

Chapter 4

Solution Procedure

4.1 Coordinate Transformation

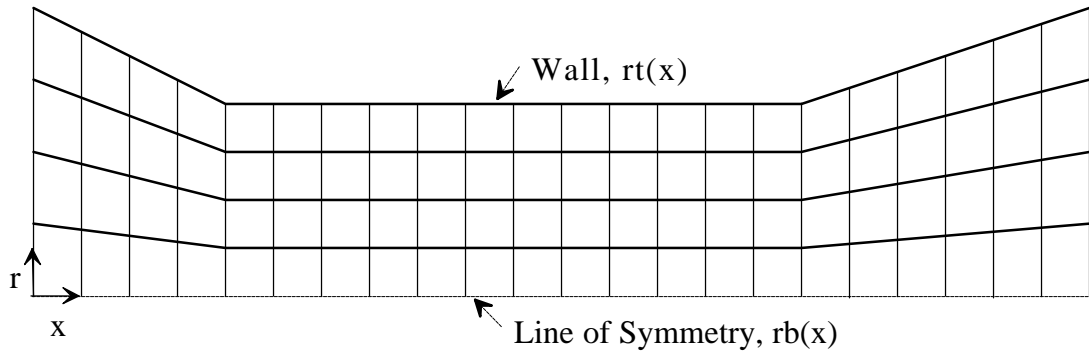
The final transformation takes the non-dimensional forms of the equations and converts them to a body fitted coordinate system appropriate to the ejector geometry. The following transformation for x and r is used and, from the chain rule of calculus, the required derivatives can easily be worked out:

$$\bar{x} = x \quad \bar{r} = \frac{r - r_b}{r_t - r_b} \quad \frac{\partial}{\partial x} = \frac{\partial}{\partial \bar{x}} - \frac{\bar{r}(r'_t - r'_b) + r'_b}{r_t - r_b} \frac{\partial}{\partial \bar{r}} \quad \frac{\partial}{\partial r} = \frac{1}{r_t - r_b} \frac{\partial}{\partial \bar{r}} \quad (4.1)$$

where r_b represents the bottom boundary of the domain and r_t the top boundary. Thus the radial domain is always between 0-1. These boundaries are functions of the axial distance, x , and thus their derivatives r'_t and r'_b are simply $d(r_t)/dx$ and $d(r_b)/dx$ which represent the slope of the top and bottom boundary. The transformation thus converts a nonrectangular grid in the physical domain to a rectangular one in the computational domain (see Fig. 4.1) and greatly facilitates the process of discretization and computer solution.

Upon performing the substitutions shown in Eqn. 4.1, the transformed conservation equations follow directly. Note that the * notation used in Sec. 2.4 has been dropped for clarity with the assumption that all variables are already dimensionless. Also note that $r'_d = r'_t - r'_b$ and $r'_d = r'_t - r'_b$.

Grid in Physical Coordinates:



Grid in Transformed Coordinates:

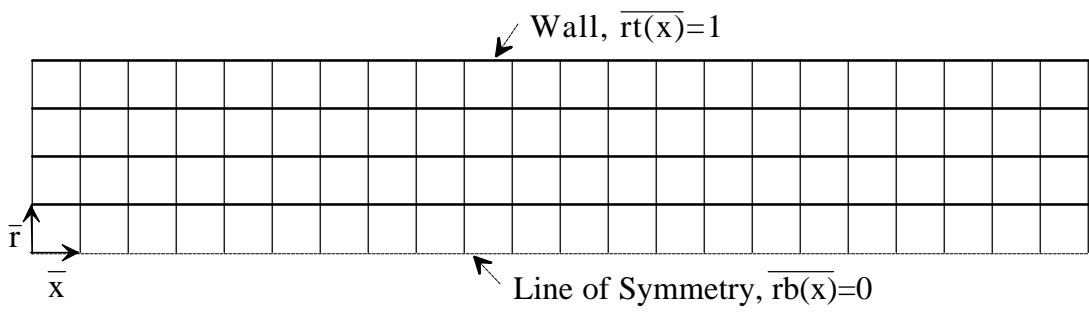


Figure 4.1 - Grid in Physical and Transformed Coordinates

Continuity:

$$\begin{aligned} \frac{\partial}{\partial \bar{x}} [r_d(\bar{r}r_d + r_b)\alpha\rho u] - \frac{\partial}{\partial \bar{r}} \left[(\bar{r}r'_d + r'_b)(\bar{r}r_d + r_b)\alpha\rho u \right] + \frac{\partial}{\partial \bar{r}} [(\bar{r}r_d + r_b)\alpha\rho v] \\ = (\bar{r}r_d^2 + r_b r_d)\Gamma \end{aligned} \quad (4.2)$$

Axial Momentum:

$$\begin{aligned} \frac{\partial}{\partial \bar{x}} \left[(\bar{r}r_d^2 + r_b r_d)\alpha\rho u^2 \right] - \frac{\partial}{\partial \bar{r}} \left[(\bar{r}r'_d + r'_b)(\bar{r}r_d + r_b)\alpha\rho u^2 \right] + \frac{\partial}{\partial \bar{r}} [(\bar{r}r_d + r_b)\alpha\rho uv] \\ = -\alpha \frac{\partial}{\partial \bar{x}} \left[(\bar{r}r_d^2 + r_b r_d)P \right] + \alpha \frac{\partial}{\partial \bar{r}} \left[(\bar{r}r'_d + r'_b)(\bar{r}r_d + r_b)P \right] + \frac{\partial}{\partial \bar{r}} \left[\frac{\alpha\mu(\bar{r}r_d + r_b)}{Re r_d} \frac{\partial u}{\partial r} \right] \\ + (\bar{r}r_d^2 + r_b r_d)(F_x + M_x) \end{aligned} \quad (4.3)$$

Radial Momentum:

$$\begin{aligned} \frac{\partial}{\partial \bar{x}} \left[(\bar{r}r_d^2 + r_b r_d)\alpha\rho uv \right] - \frac{\partial}{\partial \bar{r}} \left[(\bar{r}r'_d + r'_b)(\bar{r}r_d + r_b)\alpha\rho uv \right] + \frac{\partial}{\partial \bar{r}} [(\bar{r}r_d + r_b)\alpha\rho v^2] \\ = -(\bar{r}r_d + r_b)\alpha \frac{\partial P}{\partial \bar{r}} + \frac{\partial}{\partial \bar{r}} \left[\frac{2\alpha\mu(\bar{r}r_d + r_b)}{Re r_d} \frac{\partial v}{\partial \bar{r}} \right] + \frac{\partial}{\partial \bar{x}} \left[\frac{\alpha\mu(\bar{r}r_d + r_b)}{Re} \frac{\partial u}{\partial \bar{r}} \right] \\ - \frac{\partial}{\partial \bar{r}} \left[\frac{\alpha\mu(\bar{r}r_d + r_b)(\bar{r}r'_d + r'_b)}{Re r_d} \frac{\partial u}{\partial \bar{r}} \right] - \frac{2\mu\alpha r_d}{Re(\bar{r}r_d + r_b)^2} (\bar{r}r_d + r_b)v + (\bar{r}r_d^2 + r_b r_d)[F_r + M_r] \end{aligned} \quad (4.4)$$

Energy:

$$\begin{aligned} \frac{\partial}{\partial \bar{r}} [(\bar{r}r_d + r_b)\alpha\rho vT] + \frac{\partial}{\partial \bar{x}} \left[(\bar{r}r_d^2 + r_b r_d)\alpha\rho uT \right] - \frac{\partial}{\partial \bar{r}} \left[(\bar{r}r'_d + r'_b)(\bar{r}r_d + r_b)\alpha\rho u \right] \\ = \frac{1}{Re Pr} \frac{1}{c_p} \frac{\partial}{\partial \bar{r}} \left[\frac{(\bar{r}r_d + r_b)\alpha k}{r_d} \frac{\partial T}{\partial \bar{r}} \right] + Ec \frac{\beta u \alpha T}{c_p} \frac{\partial}{\partial \bar{x}} \left[(\bar{r}r_d^2 + r_b r_d)P \right] \\ - Ec \frac{\beta u \alpha T}{c_p} \frac{\partial}{\partial \bar{r}} \left[(\bar{r}r'_d + r'_b)(\bar{r}r_d + r_b)P \right] + \frac{(\bar{r}r_d^2 + r_b r_d)}{c_p} (E_{mt} + E_{ht} + E_{ke} + E_{wt}) \end{aligned} \quad (4.5)$$

It is these equations which are suitable for discretization.

Unlike nondimensionalization, the body fitted coordinate transformation is a required step. A simpler method which was tried for this project consisted of using a regular untransformed grid and using high viscosity cells to simulate the wall boundaries. This is a well known method and is documented by Patankar [48]. The problem with it arises for flow in diverging sections, in which the wall is thus approximated by a series of backward facing steps. It is well established that flow over a backward facing step separates, and since this problem is parabolic, it cannot handle separation. Therefore, the body fitted grid was a necessity.

4.2 Control-Volume Discretization Method

Discretization of the conservation equations is done by integrating each transformed PDE over a typical control volume, in this case a cell of rectangular shape. This well known "control volume" method produces a solution which automatically satisfies global conservation requirements over the entire duct. Since even the coarsest of grids exhibits this property, the method is not only appealing for engineering problems but allows for quicker testing of newly developed code.

Inherent in the discretization method are assumptions about the profiles of properties over the control volume. Also important is the upwind approximation in which it is assumed that the flow projects a value of an unknown downstream to the next point.

To illustrate the discretization process, the x-momentum equation will be integrated in a step-by-step fashion to show the method. The equation is first double integrated over a typical rectangular cell in the transformed coordinate domain (Figure 4.2). For the first term in Eq. 4.3 this operation is summarized as follows:

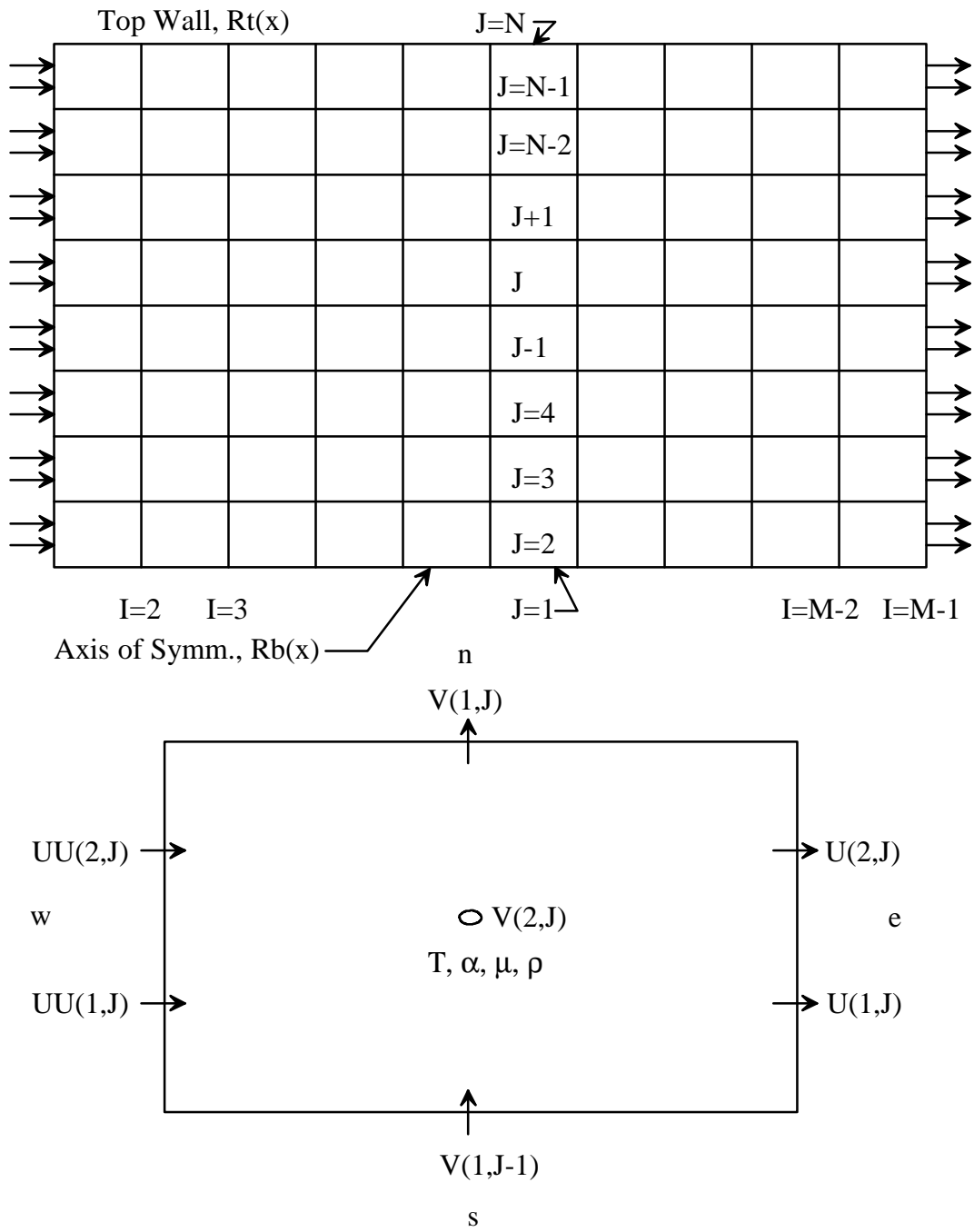


Figure 4.2 - Organization of Grid and Typical Discretization Control Volume

$$\begin{aligned}
& \int_{\bar{r}_s}^{\bar{r}_n} \int_w^e \frac{\partial}{\partial \bar{x}} \left[\left(\bar{r}r'_d + r_b r_d \right) \alpha \rho u^2 \right] d\bar{x} d\bar{r} \\
&= \int_{\bar{r}_s}^{\bar{r}_n} \left\{ \left[\left(\bar{r}r'_d + r_b r_d \right) \alpha \rho u^2 \right]_e - \left[\left(\bar{r}r'_d + r_b r_d \right) \alpha \rho u^2 \right]_w \right\} d\bar{r} \\
&= \left[\frac{1}{2} (\bar{r}_n^2 - \bar{r}_s^2) r_d (\bar{x}_e)^2 + r_b (\bar{x}_e) r_d (\bar{x}_e) (\bar{r}_n - \bar{r}_s) \right] \alpha_j \rho_j u_j^2 \\
&+ \left[\frac{1}{2} (\bar{r}_n^2 - \bar{r}_s^2) r_d (\bar{x}_w)^2 + r_b (\bar{x}_w) r_d (\bar{x}_w) (\bar{r}_n - \bar{r}_s) \right] \alpha_w \rho_w u_w^2 \\
&= F_e u_j - F_w u_w
\end{aligned} \tag{4.6}$$

Note that \bar{r}_n and \bar{r}_s simply mean the radial position of the north and south boundaries of the computational cell, in body fitted coordinates. The scripts e and w refer to the east and west boundary of the cell. Also, F_e and F_w represent the flow through the east and west faces of the boundary respectively. This later becomes a useful way to represent the convective momentum flux. From the above derivation it is clear that the physical meaning of the term is the net convective momentum flux leaving the control volume through the east and west faces.

The second term in the x-momentum equation is handled similarly:

$$\begin{aligned}
& - \int_w^e \int_{\bar{r}_s}^{\bar{r}_n} \left\{ - \frac{\partial}{\partial \bar{r}} \left[\left(\bar{r}r'_d + r'_b \right) (\bar{r}r_d + r_b) \alpha \rho u^2 \right] \right\} d\bar{r} d\bar{x} \\
&= - \int_w^e \left\{ \left[\left(\bar{r}r'_d + r'_b \right) (\bar{r}r_d + r_b) \alpha \rho u^2 \right]_n - \left[\left(\bar{r}r'_d + r'_b \right) (\bar{r}r_d + r_b) \alpha \rho u^2 \right]_s \right\} d\bar{x} \\
&= - \alpha_n \rho_n u_n^2 \int_w^e \left(\bar{r}_n r'_d + r'_b \right) (\bar{r}_n r_d + r_b) d\bar{x} + \alpha_s \rho_s u_s^2 \int_w^e \left(\bar{r}_s r'_d + r'_b \right) (\bar{r}_s r_d + r_b) d\bar{x} \\
&= F_{nu} u_n + F_{su} u_s
\end{aligned} \tag{4.7}$$

This term obviously represents convective fluxes carried by the axial component of the velocity through the north and south boundary of the control volume. The term arises because in the physical plane the control volume can be slanted upward or downward. The method for calculating u_n and u_s by the upwind method will be addressed later. Average values of α and ρ at the north and south boundary are taken as follows:

$$\alpha_n = \frac{1}{2}(\alpha_j + \alpha_{j+1}) \quad \alpha_s = \frac{1}{2}(\alpha_j + \alpha_{j-1}) \quad \rho_n = \frac{1}{2}(\rho_j + \rho_{j+1}) \quad \rho_s = \frac{1}{2}(\rho_j + \rho_{j-1}) \quad (4.8)$$

The third term is integrated as follows:

$$\begin{aligned} \int_w^e \int_s^n \frac{\partial}{\partial \bar{r}} [(\bar{r}r_d + r_b)\alpha\rho uv] d\bar{r} d\bar{x} &= \int_w^e \{ [(\bar{r}r_d + r_b)\alpha\rho uv]_n - [(\bar{r}r_d + r_b)\alpha\rho uv]_s \} d\bar{x} \\ &= \alpha_n \rho_n u_n v_n \int_w^e (\bar{r}r_d + r_b) d\bar{x} - \alpha_s \rho_s u_s v_s \int_w^e (\bar{r}r_d + r_b) d\bar{x} = F_{nv} u_n + F_{sv} u_s \end{aligned} \quad (4.9)$$

This term represents the convective flux carried by the radial velocity component. Note that the flows F_{nu} , F_{nv} , F_{su} , and F_{sv} can be combined to give the total flowrate across the control volume face. Terms can be added as follows:

$$F_n = F_{nu} + F_{nv} \quad F_s = F_{su} + F_{sv} \quad (4.10)$$

Now the convective terms across each face can be expressed in terms of the total flowrate. This simplification is useful because it allows us to develop a criterion for upwinding, a technique which enhances the solution procedure and prevents negative coefficients (to be discussed later) from arising.

Upwinding is achieved by assuming that the value of the unknown variable at a control volume face is equal to the value at the nearest upwind point. The flow thus projects the value downstream. This concept is physically realistic and, as expected, works better for strongly convective flows.

Mathematically, the upwind method states that if the flow is positive, the velocity value to be used should be the nearest point below the c.v. boundary, and if the flow is negative, the value should be the nearest point above the c.v. boundary. This amounts to:

$$\begin{aligned} &\text{if } F_n > 0 \text{ then } u_n = u_j \\ &\text{if } F_n < 0 \text{ then } u_n = u_{j+1} \end{aligned} \quad (4.11)$$

thus,

$$F_n u_n = u_j \text{MAX}[F_n, 0] - u_{j+1} \text{MAX}[-F_n, 0] \quad (4.12)$$

Similarly for the south boundary of the c.v.:

$$\begin{aligned} &\text{if } F_s > 0 \text{ then } u_s = u_j \\ &\text{if } F_s < 0 \text{ then } u_s = u_{j-1} \\ &F_s u_s = u_j \text{MAX}[F_s, 0] - u_{j-1} \text{MAX}[-F_s, 0] \end{aligned} \quad (4.13)$$

The terms in Eqn's 4.12 and 4.13 will be important in the final discretized equation.

The first pressure force term in Eqn. 4.3 is integrated as follows:

$$\begin{aligned} -\int_s^n \int_w^e \alpha \frac{\partial}{\partial \bar{x}} \left[(\bar{r}r_d^2 + r_b r_d) P \right] d\bar{x} d\bar{r} &= \int_s^n -\alpha_p \left\{ \left[(\bar{r}r_d^2 + r_b r_d) P \right]_e - \left[(\bar{r}r_d^2 + r_b r_d) P \right]_w \right\} d\bar{r} \\ &= -\alpha_p P_e RINTE + \alpha_p P_w RINTW \end{aligned} \quad (4.14)$$

This expression represents the pressure forces acting on the east and west faces of the control volume. Here it is assumed that the void fraction takes on some average value over the entire control volume (α_p). The pressure is, of course, uniform over the control volume faces because of the parabolic assumption. The terms *RINTE* and *RINTW* represent the areas of the faces in the transformed coordinate system. Several such terms are required.

The second pressure term in Eqn. 4.3 is integrated similarly:

$$\int_w^e \int_s^n \alpha \frac{\partial}{\partial \bar{r}} \left[(\bar{r}r'_d + r'_b) (\bar{r}r_d + r_b) P \right] d\bar{r} d\bar{x} \quad (4.15)$$

$$\begin{aligned}
&= \int_w^e \left\{ \alpha_p \left[\left(\bar{r}r'_d + r'_b \right) (\bar{r}r_d + r_b) P \right]_n - \alpha_p \left[\left(\bar{r}r'_d + r'_b \right) (\bar{r}r_d + r_b) P \right]_s \right\} d\bar{x} \\
&= \frac{1}{2} \alpha_p (P_e + P_w) \left\{ \int_w^e \left[\left(\bar{r}_n r'_d + r'_b \right) (\bar{r}_n r_d + r_b) \right] d\bar{x} - \int_w^e \left[\left(\bar{r}_s r'_d + r'_b \right) (\bar{r}_s r_d + r_b) \right] d\bar{x} \right\} \\
&= \frac{1}{2} \alpha_p (P_e + P_w) (RINT2N - RINT2S) \tag{4.16}
\end{aligned}$$

This term represents the axial pressure force exerted on the north and south boundary of the control volume. This term exists only because, in the physical plane, the north and south boundary is slanted to fit the ejector geometry. Note that an average pressure between the upstream and downstream values is used.

The viscous term is discretized as follows:

$$\begin{aligned}
\int_w^e \int_s^n \frac{\partial}{\partial \bar{r}} \left[\frac{\alpha \mu (\bar{r}r_d + r_b)}{Re r_d} \frac{\partial u}{\partial r} \right] d\bar{r} d\bar{x} &= \int_w^e \left\{ \left[\frac{\alpha \mu (\bar{r}r_d + r_b)}{Re r_d} \frac{\partial u}{\partial r} \right]_n - \left[\frac{\alpha \mu (\bar{r}r_d + r_b)}{Re r_d} \frac{\partial u}{\partial r} \right]_s \right\} d\bar{x} \\
&= \frac{\alpha_n \mu_n (u_{j+1} - u_j)}{Re(\bar{r}_{j+1} - \bar{r}_j)} \int_w^e \frac{\bar{r}_n r_d + r_b}{r_d} d\bar{x} - \frac{\alpha_s \mu_s (u_j - u_{j-1})}{Re(\bar{r}_j - \bar{r}_{j-1})} \int_w^e \frac{\bar{r}_s r_d + r_b}{r_d} d\bar{x} \\
&= \frac{\alpha_n \mu_n RINT4N(u_{j+1} - u_j)}{Re(\bar{r}_{j+1} - \bar{r}_j)} - \frac{\alpha_s \mu_s RINT4S(u_j - u_{j-1})}{Re(\bar{r}_j - \bar{r}_{j-1})} \\
&= CXV1(u_{j+1} - u_j) - CXV2(u_j - u_{j-1}) \tag{4.17}
\end{aligned}$$

Now it remains to discretize the interfacial source terms in Eqn. 4.3. Because source terms are often a function of the dependent variable (u in this case) it is useful to break them up into constant and varying portions. Since the discretization equations are ultimately linear, the varying portion must only allow a linear function of the dependent variable. Patankar [48] offers a detailed explanation of source term linearization.

Given these guidelines the source terms in the momentum equations are broken up as follows:

$$F_x = SCFX + SVFXu \tag{4.18}$$

From Eqn. 3.1 it is readily apparent what the values of $SCFX$ and $SVFX$ need to be:

$$\begin{aligned} SCFX &= \frac{3\rho_{cont}\alpha_{disc}C_D}{4d_{disc}} \left| \vec{V}_{other} - \vec{V} \right| u_{other} \\ SVFX &= -\frac{3\rho_{cont}\alpha_{disc}C_D}{4d_{disc}} \left| \vec{V}_{other} - \vec{V} \right| \end{aligned} \quad (4.19)$$

The values of $u_{other} - u$ used above are assumed to be known (guessed). An iterative scheme is constructed in which the guessed values become equal to the actual value of the velocity which the scheme solves for.

In general, then, integration of this source term proceeds as follows:

$$\begin{aligned} \int_w^e \int_s^n \left(\bar{r}r_d^2 + r_b r_d \right) (SCFX + SVFXu) d\bar{r}d\bar{x} &= (SCFX + SVFXu) \int_w^e \int_s^n \left(\bar{r}r_d^2 + r_b r_d \right) d\bar{r}d\bar{x} \\ (SCFX + SVFXu) \int_w^e \left[\frac{1}{2}(r_n^2 - r_s^2)r_d^2 + r_b r_d(r_n - r_s) \right] d\bar{x} &= (SCFX + SVFXu)RINT3M \end{aligned} \quad (4.20)$$

Here it is assumed that the values of $SCFX$ and $SVFX$ are constant over the control volume and, hence, can be placed outside the integral.

Similarly the second source term in the axial momentum equation can be determined:

$$\begin{aligned} M_x &= SCMX + SVMXu \\ SCMX &= MAX[\Gamma, 0]u_{other} \\ SVMX &= -MAX[\Gamma, 0] \end{aligned} \quad (4.21)$$

$$\begin{aligned} \int_w^e \int_s^n \left(\bar{r}r_d^2 + r_b r_d \right) (SCMX + SVMXu) d\bar{r}d\bar{x} &= (SCMX + SVMXu) \int_w^e \int_s^n \left(\bar{r}r_d^2 + r_b r_d \right) d\bar{r}d\bar{x} \\ (SCMX + SVMXu) \int_w^e \left[\frac{1}{2}(r_n^2 - r_s^2)r_d^2 + r_b r_d(r_n - r_s) \right] d\bar{x} &= (SCMX + SVMXu)RINT3M \end{aligned} \quad (4.22)$$

The source term discretization procedure is the same for each term, once it has been decided how it should be linearized.

Now, all the terms in the axial momentum equation have been discretized and it remains to simply put the terms together in a form compatible with a matrix solution. Without showing the required algebraic steps, the resulting linear equation is:

$$\begin{aligned}
& \{-MAX[-F_s, 0] - CXV2\}u_{j-1} \\
& + \{F_e + MAX[F_n, 0] + MAX[F_s, 0] + CXV1 + CXV2 + SVFX + SVMX\}u_j \\
& + \{-MAX[-F_n, 0] - CXV1\}u_{j+1} \\
& = F_wuu_j + P_w(\alpha_p RINTW + .5\alpha_p RINT2N - .5\alpha_p RINT2S) \\
& - P_e(\alpha_p RINTE + .5\alpha_p RINT2S - .5\alpha_p RINT2N) + SCFX + SCMX \quad (4.23)
\end{aligned}$$

Now the discretized form of the continuity equation can be introduced to simplify the above expression. In particular it is desired to eliminate F_e since it is not known because it is dependent on u . Following a procedure similar to the one shown above, the discretized continuity equation can be shown to be simply:

$$F_e - F_w + F_n + F_s = SCC \quad (4.24)$$

where $SCC = \Gamma RINT3M$.

Subtracting this equation from the coefficient of u_j in Eqn. 4.23 above, the result is:

$$\begin{aligned}
& \{-MAX[-F_s, 0] - CXV2\}u_{j-1} \\
& + \{F_w + MAX[-F_n, 0] + MAX[-F_s, 0] + CXV1 + CXV2 + SVFX + SVMX + SCC\}u_j \\
& + \{-MAX[-F_n, 0] - CXV1\}u_{j+1} \\
& = F_wuu_j + P_w\left(\alpha_p RINTW + \frac{1}{2}\alpha_p RINT2N - \frac{1}{2}\alpha_p RINT2S\right) \\
& - P_e(\alpha_p RINTE + \frac{1}{2}\alpha_p RINT2S - \frac{1}{2}\alpha_p RINT2N) + SCFX + SCMX \quad (4.25)
\end{aligned}$$

This equation can be simplified by assigning values to all the known coefficients:

$$A_j u_{j-1} + B_j u_j + C_j u_{j+1} = D_j \quad (4.26)$$

4.3 Tridiagonal Form of Equations

When this equation is written repeatedly for all the crosssectional points in the domain 2 to N-1 a tridiagonal matrix system is obtained:

$$\begin{pmatrix} B_2 & C_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ A_3 & B_3 & C_3 & 0 & 0 & 0 & 0 & 0 \\ 0 & A_4 & B_4 & C_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & A_{j-1} & B_{j-1} & C_{j-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & A_j & B_j & C_j & 0 & 0 \\ 0 & 0 & 0 & 0 & A_{j+1} & B_{j+1} & C_{j+1} & 0 \\ 0 & 0 & 0 & 0 & 0 & A_{N-2} & B_{N-2} & C_{N-2} \\ 0 & 0 & 0 & 0 & 0 & 0 & A_{N-1} & B_{N-1} \end{pmatrix} \begin{pmatrix} u_2 \\ u_3 \\ u_4 \\ u_{j-1} \\ u_j \\ u_{j+1} \\ u_{N-2} \\ u_{N-1} \end{pmatrix} = \begin{pmatrix} D_2 \\ D_3 \\ D_4 \\ D_{j-1} \\ D_j \\ D_{j+1} \\ D_{N-2} \\ D_{N-1} \end{pmatrix} \quad (4.27)$$

Note that the coefficients A_2 and C_{N-1} are equal to zero in accordance with the top and bottom boundary conditions for the domain. Therefore they are not shown in the above matrix. This system of equations is easily solved using the well known Thomas Algorithm.

4.4 IPSA Based Algorithm

For single-phase flow the parabolic solution method is relatively straightforward. In two-phase flow, however, the void fraction distribution needs to be solved for, a variable not present in single phase flow. In order to have a number of equations equal to the number of unknowns, the solution of the radial momentum equation is necessary. It is not necessary to solve this equation in a single-phase parabolic flow.

To illustrate this problem, Figure 4.3 shows the set of equations and unknowns for a small 3 by 3 computational grid. Notice that the pressure gradient for the radial momentum

Unknowns	Number of Unknowns	Equations	Number of Equations
u1	3	x1 momentum	3
dP/dx	1	continuity1	1
v1	2	continuity1	2
T1	3	energy1	3
dP/dr	3	r1 momentum	3
v2	3	r2 momentum	3
u2	3	x2 momentum	3
T2	3	energy2	3
α	3	continuity2	3
Total	24		24

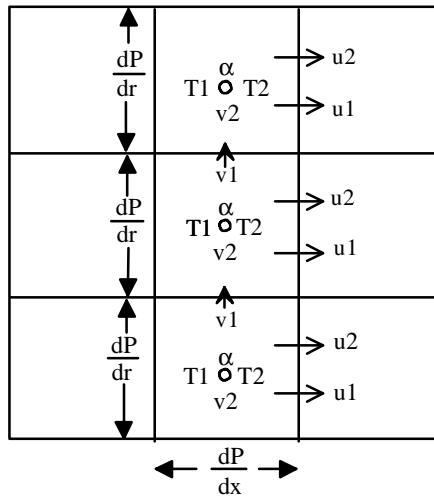


Figure 4.3 - Sample Grid for Unknowns vs. Equations

equation is considered independent of the pressure gradient in the axial momentum equation. This "uncoupling" is necessary to retain the parabolic nature of the procedure. The inaccuracy of doing this is not great as long as the radial pressure gradient is small compared to the axial gradient.

4.5 Solution of Ejector Flow Problem

Figure 4.3 also generally illustrates the procedure that the computer solution takes. First, the void fraction is guessed and the first phase equations are solved. This part of the solution is a standard parabolic procedure in which the axial pressure gradient is guessed and iterated until the global mass flowrate across the domain is satisfied. The radial momentum equation for the first phase is solved for the radial pressure gradient. Next, the second phase unknowns are solved for using the just calculated radial pressure gradient and the same guessed void fraction. Eventually the continuity equation for the second phase is solved for the void fraction distribution. The void fraction is compared to the guessed value and the procedure is iterated until the guessed void fraction equals the calculated value. This procedure is based on the IPSA procedure developed by Spalding [34].

The general procedure the solution takes for each downstream step is as follows:

1. Guess the void fraction distribution, α .
2. Guess values of u_c and v_c for the step. Generally these are taken as the upstream values.
3. Guess ΔP for the step.
4. Solve continuous phase axial momentum equation for u_c .

5. Check that global mass is conserved across the duct. If it is, continue. If not return to step 3.
6. Solve the continuous phase continuity equation for v_c .
7. Check if $u_c = u_g$ and $v_c = v_g$. If not return to step 2.
8. Solve the continuous phase r-momentum eqn. for dP/dr .
9. Guess values of u_d and v_d for the step.
10. Solve the discontinuous phase r-momentum eqn. for v_d .
11. Solve the discontinuous phase x-momentum eqn. for u_d .
12. Check if $u = u_g$, $v = v_g$. If not return to step 9.
13. Solve the discontinuous phase x-momentum eqn. for α_d .
14. Check that $\alpha_c + \alpha_d = 1$. If not return to step 1.
15. Solve the continuous and discontinuous energy equations for T_c and T_d .