acoustic and RF signals reveals unique advantages and disadvantages for each. Acoustic signals travel at a slower speed than RF, affording improved accuracy with modest clocks and equipment, and reduced interference. They are well-suited for secure environments.

However, acoustic signals suffer from limited range and narrow-beam sensors, necessitating numerous positioning nodes for practicality. RF signals, in contrast, offer greater range and omnidirectional propagation. With existing RF infrastructure in place, such as cellular sta-
1.3. Contributions

tions and Wireless Fidelity (Wi-Fi) routers, no additional installation is required, bolstering RF’s practicality. Advancements in technology, especially within 5G and beyond cellular networks, along with IEEE 802.11az Wi-Fi, have significantly enhanced distance estimation accuracy. Consequently, the forthcoming chapters delve into RF-based ranging.

The subsequent chapters present our innovative systems: OFDRA, Wi-Six, and IsoPos. In OFDRA, we propose an indoor localization system based on the 5G signal, leveraging 5G femtocells within a warehouse and integrating RIS onto drones to counter multipath and synchronization challenges. Moving to Wi-Six, we introduce a localization system where powerful 6G positioning signals dominate outdoor localization, supplemented by Wi-Fi routers atop the 6G network for indoor accuracy enhancement. Protocols facilitate cellular and Wi-Fi integration.

Lastly, IsoPos pushes boundaries by introducing a new localization system that establishes its timing mechanism. Built on the non-linear attributes of tunable quantum photonic oscillators, this system boasts high-resolution timing capabilities. The injection-locking technique on photonic oscillators ensures seamless synchronization. All five systems feature a communication model that addresses ranging-based error reduction, especially against noise, multipath, and synchronization issues. The latter part of each system’s architecture focuses on rectifying geometry-induced errors. This involves the formulation of algorithms tailored to specific system use cases, trilateration techniques, and deployment environments.

In summary, this dissertation is an odyssey into the realm of localization systems, driven by the imperative to conquer the challenges posed by the absence of GPS signals. This journey traverses the landscapes of drone navigation and the burgeoning metaverse, illuminating the evolving intersection of technology and the contemporary demands of our world. Exploring the intricacies of localization, the study unveils the elusive secondary source of error, casting a new light on the landscape. It introduces a suite of five ingenious systems - PILOT, iDROP,
OFDRA, Wi-Six, and IsoPos - each meticulously crafted to transcend error, delivering unmatched accuracy. From the resonating tones of acoustic signals to the pervasive reach of RF waves, each system is engineered with a singular aim: to withstand the challenges of error, adapting flawlessly to the complexities of drone flight and metaverse traversal. Anchored by a spirit of innovation, these systems harmonize technology, promising a future where the intricate interplay between positioning nodes, algorithms, and synchronized communication yields unprecedented precision. As the dawn of this transformative journey beckons, let us venture forth with eager anticipation into the depths of this dissertation’s narratives.

1.4 Related Work

Before we delve into the remaining chapters of this dissertation and explore them in detail, it is essential to provide a succinct overview of the relevant background and related research pertaining to the field. This contextual information proves valuable for the ensuing chapters of this dissertation. Commencing our exploration, we begin by shedding light on the realm of localization techniques, as detailed in 1.4.1. This elucidation not only unveils the available methods but also enriches our understanding through pertinent examples and intricate elucidations. Transitioning seamlessly, 1.4.2 delves into an insightful exposition on the auxiliary techniques that serve as the bedrock for ameliorating the challenges within ranging-based localization strategies. To be more specific, we embark on a comprehensive elucidation of Reconfigurable Intelligent Surfaces (RIS)–an integral technique we strategically harness to effectively address synchronization intricacies. Subsequently, our exploration widens its purview to encompass the concept of isochrons in photonic oscillators. A closer examination of this concept becomes the cornerstone of our innovative timing mechanism, an exploration that will be delved into more extensively in the forthcoming chapters. Finally, we explore the
1.4. RELATED WORK

background of the anchor placement techniques for the ranging-based localization systems in 1.4.3.

1.4.1 Localization Techniques

GPS is a prominent framework in the localization literature. It falls under the Global Navigation Satellite System (GNSS) umbrella and provides absolute location information in the form of latitude and longitude for any object that concurrently has access to at least four satellites. However, in crowded urban environments with tall skyscrapers that block the signals or indoor environments where the signal is weaker, it falls short of providing accurate localization. Numerous studies have investigated positioning in the absence of a GPS signal [2, 3, 4, 5, 6, 7, 8] and alternative techniques have been proposed for indoor localization [9, 10, 11, 12].

Vision-base

One scenario is where the target can localize itself using visual sensors (e.g., cameras) [13, 14, 15, 16, 17, 18, 19, 20] with methods such as Visual Odometry (VO) [21], Simultaneous Localization and Mapping (SLAM) [22, 23] and Optical Flow [24]. These methods can also be combined with some auxiliary techniques, such as a combination of visual methods with deep neural networks [25] or LiDAR [26]. However, in general, vision-based approaches are expensive both in terms of the necessary hardware infrastructure and computational complexity. Moreover, the accuracy of final location estimation often deteriorates due to blurring caused by the vibrations in the case of high-mobility targets such as drones.
Fingerprinting

Another substitute for GPS is using fingerprinting techniques [27]. Two well-known measurement options for this are the Channel State Information (CSI) [28, 29, 30] and Received Signal Strength (RSS) [31, 32, 33, 34, 35, 36]. These methods require additional time and computing resources to generate and maintain an offline map. This is not only a time-consuming task, but the final accuracy is sensitive to any real-time changes that have not been updated in the most recent rendition of the environment.

Ranging-base

Localization using the transmitted signals between an object and anchor node(s) is known as the ranging-based approach [37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48]. Depending on the type of sensors used, ranging methods can be based on acoustic signals [49, 50, 51] or Radio Frequency (RF) [52, 53, 54] signals. The former is typically used in short-range applications such as high-accuracy gesture tracking [49] where the distances between a target and the anchors are very small. Since the propagation speed of these signals is much slower compared to RF signals, they are easier to resolve [55]. On the other hand, acoustic signals do not scale well to larger indoor spaces because they suffer from bad coverage over longer distances, making them a poor choice for indoor localization.

Two pioneers in RF-based ranging techniques are cellular positioning [56], which excels outdoors due to its widespread coverage, and Wi-Fi positioning [54, 57, 58], well-suited for indoors due to its availability and adaptability to different environments. Cellular positioning, a concept dating back to the first generation, has evolved with each subsequent generation, leading to increasingly stringent requirements. Advancements in infrastructure and bandwidth have contributed to improved accuracy over time. Similarly, Wi-Fi posi-
tioning has progressed to the extent that ongoing projects like IEEE 802.11az are dedicated solely to achieving high-accuracy positioning within the Wi-Fi framework. These approaches cater respectively to outdoor and indoor settings, reflecting their suitability for different environments.

A localization technique can be based on a variety of measurement types such as time of arrival (TOA), angle of arrival (AOA), or time difference of arrival (TDOA). AOA estimation uses angulation as a localization technique [59]. It requires special antenna arrays [60] and high computing power to render the complex algorithms (e.g., Multiple Signal Classification (MUSIC)) needed for angle estimations. A simpler measurement method is the TOA-ranging approach. It can be used without requiring high processing power or any special antenna arrays. The measurements are translated to distance based on $d = c \times t$; where $d$ is the distance between the user and the anchor, $c$ is the propagation speed of the transmitted signal (speed of light for RF signals and sound speed for acoustic signals) and $t$ is the time of flight. Knowing the distances between multiple anchor nodes and the localization target, it becomes possible to make use of trilateration techniques to obtain the final location of a target. When dealing with TOA-based techniques, two key challenges arise: the need for precise time synchronization between anchors and the target, and the corrupting effect of multipath fading on TOA measurements due to variations in recording times at the receiver.

To solve the stringent time synchronization issue, time difference of arrival (TDOA) techniques have been proposed [61]. With this approach, instead of calculating the distance between the target and each of the anchors separately based on TOA information, the localization is conducted based on the differences in the distance by using hyperbolic lateration. While this may solve the tight synchronization requirement between a target and the anchor, there still exists a necessity for synchronization among the anchors, which can introduce errors in the final location estimation.
To adjust for this source of error, another approach is to use the round trip time (RTT) from the anchor to the target and back to the anchor, to preclude the need for tight synchronization. Instead of measuring TOA at endpoints (e.g., transmitted from the anchor node and measured at the target), with RTT the measurement occurs at the same node. Upon transmitting a signal, an anchor node awaits a response from the target in order to measure the TOA. Since the original transmitted signal and the received signal are processed at the same location, time synchronization is not required. For this reason, RTT-based ranging forms the foundation of our RIS-based localization framework.

1.4.2 Advanced Ranging-based Prerequisites

Reconfigurable Intelligent Surfaces

Wireless communication society has dealt with channel-induced challenges such as total obstruction of the transmission signal, strong noise and multipath fading. Recently, with the advancements of high-frequency communication links and the emergence of massive antenna arrays, RIS has become a realistic contender for augmenting 5G and 6G networks. RIS is a programmable surface where the reflection of the incident signal can be steered in a given direction \[62\]. This offers flexibility in indirectly controlling the propagation environment by changing the surface behavior of the RIS towards the inbound signals \[63, 64\].

RIS deployment traditionally takes place through the installation of a large reflective wall to provide communication signals to blind zones \[65, 66\]. In recent years, studies that use RIS for positioning purposes have emerged \[67, 68, 69, 70\]. Their goal is to control the propagation characteristics of the environment to the benefit of improving the positioning capabilities \[70\]. With higher frequency ranges, the size of the RIS patches is compact enough to be easily installed on smaller targets \[71\], which enables them to be used in novel ways
for positioning.

**Isochrons in Photonic oscillators**

Photonic oscillators consisting of a set of coupled semiconductor lasers can be ideally used as receiver’s clocks due to their remarkable frequency tunability as well as their compactness and implementation in photonic integrated circuits [72, 73]. A photonic oscillator is characterized by a self-sustained oscillation corresponding to a stable limit cycle of the underlying dynamical system governing its internal dynamics. The oscillation frequency is uniquely determined by the parameters of the system and may range from 100 MHz to more than 100 GHz [74, 75]. These oscillations are remarkably robust in terms of noise perturbations. Apart from their frequency spectrum, they are uniquely characterized by their isochrons’ structure that determines both their synchronization properties under a periodic signal from an external controller and their phase shift under a user-emitted pulse booting the photonic oscillator.

The notion of isochrons in oscillatory systems was originally introduced by Winfree [76] and has been heavily utilized in the context of mathematical biology and neuroscience [77]. In a technological context, later studies of electronic oscillators have utilized similar concepts and methods, but with different terminologies, without making explicit use of the notion of isochrons [78, 79], and only recently this notion has been introduced in the study of photonic oscillators in terms of their synchronization dynamics and the formation of frequency combs [80, 81, 82].
1.4.3 Optimal Sensor Placement

Finding the optimal geometry for the placement of the localization anchor nodes is a well-investigated problem in the positioning literature [83, 84, 85, 86] and wireless networks [87, 88, 89]. Most of the studies focus on finding the minimal number of anchor nodes required for positioning [90, 91]. However, fewer studies investigate the effect that the node placement can have on the final accuracy [83]. For the first issue, the different types of medium access technologies provided by the sensors can play a significant role. In cases where the system is based on low-power Bluetooth sensors, the transmission is omnidirectional and coverage is restricted by the distance and obstacles [92]. On the other hand, with ultrasound-based receivers, the beam angle of the sensors makes it difficult to find the number of nodes and their placement [90]. Optimizing the number of required anchors is mostly used for finding the number of sensors for large indoor environments with different stories and rooms. After finding the number of anchors, a second optimization problem is formulated to find the placement of sensors to minimize the localization error induced by the relative geometry between the transmitter and anchors [85]. This relative geometry between the user and the anchor nodes of the proposed localization system can significantly affect the final accuracy [83, 93].

1.5 List of Publications

The research conducted in this dissertation has resulted in the publication of several papers. In the following, I will present the papers for which I am the primary author.

Journals:

1.5. List of Publications


Conference Proceedings:


Chapter 2

PILOT: High-Precision Indoor Localization for Autonomous Drones

In many scenarios, unmanned aerial vehicles (UAVs), aka drones, need to have the capability of autonomous flying to carry out their mission successfully. In order to allow these autonomous flights, drones need to know their location constantly. Then, based on the current position and the final destination, navigation commands will be generated and drones will be guided to their destination. Localization can be easily carried out in outdoor environments using GPS signals and drone inertial measurement units (IMUs). However, such an approach is not feasible in indoor environments or GPS-denied areas. In this chapter, we propose a localization scheme for drones called PILOT (High-Precision Indoor Localization for Autonomous Drones) that is specifically designed for indoor environments. PILOT relies on ultrasonic acoustic signals to estimate the target drone’s location. In order to have a precise final estimation of the drone’s location, PILOT deploys a three-stage localization scheme. The first two stages provide robustness against the multi-path fading effect of indoor environments and mitigate the ranging error. Then, in the third stage, PILOT deploys a simple yet effective technique to reduce the localization error induced by the relative geometry between transmitters and receivers and significantly reduces the height estimation error. The performance of PILOT was assessed under different scenarios and the results indicate that PILOT achieves centimeter-level accuracy for three-dimensional localization of
drones.

2.1 Chapter Overview

Over the past few years, the global drone industry has grown progressively and the number of applications in which civilian drones play an essential role in both indoor and outdoor environments has therefore increased. There are a wide range of indoor drone applications today, ranging from recreational use cases to essential safety-of-life use cases. Examples include shipping and delivery of packages, aerial photography for capturing footage and geographical mapping, providing temporary cellular coverage in case of disasters, reconnaissance inside nuclear power plants, helping firefighters locate individuals inside burning buildings, and security surveillance inside large building complexes among others.

Drones need to fly fully or partially autonomously in all of the above applications to carry out their mission successfully. To allow such fully or partially autonomous flights, the ground control station or any other infrastructure that supports the operation of the drone needs to continuously localize and monitor the drone’s position and send this information to the navigation controller of the drone to provide the capability of autonomous navigation for the drone. Localization and monitoring the drone’s position can be easily carried out in outdoor environments using GPS and the Inertial Measurement Units (IMUs) of the drone without seeking aid from the ground controller or any other extra infrastructure to perform this task. Such an approach is, however, not feasible in indoor environments or GPS-denied areas.

Vision-based approaches are widely used for drone localization when GPS is not accessible (e.g., [21]). However, because of the vibration of the drone during flight, the accuracy of current vision-based methods is generally limited. In addition, in vision-impaired conditions (e.g., low light conditions or blockage of the line of sight), the accuracy will further deterio-
rate. In addition to all the mentioned issues, vision-based approaches are expensive both in terms of the hardware cost and the computational complexity.

In addition to vision-based methods, there are other techniques, including, Doppler-shift-based tracking (e.g., [100]), fingerprinting-based localization using received signal strength (RSS) or channel state information (CSI) (e.g., [30, 34, 35]), and localization using cellular networks (e.g., [101]). Doppler-shift-based tracking has limitations; by itself, it does not provide sufficient tracking accuracy. Fingerprinting-based techniques are too vulnerable to any changes that happen in the environment after the off-line phase of collecting the RSS or CSI information; hence, they are not capable of providing high accuracy localization. With the current technology, localization using cellular networks has accuracy in the order of tens of meters and cannot provide centimeter-level high accuracy localization. In addition, there is a good chance of having weak cellular coverage in indoor environments, which itself degrades the accuracy drastically.

Another well-known class for localization in the absence of a GPS signal is the ranging-based method. This category uses the concept behind GPS and makes an indoor positioning system that resembles how localization using GPS works, i.e., there are some fixed beacons in the room (similar to the satellites) and the localization task gets done by performing some measurement techniques on the communicated signals between these beacons and the target object. Ranging-based methods either use Radio Frequency (RF) signal (RF-based localization, e.g., [54]) or they are working based on acoustic signals (acoustic-based localization, e.g., [6]). The major problem with ranging-based methods is the degradation of localization accuracy due to (i) multi-path fading effect and (ii) the relative geometry between the transmitter(s) and receiver(s).

In this chapter, we propose a three-dimensional localization scheme for drones in GPS-denied indoor environments that is referred to as High-Precision Indoor Localization for
Autonomous Drones (PILOT). PILOT is based on the ranging-based localization approach with significant enhancement to provide centimeter-level localization accuracy for drones in indoor environments. PILOT uses ultrasound acoustic-based signals for localization. We claim that localization based on acoustic signals provides a variety of advantages over approaches based on RF. Compared to RF signals, the significantly slower propagation speed of acoustic signals allows for higher localization accuracy without the need for having expensive equipment with a high sampling rate. PILOT uses high-frequency acoustic signals, referred to as ultrasound, to prevent any interference with the propeller-generated noise of the drone or human-generated noise in the environment.

Despite most of the available indoor positioning schemes that focus merely on two-dimensional localization, PILOT not only offers localization in three dimensions, but it also proposes additional solution to alleviate the estimation error in Z-axis. Moreover, to provide an accurate location estimation for drones, PILOT leverages some techniques to counter the multi-path fading effect of indoor environments. To the best of our knowledge, PILOT is the first to make completely different and innovative use of these techniques and proposes novel improvements on them to provide a three-dimensional positioning scheme that is robust against the multi-path fading effect and offers centimeter-level accuracy for location estimation of drones in indoor environments without any dependency on GPS signals. The summary of our contributions is as follows.

- We propose PILOT, a high-accuracy three-dimensional localization scheme for drones in indoor environments that is highly resilient to noise and multi-path fading and achieves high accuracy by employing three stages.

- PILOT overcomes the multi-path fading effect of the indoor environments by employing the Frequency Hopping Spread Spectrum (FHSS) technique for signal communication, which to the best of our knowledge, is a novel approach for localizing a drone in three-dimensional
• PILOT mitigates the localization error and provides further accuracy compared to the merely ranging-based schemes by leveraging the Doppler-shift effect to measure the velocity of the drone and designs a Kalman filter for velocity and distance incorporation.

• PILOT leverages an additional ultrasound transceiver and uses the reflected ultrasonic signal bounced from the ceiling to measure the drone’s height separately and combine it with the available height measurements from the first two stages in a filter to improve the Z-axis estimation accuracy.

• Our comprehensive simulation and experimental results indicate that PILOT achieves a three-dimensional localization error of less than 1.2 cm. Also, it significantly reduces the Z-axis estimation error compared to the prior state-of-the-art.

The remainder of this chapter is structured as follows. In the next section, we will review some of the relevant works in this area. Then, in section 2.2, 2.3, and 2.4, which are the core sections of this chapter, we will thoroughly explain our scheme for localizing a drone in indoor environments. Followed by these sections, in section 2.5, we will describe our simulation test setup and showcase the preliminary performance assessment results on PILOT. Next, in section 2.6, we will be investigating the reason behind having a bad Z-axis estimation compared to the X – Y plane and we will be providing our solution to rectify this issue. Next, in section 2.7, first, we describe our experimental test setup and then provide the final evaluation results on PILOT. Finally, in section 2.8, we conclude our work.
2.2 Robust Ranging using FHSS Ultrasound Signals

To accomplish fully or partially autonomous drone navigation, the most critical task is constant localization of the drone in three dimensions with high accuracy. We explain how PILOT performs a distance estimation that is highly robust against the noise and the multi-path fading effect of an indoor environment in this section, and in the next two sections, we will discuss how it further improves the accuracy of distance estimation and performs the three-dimensional localization.

As discussed earlier, the ranging-based method is an excellent alternative for localization in the absence of GPS signals. To briefly explain this class of localization, it is required to know the three major concepts: (i) signals to be measured for ranging, (ii) the measurement method, and (iii) the localization technique. Examples of signals deployed for the measurements are RF, acoustics, ultrasound, visible light, and others. Well-known measurement methods for ranging-based localization include angle of arrival (AOA), time of arrival (TOA), time difference of arrival (TDOA), received signal strength (RSS), etc. Finally, techniques for location estimation are angulation, lateration, and fingerprinting.

RF signals propagate in the speed of light (approximately $3 \times 10^8 \, (m/s)$), which is orders of magnitude faster than the acoustics or ultrasound which they travel in the speed of sound (approximately 340 $(m/s)$); therefore, for having a high accuracy localization, RF-based systems require more expensive equipment with high sampling rate. In addition, RF signals travel through walls and ceilings, causing interference with the localization system. Finally, there are places (e.g., hospitals, military bases, etc.) where RF signals are prohibited. All being said, to provide high accuracy localization with the minimum hardware and computational cost, PILOT leverages the acoustic waveform as the signal to measure. In addition, to avoid any interference with human or propeller-generated noise in the audible ranges of
acoustic signals, PILOT uses the higher frequencies known as *ultrasound*.

AOA and angulation require special antenna arrays and applying some computationally complicated techniques such as the multiple signal classification (MUSIC) [51] which incurs high complexity calculations and makes the approach expensive both in terms of hardware cost and processing power. Merely RSS, in which the distances between the target object and the beacons are calculated based on the received power and the path loss formulas, suffer from poor accuracy. To solve this, RSS-fingerprinting methods in which the localization takes place in two steps are offered. In the first step, called the offline phase, an RSS map of the entire venue is collected. Then, in the online phase, localization takes place by comparing the measured received power with the data from the offline phase. Although this improves localization accuracy compared to RSS itself, it is too sensitive to real-time changes; therefore, it is not reliable for high accuracy localization.

PILOT uses TOA of received ultrasound signals for ranging. Compared to other measurement methods, TOA seems to be the best choice in terms of accuracy, computation’s simplicity, and implementation cost. The main challenge of relying on the TOA of the received signal is the precise time of arrival detection. In indoor environments, multi-path fading is an inevitable effect due to the reflection of the original signal from walls, ceiling, floor, and other artifacts within the room. Multi-path fading may be a significant problem for measuring the precise TOA, as several copies of the original signal with varying arrival times make it difficult to detect the exact TOA of the original signal. To minimize the multi-path effect and be able to detect the exact TOA of the original signal, PILOT leverages the FHSS technique for signal transmission.

FHSS is a well-known technique where uses different frequency hops as carrier frequencies and spreads the signal. It has applications both in military transmission mainly to avoid jamming and also civilian use, such as the well-known Bluetooth, mainly to provide multi-
user capability. We are applying this spread spectrum technique to work in our favor, i.e., we are using the hopping over different frequencies as a method to overcome multi-path fading. To the best of our knowledge, this is the first time using the FHSS technique to counter the multi-path fading effect and offer more accurate location estimation for drones.

There are two kinds of frequency hopping techniques, slow frequency hopping (SFH) and fast frequency hopping (FFH). In SFH, one or more data bits are transmitted within one hop. An advantage is that coherent data detection is possible. In the latter kind, one data bit is divided over multiple hops and coherent signal detection is complex and seldom used. PILOT uses the SFH.

Similar to the approach used in [38], in our scheme, the transmitting signal of the $k$-th drone is modulated using Binary Phase Shift Keying (BPSK) modulation and then it is spread using a sinusoidal signal with variable frequencies depending on the pseudo-random code:

$$s^{(k)}(t) = d^{(k)} \cdot pT_B(t) \cdot \sin(2\pi f_m t + \phi),$$

(2.1)

where $d^{(k)}$ is the transmitted data symbol of the ultrasonic transmitter on the $k$-th drone, the rectangular pulse $pT_B$ is equal to 1 for $0 \leq t < T_B$ and zero otherwise, $T_B$ is the data symbol duration, and $f_m$ is the set of frequencies over which the signal hops. Then the received signal is in the form of:

$$r^{(k)} = d^{(k)} \cdot pT_B(t - \tau) \cdot \sin(2\pi f_m (t - \tau) + \phi) + \mathcal{M}(t) + \mathcal{N}(t),$$

where $\tau$ is the propagation delay that we are using for calculating the distance, $\mathcal{N}(t)$ is the Gaussian noise, and $\mathcal{M}(t)$ is the multi-path effect, which can be expressed as the following
2.2. **ROBUST RANGING USING FHSS ULTRASOUND SIGNALS**

summation:

\[ M(t) = \sum_{i=1}^{N} \alpha_i \cdot s^{(k)}(t - \tau_i), \]  

(2.2)

where \( \alpha_i \) is the attenuation of path \( i \) and \( \tau_i \) is the time delay of path \( i \). As long as we make sure that the hopping speed is faster than the time delay of each path \( \tau_i \), then before the arrival of any of the reflected signals, we already have changed the frequency and different paths do not interfere with the original signal.

By eliminating the multi-path effect using FHSS technology, the received signal would be just the time-delayed transmitted signal plus noise:

\[ r^{(k)} = d^{(k)} \cdot p_{TB}(t - \tau) \cdot \sin(2\pi f_m(t - \tau) + \phi) + N(t). \]  

(2.3)

Therefore, by performing a cross-correlation between the received signal and the known transmitted signal (the one without the time delay) and by detecting the sample bit at which the peak occurs, the distance is calculated as follows:

\[ d = c_{\text{sound}} \cdot TOF = c_{\text{sound}} \cdot \frac{n_{\text{samples}}}{f_s}; \]  

(2.4)

where \( n_{\text{samples}} \) is the sample number of the maximum peak and \( f_s \) is the sampling frequency.

Fig. 2.1 depicts a simple scenario of how the transmitter generates the FHSS modulated waveform for transmission. Fig. 2.2 shows how the ultrasound receiver demodulates the FHSS waveform after it has passed through the channel. Fig. 2.3 shows how the time delay is getting measured at the receiver. Fig. 2.3 is just to depict a clear representation of how the cross-correlation process works; therefore, it uses a simple input and not the actual FHSS waveform that PILOT uses.
Figure 2.1: Illustration on steps of generating the FHSS waveform: (i) Generate a set of random bits; (ii) Convert the bits into rectangular pulses; (iii) Apply the BPSK modulation; (iv) Generate six random sets of spreading codes; (v) Generate the FHSS waveform using the BPSK signals and the spearing codes.
2.2. Robust Ranging using FHSS Ultrasound Signals

Figure 2.2: Illustration of a simple receiving scenario.

Figure 2.3: Simple illustration of how the cross-correlation function works to find the bit at which the peak occurs, which later be translated into the time delay: (i) Input Sequence: Make a random set of data, in this example, we use a triangle set; (ii) Output Sequence: Add Gaussian noise and the delay to the input sequence; (iii) Perform the cross-correlation function and find the delay based on the observed peak.
For calculating distances using ultrasonic signals, the velocity of ultrasonic signals must be known. As [102] suggested, there are two ways to determine the exact velocity. As a first alternative, the velocity of the sound can be determined from its nominal value at a certain temperature as follows:

\[ c_{\text{sound}} = 331.3 \sqrt{1 + \frac{T}{273.15}}. \]  \tag{2.5}

Eq. 2.5 uses the velocity of the sound at 0 °C as a reference in order to find the value \( c_{\text{sound}} \) in \( \text{m/s} \) at a temperature \( T \) in °C. The other way to determine the velocity is by measuring the traveling time of the ultrasound signal at a known distance. In our system, we calculated the velocity using both methods. In the first method, a digital thermometer can be deployed to measure the ambient temperature to substitute in the Eq. 2.5. On the other hand, in the second method, the \( c_{\text{sound}} \) can be constantly updated by locating a reference ultrasound speaker at a known position in the room where it constantly transmits a pilot signal to the ultrasound microphones installed at known locations in the room.

Besides temperature, humidity and wind are also effective on sound velocity. However, the influence of the humidity is negligible compared to the one of the temperature and intense wind is not applicable in our indoor setup. The effect of indoor airflow due to air-conditioning was investigated in [103, 104]. In order to reduce the influence of wind, Martin et al. in [105] suggested estimating the sound velocity continuously by measuring the travel time for a known distance in proximity and correcting the other measurements accordingly, which is the exact same technique we used as the second method of measuring the velocity of sound.
2.3 Enhancing the Robustness of FHSS Ranging

In the previous section, we proposed a method for ranging and estimating the distance between the drone and receiver beacons based on the TOA of ultrasonic signals and for overcoming the multipath, we proposed the FHSS technique. Another approach for keeping track of a drone’s movement is by estimating the relative velocity of the drone with respect to each of the receiver beacons. This can be done by measuring the received signals’ frequency shift (i.e., Doppler-shift effect) and then estimating the distance. However, the key drawback with using this method alone is that the error can increase over time, and thus, over a prolonged period, this method alone will not provide reliable and accurate tracking.

However, leveraging a Kalman filter to incorporate the data from both methods (i.e., TOA-based distance estimation and velocity estimation based on frequency shift measurement) can prevent the velocity estimation errors from increasing over time. In this approach, data collected from each of the aforementioned methods play an essential role in preserving the system’s high accuracy location estimation capability over time. On the one hand, velocity estimation based on frequency shift measurements cancels the TOA-based distance estimation measurement error and improves system accuracy. On the other hand, the data obtained from the TOA-based distance estimation at each moment is the primary source of initial data for the final distance calculation, and the method is not merely dependent on
the velocity estimate, which makes it unreliable over time. Thus, the combination of the
data from both methods keeps the system’s distance estimation highly accurate at all times.
This approach achieves greater noise robustness and provides more precise localization and
monitoring of the drone’s movement. This section first describes how PILOT estimates the
velocity using change in the frequency and then explains how it integrates distance and
velocity estimations in a Kalman filter.

### 2.3.1 Velocity Estimation by Measuring Doppler Shift

The following equation is being used to express the Doppler shift effect:

\[ F_s = \frac{v}{c} \cdot F, \quad (2.6) \]

where \( F_s \) is the amount of the frequency shift, \( v \) is the relative velocity between transmitter
and receiver, \( c \) is the speed at which the signal propagates (\( c \) would denote the speed of light
if the signal is an RF signal, and similarly, \( c \) would denote the speed of sound if the signal is
an acoustic signal), and \( F \) is the actual frequency in which the signal is transmitted. Using
Eq. 2.6, whether the receiver and transmitter move towards or away from each other can be
established by treating \( v \) as a vector in the equation.

Several studies have used Doppler shift measurements to estimate the speed and direction
of a UAV. For example, in [100], to estimate a flying UAV’s speed and direction, Shin et
al. used Doppler-shift. Nevertheless, depending solely on the Doppler shift has limitations.
For instance, it is necessary to know the original location of the target drone to be able
to estimate its location continuously at each moment. Moreover, it is almost impossible to
achieve high accuracy because of the propagation of the error in the result over time. To
overcome these limitations, some methods use Doppler shift measurements as an auxiliary
localization tool to improve the efficiency of the primary localization method [47, 50, 51]. PILOT also adopts this strategy.

To estimate the relative velocity between the target drone and each of the receiver beacons fixed throughout the room (where the drone is located), PILOT uses the following procedure:

- The ultrasound transmitter system mounted on-board the drone continuously transmits the FHSS waveform at frequency $F = f_m$.

- After receiving the signal in each of the receiver beacons, PILOT first applies Fast Fourier Transform (FFT) techniques to obtain the frequency content of the received signal and find the peak in the frequency domain and then estimates the frequency shift of the signal, $F_s$, by calculating the difference between the peak frequency of the received signal and $F$.

- An estimate of the drone’s velocity is calculated using Equation 2.6.

### 2.3.2 Combining Distance and Velocity Estimation Using a Kalman Filter

PILOT incorporates the distance estimations obtained by the FHSS-based ranging method and the estimates from Doppler shift-based velocity estimation method. The reason for this approach is to boost the accuracy and robustness of distance estimation performance against noise. There are two ways to combine these two forms of estimates: using a Kalman filter [51] or through an optimization framework [50]. In the latter method, we need to devise an optimization framework that consists of both estimates in its objective function. Then,
similar to [50], we can form an optimization framework as follows:

\[
\begin{align*}
\sum_{i \in [k-n+1 \ldots k]} & \sum_{j} \alpha(|x_i - c_j| - |x_0 - c_j| - d_{FHSS}^{i,j})^2 + \\
\sum_{i \in [k-n+2 \ldots k]} & \sum_{j} \beta(|x_i - c_j| - |x_{i-1} - c_j| - v_{Doppler}^{i-1,j})^2; \\
\end{align*}
\]  

where \( k \) is the current processing interval, \( n \) is the number of intervals used in the optimization, \( x_i \) denotes the drone’s position at the beginning of the \( i \)-th interval, \( x_0 \) denotes the reference position, \( c_j \) denotes the \( j \)-th ultrasound receiver’s position in the room, \( d_{FHSS}^{i,j} \) denotes the distance change from the reference location with respect to the \( j \)-th ultrasound receiver at the \( i \)-th interval, \( v_{Doppler}^{i,j} \) denotes the relative velocity between the drone and the \( j \)-th ultrasound receiver during the \( i \)-th interval, \( T \) is the interval duration, and \( \alpha \) and \( \beta \) are the relative weights assigned to the distance and velocity measurements. The only unknown in the optimization is the drone’s position over time (i.e., \( x_i \)).

The objective reflects the goal of finding a solution \( x_i \), which is the drone’s position, that best fits the distances from FHSS ranging and velocities from Doppler shift measurements. The first term reflects that the distance calculated based on the coordinates should match the distance estimated from the FHSS ranging. Similarly, the second term captures that the distance traveled over an interval computed from the coordinates should match the distance derived by multiplying the interval time to the velocity obtained from the Doppler shift measurements. The objective function consists of terms from multiple intervals to improve the accuracy. The formulation in Eq. 2.7 is general and not restricted just to one dimension for tracking the drone on a line, and it can support both two-dimensional and three-dimensional coordinates. In fact, \( x_i \) and \( c_j \) are both vectors whose sizes are determined by the number of dimensions.
2.3. Enhancing the Robustness of FHSS Ranging

Because this optimization problem is non-convex, there is no guarantee of convergence, and it may be slow and computationally expensive. There is some work in the literature to address this issue. Similar to the solution in [50], some changes to the optimization framework are required to make the problem convex. To simplify the objective, we create a new parameter called $D_{i,j}$ that denotes the drone’s distances to different ultrasound receiver beacons over time (i.e., replacing $|\mathbf{x}_i - \mathbf{c}_j|$ in the objective function with $D_{i,j}$), which is a convex function in terms of $D_{i,j}$. However, not all distances are feasible (i.e., there may not exist coordinates that satisfy the distance constraints). As a result, additional constraints must be derived to enforce feasibility. To begin, a convex relaxation of the original problem must be solved by treating distances as unknowns and replacing feasibility constraints with triangular inequality constraints. Triangular inequality constraints are necessary for the distances to be realizable in a low-dimensional Euclidean space. They are also sufficient in a two-dimensional space to ensure feasibility, but not in a three-dimensional space. As a result, the solution obtained in the first step must be projected into a feasible solution space in the following step. This projection is related to network embedding, which embeds network hosts in a low-dimensional space while preserving their pairwise distances to the greatest extent possible. To rectify this problem, Mao et al. [50] developed an embedding method based on the Alternating Direction Method of Multipliers (ADMM) [106] to efficiently solve the problem.

One possible benefit of this method compared to the Kalman filter is to combine the Inertial Measurement Unit (IMU) information captured from drone’s IMU sensors with the distances from FHSS ranging and velocities from Doppler shift, another term could be added to the objective function, which represents the effect of IMU measurements. However, since PILOT does not depend on the IMU measurements, there is no benefit in using the optimization framework.

We concluded that, because of its substantially higher computational complexity, the op-
timization method is insufficient, which would place an enormous burden on the PILOT processing elements. Therefore, PILOT employs the Kalman filter approach. One of our primary objectives is to ensure that the computational complexity and overhead of PILOT’s communication are low relative to the state of art. The following paragraphs will provide a brief overview of how PILOT blends distance and velocity estimates using a Kalman filter.

Let $D_k$ denote the actual distance between the ultrasound transmitter and a receiver beacon in the $k$-th window, $t$ denote the duration, $v_k$ denote the measured Doppler velocity, $n_k$ capture the error in Doppler measurements, $d_k$ denote the measured distance, and $w_k$ denote the distance measurement error. The following equations describe the relationship between these variables:

$$
D_k = D_{k-1} + v_k \cdot t + n_k
$$
$$
d_k = D_k + w_k
$$

As we mentioned before, $d_k$ and $v_k$ are from distance and velocity measurements, respectively. We can utilize the redundancy between these two measurements by using a Kalman filter to decrease the impact of noise and further enhance the accuracy of distance estimation. According to [51], the optimal distance estimation $\hat{D}_k$ is given by:

$$
\hat{D}_k = \hat{D}_{k-1} + v_k \cdot t + \frac{\hat{p}_k + q_k}{\hat{p}_k + q_k + r_k} (d_k - \hat{D}_{k-1} - v_k \cdot t),
$$

where $\hat{p}_k = \frac{r_k (\hat{p}_{k-1} + q_k)}{\hat{p}_{k-1} + q_k + r_k}$, and the variables, $q_k$ and $r_k$, denote the standard deviation for $n_k$ and $w_k$, respectively.


2.4 Three-dimensional Localization

After estimating the distance between an ultrasonic transmitter and a receiver, the next step is to localize the transmitter in three dimensions. Angulation and lateration are the two most well-known localization techniques used to estimate the target object’s position based on the angle or distance measurements respectively. As we mentioned in previous sections, due to the challenges with the angle-based measurement methods, PILOT is measuring distance, hence the lateration technique for the final three-dimensional localization of the drone.

To use trilateration for localizing an object in two dimensions, the distances between the beacons require at least three beacons for the target drone with an unknown position to localize itself in two dimensions. (a) Target drone has the distance of $d_1$ from the beacon. All the points on the circle with the radius of $d_1$ with the center of the beacon is a possible position for the drone, but no certain answer. (b) The target drone has the distance of $d_1$ to the first beacon and $d_2$ to the second beacon. Two circles intersect in two points, and these two points are the possible position for the target drone; again no certain answer. (c) The target drone has the distance of $d_1$ to the first beacon, $d_2$ to the second beacon, and $d_3$ to the third beacon. The intersection of three circles is just one point which is the position of the target drone.
object and at least three sources are needed. Similarly, in three-dimensional localization, in order to uniquely localize the target object, we need to measure the distances between the target object and at least four different sources. Fig. 2.5 explains this better. Let’s denote the distance between a transmitter and \(i\)-th receiver as \(d_i\). Also, the position of the transmitter is denoted as \([x, y, z]^T\) and the position of the \(i\)-th receiver is denoted as \([x_i, y_i, z_i]^T\). Then using trilateration rules, we have:

\[
\begin{align*}
(x_1 - x)^2 + (y_1 - y)^2 + (z_1 - z)^2 &= d_1^2 \\
(x_2 - x)^2 + (y_2 - y)^2 + (z_2 - z)^2 &= d_2^2 \\
&\quad \vdots \\
(x_n - x)^2 + (y_n - y)^2 + (z_n - z)^2 &= d_n^2
\end{align*}
\] (2.10)

After applying some mathematical manipulation, we can then simplify these quadratic equations and write them down in the linear form of \(Ax = b\) where \(A\) and \(b\) are equal to:

\[
A = \begin{bmatrix}
2(x_n - x_1) & 2(y_n - y_1) & 2(z_n - z_1) \\
2(x_n - x_2) & 2(y_n - y_2) & 2(z_n - z_2) \\
& \vdots & \vdots \\
2(x_n - x_{n-1}) & 2(y_n - y_{n-1}) & 2(z_n - z_{n-1})
\end{bmatrix},
\]

\[
b = \begin{bmatrix}
d_1^2 - d_n^2 - x_1^2 - y_1^2 - z_1^2 + x_n^2 + y_n^2 + z_n^2 \\
d_2^2 - d_n^2 - x_2^2 - y_2^2 - z_2^2 + x_n^2 + y_n^2 + z_n^2 \\
& \vdots \\
d_{n-1}^2 - d_n^2 - x_{n-1}^2 - y_{n-1}^2 - z_{n-1}^2 + x_n^2 + y_n^2 + z_n^2
\end{bmatrix}.
\]

The vector \(x = [x, y, z]^T\) which includes the coordinate of the object that need to be localized.
would be: \[ x = (A^TA)^{-1}A^Tb. \]

## 2.5 Preliminary Simulation Results

This section describes the simulation setup used for preliminary tests followed by the result analysis to evaluate the performance of the enhanced FHSS ranging.

### 2.5.1 Simulation Setup

**Transmitter**

The transmitter subsystem, which is the ultrasonic transmitters on-board the drone, generates the desired FHSS signals. We used signals in the frequency range between 25 KHz and 55 KHz for two reasons. Firstly, frequencies at or below 20 KHz need to be avoided since that range would interfere with the frequency range of a human voice. Any overlap would result in degraded performance. In addition, according to the Nyquist theorem, in order to prevent aliasing, the sampling rate must be at least twice the maximum frequency, i.e., if the system operates in the frequency range from 25 KHz to 55 KHz, then it requires that the sampling rate should be at least 110 KHz to avoid aliasing. We do not use frequencies above 55 KHz because we do not intend to deal with high frequencies to avoid both processing and equipment costs, which means that the sampling rate simply needs to be 110 KHz or more. PILOT uses the FHSS waveform. In this waveform, the frequency range is from 25 KHz to 55 KHz with 6 sub-frequency carriers located at 27.5 KHz, 32.5 KHz, 37.5 KHz, 42.5 KHz, 47.5 KHz, and 52.5 KHz and the bandwidth dedicated for each of these sub-carriers is 5 KHz. A single hop occurs within the transmission time of each data bit, and this hopping rate is fast enough (based on the multi-path analysis of the room using image theory and finding
the delay spread of the multi-path signals) to mitigate the effects of multi-path interference. We set the sampling frequency ($f_s$) to 340 KHz; this guarantees not having aliasing and also simplifies our calculations. For modulation, PILOT uses BPSK (Binary Phased Shift Keying) to take advantage of BPSK’s high noise robustness.

In [55], there is just one ultrasonic speaker on-board the drone, which is transmitting the FHSS waveform for ranging purposes. That setup works based on the assumption that an ultrasonic transmitter has an omni-directional propagation pattern. Although this assumption perfectly works in simulation tests, it will not work in real-life experiments. Unlike most RF transmitters, most ultrasonic transmitters have a relatively narrow-angle radiation beam, i.e., only receivers within the imaginary cone-shape propagation pattern in front of the transmitter can receive the signal. As we mentioned in previous sections, three-dimensional localization requires distances between the transmitter and at least four receivers. Since we put the receivers on the walls of the room, just one of them can receive the transmitted signal from an ultrasonic transmitter on-board the drone, i.e., only the receiver in front of that transmitter can receive the signal. To overcome this issue, we propose the four-leaf clover transmitter setup where the ultrasonic transmitter setup on-board the drone consists of four transmitters facing the front, back, and sides. In this way, at each moment, the same FHSS waveform is transmitted from each of these four transmitters and all the receivers in the room can receive the signal.

**Channel**

We used a Rayleigh channel model in the simulations, which considers the impact of the multi-path fading interference on the transmitted signals. Moreover, we used additive white Gaussian noise (AWGN) to show the impact of the noise in our localization process. Furthermore, we assumed that the movement of the target drone is confined to a rectangular
room with dimensions of $5 \text{ m} \times 5 \text{ m} \times 3 \text{ m}$. This is the typical dimension for indoor office spaces.

**Receiver**

The receiver sub-system, which is the four receiver beacons located at known positions in the room, cross-correlates the received signal with a reference signal, and then seeks the sample bit that makes the cross-correlation peak. After finding the sample bit in which the cross-correlation peak happens, the receiver sub-system estimates the distance from the drone to each of the receiver beacons using the following relation: 

$$d = n_{\text{samples}} \times c_{\text{sound}} / f_s,$$

where $n_{\text{samples}}$ is the sample number where the maximum cross-correlation occurs and $f_s$ is the sampling frequency. There are four receiver beacons in the room, and the same procedure is used by each of them.

To achieve the best coverage for any possible drone location in the room, we placed the four receiver beacons in positions to best ensure that at least one receiver beacon is in the line of sight and very close to the corresponding transmitter on-board the drone that is aiming at that receiver. Therefore, the receiver beacons can receive the ultrasound signal with a high signal-to-noise ratio (SNR) at every drone location in the room. Specifically, the $(x, y, z)$ coordination of the ultrasound receivers in the room is $(2.5, 0, 1.5)$, $(5, 2.5, 2.5)$, $(2.5, 5, 2)$, and $(0, 5, 3)$ where all the numbers are in meters.

### 2.5.2 Simulation Results

In this part, we show the results of our preliminary simulations to evaluate the performance of the first two stages of PILOT by measuring the error between the estimated position of the drone and its actual position. We assess the performance of the scheme by benchmarking
it with respect to a basic reference scheme that uses only FHSS-based distance estimation (before enhancement) to locate a target drone.

For the simulation tests, we ran Monte Carlo with a large enough number of iterations. Moreover, to present the final error, we averaged the error between the actual position of the drone and the estimated position over the entire flight trajectory. We ran the simulation over many different trajectories that were generated randomly. This ensures that the performance evaluation is valid over any possible drone’s trajectory, and not just for some specific scenarios. To showcase the result, we randomly chose seven of the flight trajectories and demonstrated the performance of PILOT on them. The placement of the ultrasound receivers remained the same for all of the trajectories.

In Fig. 2.6, we show a drone’s actual trajectory as well as the estimated trajectory using
2.5. Preliminary Simulation Results

Figure 2.7: Average localization error (cm) vs. SNR (dB) for $X-Y$ plane and $Z$-axis for PILOT’s first stage only (as a benchmark) and PILOT’s complete first two stages together (FHSS ranging and Doppler shift velocity estimation incorporated in the Kalman filter).

FHSS distance estimation and Doppler shift velocity estimation incorporated in the Kalman filter (the first two stages of the PILOT). This is one of the drone’s random flight trajectories in the simulation tests. The locations of the ultrasonic receivers are also indicated in this figure. The actual and estimated trajectories seem to overlap perfectly because, relative to the room’s dimensions in which the drone’s movement is confined, the localization estimation error is significantly small.

Fig. 2.7 illustrates the relationship between the localization performance and the signal-to-noise ratio (SNR) of the received signal at each of the ultrasound receivers. As predicted, the localization error is inversely proportional to the signal’s SNR value. Moreover, as is shown in this figure, deploying the second stage of the PILOT improves the localization accuracy both in the $X-Y$ plane and in the $Z$-axis. Finally, in the figure, we note that the $Z$-axis localization error is much greater than that of the $X-Y$ plane. We will thoroughly investigate this issue in the next section and show that the reason behind this is the relative...
Figure 2.8 compares the localization error between the first stage of the PILOT (FHSS ranging only) which as we mentioned before, is the benchmark, and the complete first two stages of PILOT (FHSS ranging and Doppler shift velocity estimation incorporated in the Kalman filter). As is shown in the figure, the $X - Y$ plane localization error is plotted separately from the $Z$-axis to show the massive difference between them and justify the need to implement the third stage of PILOT. As can be seen in this figure, in all the seven random drone flight trajectories, the second stage of PILOT slightly improves the localization error both for the $X - Y$ plane and the $Z$-axis; however, the $Z$-axis localization error is still more than twice of which in the $X - Y$ plane.
2.6 Height Estimation Improvement

The first two stages of PILOT provide high accuracy localization in the $X-Y$ plane; however, the $Z$-axis localization error still needs to be improved because it is much greater than that of the $X$ or $Y$ axis. Generally, the localization error for the ranging-based localization methods originates from two sources, first the error in estimating the distance between the target and each beacon, known as the ranging error, and second the error arising from the relative geometry between the target and all of the beacons. The reason for having a more significant error in the $Z$-axis compared to the $X-Y$ plane is the relative geometry between the transmitter setup on-board the target drone and the receiver beacons in the room.

Thus far, we have shown how the first two stages of PILOT abate the ranging error by deploying the FHSS communication scheme for the distance estimation task and making the scheme robust against noise and the indoor multi-path fading effect. In this section, we first thoroughly investigate the reason behind having a bad $Z$-axis estimation and then show how PILOT provides a solution to lessen the $Z$-axis estimation error, leading to a better overall three-dimensional accuracy. To the best of our knowledge, PILOT is the first scheme that provides solutions to mitigate both the ranging and the geometry-related errors and proposes highly accurate localization for drones in indoor environments.

The Cramer-Rao Bound (CRB), which is the lower bound on the position variance that can be obtained using an unbiased location estimator \cite{93}, is a useful metric for evaluating the localization accuracy. In \cite{93}, Rajagopal showed that for a two-dimensional trilateration system with an unbiased estimator, under the assumption that the range measurements are independent and have zero-mean additive Gaussian noise with constant variance $\sigma_r^2$, the CRB variance of the positional error $\sigma^2(r)$ at position $r$, as defined by $\sigma^2(r) = \sigma_x^2(r) + \sigma_y^2(r)$
is given by:

\[
\sigma(r) = \sigma_r \times \sqrt{\frac{N_b}{\sum_{k=1}^{N_b-1} \sum_{j=k+1}^{N_b} C_{kj}}},
\]  

(2.11)

where \( N_b \) is the number of beacons, \( C_{kj} = |\sin(\theta_k - \theta_j)| \), \( \theta_k \) is the angle between \( b_k \) and \( r \), and \( b_k \) is the \( k \)-th beacon.

This illustrates that the error of localization is a multiplication of the error of range measurement with another variable that is a function of the number of beacons and the angle between the beacons and the target object. This function is called Geometric Dilution of Precision (GDOP) and we have: \( \sigma(r) = \sigma_r \times GDOP \). Since CRB is directly proportional to the GDOP, GDOP can be used as a reasonable guide for measuring the accuracy of the localization \[88, 93, 107, 108\].

In general, for three-dimensional localization of an object at \([x \ y \ z]^T\) using ultrasound beacons, we have:

\[
\sqrt{\text{Var}(x) + \text{Var}(y) + \text{Var}(z) + \text{Var}(c\tau)} = GDOP \cdot \sigma_r,
\]  

(2.12)

where \( c \) here is the speed of sound and \( \tau \) is the receiver’s clock offset. Since we have synchronization between the transmitter and the ultrasound receiver beacons in our work, then the timing offset is considered to be zero; therefore:

\[
GDOP = \sqrt{\frac{\sigma_x^2 + \sigma_y^2 + \sigma_z^2}{\sigma_r^2}}.
\]  

(2.13)

The distance between the drone and each of the beacons is calculated from the following:

\[
r_i = \sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2},
\]  

(2.14)
where, as discussed earlier, $[x\ y\ z]^T$ denote the drone’s position and $[x_i\ y_i\ z_i]^T$ denote the position of the $i$-th receiver beacon. The exact $r_i$ is not known due to the ranging measurement error and that causes errors in the Eq. 2.14 solution for $[x\ y\ z]^T$. Similar to [109], we take the differential of Eq. 2.14 and disregard terms beyond first order to find a relationship between the solution errors and the ranging errors between the drone and each of the ultrasound receiver beacons in the room:

$$
\Delta r_i = \frac{\Delta x(x - x_i) + \Delta y(y - y_i) + \Delta z(z - z_i)}{\sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2}} = \Delta x \cos \alpha_i + \Delta y \cos \beta_i + \Delta z \cos \gamma_i,
$$

Let $\Delta X = [\Delta x\ \Delta y\ \Delta z]^T$ be the position error vector and $\Delta R = [\Delta r_1 \cdots \Delta r_n]^T$ be the target range error vector. Then we can define matrix $C$ as:

$$
C = \begin{bmatrix}
c_1^1 & c_2^1 & c_3^1 \\
\vdots & \vdots & \vdots \\
c_1^n & c_2^n & c_3^n
\end{bmatrix},
$$

where $[c_1^i\ c_2^i\ c_3^i] = [\cos \alpha_i \cos \beta_i \cos \gamma_i]$. Now we can write $\Delta R = C \Delta X$ and then we have $\Delta X = (C^T C)^{-1} C^T \Delta R$. We know that:

$$
\text{Cov}(\Delta X) = \mathbb{E}(\Delta X \Delta X^T) = \begin{bmatrix}
\sigma_x^2 & \sigma_{xy} & \sigma_{xz} \\
\sigma_{yx} & \sigma_y^2 & \sigma_{yz} \\
\sigma_{zx} & \sigma_{zy} & \sigma_z^2
\end{bmatrix}. \quad (2.15)
$$
If we assume that $\text{Var}(r_i) = \sigma^2_r$ and that the errors $\Delta r_i$ are uncorrelated, then:

$$
\mathbf{E}(\Delta X \Delta X^T) = \mathbf{E}((C^TC)^{-1}C^T \Delta R)((C^TC)^{-1}C^T \Delta R)^T)
$$

$$
= (C^TC)^{-1}C^T \mathbf{E}(\Delta R \Delta R^T)((C^TC)^{-1}C^T)^T
$$

$$
= (C^TC)^{-1}C^T C(C^T)^{-1}\sigma^2_r = (C^TC)^{-1}\sigma^2_r.
$$

Eq. 2.13, Eq. 2.15 and the above result show that the $(C^TC)^{-1}$ diagonal elements can be used to determine the GDOP. GDOP consists of Vertical Dilution of Precision (VDOP) and Horizontal Dilution of Precision (HDOP), i.e.,

$$
\text{GDOP} = \sqrt{\text{HDOP}^2 + \text{VDOP}^2}
$$

where $\text{HDOP} = \sqrt{\sigma^2_x + \sigma^2_y}$ is the effect of the relative geometry between transmitter setup and receivers on the $X-Y$ plane’s estimation accuracy and $\text{VDOP} = \sqrt{\sigma^2_z}$, on the other hand, indicates the impact of geometry on the $Z$-axis estimation. This explains how the $X-Y$ plane localization error can be different from the one for the $Z$-axis. The evaluation of GDOP values is shown in Table 2.1 [92],[110]. In our problem, after measuring the GDOP, HDOP, and VDOP for all the points in the room, we saw that the average of HDOP values over all the points is placed in the ”very good” category of the Table 2.1 whereas the average for VDOP is in the ”good” category. This is the reason for having a worse average $Z$-axis estimation error in comparison with the $X-Y$ average estimation error. Fig. 2.9 shows a color representation of the HDOP and GDOP values in the room. In the following, we propose our solution to solve this problem.

We use a separate ultrasonic transceiver installed on-board the drone to estimate the height continuously, then using a filter, we incorporate this measurement with the $Z$-axis estimation that is already available from the previous steps of PILOT. This significantly improves the overall $Z$-axis estimation accuracy and compensates for having a higher average VDOP compared to the average HDOP.
2.6. Height Estimation Improvement

Figure 2.9: Graphical representation of the HDOP and VDOP values in the room.

Table 2.1: Evaluation of GDOP Values

<table>
<thead>
<tr>
<th>GDOP Values</th>
<th>Evaluation of the geometry of the beacons</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 1</td>
<td>Measurements error or redundancy</td>
</tr>
<tr>
<td>1</td>
<td>Ideal</td>
</tr>
<tr>
<td>1 – 2</td>
<td>Very Good</td>
</tr>
<tr>
<td>2 – 5</td>
<td>Good</td>
</tr>
<tr>
<td>5 – 10</td>
<td>Medium</td>
</tr>
<tr>
<td>10 – 20</td>
<td>Sufficient</td>
</tr>
<tr>
<td>&gt; 20</td>
<td>Bad</td>
</tr>
</tbody>
</table>
We place the ultrasonic transceiver facing upward on-board the drone and by measuring the time of flight of the ultrasonic signal emitted from this sensor, after it is reflected from the ceiling, we can determine the distance between the drone and the ceiling. Then, we find the drone’s height at each moment by subtracting this result from the room’s height. The channel between the drone and the ceiling is typically more reliable than the one between the drone and the floor, since there are usually no error-inducing artifacts between the drone and the ceiling. Following shows the height estimation using this extra ultrasonic transceiver:

\[ h' = \frac{c_{\text{sound}} \cdot t}{2}; \quad h_{\text{drone}} = H - h', \]

where \( h' \) is the distance between the drone and the ceiling, \( t \) is the total time that takes the signal to travel from ultrasonic transceiver on-board the drone, hit the ceiling, reflecting, and is received in the ultrasonic transceiver on-board the drone, \( H \) is the room’s height, and \( h_{\text{drone}} \) is the estimation for drone’s height. The revised final location estimation of the drone is then: \( \mathbf{x} = [x, y, (w_1 \cdot z + w_2 \cdot h_{\text{drone}})]^T \), where \( w_1 \) and \( w_2 \) are the weights assigned in the filter to incorporate the estimated height from the first two stages of PILOT and the third stage respectively in order to calculate the final revised height for the drone.

The complete procedure that PILOT employs to localize a drone in a GPS-denied environment is illustrated below:

- **Drone Station:**
  
  1. Four ultrasonic transmitters continuously transmit FHSS signals for distance and velocity estimation.
  
  2. One ultrasonic transceiver continuously measures the drone’s height separately.

- **Receivers:**
1. **Time Domain Analysis**: Estimate the TOA of the received FHSS signal using the cross-correlation technique and compute the distance:

\[
d = \frac{n_{\text{samples}}}{f_s} \cdot c_{\text{sound}};
\]

2. **Frequency Domain Analysis**: Estimate the shift in the frequency of the received FHSS signal using the FFT technique and compute the velocity:

\[
v = c \cdot \frac{F_s}{F};
\]

3. **Kalman Filter**: Combine the calculated distance and velocity to smooth out the result and find the final distance between the drone and each of the receivers:

\[
D_k = D_{k-1} + v_k \cdot t + n_k,
\]

\[
d_k = D_k + w_k,
\]

\[
\dot{D}_k = \dot{D}_{k-1} + v_k \cdot t + \frac{\dot{p}_{k-1} + q_k}{p_{k-1} + q_k + r_k} (d_k - \dot{D}_{k-1} - v_k \cdot t);
\]

- **Center Station**:

1. 3D Localization of drone by applying Trilateration technique on estimated distances between the drone and receivers:

\[
x = [x \ y \ z]^T = (A^T A)^{-1} A^T b;
\]
board the drone with the calculated 3D location to improve the Z-axis estimation:

$$\mathbf{x} = [x, y, (w_1 \cdot z + w_2 \cdot h_{\text{drone}})]^T;$$

3. Send the navigation command to the drone controller.

As a graphical summary of the PILOT procedure, Fig. 2.10 represents the scheme that PILOT employs to localize the drone in three-dimensional space accurately.

## 2.7 Experimental Testbed & Performance Evaluation

The performance of PILOT was comprehensively assessed using real-life experimental tests coupled with simulations in MATLAB. In this section, first, we provide details of the experimental testbed in 2.7.1, and then showcase the evaluation results in 2.7.2.

### 2.7.1 Experimental Testbed

The experimental test setup consists of two stations, as shown in Fig. 2.11. First, which is the left part in the figure, is the drone and the system on-board it. Second, which is the right side of the figure, is the ground control station that helps input the transmitted data into the MATLAB program running on a Dell XPS 15 laptop. A Parrot Mambo Drone is the drone used for the experiment. It is a low-cost, off-the-shelf, and ultra-light drone suitable for indoor experiments and has the capacity to hold some light loads as well. The designed system mounted on-board the drone consists of an Arduino Uno micro-controller connected to an \( HC-SR04 \) sensor and a XBee S1 module. The \( HC-SR04 \) sensor is for ultrasonic distance measurement purposes and the XBee S1 module is for wireless communication with
Drone continuously transmits **FHSS** ultrasound signal (for distance and velocity estimation). Ultrasound transceiver for **additional height estimation**.

Following procedure at each of the receiver beacons

- **Compute TOA** of the received FHSS signal using **cross-correlation** and estimate the **distance**:
  \[ d = \frac{n_{samples}}{f_s} \cdot c_{sound} \]

- **Compute the shift in the frequency using FFT** of received signal and estimate the **velocity**:
  \[ v = \frac{F_x}{F} \cdot c_{sound} \]

- **Use Kalman filter** to combine the data from previous steps
  \[ D_k = D_{k-1} + v_k \cdot t + n_k \]
  \[ d_k = D_k + w_k \]

Drone’s three-dimensional localization using the estimated **Distances** and **Trilateration** techniques. Incorporate the data from height estimation to further improve Z-axis accuracy.

Figure 2.10: PILOT’s procedure for three-dimensional localization.
Figure 2.11: Parrot Mambo Drone equipped with the ultrasound transceiver system, which has the capability of wireless signal transmission on the left side, and the signal reception on the right side.

the ground controller. In the ground control unit, another Arduino Uno micro-controller connected to a XBee S1 receives the data and transfers it into the MATLAB program running on the laptop. All the experiments were conducted in a hallway inside the building with dimensions $5 \, \text{m} \times 5 \, \text{m} \times 3 \, \text{m}$.

2.7.2 Results & Evaluation

In this part, we show the results of our simulations and experiments to evaluate the performance of PILOT by measuring the error between the estimated position of the drone and its actual position. We assess the performance of the scheme by benchmarking it with respect to a basic reference scheme that uses only FHSS-based distance estimation (PILO’s first stage) to locate a target drone.

For the simulation tests, we used the same setup and configuration as the preliminary tests. Moreover, to present the final error, we averaged the error between the actual position of the drone and the estimated position over the entire flight trajectory. For the experimental tests, we flew the Parrot Mambo drone equipped with our system over different trajectories
2.7. Experimental Testbed & Performance Evaluation

Figure 2.12: Evaluating the accuracy of Z-axis estimation before and after deploying the third stage and comparison between PILOT’s overall X−Y plane estimation accuracy with PILOT’s Z-axis estimation accuracy.

in the room. Having multiple random different flight trajectories ensures that the results are valid for any trajectories and not just restricted to some specific ones. To showcase the evaluation results, we chose five random flight trajectories and presented the result for them. In each of these five random trajectories, we made the route in the room before flying the drone and used them as the ground truth, i.e., we compared the estimated location at each moment with these actual ground truth paths.

As is seen in Fig. 2.12, a comparison between the X − Y plane localization error and the Z-axis localization error before deploying the third stage of PILOT shows the significant difference between them and justifies the necessity of deployment of the third stage of PILOT. Moreover, as is shown in this figure, the third stage of the PILOT performs as expected and it improves the Z-axis localization error significantly by constantly transmitting the measured
Figure 2.13: Evaluating the performance of PILOT: overall three-dimensional localization accuracy.

data from the supplementary ultrasound transceiver on-board the drone (HC-SR04) using the XBee S1 wireless module to the receiver module connected to the Dell XPS 15 laptop and combines this information with current Z-axis estimation in a filter.

In Fig. 2.13, we compare the performance of PILOT with that of the benchmark scheme (which relies only on FHSS-based distance estimation to localize a target drone) in terms of the overall three-dimensional localization error. The average value of three-dimensional localization error for PILOT is less than 1.2 cm. As shown in the figure, the benchmark scheme’s localization error is more than twice that of PILOT. The benchmark scheme focuses on mitigating ranging-based error by deploying the FHSS communication scheme for localization. Other localization schemes proposed in the literature try to improve the localization accuracy by proposing their own techniques to mitigate the ranging-based error. However, PILOT considers both the ranging error and the error due to the relative geometry between
the transmitter and receivers and proposes solutions to mitigate both of them and further improve localization accuracy.

In Fig. 2.14a, we evaluate the performance of the full scheme PILOT by showcasing the distribution of localization errors on the $X - Y$ plane across five different trajectories. To achieve this, we present a boxplot for each trajectory. Within each box, the central red mark indicates the median, while the bottom and top edges (depicted as blue lines) represent the 25-th and 75-th percentiles, respectively. The whiskers extend to the most extreme data points that are not considered outliers, and any outliers are individually plotted using the red + marker symbol.

It is important to note that in Fig. 2.14a, we focus on displaying the distribution of localization errors on the $X - Y$ plane for each trajectory, rather than just showing the average error. This approach provides insight into both the median error and the region where the majority of errors are concentrated. Additionally, it reveals the presence of outliers and their distance from the median.

Furthermore, Fig. 2.14b presents the same information as Fig. 2.14a, with one key difference. Instead of showcasing the localization error distribution on the $X - Y$ plane, this figure represents the overall three-dimensional localization error distribution.

PILOT achieves significant improvement in comparison with other drone localization schemes in the literature [37, 38, 51, 55, 111, 112]. For instance, in [55], Famili et al. failed to solve the Z-axis estimation error and their scheme had a much greater Z-axis estimation error compared to the $X - Y$ plane estimation error. The scheme proposed by Segers et al. [37] incurs an error of 2 cm or greater in terms of localization error just for the $X - Y$ plane. Their scheme is merely for two-dimensional localization and they do not consider the complete three-dimensional localization. In [51], Mao et al. proposed an FMCW method
Figure 2.14: Evaluating the performance of the full scheme PILOT: (a) $X-Y$ plane localization error distribution; (b) Overall three-dimensional localization error distribution.
to overcome the impact of interference and multi-path; however, their system was designed to track the drone on one line, just a single dimension, whereas PILOT proposes a three-dimensional localization. As another example, in [111], O’Keefe et al. proposed a scheme for three-dimensional localization of drones which incurs an average error of 5.2 cm which is approximately five times worse than what PILOT provides.

2.8 Summary

In this chapter, we proposed PILOT, a three-dimensional localization scheme for drones in GPS-denied environments. PILOT takes advantage of the beneficial features of ultrasonic signals and develops a three-stage system to accurately estimate the drone’s position in three dimensions. In the first stage, PILOT uses an FHSS-based ranging technique to estimate the distance between the drone and each receiver beacon. The FHSS waveform guarantees robustness against noise and indoor multi-path fading. In the second stage, PILOT first estimates the relative velocity between the drone and each receiver beacon using the Doppler-shift effect. Then, it designs a Kalman filter for estimating the final distance by combining the Doppler shift-based velocity estimation with FHSS-based distance estimation to mitigate the error. In the third stage, by providing a separate height estimation using an additional ultrasonic sensor, PILOT improves the Z-axis estimation error due to the relative geometry between the transmitters and receivers. Conducting thorough simulation tests coupled with real-life experiments to evaluate PILOT’s performance shows that PILOT achieves higher localization accuracy compared to the schemes proposed in the literature.
Chapter 3

iDROP: Robust Localization for Indoor Navigation of Drones with Optimized Beacon Placement

As discussed in the previous chapter, drones in many applications need the ability to fly fully or partially autonomously to accomplish their mission. To allow these fully/partially autonomous flights, first, the drone needs to be able to locate itself constantly. Then the navigation command signal would be generated and passed on to the controller unit of the drone. In this chapter, we propose a localization scheme for drones called iDROP (Robust Localization for Indoor Navigation of Drones with Optimized Beacon Placement) that is specifically devised for GPS-denied environments (e.g., indoor spaces). Instead of GPS signals, iDROP relies on speaker-generated ultrasonic acoustic signals to enable a drone to estimate its location. In general, localization error is due to two factors: the ranging error and the error induced by relative geometry between the transmitters and the receiver. iDROP mitigates these two types of errors and provides a high-precision three-dimensional localization scheme for drones. iDROP employs a waveform that is robust against multi-path fading. Moreover, by placing beacons in optimal locations, it reduces the localization error induced by the relative geometry between the transmitters and the receiver.
3.1 Motivation

In the previous chapter, we proposed a comprehensive strategy aimed at mitigating the adverse effects of multipath fading within indoor environments. This framework not only demonstrated its efficacy in overcoming these challenges but also provided a robust and accurate localization system tailored explicitly for indoor Unmanned Aerial Vehicles (UAVs) navigation. However, this accomplishment was accompanied by the identification of two significant challenges that warrant further investigation. Firstly, our initial approach entailed situating the transmitter sensor aboard the UAV, while distributing all receiving sensors within the interior space. Consequently, the localization process occurred on the receiving end within the room, necessitating the subsequent transmission of the localization signal to the UAV. This entailed an additional communication link, potentially introducing delays and errors. Moreover, a conspicuous observation emerged—uneven accuracy in height (Z-axis) estimation compared to the estimations within the X – Y plane. This observation, consistent across all tests, was unexpected since all three axes traversed the same channel, implying a similar error range. The convergence of these challenges prompted a comprehensive reevaluation, culminating in the design of a novel system to address these issues.

The subsequent advancement in our methodology was precipitated by the aforementioned challenges. Specifically, the revised approach involved relocating the receiving sensor onto the UAV, while distributing the transmitting sensors throughout the indoor space. This innovative configuration empowered the UAV to autonomously ascertain its own location, thus circumventing the need for an additional communication link to relay positioning data. However, the transition to this new arrangement was not bereft of complexity. It ushered in a series of new challenges, most notably the demand for signal separation at the receiver. This necessitated the development of an entirely new communication model, characterized by resilience against multipath effects and noise, while concurrently enabling effective signal
separation within the receiver framework.

Upon grappling with these intricacies, an exploratory tangent emerged—probing the nuances of location accuracy across distinct axes. This exploration unearthed a hitherto uncharted facet within ranging-based localization. Beyond the inherent errors introduced by ranging complexities, an additional layer of inaccuracy was traced back to the relative geometry between transmitters and receivers. This revelation highlighted that two systems, identical in hardware and experiencing comparable ranging errors, could exhibit disparate final location accuracy based solely on sensor placement within their geometric context. Further analysis illuminated a disparity in error propagation between vertical ($Z$-axis) and horizontal ($X−Y$ plane) axes, traceable to the geometric arrangement. This insight served to rationalize the observed discrepancies in positioning accuracy. While devising a solution for stationary targets posed a relatively surmountable challenge—facilitated by identifying optimal geometric configurations to minimize relative geometry-induced errors—our focus on UAVs introduced an additional layer of complexity. The inherent mobility of UAVs compounded the conundrum, requiring the determination of sensor placements within indoor spaces that inherently minimized errors attributable to relative geometric factors. This intricate web of challenges underscores the principal motivations behind the novel system introduced within this chapter.

### 3.2 Chapter Overview

Over the past few decades, the global drone industry has expanded exponentially and the number of applications in which drones play a significant part, both in indoor and outdoor environments, has subsequently increased. There is a wide range of indoor drone applications nowadays, ranging from recreational use to life-saving matters. Examples include
reconnaissance inside nuclear power plants, helping firefighters to locate people inside burning buildings, security surveillance inside large warehouses, etc.

Drones must have fully or partially autonomous flying capability in most of the above applications to perform their task successfully. To allow these autonomous flights, first, the drone needs to localize itself constantly. Then the navigation command signal would be generated and passed on to the controller unit of the drone according to the current position and ultimate destination. In outdoor environments, drones can easily use GPS signals for self-localization; however, such an approach is not feasible in indoor spaces or GPS-denied areas.

In the absence of the GPS, vision-based methods are widely used for localization and navigation of drones (e.g., [21]). However, the accuracy of current vision-based approaches is usually limited due to the drone’s vibration during flight. In addition, the accuracy can degrade further in vision-impaired environments (e.g., low light environments). Moreover, vision-based methods are expensive both in terms of hardware price and computational complexity.

In addition to vision-based approaches, ranging-based methods are commonly deployed for indoor localization. In this category, the localization is based on the received signal information. RF-based localization (e.g., [54]) and localization based on acoustic signals (e.g., [55]) are the examples for this group. Besides the characteristic of the received signal, the arrangement of the beacons plays an important role in the localization accuracy in this group.

In this chapter, we propose iDROp (Robust Localization for Indoor Navigation of Drones with Optimized Beacon Placement), a three-dimensional localization scheme for drones in GPS-denied environments. iDROp uses ultrasonic acoustic-based signals for localization. We claim that acoustic signals have some advantages over the localization schemes based
on RF signals. Compared to RF signals, the significantly slower propagation speed of the acoustic signals allows for higher accuracy of localization. In addition, RF signals can penetrate through walls and ceilings, further degrading the accuracy of the localization. iDROP uses high-frequency acoustic signals, known as ultrasounds, to prevent any interference with human-generated or drone’s propeller noise. Moreover, iDROP develops an optimization framework to find the optimal placement for the ultrasound transmitter beacons. Following is a summary of our contributions.

- We propose iDROP, a three-dimensional localization scheme for drones in GPS-denied environments which is robust against noise and multi-path fading and provides location estimation with high accuracy.

- iDROP uses the hybrid Frequency Hopping Code Division Multiple Access (FH-CDMA) as the communication scheme to maximize robustness to noise and multi-path fading and to facilitate signal separation at the receiver.

- iDROP develops an optimization framework to reduce the height estimation error due to the relative geometry between transmitters and receivers.

- By leveraging the code division multiple access techniques, iDROP reduces the communication link used for navigation commands by placing the receiver on-board and transmitter beacons in the room.

- Our simulation and experimental results indicate that iDROP’s localization error is less than 1.5 centimeters in three-dimensional space.

The rest of this chapter is organized as follows. In the next section, we review some of the related works in this area. Then, in section 3.3 which is the first core of this chapter, we fully explain the first stage of our localization scheme for drones in no-GPS environments. In section 3.4, we describe the preliminary simulation setup and bring the results of local-
3.3 Robust Localization with Hybrid FH-CDMA Ultrasound Signals

iDROP is a novel and highly accurate three-dimensional localization scheme for drones in indoor environments which deploys two steps to reduce both sources of localization error, known as the ranging error and the error due to the relative geometry between the receiver and transmitter beacons.

This section thoroughly investigates how iDROP reduces the ranging error and increases the localization accuracy by making the system robust against noise and the indoor multi-path fading effect. Then, in section 3.5, we will elaborate how it minimizes the error due to the relative geometry and increases the overall accuracy of localization by optimizing the placement of beacons in the room.

3.3.1 Measurement Method and Technique

As we have discussed earlier in Sec. 3.2, the ranging-based localization with ultrasonic signals has some advantages over the other techniques. Hence, iDROP uses ultrasound acoustic
signals for distance estimation. Well-known measurement methods are the angle of arrival (AOA), time of arrival (TOA), time difference of arrival (TDOA), and received signal strength (RSS). Techniques for location estimation are angulation, lateration, and fingerprinting. AOA methods incur high expenses in terms of both the hardware cost and the processing power because they require special antenna arrays and complicated calculations. RSS and fingerprinting are too prone to changes in real-time and are therefore neither reliable nor highly accurate. All said, iDROP uses the trilateration technique and the TOA of the received ultrasound signals for localization. Multi-path fading is one of the main challenges of relying on the TOA of the received signal. The presence of the copied version of the original signal in the receiver makes detecting the arrival time of the original signal hard or even impossible. iDROP overcomes the multi-path fading effect by proposing a hybrid FH-CDMA communication scheme for its signal transmission.

3.3.2 Implementation Challenges

In terms of placement of the transmitter(s) and receiver(s) for a ranging-based localization, there are two general scenarios, either have the receiver(s) on-board the drone and keep the transmitter(s) in the room or vice versa. The localization calculation task takes place on the receiver side of the system; therefore, not having them on-board the drone requires another communication link for sending the final location estimation to the drone. This unnecessary communication link increases the cost, slows down the whole process, and may incur additional errors, which degrades the accuracy.

Therefore, it is better to have the receiver(s) mounting on-board the drone and the transmitter(s) in the room. In this case, having one transmitter in the room and multiple receivers on-board the drone [111] causes several problems. It adds extra weight to the drone, in-
3.3. Robust Localization with Hybrid FH-CDMA Ultrasound Signals

Figure 3.1: This figure illustrates the hybrid FH-CDMA transmission of encoded symbols over different FH channels.

creases the power consumption, and most importantly, there is not enough space between the receivers which incurs error due to the relative geometry between the transmitter and receivers and significantly degrades the accuracy of the localization.

To overcome these challenges, iDROP mounts one receiver on-board the drone and keeps all the transmitters spatially distanced from each other in the room. However, this method raises a new challenge, the need for signal separation in the receiver. The receiver requires the capability of separately detecting the TOA of each signal transmitted from a different transmitter. To rectify this matter, iDROP deploys a code division technique. It assigns a code to the transmitted signals of each of the ultrasound transmitters in the room, i.e., transmitted signals at each transmitter are encoded using a code that is orthogonal to all other transmitters’ code. Having four transmitters, iDROP generates a different orthogonal code for each transmitter using a Walsh-Hadamard matrix of size four. Data bits of each transmitter would be multiplied with one of the rows of this matrix. At the receiver side, received signals will be multiplied with all the four codes and signals from each transmitter get detected.
3.3.3 Hybrid FH-CDMA

iDROP deploys a hybrid FH-CDMA technique which, to the best of our knowledge, is the first time this technique has been applied for a localization purpose and it is the most desirable scheme to address both problems of multi-path and signal separation. The hybrid FH-CDMA is a communication scheme that combines two well-known techniques, the Frequency Hopping (FH) and the Code Division Multiple Access (CDMA). iDROP uses this method to rectify the challenge of signal separation in the receiver with the multiple access capability and at the same time, brings robustness against noise and the indoor multi-path fading using the frequency hopping technology.

In our system, as long as we make sure that the hops happened fast enough that before the first multi-path reflection arrived at the receiver, we already hopped to another carrier frequency, then we can promise a transmission that is robust against the multi-path fading. We picked the hopping rate equal to the symbol rate, which is fast enough to avoid multi-path according to the room channel characteristic.

First, at each ultrasound transmitter beacon in the room, the code assigned to that transmitter would be assigned to each data bit of that transmitter. Therefore the symbols are no longer just a bit; they are coded bits that include four bits, i.e., data-symbols of different beacons are spread with their assigned code and generated the coded symbols. Then, since the hop rate equals the symbol rate, each coded symbol is transmitted in different FH channels. Here, the coded symbols enable signal separation of different transmitters in the receiver side and the different frequency channels are for managing the multi-path fading effect of the room.

Similar to [38], in our scheme, the transmitting signal of the $i$-th transmitter is modulated using Binary Phase Shift Keying (BPSK) modulation, then encoded with its dedicated code,
and then the coded symbols are spread using a sinusoidal signal with a variable frequency depending on the pseudo-random code which is known both in transmitter and receiver side:

\[
s^{(i)}(t) = d^{(i)} \cdot c^{(i)} \cdot pT_B(t) \cdot \sin(2\pi f_m t + \phi),
\]

where \( T_B \) is the data symbol duration, \( d^{(i)} \cdot c^{(i)} \) is the transmitted symbol of the \( i \)-th ultrasonic transmitter in the room where \( d^{(i)} \) is the data bit and \( c^{(i)} \) is the dedicated code to that transmitter, the rectangular pulse \( pT_B \) is equal to 1 for \( 0 \leq t < T_B \) and zero otherwise, and \( f_m \) is the set of frequencies over which the signal hops. Then the received signal is in the form of:

\[
r = \sum_{i=1}^{4} s^{(i)}(t - \tau_i) + M + N,
\]

where \( \tau_i \) is the propagation delay from the \( i \)-th transmitter to the receiver on-board drone that we are using for calculating the distance, \( N \) is the overall Gaussian noise, and \( M \) is the summation of all the multi-path fading effects:

\[
M = \sum_{i=1}^{4} \sum_{j=1}^{N} \alpha_{ij} \cdot s^{(i)}(t - \tau_{ij}),
\]

where \( \alpha_{ij} \) is the attenuation of path \( j \) for the \( i \)-th transmitter, \( \tau_{ij} \) is the time delay of the path \( j \) for the \( i \)-th transmitter, and \( N \) is the number of multi-path signals. Multi-path is a big issue for indoor environments and we are using the frequency hopping technique to help overcome this effect. As long as we make sure that at each transmitter, the speed of hopping is faster than the time delay of all the multi-path signals corresponding to that transmitter \( (\tau_{ij}) \), then before the arrival of any of the reflected signals, we already have changed the frequency and different paths will not interfere with the original signal.
By ensuring that multi-path effects are eliminated using different FH-channels, the received signal would be only the delayed time of the transmitted signal plus noise:

\[ r = \sum_{i=1}^{4} s^{(i)}(t - \tau_i) + \mathcal{N}. \] (3.3)

By multiplying the received signal in each code related to each transmitter, the received signal from the \( i \)-th transmitter in the receiver would be in the form of:

\[ r^{(i)} = d^{(i)} \cdot p T_B(t - \tau_i) \cdot \sin(2\pi f_m(t - \tau_i) + \phi) + \mathcal{N}. \] (3.4)

Therefore, by implementing a cross-correlation between the received signal and the known transmitted signal (the one without the time delay) and detecting the sample at which the peak occurs, the distance is calculated as the following:

\[ d = \frac{n_{\text{samples}}}{f_s} \cdot c_{\text{sound}}, \] (3.5)

where \( n_{\text{samples}} \) is the sample number of the maximum peak, \( f_s \) is the sampling frequency, and \( c_{\text{sound}} \) is the speed of sound.

### 3.3.4 Three-dimensional Localization

After having successfully measured the distance between an ultrasonic transmitter and the receiver, the next step is the three-dimensional localization of the receiver. To localize an object in two dimensions using trilateration, at least distances between the object and three sources are needed. Similarly, in three-dimensional localization, it is required to have the distance between the object and at least four sources to localize the object uniquely. Let’s denote the distance between the receiver and the \( i \)-th transmitter as \( d_i \). Also, the position
of the receiver is $[x\ y\ z]^T$ (which in fact is the position of the drone) and the position of the $i$-th transmitter denotes as $[x_i\ y_i\ z_i]^T$. Then using trilateration rules, we have:

$$(x_1 - x)^2 + (y_1 - y)^2 + (z_1 - z)^2 = d_1^2$$

$$(x_2 - x)^2 + (y_2 - y)^2 + (z_2 - z)^2 = d_2^2$$

$$\vdots$$

$$(x_n - x)^2 + (y_n - y)^2 + (z_n - z)^2 = d_n^2$$

We can then simplify these quadratic equations and write them down in the form of $Ax = b$ where $A$ and $b$ are equal to:

$$A = \begin{bmatrix}
2(x_n - x_1) & 2(y_n - y_1) & 2(z_n - z_1) \\
2(x_n - x_2) & 2(y_n - y_2) & 2(z_n - z_2) \\
\vdots & \vdots & \vdots \\
2(x_n - x_{n-1}) & 2(y_n - y_{n-1}) & 2(z_n - z_{n-1})
\end{bmatrix},$$

$$b = \begin{bmatrix}
d_1^2 - d_n^2 - x_1^2 - y_1^2 - z_1^2 + x_n^2 + y_n^2 + z_n^2 \\
d_2^2 - d_n^2 - x_2^2 - y_2^2 - z_2^2 + x_n^2 + y_n^2 + z_n^2 \\
\vdots \\
d_{n-1}^2 - d_n^2 - x_{n-1}^2 - y_{n-1}^2 - z_{n-1}^2 + x_n^2 + y_n^2 + z_n^2
\end{bmatrix}.$$
bustness against the noise and the multi-path fading effect of an indoor environment. By encoding the transmitting signals of each ultrasound transmitter with a code generated using a Walsh-Hadamard matrix of size four, iDROP guarantees the capability of signal separation at the receiver side. Then, for robust transmission against noise and multi-path, iDROP uses different frequency hops to transmit encoded symbols of each transmitter. Therefore, every data symbol is spread with a complete orthogonal code while successive symbols are transmitted in different frequency hopped channels. Fig. 3.1 illustrates this better.

3.4 Preliminary Simulation Results

This section evaluates the performance of the proposed FH-CDMA localization and shows the localization error for $X$, $Y$, and $Z$ axis separately. First, in 3.4.1, we describe the test setup, and then, we show the simulation result in 3.4.2.

3.4.1 Preliminary Simulation Setup

The performance of the localization scheme proposed in section 3.3 is assessed by implementing simulation in MATLAB. Similar to [55], we locate the transmitters at the $(x, y, z)$ coordinates equal to $(2.5, 0, 1.5), (5, 2.5, 2.5), (2.5, 5, 2)$, and $(0, 5, 3)$ where all the numbers are in the meter unit. To better observe how we conduct the simulation, we break it down into three sub-systems as follows.

Transmitter

The transmitter sub-system, which in fact is the ultrasonic transmitters at the known positions in the room, generates the desired FH-CDMA signals. We used signals in the frequency
range of 20 KHz to 50 KHz for two reasons. First, to avoid inciting excessive audible noise or facing interference from human-generated voice, we pick frequencies over 20 KHz to avoid overlapping with the human voice frequency range. Any overlaps may cause interference and result in degrading the performance. On the other hand, according to the Nyquist theorem, the sampling rate needs to be at least twice the maximum frequency to avoid aliasing; hence, if the system works in the frequency range of 20 KHz to 50 KHz, then the sampling frequency needs to be at least 100 KHz. To avoid the cost of processing and equipment, dealing with high frequency is not suitable; therefore, we do not transmit above 50 KHz, which means that the sampling rate simply could be 100 KHz or more. To generate the FH-CDMA waveform successfully, iDROP uses 6 different frequency hops with 5 KHz bandwidth dedicated to each hop. Also, it assigns a code to each transmitter, so every data bit of each transmitter would first be multiplied with the code and then transmitted via one of the six frequency hops centered at frequencies 22.5 KHz, 27.5 KHz, 32.5 KHz, 37.5 KHz, 42.5 KHz, and 47.5 KHz. The codes are orthogonal to each other and made by a Walsh-Hadamard matrix of size 4. At each hop, one data symbol which already been multiplied by its code would be transmitted, so the hop rate is equal to the data bit rate (actual data bit rate before multiplying by the code), which is fast enough to mitigate the multi-path fading effect of the indoor environment. Although a sampling rate of 100 KHz would be enough for our simulation, we picked sampling frequency \( f_s \) equal to 340 KHz to make sure it would be large enough to avoid aliasing and it also helps to simplify some of our calculations. Since the throughput is not essential in our case, we use BPSK modulation which does not have a good transmission rate, but is highly robust against noise.
3.4.2 Results and Motivation

Here, we present the results of our preliminary simulations using the simulation setup we described. We assess the performance of the FH-CDMA localization by calculating the error
between the actual position of the drone and our estimated position. We have used a Monte Carlo method with an adequately large number of iterations for each simulation.

In Fig. 3.2, we show a drone’s actual trajectory as well as the estimated trajectory using just FH-CDMA localization (the first stage of the iDROP). The locations of the ultrasound speaker transmitters are also indicated in this figure. The actual and estimated trajectories seem to overlap perfectly because the localization estimation error is significantly small relative to the room’s dimensions. All the other obstacles and objects in the room; including a table, several chairs, glass windows, etc., are not shown in the figure to clearly show the drone’s flight trajectory.

Fig. 3.3 shows the relationship between FH-CDMA localization performance and the signal-to-noise ratio (SNR) of the signal received by the ultrasound receiver. The localization error is inversely proportional to the SNR value of the signal, as expected. In the figure, note that
the Z-axis localization error is much greater than that of the X or Y axis at any given SNR. By conducting more simulations with different drone trajectories, we observed that the error of Z-axis localization is always drastically more than the X−Y plane localization error. This is despite the fact that all the X, Y, and Z axes should have almost similar errors because they face a similar channel. This observation motivated us to further investigate the localization error factors and propose a scheme to improve the Z-axis localization error.

### 3.5 Enhancing the Accuracy of Localization

As we saw in the previous section, the localization error for the Z-axis is much greater than that of the X or Y axis. This is due to the relative geometry between the transmitter beacons and the target receiver. In general, localization error for ranging-based localization
is originated from two sources, first the error in estimating the distance between the target and each of the beacons, known as ranging error, and the other resulted from the relative geometry between the target and beacons.

In section 3.3, we showed how iDROP lessens the ranging error by deploying the hybrid FH-CDMA communication scheme for distance estimation and making the scheme robust against the noise and the indoor multi-path fading effect. This section shows how iDROP copes with the error induced by the relative geometry between the drone and ultrasound transmitter beacons. To the best of our knowledge, iDROP is the first scheme that provides solutions to mitigate both the ranging and the geometry-related errors and proposes highly accurate localization for drones in indoor environments.

### 3.5.1 Dilution of Precision

A useful metric for measuring the localization accuracy is the Cramer-Rao Bound (CRB) which is the lower bound on the location variance that can be achieved using an unbiased location estimator [93]. In [93], Rajagopal showed that for a 2D trilateration system with an unbiased estimator, under the assumption that the range measurements are independent and have zero-mean additive Gaussian noise with constant variance $\sigma_r^2$, the CRB variance of the positional error $\sigma^2(r)$ at position $r$, as defined by $\sigma^2(r) = \sigma_x^2(r) + \sigma_y^2(r)$ is given by:

$$\sigma(r) = \sigma_r \times \sqrt{\frac{N_b}{\sum_{i=1}^{N_b-1} \sum_{j=i+1}^{N_b} A_{ij}}}$$

where $N_b$ is the number of beacons, $A_{ij} = |\sin(\theta_i - \theta_j)|$, $\theta_i$ is the angle between $b_i$ and $r$, and $b_i$ is the $i$-th beacon.

This shows that the localization error is a multiplication of the ranging measurement error
with another term, which is the function of the number of beacons and the angle between beacons and the target object. In satellite calculations, this function is called Geometry Dilution of Precision (GDOP), therefore: $\sigma(r) = \sigma_r \times GDOP$. As CRB is directly proportional to the GDOP, we can consider GDOP as a reasonable guideline to measure the localization accuracy [88, 93, 107, 108].

In general, for 3D localization of an object at $(x, y, z)$ using ultrasound beacons, we have:

$$GDOP \cdot \sigma_r = \sqrt{Var(x) + Var(y) + Var(z) + Var(\tau)},$$

where $c$ here is the speed of sound and $\tau$ is the receiver’s clock offset. In our simulations, we assume that the transmitter and receiver use the same clock, and hence we set the timing offset to zero. Therefore, we have:

$$GDOP = \sqrt{\frac{\sigma^2_x + \sigma^2_y + \sigma^2_z}{\sigma^2_r}}. \quad (3.7)$$

Let $(x, y, z)$ denote the drone’s position and $(x_i, y_i, z_i)$ denote the position for each of the ultrasound transmitter beacons in the room. Then, the drone range to each beacon is calculated from the following:

$$r_i = \sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2}. \quad (3.8)$$

Because of the ranging measurement error, the exact $r_i$ is not known and that causes errors in the solution of Eq. 3.8 for $(x, y, z)$. To find a relationship between the solution errors and the ranging errors between the drone and each of the ultrasound transmitter beacons in the
3.5. Enhancing the Accuracy of Localization

room, similar to [109], we take the differential of Eq. 3.8 and ignore terms beyond first order:

$$
\Delta r_i = \frac{\Delta x(x - x_i) + \Delta y(y - y_i) + \Delta z(z - z_i)}{\sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2}} = \Delta x \cos \alpha_i + \Delta y \cos \beta_i + \Delta z \cos \gamma_i,
$$

where $[\cos \alpha_i \cos \beta_i \cos \gamma_i]^T$ is the unit vector pointing from the drone to the $i$-th beacon.

Let $\Delta X = [\Delta x \Delta y \Delta z]^T$ be the position error vector and $\Delta R = [\Delta r_1 \cdots \Delta r_n]^T$ be the target range error vector. Then we can define matrix $U$ as:

$$
U = \begin{bmatrix}
  u_1^1 & u_2^1 & u_3^1 \\
  \vdots & \vdots & \vdots \\
  u_1^n & u_2^n & u_3^n 
\end{bmatrix},
$$

where $[u_1^i u_2^i u_3^i] = [\cos \alpha_i \cos \beta_i \cos \gamma_i]$. Now we can write $\Delta R = U\Delta X$ and then we have $\Delta X = (U^TU)^{-1}U^T\Delta R$. We know that:

$$
\text{Cov}(\Delta X) = E(\Delta X\Delta X^T) = \begin{bmatrix}
  \sigma_x^2 & \sigma_{xy} & \sigma_{xz} \\
  \sigma_{yx} & \sigma_y^2 & \sigma_{yz} \\
  \sigma_{zx} & \sigma_{zy} & \sigma_z^2 
\end{bmatrix}, \quad (3.9)
$$

If we assume that $\text{Var}(r_i) = \sigma_r^2$ and that the errors $\Delta r_i$ are uncorrelated, then:

$$
E(\Delta X\Delta X^T) = E(((U^TU)^{-1}U^T\Delta R)((U^TU)^{-1}U^T\Delta R)^T)
$$

$$
= (U^TU)^{-1}U^TE(\Delta R\Delta R^T)((U^TU)^{-1}U^T)^T
$$

$$
= (U^TU)^{-1}U^T(UU^T)^{-1}\sigma_r^2 = (U^TU)^{-1}\sigma_r^2.
$$

Eq. 3.7, Eq. 3.9 and the above result show that the diagonal elements of the $(U^TU)^{-1}$ can be used to calculate the GDOP. GDOP consists of Vertical Dilution of Precision (VDOP)
and Horizontal Dilution of Precision (HDOP), i.e., $GDOP = \sqrt{HDOP^2 + VDOP^2}$ where HDOP represents the effect of the relative geometry between transmitters and the receiver on the $X - Y$ plane estimation accuracy and VDOP, on the other hand, shows the impact of geometry on the $Z$-axis estimation. This explains why we saw different accuracy for the $Z$-axis estimation and $X - Y$ plane estimation in our preliminary tests. Table 2.1 shows the evaluation of the GDOP values.

### 3.5.2 Optimized Beacon Placement

Here, we propose an optimization algorithm to minimize the $Z$-axis estimation error induced by the relative geometry between transmitter beacons and the receiver, while keeping the horizontal error related to geometry in an acceptable range.

Even though the optimal beacon placement for localization of a single static target in two-dimension scenarios is well-understood, the optimal placement for a mobile target in a three-dimensional space is still an open problem [86]. Finding an optimal beacon placement configuration for indoor localization to minimize the localization error due to the relative geometry between the transmitter beacons and the target receiver at any given position is a well-established NP-Hard problem [83, 84, 85, 86].

Most of the earlier localization techniques have attempted to localize the unknown object just in two-dimension scenarios, which does not consider the real-world geometrical arrangement between the target and beacons in three dimensions. In the following, we will describe our systematic approach to find the optimized beacon placement in the room to improve the $Z$-axis estimation accuracy and mitigate the overall estimation error by developing a greedy algorithm.
Problem Formulation

Find the optimal placement for a set of four ultrasound transmitter beacons with the goal of minimizing the $V D O P_{avg}$ and keeping the $H D O P_{avg}$ below a required threshold, where $V D O P_{avg}$ and $H D O P_{avg}$ are the average of calculated VDOP and HDOP for a set of four beacons on all the given positions in the drone domain. Due to the constant mobility of the drone, it is not sufficient to compute the VDOP and the HDOP just for one position. Therefore, we considered all the possible locations the drone may pass by during its flight (i.e., all the points in the drone domain) and computed the average of the VDOP and the HDOP over all those possible locations and used them in our calculations. Moreover, if we simply constructed the optimization framework to minimize the average GDOP, then there would be no guarantee that it would improve the $Z$-axis estimation accuracy; hence we used the average of the VDOP and the HDOP values. The optimization problem can be formulated as follows.

$$\min \sum_{Drone\ Domain} V D O P$$

$$s.t. \quad HD O P_{avg} < h$$

The minimization of the average VDOP is because the goal here is to find the optimized beacon placement to improve the drone’s height estimation accuracy. The consideration for keeping the average HDOP below a threshold is to have a reasonable overall 3D localization error due to the relative geometry, i.e., this constraint ensures that improving the $Z$-axis estimation is not with the cost of sacrificing the $X - Y$ plane estimation accuracy.

The acquired inputs are as follows. The drone domain, set $D$, is a subspace of the room where the drone is allowed to fly. Optimization calculations are based on the average of $V D O P$ and $H D O P$ over all the points in this domain. The beacon domain, set $B$, is acceptable locations
for beacons in the room. The entire ceiling and top half of all walls are acceptable candidates for the beacon locations. $HDOP_{avg}$ tolerance ($h$) is a constraint which dictates $HDOP_{avg}$ be smaller than $h$ and $V DOP_{avg}$ tolerance ($v$) is a constraint which dictates $V DOP_{avg}$ be smaller than $v$.

As we discussed earlier, if each measurement has the same uncertainty with zero mean and unit variance and they are uncorrelated from each other, then the aforementioned HDOP and VDOP in the above steps can be derived from the diagonal elements of the matrix $Q$ as follows:

$$Q = (U^T U)^{-1} = \begin{bmatrix}
\sigma_x^2 & \sigma_{xy} & \sigma_{xz} \\
\sigma_{yx} & \sigma_y^2 & \sigma_{yz} \\
\sigma_{zx} & \sigma_{zy} & \sigma_z^2
\end{bmatrix},$$

where $VDOP = \sqrt{\sigma_z^2}$, $HDOP = \sqrt{\sigma_x^2 + \sigma_y^2}$, and

$$U = \begin{bmatrix}
\frac{x_1-x}{r_1} & \frac{y_1-y}{r_1} & \frac{z_1-z}{r_1} \\
\frac{x_2-x}{r_2} & \frac{y_2-y}{r_2} & \frac{z_2-z}{r_2} \\
\frac{x_3-x}{r_3} & \frac{y_3-y}{r_3} & \frac{z_3-z}{r_3} \\
\frac{x_4-x}{r_4} & \frac{y_4-y}{r_4} & \frac{z_4-z}{r_4}
\end{bmatrix},$$

where $(x, y, z)$ is the drone’s position, $(x_i, y_i, z_i)$ is the location coordinate of the $i$-th ultrasound transmitter beacon, and $r_i$ represents the distance between the drone and the $i$-th ultrasound transmitter beacon.
Algorithm Design

In order to find a solution with manageable computational time and effort, we develop a greedy algorithm that is based on the class of Evolutionary Algorithms (EAs) to find the beacon placement.

Algorithm 1 Beacon Placement Evolutionary Algorithm

**Input:** Drone domain (D), Beacon domain (B), $HDOP_{avg}$ tolerance ($h$), $VDOP_{avg}$ tolerance ($v$)

**Output:** Desirable placement for a set of four beacons

1: while $VDOP_{avg} > v$ 
2:   & $HDOP_{avg} > h$ do
3:   Generate a set of P random individuals, where each individual is a set of four beacons
4:   for $i = 1$ to $i = number$ of iteration do
5:     Check the fitness of all available individuals;
6:     Kill the worst ones to keep having P individuals;
7:     Select the individuals with better fitness as Parents;
8:     Crossover each two adjacent parents and make a new offspring;
9:   end for
10: end while

In the algorithm, first, an initial set of $P = 50$ randomly-generated individuals is created. Each individual here is a set of four transmitter beacons selected randomly from the beacon domain. To avoid being trapped in a local minimal, we distributed the initial individuals in different groups: all the beacons on the ceiling, all of them on the walls, or some on the ceiling and some on the walls. We generated some random individuals for each of these groups. That is the reason we chose 50 randomly-generated individuals, because if it was less than 50, it was not able to include all the different groups, and if it was more than 50, then it made the solving time unnecessarily longer.

Then, individuals are sorted according to the fitness (cost) function and those with better fitness are chosen for reproduction. The fitness function is the average of $VDOP$ over the entire drone domain that is achieved using that specific arrangement of four beacons. Then,
the algorithm picks the first 40 individuals in the line as a parents group that will be used to reproduce new individuals. Every two adjacent of them make a new set of four beacons using a crossover technique; therefore, it would be 20 off-springs total. Next, the algorithm checks every new-generation individual according to the fitness function. Out of 70 total populations, including both the parents and off-springs, the last 20 in the line will be eliminated, so the population remains 50.

For generating a new off-spring, each set of parents includes 8 beacons total (4 beacons per parent). The crossover technique switches some of the coordinate parameters of the first four beacons with the ones from the second set of four beacons.

After the initialization, the described procedure would repeat for 100 iterations. We checked the algorithm for different iteration numbers and saw that larger ones (e.g., 1000) just make the process slower without bringing any significant improvement on the final result. Moreover, iterations less than 100 still did not provide a minimal answer, so that was why we picked 100 as the number of iterations. After that, the first individual in the line according to the fitness function is selected. If it has an average VDOP and HDOP over the entire drone domain less than \( v \) and \( h \) respectively, then that individual represents the final answer which is the beacon placement configuration in the room. Otherwise, the algorithm starts over again, from generating the 50 first new random individuals and repeating the procedure. The algorithm terminates whenever the final result satisfies the constraints.

### 3.5.3 Additional Sensor for Height Estimation

We have improved the accuracy of the \( Z \)-axis estimation by optimizing the beacon placement. However, as is seen in Fig. 3.4a, still the \( Z \)-axis estimation accuracy is slightly worse than the location estimation accuracy of the \( X - Y \) plane. To further improve the height estimation,
we use a separate ultrasonic transceiver mounted on-board the drone to continuously estimate the height. Then, using a filter, we incorporate this measurement with the Z-axis estimation that has been already available from the first step of iDROP. This significantly improves the Z-axis estimation accuracy.

We mount the ultrasonic transceiver facing upward on-board the drone and find the distance between the drone and the ceiling by calculating the time of flight of the ultrasonic signal transmitted from the sensor on-board the drone, after it is reflected from the ceiling. Then, simply by subtracting this result from the room height, we find the drone’s height at each moment. The channel between the drone and the ceiling is usually more reliable than the one between the drone and the floor because usually there are no objects between the drone and the ceiling that induce errors. The height estimation using this extra ultrasonic transceiver is then: $h_{\text{drone}} = H - d$ and $d = c_{\text{sound}} \cdot t/2$; where $d$ is the distance between the drone and the ceiling, $t$ is the total time that takes the signal to travel from ultrasonic transceiver on-board the drone and hit the ceiling, reflecting, and is received in the ultrasonic transceiver on-board the drone, $H$ is the room height, and $h_{\text{drone}}$ is the estimation for drone’s height. The final revised height estimation is then: $z_{\text{revised}} = w_1 \cdot z + w_2 \cdot h_{\text{drone}}$, where $w_1$ and $w_2$ are the weights assigned for the Z-axis estimation from the first stage of iDROP and this stage, and $z$ is the estimated height from the first stage of the iDROP.

### 3.6 Experimental Setup and Performance Evaluation

#### 3.6.1 Experimental Setup

To assess the performance of iDROP, we conducted some experimental tests coupled with MATLAB simulations. As already illustrated in Fig. 2.11 in the past chapter, the experi-
Figure 3.4: (a) Average Localization error vs. SNR (dB). (b) Z-axis and overall three-dimensional localization error for seven random trajectories before and after applying the beacon placement optimization algorithm.
3.6. EXPERIMENTAL SETUP AND PERFORMANCE EVALUATION

Table 3.1: Comparison of iDROP with Comparable Schemes.

<table>
<thead>
<tr>
<th>System</th>
<th>Signal</th>
<th>Multi-path Solution</th>
<th>3D Localization</th>
<th>Claimed Accuracy</th>
<th>Optimized Beacon Placement</th>
<th>Improved Z-axis Estimation</th>
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</thead>
<tbody>
<tr>
<td>[37]</td>
<td>Ultrasound, TOA</td>
<td>None</td>
<td>No</td>
<td>2 cm for 2D</td>
<td>No</td>
<td>No</td>
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<td>[51]</td>
<td>Acoustic, TOA</td>
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<td>5.2 cm for 3D</td>
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<td>FHSS</td>
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<td>1.4 cm for 3D</td>
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<td>No</td>
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<tr>
<td>[96]</td>
<td>Ultrasound, TOA</td>
<td>FH-CDMA</td>
<td>Yes</td>
<td>1.5 cm for 3D</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>iDROP</td>
<td>Ultrasound, TOA</td>
<td>FH-CDMA</td>
<td>Yes</td>
<td>1.2 cm for 3D</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Experimental test setup consists of two stations: first, the drone and the system on-board it, and the second one is the ground control station which helps to input the transmitted data into the MATLAB program running on a Dell XPS 15 laptop. The drone that we used for the experiment is a Parrot Mambo Drone. It is a cheap, off-the-shelf, and ultralight drone suitable for indoor experiments. Also, it has the capability of carrying some light loads. The designed system mounted on-board the drone consists of an Arduino Uno micro-controller connected to an HC-SR04 sensor for ultrasonic distance measurement purposes and a XBee S1 module for wireless communication with the ground controller. In the ground control unit, another Arduino Uno micro-controller connected to a XBee S1 receives the data and further transfers it into the MATLAB program running on the laptop. All the experiments were conducted in a hallway inside a building with dimensions 5 m × 5 m × 4 m. This hallway contains several objects and obstacles, including tables, chairs, glass windows, etc.
3.6.2 Final Results

In Fig. 3.4a, a comparison of the localization error for both the $Z$-axis and the $X-Y$ plane, in different SNRs, before and after using the proposed EA optimization framework is seen. The new $(x, y, z)$ coordination of the ultrasound transmitters in the room that is obtained from the EA optimization framework is $(4.5, 0, 2.5), (5, 4, 3.5), (1, 5, 2)$, and $(1.5, 2, 4)$ where all the numbers are in meter. As is seen in this figure, using the optimized placement for transmitters improves the $Z$-axis localization accuracy significantly. Moreover, it does not drastically degrade the $X-Y$ plane localization accuracy. The reason for developing the EA optimization framework is to find the optimal placement of transmitters in the room to mitigate the localization error due to the relative geometry between the transmitters and the target drone. More specifically, it is designed in a way to mitigate the $Z$-axis localization error without significantly degrading the $X-Y$ plane localization accuracy. This plot shows that the proposed EA optimization framework performs as is expected.

In Fig. 3.4b, the $Z$-axis localization error for seven different random drone trajectories in cases where the hybrid FH-CDMA technique is used for localization in comparison with the cases where leverage the hybrid FH-CDMA technique for localization in combination with using the new optimal coordination for transmitters, is seen. Moreover, this figure shows a similar comparison for the overall three-dimensional localization error. As depicted in this figure, for all of these random trajectories for the drone in the room, both the $Z$-axis and the overall localization error improve when the optimized beacon placement is used.

In Fig. 3.5a, the localization error of the $Z$-axis with the $X-Y$ plane is compared. This figure justifies the necessity of having the auxiliary sensor for height estimation. As is seen in this figure, even though the optimized placement for transmitters improved the $Z$-axis localization accuracy significantly, the $Z$-axis error may not be as low as the $X-Y$ plane
Figure 3.5: (a) Evaluating the performance of iDROP: Comparison between the $X-Y$ plane average estimation error and the $Z$-axis. (b) Evaluating the performance of iDROP: overall three-dimensional localization accuracy.
localization error. This figure shows how the last step of iDROP further improves the Z-axis estimation by constantly transferring the measured data from the ultrasound sensor onboard the drone (HC-SR04) to the receiver module connected to the Dell XPS 15 laptop. Therefore, iDROP successfully improves the Z-axis estimation without sacrificing the X–Y Plane localization accuracy.

In Fig. 3.5b, the performance of iDROP with that of the benchmark scheme (which relies only on FH-CDMA distance estimation to localize a target drone) in terms of the overall three-dimensional localization error is compared. The average value of three-dimensional localization error for iDROP is 1.2 cm. As is seen in the figure, the benchmark scheme’s localization error is more than twice that of iDROP. This is because the benchmark scheme merely focuses on mitigating ranging-based error by deploying the FH-CDMA communication scheme for localization. Other drone localization schemes proposed in the literature do the same and try to improve the localization accuracy by proposing their technique to mitigate the ranging-based error. However, iDROP proposes a scheme that deals with both ranging-based error and geometry-related error and in this way, it further improves the accuracy.

In Fig. 3.6a, the performance of the full scheme iDROP is evaluated by showcasing the localization error distribution for the X–Y plane over the five different trajectories. To that end, we depicted the boxplot for each trajectory. On each box, the central read mark indicates the median, and the bottom and top edges of the box (blue lines) indicate the 25-th and 75-th percentiles, respectively. The whiskers extend to the most extreme data points not considered outliers, and the outliers are plotted individually using the red + marker symbol. It is noteworthy that in Fig. 3.6a, for each trajectory, we showcase the X–Y plane localization error distribution rather than merely showing the average error. This helps us to know the median of the error as well as the region in which the majority of the error is located.
3.6. EXPERIMENTAL SETUP AND PERFORMANCE EVALUATION

Figure 3.6: Evaluating the performance of the full scheme iDROP: (a) $X - Y$ plane localization error distribution; (b) Overall three-dimensional localization error distribution.
Moreover, this shows the number of outliers and how far they are from the median.

Fig. 3.6b showcases the exact same thing as Fig. 3.6a with one difference. In this figure, instead of presenting the $X-Y$ plane localization error distribution, the overall three-dimensional localization error distribution is represented.

iDROP achieves significant improvement in comparison with other drone localization schemes in the literature [37, 38, 55, 111, 112]. For instance, in [111], O’Keefe et al. proposed a scheme that incurs an average error of 5.2 cm for three-dimensional localization for drones. The scheme proposed by Segers et al. [37] incurs an error of 2 cm or greater in terms of localization error just for the $X-Y$ plane (two-dimensional localization). Table 3.1 compares iDROP with other comparable schemes introduced in the literature.

As is seen in Table 3.1, iDROP proposes a novel solution for each of the existing challenges, and that is why it has a better overall performance in comparison with the works in the literature. For instance, in [51], Mao et al. proposed an FMCW technique to overcome the multi-path fading effect; however, their scheme was designed just for tracking a drone on a line and they did not consider three-dimensional localization and its challenges. In [55], Famili et al. proposed a multi-path robust scheme for three-dimensional localization of drones; however, their scheme had significantly low accuracy for $Z$-axis estimation. Moreover, their claimed accuracy is merely based on MATLAB simulations and they did not provide any real-life experiments with an actual drone to assess their proposed scheme. In [111], O’Keefe failed to propose an optimal solution for beacons because of their choice of system design and lack of signal separation capability. They also did not consider a multi-path-robust communication technique. Therefore, their proposed scheme had high localization errors. In another work [96], Famili et al. proposed a multi-path robust system for drones’ three-dimensional localization in indoor environments. Even though their proposed scheme fixed the signal separation challenge and eliminated the unnecessary communication
link between the drone and the transmitter beacons in the room, they failed to explain the reason behind having a bad Z-axis estimation and their system lacked the optimal beacon placement analysis. iDROP has robustness against multi-path fading and noise and provides signal separation capability. Moreover, it proposes an optimized placement for beacons in the room and improves the Z-axis estimation accuracy. Overall, iDROP provides a highly accurate 3D localization in real-time scenarios for drones.

### 3.7 Summary

In this chapter, we presented a multi-path-robust localization system with optimized beacon placement that can be used for autonomous navigation of drones in indoor environments. First, the drone can locate itself in three-dimensional space using the TOA of the received FH-CDMA ultrasound waveform. Then, to improve the Z-axis estimation accuracy, an EA optimization framework finds an optimized location for the transmitter beacons in the room. Moreover, an additional ultrasound transceiver provides a separate measurement of the drone’s height for improving the Z-axis estimation accuracy. Finally, we evaluate the performance of our proposed system by conducting experiments coupled with simulations.
Chapter 4

OFDRA: Optimal Femtocell Deployment for Accurate Indoor Positioning of RIS-Mounted AVs

The pursuit of high-accuracy localization without relying on the global positioning system (GPS) has gained significant interest in recent years. The deployment of autonomous vehicles (AVs) in diverse indoor applications exemplifies a prominent domain where the demand for a robust positioning system is evident. With the advancements in 5G and beyond radio access networks (RAN), the availability of new positioning signals presents an opportunity to deliver accurate location estimates for these applications. Nevertheless, these signals encounter substantial path losses in indoor environments. Additionally, the precise localization within existing frameworks requires stringent synchronization, which is challenging to meet. In this chapter, we propose **OFDRA: Optimal Femtocell Deployment for Accurate Indoor Positioning of RIS-Mounted AVs**, a novel positioning framework that is robust against multipath and does not require strict synchronization between anchor-anchor or anchor-target entities. The first design objective of OFDRA is the mitigation of ranging errors by leveraging a compact reconfigurable intelligent surface (RIS) mounted on top of AVs acting as a programmable mirror in a 5G network. The second design objective is to achieve optimal anchor placement in three-dimensional indoor spaces, thereby reducing the geometric dilution.
of precision (GDOP) and mitigating geometric-induced errors in the final position estimation. Our experimental verification reveals that the localization error is influenced by GDOP, encompassing both the $X-Y$ plane and $Z$-axis estimations. Through optimized anchor placement, OFDRA demonstrates a seven-fold enhancement in $Z$-axis accuracy compared to the state-of-the-art, achieving a sub-1 m three-dimensional accuracy for more than 95% of cases.

4.1 Motivation

Up until this point, across the preceding chapters, we have formulated a comprehensive system tailored for achieving high-accuracy localization of Unmanned Aerial Vehicles (UAVs) within indoor environments. To mitigate localization errors, we derived an error formula for three-dimensional ranging-based localization. This analysis revealed the existence of two distinct error sources: ranging-based errors and those induced by the relative geometry between transmitters and receivers. Our devised solution addresses both error origins, culminating in the creation of a holistic scheme that effectively addresses all forms of error and bestows centimeter-level precision to the three-dimensional localization of drones. The core mechanism involves the measurement of time of arrival (TOA) of ultrasound acoustic signals to gauge the distances separating the UAV and surrounding beacons, followed by trilateration to yield the final target location. The strategic deployment of beacons underwent meticulous optimization through an evolutionary algorithm, accounting for beacon count and placement to minimize error arising from relative geometric aspects. As a result, our proposed system achieved an impressive sub-2 centimeter position error within three-dimensional space.

Our pursuit of ultrasound-based acoustic signal localization was marked by a substantial investment of time and effort. This choice stems from several factors: (i) their propagation
at the speed of sound, a significantly slower rate than the light speed of RF signals, thereby offering enhanced accuracy through cost-effective and less intricate devices with reduced sampling rate requirements; (ii) their diminished susceptibility to co-channel interference relative to RF signals, especially in the crowded 2.4 GHz spectrum; (iii) their imperviousness to propagation through walls and ceilings, a contrast to RF signals with their superior penetration, making them prone to adjacent room interference; and (iv) instances, such as medical facilities and nuclear power plants, where RF signal deployment is proscribed, a restriction not typically applicable to acoustic signals.

Nevertheless, a salient caveat looms—requiring the installation of these ultrasound acoustic sensors within the target area for system operability. Availability of these sensors is limited in existing spaces, impeding repurposing for positioning needs. Moreover, this modality remains suited primarily for indoor settings, with its viability not extending to outdoor scenarios devoid of GPS signals. This backdrop prompts us to contemplate alternate signals viable for localization, satisfying criteria such as: (i) ubiquitous presence requiring no proprietary beacon installation, utilizing existing technology for positioning goals; (ii) applicability within outdoor environments; (iii) acceptable performance even if not achieving centimeter-level precision, as long as the accuracy approximates meters.

Foremost in this regard is Wireless Fidelity (Wi-Fi), though its utility diminishes outdoors. Cellular signals, on the other hand, emerge as a potential candidate. Historically utilized for positioning since early generations, their initial purpose lay in emergency call location. Initial techniques employed cell identification (CID), associating the user’s location with the closest base station. However, the accuracy was modest, around half a kilometer or potentially worse, contingent on cell size. Over generations, accuracy has improved, with Long-Term Evolution (LTE) achieving sub-100 meter accuracy. Yet, this falls short of our needs. Encouragingly, the recent iterations of 5G New Radio (NR) introduce the prospect
of meter-level accuracy. While still trailing ultrasound signals in precision, their widespread presence and applicability to indoor and outdoor settings positions them as a feasible choice for drone localization in GPS-absent scenarios. Hence, they warrant investigation as a surrogate technique for drone positioning in the absence of GPS signals within this chapter.

4.2 Chapter Overview

For years, precise localization and navigation have received significant attention in both military and civilian use cases. An outcome of this research has been the invention of the global navigation satellite system (GNSS) to provide localization using a constellation of satellites [113]. As a prime example within the GNSS domain, the global positioning system (GPS) facilitates navigation in outdoor environments with clear visibility between satellites and a target. However, in urban environments, particularly indoors, blockages and obstructions can compromise the reliability of GPS. Therefore, the development of alternative techniques for indoor positioning has been an open problem for decades.

The automated vehicle industry, encompassing terrestrial like automated guided vehicles (AGVs) and aerial such as unmanned aerial vehicles (UAVs), also known as drones, has experienced rapid growth [114, 115]. These high-mobility equipment variants enable diverse logistical operations within smart buildings. In large warehouses, the automation of item organization is facilitated by AGVs, such as autonomous forklifts [116], as well as UAVs equipped with radio frequency identification (RFID) tag scanners [117]. These systems rely on precise indoor localization for the efficient execution of tasks.

To provide high-accuracy localization for such use cases, substitutes for GPS have been developed. The majority include but are not limited to ranging-based approaches [94], vision-based techniques [118], and fingerprinting methods [119]. Ranging-based methods excel
in delivering precise location information due to their efficient and straightforward infrastructure deployment, making them the preferred choice for location-tracking applications. Vision-based techniques can incur high expenses, both in terms of hardware cost and software computations. Additionally, fingerprinting methods may experience a significant decline in accuracy when real-time environmental changes occur after the offline fingerprinting stage. To that end, as the foundation of our approach, we rely on ranging-based methods to gather the necessary signal information to be used in our localization framework. With this choice of measurement acquisition, the overall localization estimation error for indoor environments can be expressed as a multiplication of the ranging-based error and the geometric dilution of precision (GDOP).

For the ranging methods, errors are caused by inaccuracies that occur while capturing the signal travel time between a user and the localization system. The lack of synchronization among nodes of the localization system as well as the users, in addition to the multipath fading characteristics of indoor environments, compromises the accuracy of the signal measurements. Additionally, the poor placement of the localization anchors can yield an undesirable GDOP and inaccurate $X - Y$ plane and $Z$-axis estimations. For instance, with ranging errors in sub-2 m, with a GDOP of 10, the final positioning error ends up being in
4.2. Chapter Overview

the sub-20 m range, which is unacceptable for sensitive use cases.

In this article, we propose **OFDRA: Optimal Femtocell Deployment for Accurate Indoor Positioning of RIS-Mounted AVs**, a high-accuracy localization system for indoor environments. In contrast to the existing work which focuses mainly on two-dimensional localization [120], OFDRA operates in three dimensions. To do so, it addresses two key challenges:

**Challenge 1 - Ranging errors:** Leveraging the high bandwidths of 5G New Radio (NR), OFDRA can obtain accurate estimations of the signal time of flight (TOF). These measurements are then translated into distances and employed in a three-dimensional localization system leveraging trilateration. Additionally, in order to relax the stringent synchronization requirements between anchor-anchor or anchor-target, as well as mitigate the effects of multipath fading, OFDRA is the first to propose utilizing a compact reconfigurable intelligent surface (RIS) mounted on mobile equipment. Using this RIS, a reflection of any incident signal on the drone can be sent back to the localization anchor. This allows for more accurate calculations to be performed at the anchor for the signal TOF without requiring synchronization among the nodes.

**Challenge 2 - Geometric-induced errors in 3D:** By driving the Cramér-Rao Lower Bound (CRLB) for localization and subsequently obtaining the positioning error bound (PEB), we reveal that the localization error in trilateration techniques stems not only from ranging errors but also from a high GDOP caused by the relative geometry between user and anchor nodes. Experimental verification confirms that in three-dimensional space, Z-axis estimation errors have a greater impact on GDOP deterioration than X – Y plane errors. Unlike the existing work in literature [96] which focuses on improving the location accuracy by only mitigating the ranging errors, OFDRA considers both the ranging- and the geometric-induced errors. For the latter, OFDRA proposes a novel optimization framework to derive the optimal anchor placement. Ultimately, this enables the accurate localization
of high-mobility targets in three-dimensional spaces.

A visual representation of OFDRA is illustrated in Fig. 4.1. Our contributions are summarized below:

- We propose OFDRA, a high-accuracy localization system substitute to GPS for indoor use-cases, which unlike the state-of-the-art, considers the three dimensions instead of just two-dimensional space.

- Firstly, we take advantage of large available bandwidths and high carrier frequencies to propose compact RIS boards to be mounted on high mobility targets to mitigate the effects of multipath fading and obtain round trip travel time of 5G signals without synchronization between anchor-anchor or anchor-target.

- Secondly, by deriving the CRLB on trilateration techniques, we show that the errors in ranging measurements originate from two sources: the ranging error and the geometric-induced error. For the latter, experiments show that the $Z$-axis yields poorer estimates compared to the $X−Y$ plane.

- Thirdly, we formulate a novel optimization framework for a problem with non-polynomial (NP)-Hard characteristics and utilize evolutionary algorithms (EAs) to find the optimal placement of 5G femtocells to obtain a low GDOP. This includes the $Z$-axis estimation which is currently absent from the localization literature and is essential for targets that operate in three dimensions.

- Finally, through comprehensive evaluations, we show that OFDRA improves the accuracy in the $Z$-axis by a factor of 7 compared to the state-of-the-art and provides a final three-dimensional accuracy of sub-1 (m) for more than 95% of the scenarios.
4.3 OFDRA Stage One: Multipath-Robust Synchronization-Free RIS-based Ranging

In this section, we describe how OFDRA addresses the first challenge, mitigating ranging errors. First, in Sec. 4.3.1, we show how the RIS can be used for mitigating the effects of multipath for synchronization-free localization. Next, in Sec. 4.3.2, we derive the distance estimation’s error bound of the proposed RIS-based ranging in the 5G-NR femtocell network. Finally, in Sec. 4.3.3, we derive the trilateration formula for three-dimensional positioning.

4.3.1 RIS-ranging Framework

In this section, we briefly introduce the RIS, also known as intelligent reflecting surface (IRS), large intelligent surface (LIS), or software-controlled metasurface. We explore the metasurface-based and antenna-based RIS models and evaluate their prospects. Finally, we explain how we exploit RIS in a novel way compared to more traditional approaches to achieve a synchronization-free multipath-robust positioning scheme that is similar to RTT-based techniques.

RIS seeks to regulate the properties of incident signals (such as reflection, refraction, absorption, focusing, and polarization) through either conventional antenna arrays or metasurfaces.

**Antenna Array-based RIS:** With many cheap and passive antenna elements acting as reflectors without RF chains, it becomes possible to construct an RIS instead of the traditional massive multiple-input multiple-output (mMIMO) system. A dielectric substrate with relative permittivity $\epsilon_r$ is positioned on a metallic ground plane. The metallic patch elements are then mounted on top of this substrate to form the RIS. The inter-spacing distance in the antenna array is typically in the order of half of the incident signal’s wavelength ($\lambda/2$).
By modeling a patch antenna element as a cavity resonator, oscillations can be triggered when incoming waves are at the resonance frequency. As a result, the reflection angle of the outgoing signal can be controlled so that it equals the incident angle of the incoming signal.

**Metasurface-based RIS:** Compared to antenna arrays without RF chains, metasurface-based RISs are two-dimensional arrays comprised of electrically thin and dense elements that have the desired properties. These elements are referred to as meta-cells, unit-cells or meta-atoms. Provided that the elements of the metasurface are fixed, the same equations that are used to describe the behavior of the antenna array RIS, can also describe the signal reflected by a metasurface. The fundamental difference between the two is that the size of a meta-cell is much smaller than the wavelength of the signal, typically in the range of $\lambda/10$ to $\lambda/5$ [62]. According to the generalized Snell’s law, reflection and refraction can be achieved in other directions by distributing these unit cells with different polarizations.

Leveraging RIS, the propagation environment can be controlled by actively sensing the radio waves and reflecting them in desired directions. As illustrated in Fig. 4.2, the traditional approach of deploying an RIS involves strategically installing it as a large surface so that it can collect the electromagnetic waves emitted from the transmitters. Next, using the controllers
inside the RIS elements, the transmitted wave can be steered towards the corresponding recipient in the environment.

In contrast to the depiction in Fig. 4.2, we leverage RIS in a novel way. Instead of having a large metasurface installation, we propose a more compact and portable RIS that can be mounted on top of high-mobility targets such as drones. The scenario is depicted in Fig. 4.3. This RIS will receive and reflect signals from 5G-NR femtocells that have been installed on the ceiling facing downwards.

Femtocells are commercial-off-the-shelf (COTS) small base stations that can be easily installed to provide 5G cellular coverage for indoors with a range of 10 m to 50 m. When the 5G-NR downlink positioning reference signal (PRS) is transmitted from the femtocell, it is received by the RIS mounted on the drone. The PRS is a dedicated signal used for localization and positioning purposes. It was introduced in the third-generation partnership project (3GPP) for 4G and is also present in 5G and subsequent generations. In our scheme, the PRS strikes the RIS with an incidence angle \( \theta \) while traveling at an angle \( \phi \) with respect to the \( X-Y \) plane. In this scenario, the components of the signal on the RIS \( X-Y \) plane are given by:

\[
S_{RIS}(x, y) = f_1^T h_{gNB\rightarrow RIS}(\theta, \phi) s_{prs}(\theta, \phi);
\]

(4.1)

where \( h_{gNB\rightarrow RIS}(\theta, \phi) \) is the wireless channel between the femtocell and the RIS, \( s_{prs}(\theta, \phi) \) is the PRS sent by the femtocell, and vectors \( f_1 \) and \( f_2 \) are defined as:

\[
f_1 = [1 \ e^{jk_1 d \sin \alpha} \ e^{jk_2 d \sin \alpha} \ldots e^{jk(N-1) d \sin \alpha}]^T; \ f_2 = [1 \ e^{jk_1 d \sin \beta} \ e^{jk_2 d \sin \beta} \ldots e^{jk(M-1) d \sin \beta}]^T; \]

(4.2)

with \( k = 2\pi/\lambda \), \( \sin \alpha = \cos \theta \sin \phi \), \( \sin \beta = \sin \theta \sin \phi \), and \( d \) is the inter-spacing distance in the antenna array, for an \( N \times M \) RIS. With \( (0, 0, 0) \) as the reference point, the relative phase
shift at point \((n, m)\) on the RIS is given by:

\[
\Omega(n, m) = k(nd \sin \theta \cos \phi + md \sin \theta \sin \phi).
\] (4.3)

For a reflection in any given \(\theta_r, \phi_r\) direction, the RIS elements need to be configured as:

\[
\Omega_r(n, m) = k(nd \sin \theta_r \cos \phi_r + md \sin \theta_r \sin \phi_r);
\] (4.4)

which means that the phase change required at any point \((n, m)\) on the RIS equals:

\[
\Omega_s(n, m) = \Omega_r(n, m) - \Omega(n, m) = knd(\sin \theta_r \cos \phi_r - \sin \theta \cos \phi) + kmd(\sin \theta_r \sin \phi_r - \sin \theta \sin \phi)\) (4.5)

In our system, the goal is to control these values in a way that any incoming signal achieves a reflection on the incident path. So, we can set the values for \(\theta_r\) and \(\phi_r\) based on the \(\theta\) and \(\phi\) of the received incident signal and we have:

\[
\Omega_s(n, m) = \Omega_r(n, m) - \Omega(n, m) = -2kd(n \sin \theta \cos \phi + m \sin \theta \sin \phi).\] (4.6)

With such a reflection, we eliminate the need for stringent time synchronization between the target and the femtocell, since both the transmit and receive time will be measured at the femtocell. Furthermore, the estimated distance will be the absolute distance between the transmitter and receiver, not the difference of distances.

**Effect on Time Synchronization:** Our RIS-based framework relies on RTT measurements, which means that it does not require synchronization among the anchor nodes or the anchors and the target. Compared to observed time difference of arrival (OTDOA) techniques which need to have synchronization between the anchors, OFDRA discards this requirement and yields better localization estimates. Compared with existing RTT-based
4.3. OFDRA Stage One: Multipath-Robust Synchronization-Free RIS-based Ranging

approaches which require the target to receive, process, and respond to an incoming localization signal, OFDRA simply uses the compact RIS to reflect the signal back to the transmitting anchor. Hence, no processing on the UE side is required.

Effect on Multipath Fading Mitigation: Multipath components are copies of the original signal and they deteriorate the exact estimation of the original signal. However, with the RIS-based scheme, any copy of the original signal will be reflected back on the path it came from. Due to high path loss and absorption of the objects on the way, the multipath signals are either absorbed in their origin or reach femtocell with extremely low power, on the level of the noise. Let us consider a scenario where for the original PRS signal transmitted from the femtocell, there exist two multipath copies, one from a wall reflection and another from an object reflection. If RIS measures the TOA of the original signal and attempts to determine the distance based on this measurement, then due to the existence of these different copies, accuracy will be affected. However, based on the specific configuration used in OFDRA, RIS simply reflects back whatever it receives with the same incident angle. This means that the original signal goes back to the femtocell directly, while the multipath components are sent back toward their original reflectors.

In other words, for other multipath signals, which originate from reflections off walls, ceilings, floors, or other objects in the environment, the RIS sends them back to their respective origins and not to the femtocells. These signals may then undergo additional reflections, potentially including reflections back toward the femtocells. However, it is important to note that they have already been reflected once by the RIS and twice by their respective sources. As a result, their power is significantly reduced. This reduction in power can cause the multipath signal to get damped in its origin and not undergo any further reflection, or if there is a reflection, it will have extremely low power, in the order of the noise level, and will be disregarded by the system. We visually illustrate this concept in Fig. 4.3.
As a result of this configuration for the RIS, multipath components cannot corrupt the estimation of the original signal. This means that the only signal received back at the femtocell is the one transmitted from the femtocell and reflected off the RIS mounted on the user. It is imperative to note that even though OFDRA is designed to perform in the presence of non-line-of-sight (NLOS) multipath signals, the presence of the line-of-sight (LOS) signal between the femtocell and the RIS is still required for high-accuracy localization. This signal, in fact, is the original PRS signal transmitted from the femtocell.

After successfully obtaining the TOA of the original signal while mitigating all the multipaths, the distance between the drone/forklift and the $i$-th femtocell is then determined using $d_i = c \cdot t_i/2$; where $t_i$ is the time when the signal is received at the $i$-th femtocell. This represents the time it takes the PRS to go from the $i$-th femtocell to the RIS mounted on the target and then be reflected back to the same femtocell ($t_i = t_{\text{femtocell}_i \rightarrow \text{RIS} \rightarrow \text{femtocell}_i}$).

### 4.3.2 RIS-ranging Error Bound

Positioning can be expressed as an estimation problem where the Cramér-Rao Lower Bound (CRLB) is a common benchmark describing the best achievable theoretical performance for an unbiased estimator. It expresses a lower bound on the variance of unbiased estimators for an unknown parameter and it is equivalent to the inverse of the Fisher Information Matrix (FIM). For an unknown parameter $\theta$ to be estimated from the measurements of random variable $x$ with the probability density function $f(x; \theta)$, the variance of any unbiased estimator $\hat{\theta}$ is bounded by the inverse of the Fisher information $I(\theta)$:

$$\text{var}(\hat{\theta}) \geq \frac{1}{I(\theta)};$$  \hspace{1cm} (4.7)

where $I(\theta) = E[(\frac{\partial}{\partial \theta} \ln f(x|\theta))^2]$. 


To evaluate our 5G-NR RIS-based positioning and determine the exact relationship between effective bandwidth and localization performance, we obtain the Positioning Error Bound (PEB) from the CRLB. The PEB is the lowest error bound achievable for a positioning technique as well as the square root of the CRLB for the position estimation.

\[ PEB = \sqrt{CRLB(\hat{\zeta})} = \sqrt{tr(I^{-1}(\zeta))}; \]  \hspace{1cm} (4.8)

where \( \zeta \) is an unknown position parameter vector defined as \( (\zeta = [x \ y \ z]^T) \), \( \hat{\zeta} \) is the corresponding estimation and \( tr\{I(\zeta)\} \) is the trace of the FIM. Similar to [94], we can show

\[ PEB = \sigma(r) = \sigma_r \cdot GDOP; \]  \hspace{1cm} (4.9)

where \( \sigma_r \) is the variance of the distance estimation error between the target and the anchor, while GDOP is the Geometric Dilution of Precision as a result of the geometric-induced error due to anchor placements.

In this section, we focus on mitigating \( \sigma_r \) and shift our attention to GDOP in Sec. 4.4.1. Hence, our first goal is to derive the CRLB on the distance estimator for our proposed RIS-based ranging system. To achieve this, we start by deriving the CRLB of a one-dimensional ranging, which is the CRLB for the distance estimation between the anchor node and the user:

\[ CRLB(\hat{d}) = c \cdot CRLB(\hat{\tau}); \]  \hspace{1cm} (4.10)

where \( c \) is the speed of light, \( \hat{d} \) is the estimated distance, and \( \hat{\tau} \) is the estimated TOA of the signal.

Based on Eq. 4.10, in order to find the CRLB on \( d \), we first need to expand the CRLB on \( \tau \). For this, we start by considering the downlink PRS used by LTE as well as 5G and
beyond networks for localization measurements. PRS is an Orthogonal Frequency-Division Multiplexing (OFDM) modulated signal [61] which can be sampled as:

$$s_i(t) = \frac{1}{\sqrt{N}} \sum_{k=-N/2}^{N/2-1} S_i[k] \exp(j2\pi k\Delta f t);$$  \hspace{1cm} (4.11)$$

where $s_i(t)$ is the signal representation of the $i$-th OFDM symbol in the time domain, $S_i[k]$ is the signal allocated to the $k$-th subcarrier of the $i$-th OFDM symbol, $N$ is the total number of subcarriers in each OFDM symbol, $\Delta f = \frac{1}{NT_s}$ is the effective bandwidth and $T_s$ is the sampling time period dedicated to one symbol. After the signal passes through the channel, the receiver sees the time-delayed version of the transmitted signal and the noise as:

$$y_i[n] = s_{R,i}[n] + n_i[n];$$  \hspace{1cm} (4.12)$$

where $n_i[n] \sim \mathcal{CN}(0, \sigma_n^2)$ and $s_{R,i}[n]$ is given by:

$$s_{R,i}[n] = s_i(nT - \tau) = \frac{1}{\sqrt{N}} \sum_{k=-N/2}^{N/2-1} S_i[k] \exp(j2\pi k\Delta f (nT - \tau)).$$  \hspace{1cm} (4.13)$$

To derive the lower bound on the variance for the estimate for $\tau$, we have:

$$\text{var}(\hat{\tau}) \geq \text{CRLB}(\hat{\tau}) = \frac{1}{E[(\frac{\partial}{\partial \tau} \ln p(y|\tau))^2]};$$  \hspace{1cm} (4.14)$$

where $p(y|\tau)$ is the PDF of $y$ given $\tau$. We can simplify Eq. 4.14 into the closed-form:

$$\text{var}(\hat{\tau}) \geq \text{CRLB}(\hat{\tau}) = \frac{\sigma_n^2}{8\pi^2(\Delta f)^2 \sum_{i=0}^{N_{\text{symbol}}-1} \sum_{k=-N/2}^{N/2-1} k^2 S_i[k]^2} = \frac{1}{8\pi^2(\Delta f)^2 K};$$  \hspace{1cm} (4.15)$$

where $N_{\text{symbol}}$ is the number of the OFDM symbols used in the PRS frame structure and $K$
4.3. OFDRA Stage One: Multipath-Robust Synchronization-Free RIS-based Ranging

is:

$$\mathcal{K} = \frac{1}{\sigma_n^2} \sum_{i=0}^{N_{symb} - 1} \sum_{k=\pm N/2} k^2 S_i[k]^2. \quad (4.16)$$

In LTE, each subframe transmission time interval (TTI) is set to 1 ms, consisting of two slots. In 5G-NR, each subframe may contain 1, 2, 4, 8, 16, and 32 slots. For both LTE and 5G, each slot consists of 14 OFDM symbols [61].

Now, if we plug in Eq. 4.15 into Eq. 4.10 we can obtain the bounds for a one-dimensional ranging assuming the transmitting anchor and the target is fully synchronized:

$$CRLB(\hat{d}) = \frac{c}{8\pi^2(\Delta f)^2 \mathcal{K}}; \quad \sigma_r \geq \sqrt{CRLB(\hat{d})} = \frac{\sqrt{c}}{2\sqrt{2\pi \Delta f}} \sqrt{\frac{1}{\mathcal{K}}}. \quad (4.17)$$

To derive the ranging error bound for our proposed RIS-based method, we need to replace $s_i(nT - \tau)$ with $s_i(nT - 2\tau)$ in Eq. 4.13 because the flight time in our proposed scheme (similar to RTT) is twice the time delay in a one-dimensional ranging. Then, following the same procedure and taking the derivative with respect to $\tau$, similar to [121], we can show that: $\sigma_{RIS} = \frac{1}{2} \cdot \sigma_r$, where $\sigma_{RIS}$ is the RIS-based ranging error bound. Using Eq. 4.17, obtain the error bound for our proposed RIS-based ranging system as follows:

$$\sigma_{RIS} = \frac{1}{2} \cdot \sigma_r \geq \frac{1}{2} \sqrt{CRLB(\hat{d})} = \frac{1}{2} \cdot \frac{\sqrt{c}}{2\sqrt{2\pi \Delta f}} \sqrt{\frac{1}{\mathcal{K}}}. \quad (4.18)$$

Assuming perfect synchronization between the anchor nodes and the target is not plausible. Therefore, we proposed RIS-based ranging to fully overcome this issue by employing a technique similar to RTT. We derived the CRLB for our scheme to be able to evaluate the performance. The other candidate solution to relax the synchronization requirement is the legacy OTDOA technique. Here, we briefly derive the performance error bound for the
OTDOA technique to be able to compare it with our proposal.

For OTDOA, all transmitting anchors are assumed to be synchronized (which this is not a requirement in our scheme), but no synchronization exists between any of the anchors and the target. As a result, the estimated time delay is given by:

\[ \tau = \tau_{TOF} + \tau_{Biased}; \]  

(4.19)

where \( \tau_{TOF} \) is the TOF of the signal and \( \tau_{Biased} \) is the synchronization bias between an anchor and the target. According to Álvarez et al. [122], under the assumption that the time measurements are uncorrelated, the variance associated with the distance difference between two nodes is the sum of the variances for each node. Using this assumption, we can plug in Eq. 4.15 into Eq. 4.10 to obtain \( CRLB(\hat{d}_{OTDOA}) \) as:

\[ CRLB(\hat{d}_{OTDOA}) = \frac{c \sigma^2}{8 \pi^2 (\Delta f)^2} \cdot \left( \frac{1}{h(SNR_m)} + \frac{1}{h(SNR_n)} \right); \]  

(4.20)

where \( m, n \in \{1, 2, ..., M\} \) and \( M \) is the number of anchor nodes. For each \( m \)-th anchor we have:

\[ h(SNR_m) = \sum_{i=0}^{N_{symbol}-1} \sum_{k=-N/2}^{N/2-1} k^2 S_{i,m}[k]^2; \]  

(4.21)

where \( S_{i,m}[k] \) is the signal allocated to the \( k \)-th subcarrier of the \( i \)-th OFDM symbol between the \( m \)-th anchor and the target. From this, we can gather that both \( h(SNR_m) \) and \( h(SNR_n) \) are related to the received power from anchor \( m \) and \( n \). In the OTDOA method, we select one as the reference anchor to subtract it from the remainder. From Eq. 4.20, we can show:

\[ \sigma_{r,OTDOA} \geq \sqrt{CRLB(\hat{d}_{OTDOA})} = \frac{\sqrt{c}}{2 \sqrt{2 \pi} \Delta f} \cdot \sqrt{\frac{\sigma^2_m}{h(SNR_m)} + \frac{\sigma^2_n}{h(SNR_n)}}. \]  

(4.22)
Comparing Eq. 4.18 and Eq. 4.22, we can see that the proposed RIS-based ranging technique has a ranging error bound twice the state-of-the-art 5G-NR OTDOA method. Moreover, unlike OTDOA which requires tight synchronization between the anchor nodes, the proposed scheme requires no synchronization.

Based on Eq. 4.18 and Eq. 4.22, we can see that in all the time-based ranging measurements (OTDOA, RTT, or our proposed RIS-base method), the effective bandwidth $\Delta f$ is directly related to the accuracy of the distance estimation. As an example, increasing bandwidth by a factor of five results in the distance estimation error bound being five times smaller. Given that $PEB$ is directly proportional with $\sigma_r$ from Eq. 4.9, this leads to a decrease by a factor of five in the final $PEB$ value. As an example, compared with LTE, which had up to 20 MHz [61] channels, 5G-NR offers up to 100 MHz [61] channels in the sub-7-GHz frequency range, meaning five times better final positioning accuracy.

### 4.3.3 Trilateration

For trilateration in two dimensions, a minimum of three anchor nodes is required to accurately carry out the position estimations. For accuracy in the third dimension, an additional anchor node is required for the same process. Let us consider a scenario where a drone mounted with a compact RIS has a given distance $d_i$ from the $i$-th anchor. In this scenario, the position of the drone is $[x, y, z]^T$ and the location of the $i$-th anchor is denoted by
[x_i \ y_i \ z_i]^T$. Using trilateration, we have:

\[
(x_1 - x)^2 + (y_1 - y)^2 + (z_1 - z)^2 = d_1^2 \\
(x_2 - x)^2 + (y_2 - y)^2 + (z_2 - z)^2 = d_2^2 \\
\vdots \\
(x_n - x)^2 + (y_n - y)^2 + (z_n - z)^2 = d_n^2.
\]  

(4.23)

We can simplify these equations into the linear form \(Ax = b\), where \(A\) and \(b\) are equal to:

\[
A = \begin{bmatrix}
2(x_n - x_1) & 2(y_n - y_1) & 2(z_n - z_1) \\
2(x_n - x_2) & 2(y_n - y_2) & 2(z_n - z_2) \\
\vdots & \vdots & \vdots \\
2(x_n - x_{n-1}) & 2(y_n - y_{n-1}) & 2(z_n - z_{n-1})
\end{bmatrix}, \quad b = \begin{bmatrix}
d_1^2 - d_n^2 - x_1^2 - y_1^2 - z_1^2 + x_n^2 + y_n^2 + z_n^2 \\
d_2^2 - d_n^2 - x_2^2 - y_2^2 - z_2^2 + x_n^2 + y_n^2 + z_n^2 \\
\vdots \\
d_{n-1}^2 - d_n^2 - x_{n-1}^2 - y_{n-1}^2 - z_{n-1}^2 + x_n^2 + y_n^2 + z_n^2
\end{bmatrix}.
\]

Finally, vector \(x = [x \ y \ z]^T\) is given by: 
\[x = (A^T A)^{-1} A^T b.\]

### 4.4 OFDRA Second Stage: Optimal Configuration of Femtocells

In order to reduce the GDOP for improving localization accuracy, we propose an optimization algorithm to minimize the Z-axis estimation error induced by the relative geometry between femtocells and the target, while keeping HDOP in an acceptable range. In this section, we first present the three-dimensional positioning bounds along with the Z-axis error investigation methodology in Sec. 4.4.1. Next, the optimal femtocell placement problem is
4.4. OFDRA Second Stage: Optimal Configuration of Femtocells

formulated in Sec. 4.4.2, followed by the design of the optimization algorithm in Sec. 4.4.3.

4.4.1 Three-dimensional Positioning Bounds and $Z$-axis Error Investigation

As shown in Eq. 4.9, inaccuracy in location estimations is the result of two separate sources of error. The first is the error in estimating the distance between the user and the anchor nodes, known as the ranging error. The second results from the relative geometry between the target and the anchors.

In Sec. 4.3, we showed how OFDRA mitigates the ranging errors by utilizing a compact RIS mounted on mobile targets with 5G femtocells as anchor nodes of the localization system. Using RIS makes the system robust against multipath fading and errors induced by miss-synchronization. Here we first show that the geometric-induced error is caused by a high GDOP value as a result of poor anchor placement. Next, using OFDRA, we design an optimization framework to find the optimal anchor placement to reduce the relative geometry error between the target and the femtocells. To the best of our knowledge, OFDRA is the first scheme that provides solutions to mitigate both the ranging- and the geometric-induced errors to propose high-accuracy localization for three-dimensional indoor spaces.

As we discussed in Sec. 4.3.2, a useful metric for measuring the localization accuracy is the CRLB, a lower bound on the location variance that can be achieved using an unbiased location estimator. Under the assumption that the range measurements are independent and have zero-mean additive Gaussian noise with constant variance $\sigma_r^2$, Rajagopal [93] has shown that a two-dimensional trilateration system with an unbiased estimator, with the positional
error \( \sigma^2(r) = \sigma_x^2(r) + \sigma_y^2(r) \) at position \( r \), has the CRLB variance:

\[
\sigma(r) = \sigma_r \times \sqrt{\frac{N_f}{\sum_{i=1}^{N_f-1} \sum_{j=i+1}^{N_f} A_{ij}}};
\]

where \( N_f \) is the number of femtocells, \( A_{ij} = |\sin(\theta_i - \theta_j)| \), \( \theta_i \) is the angle between \( f_i \) and \( r \), and \( f_i \) is the \( i \)-th femtocell. This shows that the localization error is a multiplication of the ranging measurement error and the GDOP, which is the function of the number of femtocells and the angle between femtocells and the target object. As CRLB is directly proportional with GDOP, we can consider GDOP as a metric for evaluating localization accuracy \[93\].

For the three-dimensional localization of an object at \( (x, y, z) \), we have:

\[
GDOP \cdot \sigma_r = \sqrt{\text{Var}(x) + \text{Var}(y) + \text{Var}(z) + \text{Var}(c\tau)};
\]

where \( c \) is the speed of light and \( \tau \) is the receiver clock offset. In our simulations, since we rely on RTT, there is no timing offset between the transmitter and the receiver. Hence we remove \( \tau \) to obtain:

\[
GDOP = \sqrt{\frac{\sigma_x^2 + \sigma_y^2 + \sigma_z^2}{\sigma_r^2}}. \tag{4.24}
\]

If we denote the position of the drone and the femtocell as \([x \ y \ z]^T\) and \([x_i \ y_i \ z_i]^T\) respectively, then the distance between them is given by:

\[
r_i = \sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2}. \tag{4.25}
\]

As a result of the ranging measurement error, we do not know the exact \( r_i \), which causes errors in the solution of Eq. 4.25 for \([x \ y \ z]^T\). To correlate the error in the solution with
ranging errors, we take the first order differential of Eq. 4.25 as it was done in [94]:

\[ \Delta r_i = \Delta x(x - x_i) + \Delta y(y - y_i) + \Delta z(z - z_i) = \Delta x \cos \alpha_i + \Delta y \cos \beta_i + \Delta z \cos \gamma_i; \]

where \([\cos \alpha_i \cos \beta_i \cos \gamma_i]^T\) is the unit vector pointing from the drone to the \(i\)-th femtocell.

If we let \(\Delta \Theta = [\Delta x \Delta y \Delta z]^T\) be the position error vector and \(\Delta \Psi = [\Delta r_1 \cdots \Delta r_n]^T\) be the target range error vector, then we can define matrix \(\Phi\) as:

\[
\Phi = \begin{bmatrix}
\phi_1^i & \phi_2^i & \phi_3^i \\
\vdots & \vdots & \vdots \\
\phi_1^n & \phi_2^n & \phi_3^n
\end{bmatrix};
\]

where \([\phi_1^i \phi_2^i \phi_3^i] = [\cos \alpha_i \cos \beta_i \cos \gamma_i]\). Now we can write \(\Delta \Psi = \Phi \Delta \Theta\), which gives \(\Delta \Theta = (\Phi^T \Phi)^{-1} \Phi^T \Delta \Psi\). We know that:

\[
\text{Cov}(\Delta \Theta) = \text{E}(\Delta \Theta \Delta \Theta^T) = \begin{bmatrix}
\sigma_x^2 & \sigma_{xy} & \sigma_{xz} \\
\sigma_{yx} & \sigma_y^2 & \sigma_{yz} \\
\sigma_{zx} & \sigma_{zy} & \sigma_z^2
\end{bmatrix}.
\]

(4.26)

Since all the femtocells have similar characteristics and the ranging procedures between the femtocells and the RIS for distance estimation are also similar, it is reasonable to assume that they all exhibit the same variance of ranging error. Moreover, in cases where there exist slight changes in the femtocell properties or in scenarios where the communication procedures between the RIS and the femtocell are not similar for all the femtocells, the same assumption of having similar variances can still be held. For these cases, we can employ a worst-case scenario approach where we consider all variances equal to the worst variance observed among the femtocells. It is also important to note that the measurements obtained from one
femtocell are completely separated and uncorrelated from the measurements obtained from the other femtocells. Based on these observations, we can assume that $\text{Var}(r_i) = \sigma^2_r$ and that the errors $\Delta r_i$ are uncorrelated, then:

$$E(\Delta \Theta \Delta \Theta^T) = E(((\Phi^T \Phi)^{-1} \Phi^T \Delta \Psi)((\Phi^T \Phi)^{-1} \Phi^T \Delta \Psi)^T)$$

$$= (\Phi^T \Phi)^{-1} \Phi^T \Phi (\Phi \Phi^T)^{-1} \sigma^2_r = (\Phi^T \Phi)^{-1} \sigma^2_r.$$  

Combining this result with Eq. 4.24 and Eq. 4.26, we can show that the diagonal elements of $(\Phi^T \Phi)^{-1}$ can be used to calculate the GDOP.

If we break down GDOP even further, we can see that it consists of Vertical Dilution of Precision (VDOP) and Horizontal Dilution of Precision (HDOP), i.e., $\text{GDOP} = \sqrt{\text{VDOP}^2 + \text{HDOP}^2}$. The latter represents the effect of the relative geometry between femtocells and the target on the $X - Y$ plane, while the former on $Z$-axis. This explains the difference in accuracy seen for $Z$-axis and $X - Y$ plane estimations in Fig. 4.7. Table 2.1 shows the evaluation of the GDOP values.

Fig. 4.4 shows the GDOP, HDOP, and VDOP (xDOP) values for random femtocell placements in the target space. The results are for the configuration we began with in Section 4.3.1 which resulted in inaccurate $Z$-axis estimations compared to the $X - Y$ plane. xDOP values are calculated for each point in the environment. In order to accurately represent the results for a three-dimensional space, we show the values on different $z$ planes for all the $(x, y)$ points.

The top row in Fig. 4.4 shows HDOP, the middle row VDOP, and finally, the bottom row GDOP, which is calculated based on HDOP and VDOP. These results show us that, even if the HDOP values are low, with high VDOP values, the overall three-dimensional error which is related to GDOP will be high.
4.4. OFDRA Second Stage: Optimal Configuration of Femtocells

Figure 4.4: xDOP representation for the femtocell placement based on the first step of OFDRA (the benchmark which is RIS-based ranging alone with no optimal placement). The top row is for HDOP, the middle row is for VDOP, and the bottom corresponds to GDOP. In each row, left to right shows the values for corresponding xDOP on a different $z$ plane. $z$ planes goes from height of 1 m from the ground to 3 m.

Furthermore, our results highlight that the xDOP values can fluctuate across the $X - Y$ plane. Intuitively, it makes sense that different $(x, y)$ points should have different xDOP values. Moreover, as we can see in this plot, higher $z$ planes have worse xDOP compared to the ones with a lower height. This is because in our configuration, the femtocells are located close to the ceiling, meaning that higher $z$ planes have lower proximity to the femtocell anchors. In lateration-based techniques, lower proximity to the sources will increase the localization errors. For this reason, we see in Fig. 4.4 that higher $z$ planes (e.g., $z = 3 \, m$) which are closer to the anchor femtocells have worse xDOP compared to the $z$ planes that are further away from them (e.g., $z = 1 \, m$).
4.4.2 Problem Formulation for Optimal Femtocell Placement

The problem of finding an optimal anchor placement for indoor localization to minimize the error due to the relative geometry between at any given position is a well-established NP-Hard problem [83, 84, 85, 86]. Even though the optimization framework to find the optimal placement for a static object has been studied extensively, finding the optimal placement for a moving object still remains unsolved. In our framework, we propose a solution for scenarios with moving objects and the output of the optimization algorithm represents the optimal placement of the femtocells, which remain fixed for any given scenario. We solve this problem by developing a greedy algorithm to improve the Z-axis estimation accuracy and mitigate the overall estimation error. To do so, we first define $VDOP_{avg}$, $HDOP_{avg}$, and $GDOP_{avg}$ as the average calculated $VDOP$, $HDOP$, and $GDOP$ values at all the positions in the target domain for a given anchor placement. The goal of the optimization is to find the optimal placement of five femtocells in order to minimize $GDOP_{avg}$, while having constraints on both the $VDOP_{avg}$ and $HDOP_{avg}$ values. The problem can be formulated as follows.

$$\min \sum_{AV \text{ Domain}} \text{Trace}(((\Phi^T \Phi)^{-1})$$

$$s.t. \quad VDOP_{avg} < v \quad \& \quad HDOP_{avg} < h;$$

where $\text{Trace}(.)$ represents the summation of the diagonal elements of the matrix $(\Phi^T \Phi)^{-1}$, and $v$ and $h$ are the tolerance constraints $VDOP_{avg}$ and $HDOP_{avg}$, respectively.

To carry on with the solution, we define two domains for the indoor space. First is the UAV/AGV domain $AV$, a subspace of the indoor space where the target is allowed to move. Optimization calculations for $VDOP_{avg}$ and $HDOP_{avg}$ are carried out over this domain. Second is the femtocell domain $F$, the possible locations for femtocell placement. This is comprised of any point that has less than 1 m vertical distance to the ceiling, which is the
regulation no-fly zone for drones.

We showed in Sec. 4.3.2 that if each measurement has the same uncertainty with zero mean and unit variance with no correlation, then HDOP and VDOP can be derived from the diagonal elements of the matrix $Q$ as follows:

\[
Q = (\Phi^T \Phi)^{-1} = \begin{bmatrix}
\sigma_x^2 & \sigma_{xy} & \sigma_{xz} \\
\sigma_{yx} & \sigma_y^2 & \sigma_{yz} \\
\sigma_{zx} & \sigma_{zy} & \sigma_z^2
\end{bmatrix}, \quad \Phi = \begin{bmatrix}
x_1 - x_{r1} \\
x_2 - x_{r2} \\
x_3 - x_{r3} \\
x_4 - x_{r4}
y_1 - y_{r1} \\
y_2 - y_{r2} \\
y_3 - y_{r3} \\
y_4 - y_{r4}
z_1 - z_{r1} \\
z_2 - z_{r2} \\
z_3 - z_{r3} \\
z_4 - z_{r4}
\end{bmatrix}.
\]  

(4.27)

where $VDOP = \sqrt{\sigma_z^2}$, $HDOP = \sqrt{\sigma_x^2 + \sigma_y^2}$. Also $[x \ y \ z]^T$ is the target position, $[x_i \ y_i \ z_i]^T$ is the location of the $i$-th femtocell, and $r_i$ is the distance between them. We use this information in calculating the elements of the optimization problem.

### 4.4.3 Optimization Algorithm Design for Femtocell Placement

The greedy algorithm we develop to find the anchor placement is based on the evolutionary algorithm (EA) optimization class. First, an initial set of $P_T = 50$ randomly-generated individuals is created. Each individual is a set of five femtocells selected randomly from the domain $F$. To avoid being trapped in a local minimal, we distribute the initial individuals into different groups and generate random individuals for each of these groups.

These are then sorted according to the fitness (cost) function and those with better fitness are chosen for reproduction. The fitness function is the average of GDOP over the AV domain that is achieved using a given placement configuration. The algorithm picks the first $P_s = 40$ individuals in line as a parent group for reproducing the subsequent individuals. Using adjacent individuals in a crossover technique, $P_n = 20$ off-springs are created in total.
Algorithm 2 Femtocell Placement Evolutionary Algorithm

**Input:** AV domain (\(\mathcal{A} \mathcal{V}\)), Femtocell domain (\(\mathcal{F}\)), \(HDOP_{avg}\) tolerance (\(h\)), \(VDOP_{avg}\) tolerance (\(v\))

**Output:** Desirable placement for a set of five femtocells

1: while \(VDOP_{avg} > v \& HDOP_{avg} > h\) do
2: Generate a set of \(P_T\) random individuals, where each individual is a set of five femtocells
3: for \(i = 1\) to \(i = \text{iteration}\) do
4: Check the fitness of all available individuals;
5: Kill the worst \(P_k\) ones to keep having \(P_T\) individuals;
6: Select the \(P_s\) individuals with better fitness as Parents;
7: Crossover every two adjacent parents and make a new offspring;
8: end for
9: end while

To eliminate the worst individuals of a total population of 70, the algorithm checks the fitness scores of the new generation and removes the lowest \(P_k = 20\) to leave \(P_T = 50\).

For generating a new offspring, each set of parents includes 10 femtocells total (5 femtocells per parent). The crossover technique switches some of the coordinate parameters of the first five femtocells with the ones from the second set of five femtocells.

After the initialization, the described procedure repeats for \(N_{iter} = 100\) iterations at the end of which the first individual in the line according to the fitness function is selected. If \(VDOP_{avg}\) and \(HDOP_{avg}\) over the entire drone domain are less than \(v\) and \(h\) respectively, then that individual represents the final answer which is the femtocell placement configuration in the room. Otherwise, the algorithm starts by generating a fresh batch of \(P_T = 50\) random individuals and repeating the process. The algorithm terminates whenever the final result satisfies the constraints. It is imperative to acknowledge that all the \(P_T, P_s, P_n, P_k\), and \(N_{iter}\) values are derived empirically based on the given requirements for our problem. They can be easily adjusted to suit any new floor plan through a simple preprocessing calculation.

Also, it is noteworthy that the procedure can be terminated as soon as the constraints
are met, and there is no need to fulfill the remaining number of iterations. However, due to the relatively small time required to complete the $N_{iter} = 100$ iterations, we chose not to terminate the process while within the iteration loop. This allows the algorithm to retain satisfactory results and continue iterating to find similarly good alternatives. This flexibility offers the ability to choose the preferred setup based on situational requirements in the environment. In other words, the final solution of the optimization process consists of a list with $P_T$ different sets of femtocell placements. The algorithm guarantees that the first placement in this list satisfies the constraints; however, there is no guarantee for the remaining $P_T - 1$ setups. Nonetheless, among these $P_T - 1$ other configurations, there may be additional setups that meet the problem’s requirements. By allowing the iteration to complete even after finding a configuration that satisfies the constraints, we increase the chance of discovering these acceptable alternatives.

Moreover, it is worth emphasizing that the given floor plan serves as the primary and most crucial input of the optimization framework. The optimal placement of the femtocells is derived based on the specific floor plan provided. We have tested our algorithm with various floor plans, and all have yielded results that satisfy the constraints. In our presentation, we showcase the heat map specifically for the large warehouse scenario to demonstrate that our algorithm is capable of functioning effectively in diverse environments, even those with larger dimensions.

Lastly, it is crucial to highlight that the multipath fading characteristic of the channel has no impact on the optimization framework. As shown earlier, the PEB for three-dimensional localization reveals that the final localization error is the product of the ranging- and geometric-induced errors, and these two are independent of each other. Factors such as noise, multipath, and measurement device errors fall into the category of ranging-induced errors. On the other hand, geometric-induced errors are solely dependent on the relative geometry between the
user at a given location and the fixed placement of the femtocells.

4.5 Performance Evaluation

In this section, we describe our experimental setup and provide evaluation results for our testing with just RIS-based ranging-error mitigation; the results of the femtocell placement optimization; and finally, combining the RIS-based ranging error mitigation with optimal femtocell placement in indoor spaces.

4.5.1 Experimental Setup

We design our simulations in MATLAB 2022a running on a Dell Optiplex 7080 desktop. All the numbers are in meters unless explicitly stated otherwise. To generate our test environment, we used the 5G ToolboxTM in MATLAB to simulate femtocell gNBs in the room and the RIS mounted on the drone. The main blocks to design are: 5G gNB femtocell, the PRS signal, and the channel characteristic between the femtocells and the RIS-mounted AVs. We configure the PRS signal parameters for all of the femtocell gNBs in a way that no overlap exists between the transmission of different femtocells to avoid the problem of hearability.

Furthermore, we set the configuration for the physical downlink shared channel (PDSCH) parameters for all of the femtocells for the data transmission. Finally, the OFDM block will perform the modulation on the PRS signal transmitting from femtocells.

All the femtocell gNBs have the same carrier frequency, sub-carrier spacing, cyclic prefix, and frequency allocation within the OFDM resource grid. The carrier frequency in our tests is set to 3.5 GHz, known as Citizens Broadband Radio Service (CBRS), and used by the majority of the networks. Mid-band 5G (frequencies between 1 GHz to 7 GHz) provides
larger bandwidth channels compared to the low-band (below 1 GHz frequencies) which is a key factor in our framework for improved TOA estimation. Furthermore, compared with high-band 5G (above 24GHz) it can provide more coverage with less dense deployments given that it suffers less from path loss. While our proposed scheme will work seamlessly in higher frequencies, it will require denser deployments for the mmWave range.

The simulation setup that has been described up until now, was intended for a grounded UE, which only required two-dimensional localization. For an accurate testing environment in our evaluations, we had to consider additional configurations that are currently unavailable in the MATLAB 5G-NR positioning. In the case of UAVs (or forklift AGVs), without accurately estimating the height of the UE, localization loses meaning. Thus, for the three-dimensional localization, we modify the setup parameters and incorporate the Z-axis estimation into the system. We need to generate the system for our proposed RIS-based ranging approach which is similar to implementing RTT protocol for positioning in 5G-NR rel. 16, where the primary difference is the replacement of an active 5G UE with an RIS. Therefore, we had to account for the path loss that takes place from the RIS to the femtocell. Our system is designed
for indoor scenarios (e.g., Amazon warehouses), so we set the path-loss characteristics to ‘InF-DH’ which is designed for indoor factories with dense clutter and high anchor node heights [123]. This will generate an environment based on real-life channel values.

Femtocells are assumed to be located within 1 m of the ceiling over the entire indoor space in order to have a clear line of sight with the RIS on top of the drone. Here, we randomize the femtocell locations close to each other in an indoor space with the dimensions of 20 m × 20 m × 5 m. The position of a femtocell is marked by \([x_{gNB} \ y_{gNB} \ z_{gNB}]^T\) and the coordinate space of the possible placements is given by \(0 \leq x_{gNB} \leq 20; \ 0 \leq y_{gNB} \leq 20; \ 4 \leq z_{gNB} \leq 5\).

### 4.5.2 RIS-based Ranging without Optimal Femtocell Placement

In the initial simulations, we observed a drastic increase in localization error. This was due to the unadjusted OFDM sample rate. In indoor scenarios where the distances are smaller compared to a general outdoor environment, this created a narrow resolution window for the time instance of arrival in the femtocell which caused large errors in distance calculations. This issue is presented in Fig. 4.5. To resolve it, we increase the sampling rate, making the setup ready for indoor tests based on real-life indoor environment parameters.

As discussed, OFDRA is designed to operate effectively in cluttered indoor environments with multipath fading channels. The specific configuration for the RIS used in OFDRA makes it robust against the negative impacts of multipath. In all our experiments, we rigorously test OFDRA in highly cluttered factory environments with the presence of multipath to ensure its performance in crowded scenarios.

Also, it is important to note that for high-accuracy localization, the presence of the LoS is required. In other words, if the path between the RIS-mounted AV and all the femtocells
is completely blocked, the system cannot provide accurate results. In such scenarios, the location presented by our system will be based on the user’s previous location. However, given the exceptionally high update rate of our system, as soon as the blockage is removed, the system will quickly update the location and resume normal operation. Additionally, if the link between the AV and some of the femtocells is blocked (but not all of them), the system will provide the final location based on the previous location and the trilateration result with the available femtocells, which may not be as accurate.

Regarding the challenges due to the constant mobility of the user, techniques relying on the phase of arrival for distance and angle estimation are prone to errors induced by phase shifts due to the wobbling of the drone. However, OFDRA is immune to phase shifts since it utilizes TOA for distance measurement, eliminating reliance on phase information and ensuring robustness.

As shown in Fig. 4.6, increasing the OFDM sample rate yields a better time resolution and makes the RTT between the femtocells and RIS distinguishable. Using this setting for the OFDM sample rate, we obtain the results in Fig. 4.7 for $X-Y$ plane, $Z$-axis, and overall 2D-3D localization estimations. We see in Fig. 4.7a that the $X-Y$ plane estimation error is sub-1 m. On the other hand, Fig. 4.7b shows $Z$-axis estimation for the 95% of the scenarios is sub-10 m which is more than 10 times larger than the $X-Y$ plane. This naturally yields a much more accurate 2D estimation compared to 3D in Fig. 4.7c.

It goes to show that simply mitigating the ranging errors with the RIS-based ranging method is not sufficient to obtain accurate localization estimations in three dimensions for a mobile target. To that end, in the subsequent section, we address the geometric-induced error and high GDOP.
4.5.3 Optimal Femtocell Placement

We display the results from the optimization algorithm to show how it affects the GDOP. Compared with a benchmark, we show how the OFDRA framework can improve the overall three-dimensional localization accuracy by mitigating the $Z$-axis error induced by geometry while keeping the $X - Y$ plane estimation accuracy intact. Finally, we show the final location estimation results of OFDRA with both the ranging error mitigation and optimal femtocell placement.

The results for the xDOP values in the absence of GDOP optimization are displayed in Fig. 4.8. Instead of showing the xDOP values for all the $(x, y)$ points for given $z$ planes, we average them over all $z$ planes and show that final value. This allows us to consider all the
4.5. Performance Evaluation

\( \text{(a) HDOP} \)
\( \text{(b) VDOP} \)
\( \text{(c) GDOP} \)

Figure 4.9: xDOP representation for OFDRA full scheme

\[ z \] planes to make the representation more accurate. We can see that the VDOP values are significantly worst compared to the HDOPs which results in inaccurate \( Z \)-axis estimations leading to higher errors in three-dimensional localization. On the other hand, Fig. 4.9 shows the xDOP values for the optimal configuration obtained as a result of the OFDRA optimization framework. As shown in this figure, not only have the HDOP values slightly enhanced compared to the non-optimized scenario, but the VDOP values also improved significantly for a better \( Z \)-axis estimation. Hence, the GDOP of the final three-dimensional accuracy is much lower compared with the random placement result in Fig. 4.8.

Furthermore, to have a better understanding of xDOP values comparison, we generated Fig. 4.10 and Fig. 4.11 where the cumulative distribution function (CDF) on the xDOP values are plotted. First, in Fig. 4.10, we show the xDOP values for the configuration used in the first step of OFDRA without the optimal placement. Next, in Fig. 4.11, we did the same thing for the configuration derived from the optimization framework in the second stage of OFDRA. Both figures are plotted based on all the points in the AV domain.

Comparing Fig. 4.10 and Fig. 4.11, we can see that for the majority case (95% of the point), without employing the optimal deployment (second stage of OFDRA), the VDOP and GDOP values are around 100. This significantly goes down to 10 for the configuration with optimal femtocell deployment. As stated earlier, GDOP has a direct impact on the final positioning accuracy and 10 times improvement in the GDOP values can be translated to 10 times better
CHAPTER 4. OFDRA: OPTIMAL FEMTOCELL DEPLOYMENT FOR ACCURATE INDOOR POSITIONING OF RIS-MOUNTED AVs

4.5.4 OFDRA Full Scheme: RIS-based Ranging with Optimal Femtocell Placement

Now that we showed the effects of optimal placement, it is time to re-assess the localization accuracy of OFDRA leveraging this lower GDOP. Fig. 4.12 displays the improved results of location estimation compared to Fig. 4.7 when this optimal placement is used. Fig. 4.12a and Fig. 4.12b show the localization accuracy in the $X - Y$ plane and the $Z$-axis by plotting the CDF of the final error. We can see that compared with Fig. 4.7a and Fig. 4.7b, the $Z$-axis estimation has improved drastically while not sacrificing any performance in the $X - Y$ plane estimations. Compared with Fig. 4.7c, Fig. 4.12c shows that the 3D localization error probability has decreased by a factor of ten for 95% of all the points in the AV domain. This represents a decrease from 10 m errors to sub-1 m errors.
To have a better understanding of how OFDRA improves 3D localization, we additionally generate five random drone trajectories for evaluation given in Table 4.1. In scenarios 1-3, the drone follows a low, medium, and high altitude flight respectively throughout the majority of the $X - Y$ plane with no change in Z-axis. For scenario 4, it is constrained to a smaller area in the $X - Y$ plane with large variations in the Z-axis. Finally, in scenario 5, it experiences large variations in both $X - Y$ plane and Z-axis estimations.

In Fig. 4.13, we show the results for the benchmark (RIS only without optimal placement of femtocells) compared with the full OFDRA scheme which has both the RIS-assisted ranging and the optimal femtocell placement. For both the localization error and the xDOP values, we averaged them over each drone trajectory and display the $Z$-axis estimation and the overall 3D localization error. This allows us to observe how the average xDOP directly affects the overall localization error.

As is seen in Fig. 4.13, the RIS-assisted system without the optimal placement (benchmark) has an average 3D localization error of 3 m. However, when employing the full OFDRA scheme (RIS-assisted ranging in addition to the optimal placement of femtocells), this average 3D localization error is reduced by six-fold to 50 cm. Also, it is shown that for the former, the average GDOP is close to 30 whereas it becomes closer to 5 in the full scheme. This means better anchor placement, which translates to better final localization.

Overall, the results in Fig. 4.13 demonstrate that having a system with excellent ranging

<table>
<thead>
<tr>
<th>Path</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Low altitude flight with minimal Z-axis change</td>
</tr>
<tr>
<td>2</td>
<td>Medium altitude flight with minimal Z-axis change</td>
</tr>
<tr>
<td>3</td>
<td>High altitude flight with minimal Z-axis change</td>
</tr>
<tr>
<td>4</td>
<td>Variations in Z-axis with localized $X - Y$ plane variation</td>
</tr>
<tr>
<td>5</td>
<td>Large variations in both $X - Y$ plane and Z-axis</td>
</tr>
</tbody>
</table>
accuracy does not guarantee high-accuracy 3D localization when anchor placement is inadequate. Furthermore, we can again see that the $Z$-axis estimation suffers more severely from the geometry compared to the $X-Y$ plane. Therefore, using drones or any AVs which are subject to vertical changes, geometry can play a significant factor in sabotaging the final accuracy. This is why we present OFDRA, a systematic approach to handle both the ranging-based errors and more importantly, focus on handling the geometric-induced errors.

As the final evaluation, we compare the two- and three-dimensional error of positioning between the full setup of OFDRA (with the optimal placement) and the benchmark with random femtocell placement.

The proposed OFDRA is working in the C band (3.5 GHz) which is the 5G-NR sub-7 GHz range. We chose this because it provided larger bandwidth compared to legacy 4G LTE or the 5G-NR sub-1 GHz, which means better resolution for time arrival estimation and easier resolvability for multipath. On the other hand, the frequency is not too high that the 5G femtocell coverage range significantly decreases. Therefore, perfect choice for a high-accuracy localization system in large indoor environments (e.g., warehouse). Lower frequencies (and smaller bandwidths) could not provide the same accuracy and also made the RIS size much larger and infeasible to mount on the drone. On the other hand, higher frequencies (and larger bandwidths) brought better accuracy by losing the range of operation meaning for a large warehouse we require to install much more femtocells to be able to cover the entire venue.

However, as we stated earlier, OFDRA can perform in these higher frequencies (e.g., 5G-NR mmWave or 6G) as well. Since femtocells in those high frequencies do not have the same coverage as before, they are more suitable for smaller indoor environments, for example, for playing an Augmented/Virtual Reality (AR/VR) in smaller rooms or for AVs working in smaller areas but needed higher accuracy. To showcase the performance evaluation of
OFDRA, in these higher frequency ranges, we generated Fig. 4.15. For this simulation, we consider a smaller size indoor space where with not adding more femtocells, even higher frequencies still can fully cover the environment. Next, we evaluate the performance of OFDRA for three scenarios: (i) sub-7 GHz range (3.5 GHz same as before), (ii) mmWave range (42 GHz in the V band), and (iii) sub-THz range (95 GHz in the W band). Compared to sub-7 GHz, the mmWave and sub-THz have larger bandwidth available leading to better final accuracy. This was theoretically shown in Eq. 4.18, that the distance estimation performance has a direct relation with the available bandwidth meaning the larger bandwidths have better CRLB and better final distance estimations.

Moreover, to emphasize the importance of the relative geometry between the RIS-mounted AV and the femtocells, we generated Fig. 4.15a and 4.15c which are for cases without an optimal placement of femtocells. As is seen in these figures, even though the accuracy of 5G mmWave and 6G is better than 5G mid-band (effect of larger bandwidth on the ranging error), but still the 3D localization is far worse than 2D due to the effect of bad VDOP leading to a bad Z-axis estimation.

Comparing Fig. 4.15b and Fig. 4.15d with Fig. 4.15a and Fig. 4.15c, we see that even in 6G technology with extremely large bandwidth and low ranging-based error, for 3D localization, the optimal placement is a must. The main goal of our work was to show this fact and provide a solution for optimal placement. To the best of our knowledge, we are the first to propose a positioning system that considers both the wireless ranging issues and the optimal relative geometry to provide good accuracy even in 3D spaces.
4.6 Summary

In this work, we proposed OFDRA, a multipath-robust synchronization-free localization scheme for autonomous navigation of aerial/terrestrial vehicles (e.g., UAVs and AGVs) in indoor spaces (e.g., warehouses). Unlike the majority of the available indoor geolocation systems which mostly focus on two-dimensional localization, OFDRA is capable of providing accurate estimation for both the $X-Y$ plane and $Z$-axis making it suitable even for three-dimensional spaces. OFDRA utilizes RIS technology and 5G femtocells to carry out localization using trilateration on distances derived from round trip flight time of 5G PRS signals from each of the femtocells to the RIS and back to femtocells. Moreover, we showed that the localization error is not only due to the ranging-error factors such as synchronization, multipath, noise, and interference. In fact, the relative geometry between the femtocells and the user plays an important role in the final geolocation accuracy. OFDRA took into consideration this factor and proposed a novel optimization framework based on the EA class of optimization and determined the optimal configuration for femtocells to reduce the geometric-induced error and rectify the bad $Z$-axis estimation. Real-life indoor scenario evaluations confirm the strong performance of OFDRA.
Figure 4.12: Performance evaluation for the full scheme of OFDRA (RIS-based ranging with optimal placement of femtocells): (a) Horizontal estimation evaluation; (b) Vertical estimation evaluation; (c) Comparison between 2D and 3D localization

Figure 4.13: Average localization accuracy followed by the average corresponding xDOP for five sample trajectories in Table 4.1

Figure 4.14: Final three-dimensional localization performance evaluation between OFDRA and a benchmark (with no femtocell placement consideration)
Figure 4.15: Comparison between 5G sub-7 GHz, 5G mmWave, and 6G, for scenarios without optimal deployment and with optimal deployment
Chapter 5

Wi-Six: Precise Positioning in the Metaverse via Optimal Wi-Fi Router Deployment in 6G Networks

Mobile networks are evolving to accommodate a diverse range of applications. The forthcoming generation of mobile networks will provide services via isolated segments known as network slices. A prominent use case within this framework is the provision of services for Augmented/Virtual Reality (AR/VR) devices, a pivotal element in the realization of the Metaverse. Besides strict control over latency, achieving a realistic Metaverse relies on accurately pinpointing user equipment (UE) in a three-dimensional (3D) space. Moving from 5G to 6G networks, the deployments are projected to operate at even higher frequencies and in denser settings. Thus, the issue of precise indoor positioning becomes a challenge due to the nature of signals at these higher frequencies. To that end, this chapter proposes the Wi-Six framework. Wi-Six is based on the well-established Third Generation Partnership Project (3GPP) standards that rely on a combination of regular base stations and wireless fidelity (Wi-Fi) routers for outdoor and indoor communications, respectively. This combination helps pinpoint locations indoors by taking some of the load off the base stations. In this study, we highlight the importance of Wi-Six Transmission and Reception Points (TRP) when trying to accurately localize Metaverse wearable devices such as AR/VR equipment.
We tackle the challenging problem of finding the optimal TRP placement pattern in a three-dimensional space using a novel approach. Our findings demonstrate that with the proposed optimal placement strategy, we can estimate the locations with higher accuracy compared to arbitrary anchor placements. To demonstrate how the proposed framework improves the positioning estimates, we conduct experiments with simulated users. The results show that we are able to reduce 3D positioning estimation errors by a factor of 20 compared to existing technologies that do not consider optimal anchor placement.

5.1 Motivation

In the preceding chapter, we demonstrated the necessity to shift from acoustic signals to radio frequency (RF) due to the reasons we outlined. As discussed, the two primary options at hand are cellular and wireless fidelity (Wi-Fi). We explained that Wi-Fi’s application is predominantly limited to indoor usage, prompting us to favor cellular positioning which utilizes the advanced 5G New Radio (NR) technology, as the solution we proposed in the previous chapter. In this setup, outdoor localization is managed by 5G base stations, while indoor localization is addressed through the utilization of 5G femtocells.

However, the challenge with our previous approach was the suboptimal performance of cellular technology indoors, leading us to introduce femtocells to improve indoor localization. Yet, this introduced a limitation in scenarios where deploying femtocells was not feasible, resulting in compromised indoor localization. To address this, our current chapter presents a novel localization strategy that synergizes the strengths of Wi-Fi and cellular technologies. We introduce a comprehensive system architecture where the upcoming 6G cellular network caters to precise three-dimensional outdoor localization. Meanwhile, for indoor localization, we leverage the connectivity between Wi-Fi and cellular protocols. More specifically, for in-
door positioning, we opt for the IEEE 802.11az standard, recognized for its accurate Wi-Fi positioning. Notably, this front-end interface seamlessly integrates with the broader cellular network, ensuring that all final calculations remain integrated within the expansive reach of the global 6G network.

An additional point worth considering is our transition within this chapter from focusing on drone applications to the realm of Augmented/Virtual (AR/VR) and, more broadly, metaverse positioning. As we embarked on this journey, the realm of indoor localization for drones without the reliance on GPS was particularly prominent. It was within this context that we tailored our system to cater specifically to drone applications. However, a recent development has further intensified the landscape—there’s an increasing demand for highly accurate positioning solutions to cater to metaverse users. Consequently, we have chosen to dedicate both this chapter and the subsequent one to elucidate positioning use cases within the metaverse.

In essence, the differences between positioning solutions for drones and metaverse users are not extensive. Typically, our systems are designed to be inclusive, such that even when originally tailored for drone applications, they remain adaptable to other scenarios. The drone-focused design stemmed from the demanding requisites posed by the continuous movement in all three dimensions—x, y, and z—unique to drone usage. This positioning system, although applicable to general contexts, was designed with the stringent demands of drone localization in mind.

As we shift our focus from drone applications to the metaverse, it’s important to clarify that this shift does not imply exclusivity. Our discussions remain all-encompassing, spanning various use cases, including both drones and the metaverse. However, metaverse scenarios present their own set of challenges. While similar to drones, high-accuracy positioning across all dimensions is required, the metaverse presents an additional level of complexity—
demanding even greater precision. As a result, the system design in both this chapter and the subsequent one is distinctly tailored to address metaverse applications’ exacting requirements.

5.2 Chapter Overview

The next generation of mobile networks is designed to accommodate a wide range of applications with stringent Quality of Service (QoS) demands. The infrastructure for these next-generation mobile networks is built using Commercial off-the-Shelf (COTS) hardware to provide a flexible and scalable network model, catering to a variety of vertical application scenarios. Leveraging Network Functions Virtualization (NFV), network functions are no longer tethered to specific, proprietary hardware, but are instead implemented on readily available COTS servers. Leveraging Software-Defined Networking (SDN), these mobile networks can now adapt and conform to service quality rules with near-real-time responses.

The method used to cater to the different needs of various user groups is called network slicing. The network slicing technique refers to providing services to different user groups, each with different QoS needs, by leveraging separate network segments. Every network slice is built to accommodate a unique use case, which includes enhanced Mobile Broadband (eMBB), Ultra-Reliable and Low Latency Communications (URLLC), massive Machine Type Communications (mMTC), and Vehicular to Everything (V2X) communications [124].

Fig. 5.1 represents the co-existence of these network slices to create an inter-connected and intelligent environment referred to as the Metaverse [125]. In this advanced ecosystem, users will have the ability to engage with their known world through Augmented or Virtual reality (AR/VR). This unlocks unprecedented capabilities, promoting enhanced connectivity and self-governance.
At present, mobile networks are still in the early phases of their deployment. Hence, the comprehensive array of features promised by the 5G standards set forth by the Third-Generation Partnership Project (3GPP) is yet to be fully actualized in both academic and industrial settings [61, 126]. However, researchers have already begun to examine and predict the capabilities of the forthcoming Sixth Generation (6G) mobile networks [127], which will be instrumental in delivering the enhanced throughput and reduced latency vital for realizing the Metaverse [99, 128]. Besides the QoS needs, a crucial prerequisite for most Metaverse applications is precise localization for both outdoor and indoor environments [129]. Particularly for high mobility applications and AR/VR precision, three-dimensional (3D) positioning accuracy will be of the utmost importance [57, 130, 131].

The Global Positioning System (GPS) has been a leading player in localization services. However, this technology faces disruptions in indoor settings. The 30 GHz millimeter-wave
Figure 5.2: Localization in the Metaverse where 3GPP RAN is used outdoors and Wi-Six TRPs are used indoors.

(mmWave) bands and even the present sub-6 GHz signals, which offer stable outdoor coverage ranges [61], experience significant signal loss indoors. This makes it harder to rely on positioning signals. With the advent of 6G [132], it is expected that operating frequencies will surpass mmWave bands, extending up to the Terahertz (THz) range [133]. This will pose additional challenges for the use of current localization strategies in indoor environments, where high-frequency signals suffer considerable path and penetration losses. To address this issue, we introduce the Wi-Six framework, depicted in Fig. 5.2.

With the Wi-Six framework, we aim to leverage multiple Radio Access Technology (RAT) deployments, which can share a common core network and backhaul with the 3GPP RAN, in order to gather and consolidate localization information. As depicted in Fig. 5.2, we
assume that outdoor coverage is provided by 3GPP nodes, while indoor coverage is seamlessly transitioned to Wi-Six Transmission Reception Points (TRPs) that use Wi-Fi as their access technology. This setup allows for the integration of localization feedback from both outdoor and indoor anchor nodes at a central database in the core network, using a common control plane as described in the current 3GPP standardization [134].

The main focus of this chapter is achieving precision for indoor localization since RAN signals will be too weak for accurate positioning. Considering indoor environments, we can express the total location prediction error as the multiplication of distance errors (between anchors and the target) and the geometric error induced by the positioning of anchors, otherwise known as geometric dilution of precision (GDOP). For instance, let us consider a scenario where the distance error is around 5 cm. This level of precision is achievable with 6G’s mmWave frequencies, thanks to the wider bandwidth available for signals related to positioning. However, if the target has a high GDOP of 20 due to non-optimal anchor placement, this could lead to a total location error of 1 meter (i.e., $5cm \times 20 = 100cm$). This is a twentyfold escalation in location prediction errors, which could critically deteriorate the functioning of sensitive applications in potential Metaverse situations, such as AR/VR equipment, V2X, and other specific uses.

Our goal is to deliver high-accuracy localization for Metaverse applications by investigating the impact of GDOP on the accuracy of positioning within a 3D space. Prior studies [12, 94, 135] have primarily concentrated on reducing ranging errors, without considering the potential influence of anchor placement. We experimentally demonstrate that, even in the context of minimal ranging errors, high GDOP can degrade localization accuracy. By employing heatmaps to evaluate coverage, we contrast the vertical (VDOP) and horizontal (HDOP) Dilution of Precision, indicating that the former is more challenging to reduce than the latter. Our objective is to optimize the positioning of Wi-Six TRPs to minimize the
Z-axis estimation errors for a lower VDOP while ensuring high precision for $X - Y$ plane estimations for an acceptable HDOP. In our methodology, we initially derive the Cram’er-Rao Lower Bound (CRLB) for the position estimator to illustrate how the error in trilateration techniques results from a combination of ranging errors and GDOP. We then define and resolve the NP-hard optimization problem for the ideal placement of Wi-Six TRPs using an optimization framework grounded in Evolutionary Algorithms (EAs). Our contributions are as follows:

- We propose a hybrid Radio Access Technology (RAT) localization framework that combines outdoor 6G anchors with indoor Wi-Six TRPs, centralizing positioning feedback for targets within the Metaverse.

- We extract the Position Error Bound (PEB) for the Wi-Six positioning scheme and associate the overall positioning error to ranging- and geometric-induced inaccuracies.

- We highlight the impact of GDOP in a 3D setting and demonstrate that inaccuracies in $Z$-axis estimation pose a more significant source of error in indoor localization compared to $X - Y$ plane assessments.

- Addressing the NP-hard problem of optimal Wi-Six TRP placement in variable indoor space sizes, we propose an optimization approach based on EAs.

- In addition to optimizing anchor placement, we propose a Wi-Six ranging scheme based on Time of Arrival (TOA) measurements combined with Multiple Signal Classification (MUSIC).

- To effectively demonstrate the improvement of the final localization estimate, we conduct detailed simulations where Metaverse users are subject to poor and optimized
5.3 Wi-Six Core Network Model

This section provides an overview of how Wi-Six TRPs integrate with the 3GPP RAN as well as the core network.

Leveraging multiple RATs, we propose a hub in the next-generation mobile core network for processing localization information. As the deployments evolve from 5G to 6G, we assume that the core network components will preserve their functional characteristics and operate...
from cloud-based COTS servers. Hence, we recommend adopting the framework illustrated in Fig. 5.3. In this structure, we envision a central database within the core network, which handles and stores the location details of users while they navigate the indoor and outdoor Metaverse.

The depicted situation showcases the coexistence of 6G macro/micro base stations (BSs) and indoor non-3GPP wireless local area network (WLAN) nodes [134], each providing coverage for outdoor and indoor users, respectively. The conventional 3GPP BSs maintain their connection to the core network using the widely accepted standalone access (SA) mode [124]. In contrast, indoor non-3GPP access nodes rely on the non-3GPP interworking function (N3IWF) interface to facilitate their link with the access and mobility management function (AMF). In this scenario, both RAN variants report to a shared location management function (LMF) within the core network to maintain a thorough accounting of the user locations.

This configuration ensures smooth transitions for users moving between indoor and outdoor environments, allowing them to freely choose between outdoor base stations or Wi-Six TRPs. This blend of deployment strategies aligns with the current 3GPP standardization [134], alleviating the need for extra changes to the LMF application programming interface (API). Additionally, existing projects have already begun experimenting with testing environments that use highly precise deployments [136, 137].

The remaining sections of this chapter focus on examining the indoor localization aspect, where we presume the outdoor positioning is managed by the 3GPP access network.
5.4 Wi-Six Positioning

This section outlines the Wi-Six positioning framework, detailing how Wi-Six delivers real-time location information for users in the Metaverse. To this end, we first explain the ranging mechanism we implemented to approximate the distance between the user and each of the Wi-Six TRPs in Sec. 5.4.1. Following this, we discuss the trilateration scheme we utilized to determine the user’s position in a 3D space in Sec. 5.4.2. Finally, in Sec. 5.4.3, we lay the theoretical groundwork for the optimal positioning of Wi-Six TRPs to minimize GDOP.

5.4.1 Wi-Six Ranging

Here, we present the Wi-Six ranging scheme. We designed this to be utilized in an indoor multipath environment using a TOA-based distance estimation algorithm defined in the IEEE 802.11az Wi-Fi standard [138]. The ranging algorithm is employed within the Wi-Six TRPs, which leverages IEEE 802.11az as the front end. To estimate the TOA, a MUSIC super-resolution approach is utilized [139].

The 802.11az standard, known as next-generation positioning (NGP), introduces support for two high-efficiency (HE) ranging physical layer (PHY) protocol data unit (PPDU) formats: HE ranging null data packet (NDP) and HE trigger-based (TB) ranging NDP. The HE ranging NDP and HE TB ranging NDP serve as counterparts to the HE sounding NDP and HE TB feedback NDP PPDU formats specified in the previous Wi-Fi standard. Furthermore, with 802.11az, we can make use of bandwidths up to 160MHz.

The HE ranging NDP facilitates the positioning of one or more users and offers the option of employing a secure HE long training field (HE-LTF) sequence. In the case of a single-user HE ranging waveform, it includes HE-LTF symbols specifically designed for a single user...
and may also incorporate an optional secure HE-LTF sequence. Conversely, the multi-user HE ranging waveform only allows for secure HE-LTF symbols for multiple users. To enhance the accuracy of position estimation, both single-user and multi-user waveforms can include multiple repetitions of the HE-LTF symbols. In our proposed ranging scheme, we have an 802.11az network comprised of multiple Wi-Six TRPs and a single user.

To obtain distance ranging measurements between the user and each Wi-Six TRP, we capture the timestamps of the NDPs. Fig. 5.4 illustrates the measurement sounding phase between the user and a single Wi-Six TRP, showcasing the timestamps recorded at each stage of the communication process. The process is based on the fine-time measurement (FTM) protocol used in IEEE 802.11 positioning standards which is the equivalent of the round-trip time
(RTT) protocol in 3GPP cellular positioning standards, and it involves the following steps:

- The user transmits the uplink NDP (UL NDP) and records the transmission time as \( t_1 \) (UL TOD), indicating the User-to-Device Time of Departure.

- The Wi-Six TRP receives the UL NDP and captures the time \( t_2 \) (UL TOA), denoting the User-to-Device Time of Arrival.

- The Wi-Six TRP then transmits the downlink NDP (DL NDP) and records the transmission time as \( t_3 \) (DL ToD), representing the Device-to-User Time of Departure.

- The user receives the DL NDP and captures the time \( t_4 \) (DL TOA), indicating the Device-to-User Time of Arrival.

We can then combine these timestamps to calculate the RTT as follows:

\[
R_{RTT} = (t_4 - t_1) - (t_3 - t_2);
\]  

and we can estimate the distance between the user and the Wi-Six TRP based on \( d = \frac{R_{RTT}}{2} \times c \), where \( c \) is the speed of the light. Fig. 5.5 provides a visual representation of the distance ranging process. It is noteworthy that to estimate \( t_2 \) and \( t_4 \), we used MUSIC super-resolution. To achieve this, we follow these steps:

- Interpolate the missing subcarriers in the channel frequency response (CFR), assuming a uniform subcarrier spacing.

- Estimate the correlation matrix of the CFR.

- Employ spatial smoothing to decorrelate the multipaths.
Enhance the correlation matrix estimate by conducting forward-backward averaging. This process considers CFR estimates from multiple spatial streams as separate CFR snapshots and utilizes all snapshots for correlation matrix estimation.

Utilize the MUSIC algorithm. Perform eigendecomposition on the correlation matrix to separate it into signal and noise subspaces. Estimate the time-domain delay profile by identifying instances where the signal and noise subspaces are orthogonal. Here, we assume that the precise dimension of the signal subspace, which corresponds to the number of multipaths, is known.

Determine the TOA by identifying the first peak of the recovered multipaths in the estimated delay profile, considering it as the direct-line-of-sight (DLOS) path.

To evaluate the Wi-Six ranging approach’s performance in a multipath environment, we present Fig. 5.6. The plot provides a comparison between the true multipath delay profile and the Wi-Six estimated delay profile utilizing the MUSIC algorithm for a single 802.11
5.4. Wi-Six Positioning

Figure 5.6: Simulation of a single 802.11 packet transmission through a multipath channel and estimation of the delay profile and TOA using the MUSIC super-resolution technique with various dedicated bandwidth: (a) 20 MHz bandwidth; (b) 40 MHz bandwidth; (c) 160 MHz bandwidth.

link with varying dedicated bandwidths. While both 802.11ac (Wi-Fi 5) and 802.11ax (Wi-Fi 6) support bandwidths of 20 MHz, 40 MHz, 80 MHz, and 160 MHz, we specifically focus on 20 MHz and 40 MHz in Fig. 5.6(a) and Fig. 5.6(b), respectively, as they are more commonly used in these technologies. In Fig. 5.6(c), we explore the 160 MHz bandwidth, which is adopted as the bandwidth of 802.11az in our Wi-Six framework. As depicted in these figures, the increase in bandwidth has a considerable impact on the accuracy of the MUSIC procedure in detecting the exact TOA in multipath environments.

5.4.2 Wi-Six Trilateration

Within the Wi-Six model, we use Time of Arrival (TOA) measurements to calculate the distance between the Metaverse user and the Wi-Six TRPs. In contrast to existing indoor positioning studies which primarily concentrate on two-dimensional (2D) space, we seek to provide localization for a 3D area. Hence, in order to pinpoint the user’s location accurately, a minimum of four Wi-Six TRPs are required.

In our system model, \( d_i \) represents the distance between the \( i \)-th Wi-Six TRP, positioned at \( [x_i \ y_i \ z_i]^T \), and the user in the Metaverse, located at \( [x \ y \ z]^T \). Through trilateration, this
results in:

\[(x_1 - x)^2 + (y_1 - y)^2 + (z_1 - z)^2 = d_1^2\]

\[\vdots\]

\[(x_n - x)^2 + (y_n - y)^2 + (z_n - z)^2 = d_n^2.\]  \hspace{1cm} (5.2)

Simplifying this, we can represent it in the form \(Ax = b\), where \(A\) and \(b\) are:

\[
A = \begin{bmatrix}
2(x_n - x_1) & 2(y_n - y_1) & 2(z_n - z_1) \\
2(x_n - x_2) & 2(y_n - y_2) & 2(z_n - z_2) \\
\vdots & \vdots & \vdots \\
2(x_n - x_{n-1}) & 2(y_n - y_{n-1}) & 2(z_n - z_{n-1}) \\
\end{bmatrix},
\]

\[
b = \begin{bmatrix}
d_1^2 - d_n^2 - x_1^2 - y_1^2 - z_1^2 + x_n^2 + y_n^2 + z_n^2 \\
d_2^2 - d_n^2 - x_2^2 - y_2^2 - z_2^2 + x_n^2 + y_n^2 + z_n^2 \\
\vdots \\
d_{n-1}^2 - d_n^2 - x_{n-1}^2 - y_{n-1}^2 - z_{n-1}^2 + x_n^2 + y_n^2 + z_n^2
\end{bmatrix}.
\]

In the end, the position of the Metaverse user \(\mathbf{x} = [x \ y \ z]^T\), is given by \(\mathbf{x} = (A^T A)^{-1} A^T \mathbf{b}\).

### 5.4.3 Wi-Six Positioning Error Bound

In order to assess the accuracy of the final localization, we calculate the Position Error Bound (PEB) for Wi-Six. This is achieved by utilizing the lower limit on location variability, referred to as the Cramer-Rao Lower Bound (CRLB), which can be acquired using an unbiased location estimator. We assume that the computed distances are affected by zero-mean additive Gaussian noise, maintaining a fixed variance of \(\sigma_r^2\) [83], and that these measurements
are independently distributed. Given this assumption, it can be demonstrated that in a 2D trilateration system using an unbiased estimator, the variance of CRLB for positional error, represented as \( \sigma^2(r) = \sigma_x^2(r) + \sigma_y^2(r) \), at location \( r \) is expressed as:

\[
\sigma(r) = \sigma_r \times \sqrt{\frac{N_w}{\sum_{i=1}^{N_w-1} \sum_{j=i+1}^{N_w} W_{ij}}}. \tag{5.3}
\]

In this equation, \( w_i \) is the \( i \)-th Wi-Six TRP among a total of \( N_w \), \( \theta_i \) is the angle between \( w_i-r \) and \( W_{ij} = |\sin(\theta_i - \theta_j)| \).

This formula shows that the final error in location estimations is linked to a function of \( N_w \) and the angle between the TRPs, known as the GDOP, multiplied by the range-based errors \( \sigma_r \). This provides the final expression \( \sigma(r) = \sigma_r \times GDOP \) for positional estimation error. As observed from Eq.5.3, GDOP can be regarded as a key parameter that can affect location precision\[86, 107\], since it directly impacts the final estimation.

For a Metaverse user with position \((x, y, z)\), the GDOP value for 3D localization is:

\[
GDOP \cdot \sigma_r = \sqrt{\text{Var}(x) + \text{Var}(y) + \text{Var}(z) + \text{Var}(ct)}.
\]

Here, \( c \) represents the speed of light, and \( \tau \) stands for the clock delay of the receiver. If we postulate that both the transmitter and the receiver operate on the same clock, we can adjust the timing discrepancy to zero in GDOP, which results in the following:

\[
GDOP = \sqrt{\frac{\sigma_x^2 + \sigma_y^2 + \sigma_z^2}{\sigma_r^2}}. \tag{5.4}
\]

In Fig.5.7, we depict the potential impact of a high GDOP on the location estimation error. Fig.5.7a, with its better anchor placement compared to Fig. 5.7b, shows a smaller shaded
CHAPTER 5. WI-SIX: PRECISE POSITIONING IN THE METAVERSE VIA OPTIMAL WI-FI ROUTER DEPLOYMENT IN 6G NETWORKS

(a) Anchor placement for low GDOP
(b) Anchor placement for high GDOP

Figure 5.7: Effect of GDOP on location estimation error due to anchor placement

area representing the possible error region.

For a Metaverse user located at \((x, y, z)\) and Wi-Six TRP positions given by \((x_i, y_i, z_i)\),

\[
r_i = \sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2}. \tag{5.5}
\]

which represents the distance between the target and the anchor. The precise value of \(r_i\) is indeterminable due to the range errors, leading to imprecision when solving for the user’s position in Eq.\(5.5\). Our objective is to delve deeper into the relationship between the ranging error and the final localization error. To accomplish this, we differentiate Eq.\(5.5\), disregarding the terms beyond the first order \([109]\) to obtain:

\[
\Delta r_i = \frac{\Delta x(x - x_i) + \Delta y(y - y_i) + \Delta z(z - z_i)}{\sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2}} = \Delta x \cos \alpha_i + \Delta y \cos \beta_i + \Delta z \cos \gamma_i.
\]

Here, \([\cos \alpha_i \cos \beta_i \cos \gamma_i]^T\) signifies the unit vector that points from the user towards the TRP.

Denoting the position error and target range vectors as \(\Delta X = [\Delta x \ \Delta y \ \Delta z]^T\) and \(\Delta R = \ldots\)
where \([v_1^i v_2^i v_3^i] = [\cos \alpha_i \cos \beta_i \cos \gamma_i]\). This gives us \(\Delta R = V \Delta X\) and by extension \(\Delta X = (V^T V)^{-1} V^T \Delta R\). Using

\[
\text{Cov}(\Delta X) = \mathbb{E}(\Delta X \Delta X^T) = \begin{bmatrix}
\sigma_x^2 & \sigma_{xy} & \sigma_{xz} \\
\sigma_{yx} & \sigma_y^2 & \sigma_{yz} \\
\sigma_{zx} & \sigma_{zy} & \sigma_z^2
\end{bmatrix},
\]

(5.7)

we assume \(\text{Var}(r_i) = \sigma_r^2\) and the errors \(\Delta r_i\) are uncorrelated to obtain:

\[
\mathbb{E}(\Delta X \Delta X^T) = \mathbb{E}((V^T V)^{-1} V^T \Delta R)((V^T V)^{-1} V^T \Delta R)^T) = (V^T V)^{-1} V^T \mathbb{E}(\Delta R \Delta R^T)((V^T V)^{-1} V^T)^T = (V^T V)^{-1} V^T V(VV^T)^{-1} \sigma_r^2 = (V^T V)^{-1} \sigma_r^2.
\]

(5.8)

The results from Eq.5.4 and Eq.5.7, which represent the GDOP and the distance between the anchor and the user, allow us to utilize the diagonal elements of \((V^T V)^{-1}\) to calculate

\[
GDOP = \sqrt{HDOP^2 + VDOP^2}.
\]

Here, the Horizontal Dilution of Precision (HDOP) refers to errors on the \(X - Y\) plane, while the Vertical Dilution of Precision (VDOP) pertains to those along the \(Z\)-axis. The computation of the GDOP values is presented in Table 5.1.
Table 5.1: Comparing GDOP Values

<table>
<thead>
<tr>
<th>GDOP Values</th>
<th>Wi-Six TRP Geometry</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Ideal</td>
</tr>
<tr>
<td>1 – 2</td>
<td>Very Good</td>
</tr>
<tr>
<td>2 – 5</td>
<td>Good</td>
</tr>
<tr>
<td>5 – 10</td>
<td>Medium</td>
</tr>
<tr>
<td>10 – 20</td>
<td>Sufficient</td>
</tr>
<tr>
<td>&gt; 20</td>
<td>Bad</td>
</tr>
</tbody>
</table>

5.5 Wi-Six Optimization Framework

Most research in the field of localization has predominantly explored the issue in a 2D context. Yet, for users in the 6G Metaverse, a 2D analysis will not be adequate to deliver the necessary precision in location estimation. However, optimizing anchor placements in indoor settings for a 3D space is acknowledged as a challenging, NP-hard problem [83, 84, 85, 86].

The main goal of this section is to determine the best placement of Wi-Six TRPs for indoors to minimize the error caused by poor anchor placement. We include two key constraints related to precision in the $X - Y$ plane and Z-axis, all the while targeting a reduced GDOP. Notably, a minimal GDOP does not automatically guarantee low HDOP and VDOP values. As such, in our goal to achieve the best placement, we place emphasis on HDOP and VDOP values separately.

The remaining part of this section presents our formulation of the indoor anchor placement problem to achieve low HDOP and VDOP, followed by the introduction of the greedy algorithm that we employ to resolve it.
5.5.1 Problem Formulation

Within the Metaverse environment, a user, such as someone donning AR goggles, is essentially a moving target. This dynamism complicates the task of fine-tuning the GDOP, VDOP, and HDOP based on just one fixed position. To uphold the integrity of our solution, we address this challenge by considering all conceivable user positions and then calculating the average GDOP, HDOP, and VDOP values. These averaged values are denoted as $GDOP_{avg}$, $VDOP_{avg}$, and $HDOP_{avg}$ for the 3D space being analyzed. Our primary objective is to determine the best Wi-Six TRP positioning that delivers the lowest $GDOP_{avg}$, while also adhering to specific limits for $HDOP_{avg}$ and $VDOP_{avg}$. Hence, the optimization problem can be expressed as follows:

$$\text{min } \sum_{U} GDOP$$

$$s.t. \quad HDOP_{avg} < h_{tol}; \quad VDOP_{avg} < v_{tol}. \quad (5.9)$$

When designing the problem, we divide our indoor space into two distinct areas: $U$ and $S$. The $U$ zone illustrates where the Metaverse user might be, encompassing all potential user locations. For our optimization task, we evaluate over all points in $U$ to arrive at average measurements for both $VDOP_{avg}$ and $HDOP_{avg}$. On the other hand, $S$ denotes the anchor zone, highlighting all potential spots for Wi-Six TRP setup. We set individual threshold values, $(h_{tol})$ for $HDOP_{avg}$ and $(v_{tol})$ for $VDOP_{avg}$. This method allows us to control errors in both horizontal ($X-Y$ plane) and vertical ($Z$-axis) dimensions separately. Ultimately, the objective function seeks to reduce the $GDOP$ experienced by the user, by resolving Eq. 5.9, while remaining within the boundaries set by the average $HDOP$ and $VDOP$ constraints.

In Section 5.4.3, we concluded that while defining GDOP, if the measurements are uncorre-
lated with identical uncertainties, possessing zero mean and unit variance, we can leverage the elements of the matrix $Q$ to extract HDOP and VDOP as shown below.

$$Q = (V^T V)^{-1} = \begin{bmatrix} \sigma_x^2 & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_y^2 & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_z^2 \end{bmatrix}. \quad (5.10)$$

In this case, $V DOP = \sqrt{\sigma_z^2}$ and $H DOP = \sqrt{\sigma_x^2 + \sigma_y^2}$, with $V$ specified as follows:

$$V = \begin{bmatrix} \frac{x_1-x}{r_1} & \frac{y_1-y}{r_1} & \frac{z_1-z}{r_1} \\ \frac{x_2-x}{r_2} & \frac{y_2-y}{r_2} & \frac{z_2-z}{r_2} \\ \frac{x_3-x}{r_3} & \frac{y_3-y}{r_3} & \frac{z_3-z}{r_3} \\ \frac{x_4-x}{r_4} & \frac{y_4-y}{r_4} & \frac{z_4-z}{r_4} \end{bmatrix}. \quad (5.11)$$

In the matrix $V$, the coordinates $(x, y, z)$ represent the location of the Metaverse user, and $(x_i, y_i, z_i)$ are the coordinates of the $i$-th Wi-Six TRP. Here, $r_i$ specifies the distance between these two points.
5.5. Wi-Six Optimization Framework

Figure 5.9: CDF plot representation of xDOP values for various room dimensions with optimal configuration shown in Table 5.4.

5.5.2 Algorithm Design

To solve the problem efficiently and reduce computational time, we employ an EA-based greedy algorithm.

**Algorithm 3 Wi-Six TRP Placement Evolutionary Algorithm**

<table>
<thead>
<tr>
<th>Input:</th>
<th>User domain ((U)), Wi-Six TRPs domain ((S)), (HDOP_{avg}) tolerance ((h_{tol})), (VDOP_{avg}) tolerance ((v_{tol}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output:</td>
<td>Optimal placement of four Wi-Six TRPs</td>
</tr>
</tbody>
</table>

1. while \(VDOP_{avg} > v_{tol} \& HDOP_{avg} > h_{tol}\) do
2. Generate G random individuals containing four Wi-Six TRPs each
3. for \(i = 1\) to \(i = number\ of\ iteration\) do
4. Check the fitness function of the individuals;
5. Remove the lowest performers to leave G individuals;
6. Choose individuals with better fitness as parents;
7. Crossover adjacent parents and produce a new offspring;
8. end for
9. end while

Upon initialization, the algorithm generates an initial population of 10 random individuals for the Evolutionary Algorithm configuration. Each individual is composed of four randomly selected Wi-Six TRPs from the domain \(S\). These individuals are arranged into diverse groups to prevent the solution from being ensnared in local minima. The Wi-Six TRPs within the \(S\) domain can be positioned anywhere - on the ceiling, the walls, or dispersed throughout the indoor space.
The choice of $G = 10$ is optimal in this context as it allows us to incorporate all the groups in determining the final solution. With $G \geq 10$ in the initial sets, the computational complexity would rise, leading to longer convergence times. Conversely, with $G \leq 10$, it would be unfeasible to involve all the groups. Although values slightly larger than $G = 10$ can still be used, in this specific case, they do not significantly improve the accuracy of the final solution, but they do expedite its delivery.

The algorithm sorts the initially created individuals using a fitness or cost function and selects them for reproduction. The fitness function is identified as the average VDOP evaluation over the domain $U$ for a specific configuration of four Wi-Six TRPs or anchors. To proceed, eight individuals from the sorted list are chosen as parent groups to produce the next generation. Using adjacent pairings, new configurations of four Wi-Six TRPs are created via a crossover technique, resulting in four offspring. The fitness function then sifts through the combined group of 14 individuals and removes the least fit four, leaving ten in total. These new offspring consist of eight Wi-Six TRPs, four from each parent. The crossover technique is used to interchange the coordinates of the first four TRPs with those of the second set.

In other words, the algorithm is repeated 100 times using the process described above in order to arrive at the optimal solution. Increasing the number of iterations significantly beyond 100, for example, to 1000, would drastically increase the computation time while not substantially improving the results. This reveals that the balance between computational efficiency and solution accuracy is found at around 100 iterations for this particular problem and implementation.

Overall, the algorithm proceeds by selecting the individual that has the best (lowest) value of the fitness function. If this individual satisfies both the vertical and horizontal tolerance levels ($v_{tot}$ and $h_{tot}$) for the average VDOP and HDOP, respectively, it is considered the optimal solution for the placement of the Wi-Six TRPs. If it does not meet these conditions, the
5.6 Results & Evaluations

Table 5.2: Random Wi-Six TRP placement for different room sizes for benchmarking the optimal solution

<table>
<thead>
<tr>
<th>Room Dimensions</th>
<th>Wi-Six TRP # 1</th>
<th>Wi-Six TRP # 2</th>
<th>Wi-Six TRP # 3</th>
<th>Wi-Six TRP # 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small (10m × 10m × 4m)</td>
<td>(2,1,4)</td>
<td>(1,7,4)</td>
<td>(8,0,4)</td>
<td>(10,9,3.9)</td>
</tr>
<tr>
<td>Medium (15m × 15m × 4m)</td>
<td>(0,0,4)</td>
<td>(1,15,4)</td>
<td>(14,0,4)</td>
<td>(15,12,3.9)</td>
</tr>
<tr>
<td>Large (20m × 20m × 4m)</td>
<td>(1,1,4)</td>
<td>(1,19,4)</td>
<td>(18,2,4)</td>
<td>(18,20,3.9)</td>
</tr>
</tbody>
</table>

algorithm restarts with a new set of 10 individuals. This iterative process continues until the algorithm finds a solution that satisfies both the vertical and horizontal tolerances, ensuring the positioning of the Wi-Six TRPs minimizes error while satisfying these constraints.

5.6 Results & Evaluations

In this section, we present the results, which are organized into two sub-sections for clarity. Firstly, in Sec. 5.6.1, we conduct a performance evaluation of the proposed optimization algorithm. We provide results that demonstrate the significant impact of the GDOP on overall performance. Next, in Sec. 5.6.2, we first present the results for indoor positioning using Wi-Six TRPs without the Wi-Six optimal placement. This serves as a benchmark where we evaluate the performance of 3D indoor localization using Wi-Six TRPs; i.e., the IEEE 802.11az technology in the front, supported by the 6G network as the backhaul. Subsequently, we showcase the assessment results of the entire Wi-Six positioning scheme with optimal placement, highlighting the improvements achieved in indoor positioning accuracy. All simulations in this study were conducted using MATLAB 2022a, executed on a Dell OptiPlex 7080 desktop computer featuring an Intel(R) Core(TM) i9-10900K CPU and 64 GB of RAM.
5.6.1 Wi-Six Optimal Placement Assessment

Here, our goal is to demonstrate how our optimization framework manages to minimize the GDOP with low VDOP and HDOP (xDOP) values. To do so, we first provide the xDOP values for arbitrary anchor placements given in Table 5.2 and more centralized anchor placements in Table 5.3. These will illustrate how random anchor placements will lead to poor location estimates. Next, we provide the results of our optimization algorithm to show the improvement.

Here, the primary goal is to determine the optimal placement of Wi-Six TRPs, therefore, it fully depends on the room’s dimensions (i.e., the provided floor plan). To account for this, our algorithm takes the floor plan as input and provides the optimal oscillator placement as output. The performance of the proposed algorithm is assessed on three distinct floor plans, with the primary aim of demonstrating its versatility across room dimensions of varying sizes, from modest to exceedingly expansive.

First, a large office room with dimensions of $10\,\text{m} \times 10\,\text{m} \times 4\,\text{m}$ serves as an example for playing virtual games using VR devices. Next, a large conference room measuring $15\,\text{m} \times 15\,\text{m} \times 4\,\text{m}$, which is suitable for AR use cases in larger setups. Finally, the algorithm is tested on an extreme example, a $20\,\text{m} \times 20\,\text{m} \times 4\,\text{m}$ large game room for multi-user AR/VR games such as laser tag. The algorithm will converge faster for smaller spaces compared to larger environments; however, even in larger setups, the proposed algorithm achieves the optimal solution in a timely manner. Moreover, based on our results, the algorithm can work even in larger test setups (e.g., a large warehouse floor plan with dimensions of $50\,\text{m} \times 50\,\text{m} \times 4\,\text{m}$). The only additional requirement for this case is the employment of extra TRPs to maintain coverage throughout the space and an optimal solution will still be calculated, albeit with higher compute time. Given that we target Metaverse
users such as AR/VR devices, considering a room size around 10m × 10m × 4m is acceptable. However, we provide larger rooms just to show that our algorithm is not restricted to the size and floor plan of the indoor space. In fact, the algorithm effectively works in any given floor plan. All the dimensions mentioned in tables and figures are in meters unless stated otherwise.

To evaluate the xDOP results and facilitate a comparison between the non-optimized and optimal placement effects, we present the cumulative distribution function (CDF) graphs for xDOP values, both for optimal and non-optimized configurations. Specifically, Figure 5.8 illustrates the CDF for the non-optimized placement (extracted from Table 5.2), while Figure 5.9 showcases the CDF for the optimal placement (extracted from Table 5.4).

Comparing Figure 5.8 and Figure 5.9, we can see that for the majority case (95% of the point), without employing the Wi-Six optimal placement framework, the VDOP and GDOP values are around 300. This significantly goes down to 15 for the configuration with optimal Wi-Six TRPs deployment. As stated earlier, GDOP has a direct impact on the final positioning accuracy and more than 20 times improvement in the GDOP values can be translated to more than 20 times better final accuracy.

As an alternative means for assessing the performance of the Wi-Six optimal placement algorithm, we depict heatmap plots, both for the non-optimized and optimal placement. To provide a reference point against which we can evaluate the outcomes of our optimal solution, we initially calculate xDOP values for randomly positioned Wi-Six TRPs, as provided in Table 5.2, across various room sizes. To accurately depict the xDOP values for 3D space, we calculate the average of all xDOP values for all potential \((x, y)\) points across all \(z\) planes. Heatmaps enable us to provide a detailed visual representation of location estimations for both the \(X - Y\) plane and \(Z\)-axis computations, making the results more precise and easily interpretable. To demonstrate the performance of the algorithm in satisfying different
Figure 5.10: 3D xDOP representation for three different placements: Top row associates with the “Configure 1”; Middle row with the “Configure 2”; and Bottom row with the “Configure 3” in Table 5.2
constraints, we separate our results into GDOP, VDOP, and HDOP. This helps us identify whether poor location estimates result primarily from errors in the $X - Y$ plane or $Z$-axis estimates.

First, in Figure 5.10, we show the heatmap of xDOP values in a 3D plot. In this Figure, the first row represents the xDOP values (left to right: HDOP, VDOP, and GDOP) corresponding to the first configuration in Table 5.2, the second row is the result of the arrangement for the second configuration in the table, and finally, the last row in this plot showcases the third configuration for larger room size. As seen in this Figure, it is not easy to read for all the points in the environment, as some points in the front block the points in the middle part of the Figure. To enhance clarity and facilitate comparison, we present a 2D representation of Figure 5.10 as depicted in Figure 5.11. This visualization allows us to observe the xDOP values for all points in the $X - Y$ plane. Similar to Figure 5.10, these values are obtained after averaging over different $z$ planes.

The results depicted in Figure 5.11 demonstrate that, regardless of the size of the indoor space, a random anchor deployment will result in subpar xDOP values, thus negatively impacting the final localization performance. In Figure 5.11, the first row displays the xDOP values (from left to right: HDOP, VDOP, and GDOP) corresponding to the first arrangement in Table 5.2. The second row presents the results of the arrangement for the second configuration, and finally, the last row in this plot is for the third configuration for larger room sizes.

As an additional benchmark, we present the results in Figure 5.12 for random TRP placement. For this, we select one large room (with dimensions $20m \times 20m \times 4m$) and manually relocate the Wi-Six TRPs to random positions. These positions are shown in Table 5.3. The findings suggest that achieving the optimal positioning manually is unlikely, as the HDOP and VDOP values fluctuate independently. This implies that intuitively placing the anchors
Figure 5.11: xDOP representation for three different placements: Top row associates with the “Configure 1”; Middle row with the “Configure 2”; and Bottom row with the “Configure 3” in Table 5.2.
Table 5.3: Manually located Wi-Six TRPs in a large room for benchmarking the optimal solution

<table>
<thead>
<tr>
<th>Manually Placements</th>
<th>Wi-Six TRP # 1</th>
<th>Wi-Six TRP # 2</th>
<th>Wi-Six TRP # 3</th>
<th>Wi-Six TRP # 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Placement 1</td>
<td>(0,10,4)</td>
<td>(10,0,4)</td>
<td>(10,20,4)</td>
<td>(20,10,3.9)</td>
</tr>
<tr>
<td>Placement 2</td>
<td>(3,10,4)</td>
<td>(10,3,4)</td>
<td>(10,17,4)</td>
<td>(17,10,3.9)</td>
</tr>
<tr>
<td>Placement 3</td>
<td>(6,10,4)</td>
<td>(10,6,4)</td>
<td>(10,14,4)</td>
<td>(14,10,3.9)</td>
</tr>
</tbody>
</table>

centrally won’t necessarily result in an optimal configuration.

As seen in Figure 5.11 and Figure 5.12, all configurations are showing poor VDOP and GDOP values for the majority of the covered area, rendering the final positioning unreliable. Therefore, even if one could reduce the ranging errors to below a centimeter by leveraging large bandwidths (around 1.5 GHz) promised in 6G, the final 3D positioning would still lack precision. Specifically, the localization error could potentially escalate by a factor of 20, resulting in errors in the 20 cm range - a margin of error that would be unacceptable for Metaverse applications.

A key observation from Figure 5.11 and Figure 5.12 is the overall xDOP patterns. Although HDOP remains significant and broadly influences localization accuracy, the final GDOP pattern directly reflects the corresponding VDOP map. This implies that even when $X - Y$ plane estimates are spot-on, providing accurate 2D localization, this performance does not necessarily translate to three dimensions. This finding validates our decision to consider separate constraints for HDOP and VDOP during our optimization process for positioning the Wi-Six TRPs.

In our concluding set of results shown in Figure 5.13, we display the xDOP values for the optimal positioning of Wi-Six TRPs. The rows of Figure 5.13 correspond to the xDOP results derived from anchor placements provided in Table 5.4 for the three room dimensions. These
Figure 5.12: xDOP representation for three sets of manually located Wi-Six TRPs for a large room represented in Table 5.3
5.6. Results & Evaluations

Table 5.4: Optimal Wi-Six TRPs placements for different room sizes

<table>
<thead>
<tr>
<th>Configurations</th>
<th>Wi-Six TRP # 1</th>
<th>Wi-Six TRP # 2</th>
<th>Wi-Six TRP # 3</th>
<th>Wi-Six TRP # 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimal Configure 1</td>
<td>(9,9.5,4)</td>
<td>(5,5,4)</td>
<td>(0,8.5,3.5)</td>
<td>(10,3.5,3)</td>
</tr>
<tr>
<td>Optimal Configure 2</td>
<td>(13,5,14,4)</td>
<td>(7,7,4)</td>
<td>(0,12.5,3.5)</td>
<td>(15,5,3)</td>
</tr>
<tr>
<td>Optimal Configure 3</td>
<td>(14,0,3.5)</td>
<td>(20,13,2.5)</td>
<td>(1,20,3)</td>
<td>(10,5,13,4)</td>
</tr>
</tbody>
</table>

Results unequivocally illustrate that appropriate TRP placement can significantly reduce the error from GDOP in the $\sigma(r) = \sigma_r \times GDOP$ formula. Therefore, in achieving finer positioning within the Metaverse ecosystem, GDOP won’t be a major source of error if ranging errors can be diminished through the use of higher bandwidths.

Figure 5.13 also provides insight into how room dimensions impact the algorithm’s performance in larger indoor spaces. For smaller and medium-sized rooms, it is evident that the dark-shaded region representing high-accuracy localization is broader. Nevertheless, even in larger rooms, the performance remains impressive, staying below 5 for most of the area. Moreover, compared to the deployment results in Figure 5.11, the Wi-Six optimization framework manages to preserve the HDOP accuracy while notably reducing the VDOP across the room.

In Figure 5.14, we present the outcomes of the optimal placement in a 3D graph, which serves as the 3D representation of Figure 5.13. As previously stated, our preference in this chapter lies with the 2D view, achieved by averaging over all z planes, as it allows us to present all data points effectively without the representation of any point getting blocked by others. Nevertheless, we also include Figure 5.14 to facilitate a straightforward comparison with Figure 5.10 and to demonstrate the enhancements attained by the optimal placement in the 3D visualization.
Figure 5.13: xDOP representation for the optimal placement associated with “Wi-Six Solution” in Table 5.4.
Figure 5.14: xDOP 3D representation for the optimal placement associated with “Wi-Six Solution” in Table 5.4
5.6.2 Wi-Six Positioning Assessment

Here, we present the results of estimating the user’s position in an indoor multipath environment using the Wi-Six positioning framework. The performance of the positioning algorithm is evaluated and compared, both with and without the incorporation of the Wi-Six optimal placement algorithm. This evaluation allows for a comprehensive analysis of the positioning algorithm’s effectiveness and the added benefits of employing the Wi-Six optimal placement algorithm in improving performance.

As Wi-Six utilizes the 802.11az Wi-Fi in the front end for positioning, our simulation involves an 802.11az network comprised of multiple Wi-Six TRPs and a single user. Our program conducts a ranging measurement exchange for each user-TRP pair and subsequently trilaterates the user’s position using these measurements. For various environments with different floor plans and dimensions, we repeat this process for every point in the set \( U \), analyzing the performance of the proposed positioning scheme at each potential user location within the indoor environment.

For the packet transmission and reception in our simulations, we model the measurement exchange between the user and Wi-Six TRPs by conducting the steps outlined in the following.

- Generate a ranging NDP. Then, apply a delay to the NDP based on the distance between the user and the Wi-Six TRPs. These distances are derived at each location in set \( U \) and the delays include both fractional and integer sample delays.

- Propagate the waveform through an indoor TGax channel, considering different channel realizations for each packet. Then, add additive white Gaussian noise (AWGN) to the received waveform. The signal-to-noise ratio (SNR) value remains the same for all links between the user and Wi-Six TRPs.
5.6. Results & Evaluations

Figure 5.15: Overview of the simulation chain for Wi-Six positioning

- Conduct synchronization and frequency correction on the received waveform to ensure accurate demodulation. Then, demodulate the HE-LTF symbols from the waveform.

- Using the HE-LTF to estimate the channel frequency response. Then, utilize the MUSIC super-resolution algorithm to estimate the distance between the user and each Wi-Six TRP.

- Trilaterate the position of the user by employing the combined distance estimates from the aforementioned user-TRP pairs.

By following these steps, we simulate the measurement exchange process and obtain 3D position estimations for the user in the Wi-Six system. Figure 5.15 depicts the processing steps involved for each user-TRP link.

In the following, we will elaborate on the parameter values employed in our experiments to configure the waveform communicated between the user and the Wi-Six TRPs followed by the configuration values for the channel.

**Waveform Configuration:** To configure the waveform generators for each Wi-Six TRP and the user, the following parameters were considered. The channel bandwidth is set to 160 MHz, and both the Wi-Six TRPs and the user utilize 2 transmit and 2 received antennas.
Table 5.5: Waveform Configuration

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carrier Frequency</td>
<td>5 GHz</td>
</tr>
<tr>
<td>Bandwidth</td>
<td>160 MHz</td>
</tr>
<tr>
<td># of TX Antenna</td>
<td>2</td>
</tr>
<tr>
<td># of RX Antenna</td>
<td>2</td>
</tr>
<tr>
<td># of Space-Time Streams</td>
<td>3</td>
</tr>
<tr>
<td>Guard Interval</td>
<td>16 msec</td>
</tr>
</tbody>
</table>

The number of space-time streams is also set to 2 in our experiments. For the HE ranging NDP parameters of the user and the Wi-Six TRPs, the number of HE-LTF repetitions is set to 3. A guard interval of 1.6 microseconds is considered. The sampling rate used is the same as the channel bandwidth, which is 160 MHz. The entire setup is conducted on the 5 GHz carrier frequency. Moreover, for the RTT procedure, a time delay of 16 milliseconds is used between UL NDP TOA and DL NDP ToD. All these parameter values are shown in Table 5.5.

**Channel Configuration:** To configure the channel, we utilized a System object in MATLAB capable of generating two types of channels. The first type features a dominant direct path, where the Direct Line-of-Sight (DLOS) path is the strongest among all paths. The second type is a channel with a non-dominant direct path, where the DLOS path is present but not the strongest among all paths. For the channel delay profile model, we employed the following parameters:

- Breakpoint Distance: 5 m
- RMS Delay Spread: 15 nanoseconds
- Maximum Delay: 160 nanoseconds
- Rician K-factor: 0 dB
5.6. Results & Evaluations

- Number of Taps: 9
- Number of Clusters: 2

These parameters were utilized to characterize the channel’s delay profile before further processing with the bandwidth reduction factor.

In our experiments, we conduct ranging and positioning simulations for all points within set $U$. At each point, the Wi-Six TRPs and the user engage in multiple uplink and downlink packet exchanges. Initially, we estimate the ranging error between each Wi-Six TRP and the user for all points in set $U$. At each point, we employ different channel realizations and varying Additive White Gaussian Noise (AWGN) profiles. We also compare the calculated distance between each Wi-Six TRP and the user with the known distances. In other words, the first step in our simulations involves conducting Wi-Six TRP ranging, wherein we estimate the distance between each Wi-Six TRP and the user. To evaluate the ranging procedure’s performance, we position the user at every possible location in set $U$ and estimate the distance between that point and all Wi-Six TRPs. The ground truth is extracted based on the precise user and Wi-Six TRP locations, which are known. By comparing the estimated distance with the ground truth distance, we determine the accuracy of the ranging procedure.

The subsequent step in our program is to estimate the user’s position using trilateration techniques. For this process, a minimum of four Wi-Six TRPs is required for 3D positioning. The ground truth position (exact position) at each point is obtained from the ground truth distances. By comparing the estimated position with the ground truth, we determine the 3D positioning error using the following formula:

$$e_{pos} = \sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2};$$  \hspace{1cm} (5.12)
where $e_{\text{pos}}$ is the 3D positioning error, and $\Delta x$, $\Delta y$, and $\Delta z$ represent the differences between the estimated position and the ground truth in the $x$, $y$, and $z$ dimensions, respectively. This formula allows us to quantify the accuracy of the positioning technique and assess its performance in the simulation experiments.

Figure 5.17 illustrates the cumulative distribution function (CDF) of localization accuracy with the optimal placement, as determined from Table 5.4. It is evident from the Figure that regardless of the dimensions of the environment, an impressive overall 3D accuracy of less than 1 m has been attained.

To compare the Wi-Six full scheme with a benchmark, Figure 5.16 displays non-optimized configurations derived from Table 5.2. These plots serve as a contrast to the optimal scenarios.
depicted in Figure 5.17. In the non-optimized placements, 3D errors escalate to 10 m, whereas in the optimal cases, they remain below 1 m. The significance of errors exceeding 10 m in an indoor environment cannot be understated; such inaccuracies render the positioning system utterly unreliable. This underscores the criticality of employing optimal placement strategies.

In essence, even with the inherent advantages of 802.11az in 160 MHz bandwidth for positioning, which can provide distance estimation with an error of below 10 cm, failure to utilize the optimal placement of Wi-Six TRPs leads to 3D positioning errors surpassing 10 m. It is important to emphasize that contemporary state-of-the-art positioning systems often do not account for the adverse effects of geometry-induced errors. Since many of these systems predominantly focus on 2D localization, the detrimental impact of GDOP (Geometric Dilution of Precision) may not be readily discernible in their final solutions, as HDOP is not as severely affected as VDOP. However, for a comprehensive 3D positioning system, considering the effect of geometry becomes imperative based on our findings.

5.7 Summary

In this chapter, we presented a novel approach for the localization of Metaverse users, especially when conventional methods like GPS or weak 3GPP RAN signals are absent. Our method leverages the round trip time between Wi-Six TRPs and Metaverse devices, providing precise location data. A primary discovery of our research is the strong influence of the spatial configuration of the Wi-Six TRPs on localization accuracy. We demystified the composition of localization error, pointing out its dependence on both direct measurement inaccuracies and geometric properties, namely GDOP. Within GDOP, we analyzed the issue into two domains: the horizontal plane (X – Y) and the vertical plane (Z-axis). Our empirical studies revealed the notable observation that in 3D environments, users are more
susceptible to discrepancies in vertical positioning than in horizontal ones. To optimize the positioning of Wi-Six TRP anchors, we proposed an innovative optimization strategy. Our solution, based on the EA class has significantly improved vertical estimations without compromising the HDOP. Furthermore, we provide a thorough simulation of the entire Wi-Six platform for indoor setups based on real-life values. The culmination of our efforts paves the way for a more accurate and efficient positioning mechanism for the future of the Metaverse.
Chapter 6

IsoPos: Isochrons in Photonic Oscillators for Positioning

For decades, high-accuracy localization has driven the interest of the research community. Recent cases include Augmented and Virtual Reality (AR/VR), indoor robotics, and drone applications, that have led to the emergence of sub-centimeter localization requirements. This study introduces a new approach for high-accuracy localization by utilizing isochrons in injection-locked tunable quantum photonic oscillators, which we referred to as IsoPos: Isochrons in Photonic Oscillators for Positioning. The proposed paradigm shift takes advantage of photonic oscillators’ radical frequency tunability and isochron structure to offer an innovative path for measuring the Time of Arrival (TOA). To achieve precise TOA measurements, IsoPos utilizes the phase shift induced by the incoming user signal. This shift is detected by analyzing the phase response of the receiver’s photonic oscillator, which is exclusively determined by its isochron structure. Furthermore, IsoPos uses the injection-locking method as well as the nonlinear properties of injection-locked quantum photonic oscillators to achieve highly accurate phase synchronization between different positioning nodes. This contributes to a seamless three-dimensional localization devoid of errors caused by miss-synchronization. Our numerical simulations show that IsoPos achieves sub-1 mm accuracy in three-dimensional localization, surpassing the precision of existing positioning systems by at least one order of magnitude.
6.1 Motivation

As we transitioned from drone applications to the metaverse in the preceding chapter, our objective was to enhance the precision of the positioning system. We have thus far addressed the majority of prerequisites essential for crafting a high-precision localization system using ranging-based methodologies. This spanned a comprehensive exploration of the intrinsic benefits inherent to 5G New Radio (NR) and future iterations, such as the sixth generation, in providing augmented bandwidth for positioning and improving distance estimation accuracy. Additionally, we delved into leveraging the advanced positioning capabilities of IEEE 802.11az in Wireless Fidelity (Wi-Fi) technologies. In tandem with the investigated signals—5G, 6G, and Wi-Fi—which contribute to reduced ranging error, we’ve unveiled the significance of the relative geometry between users and positioning nodes. To address this, we’ve introduced an optimal placement algorithm aimed at mitigating geometry-induced errors.

In the current chapter, we extend our boundaries to a fresh perspective. A pivotal aspect of a high-precision ranging-based positioning system revolves around the efficacy of distance estimation. Various timing mechanisms are employed to gauge the Time of Arrival (TOA) and derive distance estimates. However, these timing protocol mechanisms have inherent limitations. Our focus in this chapter gravitates toward developing an innovative, super high-resolution timing mechanism that significantly enhances distance accuracy. To achieve this, we pivot away from conventional acoustic or radio frequency (RF) systems like 5G and Wi-Fi to quantum photonic oscillators—in simpler terms, lasers. For the first time in this dissertation and pioneering across existing literature, we propose harnessing the nonlinear properties of tunable quantum photonic oscillators to introduce an inventive timing mechanism. We then integrate this distance information into a Time Difference of Arrival (TDOA) positioning system.
Parallelly, within ranging-based positioning, two critical challenges operate behind the scenes, exerting considerable influence on localization accuracy—as elucidated in prior chapters. These challenges encompass synchronization between positioning system entities and synchronization between the positioning system and the user. In earlier chapters, we operated under the assumption of perfect synchronization—between users and positioning systems, as well as among positioning system nodes. However, this assumption doesn’t hold true in real-world scenarios, particularly concerning wireless synchronization between mobile users and positioning nodes. Most existing technologies employ diverse schemes to address this synchronization disparity. The TDOA technique emerges as a well-recognized approach that circumvents the synchronization challenge between users and positioning nodes, albeit it necessitates stringent synchronization between the nodes themselves.

Over the past two chapters, we harnessed techniques reliant on Round Trip Time (RTT) or analogous protocols. Notably, the TDOA technique, although prevalent in current technologies, hinges significantly on strict synchronization between positioning nodes. In this chapter, we take a pivotal step by integrating the TDOA technique into our positioning systems. Through the injection locking technique of photonic oscillators, we establish a remarkably precise synchronization mechanism among positioning nodes, notably enhancing the performance of the TDOA technique.

### 6.2 Chapter Overview

High-precision positioning is crucial for the development of indoor Location-Based Services (LBSs) like emergency response, asset tracking, and autonomous robotic systems [140, 141]. This is especially true for futuristic innovations like Augmented and Virtual reality (AR/VR) [142]. Despite its crucial role in outdoor positioning and navigation, the Global
Positioning System (GPS) encounters substantial limitations when used indoors [143]. This drawback mainly arises from environmental disturbances, including obstruction of satellite signals by buildings and other constructions. Even when GPS signals can be received indoors, they often lack the precision required to offer accurate positioning information [144]. As a result, GPS technology fails to deliver optimal performance when deployed indoors.

Vision-based techniques are frequently utilized as the primary technology for high-accuracy indoor localization/tracking due to their superior accuracy [145]. However, such methods face practical constraints such as sensitivity to lighting conditions, occlusion, and high computational complexity. Hence, the effectiveness of vision-based approaches is limited, especially in visually impaired environments [50]. On the other hand, indoor positioning can also be achieved using ranging-based techniques, which involve utilizing radio frequency (RF) signals, among others. However, the accuracy of such technologies is contingent on the precision of Time of Arrival (TOA) or Angle of Arrival (AOA) measurements [94].

The accuracy of time measurements in ranging-based techniques hinges on measuring the RF signal after it has propagated through the environment. As a result, any degradation of signal quality, including the presence of noise, interference, and multipath fading can significantly undermine the accuracy. More importantly, limitations in receiver resolution drastically affect the accuracy of time measurements, thereby, compromising the overall positioning accuracy [94]. Ultimately, existing systems for obtaining TOA suffer not only from these limitations but also lack a mechanism for measuring time with high resolution, resulting in inadequate localization accuracy.

One of the primary contributions of our work is the introduction of a novel timing mechanism called **IPT: Isochron Phase-Shift Timing**. IPT accurately calculates the phase shift resulting from the isochron structure of an oscillator, enabling highly precise measurement of the TOA for incoming RF signals. These precise time measurements play a crucial role in
Figure 6.1: System overview showing user connection to the RF interface of IsoPos to boot the follower oscillators by generating excitation pulses. Precise synchronization is achieved by periodic pulses generated through the controller oscillator passed to the followers through optical fiber links.

Our three-dimensional localization system, IsoPos: Isochrons in Photonic Oscillators for Positioning. IsoPos utilizes these accurate TOA measurements to determine the distances between the user and each positioning node, which are later employed in a Time Difference of Arrival (TDOA) scheme for localization. Another key contribution of our work is the introduction of a novel synchronization scheme named ILS: Injection-Locking Synchronization. By leveraging ILS, we achieve precise synchronization among all the positioning nodes within IsoPos. This synchronization ensures that all nodes share the same clock for their measurements, further enhancing the accuracy and reliability of the system.

Novel Timing Mechanism: We introduce IPT, a novel approach for timing measurements with extremely high resolution leveraging isochrons in photonic oscillators. This approach is based on the concept of isochrons, originally introduced in the context of mathematical biology [76] and closely related to any robust self-sustained oscillation [146] occurring in physical
or man-made systems. Such oscillations, also known as *Limit Cycles* in the terminology of the theory of dynamical systems, can serve as clocks with an extremely precise period. Their periods can be uniquely partitioned, not necessarily uniformly. Each partition is known as an isochron and the union of these isochrons provides the sub-cycle time resolution. Isochrons dictate the system’s phase response to external stimulations having the form of either a single pulse or a periodic pulsatile sequence. The phase response determines the induced phase shift of the clock due to an incoming pulse, in the first case, and the synchronization properties of the clock with the periodic sequence, in the second case [77]. It is only recently that these concepts have been introduced in photonic oscillators consisting of two coupled lasers in a controller-follower configuration [80, 81]. Such an optically injected laser is well-known for its tunable self-sustained oscillations and widely used in various applications including secure chaos [147] and quantum communications [148, 149]. The potential of utilizing photonic oscillators along with their isochrons’ structure for precise time is discussed for the first time in this work.

**Novel Synchronization Approach:** We propose ILS, a novel synchronization approach that employs optical injection locking to implement precise time synchronization between different positioning anchors, a significant challenge in many available architectures. This synchronization ensures that all measured TOAs at various nodes have the same time bias compared to the user, enabling the utilization of the TDOA technique for three-dimensional positioning.

**Localization System:** We construct our localization system, IsoPos, by utilizing the IPT mechanism for timing and the ILS protocol for anchor synchronization. IsoPos offers greater flexibility compared to existing localization systems, as it provides time measurements with extremely high resolution independent of received RF signal characteristics. For the IPT mechanism in IsoPos, the transmitted signal from the user only triggers the system to gen-
erate excitation pulses, while TOA measurements are derived from the phase response of the photonic oscillators’ behavior to these pulses, determined by the isochrons’ structure. A schematic overview of the IsoPos mechanism is illustrated in Figure 6.1.

Furthermore, we demonstrate that the Positioning Error Bound (PEB) of IsoPos is affected by ranging- and geometry-induced errors. Utilizing IPT and ILS mechanisms, IsoPos reduces the ranging-based errors substantially. The geometry-induced errors, on the other hand, are caused by the relative geometry between the positioning nodes and the user. Finding an optimal anchor placement that encompasses all possible user locations in a three-dimensional space is a known NP-hard problem \[94\]. To counter the adverse impact of relative geometry, we propose an optimization framework based on evolutionary algorithms (EAs). Our findings indicate that our proposed system leads to precise localization and tracking with sub-1 mm accuracy in all three dimensions, which is an order of magnitude improvement over existing systems that offer centimeter-level accuracy. These results demonstrate the potential of IsoPos as a highly accurate localization solution for real-world applications.

Our primary contributions are detailed below.

- We present IsoPos, a high-accuracy three-dimensional localization system that utilizes the isochron’s structure of photonic oscillators and injection locking scheme to achieve precise results.

- IsoPos leverages the proposed IPT protocol for timing, which is an innovative method for measuring TOA that leads to distance estimation with an accuracy of sub-tenths of a millimeter.

- To accurately measure TOA, IPT calculates the induced phase shift in the oscillation of the receiver by determining the intricate structure of its isochrons. These structural results are used to calculate the system’s response to instantaneously generated
excitation pulses upon receiving the user’s RF signal.

- IsoPos utilizes the proposed ILS protocol for synchronization. ILS leverages the highly tunable, robust limit cycles established by injection-locked semiconductor lasers and the synchronization properties of such photonic oscillators to achieve precise synchronization between various positioning nodes.

- We derive the PEB for IsoPos and attribute the positioning error to ranging- and geometry-induced errors. We design an optimization framework to address the NP-hard problem of mitigating geometry-induced errors.

- Using commercially available semiconductor laser parameters, we design a comprehensive evaluation campaign to assess the performance of our proposed system. The numerical results confirm that IsoPos achieves sub-millimeter localization accuracy in three dimensions.

### 6.3 System Model

In this section, we provide a concise overview of the proposed localization approach, which is broken down into three functional blocks for clarity. In the following, we will elaborate on each block. The first two blocks mainly cover the IPT mechanism and timing procedure, while the third block covers the ILS protocol and synchronization. Figure 6.1 provides a graphical representation of the system.

**User RF Transmission:** To initiate TOA measurement, we use the RF signal transmission between the user and the proposed system. This signal is denoted as $s(t)$. Due to the fact that the TOA measurement in our proposed approach does not require demodulation of $s(t)$, i.e., it is not based on the content of the received signal, we are able to design it freely
and robustly against environmental influences. Because the receiver knows $s(t)$, it ensures that only this signal can activate the system, while rejecting all other signals. This design approach enhances system robustness against destructive environmental impacts.

**RF Interface & Follower Oscillators:** The initial part of this block is the RF interface or booting block, which serves as an intermediary between our system and the physical world. It includes the control system for booting the follower oscillators, as depicted in Figure 6.1. Upon the arrival of the designed $s(t)$, the RF interface’s sole responsibility is to instantly boot the system. In other words, it causes the system to generate the necessary excitation pulses internally and feed them through optical fibers to the follower oscillators. The Phase Transition Curve (PTC) quantifies the effect of an excitation pulse on each oscillator. Its shape is determined by the structure of the system’s isochrons, and the induced phase shift can be used to determine the time instance at which the excitation pulse is generated. Since the booting block starts instantaneously after receiving the RF signal from the user, the measured time instance is the same as the TOA of the received RF signal. At the stage where the excitation pulses are fed into the follower oscillators, the amplitude of the excitation pulse can be set to be the same for all oscillators. It is noteworthy that this is independent of the received RF signal strength or quality which can be different due to attenuation along different paths.

**Controller Oscillator:** The last block of the system is the controller oscillator which is in charge of synchronizing the phase of all the follower oscillators. This block leverages the combination of the injection locking mechanism with the synchronization properties of the photonic oscillator under an externally modulated periodic signal. This allows precise synchronization among all follower oscillators and ensures that their initial phases are identical. As a result, all the phase shifts caused by the excitation pulses share a common frame of reference enabling the system to perform precise positioning based on time differences of ar-
rivals. The stable phase-locking conditions for the amplitude and frequency of the external synchronizing signal are determined by the structure and phase response of the oscillator’s isochrons. Notably, synchronization of the follower oscillators is achieved by a periodic sequence of excitations modulating the injection rate of the controller oscillator, whereas for the measurement of TOA, only the effect of a single excitation pulse on each follower oscillator is required.

### 6.4 Synchronization Protocol and Time-measurement Mechanism

Here, we present the ILS protocol used for synchronization and the IPT mechanism utilized for measuring TOA, which are the core contributions of IsoPos system and the foundations of our study. The following subsections delve into the essential preliminaries and in-depth approach details concerning photonic oscillators, isochrons, and time measurement concepts.

#### 6.4.1 Photonic Oscillator Model

A fundamental tunable photonic oscillator consists of two coupled semiconductor lasers in a controller-follower configuration, where the output of the former is optically injected into the latter. The semi-classical description of this system is given by the following set of differential equations for the complex amplitude $E$ of the optical field $(E_{\text{opt}}(\tau) = E(\tau)e^{i\omega_0\tau})$
6.4. SYNCHRONIZATION PROTOCOL AND TIME-MEASUREMENT MECHANISM

and the carrier density $n$

$$\frac{dE}{d\tau} = \frac{\Gamma G_N}{2} (1 + i\alpha) nE + \kappa E_{in} e^{i\nu \tau}$$

$$\frac{dn}{d\tau} = \frac{J - J_{th}}{e} - \frac{n}{\tau_s} \left( \frac{1}{\Gamma \tau_p} + G_N n \right) |E|^2$$

where the optical field has been renormalized such that the power $|E|^2$ represents the number of photons in the active layer, the injected monochromatic optical field is $E_{in}(\tau) = E_{in}(\tau)e^{i\omega_{in}\tau}$, and $\tau$ denotes real time. The parameter $\Gamma$ is the confinement factor, $\alpha$ is the linewidth enhancement factor, $J$ is the pump current, whereas $J_{th}$ is the threshold current, $e$ is the elementary charge, $\tau_p$ and $\tau_s$ are the photon and carrier lifetimes, respectively—typically measured in psec—, and $G_N$ is the gain coefficient at transparency. The parameters $\kappa$ and $\nu$ correspond to the injected field and denote the injection rate and the detuning between the injected signal and the follower laser, respectively. By introducing the new time $t$ and the new dependent variables $x, \ y, \ Z$

$$t \equiv \frac{\tau}{\tau_p}, \ x \equiv \text{Re} \left( \sqrt{\frac{\tau_s G_N}{2}} E \right), \ y \equiv \text{Im} \left( \sqrt{\frac{\tau_s G_N}{2}} E \right), \ Z \equiv \frac{\Gamma G_N \tau_p}{2} n;$$
a set of dimensionless nonlinear differential equations for the normalized complex electric field $\tilde{E} = x + iy$ and the normalized excess carrier density $Z$ can be obtained;

\[
\frac{dx}{dt} = (x - \alpha y)Z + \Omega y + \eta \\
\frac{dy}{dt} = (y + \alpha x)Z - \Omega x \\
\frac{dZ}{dt} = P - Z - (1 + 2Z)(x^2 + y^2)
\]  

where

\[ T \equiv \frac{\tau_s}{\tau_p}, \quad P \equiv \frac{\tau_s \tau_p G_N \Gamma}{2} \left( \frac{J - J_{th}}{e} \right), \quad \eta \equiv \sqrt{\frac{\tau_s G_N \tau_p \kappa E_{in}}{2}}, \quad \Omega \equiv \nu \tau_p. \]

$T$ is the ratio of carrier to photon lifetimes, $P$ is the normalized excess electrical pumping rate of the follower laser, $\eta$ the normalized injection rate, and $\Omega$ the normalized detuning between the frequency of the controller laser and the frequency of the free-running follower laser [150].

As a nonlinear system, this setup is known to have a rich set of dynamical features ranging from stable steady states, and limit cycles born out of Hopf bifurcations, to chaotic outputs [1]. For our purpose, we focus on the well-defined parameter range where this system supports stable limit cycles corresponding to self-sustained oscillations.

Each limit cycle is characterized by its period $T_{lc}$ (in the following analysis $T_{lc}$ has been renormalized to 1), its spectral content (discrete spectral lines at integer multiples of $f_{lc} = T_{lc}^{-1}$, since it is in general non-harmonic) and by the rate of convergence of nearby initial conditions towards the limit cycle.

Moreover, the limit cycle is characterized by a phase variable parameterizing each point of
6.4. Synchronization Protocol and Time-Measurement Mechanism

Figure 6.2: (a) Definition of phase function on the limit cycle; $\theta$ is measured in units of $T_{lc}$. The closed curve corresponds to the limit cycle, while the black points correspond to the unstable equilibria of the system. (b) Isochrons' structure in the phase space of the system. (c) Asymptotic phase on the section $Z = 0$ of the phase space. The closed continuous curve corresponds to the projection of the limit cycle of the system on this section, while the closed dashed curve corresponds to the projection of the perturbed initial conditions on this section. $B_0$: point on the limit cycle, $B$: perturbed initial point of the system.

the cycle as represented in Figure 6.2(a). The concept of the phase can also be extended outside the limit cycle by introducing the asymptotic phase function\[76\], which is defined as the relative phase with which an initial condition ends up in the limit cycle.

Isochrons are defined as the locus of the initial conditions, within the basin of attraction of the limit cycle, that have the same asymptotic phase\[76\], and partition the phase space as shown in Figure 6.2(b)-6.2(c). The structure of the isochrons determines the phase response \[146\] of the limit cycle to a pulse kick of amplitude $A$ that moves the system to an initial condition outside the limit cycle as illustrated in Figure 6.2(c). The relation between the phase of the oscillation at the time when the pulse arrives and the new (asymptotic) phase is provided by the Phase Transition Curve (PTC)

\[
PTC(\theta) = \theta_{\text{new}} \mod 1, \tag{6.3}
\]
Table 6.1: Parameter values for a semiconductor laser [1].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>linewidth enhancement factor</td>
<td>2.0</td>
</tr>
<tr>
<td>carrier lifetime</td>
<td>250 psec</td>
</tr>
<tr>
<td>photon lifetime</td>
<td>2 psec</td>
</tr>
<tr>
<td>detuning</td>
<td>5 GHz</td>
</tr>
<tr>
<td>injection</td>
<td>0.0743</td>
</tr>
<tr>
<td>period of limit cycle</td>
<td>0.2 – 0.08 nsec</td>
</tr>
</tbody>
</table>

or the Phase Response Curve (PRC)

\[ PRC(\theta) = \theta_{\text{new}} - \theta. \]  

(6.4)

Depending on the amplitude of the kick, the PTC can be either invertible (Type 1) or non-invertible (Type 0) as shown in Figure 6.3. For the realistic values exhibited in Table 6.1, the laser exhibits a stable intensity oscillation, characterized by the presence of a periodic orbit that exhibits a decreasing period in response to an increase in the injection strength. As an example, we select a moderate injection strength \( \eta = 0.04 \) that corresponds to \( T_{lc} \approx 0.1 \) nsec.

The isochrons of the system are computed as level sets of the time average of an observable \( f \) along the trajectories of the system [151]:

\[ f^*_{\omega} = \lim_{T \to \infty} \frac{1}{T} \int_0^T f \circ \phi^\tau(x, y, Z)e^{-i\omega \tau} d\tau, \]  

(6.5)

where \( \omega \) represents the dimensionless angular frequency of the limit cycle, \( f = x^2 + y^2 \equiv I \), i.e., the dimensionless intensity of system (6.2), and \( \phi : \mathbb{R}^+ \times \mathbb{R}^3 \to \mathbb{R}^3 \), i.e. \( \phi(t, x, y, Z) = \phi^t(x, y, Z) \), is the flow describing the dynamics of (6.2). This integral is evaluated numerically over a finite time horizon, \( T = kT_{lc}, k \in \mathbb{Z} \). Longer time horizons lead to better convergence towards the limit cycle, resulting in a more accurate calculation of the asymptotic phase.
6.4. SYNCHRONIZATION PROTOCOL AND TIME-MEASUREMENT MECHANISM

Figure 6.3: Phase transition curves. Type 1 PTC: invertible; small-amplitude excitation. Type 0 PTC: non-invertible; large-amplitude excitation.

6.4.2 Follower Oscillators Synchronization

IsoPos employs the TDOA technique to address the lack of synchronization that arises between the user and the localization system. As such, it is imperative to establish perfect synchronization among the anchors in the localization system. However, achieving perfect synchronization remains a significant challenge in the current state-of-the-art, as even minute deviations can have a substantial impact on the overall accuracy of the system. To that end, we propose the ILS mechanism that uses the controller-follower phase-lock injection scheme. Based on that, the injected power is modulated with a synchronization signal consisting of a train of periodic pulsatile stimulations of magnitude $A$ delivered with a period $T_s$ (in units of $T_{lc}$),

$$
\eta(t) \rightarrow \eta + A \sum_{n=1}^{m} \delta(t - n T_s). \quad (6.6)
$$

The phase $\theta_{n+1} \in [0, 1)$ at the moment of every stimulus ($n + 1$) is

$$
\theta_{n+1} = [PTC(\theta_n, A) + T_s] \mod 1, \quad (6.7)
$$
where \( \theta_n \) is the phase of the system before the stimulus. This equation defines a Poincaré mapping of the interval \([0, 1)\) to itself, i.e., a circle map that governs the synchronization dynamics of the driven system. The fixed points \( \theta^* \) of this mapping correspond to phase locking and are given by the equation

\[
PRC(\theta^*) = 1 - T_s,
\]

expressing that synchronization is achieved when the stimulated phase kick compensates for the frequency detuning between the periods of the limit cycle and the synchronization signal. The stability condition of the fixed point, and therefore the synchronization process, is determined by the slope of the PRC as follows

\[
-2 < PRC'(\theta^*) < 0.
\]

The stability of the fixed point is related to the robustness of the synchronization process under the presence of noise and/or small parameter deviations. The slope value \( \mu = PRC'(\theta^*) = PTC'(\theta^*) - 1 \) is equivalent to the characteristic multiplier governing the convergence of nearby initial conditions to the fixed point. Synchronization dynamics of the Poincaré mapping towards a fixed point corresponding to a phase-locked state are depicted in Figure 6.4. Given the parameter values specified in Table 6.1, it is possible to achieve synchronization among the four follower oscillators, having different (random) initial phases, through a sequence of 30 pulsatile stimulations of amplitude \( A = 0.35 \) delivered every \( 1.105 \) nsec. It is worth mentioning that the above analysis ensures that the system evolves to the desired phase-synchronized periodic state which is appropriate for its function as a clock, and not to chaotic states existing outside the domain of the above stability conditions [80, 81].
6.4. SYNCHRONIZATION PROTOCOL AND TIME-MEASUREMENT MECHANISM

Figure 6.4: (a) Phase response curve corresponding to a stimulation of amplitude $A = 0.35$ of the limit cycle in Figure 6.2(a). The continuous part of the PRC indicates the region of existence of stable fixed points, i.e., (6.9) is satisfied. (b) Cobweb diagram for the evolution of the four initial conditions to a stable fixed point of the circle map (6.7). (c), (d) Convergence of $\theta_n$—orbits with varying initial conditions to a stable fixed point of Eq. (6.7), meaning that phase locking is achieved.

6.4.3 Fine Time Measurements

Here we outline the IPT mechanism to demonstrate how the TOA measurements work in IsoPos. With each follower oscillator phase-synchronized using ILS protocol under the action of the controller synchronization signal, a pulse is received from the boot block at a different TOA in each oscillator, depending on its distance from the user. This signal pulse arrives when the oscillator’s cycle is at a specific phase $\theta_{in}$ and kicks the oscillator to a new phase $\theta_{out}$, with the two phases related through the PTC. The new phase $\theta_{out}$ can be readily exploited for determining the phase upon pulse arrival $\theta_{in}$ when the PTC is invertible (Type 1), as in Figure 6.3, and in regions where multistability [81] does not take place, namely for relatively small pulse amplitudes. This condition dictates the common amplitude of the pulse that is fed to the follower oscillators from the boot block. The TOA of the user signal in each oscillator is calculated from the $\theta_{in}$ with the PTC given as a lookup table. Based on the application, either this time is in the same cycle that we know or a number of cycles have to be added to it based on the coarse measurement.
6.4.4 Time Scales and Accuracy

The resolution and the accuracy of the phase and time measurements depend on three characteristics: (a) the frequency of the limit cycle $f_{lc}$, (b) the convergence rate of excitations towards the stable limit cycle, and (c) the slope $\mu$ of the PRC at the fixed point.

The frequency $f_{lc}$ determines the coarse time unit whereas the final time and, consequently, the localization resolution is determined by the division of the oscillation cycle into a discrete set of isochrons. Since there are no inherent restrictions on the level of discretization of the continuous isochrons’ structure, an exceptionally high resolution can be achieved without resorting to extremely high frequencies that would raise technological requirements. The convergence rate towards the limit cycle dictates the time required for the system to return to the oscillation with the new asymptotic phase after receiving a user signal. This rate indicates the limit cycle’s robustness and imposes a restriction on the update speed for user positioning. The slope $\mu$ provides a measure of the convergence speed of the synchronization process for the follower oscillators. It is notable that the remarkable tunability of the optically injected system allows for appropriate parameter selections in order to have sufficiently large frequencies $f_{lc}$ ranging from 100 MHz to larger than 100 GHz [74, 75] as well as desired convergence rates for negligibly small waiting times, at the order of nanoseconds (nsec), and sufficiently high update rates for user positioning, even when moving with a high speed.

6.5 Three-dimensional Positioning

First, Sec. 6.5.1 introduces the three-dimensional localization method used by IsoPos. Following that, in Sec. 6.5.2, the PEB of the proposed scheme is derived.
6.5. THREE-DIMENSIONAL POSITIONING

6.5.1 Trilateration Localization

Following the successful measurement of the TOA at each of the follower oscillators, which is based on their phase response to the periodic impulse excitation signal, the user’s final location in three-dimensional space is determined. To accomplish this, the times must be converted to distances and the distances must be trilaterated.

The RF signal transmitted by the target user and the localization system (i.e., the follower oscillators) are not synchronized. However, due to the controller injection-locking procedure, all of the follower oscillators have synchronized phases, implying that all of the measured TOAs in different follower oscillators have a synchronized clock. This means that all of the received times have the same synchronization bias, and the distances between the user and follower oscillators can be written as follows

\[ r_i = c \times (t_i - t_T + \beta) = c \times (\tau_i + \beta), \quad (6.10) \]

where \( r_i \) represents the corresponding distance between the user and the \( i \)-th follower oscillator, \( t_i \) is the received time at the \( i \)-th follower oscillator, \( t_T \) is the transmit time, i.e., the time that the signal left the RF transmitter on the user, \( \beta \) is the synchronization bias between the transmitter clock on the user and any of the follower oscillators, \( \tau_i \) is the propagation time delay between the RF transmitter on the user and the \( i \)-th receiver follower oscillator, and \( c \) is the speed of light; \( i \in \{0, \cdots, N - 1\} \), where \( N \) is the number of follower oscillators which equals four in our design. For three-dimensional localization the minimum number of four positioning nodes is required.

The actual distances are unknown due to the \( \beta \) synchronization bias between the transmitter and each of the receivers, as shown in Eq. \((6.10)\). However, \( \beta \) can be eliminated if we take
one of the follower oscillators as the reference and subtract the rest from it as shown below

\[ r_i - r_0 = c \times (t_i - t_0), \quad (6.11) \]

where \( i \in \{1, \cdots, N - 1\} \), and the precise measurement of \( t_0 \) and all the remaining \( t_i \) values are available. A hyperboloid is the geometrical representation of a set of points in three-dimensional space with a constant distance subtraction to known points (the foci). The user’s three-dimensional location is represented by the intersection of all these hyperboloids. The mathematical model for finding the intersection is as follows

\[
\begin{bmatrix} x \\ y \\ z \end{bmatrix}^T = \min_e (x, y, z), \quad (6.12)
\]

where \( \begin{bmatrix} x \\ y \\ z \end{bmatrix}^T \) is the location of the user in a Cartesian coordinate system and \( e(x, y, z) \) is defined as

\[
e(x, y, z) = \sum_{i=1}^{N-1} \left\{ (r_i - r_0) - \sqrt{(x_i - x_0)^2 + (y_i - y_0)^2 + (z_i - z_0)^2} \right\}, \quad (6.13)
\]

where \( \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}^T \) values are the Cartesian coordinates of the \( i \)-th receiver follower oscillator.

### 6.5.2 Positioning Error Bound

In this part, we calculate the PEB for the proposed localization system. As previously stated, an accurate estimate of \( r_i \) is not available. However, the precise clock synchronization among the positioning system’s follower oscillators, achieved through injection phase-locking, enables the difference \( r_i - r_j \) to be computed with high accuracy. The reference oscillator is
6.5. Three-dimensional Positioning

$r_0$ for the remainder of the process, and the $r_i - r_0$ values are calculated as follows

$$r_i - r_0 = \sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2} - \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2}. \quad (6.14)$$

Because of the ranging measurement errors, the precise $r_i - r_0$ is unknown, resulting in errors when solving for $[x \ y \ z]^T$ in Eq. (6.14). The variance of the three-dimensional location estimator is required to find a correlation between the overall three-dimensional location error, $\sigma_T(x, y, z)$, and the distance estimation ranging errors, $\sigma_{r_i}$, that come from the measurement devices

$$\sigma_T(x, y, z) = \sqrt{\sigma_x^2 + \sigma_y^2 + \sigma_z^2}, \quad (6.15)$$

where $(\sigma_x^2, \sigma_y^2, \sigma_z^2)$ are the variances of the error for $x-$, $y-$, and $z-$axis estimation, respectively. Let $\Delta X = [\Delta x \ \Delta y \ \Delta z]^T$ be the derivative on the overall $[x \ y \ z]^T$ estimations; then we have

$$\text{Cov}(\Delta X) = E(\Delta X \Delta X^T) = \begin{bmatrix} \sigma_x^2 & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_y^2 & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_z^2 \end{bmatrix}. \quad (6.16)$$

As a result, for the variance of the overall location estimation, based on Eq. (6.15), we have

$$\sigma_T^2(x, y, z) = \text{Trace} \left( E(\Delta X \Delta X^T) \right), \quad (6.17)$$
and \( \text{Trace}(. \) denotes the sum of the diagonal elements of the matrix. Then, the correlation between \( \sigma^2_T(x, y, z) \) and the \( \sigma_{r_i}^2 \) is calculated. For this, we differentiate Eq. \((6.14)\), that is

\[
\frac{\Delta x(x - x_i) + \Delta y(y - y_i) + \Delta z(z - z_i)}{\sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2}} - \frac{\Delta x(x - x_0) + \Delta y(y - y_0) + \Delta z(z - z_0)}{\sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2}}, \tag{6.18}
\]

where second- and higher-order terms have been neglected. For the localization system with \( N \) follower oscillators, Eq. \((6.18)\) can be written as

\[
\Delta r_i - \Delta r_0 = \Psi \Delta X,
\]

or equivalently

\[
\Delta X = (\Psi^T \Psi)^{-1} \Psi^T (\Delta R_i - \Delta R_0);
\]

where \( \Delta R_i = [\Delta r_1 \cdots \Delta r_{N-1}]^T \), \( \Delta R_0 = [\Delta r_0 \cdots \Delta r_0]^T \), and

\[
\Psi = \begin{bmatrix}
\frac{x-x_1}{r_1} & \frac{x-x_0}{r_0} & \frac{y-y_1}{r_1} & \frac{y-y_0}{r_0} & \frac{z-z_1}{r_1} & \frac{z-z_0}{r_0} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\frac{x-x_{N-1}}{r_{N-1}} & \frac{x-x_0}{r_0} & \frac{y-y_{N-1}}{r_{N-1}} & \frac{y-y_0}{r_0} & \frac{z-z_{N-1}}{r_{N-1}} & \frac{z-z_0}{r_0}
\end{bmatrix}.
\]

Without loss of generality, we can assume \( \text{Var}(r_i) = \sigma_r^2 \) and the errors \( \Delta r_i \) to be uncorrelated; therefore, we have

\[
\text{Cov}(\Delta X) = ((\Psi^T \Psi)^{-1} \Psi^T) \left( I_{N-1} + J_{N-1} \right) ((\Psi^T \Psi)^{-1} \Psi^T)^T \sigma_r^2,
\]

where \( I_{N-1} \) is the identity matrix of size \( (N-1) \times (N-1) \) and \( J_{N-1} \) is the \( (N-1) \times (N-1) \) matrix with all its entries equal to one. Based on Eq. \((6.17)\) the variance of the three-dimensional location estimator is given by

\[
\sigma^2_T(x, y, z) = G(x, y, z) \cdot \sigma_r^2, \tag{6.19}
\]
where $G(x,y,z)$ is defined as

$$G(x,y,z) = \text{Trace} \left( \Phi \left( I_{N-1} + J_{N-1} \right) \Phi^T \right) ; \quad \Phi = (\Psi^T \Psi)^{-1} \Psi^T.$$

(6.20)

As a result, two factors influence overall location accuracy: (i) the geometry-induced error $G(x,y,z)$, which is primarily tied to the relative geometry between the elements of the localization system and the user; and (ii) the ranging-error $\sigma_r^2$, which is caused by ranging measurements errors.

### 6.6 Optimal Placement of Follower Oscillators

The previous section illustrated that accurate three-dimensional user localization cannot be achieved only with precise time and distance estimation. To attain high-accuracy three-dimensional localization, the configuration of the localization system components is also essential.

The primary goal of current state-of-the-art research is to reduce ranging-induced errors while ignoring the geometry-induced source. Even though advanced commercial geolocation systems take this into account when making calculations, their effects still cannot be completely eliminated. This is because it is inherently difficult to determine the ideal anchor placement in three-dimensional space for a moving user in order to minimize geometric-induced errors at all points. This is an NP-hard open problem.

Additionally, $G(x,y)$ is typically not excessively dependent upon various deployments. The majority of the available literature studies two-dimensional localization. Therefore, since the users do not move along the $Z-$axis, the negative effect of geometry is not obvious in their final solutions. Nevertheless, based on our experiments, $G(z)$ is significantly more sensitive
to the deployment of positioning nodes, and a random placement may result in a very high
$G(z)$, resulting in an unreliable height estimation.

We present an optimization algorithm for determining the optimal deployment of follower
oscillators for any given floor plan in order to alleviate the geometry-induced localization
error. By constraining average $G(x, y)$ and $G(z)$, the proposed algorithm improves estimation
accuracy on the $X – Y$ plane (horizontal) and $Z-$axis (vertical), respectively. We made use
of two constraints on $G(x, y)$ and $G(z)$ in order to reach a solution to the NP-Hard problem
in a timely manner. Once these two constraints are satisfied, the placement configuration
with the smallest $G(x, y, z)$ value represents the ultimate solution. Although there may be
other solutions with similar or lower values of $G(x, y, z)$, our primary objective is to guarantee
that the relative geometry of the anchor placement does not significantly impact the location
estimation for any point in the room. As a result, finding a placement that meets the desired
thresholds for $G(x, y)$ and $G(z)$ satisfies our ultimate objective. This subtle but significant
factor accelerates our optimization algorithm while preserving our objective of minimizing
the effect of geometry on the overall location estimation of all points in the space.

In the remainder of this section, we explain the detail of our proposed optimization frame-
work. In Sec. 6.6.1, we formulate the NP-Hard optimal placement problem followed by the
illustration of the optimization algorithm’s mechanism in Sec. 6.6.2.

### 6.6.1 Problem Formulation

Given the user’s constant mobility, it is insufficient to compute $G(x, y, z)$, $G(x)$, and $G(z)$
for a single position. As a result, we calculated the average of the $G(x, y, z)$, $G(x, y)$, and
$G(z)$ for all possible indoor locations based on the provided floor plan.

Our primary goal is to determine the optimal placement for a set of four follower oscillators
by minimizing the average of \( G(x, y, z) \) while keeping the averages of \( G(x) \) and \( G(z) \) below the required thresholds. The concluding optimization framework can be expressed as follows

\[
\min \sum_{U} \text{Trace} \left( \Phi (\mathbb{I}_{N-1} + \mathbb{J}_{N-1}) \Phi^T \right) \\
\text{s.t. } G(x, y) < h_T \text{; } G(z) < v_T
\]

where \( U \) is the user domain which is a subspace of the indoor environment that includes all the possible user positions, \( \Phi \) comes from Eq. (6.20), \( h_T \), and \( v_T \) are the threshold values for \( \overline{G(x, y)} \) and \( \overline{G(z)} \), respectively; \( \overline{G} \) represents the average of \( G \).

Our primary objective is to minimize \( \overline{G(x, y, z)} \) to determine the optimized anchor placement for mitigating the negative impact of relative geometry and improving localization accuracy. In the meantime, to maintain low horizontal and vertical estimation errors, we enforce the constraints for \( \overline{G(x, y)} \) and \( \overline{G(z)} \) respectively. This ensures that enhancing the overall localization accuracy is done by improving both the \( X-Y \) plane estimation and \( Z \)-axis estimation. Optimization calculations are conducted over all the points in \( U \).

The positioning system nodes (i.e., the follower oscillators) may be located anywhere within the anchor domain, denoted by set \( A \). This includes the entire ceiling as well as the top portion of each side wall.

### 6.6.2 Optimization Framework

We construct an algorithm that is based on the evolutionary algorithms (EAs) class to find a solution with the least amount of computation time.

In our EA setup, an initial set of \( P_T = 10 \) random individuals is first produced. Four follower oscillators, chosen at random from the domain \( A \), make up each individual. We divide the
Algorithm 4 Follower Oscillators Deployment Algorithm

**Input:** User domain (\(U\)), Anchor domain (\(A\)), \(G(x, y)\) threshold (\(h_T\)), \(G(z)\) threshold (\(v_T\))

**Output:** Desirable placement for a set of four follower oscillators

1. while \(G(z) > v_T \) & \(G(x, y) > h_T\) do
2. Generate a set of \(P_T\) random individuals, where each individual is a set of four follower oscillator anchors
3. for \(i = 1\) to \(i = \text{number of iteration}\) do
4. Check the fitness of all available individuals;
5. Kill the worst ones to keep having \(P_T\) individuals;
6. Select the individuals with better fitness as parents;
7. Crossover every two adjacent parents, make an offspring;
8. end for
9. end while

individuals into various groups to prevent getting caught in local minima.

We chose \(P_T = 10\) so that we could include all of the different groups. We would not have been able to be all-inclusive with less than 10, and more than 10 would have added unnecessary computing time to the problem. We avoided including extra individuals to speed up the determination of the final solution, even though it appeared that a slightly larger set size would have been acceptable.

Following generation, the individuals are sorted according to the fitness (cost) function and selected for reproduction based on the outcome. The fitness function is the \(G(x, y, z)\) over the entire set \(U\) obtained by arranging four follower oscillators in a specific manner. The algorithm then selects the first \(P_s = 8\) individuals to serve as a parent group for the reproduction of new individuals. Using a crossover technique, every two adjacent pairs create a new set of four follower oscillators, resulting in a total of 4 offspring. The algorithm then checks the fitness function of each new individual once more. The last \(P_k = 4\) in terms of fitness are eliminated from the total population of \(P_n = 14\), including both parents and offspring, leaving 10 as the number of the final individuals at the end of each stage.

Each set of parents includes eight follower oscillators in total, four per parent, for generating
Table 6.2: Follower oscillator placement for different room sizes with optimal solution

<table>
<thead>
<tr>
<th>Room Dimensions</th>
<th>Osc. # 1</th>
<th>Osc. # 2</th>
<th>Osc. # 3</th>
<th>Osc. # 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Office Room (5m × 5m × 4m)</td>
<td>(0,1,2)</td>
<td>(5,1,2)</td>
<td>(5,5,3)</td>
<td>(2,2,4)</td>
</tr>
<tr>
<td>Conference Room (10m × 10m × 4m)</td>
<td>(7,3,4)</td>
<td>(6,1,4)</td>
<td>(0,9,4)</td>
<td>(8,0,2)</td>
</tr>
<tr>
<td>Game Room (20m × 20m × 4m)</td>
<td>(19,10,4)</td>
<td>(3,8,4)</td>
<td>(8,20,3)</td>
<td>(2,0,2)</td>
</tr>
</tbody>
</table>

new offspring. The crossover technique replaces the first four follower oscillators' coordinate parameters with those from the second set.

We concluded that 100 iterations of the described process were sufficient to reach an optimal solution after testing the algorithm for different iteration numbers. A higher number of iterations (e.g., 1000) simply lengthens the process without improving the final result significantly. Furthermore, a smaller number of iterations (less than 100) were unable to offer a minimal solution.

Following the completion of the iterations, the first individual in the line according to the fitness function is chosen. If it has a $G(x, y)$ and $G(z)$ over the entire set $\mathbb{U}$ that is less than $h_T$ and $v_T$, respectively, it represents the final solution as the four follower oscillator placement configuration. Otherwise, the algorithm restarts the procedure by generating a new set of $P_T = 10$ individuals. When the final result satisfies the constraints, the algorithm terminates.

### 6.7 Performance Evaluation

This section includes an assessment of the optimal anchor placement algorithm in Sec. 6.7.1, followed by a comprehensive evaluation of the entire IsoPos system in Sec. 6.7.2.
Figure 6.5: Illustration of \( G(.) \) values comparison between the random and optimal placement for various indoor space dimensions.

Figure 6.6: CDF plot representation of \( G(.) \) values for various room dimensions with optimal configuration shown in Table 6.2.
Table 6.3: Follower oscillator placement for different room sizes with a random configuration to benchmark the optimal solution

<table>
<thead>
<tr>
<th>Room Dimensions</th>
<th>Osc. # 1</th>
<th>Osc. # 2</th>
<th>Osc. # 3</th>
<th>Osc. # 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Office Room (5m × 5m × 4m)</td>
<td>(3,4,4)</td>
<td>(3,2,4)</td>
<td>(3,1,4)</td>
<td>(4,1,4)</td>
</tr>
<tr>
<td>Conference Room (10m × 10m × 4m)</td>
<td>(6,10,4)</td>
<td>(2,7,4)</td>
<td>(3,7,4)</td>
<td>(7,1,4)</td>
</tr>
<tr>
<td>Game Room (20m × 20m × 4m)</td>
<td>(18,10,4)</td>
<td>(14,3,4)</td>
<td>(19,11,4)</td>
<td>(14,1,4)</td>
</tr>
</tbody>
</table>

Figure 6.7: CDF plot representation of $G(.)$ values for various room dimensions with random configuration represented in Table 6.3.

### 6.7.1 Optimal Placement Algorithm Assessment

By displaying performance results and contrasting them with a benchmark that ignores the optimal placement, the usefulness of our suggested optimization algorithm is evaluated.

Because the primary goal is to identify the optimal placement of follower oscillators, the solution is entirely dependent on the dimensions of the room (i.e., the provided floor plan). As a result, we devised our algorithm to take the floor plan as input and produce the optimal placement of the oscillators as output.

The effectiveness of the algorithm is evaluated on three distinct floor plans to showcase its adaptability to different room dimensions, ranging from modest to extremely large sizes. The first example involves a typical office room measuring $5 \text{ m} \times 5 \text{ m} \times 4 \text{ m}$ acts as an example for playing virtual games on VR devices. The second example is a large conference
room with dimensions of $10\ m \times 10\ m \times 4\ m$, which is suitable for AR applications in larger setups. Lastly, the algorithm is put to the test on an extreme example, a large game room measuring $20\ m \times 20\ m \times 4\ m$ that can accommodate multi-user AR/VR games such as laser tag.

Figure 6.5 compares the $G(.)$ values of optimal and random placements. Non-optimized random configurations and the optimal placements are listed in Table 6.3 and Table 6.2, respectively, where all the numbers are demonstrated in meters.

As shown in the figure, the algorithm consistently produces satisfactory results across all scenarios tested. Furthermore, the provided figure emphasizes the importance of using the proposed optimization algorithm for follower oscillator placement rather than a random approach, due to the significant impact of $G(x, y, z)$ values on overall accuracy. It should be noted that the values of $G(x, y, z)$ differ for each $(x, y, z)$ point in the room. As a result, displaying values for all points in three dimensions necessitates a four-dimensional plot. To avoid this, in Figure 6.5, we take an average of all the $z$ planes and display the final result on the $X-Y$ plane. The proposed algorithm is expected to perform well, particularly for smaller dimensions. However, it demonstrated its ability to generate optimal solutions in a timely manner even for significantly larger dimensions.

Figure 6.6 presents the cumulative distribution function (CDF) of $G(.)$ values for the optimal placement in the mentioned three distinct indoor environments. These optimal placements are enumerated in Table 6.2. The principal aim of the analysis in this figure is to demonstrate the number of points in set $\mathbb{U}$ that have $G(x, y, z)$ values lower than a specific threshold. The optimization problem’s objective was to guarantee that the majority of the points had their $G(x, y, z)$ lower than 20. We set a threshold of 20 given the system’s accuracy in providing distance information and the requirement to keep overall three-dimensional accuracy below 1 mm. As depicted in the figure, this objective has been attained for all the scenarios.
For instance, in a typical office room, more than 95% of the points in $\mathbb{U}$ have $G(x, y, z)$ values less than 8. In the case of a large conference room, this value is 14, and for a large game room, it is 12. These values are all below 20, confirming that the proposed optimization framework performed as anticipated.

In order to establish a benchmark and underscore the crucial nature of optimal placement requirements, we present in Figure 6.7 the identical plots as depicted in Figure 6.6, except with a random placement configuration, as listed in Table 6.3. As can be observed in this figure, the $G(x, y, z)$ values surge to several hundred, rather than being predominantly below 20 for the majority of the points in set $\mathbb{U}$. This implies that even if a highly precise system is employed to measure time (and, by extension, distance), the final three-dimensional accuracy may be unreliable owing to the large $G(x, y, z)$ values.

### 6.7.2 Final Assessment

In summary, our study has three main goals. To begin, we propose a novel method for measuring time with high precision using isochrons in photonic oscillators. The accuracy is determined by the oscillation frequency as well as the resolution of the oscillation cycle in terms of isochrons. As shown in Figure 6.8, the accuracy of the phase shift measurement induced by an incoming pulse is proportional to the number of cycles $k$ required for the excitation to relax on the system’s stable limit cycle. The spatial resolution $\Delta x_{rms}$ is proportional to the phase measurement error $\Delta \theta_{rms}$ as

$$
\Delta x_{rms} = \Delta \theta_{rms} \frac{c}{f_{lc}},
$$

where the first term clearly demonstrates the benefit of measuring the phase within a cycle versus measuring whole cycles. As an example, for $f_{lc} = 3$ GHz or $f_{lc} = 30$ GHz the
spatial resolution due to time measurement is $\Delta x_{\text{rms}} = 10^{-4} \, m$ or $\Delta x_{\text{rms}} = 10^{-5} \, m$ for a $\Delta \theta_{\text{rms}} = 10^{-3}$ achieved after $k \simeq 50$ cycles corresponding to a negligible time span, smaller than $10^{-8} \, \text{sec}$.

Second, we look into the viability of using injection-locking technology to achieve precise clock synchronization, which is critical for TDOA systems. For the clock synchronization, the nonlinear characteristic of the synchronization mechanism ensures an exponential convergence rate and a precise phase locking of the receivers’ clocks as presented in Figure 6.4(c)-6.4(d). Finally, we improve three-dimensional localization accuracy by introducing $G(x, y, z)$ and creating an optimization framework to reduce geometry-induced error in overall location estimation.

The CDF of localization accuracy with the optimal placement is shown in Figure 6.9, which is based on Table 6.2. As shown in the figure, regardless of the environment’s dimensions, an overall three-dimensional accuracy of less than 1 mm is achieved. These plots are created using the parameter values discussed in Sec. 6.4s core, which were used to develop our system.

To establish a baseline, we show non-optimized placement configurations in Figure 6.10, as listed in Table 6.3. These plots can be compared to the optimal plots shown in Figure 6.9. The three-dimensional error increases to 1 cm in the non-optimized case, but remains primarily below 1 mm in the optimized case. This emphasizes the importance of proper anchor placements.

We plot the localization accuracy for different lasers’ realistic parameter values using our optimal placement approach, in Figure 6.11. The frequency of the limit cycle $f_{\text{lc}}$ and the phase measurement error $\Delta \theta_{\text{rms}}$ are of particular interest. Smaller values for $f_{\text{lc}}$ result in performance degradation, as shown in Figure 6.11(a). When we change the $f_{\text{lc}} = 30 \, \text{GHz}$ to $f_{\text{lc}} = 3 \, \text{GHz}$ for a fixed value of $\Delta \theta_{\text{rms}} = 10^{-3}$, the localization error increases from 1 mm
to 1 cm. This gives us design flexibility in determining the frequency of the limit cycle based on the use case. Higher frequencies are better suited for applications requiring greater localization accuracy (sub-1 mm), such as finger tracking. Lower frequencies, on the other hand, can be used for applications requiring sub-cm accuracy, such as autonomous indoor navigation of drones in large warehouses. Furthermore, in Figure 6.11(b), we investigate the trade-off between the system’s update rate and the overall accuracy: waiting for longer cycles enhances the overall accuracy; for instance, \( k > 50 \) which is the plot with \( \Delta \theta_{rms} = \frac{1}{2} \times 10^{-3} \). This means that the update rate will be slowed by a few nanoseconds. Longer relaxation times, in general, result in better resolution for phase measurements, which leads to better overall three-dimensional accuracy. As shown in Figure 6.8(b), increasing the relaxation time to a hundred cycles (\( k = 100 \)) improves phase shift resolution significantly (\( \Delta \theta_{rms} = 10^{-5} \)), which is hundred times better than \( k = 50 \). This means that for lower frequencies, we can achieve high-accuracy localization by using a longer relaxation time (slightly slower update rate).

### 6.8 Summary

**Conclusion:** In this article, we propose a novel positioning system, named IsoPos, that utilizes ideas from tunable photonic oscillators and their isochrons’ structure to enhance localization accuracy. To that end, we introduce IPT mechanism, a novel approach for measuring the TOA with extremely high resolution. IPT exploits the phase shift in the receiver’s oscillation—determined by its isochrons’ structure—induced by excitation pulses. To attain precise synchronization among different positioning nodes, which is a challenge in current localization architectures, we propose ILS protocol, which employs the injection locking of tunable photonic oscillators. In addition, we present an optimization framework to
mitigate the adverse impact of geometry on location accuracy. Our comprehensive numerical results show that the proposed system achieves sub-1 mm three-dimensional localization accuracy across different scenarios.

**Future Work:** The chief goal of this work was to introduce a novel high-resolution TOA measurement and precise time synchronization approach. These were the foundational elements of our proposed positioning system. We intend to focus our future research primarily on the RF transmission aspect and investigate the possibility of mitigating potential channel-induced negative impacts, such as multipath, noise, and interference, through the robust selection of $s(t)$. With that objective in mind, we plan to examine the most effective design of $s(t)$ as well as the feasibility of using non-line-of-sight (NLoS) information in large-scale scenarios. This is significant when the existence of LoS is uncertain. Furthermore, we are developing a testbed that will include quantum well and external cavity lasers. The end goal we aim to achieve is the development of a chip-level model that can confirm the effectiveness of the suggested system in practical situations, such as delivering accurate hand-tracking for AR glasses.
6.8. Summary

(a) Each PRC\(_k(\theta)\) indicates the phase response of the system to an external excitation of fixed amplitude for varying relaxation intervals corresponding to integer multiples of \(T_{lc}\), i.e. \(kT_{lc}\). (b) The root-mean-square error of the phase response for varying relaxation intervals.

Figure 6.9: CDF plots representation of three- and two- dimensional localization accuracy as well as the \(Z\)–axis estimation for various room dimensions with the optimal configuration shown in Table 6.2.
Figure 6.10: CDF plots representation of three- and two-dimensional localization accuracy as well as the Z-axis estimation for various room dimensions with a random non-optimized configuration shown in Table 6.3.

Figure 6.11: Three-dimensional localization accuracy evaluations for different laser’s parameter values: (a) Effect of $f_{lc}$ for the fixed value of $\Delta \theta_{rms} = 10^{-3}$ ($k \simeq 50$ cycles); (b) Effect of $\Delta \theta_{rms}$ for the fixed value of $f_{lc} = 30$ GHz.
Chapter 7

Conclusion

Main Focus: The central theme of this dissertation revolves around the design of high-precision localization systems operating without GPS signals. The primary focus lies on indoor localization, a challenging scenario often devoid of reliable GPS coverage. Our systems are tailored to cater to diverse indoor environments, ranging from small office spaces or compact setups to expansive gaming arenas or the intricate logistics of sprawling warehouses. While adaptable to various indoor contexts, our systems are particularly refined for drone operations and metaverse applications.

It is pivotal to recognize that this specialization does not restrict the scope of our systems. Although they adeptly address general use cases that necessitate moderate resolution or two-dimensional positioning, their prowess is further highlighted in the rigorous demands of drone navigation and metaverse scenarios. These domains demand heightened accuracy and three-dimensional positioning due to their incessant movement across all dimensions. As drones initially captured attention and the metaverse subsequently exploded in popularity, the significance of precise localization for these applications is underscored. Autonomous drone navigation, for instance, hinges on accurate positioning, with prominent companies spearheading drone-based autonomous deliveries and warehouse management. Similarly, the immersive essence of Augmented/Virtual Reality (AR/VR) games within the metaverse critically hinges on precise localization. This contextual backdrop intensifies the relevance of our research and accentuates the role of high-accuracy positioning in shaping these trans-
formative technological frontiers.

**Choice of Ranging-Based Techniques:** Amid various localization approaches in GPS-absent environments, including vision-based, fingerprinting, and ranging-based techniques, we gravitate toward the latter. This preference is rooted in their resilience to real-time environmental changes, setting them apart from fingerprinting alternatives susceptible to fluctuations. Moreover, ranging-based methods showcase robust performance even in dimly lit conditions, an advantage that positions them favorably compared to vision-based methodologies.

Although vision-based methods exhibit exceptional precision, they are not devoid of limitations. In the context of drone navigation, the inherent instability and motion-induced wobbling of drones can lead to image blurring, rendering vision-based methods ill-suited for precision tasks. Furthermore, these methods often incur significant costs, necessitating elaborate calculations and expensive equipment. While vision-based schemes may excel in applications like AR/VR hand tracking, they flounder in scenarios with limited lighting conditions. Ranging-based techniques, in contrast, manifest as sturdy and dependable solutions, showcasing the capacity to provide accurate measurements in challenging scenarios.

**Two Sources of Positioning Errors:** At the heart of our methodology lies the interaction between two sets of sensors: those that are positioning nodes (beacons/anchors) and those integrated into user devices. This synergy facilitates time of flight (TOF) determination, a fundamental parameter for precise distance estimation—fundamental for trilateration-based three-dimensional positioning. Our work is fundamentally grounded in the comprehensive analysis of positioning errors, primarily stemming from two sources: ranging errors and geometry-induced errors. Ranging errors encapsulate a spectrum of factors, including imprecise measurements, channel quality, noise, the influence of multipath signals, and others.
Deciding between acoustic and RF signals assumes pivotal importance, as it can directly influence the calibration of ranging errors. Variables such as signal type, bandwidth, and antenna configuration collectively impact distance estimation accuracy. Geometry-induced errors, in contrast, remain independent of signal quality, instead hinging on the spatial configuration between sensors and users. A pivotal aim of this dissertation is to underscore the importance of addressing geometry-induced errors, often overshadowed by the prevailing focus on mitigating ranging errors. Amidst the extensive efforts to enhance ranging-based accuracy through signal modulation techniques, we emphasize the critical nature of rectifying geometry-induced errors.

**Holistic Approach to Error Mitigation:** At the core of our research lies a dual-pronged strategy that addresses both ranging errors and geometry-induced errors comprehensively. Our five proposed systems—PILOT, iDROP, OFDRA, Wi-Six, and IsoPos—exemplify this holistic approach. Each system embodies a two-phase structure: the initial phase is dedicated to countering ranging errors, thereby elevating distance estimation accuracy, while the subsequent phase is focused on strategies to minimize geometry-induced errors. This intricate interplay between sources of error ensures the robustness and high precision of our systems.

**Significance of Geometry-Induced Errors:** This dissertation underscores the paramount importance of addressing geometry-induced errors, a facet often overshadowed by the prevailing focus on enhancing ranging-based precision. Our commitment to rectify this imbalance is evident in the comprehensive nature of our systems. These systems, meticulously tailored to address both ranging errors and geometry-induced errors, emerge as pioneers poised to redefine the landscape of drone navigation precision and metaverse experiences. The dual-phase approach implemented by each system—skillfully contending with both sources of error—ensures their adaptability in navigating intricate error scenarios, thereby furnishing
unparalleled accuracy in localization solutions.

**Overview of the Five Systems:** The preliminary focus on acoustic ultrasound signals in the first two chapters emerges as a strategic choice. These chapters are positioned to address multipath challenges, a prevailing concern in drone localization within indoor environments. Here, PILOT employs the Frequency Hopping Spread Spectrum (FHSS) technique for signal communication, while iDROP capitalizes on the Frequency-Hopping Code Division Multiple Access (FH-CDMA) technique. This serves as a stark contrast to RF signals, demonstrating the pros and cons intrinsic to both domains.

Acoustic signals, characterized by a more deliberate propagation speed compared to the rapid pace of RF signals, offer a compelling advantage. This deliberate pace ensures superior accuracy despite potentially slower clocks and more budget-friendly equipment. Moreover, acoustic signals exhibit a marked reduction in interference from external sources, rendering them suitable even within secure environments. However, their limited range and narrow beam patterns present challenges—necessitating a dense deployment of positioning nodes, a constraint that impedes practicality. In contrast, RF signals encompass the advantages of extended range and omnidirectional propagation, enriched by the existing infrastructure provided by cellular stations and wireless fidelity (Wi-Fi) routers. This renders RF signals a pragmatic choice, especially in light of advancements within 5G, 6G, and IEEE 802.11az standards.

The subsequent chapters—dedicated to RF-based ranging—bear testimony to this evolution. OFDRA introduces an innovative indoor localization system that harnesses 5G femtocells and deploys Reconfigurable Intelligent Surfaces (RIS) atop drones. This dynamic pairing counters multipath effects and synchronization challenges, setting new benchmarks for warehouse localization. Transitioning outdoors, Wi-Six introduces an ingenious architecture integrating potent 6G positioning signals in open environments. Indoors, the system cleverly
incorporates Wi-Fi routers atop the 6G network, attenuating errors due to weaker 6G signals. By adeptly integrating cellular and Wi-Fi protocols, Wi-Six exemplifies the harmonious coexistence of technologies.

Intriguingly, the final chapter–IsoPos–ushers in an era of self-sufficiency. By deviating from conventional infrastructural dependencies, IsoPos introduces a proprietary localization system centered around tunable quantum photonic oscillators. These oscillators, supported by injection-locking techniques, revolutionize timing mechanisms and offer unprecedented synchronization precision. This audacious step attests to our commitment to pushing boundaries and reshaping localization paradigms.

In all five systems, meticulous communication models establish a foundation for error reduction, meticulously addressing challenges like noise, multipath, and synchronization issues. However, our innovation transcends the reduction of ranging-based errors. The latter portions of each system are dedicated to addressing geometry-induced errors, through algorithmic innovation informed by specific use cases, trilateration techniques, and deployment environments.

**A Holistic Vision:** In summation, this dissertation embarks on a comprehensive journey, highlighting the pivotal role of accurate localization within drone navigation and metaverse experiences. Our five systems—the result of meticulous design and strategic implementation—encompass acoustic signals, 5G/6G, Wi-Fi, and injection-locked quantum photonic oscillators. Each solution undergoes in-depth exploration, a testament to our commitment to unraveling the complexities of high-accuracy positioning. This thesis bridges theoretical insights and pragmatic solutions, offering a nuanced perspective on the landscape of precision localization.

Throughout this journey, my personal insights illuminate the following key revelations.
Among the options for accurate positioning in GPS-deficient environments, ranging-based solutions emerged as a beacon of promise, outshining alternatives such as vision-based or fingerprinting methods. Within the realm of ranging-based techniques, the intricacies of error sources assumed paramount significance. Current state-of-the-art innovations, though laudable in their approach to mitigating ranging-based errors, often overlook the equally critical geometry-induced errors. This oversight can be attributed to their emphasis on two-dimensional localization, where geometry-induced errors remain less conspicuous. However, a profound shift occurs in three-dimensional scenarios. The $Z$-axis estimation is significantly marred by geometric distortions, underscoring the pivotal relevance of addressing geometry-induced errors alongside their ranging-based counterparts.

Acoustic signals, for all their merits, find their niche in short-range applications, such as high-precision motion tracking within close proximity to positioning nodes. In contrast, the realm of RF signals presents a broad canvas of opportunities, demonstrating immense potential across various contexts. Among the gamut of RF-based techniques, the ascendancy of Wi-Fi-based and cellular-based positioning holds sway. Particularly noteworthy, the IEEE 802.11az standard is tailored to precision positioning, while 6G’s prominence in cellular networks augurs well for future positioning solutions. The outdoor landscape echoes the potential ascendancy of 6G, while indoors, the ubiquity of Wi-Fi infrastructure positions it as a strong contender. The strategic amalgamation of Wi-Fi indoors and cellular outdoors is poised to usher in an era of universal positioning systems—an innovation poised to shape future horizons.

These insights coalesce into my conclusive reflections and a gaze into the future’s panorama. Irrespective of the unfolding events, the surging importance of accurate positioning becomes evident as the metaverse and virtual reality attain newfound tangibility. The verisimilitude of these immersive realms hinges upon accurate positioning, solidifying its pivotal role in the
yet-unwritten chapters of our technological odyssey.
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