DETERMINISTIC INVENTORY MODELS
WITH
MULTIPLE CONSTRAINTS

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INTRODUCTION

Although unconstrained inventory models have received much attention, the importance of constrained inventory models cannot be overemphasized. Few firms are blessed with infinite resources.

The constraints in inventory system deal with resources that in some way place limitations on the theoretical optimal policy.[1] When determining the policy, a straightforward application of the theoretical optimal formula may lead to impractical results. These may arise from a violation of the restrictions on available resources, which the theoretical optimal policy does not consider.

This research is directed towards improved management by recognizing a real world situation. The problem is to minimize the total cost of the inventory system in the face of multiple constraints. This suggests techniques of optimization such as dynamic programming, direct enumeration, or Lagrangian multipliers. The Lagrangian multiplier method is selected for its convenience.

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[1]Chapter II deals with the derivation of theoretical optimal policy. It is abstracted from Reference 3, Section 10.7.

The assumptions underlying this treatise are:

1) the models are deterministic in that both demand and procurement lead time are fixed.

2) only the single-item, single-source system is discussed in that only one item is to be procured from a predetermined source.[1]

SURVEY OF THE LITERATURE

The purpose of this survey was to review the literature available on the application of the Lagrangian multiplier technique to the inventory models. The findings are abstracted in the paragraphs which follow.

Hadley and Whitin (Ref. 5-Page 433)[2] give the mathematical development of the Lagrangian multiplier technique and discuss the interpretation of the Lagrangian multipliers. They also discuss Newton's method of iteration.

Banks (Ref. 1) illustrates the general model[3] subject to only a warehouse constraint. The inventory system under study is the multiple-item, single-source and the method of

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[1]Other systems are the single-item, multiple-source, the multiple-item, single source, and multiple-item, multiple-source presented in Reference 4.

[2]The sources are referred to as (Ref. X-Page Y) where X represents reference number as listed in bibliography and Y represents page number wherever applicable.

[3]See Figure number one.

determining the Lagrangian multiplier is by iteration. Parsons (Ref. 8-Page 360) discusses a set up time constraint, a capital constraint, a number of orders constraint and a warehouse constraint with each acting separately. He concludes that the Lagrangian multiplier technique can also be used when several compatible restrictions are simultaneously active. He, however, concludes that an explicit solution for the Lagrangian multipliers very often becomes impossible to find and some search technique must be used.

Churchman, Ackoff and Arnoff (Ref. 2-Page 255) discuss the special model[1] under single linear and non-linear constraints for the multiple-item, single-source system. They also discuss the same model with two constraints. The method of determining the values for the Lagrangian multipliers is by trial and error. They point out that there need not always be a solution to the problem of inventory model under multiple constraints. Also, given the existence of a solution they feel the need of an efficient method of determining the Lagrangian multipliers.

The literature survey indicated a need to,

1) use the general model for analysis with multiple constraints,

2) determine the Lagrangian multipliers explicitly,

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[1]See Figure number two.

3) clearly indicate the combinations of compatible and incompatible constraints.

The author believes that the research presented in this treatise overcomes these difficulties for the single-item, single-source inventory system.

CHAPTER II

THE GENERAL MODEL

The main purpose for holding inventory is to make demand independent of supply. This treatise concerns itself with the single-item, single-source procurement and inventory process which may be described as follows. A stock of certain item is maintained to meet demand. When the number of items on hand and on order falls to a predetermined level, action is initiated to procure a replenishment quantity from the single predetermined source. The objective of this chapter is to present a general model which may be used to determine the procurement level and the procurement quantity so that the total cost associated with the procurement and inventory system will be minimized.

GENERAL SYMBOLISM

The effectiveness function for the system described above may be expressed as follows:

$$E = f(x,y)$$

Where E = measure of effectiveness sought (minimize total system cost).

x = policy variables under direct control of the decision maker of when to procure and how much to procure.

y = the parameters not directly under control of the decision maker of procurement lead time, replenishment rate, item cost, procurement cost, demand, holding cost, and shortage cost.

The following symbolism will be adopted:

TC = total system cost per period

L = procurement level

Q = procurement quantity

D = demand rate in units per period

T = lead time in periods

P = number of periods per cycle

R = replenishment rate in units per period

C_i = item cost per unit

C_p = procurement cost per procurement

C_h = holding cost per unit per period

C_s = shortage cost per unit short per period.

MODEL FORMULATION

If it is assumed that demand for the item is deterministic, that replenishment rate is finite ($R > D$), that shortage cost is finite, and that unsatisfied demand is not lost; the inventory process may be graphically represented as in Figure 1.

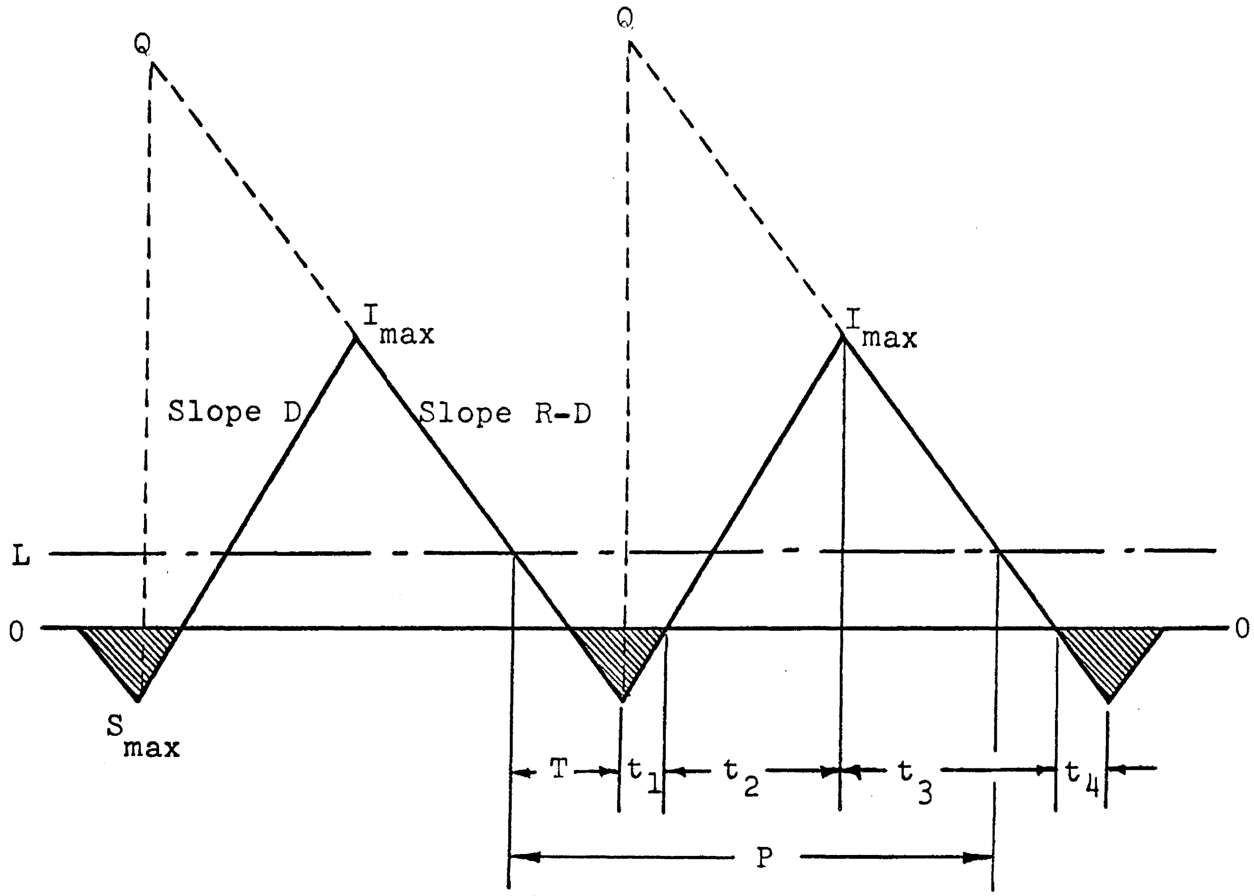


Figure 1. The General Model

The total system cost per period will be the sum of the item cost per period (IC), the procurement cost per period (PC), the holding cost per period (HC), and the shortage cost per period (SC);

$$TC = IC + PC + HC + SC.$$

The mathematical expressions will be developed for each component of the total cost and added together to get the total system cost per period.

<u>Item cost</u>. Item cost per period will be the item cost per unit times the demand in units per period,

$$IC = C_1 D \ . \tag{2.1}$$

<u>Procurement cost</u>. The procurement cost for the period will be cost per procurement divided by the number of periods per cycle,

$$PC = C_p/P \ . \tag{2.2}$$

From Figure 1,

$$\text{slope } D = Q/P,$$
$$P = Q/D \ . \tag{2.3}$$

Substituting Equation (2.3) into Equation (2.2) gives

$$PC = \frac{C_p D}{Q} \ . \tag{2.4}$$

<u>Holding cost.</u> Holding cost for the period will be the holding cost per unit per period times the average number of units in stock ($\frac{I}{P}$) for the period,

$$HC = \frac{C_h I}{P} \tag{2.5}$$

The expression for I can be derived as follows: the net rate of accumulation during time $(t_1 + t_2)$ is $(R - D)$. The maximum accumulation is designated as I_{max} in Figure 1. The following algebraic expressions are evident:

$$(t_1 + t_2)(R-D) = (t_3 + t_4)D \tag{2.6}$$

$$(t_1 + t_2) = Q/R \tag{2.7}$$

$$t_3 + t_4 = \frac{I_{max} + DT-L}{D} . \tag{2.8}$$

From Equation (2.6), Equation (2.7), and Equation (2.8)

$$I_{max} + DT-L = \frac{Q}{R}(R-D)$$

$$I_{max} = \frac{Q}{R}(R-D) + L-DT$$

$$= Q(1-\frac{D}{R}) + L-DT . \tag{2.9}$$

The total number of unit periods of stock on hand during the inventory cycle is

$$I = \frac{I_{max}}{2}(t_2 + t_3)$$

$$= \frac{I_{max}}{2} \left[\frac{I_{max}}{R-D} + \frac{I_{max}}{D}\right]$$

$$= \frac{I^2_{max}}{2} \left[\frac{1}{R-D} + \frac{1}{D}\right] .$$

Substituting Equation (2.9) for I_{max} gives

$$I = \frac{[Q(1-D/R) + L-DT]^2}{2} \left[\frac{1}{R-D} + \frac{1}{D}\right] . \qquad (2.10)$$

Substituting Equation (2.10) in Equation (2.5) gives

$$HC = \frac{C_h D}{Q} \left[\frac{\{Q(1 - D/R) + L-DT\}^2}{2} \left(\frac{1}{R-D} + \frac{1}{D}\right)\right] .$$

But since

$$\frac{D}{Q} \left(\frac{1}{R-D} + \frac{1}{D}\right) = \frac{1}{Q(1 - D/R)} , \qquad (2.11)$$

$$HC = \frac{C_h}{2Q(1 - D/R)} \{Q(1 - \frac{D}{R}) + L-DT\}^2 . \qquad (2.12)$$

<u>Shortage cost.</u> Shortage cost for the period will be shortage cost per unit per period times the average number of units short for the period (S),

$$SC = \frac{C_s S}{P}$$

The total number of unit periods of shortage during the cycle is

$$S = \frac{S_{max}}{2} (t_1 + t_4)$$

$$= \frac{S_{max}}{2} [\frac{S_{max}}{R-D} + \frac{S_{max}}{D}] .$$

But since $S_{max} = DT-L$

$$S = \frac{(DT-L)^2}{2} [\frac{1}{R-D} + \frac{1}{D}] . \qquad (2.14)$$

Substituting Equation (2.14) in Equation (2.13) gives

$$SC = \frac{C_s D}{Q} [\frac{(DT-L)^2}{2} (\frac{1}{R-D} + \frac{1}{D})] .$$

Using Equation (2.11)

$$SC = \frac{C_s (DT-L)^2}{2Q(1 - D/R)} . \qquad (2.15)$$

Total system cost. The total system cost per period for the manufacturing model will be a summation of the four cost components developed in Equation (2.1), Equation (2.4), Equation (2.12), and Equation (2.15). Thus,

$$TC = C_1 D + \frac{C_p D}{Q} + \frac{C_h}{2Q(1-D/R)} [Q(1 - \frac{D}{R}) + L-DT]^2$$
$$+ \frac{C_s (DT-L)^2}{2Q(1 - D/R)} . \qquad (2.16)$$

Equation (2.16) may be modified as follows:

$$TC = C_1 D + \frac{C_p D}{Q} + \frac{C_h Q(1-D/R)}{2} - C_h (DT-L)$$
$$+ \frac{(C_h + C_s)(DT-L)^2}{2Q(1-D/R)} . \qquad (2.17)$$

OPTIMIZATION

The total system cost is a function of two independent variables, L and Q. Taking the partial derivatives of TC with respect to Q and with respect to (DT-L) and setting the results equal to zero gives

$$\frac{\partial TC}{\partial Q} = -\frac{C_pD}{Q^2} + \frac{C_h(1-D/R)}{2} - \frac{(C_h + C_s)(DT-L)^2}{2Q^2(1 - D/R)} = 0 \qquad (2.18)$$

$$\frac{\partial TC}{\partial (DT-L)} = -C_h + \frac{C_h(DT-L)}{Q(1-D/R)} + \frac{C_s(DT-L)}{Q(1-D/R)} = 0 \qquad (2.19)$$

From Equation (2.19)

$$\frac{DT-L}{Q} = \frac{C_h(1 - D/R)}{C_h + C_s} . \qquad (2.20)$$

Substituting Equation (2.20) in Equation (2.18) gives

$$-\frac{C_pD}{Q^2} + \frac{C_h(1-D/R)}{2} - \frac{C_h{}^3(1-D/R)}{2(C_h + C_s)^2} - \frac{C_sC_h{}^2(1-D/R)}{2(C_h + C_s)} = 0.$$

$$\frac{C_pD}{Q^2} = \frac{C_hC_s(1-D/R)}{2(C_h + C_s)}$$

i.e.

$$Q = \sqrt{\frac{1}{1-D/R}}\sqrt{\frac{2C_pD}{C_h} + \frac{2C_pD}{C_s}} . \qquad (2.21)$$

The minimum cost procurement level may be derived from Equation (2.20) as

$$L = DT - \frac{C_h(1 - D/R)}{C_h + C_s} \sqrt{\frac{1}{1-D/R}} \sqrt{\frac{2C_pD}{C_h} + \frac{2C_pD}{C_s}}$$

$$= DT - \sqrt{1-D/R} \sqrt{\frac{2C_pD}{C_s(1 + C_s/C_h)}} \; . \qquad (2.22)$$

Equation (2.21) and Equation (2.22) may now be substituted back into the total cost equation to give an expression for minimum total system cost. The result is

$$TC_{min} = C_1D + \sqrt{\frac{1}{1-D/R}} \sqrt{\frac{2C_pC_hC_sD}{C_h + C_s}} \; . \qquad (2.23)$$

CHAPTER III

THE CONSTRAINED GENERAL MODEL

In this chapter following constraints will be discussed in relation with the general model:

Constraint one: There is a limitation on the maximum warehouse space available.

Constraint two: An upper limit exists on the maximum number of orders that can be placed per period.

Constraint three: The maximum amount of capital invested in the inventory is limited.

Constraint four: There is a limitation on the number of set ups that can be made per period.

DIMENSIONAL ANALYSIS OF CONSTRAINTS

In this section the nature of each constraint is examined in order to make sure that the expression developed for each constraint is correct. A dimensional analysis is carried out using the following fundamental quantities in the analysis:

1) Time in periods is denoted by [T]

   Time in hours is denoted by [t]

2) Unit of inventory is denoted by [N]

3) Unit of money is denoted by [$]

4) Unit of length is denoted by [L]

and 5) Dimensionless quantities are denoted by [1] .

Warehouse restriction. Each item that is inventoried consumes scarce warehouse space. The fact that acquiring warehouse space reduces working capital of the firm puts limitations on the maximum warehouse space available. Let,

I_{max} = maximum units of inventory during the cycle

$= Q(1 - \frac{D}{R}) + L-DT$

K = fraction of I_{max} in storage (for average inventory this will be 0.5)

w = space consumed by each unit in square feet per unit per period

and S = total warehouse space available in square feet per period.

Therefore, total space consumed $\leq$ total space available,

or $$KwI_{max} \leq S. \qquad (3.1)$$

Writing in dimensional form

$$[1] \frac{[L^2]}{[N][T]} [N] \leq \frac{[L^2]}{[T]}$$

$$\frac{[L^2]}{[T]} \leq \frac{[L^2]}{[T]} .$$

Restriction on number of orders. Placing the order involves secretarial and accounting activities, follow up, and receiving. Since these activities consume scarce management resources of the firm, many firms put a restriction on the maximum amount of orders that can be placed per period. Let,

D = demand in units per period

Q = procurement quantity in units

A = maximum number of orders per period.

Therefore, the maximum number of orders placed $\leq$ maximum number of orders available, or

$$\frac{D}{Q} \leq A \; . \tag{3.2}$$

Writing in dimensional form

$$\frac{[N]}{[T]} \quad \frac{[1]}{[N]} \leq \frac{[1]}{[T]}$$

$$\frac{[1]}{[T]} \leq \frac{[1]}{[T]} \; .$$

<u>Capital restriction.</u> Inventory consumes scarce capital and hence an upper limit is normally placed on the capital that can be invested in the inventory. Let,

I_{max} = maximum units in inventory during the cycle

C_1 = item cost in dollars per unit

C = maximum capital available in dollars.

Therefore, the maximum investment in inventory $\leq$ maximum capital available for inventory, or

$$I_{max} C_1 \leq C \tag{3.3}$$

Writing in dimensional form

$$[N] \frac{[\$]}{[N]} \leq [\$]$$

$$[\$] \leq [\$].$$

<u>Set up time restriction</u>. Manufacturing usually starts with the activity of set up. This includes setting up tools, changing jigs and fixtures, and other similar operations to start production. Since a limited number of set up men are available on each shop floor the total set up time on the floor should not exceed their capacity. Let,

D = demand in units per period

Q = procurement quantity in units

t = time required per set up in hours

T' = total time available in hours per period.

The number of periods per cycle was given by Equation (2.3) as $P = Q/D$. Thus, the number of cycles per period is D/Q. Therefore, the total set up time required $\leq$ available set up time, or

$$\frac{D}{Q} t \leq T' . \tag{3.4}$$

Writing in dimensional form

$$\frac{[N]/[T]}{[N]} [t] \leq \frac{[t]}{[T]}$$

$$\frac{[t]}{[T]} \le \frac{[t]}{[T]}$$

The dimensional analysis proves that Equation (3.1), Equation (3.2), Equation (3.3) and Equation (3.4) are dimensionally balanced.

THE GENERAL MODEL WITH TWO CONSTRAINTS

Consider first the case where there is an upper limit S on the square feet of warehouse space, and an upper limit A on the maximum number of orders that can be placed per period. Both of these constraints are assumed to act simultaneously. The Lagrangian multiplier technique can be used to determine an optimum on the boundary for this situation. The problem requires the optimization of

$$TC(Q,L) = C_i D + \frac{C_p D}{Q} + \frac{C_h Q(1-D/R)}{2} - C_h(DT-L) + \frac{(C_h + C_s)(DT-L)^2}{2Q(1-D/R)}, \text{ from Equation (2.17)}$$

subject to the following constraint conditions:

$$KwI_{max} \le S \quad \text{from Equation (3.1)}$$
$$D/Q \le A, \quad \text{from Equation (3.2)}$$

Let,

λ_1 be such that $\lambda_1 < 0$ for $(S - KwI_{max} = 0)$, and $\lambda_1 = 0$ for $(S - KwI_{max} > 0)$. Then,

$$\lambda_1 (S - KwI_{max}) = 0$$

or $$\lambda_1 [S - Kw \{Q(1 - \frac{D}{R}) - (DT-L)\}] = 0 . \qquad (3.5)$$

Similarly let,

λ_2 be such that $\lambda_2 > 0$ for $(A - \frac{D}{Q} = 0)$, and $\lambda_2 = 0$ for $(A - \frac{D}{Q} > 0)$. Then,

$$\lambda_2 (A - \frac{D}{Q}) = 0. \qquad (3.6)$$

Equation (2.17), Equation (3.5), and Equation (3.6) may be added to give the total cost function as:

$$TC = C_1 D + \frac{C_p D}{Q} + \frac{C_h Q(1 - D/R)}{2} - C_h (DT-L) + \frac{(C_h + C_s)(DT-L)^2}{2Q(1-D/R)} + \lambda_1 [S - Kw \{Q(1 - \frac{D}{R}) - (DT-L)\}] + \lambda_2 (A - \frac{D}{Q}) . \qquad (3.7)$$

In Equation (3.7) there are four unknowns; Q, L, λ_1, and λ_2. Four independent equations must be obtained to get the explicit values of these quantities. This can be done by differentiating Equation (3.7) with respect to each of these unknown quantities and equating the resulting expressions to zero. These four equations then can be solved simultaneously to get explicit values of Q, L, λ_1 and λ_2. Differentiating Equation (3.7) with respect to Q gives

$$\frac{\partial TC}{\partial Q} = -\frac{C_p D}{Q^2} + \frac{C_h(1-D/R)}{2} - \frac{(C_h + C_s)(DT-L)^2}{2Q^2(1-D/R)}$$

$$-\lambda_1 Kw + \lambda_2 \frac{D}{Q^2} = 0 . \tag{3.8}$$

Differentiating Equation (3.7) with respect to (DT-L) gives

$$\frac{\partial TC}{\partial (DT-L)} = -C_h + \frac{2(C_h + C_s)(DT-L)}{2Q(1-D/R)} + \lambda_1(-Kw)(-1) = 0. \tag{3.9}$$

Differentiating Equation (3.7) with respect to λ_1 gives

$$\frac{\partial TC}{\partial \lambda_1} = S - Kw \{Q(1 - \frac{D}{R}) - (DT-L)\} = 0. \tag{3.10}$$

And differentiating Equation (3.7) with respect to λ_2 gives

$$\frac{\partial TC}{\partial \lambda_2} = A - \frac{D}{Q} = 0 . \tag{3.11}$$

From Equation (3.11)

$$A = \frac{D}{Q} \qquad \text{or} \qquad Q = \frac{D}{A} . \tag{3.12}$$

Substituting for Q in Equation (3.10) from Equation (3.12),

$$S = Kw \{\frac{D}{A} (1 - \frac{D}{R}) - (DT-L)\}$$

$$\frac{S}{Kw} = \frac{D}{A} (1 - \frac{D}{R}) - (DT-L)$$

$$(DT-L) = \frac{D}{A} (1 - \frac{D}{R}) - \frac{S}{Kw} \tag{3.13}$$

$$L = DT - \frac{D}{A}\left(1 - \frac{D}{R}\right) + \frac{S}{Kw} . \tag{3.14}$$

From Equation (3.9)

$$\lambda_1 = \frac{C_h}{Kw} - \frac{(C_h + C_s)(DT-L)}{KwQ(1-D/R)} .$$

Substituting for Q and (DT-L), Equation (3.12) and Equation (3.13) respectively, in the above expression gives

$$\lambda_1 = \frac{C_h}{Kw} - \frac{(C_h + C_s)\left(\frac{D}{A}\left(1 - \frac{D}{R}\right) - \frac{S}{Kw}\right)}{Kw\,\frac{D}{A}\,(1 - D/R)} .$$

Simplifying further,

$$\lambda_1 = \frac{C_h}{Kw} - \frac{(C_h + C_s)}{Kw} + \frac{SA(C_h + C_s)}{K^2w^2D(1-D/R)}$$

$$= -\frac{C_s}{Kw} + \frac{SA(C_h + C_s)}{K^2w^2D(1-D/R)} . \tag{3.15}$$

From Equation (3.8)

$$-\lambda_2\,\frac{D}{Q^2} = -\frac{C_pD}{Q^2} + \frac{C_h(1-D/R)}{2} - \frac{(C_h + C_s)(DT-L)^2}{2Q^2(1-D/R)} - \lambda_1 Kw\left(1-\frac{D}{R}\right).$$

Multiplying both sides by $(-Q^2/D)$ gives

$$\lambda_2 = C_p - \frac{C_hQ^2(1-D/R)}{2D} + \frac{(C_h + C_s)(DT-L)^2}{2Q^2(1-D/R)}\;\frac{Q^2}{D}$$

$$+ \lambda_1 Kw\left(1 - \frac{D}{R}\right)\;\frac{Q^2}{D} .$$

Substituting for Q and (DT-L) from Equation (3.12) and Equation (3.13) respectively, λ_2 reduces to

$$\lambda_2 = C_p - \frac{C_h \frac{D^2}{A^2}(1-D/R)}{2D} + \frac{(C_h + C_s)}{2D(1-D/R)} \{\frac{D}{A}(1 - \frac{D}{R}) - \frac{S}{Kw}\}^2$$

$$+ Kw \frac{D^2/A^2}{D} (1 - \frac{D}{R})\{- \frac{C_s}{Kw} + \frac{SA(C_h + C_s)}{K^2w^2D(1-D/R)} \}$$

$$= C_p - \frac{C_hD(1-D/R)}{2A^2} + \frac{(C_h + C_s)}{2D(1-D/R)} \{\frac{D^2}{A^2} (1 - \frac{D}{R})^2$$

$$- 2 \frac{D}{A} (1 - \frac{D}{R}) \frac{S}{Kw} + \frac{S^2}{K^2w^2}\} + \frac{KwD}{A^2} (1 - \frac{D}{R})\{- \frac{C_s}{Kw}$$

$$+ \frac{SA(C_h + C_s)}{K^2w^2D(1-D/R)} \} .$$

Simplifying further,

$$\lambda_2 = C_p - \frac{C_hD(1-D/R)}{2A^2} + \frac{(C_h + C_s)D(1-D/R)}{2A^2} - \frac{(C_h + C_s)S}{AKw}$$

$$+ \frac{(C_h + C_s)S^2}{2K^2w^2D(1-D/R)} - \frac{C_sD(1-D/R)}{A^2} \cdot \frac{2}{2} + \frac{(C_h + C_s)S}{KwA}$$

$$= C_p + \frac{-C_hD(1-D/R) + (C_h + C_s)D(1-D/R) - 2C_sD(1-D/R)}{2A^2}$$

$$+ \frac{-(C_h + C_s)S + (C_h + C_s)S}{KwA} + \frac{(C_h + C_s)S^2}{2K^2w^2D(1-D/R)}$$

$$= C_p - \frac{C_sD(1-D/R)}{2A^2} + \frac{(C_h + C_s)S^2}{2K^2w^2D(1-D/R)} . \qquad (3.16)$$

Thus, by solving Equation (3.11), Equation (3.10), Equation (3.9) and Equation (3.8) simultaneously, explicit solution for policy variables Q and L can be obtained as shown in Equation (3.12) and Equation (3.14). Values for the Lagrangian multipliers can be obtained from Equation (3.15) and Equation (3.16).

INTERPRETATION OF THE LAGRANGIAN MULTIPLIERS

The above optimization with Lagrangian multipliers has an interesting economic interpretation. In the above discussion TC represents the average total cost per period and the constraints represent limitations on the physical resources such as warehouse space and number of orders. Then by its dimensions, λ_1 and λ_2 must be the values of the respective resources. Intuitively we see that

$$\frac{\partial TC}{\partial[S - Kw(Q(1-D/R) - (DT-L))]} = \lambda_1,$$

which means that λ_1 is the amount by which the total cost will increase by adding one additional unit of warehouse space. This arguement can be generalized for λ_2 in particular and λ_n in general. The Lagrangian multipliers can thus be considered to be the imputed values or shadow prices of the resources.

ILLUSTRATIVE EXAMPLE

The use of the Lagrangian multiplier as an optimization technique will now be illustrated with an example. The warehouse constraint and number of orders constraint will be assumed to act simultaneously. The general model will be used. Let,

D = 8 units per period

T = 10 periods

R = 16 units per period

C_i = \$ 4.50 per unit

C_p = \$80.00 per procurement

C_h = \$ 0.20 per unit per period

C_s = \$ 0.10 per unit short per period.

Also let,

K = 1

w = 10 square feet per unit per period

S = 250 square feet per period

and A = 0.04 order per period.

<u>Step one - calculate policy variables Q and L.</u> By using Equation (2.21) and Equation (2.22) respectively.

$$Q = \sqrt{\frac{1}{1-8/16}} \sqrt{\frac{2(80)(8)}{0.20} + \frac{2(80)(8)}{0.10}}$$

$$= \sqrt{2} \sqrt{6400 + 12800}$$

$$= \sqrt{38400} = 196 \text{ units}$$

$$L = 8(10) - \sqrt{1 - \frac{8}{16}}\ \sqrt{\frac{2(180)(8)}{0.10[1 + (0.10/0.20)]}}$$

$$= 80 - \sqrt{1/2}\ \sqrt{8540}$$

$$= 80 - 65.3 \qquad = 15 \text{ units}$$

Also, $I_{max} = \{196(1 - \frac{8}{16}) + 15 - (8)(10)\}$

$$= \{98 + 15 - 80\} \qquad = 33 \text{ units}$$

$$TC_{min} = \$4.50)(8) + \sqrt{1 - \frac{8}{16}}\sqrt{\frac{2(80.00)(0.20)(0.10)(8)}{(0.20 + 0.10)}}$$

$$= \$\ 36 + \sqrt{1/2}\sqrt{85.4}$$

$$= \$\ 36 + 6.53 \qquad = \$42.53 \text{ per period}$$

Step two - check if constraints are active. The warehouse constraint is

$$K w I_{max} \leq S$$

$$(1)(10)(33) \leq 250$$

$$330 \leq 250 .$$

Since the space required for the inventory is more than the available, the constraint is active. The number of orders constraint is,

$$\frac{D}{Q} \leq A$$

$$\frac{8}{196} \leq 0.04$$

$0.0408 \leq 0.04$

Since number of order placed per period is higher than the allowed figure, the constraint is active.

Since both the constraints are active, previous analysis can be applied directly. Equation (3.12), Equation (3.14), Equation (2.9), Equation (2.17), Equation (3.15) and Equation (3.16) will be used.

Step three - policy variables under constraints.

$$Q = \frac{D}{A} = \frac{8}{0.04} = 200 \text{ units from Equation (3.12)}$$

$$L = DT - \frac{D}{A}\left(1 - \frac{D}{R}\right) + \frac{S}{Kw} \quad \text{from Equation (3.14)}$$

$$= (8)(10) - \frac{8}{0.04}\left(1 - \frac{8}{16}\right) + \frac{250}{(1)(10)}$$

$$= 80 - 100 + 25 \qquad = +5$$

$$I_{max} = Q\left(1 - \frac{D}{R}\right) + L - DT \quad \text{from Equation (2.9)}$$

$$= 200(1 - 8/16) + 5 - (8)(10)$$

$$= 100 + 5 - 80 \qquad = 25$$

$$TC_{actual} = C_i D + \frac{C_p D}{Q} + \frac{C_h Q(1-D/R)}{2} - C_h(DT-L) + \frac{(C_h + C_s)(DT-L)^2}{2Q(1-D/R)} \quad \text{from Equation (2.17)}$$

$$= (4.50)(8) + \frac{(80)(8)}{200} + \frac{(0.20)(200)(1-8/16)}{2}$$

$$-(0.20)\{(8)(10)-5\} + \frac{(0.20 + 0.10)\{(8)(10)-5\}^2}{(2)(200)(1-8/16)}$$

$$= 36.00 + 3.20 + 10.00 - 15.00 + 8.44$$

$$= 42.64$$

Therefore, penalty due to constraints

$$= TC_{actual} - TC_{min}$$

$$= \$42.64 - \$42.53$$

$$= \$\ 0.11 \text{ per period.}$$

Shadow prices of these resources can be calculated as follows:

$$\lambda_1 = -\frac{C_s}{Kw} + \frac{SA(C_h + C_s)}{K^2w^2D(1-D/R)} \quad \text{from Equation (3.15)}$$

$$= \$ - \frac{0.10}{(1)(10)} + \frac{(250)(0.04)(0.20 + 0.10)}{(1)^2(10)^2(8)(1-8/16)}$$

$$= \$ - 0.01 + 0.0075$$

$$= \$ - 0.0025 \text{ per additional square foot.}$$

and $$\lambda_2 = C_p - \frac{C_sD(1-D/R)}{2A^2} + \frac{(C_h + C_s)S^2}{2K^2w^2D(1-D/R)} \quad \text{from Equation (3.16)}$$

$$= \$80 - \frac{(0.10)(8)(1-8/16)}{(2)(0.04)^2} + \frac{(0.20 + 0.10)(250)^2}{(2)(1)^2(10)^2(8)(1-8/16)}$$

$$= \$80 - 125 + 23.4$$

$$= \$21.6 \text{ per additional order.}$$

Step four - analysis of results. The requirements to satisfy the warehouse constraint and the number of orders constraint are somewhat contradictory. The warehouse constraint requires that I_{max} (which directly increases with

Q but is not a function of Q alone) should be as small as possible to satisfy the warehouse constraint, while the number of orders constraint requires that Q should be as large as permissible by the constraint.

The procurement quantity (Q) increases from 196 in the unconstrained condition to 200 in the constrained condition. This apparently is undesirable because this will increase I_{max} which is given by $\{Q(1 - \frac{D}{R}) + L-DT\}$. As can be seen from this, I_{max} is a function of both Q and L. In the above example L decreases from 15 to 5 although Q increases. This increase in Q, and decrease in L, finally result in reducing the I_{max} from 33 to 25 to satisfy the warehouse constraint.

Close inspection of the results will reveal that both constraints are satisfied on the boundary. Warehouse space required for the constrained condition is

$$S = KwI_{max} = (1)(10)(25) = 250 \text{ square feet.}$$

Number of order placed for constrained condition is

$$A = \frac{D}{Q} = \frac{8}{200} = 0.04.$$

These values of S and A are on the boundaries of the constraints. The constraints result in the deviation of actual policy from the theoretical optimal policy. A penalty of $0.11 per period is incurred.

CHAPTER IV

THE MODELS WITH MORE THAN TWO CONSTRAINTS

In the previous chapter explicit solutions for the policy variables and the Lagrangian multipliers were obtained for the general model subject to two constraints. In this chapter all four constraints discussed previously will be assumed to act simultaneously. For ease of illustration a special model (Figure 2) which can be obtained by putting $R = \infty$, $C_s = \infty$ and $(DT) = L$ in Equation (2.17) will be used to begin the presentation.

THE SPECIAL MODEL

The total cost equation for this model becomes:

$$TC = C_1 D + \frac{C_p D}{Q} + \frac{C_h Q}{2} \; . \qquad (4.1)$$

And the constraint conditions can be written as follows:

1) warehouse constraint, $KwQ \leq S$ (4.2)
2) number of orders constraint, $(D/Q) \leq A$ (4.3)
3) capital constraint, $QC_1 \leq C$ (4.4)
4) set up time constraint, $(D/Q)t \leq T'$. (4.5)

The total cost function then can be formulated by adding the optimization function and constraint conditions according to the Lagrangian multiplier technique as follows:

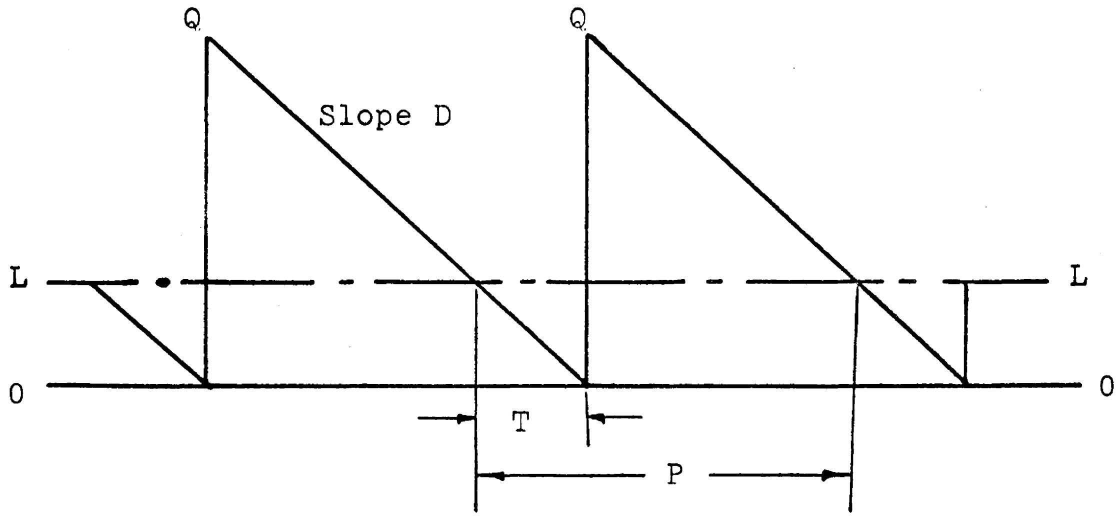


Figure 2: The Special Model

$$TC(Q) = C_1D + \frac{C_pD}{Q} + \frac{C_hQ}{2} + \lambda_1(S-KwQ)$$

$$+ \lambda_2(A - \frac{D}{Q}) + \lambda_3(C-QC_1) + \lambda_4(T' - \frac{D}{Q}t). \quad (4.6)$$

OPTIMIZATION

Explicit solutions for Q, λ_1, λ_2, λ_3 and λ_4 requires that five independent equations be obtained and solved simultaneously. The five equations are as follows:

$$\frac{\partial TC}{\partial Q} = - \frac{C_pD}{Q^2} + \frac{C_h}{2} - \lambda_1Kw + \lambda_2\frac{D}{Q^2} - \lambda_3C_1 + \lambda_4\frac{Dt}{Q^2} = 0 \quad (4.7)$$

$$\frac{\partial TC}{\partial \lambda_1} = S - KwQ = 0 \quad \text{or } S^2 - K^2w^2Q^2 = 0 \quad (4.8)$$

$$\frac{\partial TC}{\partial \lambda_2} = A - \frac{D}{Q} = 0 \quad \text{or } A^2 - \frac{D^2}{Q^2} = 0 \quad (4.9)$$

$$\frac{\partial TC}{\partial \lambda_3} = C - QC_1 = 0 \quad \text{or } C^2 - Q^2C_1{}^2 = 0 \quad (4.10)$$

$$\frac{\partial TC}{\partial \lambda_4} = T' - \frac{D}{Q}t = 0 \quad \text{or } T'^2 - \frac{D^2}{Q^2}t^2 = 0. \quad (4.11)$$

These five equations can be solved simultaneously. From Equation (4.7),

$$\frac{1}{Q^2}(-C_pD + \lambda_2D + \lambda_4Dt) = -\frac{C_h}{2} + \lambda_1Kw + \lambda_3C_1$$

$$\text{or} \quad Q^2 = \frac{(-C_pD + \lambda_2D + \lambda_4Dt)}{(-\frac{C_h}{2} + \lambda_1Kw + \lambda_3C_1)}.$$

Multiplying both numerator and denominator by (-2) gives

$$Q^2 = \frac{2C_pD - 2\lambda_2D - 2\lambda_4Dt}{C_h - 2\lambda_1Kw - 2\lambda_3C_1} \, . \tag{4.12}$$

This value of Q^2 may be substituted back into Equation (4.8), Equation (4.9), Equation (4.10), and Equation (4.11) to obtain the following expressions:

$$S^2(C_h-2\lambda_1Kw-2\lambda_3C_1)-K^2w^2(2C_pD-2\lambda_2D-2\lambda_4Dt) = 0 \tag{4.13}$$

$$A^2(2C_pD-2\lambda_2D-2\lambda_4Dt)-D^2(C_h-2\lambda_1Kw-2\lambda_3C_1) = 0 \tag{4.14}$$

$$C^2(C_h-2\lambda_1Kw-2\lambda_3C_1)-C_1{}^2(2C_pD-2\lambda_2D-2\lambda_4Dt) = 0 \tag{4.15}$$

$$T'^2(2C_pD-2\lambda_2D-2\lambda_4Dt)-D^2t^2(C_h-2\lambda_1Kw-2\lambda_3C_1) = 0 \tag{4.16}$$

Equations (4.13) through (4.15) may be further rearranged as follows

$$-2S^2Kw\lambda_1+2K^2w^2D\lambda_2-2S^2C_1\lambda_3+2K^2w^2Dt\lambda_4=-S^2C_h+2K^2w^2C_pD$$

$$-2D^2Kw\lambda_1+2A^2D\lambda_2-2D^2C_1\lambda_3+2A^2Dt\lambda_4=-D^2C_h+2A^2C_pD$$

$$-2C^2Kw\lambda_1+2C_1{}^2D\lambda_2-2C^2C_1\lambda_3+2C_1{}^2Dt\lambda_4=-C^2C_h+2C_1{}^2C_pD$$

$$-2D^2t^2Kw\lambda_1 \quad +2T'^2D\lambda_2-2D^2t^2C_1\lambda_3+2T'^2Dt\lambda_4=-D^2t^2C_h+2T'^2C_pD$$

A matrix can be formulated from these equations as follows

$$\begin{array}{ccccc}
-2S^2\ Kw & 2K^2w^2D & 2S^2\ C_1 & 2K^2w^2Dt & (-S^2\ C_h+2K^2w^2C_pD) \\
-2D^2\ Kw & 2A^2\ D & 2D^2\ C_1 & 2A^2\ Dt & (-D^2\ C_h+2A^2\ C_pD) \\
-2C^2\ Kw & 2C_1{}^2\ D & 2C^2\ C_1 & 2C_1{}^2\ Dt & (-C^2\ C_h+2C_1{}^2\ C_pD) \\
-2D^2t^2Kw & 2T'^2\ D & 2D^2t^2C_1 & 2T'^2\ Dt & (-D^2t^2C_h+2T'^2\ C_pD)
\end{array}$$

A careful study of the matrix shows that column one and column three are proportional with a ratio of proportionality of Kw/C_1. Likewise, column two and column four have a ratio of proportionality of D/Dt. The proportional rows or columns in the matrix mean incompatibility if right hand side vector is not in the same proportion. The matrix having proportional columns or rows or having all elements of one or more rows or columns equal to zero is a <u>singular matrix</u>. A singular matrix has no solution.

For the four constraints under consideration the matrix will always be singular. This situation has to do with the inequalities of the constraints themselves. The left hand sides of Inequation (4.2) and Inequation (4.4) are proportional to each other, the ratio of proportionality being Kw/C_1. Likewise the left hand sides of Inequation (4.3) and Inequation (4.5) have a ratio of proportionality of D/Dt. Thus, although there are four unknowns with four equations, explicit values cannot be obtained due to the presence of incompatibility.

INCOMPATIBLE CONSTRAINTS

The special model. The procurement quantities for different constraints, when only one of them is active at a time, can be determined by Equation (4.8), Equation (4.9), Equation (4.10), and Equation (4.11). The procurement quantity with the warehouse constraint is S/Kw, with the number of orders constraint is D/A, with the capital constraint is C/C_i, and with the set up time constraint is Dt/T'. If more than one constraint are acting simultaneously, there will never be one value of Q which will satisfy all constraints. Although explicit values of the Lagrangian multipliers can be determined, the procurement quantity will always come out to be indeterminate. The only approach to such a problem will be the trial and error method given by Churchman (Ref. 2-Page 269).

The general model. The four constraints with the general model can be subdivided into two incompatible groups the incompatibility being present within the group itself.

Group A: [Warehouse + Capital].

Group B: [Number of orders + Set up time].

Any other combination of constraints including either (or both) of these groups in combination with the remaining constraints, will also be incompatible. These incompatible combinations are listed below.

1) [Warehouse + Capital] + Number of orders

2) [Warehouse + Capital] + Set up time

3) [Number of orders + Set up time] + Warehouse

4) [Number of orders + Set up time] + Capital

5) [Number of orders + Set up time] + [Capital + Warehouse].

OPERATING POLICY UNDER INCOMPATIBILITY

The special model. It can be shown that no explicit solution exists for multiple constraints for this model. Let number of orders constraint and the capital constraint act simultaneously. Then, the optimization function can be formed as:

$$TC(Q) = C_iD + \frac{C_pD}{Q} + \frac{C_hQ}{2} + \lambda_2(A - \frac{D}{Q}) + \lambda_3(C - QC_i) \qquad (4.17)$$

Differentiating Equation (4.17) with respect to Q gives

$$\frac{\partial TC}{\partial Q} = - \frac{C_pD}{Q^2} + \frac{C_h}{2} + \frac{\lambda 2^D}{Q^2} - \lambda_3C_i = 0$$

$$\frac{1}{Q^2}(- C_pD + \lambda_2D) = - \frac{C_h}{2} + \lambda_3C_i$$

$$Q^2 = \frac{2C_pD - 2\lambda_2D}{C_h - 2\lambda_3C_i} \; . \qquad (4.18)$$

Equation (4.18) can now be substituted in Equation (4.9) and Equation (4.10) to give

$$A^2(2C_pD - 2\lambda_2D) - D^2(C_h - 2\lambda_3C_1) = 0$$

$$C^2(C_h - 2\lambda_3C_1) - C_1{}^2(2C_pD - 2\lambda_2D) = 0$$

Solving these two equations simultaneously gives

$$\lambda_2 = C_p \quad \text{and} \quad \lambda_3 = C_h/2C_1.$$

Substituting these values of λ_2 and λ_3 back into Equation (4.18) gives

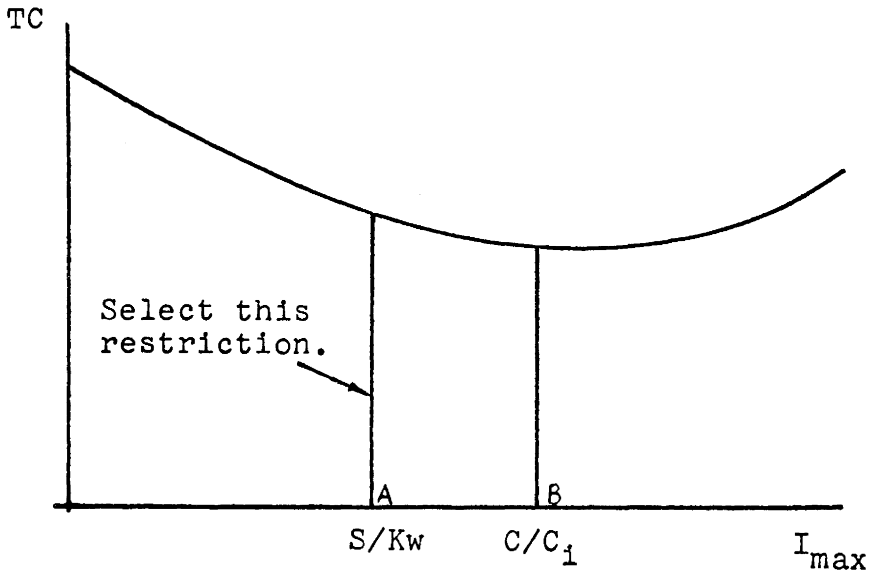
$$Q^2 = \frac{2C_pD - 2(C_p)D}{C_h - 2\cdot\frac{C_h}{2C_1}\cdot C_1} = \frac{0}{0}$$

which is indeterminate.

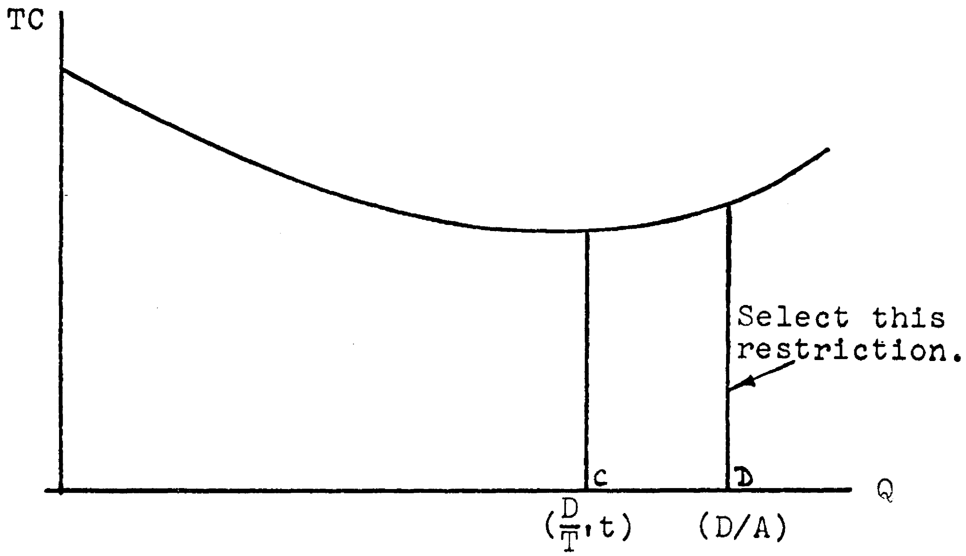
Thus, whenever more than one constraint are acting the problem will become either incompatible or indeterminate and the solution may only be obtained by the trial and error method.

The general model. An optimum solution cannot easily be obtained when incompatibility exists. However, the following scheme will be useful in obtaining the next best solution. It will be assumed that all constraints are rigid and cannot be violated.

Group A (warehouse constraint and capital constraint). The I_{max} calculated to satisfy a warehouse constraint will be S/Kw while that for a capital constraint will be C/C_1. Since, in general S/Kw is not equal to C/C_1, there can never



I. Group A constraints



II. Group B constraints

Figure 3: Selection of Proper Constraints under incompatibility.

be a single value of I_{max} which will satisfy both of these constraints at their boundaries. In this case, the alternative would be to satisfy the constraint requiring a lower value of I_{max} so that the other constraint will automatically be satisfied.

Constraints for this group are sketched in the upper half of Figure 3 assuming that S/Kw is less than C/C_i. Any attempt to increase I_{max} beyond point A will violate the warehouse restriction.

Rule 1 - Select the constraint requiring the lower I_{max} between C/C_i and S/Kw.

Group B (number of orders constraint and set up time constraint): The procurement quantity calculated to satisfy the number of orders constraint will be D/A, while that for set up time constraint will be $(D/T')(t)$. Since, in general, D/A is not equal to $(D/T')(t)$ the single procurement quantity can never satisfy both of these constraints on their boundaries. The same argument as for I_{max} in Group A may be applied, except that the larger value of Q should be chosen because Q is in the denominator of Inequality (4.3) and Inequality (4.5).

Constraints for this group are sketched in the lower half of Figure 3, assuming D/A is greater than $(D/T')(t)$. Any attempt to move towards the left of point D will violate the restriction.

Rule 2 - Select the constraint requiring larger procurement quantity between D/A and (D/T')(t).

GENERAL PROCEDURE UNDER INCOMPATIBILITY WITH THE GENERAL MODEL

<u>Step one</u>. Calculate Q, L, I_{max}, and TC_{min} disregarding constraints.

<u>Step two</u>. Check for the active constraints.

<u>Step three</u>. Select proper constraints from group A and group B.

<u>Step four</u>. Solve for the constraints in step 3.

The above procedure will be illustrated by solving an example. All data for the example of Chapter III will apply to this problem. In addition, the following data will be assumed:

t = 4 hours per set up

T' = 0.1615 hours per period

C = 135 dollars.

<u>Step one - calculate Q, L, I_{max} and TC_{min} without any constraint</u>. From the example of Chapter III,

Q = 196 units

L = 15 units

I_{max} = 33 units

TC_{min} = \$42.53 per period.

<u>Step two - check for active constraints.</u> The warehouse constraint is

$$KwI_{max} \leq S$$
$$(1)(10)(33) \leq 250$$
$$330 \leq 250.$$

Since the space required for the inventory is more than the available, the constraint is active.

The number of orders constraint is

$$D/Q \leq A$$
$$8/196 \leq 0.04$$
$$0.0408 \leq 0.04.$$

Since the number of order placed per period is more than the allowed figure, the constraint is active.

The capital constraint is

$$I_{max}C_i \leq C$$
$$(33)(4.5) \leq 135$$
$$148.5 \leq 135.$$

Since the capital required is more than the allowed limit, the constraint is active.

The number of set ups constraint is

$$(D/Q)(t) \leq T'$$

$$(8/196)(4) \leq 0.1615$$

$$0.1635 \leq 0.1615$$

Since the required set up time is more than the available, the constraint is active.

As discussed before, since all four constraints are active the incompatibility will exist. To avoid this, a screening procedure must be applied.

Step three - select proper constraints. The proper constraints will be selected from group A and group B as follows:

Group A. I_{max} as calculated from the availability of resources for the warehouse constraint is 250/(1)(10) or 25 units and that for the capital constraint is 135/4.5 or 30 units. Since 25 is less than 30, the warehouse constraint must be selected according to "Rule one".

Group B. Q calculated from the availability of resources for the number of orders constraint is 8/0.04 or 200 and that for the set up time constraint (8/0.1615)(4) or 198. Since the procurement quantity for number of orders constraint is higher, it will be selected according to "Rule two".

Step four - solve for the constraints in step 3. The problem now reduces to the general model with the warehouse constraint and the number of orders constraint which has already been solved under illustrative example of Chapter III. The procurement quantity was 200 units and I_{max} was 25 units.

It can be shown that the above technique not only avoids incompatibility but also satisfies all four constraints. Thus,

for warehouse constraint	(1)(10)(25)	$\leq$	250
	250	=	250
for number of order constraint	8/200	$\leq$	0.04
	0.04	=	0.04
for capital constraint	(25)(4.50)	$\leq$	135
	112.5	<	135
for set up time constraint	(8/200)(4)	$\leq$	0.1615
	0.1600	<	0.1615.

CHAPTER V

SUMMARY AND CONCLUSIONS

The single-item, single-source inventory system was treated with multiple constraints for the general model and the special model. The Lagrangian multiplier technique was used for optimization. The findings can be summarized as follows:

<u>General model with two constraints.</u> The general model under the warehouse constraint and the number of orders constraint was discussed in Chapter III. The values of the Lagrangian multipliers were determined explicitly as,

$$\lambda_{\text{warehouse}} = \frac{C_h}{Kw} - \frac{SA(C_h + C_s)}{K^2w^2D(1-D/R)}$$

$$\lambda_{\text{number of orders}} = C_p - \frac{C_sD(1-D/R)}{2A^2} + \frac{(C_h + C_s)S^2}{2K^2w^2D(1-D/R)}$$

The policy variables were also determined explicitly as,

$$Q = D/A$$

$$L = DT - (D/A)(1-D/R) + S/Kw.$$

Similar expressions can be developed when the capital constraint and the set up time constraint were acting simultaneously.

<u>General model with more than two constraints</u>. It was shown that incompatibility existed whenever more than two constraints were active. The four constraints with the general model were subdivided into two groups of incompatible constraints, the incompatibility being present within the group itself.

<u>Group A</u>. [Warehouse + Capital].

<u>Group B</u>. [Number of orders + Set up time].

All other incompatible combinations which can be formed by combining either (or both) of these groups in combination with the remaining constraints were also listed in Chapter IV.

Two rules were developed to help select the proper constraints when incompatibility existed. These rules were successful not only in avoiding incompatibility but also in satisfying all the constraints simultaneously.

For all remaining compatible constraints, it was concluded that Lagrangian multipliers can be determined explicitly.

<u>Special model with more than two constraints</u>. The problem of incompatibility did exist in this case also but an additional difficulty of indeterminate Q was experienced whenever two or more constraints acted simultaneously. It was concluded that the explicit solution for Q could be obtained only when the model was subject to one constraint

at a time. For all other cases it was concluded that the procurement quantity and the Lagrangian multipliers could be determined by trial and error method only.

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DETERMINISTIC INVENTORY MODELS WITH MULTIPLE CONSTRAINTS

by
Balkisan Punamchand Kacholiya

ABSTRACT

This research is directed towards the minimization of the total cost of the single-item, single-source, deterministic inventory system in the face of multiple constraints. The Lagrangian multiplier technique is used for optimization.

Two models, the general model and the special model, with multiple constraints are studied. For the special model the solution becomes indeterminate and/or incompatible whenever more than one constraint is active and the trial and error method is suggested for this situation. For the general model, the constraints are classified into two basic groups of incompatible constraints. Also, other possible groups of incompatible constraints are listed. A sample solution for one group, out of many possible groups, of compatible constraints is presented. Illustrative examples are given.