Chapter 2

Analytical Structural Vibration Prediction

Vibration prediction, as it relates to this study, is the ability to "predict" the vibration isolation achieved by a mount through using the results of a single experiment. An explanation of structural vibration prediction, along with a previous simulation approach and an alternative prediction method, are discussed in this chapter.

2.1 Vibration Prediction

The level of vibrations due to various types of excitation in a structure can be measured or approximated experimentally or analytically. The experimental process has generally been the accepted method, while analytical techniques such as finite element methods are typically used to estimate the structural vibrations to a given input. Experimental analysis is often used to verify the model results for selected structure arrangements. A clear advantage to the analytical process is the flexibility of making "what if" investigations much faster and easier than altering actual structures and performing experiments. Accurate prediction of structural vibration response, however, requires large models that are often expensive to prepare and computationally intensive. Smaller models, although relatively easy to prepare and run, can result in significant inaccuracies, rendering the results relatively useless.

In this study, it is desired to predict the vibration isolation effect of various mounting systems in a realistic structure, such as a locomotive cab, through analytical modeling and experimental testing. Specifically, it is desired to know how much more vibration isolation a softer mount can provide, and conversely, how much the vibrations increase due to a stiffer mount. By using the results of a single experiment on an isolated cab, a prediction can be made about the level of vibrations on or inside the cab with a different isolator. To predict the isolation effect of a different set of mounts, results of the single experiment are used to extract the necessary structural characteristics, such as mechanical impedance. The information is then used in conjunction with new mount characteristics to predict the vibration levels due to the new mount.

2.2 Previous Simulation Approach

In a previous study, an attempt was made to combine selected test results with analytical models of soft mounts to estimate vibration transmission due to different mounts [14]. The model was designed to include the mount transmissibility to predict the vibration isolation at various dynamic frequencies, assuming the single-degree-of-freedom configuration, shown in Fig. 2.1. The mount transmissibility magnitude T was expressed as

$$T = \left|\frac{Y}{X}\right| = \sqrt{\frac{k^2 + (c\omega)^2}{(k - m\omega^2)^2 + (c\omega)^2}} \qquad (2.1)$$

where

$$\omega = \sqrt{\frac{k}{m}} \qquad (2.2)$$

represents the natural frequency of the system. Parameters k and c represent the mount stiffness and damping, respectively, while the suspended mass is represented by m. Finally, X and Y represent the input and output from the single-degree-of-freedom configuration, respectively.

A simplified representation of the cab in a soft-mounted configuration is shown in Fig. 2.1. The excitation *X* is assumed to include the structural dynamics of the base and suspended structure, and therefore can be represented by actual data collected at the base of the mount.

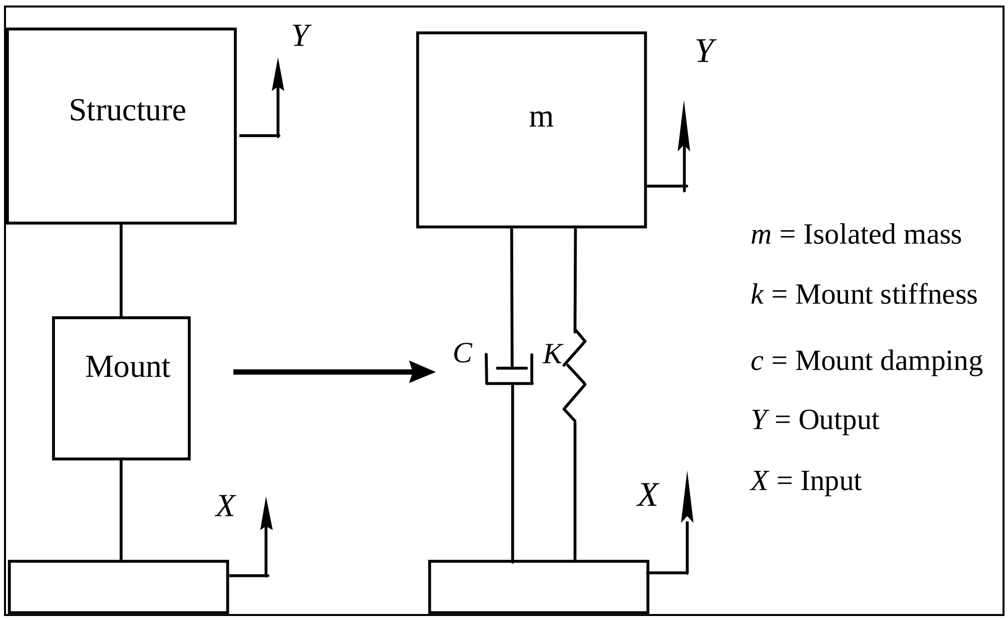

Figure 2.1 Single-Degree-of-Freedom Representation of a Mounted Structure

Therefore, knowing the transmissibility ratio of the mount, as expressed in Eq. (2.1), and the actual excitation *X* at the base of the mount, the structural response can be estimated by

$$|Y| = T * |X| \tag{2.3}$$

The evaluation of this approach was accomplished by using MATLAB to calculate the transmissibility, *T*, and the response, *Y*, for a given input, *X*, and mounting parameters *m*, *c*, and *k*. The results from the program were compared with actual experimental results, as shown in Figs. 2.2 and 2.3. Figures 2.2 and 2.3 show a comparison between the test and simulation results using an elastomeric mount with a vertical stiffness of 10,000 lb/in and a damping ratio, ξ, of 0.05 and 0.90, respectively.

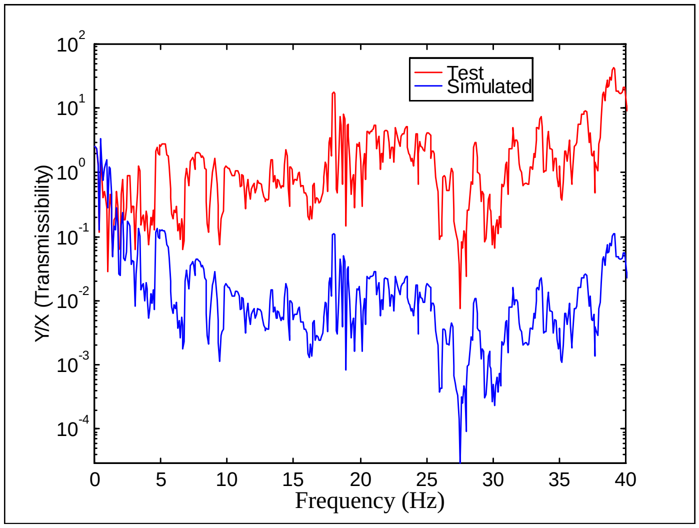

Figure 2.2 Comparison Between Test and Simulation Results for ζ = 0.05

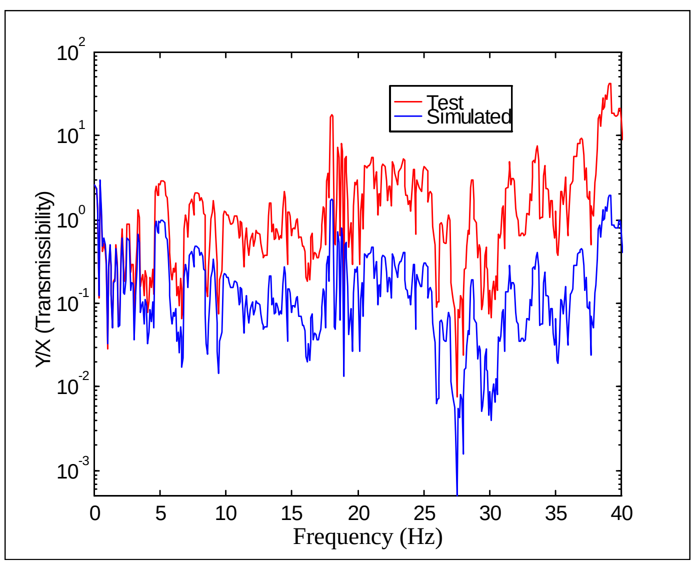

Figure 2.3 Comparison Between Test and Simulation Results for ζ = 0.90

This simulation approach did not work due to the lack of representation of the structural dynamics and mount complexities in the model. The excitation input *X* to the structure does not represent the structural dynamics. It was assumed in this approach that the excitation input to the cab would remain constant and therefore could be used in conjunction with the transmissibility of the mounts to solve for the response of the cab. However, due to the coupling and resulting dynamics between the cab and sill structures, the excitation input does not remain constant. Further, the actual transmissibility across the mounts, shown in Fig. 2.4, was not represented in the simulation model of the mount, as shown in Fig. 2.5. The vast difference between the measured and modeled transmissibilities in Figs. 2.4 and 2.5 caused the significant errors between the predicted and measured structure responses, shown in Figs. 2.2 and 2.3.

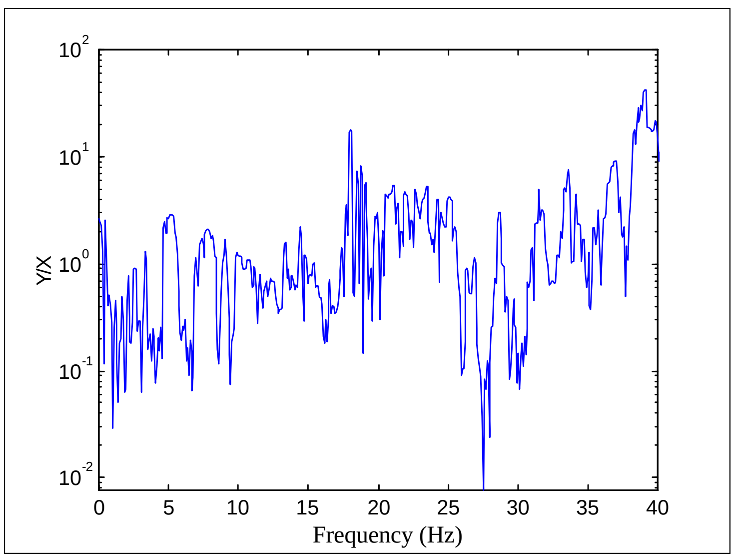

Figure 2.4 Measured Mount Transmissibility

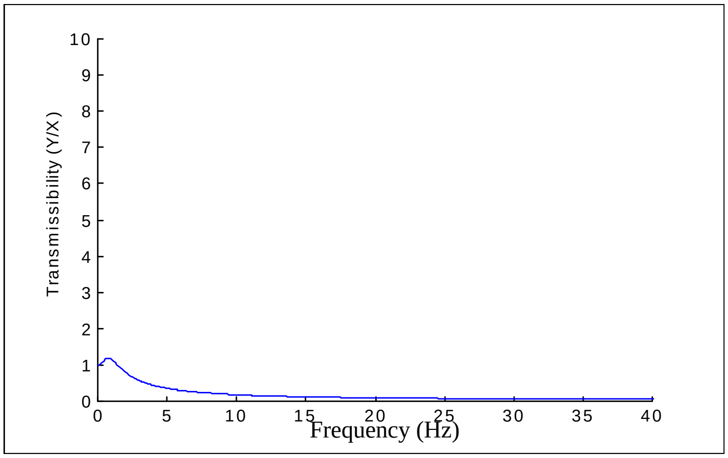

Figure 2.5 Mount Transmissibility Included in Simulation

2.3 New Simulation Approach

The approach for this study is based on solving for the mechanical impedance of the cab and sill, and using the results in conjunction with experimental data to approximate or predict structural vibrations.

The mechanical impedance of an element is defined as the ratio of the driving force acting on the element to the resulting velocity [15]. Experimental measurements of mechanical impedance for an element cannot be made due to the constraint requirements. Because such constraints are not possible for experimental analysis, mechanical impedance of a system can be approximated analytically.

Similar to the previous study, a model representing the mount system was created, as shown in Fig. 2.6. The parameters Z_s, Z_m, and Z_e represent the cab impedance, the mount impedance, and the sill impedance, respectively.

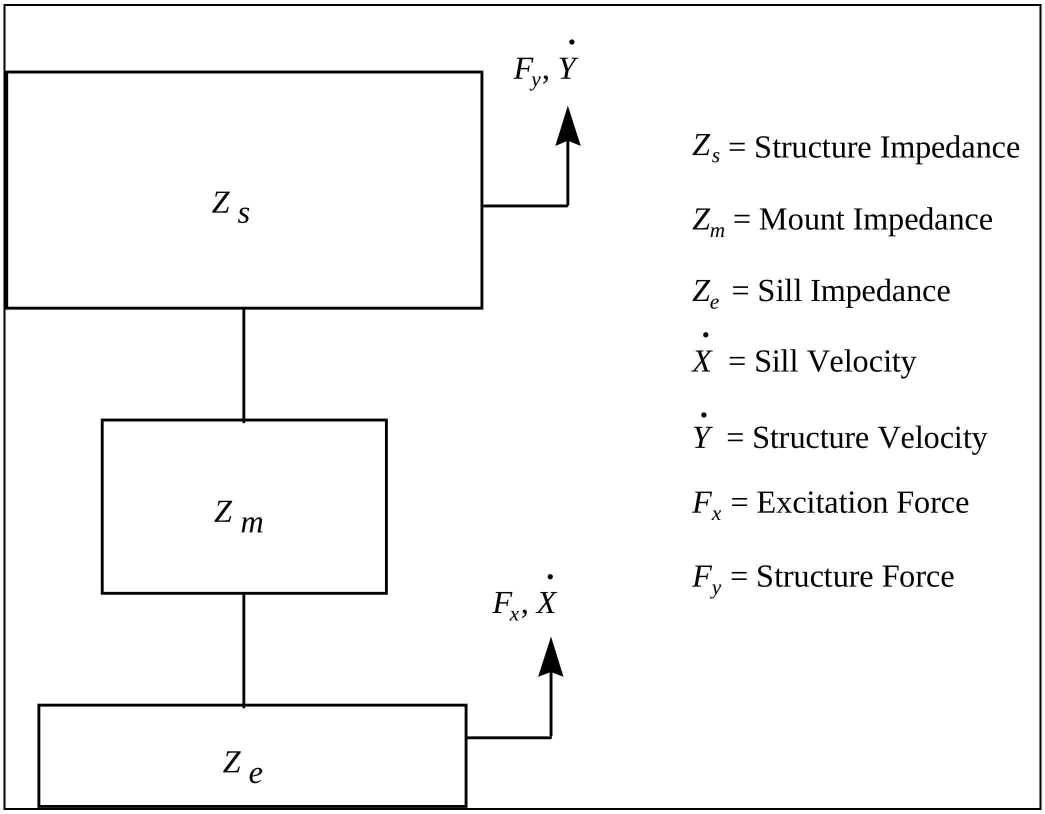

Figure 2.6 Mechanical Impedance of a Base-Excited System

The equations for Z_e and Z_s can be written as

$$Z_e = \frac{F_x}{\dot{X}} + \frac{Z_m \dot{Y}}{\dot{X}} - Z_m \tag{2.4}$$

$$Z_s = \frac{Z_m \dot{X}}{\dot{Y}} - Z_m \tag{2.5}$$

based on the summation of forces in Fig. 2.6, where F_y=0. Modeling the elastomeric mount as a parallel spring and damper, as shown in Fig. 2.1, the impedance of the mount can be represented by

$$Z_m = C + \frac{k}{jw} \quad (2.6)$$

where

$$C = 2*\xi*m*\omega_n = 2*\xi*\sqrt{(k*m)} \quad (2.7)$$

Parameters k and c represent the mount stiffness and damping respectively, while the cab mass distributed onto each elastomeric mount is represented by m. Finally, ξ is used to represent the damping coefficient of the mount.

Once the sill, structure, and mount impedances are known, Eq. (2.4-2.5) can be solved for the sill and structure velocities. The resulting velocity equations are

$$\text{Sill Velocity} = \dot{X} = \left(\frac{Z_s + Z_m}{Z_m}\right)\left(\frac{F_x}{(Z_m + Z_e)\left(\frac{Z_s + Z_m}{Z_m}\right) - Z_m}\right) \quad (2.8)$$

$$\text{Structure Velocity} = \dot{Y} = \left(\frac{F_x}{(Z_m + Z_e)\left(\frac{Z_s + Z_m}{Z_m}\right) - Z_m}\right) \quad (2.9)$$

In review, the simulation approach is used to approximate or predict structural velocities of a locomotive cab. A known input is used to introduce energy into the structure. The force applied to the structure from the input is collected, along with the sill and structure velocities. The results, $\dot{X}, \dot{Y}$, and F_x are used to determine the structural and mount impedance using Eqs. (2.4-2.6). Next, the impedance characteristics due to a new mount are solved using Eq. (2.6). Assuming that the mechanical impedance of the structure and sill do not change from using new mounts, Z_e, Z_s, and Z_m can be used to solve for the structure and sill velocities from Eqs. (2.4-2.5).

2.3.1 Analytical Model

The simulation approach that was discussed in the previous section was used to develop an analytical modeling scheme in MATLAB. The program was written to use experimental data along with analytical approximations to predict structural velocities. In Fig. 2.7, a step-by-step process of the simulation is reviewed. The columns "Experiment," "Model," and "Prediction" will be discussed separately in the following sections.

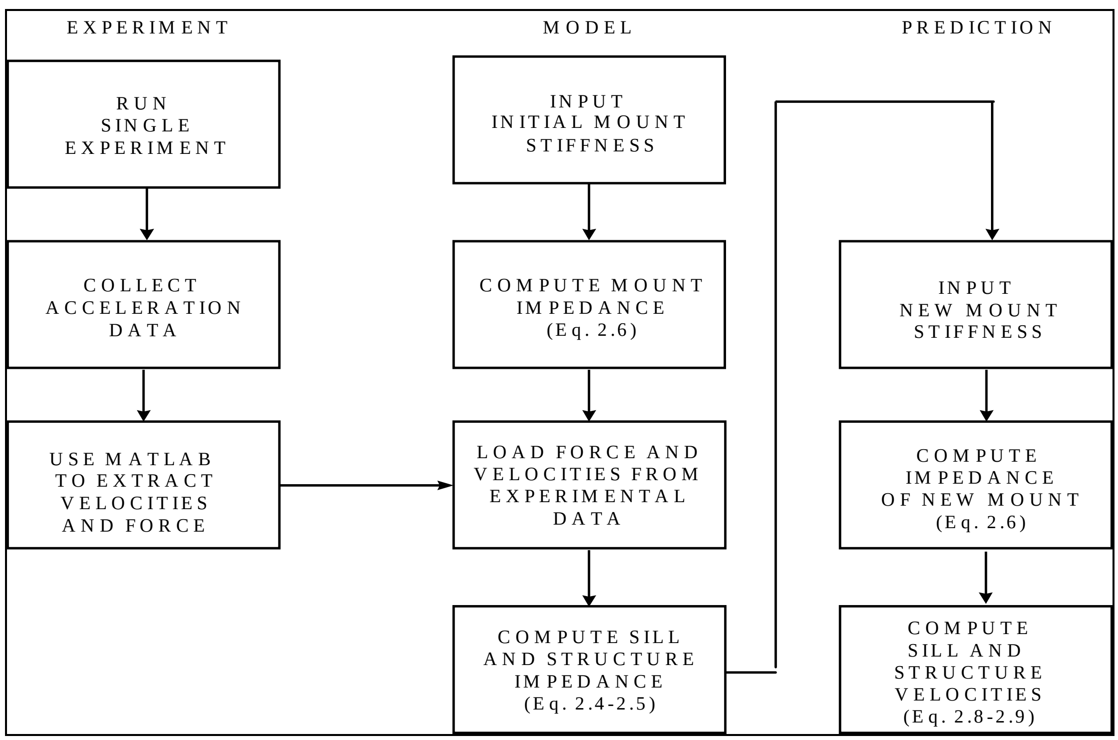

Figure 2.7 Simulation Process Overview

2.3.1.1 Experiment

First, a single experiment on the structure is run by introducing energy into the structure and collecting acceleration data at key locations using a MATLAB code. The force input from an actuation system is also measured in the experiment. To minimize numerical errors, accelerations are converted into velocities in the frequency domain by dividing the accelerations by jω. Numerical integration in the time domain can also provide velocity results, except that the percent error in the approximations would be greater than the followed approach. The results of the code are exported into data files to be used by the model code. This part of the process is labeled on the first column of Fig. 2.7 as "Experiment."

The force input, F_x, from the actuator is collected in the experiment and is shown in Fig. 2.8. The force input is collected in conjunction with the acceleration data at key locations and the results are sent to the model.

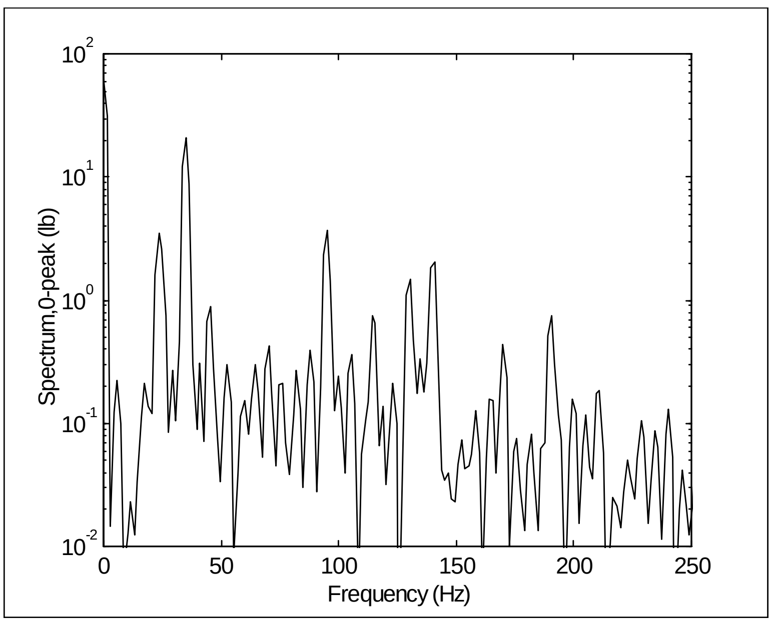

Figure 2.8 Force Plot from the Actuator

2.3.1.2 Model

The next column, titled "Model," deals with another MATLAB code which is used to complete the velocity approximations. The first step of the program requires the input of the vertical mount stiffness value from the single experiment to calculate the impedance of the mounts as a function of the frequency. The vertical stiffness value for the mount is determined by reviewing the specification sheet on the mount [16]. The specification sheet provides information on the load and deflection values of the mount. The vertical stiffness is determined by the ratio of the load over the deflection. Next, the sill and structure velocities, along with the force results of the single experiment, are loaded into MATLAB. Using the appropriate equations, the data is used to determine the sill and structure impedances.

2.3.1.3 Prediction

The final column listed in Fig. 2.7 is labeled "Prediction." The first step toward predicting the sill and structure velocities is to determine the change in impedance from using a mount having a higher or lower vertical stiffness. Inputting the vertical stiffness of the mount results in the computation of its impedance as a function of frequency. Finally, the new mount impedance results are used in conjunction with the sill and structure impedances to predict the velocities of the structures.