The Resolution and Structure of High Reynolds Number Turbulent Boundary Layers Over Rough and Smooth Walls in Pressure Gradient

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(ABSTRACT)

The velocity fields of high Reynolds number, turbulent, wall boundary layers in non-equilibrium pressure gradients are experimentally investigated. Experiments in two wall configurations were performed; one with a hydrodynamically smooth test wall composed of flat aluminum panels, and the other with a rough surface consisting of 2 mm tall, staggered, circular cylindrical elements. A representative set of pressure gradient distributions were generated on the research wall by a systematically rotated NACA 0012 airfoil placed in a wind tunnel section to determine the functional dependence of the boundary layer formation on pressure gradient. Particle image velocimetry (PIV) was the primary measurement technique used to determine time-resolved features of the velocity flow field.

It is shown that regardless of wall condition and Reynolds number, the non-equilibrium turbulent boundary layers exhibit increasingly non-local behavior with streamwise development. This is apparent as a lag to the pressure gradient distribution observed in the streamwise developing integrated boundary layer parameters. These "history effects" are also prevalent in mean velocity profiles which are exhibited as a cross-over of the favorable and adverse pressure gradient profiles in the logarithmic layer. Similar cross-over points are observed in the Reynolds shear and normal stresses, particularly at the streamwise station downstream of the pressure gradient switch. The primary effect of the rough wall is to increase the magnitude of flow scales, and, while they exhibit the same qualitative history effects as the smooth wall, the rough wall flows show an earlier relaxation to equilibrium. Despite inherent uncertainties of indirect skin friction methods for the rough wall, the effective sandgrain roughness parameter k_s does not show a functional dependency to pressure gradient history. An evaluation of the wall-similarity hypothesis solely based on boundary layer thickness to roughness parameter ratios ($/k_s$) is insufficient and additional parameters such as pressure gradient histories, local roughness Reynolds numbers, and bias uncertainties due to instrument spatial resolution must be considered.

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(GENERAL AUDIENCE ABSTRACT)

In the interface between a surface and a moving fluid is the boundary layer where high shear and viscous stresses cause the bulk velocity to decrease to zero. When turbulent, this region of fluid is characterized by random, chaotic, and fluctuating motions of varying sizes. Parameters such as pressure gradients and geometric irregularities of the surface, referred to as roughness, can increase fluctuating pressures and velocities within the boundary layer and cause unwanted noise, vibration, and increased drag. Although many studies have evaluated boundary layers with either roughness or pressure gradient independently, most surfaces in practical application are susceptible to the compounding influences of both of these parameters. Thus, it is necessary to expand the current knowledge database to include complex flow fields necessary to improve data driven modeling and vehicle design.

This study focuses on experimental observations of the turbulent velocity field developing in both a rough and smooth wall boundary layer that is induced to a family of bi-directional pressure gradients generated by the pressure field of a rotating airfoil inside in a wind tunnel. Through statistical observations of the velocity field it was found that the varying pressure gradients caused the flow to develop non-local dependencies such that the response of the downstream boundary layer was dependent on the upstream flow history. The principal effect of roughness was to increase the magnitude of turbulent scales, but to show the same qualitative response to the pressure gradient history as seen in a smooth wall flow. However, direct comparison of rough and smooth wall turbulence statistics by means of the "wall-similarity hypothesis" requires careful consideration of multiple parameters including these flow histories, scales prescribed by roughness parameters, and bias errors from experiment under-resolution of the velocity field.

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Getting a PhD is like turbulence; it's chaotic, random, and dispersed with challenges of many sizes. Although, at times the experience was rough, there were many people who made these seemingly adverse times turn favorable. I am forever grateful to have them in my life and for making this adventure wonderfully unforgettable.

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Thank you,

Vidya

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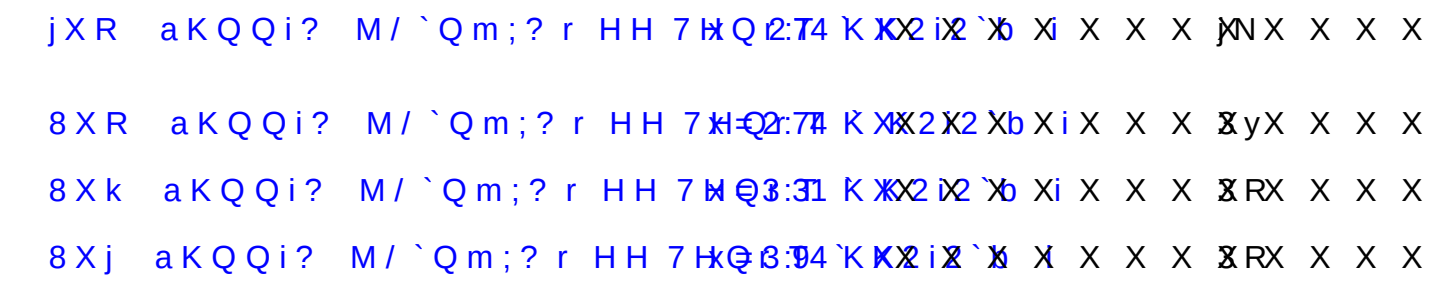

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iBQMb- T `iB+mH `Hv bBKTHB7vBM; `Qm;?M2bb KQ/2HBM; iQ #2
i?2`2 Bb M BMbm77B+B2Mi mM/2`bi M/BM; Q7 i?2 KmHiBp `B #H
`Qm;? M/ bKQQi? r HH 7HQrb BM MQM@2[mBHB#`BmK 7HQr 7B2H
F2v 72 im`2b Q7 im`#mH2M+2 bi`m+im`2b 2pQHpBM; BM MQM@2[
M/ 2p Hm i2b i?2 2tT2`BK2Mi K2 bm`2K2Mi HBKBi iBQMb BM ++
Q7 BMi2`2biX

RXk ii`B#miBQMb

h?2 rQ`F T`2b2Mi2/ BM i?Bb /Bbb2`i iBQM r b +QKTH2i2/ BM +QHH
H2 ;m2 M/ irQ `2b2 `+? K2MiQ`b K2MiBQM2/ #2HQrX

.`X . MB2H CX 6? Bib+?2M +HQb2 +QHH #Q` iQ` QM i?Bb T`QD2+i
7Q` HH +QKTmi iBQM H M/ 2tT2`BK2Mi H rQ`F Q7 i?2 7Hm+im iB
BM i?2 2tT2`BK2Mi /2bB;M- b2imT- M/ / i +[mBbBiBQMX >2 Bb
HBbi2/ DQm`M H M/ +QM72`2M+2 Tm#HB+ iBQMb- M/ ?Bb /Bbb2`
T`2bbm`2 7B2H/ T`QpB/2b +QKT MBQM T2`bT2+iBp2 iQ T`2b2Mi
p2HQ+Biv K2 bm`2K2Mib K /2 Q7 i?2 b K2 7HQr 7B2H/X

.`X qBHHB K CX .2p2M.TX EX hQ// GQ2 bi?2 p2/ b i?2 T`QD2+i T`BM
BMp2biB; iQ`b M/ b2+m`2/ i?2 ;` Mi M/ `2b2 +? 7mM/BM; 7`QK i?
h?2v b2`p2/ b +Q@+? B`b 7Q` i?2 `2b2 `+? /pBbQ`v +QKKBii22 M
rQ`F T`2b2Mi2/ BM #Qi? BM i?Bb /Bbb2`i iBQM M/ BM `iB+H2b
T22`@`2pB2r2/ Tm#HB+ iBQMX

RXj a+QT2 M/ BK Q7 S`2b2Mi qQ`F

h?2 K BM Q#D2+iBp2 Q7 i?Bb rQ`F Bb iQ BKT`Qp2 i?2 7mM/ K2Mi H
M/ K2 bm`2K2Mi HBKBi iBQMb Q7 p2HQ+Biv 7B2H/b Q7 MQM@2[m
Qp2` `Qm;? M/ bKQQi? bm`7 +2bX aT2+B7B+ HHv- i?Bb 2tT2`BK
BMp2biB; iBM; 7HQr /2p2HQTBM; Qp2` i?2 /B772`BM; r HH +QM/B

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T`2bbm`2 ;` /B2Mi 7B2H/ ;2M2` i2/ #v i?2 #HQ+F ;2 Q7 L * yyR
b2+iBQMX h?Bb T`2b2Mi rQ`F ? b i?`22 bT2+B7B+ BKb r?B+? `2

RXkM/2`bi M/ i?2 HBKBi iBQMb Q7 KmHiB@TQBMiK2 bm`2K2Mi i2
;2 p2HQ+BK2i`v- BM ++m` i2Hv K2 bm`BM;im`#mH2M+2 bi iB
#QmM/ `v H v2`b Qp2``Qm;? M/ bKQQi? bm`7 +2bX h?Bb Q#D

U V2p2 H ii2Mim iBQM Q7 im`#mH2M+2 K ;MBim/2b rBi? +Q
HmiBQM K2 bm`2K2MibX

U#Vbb2bb p `B iBQMb BM i?2 H2p2H Q7 im`#mH2M+2 ii2Mm iB
iBQMb M/ HQ+ H _2vMQH/b MmK#2`X

U+1VbiBK i2 H2p2H Q7 #B b mM+2`i BMiv bbQ+B i2/ rBi? SAc

kX*? HH2M;2 i?2 p HB/Biv Q7 i?2 r HH@bBKBH `Biv ?vTQi?2bBb
M/ bKQQi? r HH 7HQrb- T `iB+mH `Hv T2`i BMBM;iQ 7HQr /2p
T`2bbm`2 ;` /B2Mi 7B2H/X aT2+B7B+ HHv- r2 rBHH b22F iQ-

U VQKT `2 K2 M p2HQ+Biv M/ im`#mH2M+2 bi`2bb T`Q7BH2
mbBM; +QMp2MiBQM H 7Q`Kb Q7 b+ HBM;X

U#Vbb2bb MQM@HQ+ H i2M/2M+B2b Q7 i?2 7HQr /2p2HQTK2M
Q7 7HQr #2? pBQ`bX

U+1Vm;;2bi KmHiBp `B #H2 b2i Q7 T ` K2i2`b iQ BKT`Qp2 2tT2
iQ b iBb7v r HH@bBKBH `BivX

jX_2p2 H i?2 /Bbi`B#miBQM M/ +QMi2Mi Q7 2M2`;2iB+ im`#mH2
bKQQi? r HH #QmM/ `v H v2` BM/m+2/ #v MQM@2[mBHB#`B
TT`Q +? iQ BKT`Qp2 i?Bb mM/2`bi M/BM; Bb iQ,

U VQKT `2 K2 M bi iBbiB+b Q7 i?2 `Qm;? M/ bKQQi? r HH iQ B
rBi? HQ+ H _2vMQH/b MmK#2`X

U#1VT2+B7v `2;BQMb Q7 bT2+B7B+ BMi2`2bi rBi?BM i?2 #QmM
i?2 _2vMQH/b bi`2bb T`Q7BH2bX

U+XVm HBi iBp2Hv +QKT `2 i?2 BMbi Mi M2Qmb p2HQ+Biv 7B2
i?Q;QM H /2+QKTQbBiBQM iQ `2p2 H b? T2 M/ /Bbi`B#miB
/2p2HQTBM; i bT2+B7B+ r HH@MQ`K H /Bbi M+2bX

RX9 +?B2p2K2Mib

- Ç _2; `/H2bb Q7 r HH +QM/BiBQM M/ _2vMQH/b MmK#2` i?2 7HQ #mH2Mi #QmM/ `v H v2` 2t?B#Bib BM+`2 bBM;Hv MQM@HQ+ p2HQTK2MiX h?Bb Bb TT `2Mi b H ; BM i?2 `2bTQMb2 Q7 i BMi2;` i2/ #QmM/ `v H v2` T ` K2i2`b UBX2X bFBM 7`B+iBQM TH +2K2Mi i?B+FM2bbV iQ i?2 T`2bbm`2 ;` /B2Mi /Bbi`B#miBQ iQ b ô?BbiQ`v 2772+iXô

- Ç h?2`2 Bb +H2 ` 2pB/2M+2 i? i `Qm;? r HH #QmM/ `v H v2`b `2/m 2[mBHB#`BmK BM MQM@2[mBHB#`BmK 7HQrbX

- Ç >BbiQ`v 2772+ib `2 HbQ T`2p H2Mi BM i?2 K2 M p2HQ+Biv M AM i?2 K2 M p2HQ+Biv T`Q7BH2b-i?Bb Bb /2KQMbi` i2/ b ô+` /p2`b2 T`2bbm`2 ;` /B2Mi T`Q7BH2b BM i?2 HQ; `Bi?KB+ H v2` Q#b2`p2/ BM i?2 _2vMQH/b b?2 ` M/ MQ`K H bi`2bb2b-T `iB+m iBQM /QrMbi`2 K Q7 i?2 T`2bbm`2 ;` /B2Mi brBi+?X h?Bb #2? p #Q`2 r?B+? ;Bp2b BM/B+ iBQM Q7 H `;2 Qmir `/ i` MbTQ`i Q7 i?2b2 H v2`bX

- Ç 6Q` `Qm;? r HH 7HQrb- BM/B`2+i K2i?Q/b iQ /2i2`KBM2 bFBM `2;`2bbBQM 7Bib Q7 i?2 K2 M p2HQ+Biv T`Q7BH2b- b?Qr ?B;1 HQ; `Bi?KB+ `2;BQM QM i?2 +QKTmi2/ b?2 ` bi`2bb2bX .B`2+i K `2 M2+2bb `v iQ ++m` i2Hv +QKT `2 r HH +QM/BiBQMb QM i?

- Ç h?2 p HB/Biv Q7 r HH@bBKBH `Biv Bb `2H2p Mi iQ ?m;2 ` M;2 TTHB+ iBQMb-#mii?2 T`2b2Mi bim/v? b`2p2 H2/i? iKmHiBT ?BbiQ`B2b-#QmM/ `v H v2` i?B+FM2bb iQ `Qm;?M2bb T ` K2i2 MmK#2`b Kmbi #2 K i+?2/ iQ M 2[mBp H2Mi bKQQi? r HH + b2 QM i?2 # bBb Q7 i?Bb ?vTQi?2bBbX .2bTBi2 BM?2`2Mi mM+2`i K2i?Q/b 7Q` i?2 `Qm;? r HH- i?2 2772+iBp2 b M/;` BM `Qm;?M rBi? T`2bbm`2 ;` /B2Mi ?BbiQ`vX

- Ç aT iB H `2bQHmiBQM HBKBi iBQMb Q7 SAo 2tT2`BK2Mib + mb2 im`#mH2M+2 bi iBbiB+bX h?2`2 Bb MQ /Bb+2`MB#H2 BKT +i Q` `2bmHiBM; K2 M p2HQ+Biv T`Q7BH2bX >Qr2p2`- _2vMQH/b b

rBi? +Q `b2` `2bQHmiBQMb- rBi? T `iB+mH ` BKT +ib iQ i?2 r
M2MibX h?Bb `2/m+iBQM r b b?QrM iQ #2 /m2 iQ i?2 bK HH2`
Q++m``BM; i?2 r HH@MQ`K H /B`2+iBQM b QTTQb2/ iQ i?2 2
Q++m``BM; BM i?2 T`2/QKBM MiHv bi`2 KrBb2 bi`m+im`2bX

- 7Bt2/ bT iB H `2bQHmiBQM T`2b2Mib HBKBi iBQMb 7Q` bim/
/BiBQMb `2 +QKT `2/X h?Bb Bb #2+ mb2 i?2 ` M;2 Q7 b+ H2b
i?2 HQ+ H _2vMQH/b MmK#2`- ? b i?`2b?QH/ Q7 /2i2+iBQM #
QHmiBQMX h?Bb HBKBi iBQM `2bmHib BM ?B;?2` /BbT `Biv
Q7 K2 bm`2K2Mib K /2 rBi? +Q `b2 M/ 7BM2 `2bQHmiBQM r?
/p2`b2 T`2bbm`2 ;` /B2Mi- /BbT `Biv i? i Bb +QKT ` #Hv HQr
x2`Q Q` 7 pQ` #H2 T`2bbm`2 ;` /B2MiX

- +QKT `BbQM Q7 i?2 b K2 p `B #H2 [m MiBiB2b #2ir22M i?2
#QmM/ `v H v2`b b?Qri? ii?2 +Q `b2 `2bQHmiBQM+ M ii2Mm
bi`2bb2b 7Q` #Qi? `Qm;? M/ bKQQi? r HH 7HQrbX >Qr2p2`- i?
iBQM Bb KQ`2 T`2p H2Mi BM i?2 bKQQi? r HH 7HQr- r?BH2 i?2
m iBQM i +2`i BM HQ+ H _2vMQH/b MmK#2`bX L2p2`i?2H2bb
mM/2`@`2bQHmiBQM Q7 MQM@/QKBM Mi bi`2bb +QKTQM2Mib
`Qm;? M/ bKQQi? r HH 2tT2`BK2MibX

- Zm HBi iBp2 M HvbBb Q7 i?2 BMbi Mi M2Qmb bKQQi? r HH p
Q7 hQrMb2M/öb ii +?2/ 2//v ?vTQi?2bBb rBi? i?2 2MH `;2K2
r HH@MQ`K H /Bbi M+2 mM/2` /p2`b2 T`2bbm`2 ;` /B2Mi +QM
- "Qi? bT2+i` H T`QT2` Q`i?Q;QM H /2+QKTQbBiBQM QM i?2 p2
iBQM Q7 i?2 BMbi Mi M2Qmb 7HQr 7B2H/ b?Qrb i? i- Hi?Qm;?
`2bTQM/b iQ i?2 T`2bbm`2 ;` /B2Mi ?BbiQ`B2b b i?2 bKQQi? r
/B772`2M+2b BM i?2 p2HQ+Biv 7B2H/ bi`m+im`2 M/ b? T2- ;
K i+? KmHiBTH2 p `B #H2b 7Q` +QKT `BbQM Q7 `Qm;? M/ bKQ
i?2 # bBb Q7 r HH@bBKBH `BivX

RX8 ai`m+im`2 M/ *QMi2Mi

h?Bb /Bbb2`i iBQM Bb Q`; MBx2/ b 7QHHQrbX

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*? Ti2` R BMi`Q/m+2b i?2 `2b2 `+? T`QD2+i M/ KQiBp iBQM M/ bmK
7`QK i?2 /Bbb2`i iBQM rQ`FX

*? Ti2` T`QpB/2b bmKK `v Q7 i?2 7mM/ K2Mi H 7HQr T?vbB+b Q
H v2` b r2HH b `2pB2r Q7 i?2 `2H2p Mi HBi2` im`2 T2`i BMBM;
bKQQi? r HH +QM/BiBQMbX

*? Ti2` j+QMi BMb K Mmb+`BTi iBiH2/ óaT iB H _2bQHmiBQM G
;2 o2HQ+BK2i`v BM >B;? _2vMQH/b LmK#2` hm`#mH2Mi "QmM/ `v
7Q` T22`@`2pT2`BiQ2Mib BXMh?Bm+?bTi2` T`QpB/2b M 2p Hm iBQM
K2Mi HBKBi iBQMb Q7 T `iB+H2 BK ;2 p2HQ+BK2i`v BM `2bQHpBM
BMbi`mK2Mi bT iB H `2bQHmiBQMX

*? Ti2` 9+QMi BMb K Mmb+`BTi iBiH2/ ó>BbiQ`v 1772+ib M/ q
1[mBHB#`BmK hm`#mH2Mi "QmM/ `v G v2`b BM o `vBM; S`2bbm`2
aKQQi? am`7 +2bô iQ #2 bm#K2`iM/ iBQM?2 AMK2`iM/ iBQM?2 H CQm`M H 7Q`X>2 i M/
h?2 +QMi2Mib Q7 i?Bb +? Ti2` BMp2biB; i2 i?2 r HH@bBKBH `Biv
#2? pBQ`b Q7 `Qm;? M/ bKQQi? r HH im`#mH2Mi #QmM/ `v H v2`b

*? Ti2` 8+QMi BMb K Mmb+`BTi iBiH2/ ó M HvbBb Q7 *Q?2`2Mi ai
q HH hm`#mH2Mi "QmM/ `v G v2` BM S`2bbm`2 :` /B2MiXô h?Bb +?
+Q?2`2Mi bi`m+im`2b B/2MiB7B2/ mbBM; i?2 bT2+i` H T`QT2` Q`i
i2+?MB[m2 TTHB2/ iQ i?2 iBK2@`2bQHp2/ SAo / i K2 bm`2/ Qp2`

*? Ti2` e T`QpB/2b bmKK `v Q7 i?2 /Bbb2`i iBQM rQ`F M/ /Bb+mb
+QMiBMm2/ BMp2biB; iBQMX

h?2 7Q`K iiBM; Q7 i?Bb /Bbb2`i iBQM p `B2b +`Qbb +? Ti2`b b
bm#KBbbBQM `2 BM i?2 7Q`K i T`2b+`B#2/ #v i?2 Tm#HBb?2`X

*? Ti2` k

" +F;`QmM/ M/ GBi2` im`2 _2pB

kXR hm`#mH2Mi "QmM/ `v G v2`b Pp2` aKQQ

h?Bb b2+iBQM rBHH T`QpB/2 7mM/ K2Mi H `2pB2r Q7 i?2 7HQr T
`2H i2/iQ im`#mH2Mi #QmM/ `v H v2`b- T `iB+mH `Hv Qp2` bKQQi
/Bb+mbbBM; im`#mH2Mi #QmM/ `v H v2`b Qp2` `Qm;? r HHb rBH
2t KBM2/ BM i?Bb b2+iBQMX

kXRXR 6mM/ K2Mi Hb Q7 i?2 "QmM/ `v G v2`

b 7HQr T bb2b Qp2` #Q/v- i?2 BMi2`7 +2 Q7 i?2 bQHB/ bm`7 +2
?B;? pBb+Qmb M/ b?2 ` 7`B+iBQM H 7Q`+2bc i?Bb `2;BQM Bb FM
KQH2+mH ` H2p2H- i?2`2 Bb biBHH 7`B+iBQM QM i?2 bm`7 +2 `2b
i?2 7HQr rBi?BM i?2 #QmM/ `v H v2` Bb bHQr2` i? M i? i Q7 i?2 7`2
7HmB/ HQb2b HH Bib KQK2MimK- i?mb `2bmHiBM; BM x2`Q@7HQ
óMQ@bHBT +QM/BiBQM-ô M/ Bb `2T`2b2Mi2/eX b h?2 7HQr Q i?2 7HmB
2/;2 Q7 i?2 `2;BQM Q7 ?B;? b?2 ` rBHH 7QHHQr p Hm2 /2T2M/2M
ivTB+ HHv /2i2`KBM2/ #v i?2 BM7HQr p2HQ+Biv +QM/BiBQMb Q` i?
Q7 T`2bbm`2 ;` /B2MibVX h?2 TQBMi i r?B+? i?2 p2HQ+Biv T`Q7B
7`22bi`2 K Bb `272``2/iQ b i?2 #QmM/ `v H v2`b T QBMi b TT`Q
b i?2 r HH@MQ`K H /Bbi M+2 +Q``2bTQM/BM; iQ i?2 TQBMi i r?B
NNW Q7 i?2 2/;2 Up2 HQ+Biv-

6B;m`2 kXR, J2 M p2HQ+Biv T`Q7BH2 rBi? `2;BQMb BM/B+ i2/ 7Q
#mH2Mi #QmM/ `v H v2` Qp2` `Qm;? M/ bKQQi? r/HTiX /.7ì QTKQBMi
oBb?r M i? M3)ic aT (H/BM; T`Q7BHV2X b?QrM b U

kXRXk "QmM/ `v G v2` _2;BQMb a+ HBM; S ` K2i2`b

Hi?Qm;? i?BM b?2 ` H v2`- i?2 #QmM/ `v H v2` Bib2H7 + M #2 /2+
r?2`2 i?2 7HQr +QM/BiBQMb /B+i i2 +2`i BM #2? pBQ`b Q7 i?2 7H
K ;MBim/2 Q7 im`#mH2Mi 2//B2b M/ `2H iBp2 p2HQ+BiB2b /2T2M
i?2 7`22bi`2 K Q` i?2 bm`7 +2X h?2 +H bbB+ H b2T ` iBQM Q7 i?2
T`Q7BH2 BM BMM2` +QQ`/BM kXR h?2 b?2;BQMb 6B;m`2 Q7i2M B/2MiB
b+ HBM; T ` K2i2`b- +H bbB+ HHv #`QF2M /QrM BMiQ i?2 óBMM2
+QHH Tb2 i?2 T`Q7BH2b iQ bBM;H2 +m`p2 BM i?2 `2bT2+iBp2 `2;
r HH- Bb H `;2Hv 772+i2/ #v pBb+QbBiv M/ i?2 p2HQ+BMv b+ H2b
i?2 H2M;i?b b+ H2 QM i?2 T`2 K2B+iBQM p2HQ+Biv Bb /27BM2/ BM
b?2 ` bi`2 bM/ i?2 7HmB/ /2bMbBiv

$$u = \frac{r}{w}$$

UkXRV

h?2 `2;BQM +HQb2bi iQ i?2 r HH Bb `272``2/iQ b i?2 HBM2 ` bm#H
Bb H `;2Hv /QKBM i2/ #v /BbbBT iBQM /m2 iQ i?2 Qp2`r?2HKBM;
M/ i?2 K2 M p2HQ+Biv Bb /B`2+iHv T`QTQ`iBQM H iQ i?2 /Bbi M+

lYSYi ª ã¶é #ñ é²ž -Hž-¶ Q&¶ bèññ Ñržãã N

r v 7`QKi?2 r HH-i?2 p2HQ+Biv ;` /B2Mi bi `ib #2+QKBM; BM/2
2772+ib M2 ` i?2 r HH M/ i?2 HBM2 ` bm#H v2` K2`;2b BMiQ i?2 H
Q7 MóQp2`H Tô Q` ó#m772`ô H v2`X >2`2-i?2 K2 M p2HQ+Biv
#Qi?i?2 HBM2 ` bm#H v2` M/ i?2 HQ; `Bi?KB+ `2;BQMbX b Bib M
BM i?2 HQ; `Bi?KB+ `2;BQM 7QHHQrb HQ; `Bi?KB+ T`Q7BH2 /27
BM 2[m kX9Qr2 T ` K2i2`b rBi? bm+2`b+2`bMQbKQH7BóX2/ QM i?2 BM
b+ HBM; T ` K2i2`bX h?2p+QMbiMiM +QMbi MiV M/ * ivTB+ HHv
`QmM=0:41 M/C = 5:0X 6m`i?2` bm#/BpBbBQMb Q7 i?2 #QmM/ `v H
iQTB+ Q7 BMi2`2bi BM HBi2` im`2X 6Q` BMbi M+2- BM i?2B` rQ`F
im`#mH2Mi #QmM/ `v H v2`-:2Q`2i MH*Bb?2/HQ2;(BQM + HH2/i?2
#Qp2 r?B+? im`#mH2M+2 /BbbBT iBQM Bb /QKBM i2/#v BM2`iB H
2772+ib `2/Q8)BMm?2iv(/2b+`B#2i?2K2bQH v2` b MQi?2`Qp2`H T
BMiQ i?2 Qmi2` H v2` Q7 i?2 #QmM/ `v H v2`X >Qr2p2X-Ri2 /BpBb
mb2/BM T` +iB+2 7Q` Tm`TQb2b Q7 KQbi bim/B2b-BM+Hm/BM; i?

h?2 Qmi2` `2;BQM Bb b+ H2/QM T ` K2i2`b BM/B+ iBp2 Q7 i?2 H `;
r?B+? `2 i?2 2/;2Ue2HMQ+B2v#QmM/ `v H v2`Xi2BM+F?B2bb2-;BQM-i?2 I
Q7 i?2 r HH K bb2b M //BiBQM H i2`K /2b+`B#BM; i?2 r F2 2772
Bb `2M K2/ i?2 óH r Q7 i?2 r F2ô kX9Q7BM`BiM2[imQiBmQ+2/ #v *QH2b-B
i?2 r F2 T ` K2i2` Bb 7mM+iBQM Q7 r HH /kX9(kR+X2 b?QrM BM 2[m

$$\overline{u^+} = y^+ \qquad \text{(kXkV)}$$

$$\overline{u^+} = \frac{1}{\kappa}\ln(y^+) + C \qquad \text{(kXjV)}$$

$$\overline{u^+} = \frac{1}{\kappa} + C + \frac{1}{\kappa}W\left(\frac{y}{\delta}\right) \qquad \text{(kX9 V)}$$

q?2`2

$$W(0) = 0; W(1) = 0; \int_0^{\frac{y}{\delta}} dW = 1 \qquad \text{(kX9\#V)}$$

Ry +Ñžÿ ¶ lY #ž«áÌ ñ é²žé² HÔ ¶ ž ¶ `¶&

kXRXj :Qp2`MBM; 1[m iBQMb 7Q` hm`#mH2Mi 6HQrb

h?2 _2vMQH/b@ p2` ;2/ L pB2`@aiQF2b 1[m iBQMb

h?2 L pB2`@aiQF2b 2[m iBQMb- r?B+? 2Mi BH i?2 +QMb2`p iBQM
b i?2 ;Qp2`MBM; 2[m iBQMb Q7 KQiBQM 7Q` 7HmB/ 7HQrbX q2 r
irQ@/BK2MbBQM H K2 M 7HQrV- rMi?2 F2MBMiUiB+bBk2/biv
+QMbi MiX h?mb- i?2 L pB2`@aiQF2b 2[m iBQMb `2 bBKTHB7B2
7`QK b b?QrM #2HQr,

$$\frac{\partial \mu}{\partial x_i} = 0 \qquad \text{UkX8V}$$

$$\frac{\partial \mu}{\partial t} + \frac{\partial \mu u_j}{\partial x_j} = -\frac{1}{}\frac{\partial p}{\partial x_i} + \frac{\partial^2 u_i}{\partial x_j^2} \qquad \text{UkXeV}$$

q?2`p2 Bb i?2 BMbi Mi M2Qmb 7HmBb/ i?2 bBKTHB7B2/ +QQ`/BM i2 /B`2+iBQMb `2bT2+iBp2HvX

h?2 7HQr QmibB/2 i?2 #QmM/ `v H v2` UBX2X i?2 7`22bi`2 KV Bb
bBKTHB7B+ iBQM iQ i?2 L pB2`@aiQF2b 2[m iBQMb + M #2K /2 b
p iBQM Q7 KQK2MimK Bb B/2MiB+ HHv x2`Qc i?2b2 2[m iBQMb 7Q
i?2 1mH2` 2[m iBQMbX h?2 HBKBi iBQMb Q7 b iBb7vBM; #Qi? i?2
`v H v2` M/ i?2 BMpBb+B/ 7`22bi`2 K TT`QtBK iBQMb iQ i?2 L p
`2+Q;MBx2/ #v Gm/rBM; S` M/iH BM RNy8- r?Q `2 bQM2/ i? i7Q` 7
_2vMQH/b MmK#2`- i?2 7HQr +QmH/ #2 bm#/BpB/2/ BMiQ `2;BQM
H v2`V- M/ i?2 7`22bi`2 K (

.m2 iQ i?2 mMbi2 /v M im`2 Q7 im`#mH2M+2- Bi Bb /B77B+mHi iQ
QM i?2 7HQr 7B2H/ BM i2`Kb Q7 Bib BMbi Mi M2Qmb p Hm2bX M
iBK2@ p2` ;2 Q7 im`#mH2M+2 T`QT2`iB2b `2 bi iBQM `vX aQ- Bi
#QmM/ `v H v2` 2[m iBQMb b i?2 2tT2+i2/ p Hm2 Q7 Bib _2vMQH
b2T ` i2b i?2 BMbi Mi M2Qmb +QKTQM2Mib BMiQ Bib K2 M U/2MQ
M Qp2`# `V M/ 7Hm+im iBM; U/2MQi2/ rBi? M TQbi`QT?2V T `ib
/2+QKTQbBiBQM 7Q` i?2 p2HuQ=+Bivu Bi L`Bi2M i?-i i?2 iBK2@ p2`

Q7 7Hm+im iBM;+QKTQM2Mi Bb x2`Q-i?2 2tT2+i2/ p Hm2 Q7 i?2
irQ@/BK2MbBQM H-+QMb2`p iBQM Q7 K bb M/ KQK2MimK Bb bBi

$$\frac{\partial U_i}{\partial x_i} = 0 \qquad (1)$$

$$U_i \frac{\partial U_j}{\partial x_i} = -\frac{1}{\rho}\frac{\partial P}{\partial x_j} + \frac{\partial \overline{u_i^0 u_j^0}}{\partial x_i} + \nu \frac{\partial^2 U_i}{\partial x_i^2} \qquad (2)$$

h?Bb 7Q`K Q7 i?2 BM+QKT`2bbB#H2 +QMb2`p iBQM H rb Bb `272`
L pB2`@aiQF2b U_ LaV 2[m iBQMbX LQi2 i? i i?2 /2+QKTQbBiBQ
THB+BiHv BMi`Q/m+2b i2`K 7Q` $\overline{u_i^0 u_j^0}$-p2HbQ+FBMQ+Q`b2iH2iB2vMbQH/b
M/ MQ`K H bi`2bb2bX h?2 i` +2 Q7 i?2 /B ;QM Hb Q7 i?2 _2vMQH
b i?2 im`#mH2Mi FBM2iBk+ 2MM/2Bbv/Q7 BM2/ BkX#2[miBQM

$$k = \frac{1}{2}\overline{u_i^2} \qquad (3)$$

>Qr2p2`-rBi? i?Bb //BiBQM H _2vMQH/b bi`2bb i2`K-i?2`2 `2 M
FMQrMb M/ BMbm77B+B2Mi 2[m iBQMb r?2M ii2KTiBM;iQ bQHp2
aiQF2b 2[m iBQMb-i?2 /BbT `Biv 7Q` r?B+? QMHv BM+`2 b2b 7Q
`272``2/iQ b i?2 +HQbm`2 T`Q#H2KX h?2 bQHmiBQM iQ i?2 +HQ
`B2iv Q7 im`#mH2M+2 KQ/2Hb i? i K F2 bbmKTiBQMb rBi? `2;`/b
M/ i?2 hE1X h?mb-im`#mH2M+2 KQ/2Hb-bm+? b i?2 "QmbbBM2
S` M/iHöb KBtBM;@H2M;i? KQ/2H Kmbi #2 BKTH2K2Mi2/ iQ bQHp
bi iBbiB+ H 2p Hm iBQM Q7 i?2 L dyBX2 JQ`a2QF2+bi2[miBMQ/Mbp(M+2/
#mH2M+2 KQ/2Hb 2tBbi UBX2X-kTakh`-i@i+HK-`i?b2-/JB2Mi2`bbBQM 7Q
Bb #2vQM/i?2 b+QT2 Q7 i?Bb S?.X

h?2 hm`#mH2Mi "QmM/ `v G v2` 1[m iBQMb

b i?2 #QmM/ `v H v2` 2M+QKT bb2b QMHv i?BM `2;BQM Q7 i?2
+ M #2 7m`i?2` bBKTHB7B2/#v K2 Mb Q7 b+ H2@ M HvbBb- K2
Q7 i?2 p `B #H2b rBi? `2bT2+i iQ 7Bt2/`272`2M+2 p Hm2b-bm+?
p2HQU1BivM/ H `;2bi H2M;i? BM H2+b? +QKTQM2Mi /B`2+iBQM- M/

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i`BpB H i2`KbX h?2 #QmM/ `v- HBbv2 F i2?B+QM2bi2 T`2/QKBM Mi H2
BMi?2r HH@MQ`K H/B`2+iBQM-r?B+? Bb biBHH bK HH T2`+2Mi
LX h?mb-i?2 MQ`K HBx2/p `B #Hb2mbT-2`2bM+QB2T/ir-Bi2U_i/=2 U_iBUM2-/ b
$u_i = u_i^0/U_1 - x_1 = x_1/L - x_3 = x_3/L - x_2 = x_2/ - MP P = P/U_1^2$ X TTHvBM; i?2b2
MQ`K HBx iBQMb iQ i?2 _ La2[m iBQM 7Q`+QMiBMmBiv,

$$\frac{@U_1}{@x_1} + \frac{L}{}\frac{@U_2}{@x_2} + \frac{@U_3}{@x_3} = 0 \qquad (UkXRyV)$$

qBi? i?Bb MQ`K HBx iBQM Bi BbbU_1QMMU_3i?Qvii?2``2BT2Q`QM+2 p2HQ+
QMi?2Q`/2` Q7U_2 pM2HiQ+Biv +QKTQM2MtU_1BbLQXMhi?2MQQ/2` HBx iBQ
T2`7Q`K2/ M/bBKTHyBK2Q`2/7MQmK?2[m iBQM Bb b?QrM #2HQr,

$$U_1\frac{@U_1}{@x_1} + U_2\frac{L}{}\frac{@U_1}{@x_2} + U_3\frac{@U_1}{@x_3} = \frac{@P}{@x_1} \frac{@\overline{u_1^2}}{@x_1} \frac{L}{}\frac{@\overline{u_1u_2}}{@x_2} \frac{@\overline{u_1u_3}}{@x_3} + \frac{}{U_1\,L} \frac{@^2U_1}{@x_1^2} + \frac{L^2}{2}\frac{@^2U_1}{@x_2^2} + \frac{@^2U_1}{@x_3^2} \qquad (UkXRRV)$$

bQ#b2`p2/2 `HB2` 7Q` kXRyV2b2 iBQMi?2 +QMp2+iBp2 i2`Kb M/ i?2
;`/B2Mi2`K BMK2X[mB2bQ7 Q`/2` RX$\overline{u_1^2}$ QM2$\overline{u_3^2}$`-2?b2K HHr?2M
MQM@/BK2MbBQM HBx2/QMi?2 +? ` +i2`BbiB+ p2HQ+Biv M/+M
Q7U_1L Bb MQi2/iQ #2 i?2 BMp2`b2 Q7 i?2 _2vMQH/b MmK#2`- MQ
/2b+`B#BM;i?2 ` iBQ Q7 BM2`iB H iQ pBb+Qmb 7Q`+2bX 6Q` H `;2
i?2 + b2 7Q` im`#mH2Mi 7HQrb M Hvx2/BMi?Bb rQ`F-i?2 7B`bi
+QMbB/2`2/M2;HB;#H2r?2M +QKT`2/iQ i2`Kb MQ`K HBx2/QMi?
b K2 T`Q+2bbx$_3$7kQQK2MimK2[m iBQM-i?2 bi`2 KrBb2 M/bT MrBb2
TT`QtBK i2/ b,

$$U_1\frac{@U_1}{@x_1} + U_2\frac{@U_1}{@x_2} + U_3\frac{@U_1}{@x_3} \quad \frac{1}{}\frac{@P}{@x_1} \frac{@\overline{u_1^0u_2^0}}{@x_2} + \frac{@^2U_1}{@x_2^2} \qquad (UkXRkV)$$

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$$U_1\frac{\partial U_2}{\partial x_1}+U_2\frac{\partial U_2}{\partial x_2}+U_3\frac{\partial U_2}{\partial x_3}\quad -\frac{1}{\rho}\frac{\partial P}{\partial x_2}\quad \frac{\partial \overline{u_2^0 u_3^0}}{\partial x_2}+\quad \frac{\partial^2 U_3}{\partial x_2^2}\qquad \text{UkXRjV}$$

h?2`2 `2 bHB;?i/B772`2Mi 7Q` i?2 iBQM /m2 iQ i?2 H2M;i? b+
iBQMb M/ /B772`2Mi T`Q+2bb 7Q` b+ H2 M HvbBKQ2MmKX h?2
2[m iBQM Bb ;Bp2M b,

$$U_1\frac{\partial U_2}{\partial x_1}+U_2\frac{1}{\rho}\frac{\partial U_2}{\partial x_2}+U_3\frac{\partial U_2}{\partial x_3}=\quad \frac{1}{\rho}\frac{\partial P}{\partial x_2}\quad \frac{\partial \overline{u_1 u_2}}{\partial x_1}\quad \frac{1}{\rho}\frac{\partial \overline{u_2^2}}{\partial x_2}\quad \frac{\partial \overline{u_2 u_3}}{\partial x_3}+$$
$$\overline{U_1}\quad \frac{\partial^2 U_2}{\partial x_1^2}+\frac{1}{\rho^2}\frac{\partial^2 U_2}{\partial x_2^2}+\frac{\partial^2 U_2}{\partial x_3^2}\qquad \text{UkXR9V}$$

>2`2- i?2 T`2bbm`2 ;` /B2Mi M/ r HH@MQ`K H _2vMQH/b bi`2bb
Q`/2`7 i?2v `2 bB;MB7B+ MiX 6Q` ?B;? _2vMQH/b MmK#2` 7HQ
BMbB;MB7B+ Mi BM +QKT`BbQM iQ i?2 T`2bbm`2 ;` /B2Mi M/ r H
i?2 # bBb Q7 i?2 `2H iBp2 BMbB;MB7B+ M+2 Q7 bK HH i2`Kb BM
#QmM/ `v H v2`- i?2 vBKQ2MbBQM Q7 i?2 [m iBQM Bb bBKTHB7B2/ M/ T

$$0\quad \frac{1}{\rho}\frac{\partial P}{\partial x_2}\quad \frac{\partial \overline{u_2^2}}{\partial x_2}\qquad \text{UkXR8V}$$

hQ;2i?2`- 2[mkXiBQMRkXR8MkXRj 2`72``2/iQ bi?2 ô#QmM/`v H v
iBQMbXô

LQri? ir2? p27mHHv/Bb+mbb2/i?2 TT`QtBK iBQMbK /27Q`i?2
r2+ M/Bb+mbb+? ` +i2`BbiB+b Q7 i?2 #QmM/`v H v2` M/[m MiB
Bib T`QT2`iB2bX A7 +QMi`QH pQHmK2 Bb i F2M +`Qbb M BM7BM
2ti2M/b BM i?2 r HH@MQ`K H /B`2+iBQM- r2+ M b22 i? i i?2 #Qm
MQ@bHBT+QM/BiBQM i i?2 r HH M/7QHHQr bKQQi?;` /B2Mi m
bi`2 K+QM/BiBQMb i i?2 #QmM/`v H v2` 2/;2X .m2 iQ pBb+Qmb 2
i?`Qm;? i?2 #QmM/`v H v2` +QMi`QH pQHmK2 HbQ`2bmHib BM H
+QKT`2/iQ i? i Q7 M B/2 H-BMpBb+B/7HQrX h?2b2 HQbb2b `2
i?B+FM2bb i?2 /BbTH +2K2MiX?B+2bM2 2bK TT2` BM i?2 BMi2;` I

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i?2 r HH@T ` HH2H KQK2MimK 2[m kXRXeM ;Bp2M BM 2[m iBQM

$$\frac{@}{@x_1} + \frac{1}{U_e}(2 +) = \frac{C_f}{2} \qquad \text{UkXRe V}$$

q?2`2

$$= \int_0^{} 1 \quad \frac{\overline{U_1}}{U_e} \quad \frac{\overline{U_1}}{U_e} dx_2 \qquad \text{UkXRe\#V}$$

M/

$$= \int_0^{} 1 \quad \frac{\overline{U_1}}{U_e} \quad dx_2 \qquad \text{UkXRe+V}$$

q?2`2 i?2 Qp2`HBM2 BM/B+ i2biBK2@ p2` ;2/ p Hm2bX //BiBQM
K2MimK BMi2;` H 2[m iBQM `Cf i?2 bF2 7 2p UeBQMBiv?2` /Bb+mb@
bBQM `2; `/BM; i?2 KQK2MimK BMi2;` H K2i?Q/iQ/2i2`KBM2 bFBM
K2 M p2HQ+Biv T`Q7BH2b Bb kXTHXQ"Bi?2/MB/M+bM2+#2QMvbB+ HHv BM
T`2i2/ b i?2 HQbb Q7 KQK2MimK M/ i?2 /BbTH +2K2Mi Q7 K bb-`
7HQrX 1bb2MiB HHv- i?2v `2 i?2 /Bbi M+2b i? i i?2 BMpBb+B/ 7
/BbTHi+2T`Q/m+2 i?2 b K2 K bb M/ KQK2MimK b i?2 pBb+Qmb 7
/2TB+i2/ BMkXBr,m2`2 i?2 `2/ b? /2/ `2;BQMb b?Qr 2[mBp H2Mi K bb
T`QT2`iB2b #2ir22M i?2 pBb+Qmb M//BbTH +2/ BMpBb+B/ T`Q7BH
T?vbB+ H BKT +ibQ7 b?2 ` bi`2bb2b QM 7HQr M2 ` bm`7 +2-i?E
BMpbB+B/ 7HQr #2? pBQ` HHQrb mb iQ 7 B`Hv +QKT `2 i?2 K bb
p `vBM; 2ti2`M H T2`im`# iBQMb Q7 i?2 #QmM/ `v H v2`X

kXRX9 *? ` +i2`BbiB+b Q7 hm`#mH2Mi "QmM/ `v G v

h?2 7QHHQrBM; b2+iBQM rBHH /Bb+mbb i?2 +? ` +i2`BbiB+b Q7 im
`v H v2`X i bm77B+B2MiHv ?B;? _2vMQH/b MmK#2`- `272``2/
MmK#2`- i?2 #QmM/ `v H v2` 7HQr i` MbBiBQMb 7`QK H KBM ` UE
7QHHQr bKQQi? T ii2`MV iQ im`#mH2Mi UBX2X- 7HQr rBi? M BM
iB+ H KQiBQMV b b?QrM BM i?2 kbXj ?2?2 i_2vMQH/b; mM`2K#2` Bb /27
i?2 ` iBQ Q7 i?2 BM2`iB H 7Q`+2b iQ pBb+Qmb 7Q`+2b M/ Bb 7mM
L- M/ p2HUQX+Biv

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6B;m`2 kXk, a+?2K iB+ Q7 K2 M p2HQ+Biv T`Q7BH2 BM #QmM
2[mBp H2Mi ó/BbTH +2/ô BMpBb+B/ T`Q7BH2 U`B;?iVX _2/ b? /2/
KQK2MimK T`QT2`iB2b #2ir22M i?2 pBb+Qmb M/ /BbTH +2/ BMpB

6B;m`2 kXj, a+?2K iB+ Q7 bi`2 KrBb2 /2p2HQTBM; #QmM/ `v H v2
i` MbBiBQMb 7`QK H KBM `iQ im`#mH2Mi 7HQrX

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$$Re = \frac{UL}{\nu}$$

UkXRdV

AM #QmM/ `v H v2` TTHB+ iBQM- i?2 +? ` +i2`BbiB+ _2vMQH/b M
Q7 i?2 BMi2;` i2/ #QmM/ `v H v2` T ` K2i2`bX/ 2eM+ kXR2eM/ kXR2eM/ 6BQM2 2Mm@iBQ
bi M+2- KQK2MimK@i?B+FM2bb # b2/ _2vMQH/b MmK#2` rQmH/ i
b+ H2- bM/ i?2 +? ` +i2`BbiB+ p2HQ+Biv iQ #2 i?2 HQ+ H 7`22bi`2

AM ;2M2` H- i?2 /BbiBM+iBp2 [m HBi iBp2 +? ` +i2`BbiB+b Q7 im`

RX2M2` H brB`HBM; M im`2 Q7 i?2 7HQr /m2 iQ +QHH2+iBQM Q
HmKTb Q7 pQ`iB+ H 7HmB/X

U Xh?2b2 2//B2b + M p `v BM bBx2 M/ p2HQ+Biv K ;MBim/2b
im`#mH2Mi #QmM/ `v H v2`X h?2 `2H iBQM Q7 i?2 bBx2 M/
iQ hQrMb2M/öb ii +?2/ 2//v ?vTQi?2bBb r?B+? bi i2b i? i i
`2 T`QTQ`iBQM H iQ i?2 /Bbi3MeX2 7`QK i?2 r HH (

U#hv?2`2 Bb KBtBM; #2ir22M `2;BQMb- i?mb `2bmHiBM; BM i?2
imK- M/ 2M2`;v QM Km+? H `;2` b+ H2b8 2X? M H KBM `7HQr

kXh?2 `2;BQM Q7 im`#mH2M+2 M/ i?2 BMpBb+B/ 7`22bi`2 K `2
/2b+`B#2/ mbBM; i?2 #QmM/ `v H v2` 2[m iBQMbX

jXhm`#mH2M+2 Bb M mMbi2 /v T`Q+2bb- M/ Bb biQ+? biB+ M/`
i?`22@/BK2MeX BQM H (

9Xhm`#mH2Mi #QmM/ `v H v2`b ? p2 ?B;?2` bFBM 7`B+iBQM i? M
;2M2` HHv KQ`2 mMbi #H2X "2+ mb2 Q7 i?2 ?B;?2` b?2 `M2 `
T`Q7BH2b BM im`#mH2Mi 7HQr ? p2 Km+? bi22T2` p2HQ+Biv
r HH@MQ`K H8/9Bbi eiXM+2 (

h?2 #2? pBQ`rBi?BM im`#mH2Mi #QmM/ `v H v2` Bb /2T2M/2Mi Q
Q7 bi`m+im`2b /2p2HQTBM; BM i?2 BMM2` M/ Qmi2` `2;BQMbX h
_2vMQH/b MmK#2`c i?2 ?B;?2` i?2 _2vMQH/b MmK#2`- i?2 rB/2` i
bK HH2bi M/ H `;2bi b+ H2b Q7 im`#mH2M+2X h?2 bK HH2bi Q7 i
KQ;Q`Qp b+ H2b Q` i?2 ô/BbbBT iBp2 b+ H2b-ô 7Q` r?B+? i?2 pBb
M/ i?2 im`#mH2Mi FBM2iB+ 2M2`;v 8yX/ BMbB?2 H2/`B2Mi2QM?2Qi7(i?2 b

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bT2+i`mK `2 2//B2bi? i `2rBi?BMi?2 ô2M2`;v+QMi BMBM;` M;2
i?2 H `;2bi H2M;i?b+ H2 Bb QMi?2 Q`/2` Q7 KmHiBTH2 #QmM/ `v
2M2`;v@+QMi BMBM; M/ /BbbBT iBp2b+ H2b Bb i?2 ôBM2`iB H
T`BK `BHv /2i2`KBM2/ #v 8yX2`iB2HB2M7 2+ibQM M/ T`Q/m+iBQM
p `BQmb b+ H2b ? b #22M iQTB+ Q7 BMi2`2bi bBM+2 i?2 `2+Q;M
b+ H2 bBx2X G2rBb 6`v _B+? `/bQMr b i?2 7B`bi iQ `2H i2 i?2 bBx
/Bbi`B#miBQMb- M B/2 r?B+? Bb `272``2 8jQ9X a2Kô2QM2 i?2+2 bHv
2tT2`BK2Mib Q7 7HQr #2ir22M +QM+2Mi`B+ +vHBM/2`b #v :X AX
bi`m+im`2b i? ir2`2 +? ` +i2`Bx2/ #v i?2 pBb+Qmb M/ BM2`iB H
H v2`X M BMi2`K2/B i2 H2M;i?b+ H2-FMQrM b i?2 h vHQ` KB+`
2//B2b r?2`2 pBb+Qmb 7Q`+2b /QKBMi3dyXQph2i?BQM2;iB?2H7QB+Qmb(b-
Q7 im`#mH2M+2 `2 /BbT2`b2/ i?`Qm;?Qmi #QmM/ `v H v2`-p `B
i?2 /QKBM Mi b+ H2b Q7 im`#mH2M+2 Q++m``BM;rBi?BM 2 +? `2
#2 b+ H2/ b 7mM+iBQM Q7 r HH /Bbi M+2- M B/2 i? iBb FMQrM
?vTQi?2bdBXbh(?2 bK HH2` b+ H2 2//B2b `2 ;`2 iHv 772+i2/ #v pBb
/QKBM Mi BM `2;BQMb M2 ` i?2 r HH-#mi r HH +QM/BiBQMb- bm+
i?2 iQi H bi`2bb2b #v BM+`2 bBM; i?2 (kpX`hH2 2tH2Mb?Q i?B+2bi?
r HH +QM/BiBQM +QMiBMm2b iQ 772+i i?2 Qmi2` `2;BQMb Bb MQ
72r bim/B2b bm;;2bi i?2 pBb+Qmb 7Q`+2b `2K BM /QKBM Mi #Q
` i?2`i? M #2 bQH2 7mM+iBQM Q7 r HH /Bbi M+2- Bb KQ`2 bQ
BM2`iB H9eQ)+2i?Bb rBHH #2 2tTHQ`2/ BM 7mkXX2?22K BHQBMvQ7i
i?2 2M2`;v rBi?BM #QmM/ `v H v2` Bb +QMi BM2/ rBi?BM i?2 H `;
bB;MB7B+ MiHv BM7Hm2M+2/ #v i?2 BM2`iB H 7Q`+2b bm+? b T`2
`BmK im`#mH2Mi #QmM/ `v H v2`? b T`QMQmM+2/ M/ BKK2/B i2
M/ `2bmHiBM; 2//v bi`m+im`2b- Q7 i?2 BMM2` `2;BQMX S `i Q7 i?
BM; Q7 i?2 /BbT2`bBQM Q7 b+ H2b M/ i?2B` BKT +ib QM i?2 Qp2`
7`QK i?2 /B77B+mHiv BM /B`2+iHv K2 bm`BM; i?2 r?QH2 ` M;2 b+ I
HBKBi iBQMb Q7 2tT2`BK2Mi? `/r `2X a2p2` H BMp2biB; iBQMb-
Q7 i?2 T`2b2Mi /Bbb2`i iBQM rQ`F-? p2 7QmM/ i? i i?2 bT iB H `2I
BK2Mi? `/r `2 + M BM /p2`i2MiHv KBb`2T`2b2Mi i?2 i`m2 ` M;2 Q
M/ i?2 2M2`;v +QMi BMBM; b+ H2b-bBKTHv #2+ mb2 i?2 ? `/r `2
2M2`;v +QMi BM2/ #2HQr i?2 HBjK8dkQ7 BXb `2bQHmiBQM (

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kXRX8 *H bb2b Q7 hm`#mH2Mi 1//B2b

h?2 T`2pBQmb b2+iBQM BMi`Q/m+2/ i?2 B/2 Q7 i?2 ô2M2`;v + b+
#miBQM Q7 2M2`;v i? i /2T2M/b QM i?2 bBx2 Q7 i?2 2//v bi`m+im
K2i?Q/iQ mM/2`bi M/BM; im`#mH2Mi 7HQrb Bb iQ Q#b2`p2 M/ M
bi`m+im`2b iQ i?2 /2p2HQTBM; 7HQr 7B2H/b- M/ ?Qr i?2 p `BQmb
2 +? Qi?2` iQ 2Bi?2` +QMi`B#mi2 iQ Q` #`2 F /QrM i?2 2M2`;v +QM
2` `+?v Q7 2//v +H bbB7B+ iBQMb Bb [m HBi iBp2Hv mb27mH K2
Q7 im`#mH2Mi b+ H2b rBi?BM #QmM/ `v H v2`X h?2 KQbi /BbiBM
i?2 ?Q`b2b?Q2 pQ`i2t- 7B`bi BMi`Q/m+2/ i?2 B/2 Q7 i?2 bBx2 /
b? T2/ pQ`i2t 7BH K2Mi- r?2`2 i?2 H2;b Q7 i?2 7BH K2Mi `2 Q`B2
`2+iBQMX .2T2M/BM; QM i?2 2t +i bBx2- 2HQM; iBQM- M/ b2T `
`+?- i?Bb 72 im`2 b? T2 ? b #22M /m##2/ #v K Mv i2`Kb BM+Hm/
b b?QrM BMkXR9; m? 2 7HQr pBbm HBx iBQMdQ`E#BM2QKBMbQiM (
Qi?2`b- ? p2 b?QrM i?2 /QKBM M+2 Q7 ? B`TBMb BM i?2 r F2 Q` Q
H v2`- i?mb bbQ+B iBM; i?2b2 bi`m+im`2b b i?2 H `;2`- 2M2`;v +
+2Ti Q7 i?2 ? B`TBM pQ`i2t ? b bBM+2 #22M 2p Hm i2/ M/ 2tT2`BK
2t?B#Bi2/ BM im`#mH2Mk3j7HQrbyeyNkX h?2 ? B`TBM T ` /B;K 2Mi
bi`2 KrBb2fr HH@MQ`K H +`Qbb@b2+iBQM Q7 i?2 7HQr- r?2`2 i
+QKT +i pQ`iB+ H `2;BQM Ui?2 ?2 /V- `2;BQM Q7 2D2+iBQM UBX
T2`T2M/B+mH ` iQ i?2 TH M2 Q7 i?2 pQ`i2t ?2 /- M/ HQr@bT22/
/B`2+iBQM(X GQr@KQK2MimK- ó[m bBbi`2 KrBb2ô pQ`iB+2b 2tBI
7QHHQr2/ #v KBtim`2 Q7 #Qi? 72 im`2b BM i?2 HQ; `Bi?KB+ H v2
TH v ?m;2 `QH2 BM i?2 ?B2` `+?v Q7 ? B`TBM T +F2ib TT2 `BM
H v2`c i?mb- i?2v H `;2Hv 7QHHQr i?2 T ` K2i2`b bbQ+B i2/ rBi?
6Q` BMbi M+2- rBi?BM i?2 ? B`TBM T ` /B;K- i?2`2 2tBbib i?2 ó?Q
H `;2`- rB/2` ?2 /- i? i ;2ib 2HQM; i2/ #v b?2 ` bi`2bb2b BMiQ bi`2
KQK2MimK BMiQ b? T2 `2b2K#HBM; i?2 ? B`TBM pQ`i2t +HQb2`
r?B+? Bb b22M BkYX6B2im`2bj(HHm/2/ 2 `HB2`- i?2 B/2MiB7B+ iBQM
#Bi `#Bi` `v M/ mT 7Q` Dm/;2K2Mi- M/ i?2 2t +i /2p2HQTK2Mi Q7 i
iQ #2 mMBp2`b HHv ;`22/ mTQMX h?mb- [m MiBi iBp2 B/2MiB7B+
imK i` MbTQ`i M/ hE1 `2 M2+2bb `v- bm+? b KQ/2H Q`/2`@`2/m
2//B2b `2 i?2 T`BM+BT H K2+? MBbKb 7Q` KQK2MimK i` MbTQ`i
M/ T`Q/m+2 M/ /BbbBT i2 im`#mH2Mi FBM2iB+ 2M2`;v UhE1VX

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6B;m`2 kX9, 6`QK H27i iQ `B;?i- i` MbBiBQM Q7 ?Q`b2b?Q2 pQ`
>2 / M/ " M/vQT /?) v v (

h?2 K2+? MBbKb ;2M2` iBM; i?2b2 bi`m+im`2b ? p2 #22M rB/2Hv /
2i HX(bm;;2bib i? i BMBiB H ? B`TBM pQ`iB+2b `2 7Q`K2/#v b2H
r?B+? i?2M +QMM2+i M/ ;2M2` i2 i?2 b2+QM/ `v M/ i2`i3 `v bi`m
`272`b iQ i?Bb #2? pBQ` b óQ77bT`BM;ô pQ`iB+2b M/ i? i HQr@b
iQ i?2 /2p2HQTK2Mi Q7 i?2 ? B`TBM pQ`iB+2bX >Qr2p2`- i?2`2 `2
bQm`+2 Q7 pQ`i2t ;2M2` iBQM X8 mbK2 Ma xQ 5 BQr2i? Hi i?2 BMbi #BH
bm#b2[m2Mi #`2 F@mT Q7 HQr@bT22/ bi`2 Fb `2bmHi BM i?2 ;2M2
i?2 r HHX _2; `/H2bb Q7 i?2 ;`22K2Mi #2ir22M i?2b2 bim/B2b- i?2
;2M2` iBQM Q7 bi`2 KrBb2 pQ`iB+2b TT2 `b iQ #2 BM/2T2M/2Mi
?Qr2p2` i?2 b+ H2 M/ /Bbi`B#miBQM Q7 i?2 pQ`iB+2b `2X 92)X2T2M/2
_2+2Mi bim/B2b Q7 im`#mH2M+2 bi`m+im`2b ? p2 ii2KTi2/ iQ B/
QM 2M2`;v bB;M im`2b mbBM; bT iB H M/ i2KTQ` H +Q``2H iBQM
+QKTQM2Mi M HvbBb bm+? b T`QT2` Q`i?Q;QM H /2+QKTQbBiBQ
M/ HBM2 ` biQ+? biB+ 2biBK iBQM UGa1V ? p2 #22M T2`7Q`K2/
b?QrBM; 2M2`;2iB+ bi`m+im`2b +Q``2bTQ Me/BR3;)X Q H `;2 b+ H2 KQ

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6B;m`2 kX8, hE1 #m/;2iBM 7H iTH i2 bKQQi? r HH im`#mH2Mi #Q
aT H 3iR(

kXRXe hm`#mH2M+2 1[m iBQMb 7Q` ai`2bb M/ h` MbT

b b?QrM BM kX2+RiXBjQBMb +QMp2MiBQM iQ i`2 i i?2 #QmM/ `v H v2`
_2vMQH/b /2+QKTQb2/ 7Q`K /m2 iQ i?2 mMbi2 /v M im`2 Q7 im`#mH
rBi? i?2 K2 M M/ 7Hm+im iBM; +QKTQM2Mib Q7 i?2 p2HQ+Biv M/
`2bmHi2/ BM i?2 _2vMQH/b@ p2` ;2/ L pB2`@aiQF2b U_kX3dV 2[m
M/kXX3 h?2 iBK2 2pQHmiBQM Q7 i?2 _2vMQH/b bi`2bb i` MbTQ`i`2
iQ p `BQmb 7Q`+2b `2H i2/ iQ im`#mH2M+2 T`Q/m+iBQM- /BbbBT i
2[m iBQM Bb /2`Bp2/ 7`QK i?2 L pB2`@aiQF2b #v bm#i` +iBM; i?2
i?2 +Q``2H iBQM rBi? p2HQ+BivX h?2 2t +i _2vMQH/b bi`2bb i` Mb

$$\underbrace{\frac{D\overline{u_i u_j}}{Dt}}_{\text{J2 M *QMp2+iBQM}} + \underbrace{\frac{\partial}{\partial x_k}\overline{u_i u_j u_k}}_{\text{hm #mH2Mi *QMp2+iBQM}} = \underbrace{\overline{u_i u_k}\frac{\partial U_j}{\partial x_k} + \overline{u_j u_k}\frac{\partial U_i}{\partial x_k}}_{\text{S`Q/m+iBQM}} + \underbrace{r^2\overline{u_i u_j}}_{\text{oBb+Qmb .B77mbBQM}}$$

$$\underbrace{\overline{-u_i\frac{\partial p}{\partial x_j} + u_j\frac{\partial p}{\partial x_i}}}_{\text{S`2bbm`2 h` MbTQ`BbbBT iBQM}} \quad 2\,\overline{\frac{\partial \mu}{\partial x_k}\frac{\partial u_i}{\partial x_k}} \quad \text{UkXR3V}$$

h?2 # H M+2 #2ir22M i?2 T`Q/m+iBQM M/ /BbbBT iBQM i2`Kb `
M 2[mBHB#`BmK M/ MQM@2[mBHB#`BmK 7HQr- r?2`2 BM i?2 7
2[mBp H2Mi- M/ i?2 M2i` i2 Q7 2M2`;v ; BM Q` HQbb Bb x2`QX

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h?2 hE1 # H M+2 7Q` im`#mH2Mi bKQQi?@r HH- M/ Qp2` HH- i?2 T`Q/m+iBQM K ;MBim/2b `2 /2T2M/2Mi QM

kXRXd aFBM 6`B+iBQM QM aKQQi? q HHb

h?2 bFBM 7`B+iBQM Bb `;m #Hv QM2 Q7 i?2 KQbi BKTQ`i Mi T` Q7 i?2 #QmM/ `v H v2`X P#pBQmbHv- /B`2+iK2 bm`2K2Mi Q7 bFBM Hr vb 72 bB#H2-2bT2+B HHv BM i?2 + b2 Q7 7HQrb rBi? `Qm;?M2bb im#2b- M/ Qi?2` BMbi`mK2Mib ? p2 #2 kM9 mb2/X BM 72pQr- im2b K2i?Q/b `2 2Bi?2` BMi`mbBp2- Q` MQi 72 bB#H2 rBi? `Qm;?M2bb Q7 +QKTmiBM; bFBM 7`B+iBQM mbBM; 2tT2`BK2Mi H p2HQ+Biv K TT`Q +?2b `2 2tTHQ`2/ BM HBi2` im`2 iQ /2i2`KBM2 i?2 bFBM 7`B (RkRR9Rdk9jy-ede3NJX PM2 Bb mbBM; /B`2+iK MBTmH iBQM Q7 i?2 +QMb2`p iBQM Q7 KQK2MiXvX U2Q`imB@MH2Mi 7HQrb BM x2`Q Q` ;` /B2Mi- i?2 +QMp2MiBQM H *H mb2`@+? `iK2i?Q/ Bb Q7i2M BK /p Mi ;2 Q7 i?2 H r Q7 i?2 r HH iQ `2;`2bbBp2Hv 7BiK2 M p2HQ+B `2;BQMX P7i2M 2KTB`B+ H 7Bib Q7 i?2 H r Q7 i?2 r HH bm+? b i?2 T`Q7BH2 `2 mb2/ BM i?2 *H kmb2 3@X+ ?Qir2 i?2 Bb K2i?Q/ 7 BHb B T`2bbm`2 ;` /B2Mi+QM/BiBQMb- Q7i2M mM/2`@T`2/B+iBM; `2bmH 7Q` MQM@2[mBHB#kj9XKb+QBMH/Bi#B2Q/MBb+(mbb2/ BM i?2 H i2` b2+iB K2i?Q/b Q7 bFBM 7`B+iBQM /2`Bp iBQM mbBM; `2;`2bbBQM 7Bib M/fQ` MQM@2[mBHB#`BmkjX7bmQ;r2+bQMMBiBQMvbX+(H K2i?Q/ K FBM _2vMQH/b b?2 `bi`2bb T`Q7BH2b M/ bi`2 KrBb2;` /B2Mib i? i TT Q7 i?2 _2vMQH/b@ p2` ;2/ bi`2 KrBb2 KQK2MimK2[m iBQMX h?BM `2bmHiBM ++m` i2 bFBM 7`B+iBQM 2biBK i2b TTHB2/iQ 7HQrb C T`2bbm`2 ;`N/BX2Mi (

kXk _Qm;? q HH hm`#mH2Mi "QmM/ `v G v2`b

Hi?Qm;? i?2 MmK#2` Q7 bim/B2b `2; `/BM; bKQQi? r HH im`#mH2M i? M i?Qb2 BM `Qm;? r HH im`#mH2M+2-`2 H 7HQrb Q7 T` +iB+ H /`QM KB+ HHv bKQQi? /m2 iQ 2p2M K +`Qb+QTB+ bm`7 +2 /27B+B `Qm;?M2bb bm+? b B+BM; QM M B`+` 7irBM; Q` #BQHQ;B+ H /2 _Qm;?M2bb + M +QMi`B#mi2 iQ BM+`2 b2//` ; M/mMr Mi2/MQBb2 im iBQMbX h?mb- mM/2`bi M/BM; i?2 im`#mH2Mi 7HQr #2? pBQ` T`QpB/2 BMbB;?i QM #2ii2`BM; p2?B+H2 /2bB;M M/QT2` iBQM m iBQM T`QpB/2b BMbB;?i QM i?2 +m``2Mi FMQrH2/;2 Q7 i?2 #2? pB BM `Qm;? r HH +QM/BiBQMbX h?2 2 `HB2bi T22`@`2pB2r2/ `2b2 H2M+2 Bb #v CQ? MM LBFm` /b2 Me ÿXBA MBi T2 b2 T2 T2 K2 M2 M (b- LBF 2tTHQ`2/ i?2 2772+iQ7 /B772`2Mi b M/@;` BM bm`7 +2b QM i?2 # H v2`X >2 2bi #HBb?2/ [m MiBiv `272``2/ iQ b i?2 ó2772+iBp2 rQmH/ 2bb2MiB HHv b2`p2 b +QKT `BbQM 7 +iQ` 7Q` Qi?2` `Qm ` K2i2`Bb K2 bm`2 Q7 i?2 `Qm;?M2bb /Bb`mTiBQM iQ i?2 K2 M p2 +QKT `BbQM Q7 bKQQi? M/ `Qm;? r HH K2 M p2HQ+Biv T`Q7BH2 Bb ++QmMi2/ 7Q` BM i?2 KQ/B7B2/ H r Q7 i?2 r HH 7Q` `Qm;?M2b

$$U^+ = \frac{1}{\kappa}\ln(y^+) + C \qquad U^+ \qquad \text{UkXRNV}$$

q?2`2 $U^+$ Bb /QrMr `/b?B7iQ7 i?2 K2 M p2HQ+Biv T`Q7BH2 MQ`K H +Q``2bTQM/BM; iQ i?2 BM+`2 b2 BM bFBM 7`B+iBQM Ub22 i?2 b?E T`Q7BH2 rBi? `2bT2+i iQ i?2 bKQQi? kXVRXH hT?Qb7B2H2K-BM2 7B2`;m2/2iQ i?2 `Qm;?M2bb 7mM+iBQM r b BMi`Q/m+2/ ky BM M2 T2KJ(Q BMbHv#Q* H r`Bii2M BM i2`Kb Q7 i?2 2772+iBp2 b M/;` BM `Qm;?M2bb b,

$$U^+ = \frac{1}{\kappa}\ln(k_s^+) \quad 3:5 \qquad \text{UkXkyV}$$

k_s^+ Bb HbQ mb2/iQ /27BM2 i?2 `Qm;?M2bb _2vMQH/b MmK#2` r?B+? i?2 `Qm;?M2bb /Bb`mTib i?2 7HQr M/Bb /27BM2/ b 7QHHQrb,

$$k_s^+ = k_s u / \qquad \text{UkXkRV}$$

`2 Qp2` Bib TH M7Q`/k2 7B2M-2/ BMkX[kk2iBQ MQ7 i?2 `Qm;?M2bb 2H2
(N-RyjN-9d)X

$$= \frac{A_f}{A_w} \qquad \text{UkXkkV}$$

Hi?Qm;? + ``vBM; ?B;?2` K ;MBim/2b Q7 2M2`;v- i?2 bi`m+im`2b
Q7 i?2 `Qm;? r HH TT2 `b iQ #2 +QMbBbi2Mi rBi? i?2 r HH@bBKB
H `;2@b+ H2 bi`m+im`2b Q7 i?2 `Qm;? r HH #QmM/ `v H v2` 7QHHQ
bKQQi? r HH #QmMjR-9d8H8vd23`(38X hvTB+ HHv- i?2b2 `2 H `;2@b+
pQ`iB+2bX h?mb- BM i?2 + b2 Q7 7HQrb rBi? T`2bbm`2 ;` /B2Mi- i?
i?2 bi`m+im`2b TT2 `BM; BM i?2 Qmi2` `2;BQMbX

kXj hm`#mH2Mi "QmM/ `v G v2`b BM S`2bbm`

h?Bb b2+iBQM rBHH /2b+`B#2 i?2 #2? pBQ`b /B772`2MiB iBM; 7H
T`2bbm`2 ;` /B2MiX h?2 /BbiBM+iBQM #2ir22M M 2[mBHB#`BmK
iBQMb Bb HbQ BMi`Q/m+2/ M/ ?Qri?2b2 i2`Kb `2H i2 iQ 7HQrb rB
iBQM H T`2bbm`2 ;` /B2MiX *m``2Mi HBi2` im`2 Bb #B b2/ iQr `/b
2pQHpBM; rBi? i?2 b K2 bB;M Q` x2`Q T`2bbm`2 ;` /B2Mi +QM/BiB
#22M i?Q`Qm;?Hv `2pB2r2/X Hi?Qm;?- `2H2p Mi 7HQrb Qp2` p2?
2M+2 i?2 2772+i Q7 +QKTQmM/BM; Q` KBtBM; T`2bbm`2 ;` /B2Mi +
7mM/ K2Mi H mM/2`bi M/BM; Q7 7HQrb rBi? p `vBM; T`2bbm`2 ;` /

h?2 +QM+2Ti Q7 [m MiB7vBM; i?2 T`2bbm`2 ;` /B2Mi?-QM7B#2bi` +2
2bi #HBb?2/ i?2 Tr`?K-2?2Bb K2 bm`2 Q7 i?2 K ;MBim/2 Q7 i?2 T`2
TTHB2/ QM i?2 7HQrX

$$= -\frac{dp}{dx} \qquad \text{UkXkjV}$$

Ai Bb i?2 ` iBQ Q7 i?2 T`2bbm`2 M/ i?2 b?2 ` 7Q`+2 +iBM; QM i?2
< 0- 7HQrb `2 BM 7 pQ` #H2 T`2bbm`2=;0-/B72HMirb6S2VBXM6xQ2`Q
T`2bbm`2 ;` /B2Mi UwS>VX- 7MH/7rQ` `2 BM /p2`b2 T`2bbm`2 ;` /B2
*H mb2` 2bi #HBb?2/ i? i 7Q` +QMbi Mi T ` K2i2`- i?2 Qmi2`@`2

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+ M #2 b2H7@T`2b2`pBM;- M/ i?mb +QMb kyX2 h2?/2iQm#2 bBMQ2M[Q7 Hb2
bBKBH `Biv +`Qbb i?2 2MiB`2 #QmM/ `v H v2` +QK2b BMiQ [m2bi
+`2 iBM; +`Bi2`BQM i? i i?2 bBKBH `Biv H rb `2 T`2b2`p2/ 7Q`
rQmH/ i?mb 2M+QKT bb H `;2 `2 Q7 kyiX2 #QmM/ bBvQHv2Q7(b2H7@
M/ 2[mBHB#`BmK +QM/BiBQMb BM im`#mH2Mi kXX9 `2 2tTHQ`2/ 7m`

kXjXR w2`Q@S`2bbm`2 :` /B2Mi UwS:V

h?2 bBKTH2bi Q7 7HQrb Bb i?2 x2`Q@T`2bbm`2 ;` /B2Mi im`#mH
QMvKQmb rBi? 7HQr Qp2` 7H i TH i2- M/ Bb +QMbB/2`2/ + MQ
2[mBHB#`BmK +QM/BiBQMb `2 7`2[m2MiHv +QKT `2/X PM2 Q7 i
i?2 7H i TH i2 #QmM/ `v H v2` r b 2tTHQ`2/ #v S?BHHBT EH2# MQ
M2KQK2i`viQ K2 bm`2 i?2 im`#mH2Mi 2M2`;v- b?2 `bi`2bb- M//B
;m2/ ; BMbii?2 +QM+2TiQ7 7HQ E+H2 #BMQi7 Qbv (MBbQi`QTv 7 +iQ`b
iQ #2 mb2/ BM KQ/2HHBM; Mj/8 2X Th2B7+H i 2 7H7iQ2`#bQ(mM/ `v H v2` 7C
#2? pBQ`b H`2 /vQmiHkXX2 bBMB#2+MB;QMmM/ `v H v2`im`#mH2M+
B+ H 7Q`KX h?2 BMM2` `2;BQM +QMi BMb i?2 KQbi T`Q/m+iBp2 M
BMi2` +iBQM Q7 i?2 bK HH2bib+ H2b Q7 im`#mH2M+2 M/ i?2 + b+
`2;BQM pQkj)X+QQ`F #v .2 :` 77 Mkj)1TiQMB(/2/ i?2 `2b2 `+? +QKKm
rBi? r2HH@/Q+mK2Mi2/ / i # b2 Q7 M 2[mBHB#`BmK- ?B;? _2vM
7H i@TH i2 im`#mH2Mi #QmM/ `v H v2` / i mbBM;H b2`@.QTTH2`
b?Qr2/ _2vMQH/b MmK#2` BM/2T2M/2M+v Q7 BMM2` b+ HBM; 7Q`
p2HQ+Biv- M/ hE1- b r2HH b bBM;H2/ Qmi i?2 +H bbB+ H HQ; H
b H T`Q7BH2 Q7 i?2 K2 M p2HQ+Biv BM i?2 BMM2` `2;BQM rBi? b
MmK#2` ` M;2- bm;;2biBM; i? i 7Q` i?2 wS: #QmM/ `v H v2`- i?2 H
M ++m` i2- BM/2T2M/2Mi- /B`2+iK2 bm`2jkX2Mi Q7 i?2 b?2 ` bi`2b

kXjXk 6 pQ` #H2 S`2bbm`2 :` /B2Mi U6S:V

7 pQ` #H2 T`2bbm`2 ;` /B2Mi Bb +? ` +i2`Bx2/ #v i?2 /2+`2 bBM;
BM+`2 bBM; p2HQ+Biv 7B2H/ ;` /B2Mib- `2bmHiBM; BM M BM+`2 b
7 pQ` #H2 T`2bbm`2 ;` /B2Mib + M ++2H2` i2 i?2 7HQr iQ M 2ti2M
(9)X :2M2` HHv- QM BMM2`@b+ HBM;b- i?2 T`2bbm`2 ;` /B2Mi `2

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2t?B#Bi p `B iBQMb BM i?2 Qmi2` `2;BQMb Q7 i?2 im`#mH2Mi #Q
;` /B2Mi 7HQr- 72 im`2 Q7 bB;MB7B+ MiBMi2`2biBbi?22tBbi2M+
?B;?@KQK2MimK 7HmB/ iQ i?2 r HH M/ 2D2+i HQr KQK2MimK 7Hr
M/ > M` kN (HbQ 7QmM/ i? ii?2 /B772`2M+2 Q7 M2i im`#mH2M+2
r HH `2;BQM #2ir22M i?2 7 pQ` #H2 M/ x2`Q T`2bbm`2 ;` /B2Mi +Q
BKTHB2b i? ii?2 /B772`2M+2 #2ir22M i?2 /BbbBT iBQM M/ i?2 T`Q
`2K BMb +QMbi Mi- 2t+2Ti 7Q` ii?2 T2 F- r?2`2 i?2 BM2`iB H 7Q
H2p2H Q7 T`Q/m+iBQMX Pp2` HH- i?2 6S: + b2 ? b HQr2` im`#mH
Q7 i?2 #QmM/ `v H v2` i? M 7HQrb BM 2Bi?2` x2`Q Q` mMB@/B`2+
/m2 iQ i?2 ?B;?2` BM2 kNX B H 7Q`+2b (

AMp2biB; iBQMb #v b2p2` H `2b2 `+?2`b ? p2 b?QrM i? irBi?i?2 _2
BM KQbi Q7 i?2 Qmi2` H v2`- M/ +QHH Tb2 BMi?2 Qmi2` `2;BQMb r
#mi i?2 K BM /Bb+`2T M+v +Q kyN XM L2 ` ii?2 r HHHH (i?2`2 Bb M BM+
i?2 _2vMQH/b b?2 ` bi`2bb BM i?2 /QrMbi`2 K T `ib Q7 i?2 7 pQ` #
BM+`2 b2 Q7 im`#mH2M+2 M2 ` i?2 r HH BM/B+ i2b bHQr /B77mb
i?2 Qmi2` `2 kyN BXQM ?2 2tBbi2M+2 Q7 HQr@bT22/ bi`2 Fb +QMi`B#m
/vM KB+b BM i?2 r HH@`2;BQM ? p2 #22M Q#b2`p2/ kRk3 pkX B>mi+?pB2M bH 6
M/ J `mbBR+Q(#b2`p2/ i? i i?2 6S: ? b Km+? HQM;2` óK2 M/2`BM;
HQ; `Bi?KB+ `2;BQM r?2M +QKT `2/ rBi? i?2 wS: 7HQrb- +QMbBb
p2HQ+Biv 7B2H/b b?QrM BM i?2 bi`m+im`2b #vMCQbmBv Q7 Qxm(M/ `v H v
6S:- CQb?B kyN 7HQXm(M/ i? i Hi?Qm;? i?2 HQ+ HHv@b+ H2/ _2vMQH
_2vMQH/b MmK#2` BM/2T2M/2Mi M/ r2`2 r2 F2`i? M i?2 wS: bi`2bb
b?Qr2/ ?B;?2` K ;MBim/2b Q7 im`#mH2M+2 M2 ` i?2 r HH M/ HQr2
`2;BQMX h?Bb r b /m2 iQ ?B;?2` bi`2 KrBb2 M/ r HH@MQ`K H p2H
BM r2 F2`im`#mH2M+2 bi`m+im`2b r?2M +QKT `2/ rBi? i?2 wS: #Qr
BM 7 pQ` #H2 T`2bbm`2 ;` /B2Mi- i?2 bi`m+im`2b `2 KQ`2 BM+H
i?2 r HH- /m2 iQ i?2 /BbiQ`iBQM bbQ+B i2/ rBi? kyN 2`.iB2HiQ7Q`+2
i?2 ?B;?2` ++2H2` iBQM- i?2 6S: Bb HbQ bbQ+B i2/ rBi? H `;2@
?Q`BxQMi H i` BMb Q7 pQ`iB+2bX Hi?Qm;? K Mv bim/B2b 2tBbi Q
;` /B2Mi- Km+? Q7 i?Bb rQ`F Bb BM 2[mBHB#`BmK Q kRk3 Qk Mj 3?X2 p2`;2 `
1tBbiBM; HBi2` im`2 ? p2 HbQ /Bb+mbb2/ i?2 i` MbBiBQM 7`QK
M BMbm77B+B2Mi MmK#2` Q7 BM@/2Ti? bim/B2b i? i ? p2 2tTHQ
i?2v H2 p2 iQ [m2biBQM i?2 +QM+2Ti Q7 r HH@bBKBH `Biv M/ +
T ` K2i2`b `2H2p Mi iQ MQM@2[mXBHB#`BmK 7HQrb (

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kXjXj /p2`b2 S`2bbm`2 :` /B2Mi U S:V

b /p2`b2 T`2bbm`2 ;` /B2Mib `2 ivTB+ HHv +QMbB/2`2/ b mM/2b
T` +iB+ H BMi2`2bi- b i?2v `2 bbQ+B i2/ rBi? M BM+`2 b2 BM 7
7HQr- M/ ?B;?2`/` ;X aQK2 +QMi` `v #2? pBQ`b `2 b22M BM i?2
i?2 6S: +QM/BiBQMX 6Q` BMbi M+2- K2 M 7HQr Bb /2+2H2` i2/ M
i?2 im`#mH2Mi b?2 ` bi`2bbX AM M S:- i?2 H `;2@b+ H2 im`#mH
BM i?2 Qmi2` `2;BQMb Q7 i?2 7HQr U b QTTQb2/ iQ i?2 6S: r?2`2
bK HH@b+ H2 M/ +QM7BM2/ +HQb2` iQ i?2 r HH /m2 iQ ?B;?2` B
BM M S:- i?2 b+ H2b M/ bi`m+im`2b `2 KQ`2 2M2`;Bx2/ i?`Qm;
(j3-j3)X h?2 S:T`Q7BH2 Bb /BbiBM;mBb?2/ #v i?2 T`2b2M+2 Q7 b2
BM i?2 HQ; `Bi?KB+ `2;BQMX bB/2 7`QK KBH/ T`2bbm`2 ;` /B2M
i? ii?2 S:2772+iQM K2 M p2HQ+Biv T`QT ; i2b /QrM iQ i?2 HQ;
TQBMi +H bbB+ H `2;`2bbBQM 7Bib mbBM;*H mb2` K2i?Q/ `2 MQ
bFBM 7`B+ BM Q M?2 /p2`b2 T`2bbm`2 ;` /B2Mi K2 M p2HQ+Biv + M
i?`22 `2;BQMb /m2 iQ i?2Q`2iB+ H M/ 2Ud2 BK2 M+HBQmMbB(bi2
rBi? hE1 T2 Fb Q++m``BM; rBi? /Bbik MX+2 M`QbKim/2v rQH+QMbi Mi w
S:- M/ 6S:- > `mM j3) bHXQ(2/ b?Qrb i? i7Q` HH T`2bbm`2 ;` /B2
i?2 T`Q/m+iBQM H2p2Hb ` TB/Hv /2+`2 b2 BM i?2 BMM2` `2;BQMX
i?2 Qmi2` `2;BQM ? p2 HBiiH2 BKT +i iQ i?2 M2 e3 HXa2v2MQH/b
bim/B2b ? p2 b?QrM i? i i?2 im`#mH2Mi FBM2iB+ 2M2`;v BM i?2 Q
H v2` Bb BM+`2 b2 BM NX S?2 b BM/B+ i2b i? ii?2 H `;2@b+ H2 KQ
+? M;2/ BM #2? pBQ` Q` +? ` +i2` #mi? p2 bBKTHv BM+`2 b2/ BM
S: (j3)X

kX9 LQM@1[mBHB#`BmK hm`#mH2Mi "QmM/

hQ mM/2`bi M/ i?2 +QM+2Ti M/ #2? pBQ`b Q7 7HQrb BM MQM@2
2bi #HBb?r? iBb +QMbB/2`2/ iQ #2 7HQr BM 2[mBHB#`BmKX 1[m
bi i2b BM r?B+? 7HQr T`QT2`iB2b `2 QMHv /2T2M/2Mi QM HQ+ H 7
K2 M p2HQ+Biv M/ im`#mH2M+2[m MiBiB2b rBi? bi`2 KrBb2 TQbB
QM i?2 # bBb Q7 b2H7@bBKBH `BivX AM M 2[mBHB#`BmK 7HQr- i
iQ M 2[m iBQM r?2`2 i?2 T`Q/m+iBQM M/ /BbbBT iBQM 2M2`;v i2

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2[m iBQM X AM i?2 MQM@2[mBHB#`BmK 7HQr- i?2 BK# H M+2 Q7 i
i2`Kb ; BMbi i?2 /BbbBT iBp2 7Q`+2b `2bmHib BM MBM+`2 b2 Q7
h?2 HQ+ H 7HQr +QM/BiBQMb `2 MQr HbQ /2T2M/2Mi QM i?2 ?Bb
? b 2tT2`B2M+2/- Q7 r?B+? i?2 2772+ib T2`bBbi M/ 772+i i?2 H
bim/B2b ? p2 b?QrM i? i ;Bp2M bm77B+B2Mi bi`2 KrBb2 /Bbi M+2
+QM/BiBQMb- MQM@2[mBHB#`BmK 7HQr rBHH 2p2Mi HHv `2H
e-9j-99) X Hi?Qm;? i?2 `2H t iBQM iBK2 Bb bQK2r? i /2T2M/2Mi QM i
i?2 T2`im`# iBQM rBi?BM i?2 /2p2HQTBM; 7HQr- bim/B2b Q7 `Qm;?
2bi #HBb?2/ i? i /Bbi M+2 iQ 2(j-8-9-d) X A M Q7 QM@2[mBHB#`BmK
;` /B2Mi- "Q#F2) 7 QHmX M(/ i?Bb bi`2 KrBb2 H2M;i? `2[mB`2K2Mi 7Q`
#2 5 X

h?2 *H mb2` T`2bbm`2 ;` /B2Mi 7HQr i?2 b Q7i2M #22M mb2/ iQ #2
bm`2 Q7 i?2 MQM@2[mBHB#`BmK T`2bbm`2 ;` /B2Mi 7HQr #2? pBQ
bm;;2bi2/ i? iB7 i?2 T`2bbm`2 ;` /B2Mi MQM@/BK2MbBQM HBx2/ Q
i?2M i?2 7HQr Bb BM 2[mBHB#`BmK T`2bbm`2 ;` /B2Mi MQM@/BK2MbBQM
7QHHQr2/ *H mb2`öbrQ`F M/ //2/ M //BiBQM H +`Bi2`BQM iQ /2
? pBM; TQr2` H r`2H iBQM 7Q`i?2 2/;2 p2HQ+Biv /2p2HQTBM; QM
BM bim/B2b Q7 #B@/B`2+iBQM H T`2bbm`2 ;` /B2Mi 7HQr #2? pBQ`
? p2 b?QrM i?2 2tBbi2M+2 Q7 ?BbiQ`v 2772+ib /QrMbi`2 K Q7 T`2
Bb 2pB/2M+2/ #v /2H v2/ `2bTQMb2 iQ i?2 T`2bbm`2 ;` /B2Mi brBi
H v2` +m`p2b b?Qr +`QbbQp2`i? iQ++m`b /QrMbi`2 K Q7 i?2 S: b
H2M+2 bi`2bb2b rBi?`2bTQMb2 iQ T2 Fb Q7 _2vMQH/b b?2 `bi`2b
BM i?2 #QmM/ `v H v2`X 3 BM2m2Qb M2Bix2H X?(2 2tBbi2M+2 Q7 ?BbiQ`
MQi2/ i? ii?2 2772+ib 7`QK S: Kmbi #2 /B772`2MiB i2/ 7`QK i?2 ó
rBM/ imMM2H 7HQr M/ +H BK i? ii?Bb 7BM/BM; Bb QMHv `2H2p M
i?2 bim/v Q7 MQM@+QMbi Mi R3 :b ?'Q#2F72 i2ii i?X #QmM/ `v H v2`
MQi M2+2bb `BHv /2p2HQT# b2/ QM HQ+ H T`2bbm`2 ;` /B2Mi +Q
Q7 i?2 bi`2 KrBb2 T`2bbm`2 ;` /B2Mi /2p2HQTK2MiX 1bb2MiB HH
H v2` b22Kbi Q / TiiQi?2 HQ+ H T`2bbm`2 ;` /B2Mi +QM/BiBQMb-
+QMiBMm2iQ b?Qr #2? pBQ` BM7Hm2M+2/ #v i?2 mTbi`2 K bi iBQ

6Q` i?2 KQbi T `i- bim/B2b i? i? p2 2tTHQ`2/ i?2 2772+ib Q7 T`2bb
/2MiHv- (ekNj8-9) Q` Q7 wS: M/ 6S: 7HQrb i` MbBi B8Q M;?2Q2S: (
bim/B2b- HQM; rBi?i?2 +m``2MirQ`F-? p2 +QMbBbi2MiHv b22M

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BM i?2 Qmi2` `2;BQM Q7 i?2 #QmM/ `v H v2` r?2M i?2 7HQr i` MbBi
rQ`F ? b T`QpB/2/ M BM@/2Ti? 2tTHQ` iBQM Q7 i?2 i` MbBiBQM 7
2MiX b K2MiBQM2k XBkM? 2K+2 BQmM2 Q7 MQM@2[mBHB#`BmK +QM/Bi
i?2 b iBb7 +iBQM Q7 r HH@bBKBH `Biv bm+? i? i i?2 Qmi2` `2;BQ
iQ i?2 2[mBp H2Mi +QM/BiBQMb BM bKQQi? r HH 7HQrX MmK#
`Qm;?M2bb 2772+ib `2K BM BM/2T2M/2Mi Q7 T`2bbm`2 ;` /B2Mi-
r HH@bBKBH `BivX "mi 2MQm;? bim/B2b- bm+? dbbim?;i2Hv im?#i2`iB
i?2 +m``2Mi 2p Hm iBQM Q7 r HH bBKBH `Biv iQ /22K 7HQr BM 2
//BiBQM T ` K2i2`b `2H iBM; iQ i?2 T`2bbm`2 ;` /B2Mi +QM/BiBQ

kX8 amKK `v M/ : Tb BM GBi2` im`2

h?2 bmKK `v Q7 HBi2` im`2 i?mb 7 ` ? b b?QrM i? i i?2`2 `2 K Mv
i?2 #2? pBQ` Q7 i?2 im`#mH2Mi #QmM/ `v H v2`- M K2Hv r HH +Q
T`2bbm`2 ;` /B2MiX h?2b2 p `B #H2b ? p2 #22M BM/2T2M/2MiHv
#mi 7HQrb Q7 T` +iB+ H BMi2`2bi `2 KQ`2 +QKTH2t M/ Q7i2M +QM
2M+2b Q7 p `vBM; T`2bbm`2 ;` /B2Mi ?BbiQ`B2b M/ r HH +QM/BiB
#Qi? HQ+ H M/ mTbi`2 K 7HQr /2p2HQTK2Mi- i?2 7HQr Bb +QMbB
h?2`2 Bb H +F Q7 bm77B+B2Mi mM/2`bi M/BM; QM i?2 +QKTQmM
M/ MQM@2[mBHB#`BmK T`2bbm`2 ;` /B2Mi QM i?2 p2HQ+Biv 7B2

h?2 +m``2Mi `` v Q7 HBi2` im`2 ? b T`QpB/2/ bi`QM; 7QmM/ iB
BM/2T2M/2Mi p `B #H2bc BiBb +QKKQMHv ++2Ti2/i? i- +QKT `2/
;` /B2Mi 7HQr UBX2X 7HQr +`Qbb bKQQi? 7H iTH i2V- i?2 2772+
iQ BM+`2 b2 i?2 im`#mH2M+2 BMi2MbBiv M/ Qmi2` H v2` b+ HBM;
Q7 7 pQ` #H2 T`2bbm`2 ;` /B2Mi Bb iQ ++2H2` i2 i?2 7HQr- BM
`2/m+2 i?2 im`#mH2M+2 BMi2MbBiB2b bm+? i? i i?2 7HQr TT`Q +

AiBb HbQ +QKKQMHv Q#b2`p2/ i? i i?2 2772+i Q7 `Qm;?M2bb Bb i
#QmM/ `v H v2` M/ Bib +? ` +i2`BbiB+ T`QT2`iB2b +QKT `2/ iQ i
`2 /BbTmi2/ QTBMBQMb `2; `/BM; i?2 +QKT ` iBp2 /B772`2M+2b #
K2 Mb Q7 i?2 Q7 i?2 r HH@bBKBH `Biv ?vTQi?2bBb- r?B+? Bb i?2
`Qm;?M2bb 2772+ib `2 HBKBi2/ rBi?BM i?2 BKK2/B i2 pB+BMBiv
i? i QmibB/2 Q7 i?Bb `2;BQM- i?2 #QmM/ `v H v2` #2? p2b b M 2

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i?2 b K2 Qmi2` H v2` T`QT2`iB2bX Jm+? Q7 i?Bb /2# i2 +QK2b 7`Q
B7vBM; 7HQr `2;BK2b M/ `Qm;?M2bb T`QT2`iB2b iQ r?B+? i?2 r H
TTHB2/X aQK2 bim/B2b bm;;2bi #QmM/b # b2/ QM i?2 T?vbB+ H ;2
`Qm;?M2bb 2H2K2MirBi? `2bT2+iiQ i?2 Qp2` HH #QmM/ `v H v2` i
bm;;2bi b+ H2 ` iBQb # b2/ QM i?2 +? ` +i2`Bx iBQM Q7 i?2 7HQr
T ` K2i2` 7Q` `Qm;? r HHb Bb Q7i2M i F2M iQ #2 /BK2MbBQMH2bb
T ` K2k2`# b2/ QM i?2 2 `Hv bim/B2b Q7 TB+ 2X H Qr 2 #2 v L B? mH H 2M (
`Bb2 BM T` +iB+2 b Bi Bb /B7 7B+ Q2 bBK B2 QM7B? 2K2M2` ;Bp2M
`Qm;?M2bb ;2QK2i`v- M/ BM i?2 #b2M+2 Q7 /B`2+i K2 bm`2K2M
BMi`Q/m+2b mM+2`i BMiv r?2M BM/B`2+iHv /2i2`KBMBM; Bib K ;M
7HQr +QKT `BbQM #2ir22M `Qm;? M/ bKQQi? r HH Bb HbQ mM+
/Bb+mbbBQM QM MQM@HQ+ H /2T2M/2M+B2b 7Q` #Qi? `Qm;? M/

AM //BiBQM iQ M mM/2`bi M/BM; Q7 i?2b2 7mM/ K2Mi H 7HQr #2
7B+B2Mi `2b2 `+? `2; `/BM; HBKBi iBQMb Q7 KmHiB@TQBMi K2 b
S `iB+H2 BK ;2 p2HQ+BK2i`v USAoV Bb BM+`2 bBM;Hv #2+QKBM
MB[m2 7Q` im`#mH2M+2 bim/B2b- i?mb M bb2bbK2Mi Q7 i?2 #B
mM/2`@`2bQHp2/ K2 bm`2K2Mib Bb M2+2bb `vX aT iB H `2bQHmi
ii2Mm i2 K2 bm`2K2Mib Q7 i?2 im`#mH2M+2 bi iBbiB+bX HbQ- i
bm`2K2Mib K /2 Qp2` `Qm;? r HH p2`bmb bKQQi? r HH `2 MQi /
MBbQi`QTB+ im`#mH2Mi #QmM/ `v- i?2 b+ H2b BM i?2 /QKBM Mi
iBQM `2 Km+? H `;2` p2`bmb BM i?2 r HH@MQ`K H/B`2+iBQMb- M
QM i?2 K2 bm`2/[m MiBiB2bX o `B iBQMb BM +QKTQM2Mi@rBb2
BMi2`T`2i iBQMb Q7 +QKT ` iBp2 bi iBbiB+b #2ir22M 7HQr +QM/B
bKQQi? M/ `Qm;? r HH /Bb+`2T M+B2b +QK2 7`QK p `B iBQMb BM
Q` B7 i?2`2 `2 7 Hb2 Dm/;2K2Mib BM i?2 K2 bm`2K2Mi Q7 i?2 7HQ
i?Bb ? b K DQ` BKTHB+ iBQM r?2M +QKT `BM; im`#mH2Mi bi iBb
MmK#2`b M/ r HH +QM/BiBQMbX

h?2`2 `2 i?`22 K BM ; Tb BM i?2 HBi2` im`2 i? i `2 //`2bb2/ BM i?

RXM BKT`Qp2/ 7mM/ K2Mi H mM/2`bi M/BM; Q7 i?2 +QK#BM2/ 2
MQM@2[mBHB#`BmK T`2bbm`2 ;` /B2Mib QM i?2 p2HQ+Biv 7B

kXh?2 HBKBi iBQMb Q7 T` +iB+ H K2 bm`2K2Mib BM `2T`2b2MiBM
+QKT ` iBp2 Tm`TQb2bX

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jXh?2 p HB/Biv Q7 i?2 r HH@bBKBH `Biv ?vTQi?2bBb 7Q` `Qm;?
bm`2 ;` /B2Mi ?BbiQ`B2b M/ +QKT ` iBp2 /B772`2M+2b #2ir2
im`#mH2M+2 +? ` +i2`BbiB+bX

*? Ti2` j

aT iB H _2bQHmiBQM GBKBi iB
S `iB+H2 AK ;2 o2HQ+BK2i`v BM
_2vMQH/b LmK#2` hm`#mH2Mi
"QmM/ `v G v2`b

h?2 +QMi2Mib Q7 i?Bb +? Ti2` +QMi BM K Mmb+`BTi BM i?2 aT`BM
i? i ? b #22M bm#KBii2/ M/ BM@`2pB21 rt7Q `BK2Mib +BM6QHMiQ/X 2
h?Bb +? Ti2` T`2b2Mib M BMp2biB; iBQM Q7 i?2 2772+i Q7 mM/
bm`2/ rBi? T `iB+H2 BK ;2 p2HQ+BK2i`v mM/2`@`2bQHmiBQM QM
im`#mH2M+2 bi iBbiB+bX

*? Ti2` 9

>BbiQ`v 1772+ib M/ q HH aBKB
LQM@1[mBHB#`BmK hm`#mH2M
G v2`b BM o `vBM; S`2bbm`2 :`
Pp2` _Qm;? M/ aKQQi? am`7 +2

h?2 +QMi2Mib Q7 i?Bb +? Ti2` +QMi BM K Mmb+`BTi BM i?2 1Hb2
iQ #2 bm#KBii2/ M/ BM@`2pB2r 7Q` 2mM HB+ iBQM iQ i?2 H Q7 >2 i
6HQ Xrh?Bb rQ`F r b BMpBi2/ iQ Tm#HB+ iBQM iQ i?2 DQm`M H 7i2
AMi2`M iBQM H avKTQbBmK QM hm`#mH2M+2 M/ a?2 ` 6HQr S?2M
T`2b2Mib M 2p Hm iBQM Q7 i?2 T ` K2i2`b M2+2bb `v iQ /2i2`KB
bBKBH `Biv ?vTQi?2bBb 7Q` `Qm;? r HH im`#mH2Mi #QmM/ `v H v
;` /B2MiX