

Two Modifications of the $k\text{-}kL\text{-}\omega$ Transition Model based on Pohlhausen and Falkner-Skan Profiles

J. Fürst¹ M. Islam² J. Přihoda³ D. Wood²

¹Department of Technical Mathematics
FME CTU in Prague

²Department of Mechanical and Manufacturing Engineering
Schulich School of Engineering, University of Calgary

³Institute of Thermomechanics AS CR, v.v.i.
Academy of Sciences of the Czech Republic

Contents

Two
Modifications of
the $k\text{-}k_L\text{-}\omega$
Transition Model
based on
Pohlhausen and
Falkner-Skan
Profiles

Fürst, Islam,
Pøihoda, Wood

Introduction

Modifications

Computational
Domain and the
Mesh

Details of the
OpenFOAM
Cases

Concluding
Remarks

Acknowledgment

References

References

- 1 Introduction
- 2 Modifications
- 3 Computational Domain and the Mesh
- 4 Details of the OpenFOAM Cases
- 5 Concluding Remarks

Introduction

Two
Modifications of
the $k-k_L-\omega$
Transition Model
based on
Pohlhausen and
Falkner-Skan
Profiles

Fürst, Islam,
Pøihoda, Wood

Introduction

Modifications

Computational
Domain and the
Mesh

Details of the
OpenFOAM
Cases

Concluding
Remarks

Acknowledgment

References

References

- We present modifications of the three-equation $k - k_L - \omega$ eddy viscosity model proposed by Walters and Cokljat (Walters and Cokljat, 2008) for adverse pressure gradient flows.
- The model solves equations for the turbulent kinetic energy, k , the laminar kinetic energy, k_L , and the time scale $\omega = \epsilon/k$, where ϵ is the turbulent dissipation.
- The k_L equation describes the low-frequency laminar oscillations in the pre-transitional flow and allows prediction of natural transition.
- The model uses only local information and is therefore compatible with modern CFD codes including bypass transition. data for zero pressure gradient flows (see e.g. (Walters and Cokljat, 2008)) but predicts late transition for adverse pressure gradient flows at low free-stream turbulence levels. (Fürst et al., 2013).

Applications

Two Modifications of the $k\text{-}k_L\text{-}\omega$ Transition Model based on Pohlhausen and Falkner-Skan Profiles

Fürst, Islam, Pøihoda, Wood

Introduction

Modifications

Computational Domain and the Mesh

Details of the OpenFOAM Cases

Concluding Remarks

Acknowledgment

References

References

- Solidity effects on the lift and drag of blade elements modeled as a cascade. This can't be done with Xfoil and wind tunnel measurements are difficult.
- Airfoil lift and drag for Reynolds number, Re , down to 10,000 for medium and small wind turbine analysis, including multidimensional blade optimization.
- For these applications, prediction of transition can be more important than the turbulence model.
- This is work in progress.

Modifications

Two
Modifications of
the k - k_L - ω
Transition Model
based on
Pohlhausen and
Falkner-Skan
Profiles

Fürst, Islam,
Pøihoda, Wood

Introduction

Modifications

Computational
Domain and the
Mesh

Details of the
OpenFOAM
Cases

Concluding
Remarks

Acknowledgment

References

References

- The original model shows very good agreement with experimental data for zero pressure gradient flows (see e.g. (Walters and Cokljat, 2008)) but predicts late transition for adverse pressure gradient flows at low free-stream turbulence levels. (Fürst et al., 2013).
- Both the stability analysis and experiments show that the pressure gradient has a big influence on transition (Schlichting and Gersten, 2003).
- The production of k_L and transition are initiated when the vorticity Reynolds number, Re_Ω , exceeds some thresholds. Our modifications make these thresholds dependent on the dimensionless pressure gradient.
- The threshold for the production of Tollmien-Schlichting waves was obtained using linear stability limits for the Orr-Sommerfeld equation using Pohlhausen (Schlichting and Ulrich, 1942) and Falkner-Skan (Wazzan et al., 1968) profiles for the laminar boundary layer.

Modifications

Thresholds

Two
Modifications of
the $k\text{-}k_L\text{-}\omega$
Transition Model
based on
Pohlhausen and
Falkner-Skan
Profiles

Fürst, Islam,
Pøihoda, Wood

Introduction

Modifications

Computational
Domain and the
Mesh

Details of the
OpenFOAM
Cases

Concluding
Remarks

Acknowledgment

References

References

- In both cases, the threshold decreases with increasing adverse pressure gradients.
- A similar modification is used for the second threshold controlling the transfer of laminar kinetic energy to turbulent kinetic energy (and hence the natural transition process).
- The price for these modifications is that the model requires local free-stream velocity which can be obtained relatively easily from the static pressure and total pressure difference.

Modifications

Pohlhausen

Two
Modifications of
the $k\text{-}\kappa\text{-}\omega$
Transition Model
based on
Pohlhausen and
Falkner-Skan
Profiles

Fürst, Islam,
Pøihoda, Wood

Introduction

Modifications

Computational
Domain and the
Mesh

Details of the
OpenFOAM
Cases

Concluding
Remarks

Acknowledgment

References

References

- Pohlhausen profiles: $U(y)/U_e = 2\eta - 2\eta^3 + \eta^4 + \frac{\Lambda}{6}\eta(1 - \eta)^3$,
with $\eta = y/\delta$ and $\Lambda = \frac{\delta^2}{\nu} \frac{dU_e}{dx}$.
- Linear stability limit: based on Orr-Sommerfeld eq., calculation
by (Schlichting and Ulrich, 1942) $\Rightarrow Re_{ind} = f(\Lambda)$.
- It is difficult to obtain δ (and Λ) in general CFD codes, but one
can express $\Lambda = \Lambda(L)$ where $L = Re_{\Omega, max}^2 \frac{\nu}{U_e^2} \frac{dU_e}{dx}$.
- “Almost local” formulation for APG flows is obtained by
replacing $Re_{\Omega, max}$ with Re_{Ω}

New correlation for the onset of instability $C_{TS, crit}$

$$C_{TS, crit}^{APG} = \frac{536.4}{1 - 8.963L}, \text{ for } -1.5 \leq L \leq 0.$$

IN original model $C_{TS, crit} = 1000$.

New correlation for natural transition $C_{NAT, crit}$

$$C_{NAT, crit}^{APG} = \frac{1250}{1 - 8.963L}, \text{ for } -1.5 \leq L \leq 0.$$

In original model $C_{NAT, crit} = 1250$.

Modifications

Falkner-Skan profiles

Two
Modifications of
the $k\text{-}\kappa\text{-}\omega$
Transition Model
based on
Pohlhausen and
Falkner-Skan
Profiles

Fürst, Islam,
Pøihoda, Wood

Introduction

Modifications

Computational
Domain and the
Mesh

Details of the
OpenFOAM
Cases

Concluding
Remarks

Acknowledgment

References

References

- Linear stability limit calculated by (Wazzan et al., 1968),

$$Re_{\theta,ind} = f(\lambda), \text{ where } \lambda = Re_{\theta}^2 \frac{\nu}{U_e^2} \frac{dU_e}{dx}.$$

$$Re_{\theta,ind} \approx \frac{200.69}{1 - 55.287\lambda + 3.4992 \cdot 10^5 \lambda^4}.$$

- Using Falkner-Skan profiles one can get (for $-1.5 < L \leq 0$)

$$C := \frac{Re_{\Omega,max}}{Re_{\theta}} \approx 2.1884 (1 - 0.95419L - 0.13183L^2).$$

New correlation for the onset of instability $C_{TS,crit}$

$$C_{TS,crit}^{APG}(L) := Re_{\Omega,ind} = C Re_{\theta,crit}(L/C^2).$$

In original model $C_{TS,crit} = 1000$.

New correlation for natural transition $C_{NAT,crit}$

$$C_{NAT,crit}^{APG} := (C_{TS,crit}^{APG}(L) / C_{TS,crit}^{APG}(0)) C_{NAT,crit}$$

In original model $C_{NAT,crit} = 1250$.

Computational Domain and the Mesh

O-Type grid

- An O-Type grid was used as it is more appropriate for CFD analysis of airfoils with blunt trailing edges.

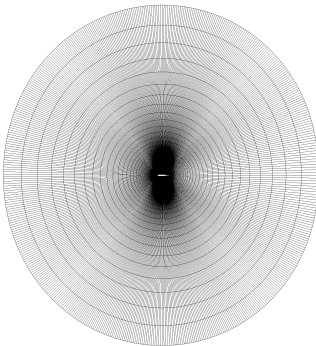


Figure: NACA 0012

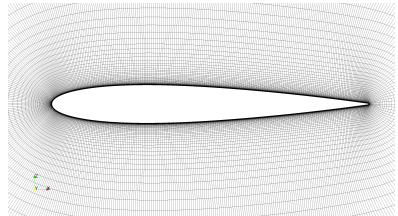


Figure: Close-up NACA 0012

Two
Modifications of
the $k\text{-}\kappa\text{-}\omega$
Transition Model
based on
Pohlhausen and
Falkner-Skan
Profiles

Fürst, Islam,
Pøihoda, Wood

Introduction

Modifications

Computational
Domain and the
Mesh

Details of the
OpenFOAM
Cases

Concluding
Remarks

Acknowledgment

References

References

Computational Domain and the Mesh

Mesh around the Leading and Trailing Edges of the Airfoils

Two
Modifications of
the $k\text{-}\kappa\text{-}\omega$
Transition Model
based on
Pohlhausen and
Falkner-Skan
Profiles

Fürst, Islam,
Pøihoda, Wood

Introduction

Modifications

Computational
Domain and the
Mesh

Details of the
OpenFOAM
Cases

Concluding
Remarks

Acknowledgment

References

References

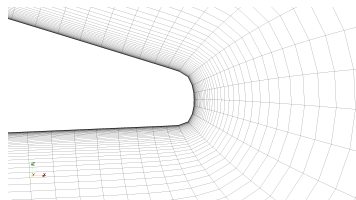
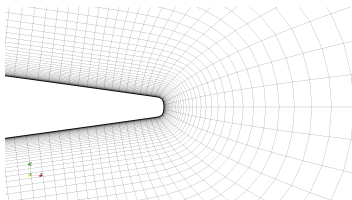
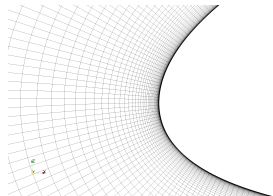
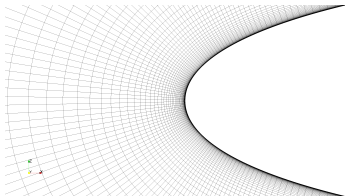


Figure: (a) NACA 0012

Figure: (b) NACA 4415

Details of the OpenFOAM Cases

Two
Modifications of
the $k\text{-}k\text{-}\omega$
Transition Model
based on
Pohlhausen and
Falkner-Skan
Profiles

Fürst, Islam,
Pøihoda, Wood

Introduction

Modifications

Computational
Domain and the
Mesh

Details of the
OpenFOAM
Cases

Concluding
Remarks

Acknowledgment

References

References

The modified $k\text{-}k\text{-}\omega$ model with pressure-gradient sensitive thresholds was implemented in OpenFOAM and validated for several test cases including:

- NACA 0012 at Reynolds number $Re = 600,000$ and free-stream turbulence level $Tu = 0.3\%$ corresponding to experimental data of Lee and Kang (Lee and Kang, 2000) at angle of attack, $\alpha = 0^\circ$, and
- NACA 4415 at $Re = 700,000$ with $Tu = 0.02\%$, also at $\alpha = 0^\circ$.

Results

NACA 0012 at $Re = 600,000$, $Tu = 0.3\%$, $\alpha = 0^\circ$

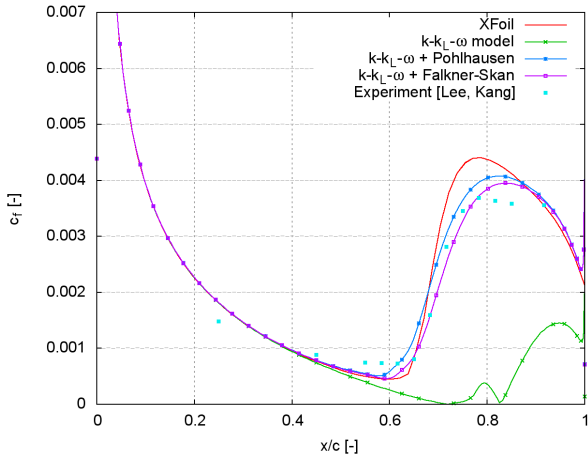


Figure: Local friction coefficient

Two Modifications of the k-k_L- ω Transition Model based on Pohlhausen and Falkner-Skan Profiles

Fürst, Islam, Pohlhoda, Wood

Introduction

Modifications

Computational Domain and the Mesh

Details of the OpenFOAM Cases

Concluding Remarks

Acknowledgment

References

References

Results

NACA 4415 at $Re = 700,000$, $Tu = 0.02\%$, $\alpha = 0^\circ$

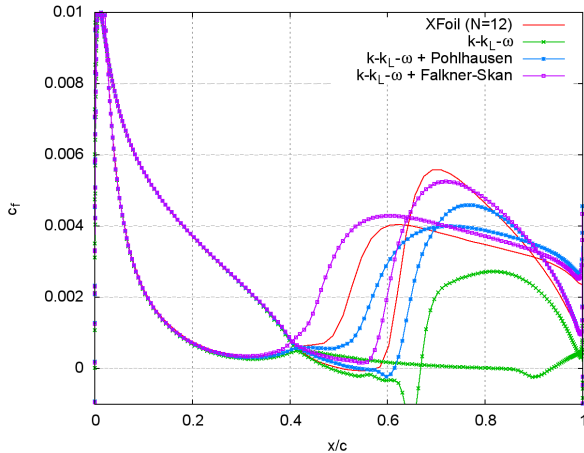


Figure: Local friction coefficient

Two Modifications of the $k-k_L-\omega$ Transition Model based on Pohlhausen and Falkner-Skan Profiles

Fürst, Islam, Pøihoda, Wood

Introduction

Modifications

Computational Domain and the Mesh

Details of the OpenFOAM Cases

Concluding Remarks

Acknowledgment

References

References

Results

Table: Lift and Drag Coefficients for NACA 0012 at $\alpha=0^\circ$

	C_l	C_d
Original $k - k_L - \omega$	-2.5×10^{-7}	0.0057
$k - k_L - \omega$ with Falkner-Skan	-4.2×10^{-7}	0.00693
$k - k_L - \omega$ with Pohlhausen	-3.2×10^{-8}	0.00708
McCroskey (1987) data correlation	0	0.00685

Table: Lift and Drag Coefficients for NACA 4415 at $\alpha=0^\circ$

	C_l	C_d
Original $k - k_L - \omega$	0.442	0.00778
$k - k_L - \omega$ with Falkner-Skan	0.454	0.00996
$k - k_L - \omega$ with Pohlhausen	0.455	0.00948
Miley (1982)	0.45	0.0094
Hoffmann et al. (1996) ($Re = 750,000$)	0.43	0.0081

Two

Modifications of
the $k-k_L-\omega$
Transition Model
based on
Pohlhausen and
Falkner-Skan
Profiles

Fürst, Islam,
Pøihoda, Wood

Introduction

Modifications

Computational
Domain and the
Mesh

Details of the
OpenFOAM
Cases

Concluding
Remarks

Acknowledgment

References

References

Concluding Remarks

Two Modifications of the $k\text{-}k\text{-}\omega$ Transition Model based on Pohlhausen and Falkner-Skan Profiles

Fürst, Islam, Pøihoda, Wood

Introduction

Modifications

Computational Domain and the Mesh

Details of the OpenFOAM Cases

Concluding Remarks

Acknowledgment

References

References

- The original model without pressure-gradient modification predicts delayed transition and separation at least on the NACA 4415.
- Both the new correlations based on Pohlhausen and Falkner-Skan profiles agree with the experimental data for the NACA 0012, as well as with the results from the of Xfoil.
- The Falkner-Skan based correlation predicts the transition slightly earlier than the Pohlhausen based one.
- Despite these differences, both modifications show clear improvement in comparison with the original model.

Acknowledgment

Two
Modifications of
the $k\text{-}k_L\text{-}\omega$
Transition Model
based on
Pohlhausen and
Falkner-Skan
Profiles

Fürst, Islam,
Pøihoda, Wood

Introduction

Modifications

Computational
Domain and the
Mesh

Details of the
OpenFOAM
Cases

Concluding
Remarks

Acknowledgment

References

References

- Preparation of the article was partly supported by the **Canadian Natural Sciences and Engineering Research Council (NSERC)** and the **ENMAX Corporation** under the Industrial Research Chairs program.

- Fürst, J., Přihoda, J., and Straka, P. (2013). Numerical simulation of transitional flows. *Computing*, 95(SUPPL.1):S163–S182.
- Hoffmann, M., Ramsay, R. R., and Gregorek, G. M. (1996). Effects of grit roughness and pitch oscillations on the naca 4415 airfoil. Technical report, OSU NREL/TP-442-7815.
- Lee, H. and Kang, S.-H. (2000). Flow characteristics of transitional boundary layers on an airfoil in wakes. *Journal of Fluids Engineering*, 122(3):522–532.
- McCroskey, W. J. (1987). A critical assessment of wind tunnel results for the naca 0012 airfoil. Technical report, NASA Tech Memorandum 100019.
- Miley, S. J. (1982). A Catalog of Low Reynolds Number Airfoil Data for Wind Turbine Applications. Technical report, US Dept Energy.
- Schlichting, H. and Gersten, K. (2003). *Boundary-Layer Theory*. Springer.

- Schlichting, H. and Ulrich, A. (1942). Zur Berechnung des Umschlages laminar-turbulent. *Jb. dt. Luftfahrtforschung* 1, 1:8–35.
- Walters, D. K. and Cokljat, D. (2008). A Three-Equation Eddy-Viscosity Model for Reynolds-Averaged Navier–Stokes Simulations of Transitional Flow. *Journal of Fluids Engineering*, 130:121401–1 121401–6.
- Wazzan, A. R., Okamura, T. T., and Smith, A. M. O. (1968). Spatial and Temporal Stability Charts for the Falkner-Skan Boundary-Layer Profiles. Technical report, Douglas Aircraft Co.