Chromatic Dispersion Measurement

in Single-Mode Optical Fibers

by Acousto-optic Modulation and Phase Detection

by

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(ABSTRACT)

A system to measure chromatic dispersion in single-mode optical fibers with near point wavelength resolution is described. Differential propagation constant data for the test fiber is collected by using an acousto-optic modulator to frequency-shift laser light passed to an optical heterodyne. Theoretical developments include a frequency-domain calculation technique for interpreting empirical results. The system is proven infeasible, however, due to inherent noise sensitivity.

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Of greatest help throughout has been my advisor, Dr. R. O. Claus. Without his initiative, this research would not have begun. Without his continuing support and guidance, it would not have reached its conclusion.

Several gentlemen of Siecor Corporation have provided valuable technical advice and encouragement. I am speaking particularly of Dr. C. Eoll, A. Garg, Dr. D. Vokey, and Dr. D. Schicketanz. Dr. Eoll further served on my graduate committee. Thanks also to Siecor for their generous funding of this research.

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The nifty illustrations are all by R. Rogers.

Perhaps less visibly, others have contributed "behind the scenes" in preparing me to take on a Master's thesis in the first place. Most of this credit belongs to my family. However, I would also like to recognize those who struggled to improve my writing style; it wasn't all wasted time.

Table of Contents

List of Illustrations

1.0 Introduction

1.1 Dispersion: General Considerations

An optical fiber is not a perfect transmission medium. Signals passing through one are subject to dispersion, a distorting mechanism present in all waveguides. In a simple model, suppose that photons entering the fiber simultaneously may reach the opposite end at slightly different times. A pulse of light propagating in the fiber will not retain its initial shape, but will spread out and blur at the edges, gradually overlapping with neighboring pulses. This effect tends to limit the bandwidth and distance between repeater stations in digital communications systems.

Dispersion is the net result of several mechanisms, categorized as either intermodal or intramodal. Intramodal dispersion manifests itself as a variation in light group velocity with wavelength; it is also called "chromatic" dispersion. If fiber systems were to use truly monochromatic sources, all of the light would have the same wavelength (neglecting modulation effects) and would propagate at uniform velocity; chromatic dispersion would not be a concern. In practice, however, LEDs and even lasers have nonzero spectral widths, and the component wavelengths making up a signal separate.

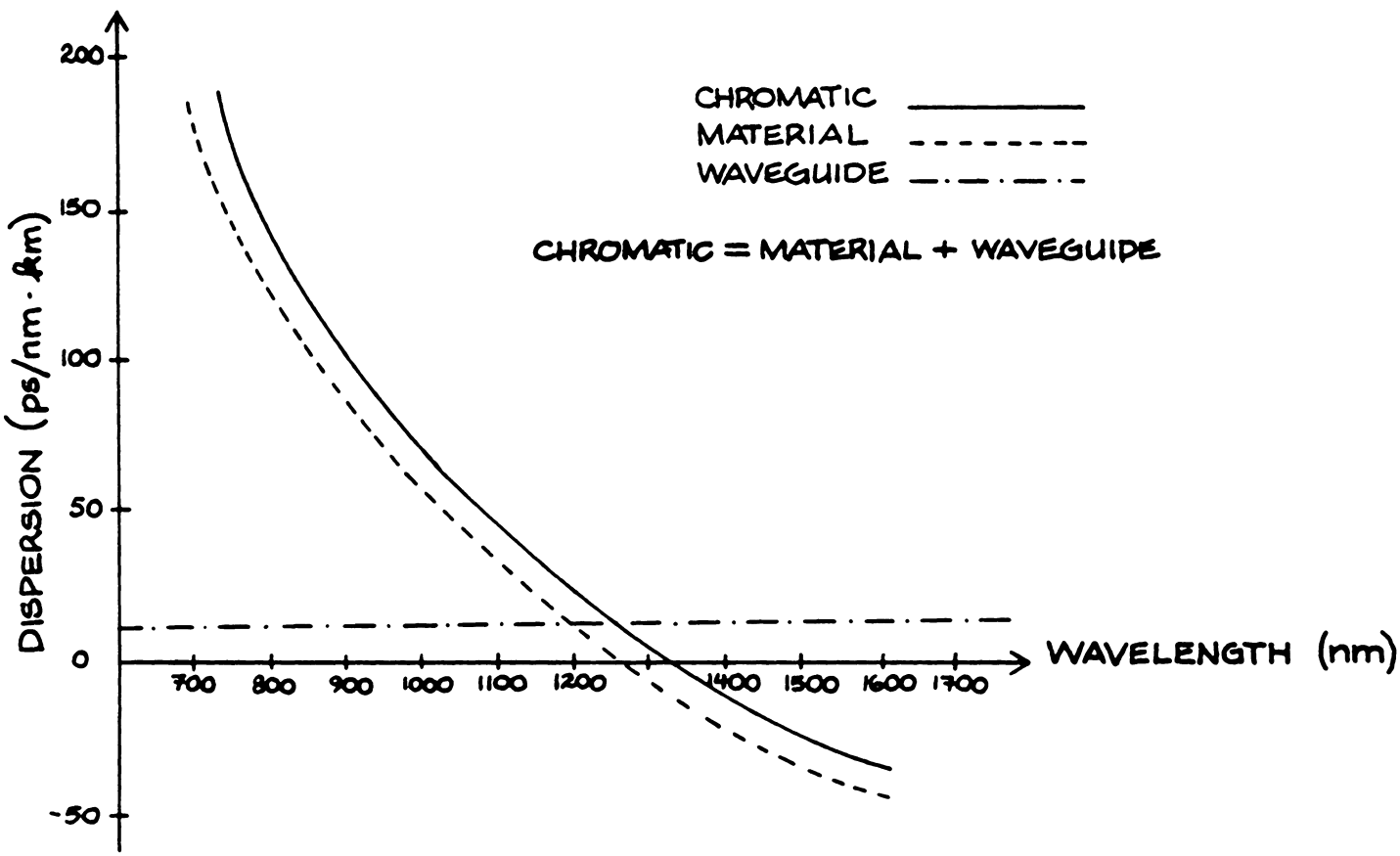

Figure 1. Typical values of intramodal dispersion components

Intramodal dispersion may itself be subdivided into waveguide and material dispersion; Figure 1 on page 2 shows approximate typical values. The former is due to purely geometric properties of the fiber such as core diameter and refractive index profile, and is only mildly wavelength dependent. Theoretical considerations show that it is typically dominated by material dispersion [1]. As the name implies, material dispersion is characteristic of the substance from which the fiber is made. It arises from a variation in refractive index, n, with wavelength, and may be estimated from measurements in bulk glass. However, the fiber drawing process stresses the material and may introduce random anomalies, so measurements on actual fibers are preferred.

Waveguide and material dispersion are usually assumed independent in system design, their magnitudes simply summed to approximate the net effect. This is not strictly correct however, the true interaction being complex and poorly understood. Fortunately, there is no qualitative difference between their effects on a signal, and the two are adequately summarized by a single intramodal dispersion measurement or specification. It is that collective measurement that is the object of this research.

In the mid 1970s it was predicted that waveguide and material dispersion would cancel somewhere in the wavelength range of 1200 to 1300 nm, where the latter crosses through a zero point [2]. Practical sources and detectors were not yet available in this wavelength region, but the zero chromatic dispersion (ZCD) point was recognized for its considerable practical importance. Very high transmission rates are possible at (or near) this wavelength, even with relatively broad spectrum LED sources. As practical devices became available at longer wavelengths, designers began to consider these dispersion advantages seriously, putting them to use in high-performance communications systems. Though material dispersion is more or less fixed for a given fiber composition, it was later realized that the precise wavelength of ZCD could be shifted to any point in the range 1300 to 1600 nm through careful choice of the fiber parameters governing waveguide dispersion [3]. One may take advantage of this knowledge by theoretically matching ZCD wavelength to the point of minimum fiber attenuation, simultaneously achieving optimum performance on both counts.

Intermodal dispersion describes wavelength-independent pulse spreading in multimode fibers. The chief theoretical advantage of single-mode fibers is freedom from this effect; "dispersion" and

"chromatic dispersion" are made synonomous. Since single-mode fibers are the exclusive focus of this research, references to "dispersion" can be taken to mean intramodal unless specifically stated otherwise.

1.2 Dispersion Defined

Having described dispersion qualitatively, we are now in a position to consider a more formal mathematical treatment. Let τ represent the fiber transit time or "group delay", i.e. the time it takes light energy to pass through a fiber of length L. Then chromatic dispersion is defined as the slope of τ versus light wavelength, λ. It is usually given normalized to a 1 km fiber length:

$$D \equiv \frac{1}{L}\frac{\partial \tau}{\partial \lambda} \tag{1.2.1}$$

The reason for the derivative in this definition may not be immediately obvious. It has to do with the way a dispersion specification is normally used in calculating system performance. The pulse broadening, $\Delta\tau$, to be expected in a given fiber at a particular wavelength is directly proportional to both fiber length and source spectral width, $\Delta\lambda$. The constant of proportionality is taken to be the chromatic dispersion specification at the center wavelength of the source:

$$\Delta\tau = DL(\Delta\lambda)$$

The system designer combines this information with the data transmission rate and other system risetime characteristics to evaluate performance.

1.3 Overview of Research

The definition of chromatic dispersion is based on a derivative with respect to wavelength; any measurement system will necessarily entail empirical data taken at more than one point on the wavelength axis. Unfortunately, dispersion is wavelength dependent, making the use of a wide interval undesirable. Simply put, the result is no longer a point slope measurement, but rather some approximation to slope over the interval. The measurement systems described in the literature either plug empirical data into a numerical differentiation formula or fit them to a curve, from which the slope is found analytically. Most rely on data taken several nanometers apart in wavelength, and many use intervals of tens of nanometers. The technique developed in this research is an attempt to make the measurement with close to point wavelength resolution by carrying out the entire procedure in the frequency domain. Narrow resolution depends on the fact that the range of light wavelengths used in fiber optics corresponds to very high frequencies, so high that it is usually inconvenient to specify them explicitly. A frequency change of Megahertz is orders of magnitude smaller; introducing such a shift has negligible effect on the wavelength axis. Thus, even though such frequency deviations from the nominal measurement point are used, the calculated result may be considered representative of a single wavelength.

Chapter two reviews chromatic dispersion measurement systems described in the literature over the past fifteen years. Chapter three describes in some detail the technique developed in this research, then summarizes both empirical and theoretical results to draw some conclusions about the method's practicality. Finally, chapter four presents a concise summary of the research.

2.0 Review of the Literature

The literature on chromatic dispersion measurement systems shows clear progressive trends toward high accuracy, broad wavelength coverage, and empirical simplicity. Over time, attention has shifted to longer wavelengths, paralleling developments in practical fiber communication systems. Several comprehensive reviews exist, notably [4] and [5]. Both of these trace important developments and include extensive theoretical discussions on all forms of dispersion.

Practical interest in chromatic dispersion arises from a pragmatic concern over the the broadening of light pulses in communication systems; an obvious measurement scheme might be to measure this broadening directly at the wavelength of interest. That is, monitor light pulses at the fiber input and output, and simply report a comparison. Though this approach has appeared in the literature, it fails to distinguish between intramodal and intermodal effects in multimode fibers, forcing the experimenter to estimate their relative contributions [6]. Where attention is strictly limited to the single-mode case, this approach has occasionally proven useful as a simple check for theoretical results, but does not give high precision [7].

2.1 *Measurement in the Time Domain*

Time domain techniques are based on a simple idea: find the derivative appearing in the defining equation (1.2.1) directly by measuring fiber group delay, τ, at several closely spaced wavelengths and then performing numerical differentiation. Figure 2 on page 8 shows an apparatus designed to get this information. The oscilloscope trigger is derived either from a detector at the fiber input (as shown) or from the pulse generator used to drive the light source. This signal passes through a variable delay line, adjusted so that the detected pulse at the fiber output is displayed. As the source wavelength is varied, differences in τ appear as horizontal displacements on the screen. Alternatively, a calibrated delay line may be adjusted as λ is varied to keep the horizontal position of this pulse on the screen constant. This method works even in multimode fibers, since variations in τ with λ are due only to intramodal dispersion. Intermodal dispersion widens pulses of all wavelengths equally, and is not of concern unless it becomes difficult to accurately distinguish the time shift of the wider pulses.

To get the derivative information, a variable wavelength light source is needed. For example, one early system used a krypton laser tunable to three wavelengths [8]. Another made use of two separate lasers [9]. However, the paucity of appropriate light sources at closely spaced wavelengths proved a difficult problem at first.

An important innovation, first appearing in 1974, was the introduction of a Raman cell in the source. A Raman cell shifts the wavelength of incoming light either up or down by interaction with optical phonons in the cell material. The longer wavelengths or "Stokes lines" are typically of interest. In long cells, multiple effects are possible, each line acting as a pump for its successor. The first such Raman cell measurement system used cells of benzene or water to provide wavelengths between 694 and 930 nm. This extended the range of coverage, but failed to reach the near infrared region of ZCD [10].

The first experimental measurements on both sides of this point were not made until 1977 [11]. In this system, the Raman cell was itself a single mode fiber 175 m long. The fiber material is not

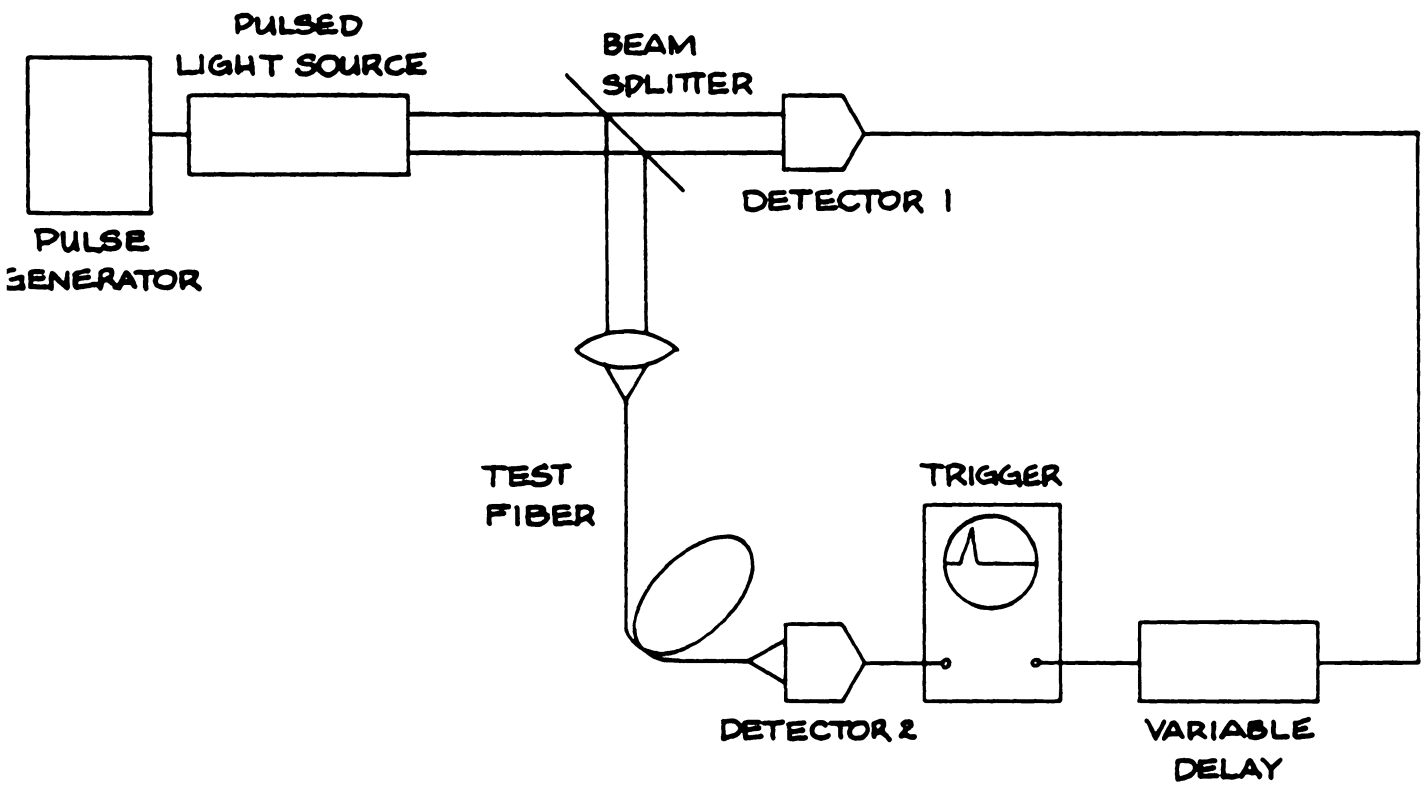

Figure 2. Simple time domain dispersion measurement apparatus

normally suitable for this purpose, but the long interaction length provided up to fifth order Stokes interaction, covering 1120 to 1510 nm. This system was later automated for efficient routine use in a Universal Fiber-Optic measurement system [12]. Distortionless pulse transmission was first demonstrated in 1979 [13].

An interesting variant of the fiber Raman technique appeared in 1980 [14]. The fiber under test is used as the Raman cell. A pulsed laser excites several Stokes lines, which are reflected by a mirror back into the fiber to the source. If the returning pulses are exactly synchronized with the next pump pulse, oscillation occurs. The mirror position needed to satisfy this condition is a direct indication of τ, a precise measurement since 30 cm of adjustment corresponds to roughly 1 ns of delay time.

Other innovations in time domain measurements have also focused on the light source used. These now include a lithium niobate parametric oscillator [15], temperature-controlled laser diodes [16], and a pulsed white-light source followed by a monochromator [17].

More recently, the concern has been to simplify the apparatus and measurement procedures for field use. For example, a simple and compact system may be built with a fiber coupler [18]. Another approach makes use of theoretically derived information to reduce the number of different laser sources needed to characterize a fiber near ZCD from three to two [19].

Time domain techniques are flexible in terms of the length of the fiber sample needed, the literature citing examples from 33m to over 10 km. Of course, better time resolution is possible in longer fibers. The attainable limit is on the order of 0.1 ns. However, this depends on generating and detecting very short light pulses, which in turn requires sophisticated and expensive equipment.

2.2 *Measurement in the Frequency Domain*

Frequency domain techniques also require high frequency signal generation, modulation and detection, and so are not empirically simpler or more economical than time-domain methods. However, they do give higher accuracy.

A novel and elegant technique appeared in 1978 [20]. A frequency generator drives an LED, modulating all wavelengths in its spectrum together. This light passes through a fiber, and due to chromatic dispersion the component wavelengths lose synchronism, no longer summing directly. As the modulation frequency is raised, the time differences become larger relative to the signal period, and the measured signal strength drops. By assuming the LED spectrum to be Gaussian, the authors were able mathematically to relate the loss of signal to chromatic dispersion at the center wavelength of the LED. This approach has also been adapted for use with laser sources, appearing in both single-laser [21] and two-laser [22] versions.

Other frequency domain systems differ from time domain only by finding group delay, τ, indirectly. Again, a variable wavelength source is needed, but is amplitude modulated at constant frequency and treated as if monochromatic. The phase delay of the modulation signal from fiber input to fiber output indicates τ. In practice, a pulsed source may be used if the modulation frequency is raised to the point that many periods occur in each pulse, creating short "steady-state" intervals for taking data. This procedure was adopted in [23], in which the steady-state signal was imposed on the light by an external modulator. The phase delay in even high-loss fibers was measured accurately by innovative use of degenerate Lissajous patterns. In a later article, the authors estimated the equivalent time resolution of this technique to be 17 ps [24].

Simplifications in apparatus and procedure again appeared. By making use of the extreme spectrum tails, four LEDs were made to cover the wavelength range 750 to 1350 nm [25]. In this case, the modulation was imposed directly on the drive current; a frequency of 32 MHz gave equivalent time resolution of 8 ps. The system was later extended to longer wavelengths [26]. For field use, several laser diodes may be connected into a similar system by a fiber coupler [27].

2.3 *Measurement Based on Light Interferometry*

Interferometric measurement systems give even greater time resolution without high frequency modulation or detection by using a partially coherent source. Light from such a source is made up of short wave trains, and will only form a clear interference pattern if the lengths of the interferometer arms are matched to within its characteristic "coherence length". One arm contains a fiber; due to dispersion, the effective optical length of this arm varies with wavelength. The other arm serves as a reference. Matching is accomplished either by varying the reference length or by scanning across the source spectrum until the interference pattern visibility peaks, permitting one to infer group delay by noting the length of the reference path. The drawback to these systems is that they have so far only been demonstrated on short fiber samples (4 m or less). However, a recent study indicates that averaging a 4 m sample from each end of a fiber comes close to describing dispersion of the fiber taken as a whole [28].

First appearing in 1981 [29], the equivalent time resolution of the interferometric approach is sensitively dependent on the degree of source coherence. Approximately 1 ps equivalent resolution was claimed for one system that made use of six discrete LEDs over the wavelength range 700 to 1100 nm [30]. Others have bettered this figure with a fiber Raman cell [31] or halogen lamps [32,33], the latter giving equivalent resolution on the order of 0.1 ps.

At least one paper, however, has criticized these approaches for the subjectivity and human error introduced in finding the point of maximum fringe visibility [34]. The authors describe a numerical procedure for extracting this information from the apparatus by computer, which they eventually hope to integrate into a completely automated measurement system.

2.4 *Other Measurement Techniques*

Other novel measurement schemes have occasionally appeared. For example, in [35], a variable-wavelength light beam incident on the side of a cylinder (i.e. unclad fiber) reveals its refractive index through an analysis of the backscatter pattern. However, applicability is suspect, as real fibers are not perfectly cylindrical or homogeneous. Also, the system involves only a 1 mm long cross-section, and nobody has ever studied the axial uniformity of fibers on this scale.

Optical auto- and cross-correlations are used to find the fiber transfer function (and thus dispersion) in [36]. However, the authors do not explicitly state the achieved resolution, and the indirect technique seems needlessly complex mathematically.

3.0 The Dispersion Measurement System

3.1 General Description

The experimental set-up is shown in Figure 3. The laser source fixes the location of measurement on the wavelength axis, and also provides the reference (radian) frequency, ω. A beamsplitter samples this reference, passing half of the light to the acousto-optic modulator (AOM). The signal generator drives the AOM at a single frequency; again expressing in radian terms, we may denote it by $\Delta\omega$. Some of the laser light passes through the AOM cell undisturbed, but the rest becomes shifted up or down in frequency by whole multiples of $\Delta\omega$. The resultant output consists of several beams each of constant frequency, separated by small differences in exit angle. Immediately to either side of the undeflected laser light are the "first-order" beams, both shifted in frequency by exactly $\Delta\omega$, one up, one down. We may choose to work with either of these, though the ensuing discussion assumes that the upshifted beam is selected.

A second beamsplitter recombines the first-order shifted light with the reference beam. Just as with sound waves of slightly different frequencies, the resulting interference exhibits "beating." More precisely, there is a variation in the light intensity at the difference frequency, $\Delta\omega$. Detector 1, at the fiber input, passes this signal to one channel of the oscilloscope.

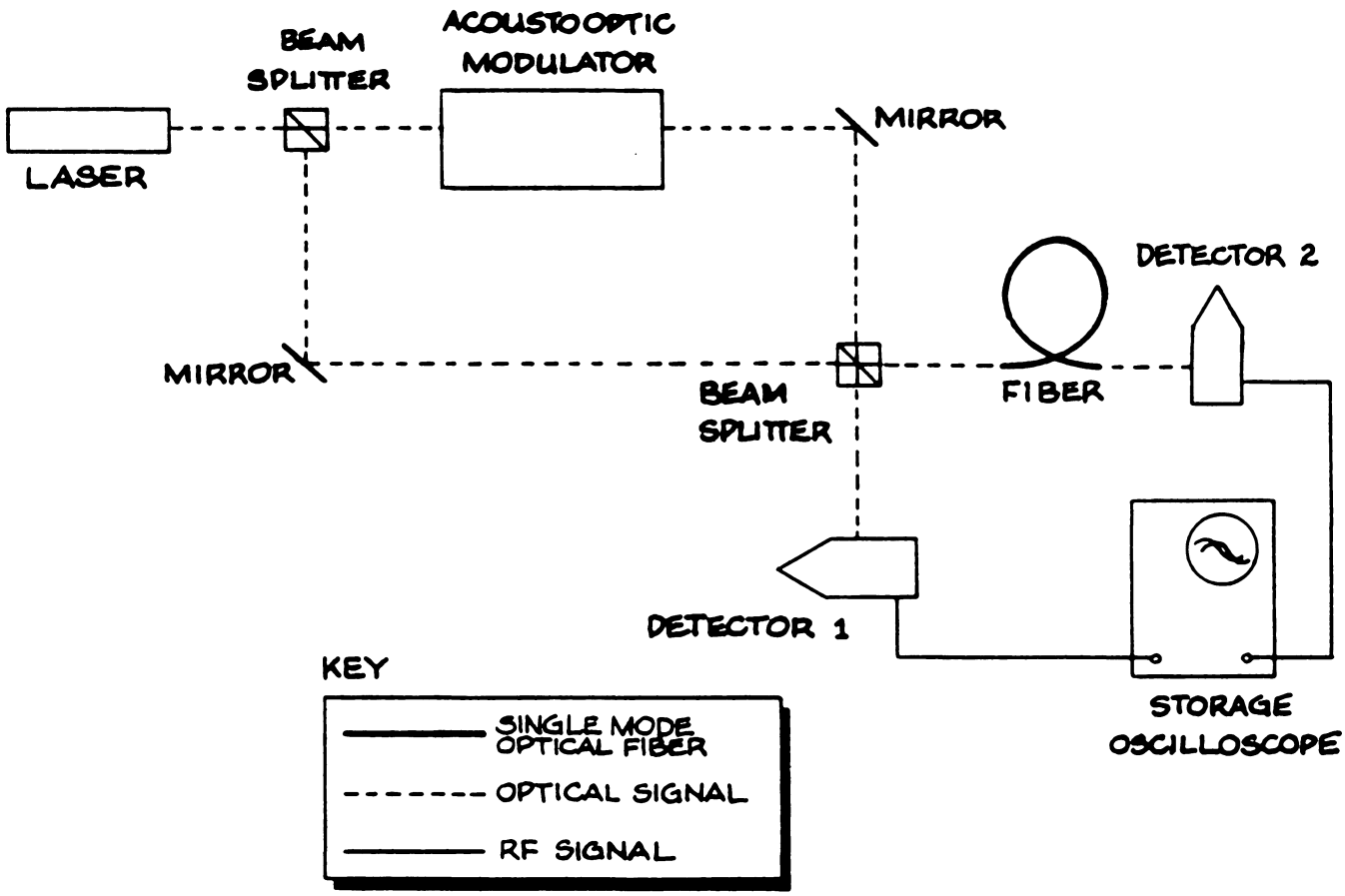

Figure 3. Dispersion measurement system

The two interfering beams are also coupled into the end of the single-mode fiber under test. Inside the fiber, the reference laser light takes on propagation constant β_1. Though practically indistinguishable in wavelength, the two beams are *not* identical in this respect, and the shifted beam takes on a different propagation constant, β_2. The beams still interfere, but the exact "beat" wavelength is determined by the details of the beta difference; it is dispersion dependent. This, of course, is the basis of the measurement.

The two beams reach the other end of the fiber, albeit attenuated, and again interfere as when first mixed. Detector 2 passes this signal to the second oscilloscope channel, where it may be compared with the output of detector 1. The relative phase of these two signals provides all the information needed to find the beta difference inside the fiber. By repeating this procedure at a number of different AOM driver frequencies ($\Delta\omega$'s), we may in principle accumulate enough information to calculate the chromatic dispersion.

3.2 Basic Theory

Inside the fiber, we may write the electric field amplitude of the reference laser light as follows:

$$E_1 = \alpha(x)\, E_0 \exp i\,[\omega t - \beta_1 x + \theta_1] \tag{3.2.1}$$

Where $\alpha(x)$ represents the light attenuation as a function of distance and θ_1 is the arbitrary phase of the light at the fiber input ($x = 0$).

Similarly, we may write the electric field amplitude of the first-order shifted beam. The frequency differs by $\Delta\omega$:

$$E_2 = \alpha(x)\, E_0 \exp i\,[(\omega + \Delta\omega)t - \beta_2 x + \theta_2] \tag{3.2.2}$$

θ_2 is again an arbitrary phase term.

The total field in the fiber is a superposition of these two beams, and may be considered as the sum of their component amplitudes. Adding equations 3.2.1 and 3.2.2, we get:

$$E = \alpha(x)\, E_0 \exp i(\omega t)[\exp i(-\beta_1 x + \theta_1) + \exp i(\Delta\omega t - \beta_2 x + \theta_2)] \tag{3.2.3}$$

The light intensity from this beam is given by:

$$H = (1/2)EE^* = \alpha^2(x)\, E_0^2 [1 + \cos(\Delta\omega t - (\beta_2 - \beta_1)x + (\theta_2 - \theta_1))] \tag{3.2.4}$$

Note that this signal is sinusoidal, varying in time at the AOM driver frequency.

We may now find the signal at each end of the fiber by substituting the appropriate x values into equation 3.2.4. Consider the input end first ($x = 0$). There is no attenuation at this point, so we let $\alpha(0)$ be unity.

$$H_{in} = E_0^2 [1 + \cos(\Delta\omega t + (\theta_2 - \theta_1))] \tag{3.2.5}$$

Next, consider the fiber output ($x = L$). This time, we must include attenuation. We may denote it by $\alpha(L) = \alpha$.

$$H_{out} = \alpha^2 E_0^2 [1 + \cos(\Delta\omega t - (\beta_2 - \beta_1)L + (\theta_2 - \theta_1))] \tag{3.2.6}$$

Comparing equations 3.2.5 and 3.2.6, it is clear that the light intensities at fiber input and output are substantially the same. Neglecting attenuation, the only difference between them is phase. The output phase must lag behind the input; the amount of lag is:

$$\Theta_{lag} = \arg\,(H_{in}) - \arg\,(H_{out}) = (\beta_2 - \beta_1)L \tag{3.2.7}$$

This phase difference is directly proportional to the fiber length, L, and the difference in propagation constant at the two frequencies. This relationship will be important in relating the phase observations to dispersion.

Supposing the fiber length known, the relative phase may be interpreted as a differential beta measurement. This suggests one possible means of relating the empirical data to dispersion. In most media, we could simply relate beta, or propagation constant, to the refractive index. In the fiber, this relationship is complicated by waveguide effects (remember "waveguide" dispersion); however, it is possible to treat the net result as an "effective" index. The differential beta measurement may then be considered as differential index data, and the calculation would proceed as in the standard technique of calculating material dispersion from index measurements in bulk glass.

Pursuing this thought farther, we might translate the measured relative phase to index terms. Denote the effective indices corresponding to β_1 and β_2 by n_1 and n_2, respectively. The following substitutions are helpful:

$$\beta_1 = \frac{2\pi n_1}{\lambda_1}$$

$$\beta_2 = \frac{2\pi n_2}{\lambda_2}$$

$$\lambda_1 = \frac{c}{f_1} = \frac{2\pi c}{\omega}$$

$$\lambda_2 = \frac{c}{f_2} = \frac{2\pi c}{(\omega + \Delta\omega)}$$

We then arrive at the desired expression:

$$\Theta_{lag} = \frac{L\,[(n_2 - n_1)\omega + n_2\Delta\omega]}{c} \tag{3.2.8}$$

The phase lag depends on two terms: one proportional to the frequency times the index difference, the second proportional to the index times the frequency difference. If we could separate out the first of these, it would provide the differential data needed for a dispersion calculation. Handling the second term is problematic, though, as the index at one of the frequencies must be known to high accuracy if this term is to be deducted from the sum.

Fortunately, there is an alternative approach that uses the differential beta information directly.

3.3 *Calculation of Results*

As described in chapter 2, existing measurement schemes tend to involve numerical differentiation with respect to wavelength. The data available from this technique are inherently frequency domain beta measurements. This section presents a derivation for an alternative means of calculating dispersion directly in the frequency domain. Taking this approach is much better than translating the data to wavelength domain and applying the standard formulas for several reasons. First, the AOM frequencies used translate to very small wavelength differences; high precision computer differentiation routines would be needed to avoid losing all significant digits in a wavelength-based computation. The alternative formula derived below involves derivatives with respect to frequency only; the problem is eliminated since the frequency differences used are well known to the experimenter. Second, numerical differentiation formulas are based on the assumption that data are available at equal intervals on the independent axis; an anticipated translation to wavelength domain would necessitate taking data at unequal frequency spacings, and these intervals would in turn depend on the wavelength of the source. Finally, the formula derived in this section will eventually yield insight into the behavior of this technique in the presence of noise.

Chromatic dispersion, as defined in section 1.2, is the derivative of group delay versus wavelength, normalized to unit fiber length. We may take this as the starting point:

$$D = \frac{1}{L}\frac{\partial \tau}{\partial \lambda} \tag{3.3.1}$$

By making some carefully chosen substitutions, we proceed to transform the formula. Group delay, τ, is the quotient of fiber length and group velocity. This may in turn be related to the propagation constant, β, and radian frequency, ω :

$$\tau = \frac{L}{v_g} = \frac{L}{(\partial\omega/\partial\beta)} = L\frac{\partial\beta}{\partial\omega} \tag{3.3.2}$$

Wavelength, λ, is the quotient of the speed of light and frequency. Again we may form an expression in ω:

$$\lambda = \frac{c}{f} = \frac{2\pi c}{\omega} \tag{3.3.3}$$

Next, we make use of the two identities (3.3.2) and (3.3.3) and the chain rule of calculus to re-express the derivative in equation (3.3.1).

$$\frac{\partial\tau}{\partial\lambda} = \frac{\partial\tau/\partial\omega}{\partial\lambda/\partial\omega} = \frac{L(\partial^2\beta/\partial\omega^2)}{-2\pi c/\omega^2}$$

We may now substitute this result back into equation (3.3.1) to relate dispersion to a normalized derivative in the (radian) frequency domain:

$$D = \frac{1}{L} L\frac{\partial^2\beta/\partial\omega^2}{-2\pi c/\omega^2} = \frac{-\omega^2}{2\pi c}\frac{\partial^2\beta}{\partial\omega^2} \tag{3.3.4}$$

For convenience, we may translate this result so that frequency is expressed directly in Hertz. A simple identity and the chain rule of calculus again provide the necessary substitutions:

$$\omega = 2\pi f$$

$$\frac{\partial\beta}{\partial\omega} = \frac{\partial\beta}{\partial f}\frac{\partial f}{\partial\omega} = \frac{1}{2\pi}\frac{\partial\beta}{\partial f}$$

$$\frac{\partial^2\beta}{\partial\omega^2} = \frac{1}{2\pi}\frac{\partial^2\beta}{\partial f^2}\frac{\partial f}{\partial\omega} = \left(\frac{1}{2\pi}\right)^2\frac{\partial^2\beta}{\partial f^2} \tag{3.3.5}$$

When (3.3.5) is applied to (3.3.4), the dispersion equation becomes:

$$D = \frac{-f^2}{2\pi c}\frac{\partial^2\beta}{\partial f^2} \tag{3.3.6}$$

The derivative appearing in this equation is in principle readily computed from the available data, and is then simply related to dispersion by a known linear factor.

3.4 *Practical Details*

The discussion has so far glossed over a number of details related to actually implementing the apparatus and interpreting experimental observations. These need to be individually addressed in some detail before any measurements are possible.

The equations appearing in the theoretical discussion of section 3.2 assume that the intensities of the reference and shifted beams are identical. In practice, the stronger beam could be attenuated until this condition is met, but such a procedure turns out to be unnecessary. If the intensities are mismatched, only a constant term is added to the detected signals; there is no effect on the phase of the rf interference.

Referring again to Figure 3 on page 14, it is clear that the interference pattern of reference and shifted beams will undergo some nonzero phase change in the air gaps between the fiber ends and the detectors. Since we are interested in the phase change due strictly to the fiber, it is important either to account for the additional phase or to eliminate this nuisance effect. Fortunately, there is a simple solution: simply match the total length of the air gaps included between the second beamsplitter and each detector. The measurement is a differential one; a constant phase added to both detected signals will be subtracted off in finding the phase difference between them.

A similar discussion applies to the phase introduced in electronically amplifying the signal at each detector. Again, due to the differential nature of the measurement, it is not important to know

the precise phase characteristics of the amplifiers, as long as they are the same. Note that the signals have identical frequency.

The refractive index interpretation of results was rejected because it explicitly involved the value of n at the measurement wavelength, information which might not be known to any high level of accuracy. The formula derived in the last part involves only a beta derivative. This does not appear to involve any absolute beta values until we consider that a numerical differentiation formula such as the one derived in appendix A must be used. As it turns out, differential data may be substituted for the absolute ordinate values appearing in the formula, and this is relatively simple to justify. The beta differences, as measured, are always referred to the same reference: β_1, the propagation constant at the reference (laser) wavelength. We may model this mathematically by supposing that some constant term has been subtracted from each measured value. Referring to appendix A again, it is as though another constant term were inserted into each of the original five Taylor series expansions. Taking the specified linear combination cancels these terms, just as it cancels the constant $f(a)$. The result is still a second derivative term plus the (same) inevitable error; using differential data will not have any influence on the calculated value.

In the same vein, the true relative phase of the two signals will include whole multiples of 2π radians that cannot be measured directly, but will have to be inferred from the (known) length of the fiber under test. An uncertainty of one or more whole wavelengths is entirely plausible if L is large enough. Again, the differential nature of the technique comes into play. Any assumed whole number of 2π shifts inside the fiber is irrelevant, as long as the five recorded phase differences accurately reflect the *change* in relative phase as the AOM driver frequency is varied. The constant term will again be eliminated in the numerical differentiation process. The problem of measuring absolute phase lag again simplifies to the much simpler differential case.

3.5 *Demonstration System*

The apparatus of Figure 3 on page 14 was constructed using a Helium-Neon laser (λ_0 = 632.8 nm). The plan was to prove the technique with this equipment before moving on to longer wavelengths. In addition to the obvious advantage of working with visible light, the greater dispersion expected at this wavelength (see Figure 1 on page 2) should have led to more easily discernible effects. The sample of fiber used was 28 m long.

The overlapping ranges of the available signal generator and AOM limited the modulation frequencies available to the interval 30 to 40 MHz. Since five phase measurements were needed at equal frequency intervals, 2 MHz spacing was chosen. A frequency counter was also connected to the signal generator to ensure spacing uniformity, as the numerical differentiation procedure depends on this assumption for validity. To gauge repeatability, data was actually taken at ten modulation frequencies, from 31 to 40 MHz, at 1 MHz intervals. The data points were then assigned alternately to two sets, each of five points 2 MHz apart. Both data sets were expected to reflect dispersion at the same nominal (laser) wavelength, and therefore to substantially agree when put independently through the calculation procedure.

Correct alignment of the two interfering beams proved to be crucial. Detected signal strength was strongly influenced by even slight misadjustments of the mirrors, beamsplitters, lens, and fiber end. The signal at the fiber output was particularly elusive thanks to coupling losses at the fiber input. Moreover, the alignment of the first-order shifted beam from the AOM varies with modulation frequency, so it was necessary to re-adjust the apparatus slightly at each new value of $\Delta\omega$ to regain acceptable signal levels. Fortunately, once the initial alignment was established, the minor adjustments required were easily carried out with an eye to maximizing displayed signal amplitude on the oscilloscope. Isolating the apparatus from vibration was also difficult, and a chief source of experimental error.

The storage oscilloscope provided limited signal processing capabilities, and each signal waveform used in the relative phase measurements was averaged over 1000 iterations. Though it

was not possible to acquire both signals simultaneously, detector 1 provided the oscilloscope trigger throughout. At five nanoseconds per division, 1024 points per sample, time resolution was limited to 0.05 ns. In practice, however, relative time displacement of the two signals proved repeatable to only about .3 ns. At 40 MHz, this represents a relative phase uncertainty of approximately 4 degrees.

Though several data sets were taken with this apparatus, calculated results were not only wildly inconsistent but differed from the anticipated value by many orders of magnitude. At first, suspicions pointed to an order-of-magnitude problem in representing or expressing intermediate quantities within the computer program used (reprinted in appendix B). However, it later became evident that the difficulties lay in the noise sensitivity inherent in the measurement system.

3.6 Uncertainty Analysis

In section 3.3, I showed that the dispersion equation can be re-expressed as:

$$D = \frac{-f^2}{2\pi c}\frac{\partial^2\beta}{\partial f^2} \tag{3.6.1}$$

The calculated dispersion is linearly related to an experimentally determined second derivative. We may take it then that the uncertainty in the result is simply related to uncertainty in this derivative by the same linear factor. We denote uncertainty by δ to get:

$$\delta D = \frac{-f^2}{2\pi c}\,\delta\left(\frac{\partial^2\beta}{\partial f^2}\right) \tag{3.6.2}$$

To estimate the uncertainty in the derivative, we must then consider the details of numerical differentiation. A representative five-point formula is derived in appendix A. It is repeated here for convenience:

$$f''(a) \cong \frac{-f(a+2h) + 16f(a+h) - 30f(a) + 16f(a-h) - f(a-2h)}{12h^2} \qquad (3.6.3)$$

We must assume worst case: errors in all numerator terms sum without cancellation. Adding the absolute values of these coefficients gives a total of 64. Substituting this result into equation 3.6.2, we now have:

$$\delta D = \frac{-f^2}{2\pi c} \frac{64\,(\delta\beta)}{12h^2} \qquad (3.6.4)$$

At this point, it is helpful to substitute representative values for h and f. The symbol h represents the frequency interval separating the five phase measurements (in the context of this research, 2 MHz). Supposing the light source to be a Helium-Neon laser (λ = 632.8 nm), we substitute the corresponding value of f to get:

$$\delta D = (-1.59x10^{11}\ ps/nm)(\delta\beta) \qquad (3.6.5)$$

There is a straightforward interpretation of this result: consider the uncertainty in the β measurement to be expressed per kilometer of fiber. This will put the expression in conventional units. For example, for the four degree phase uncertainty cited above in the demonstration system with 28 m of fiber, we get $\delta\beta$ = 2.5 rad/km. If we accept this result, the uncertainty of equation 3.6.5 becomes $\delta D = -4x10^{11}$ ps/nm·km. Considering that the actual value we are trying to measure is approximately 300 ps/nm·km, it is immediately clear that the uncertainty in D is many orders of magnitude too high. In fact, the uncertainty overwhelms and completely obscures any useful data contained in the experimental observations.

Suppose that by a combination of electrical and mechanical noise filtering, the experimental uncertainty could be brought down to the limit of oscilloscope resolution. The resultant uncertainty from equation 3.6.5 would then drop to $\delta D = -7x10^{10}$ ps/nm·km. Again considering the true magnitude of D, it appears that the requisite measurement precision is not attainable; an improvement of at least ten or eleven orders of magnitude is needed.

Is there any recourse? We have control over several of the parameters influencing the calculated error in equation 3.6.2. They are not all independent; however, we may consider the options individually:

1. We might use this method only in very long fibers, distributing uncertainty in relative phase over several kilometers to reduce uncertainty in β. Unfortunately, fiber attenuation will limit us long before increased L could make a significant difference.

2. The light frequency appears as a squared term. Increasing the wavelength to the near-infrared reduces f, but only gives an overall improvement of roughly one order of magnitude. Again, this is not enough. Note also that frequency is not a freely-varying parameter. The measurement is wavelength dependent, and applies only to the nominal point at which the data is taken; the range of interest is not negotiable.

3. Perhaps the worst case assumption that gave the factor of 64 in the numerator is unduly pessimistic. However, we cannot seek orders of magnitude improvement here either.

4. We could derive and use another numerical differentiation formula. However, the formula cited contains all of the second derivative information available from the five sample points. We could include more points, but the AOM has already been modulated over most of its useful range.

5. The only other parameter we may conceivably vary is h, the frequency interval by which the beta measurements are separated. But, as noted above, the useful range of the AOM is basically exhausted. A second derivative calculation is limited to a minimum of three terms; we could drop two out of five and increase h by a factor of 2. Unfortunately, a three point numerical differentiation calculation has higher inherent error since it is possible to cancel fewer Taylor series terms (see appendix A). A factor of 2 improvement, even when squared, is inadequate in any case.

Apparently, what is needed to solve this impasse is a greatly extended range of feasible modulation frequencies. Given this, one could trade off increased h against wavelength resolution. But the necessary improvement is substantial; many orders of magnitude at minimum. Also, the AOM is not the only device that must operate over the new range: a complementary signal generator, oscilloscope, and detector/amplifiers would be needed. Constraints of existing devices seem insurmountable for the present.

4.0 Summary and Conclusions

The measurement system developed in this research was designed to find chromatic dispersion in single-mode optical fibers. Unlike other techniques described in the literature, this system was to give virtually point wavelength resolution. To demonstrate the concept, the apparatus was constructed using a Helium-Neon laser source, but did not lead to acceptable results. Insight gained from this experiment eventually led to an analysis showing that the system suffers from a fundamental defect: high sensitivity to noise.

To become acceptably robust, the procedure must sacrifice some of the anticipated wavelength resolution by raising the acoustooptic modulation frequencies several orders of magnitude; a possibility not available given the current state of the art. Perhaps the resolution of existing systems *can* be improved; it *cannot* be improved to the extreme degree that we had hoped.

5.0 References

1. D. Gloge "Dispersion in weakly guiding fibers" *Applied Optics* 10, 2442 (1971).

2. D. N. Payne and W. A. Gambling "Zero material dispersion in optical fibres" *Electronics Letters* 11, 176 (1975).

3. L. G. Cohen, C. Lin and W. G. French "Tailoring zero chromatic dispersion into the 1.5-1.6 μm low loss spectral region of single-mode fibers" *Electronics Letters* 15, 334 (1979).

4. D. Marcuse *Principles of Optical Fiber Measurements* New York: Academic Press 1981.

5. G. Cancellieri and U. Ravaioli *Measurements of Optical Fibers and Devices: Theory and Experiments* Dedham, MA: Artech House 1984.

6. D. Gloge, A. R. Tynes, M. A. Duguay, and J. W. Hansen "Picosecond pulse distortion in optical fibers" *IEEE Journal of Quantum Electronics* QE-8, 217 (1972).

7. A. Kawana, M. Kawachi, T. Miyashita, M. Saruwatini, K. Asatini, J. Yamada, and K. Oe "Pulse broadening in long-span single-mode fibers around a material-dispersion-free wavelength" *Optics Letters* 2, 106 (1978).

8. D. Gloge and E. I. Chinnock "Fiber-dispersion measurements using a mode-locked krypton laser" *IEEE Journal of Quantum Electronics* QE-8, 852 (1972).

9. D. Gloge, E. L. Chinnock and T. P. Lee "GaAs twin laser set up to measure mode and material dispersion in optical fibres" *Applied Optics* 13, 261 (1974)

10. B. Luther-Davies, D. N. Payne, and W. A. Gambling "Evaluation of material dispersion in low loss phosphosilicate core optical fibres" *Optics Communications* 13, 84 (1975).

11. L. G. Cohen and C. Lin "Pulse delay measurements in the zero material dispersion wavelength region for optical fibers" *Applied Optics* 16, 3136 (1977).

12. L. G. Cohen and C. Lin "A universal fiber-optic (UFO) measurement system based on a near-IR fiber Raman laser" *IEEE Journal of Quantum Electronics* QE-14, 855 (1978).

13. D. M. Bloom, L. F. Mollenauer, C. Lin, D. W. Taylor, and A. M. DelGaudio, "Direct demonstration of distortionless picosecond-pulse propagation in kilometer-length optical fibers" *Optics Letters* 4, 297 (1979).

14. C. Lin, L. G. Cohen, W. G. French, and H. M. Presby "Measuring dispersion in single-mode fibers in the 1.1-1.3 μm spectral region - a pulse synchronization technique" *IEEE Journal of Quantum Electronics* QE-16, 33 (1980).

15. D. N. Payne and A. H. Hartog "Determination of the wavelength of zero material dispersion in optical fibres by pulse-delay measurements" *Electronics Letters* 13, 627 (1977).

16. T. Miyashita, M. Horiguchi, and A. Kawana "Wavelength dispersion in a single-mode fibre" *Electronics Letters* 13, 227 (1977).

17. M. Horiguchi, Y. Ohmori, and T. Miya "Evaluation of material dispersion using a nanosecond optical pulse radiator" *Applied Optics* 18, 2223 (1979).

18. C. Lin, A. R. Tynes, A. Tomita, P. L. Liu, and D. L. Philen "Chromatic dispersion measurements in single-mode fibers using picosecond InGaAsP injection lasers in the 1.2- to 1.5- μm spectral region" *The Bell System Technical Journal* 62, 457 (1983).

19. D. W. Schicketanz and C. K. Eoll "Dispersion measurement using only two wavelengths" *Electronics Letters* 22, 209 (1986).

20. T. Tanifuji and M. Ikeda "Simple method for measuring material dispersion in optical fibres" *Electronics Letters* 14, 367 (1978).

21. J. Yamada "Simple dispersion measurement of a long-span, single-mode fibre using the longitudinal mode spacing of a semiconductor laser" *Optical and Quantum Electronics* 14, 183 (1982).

22. R. Rao "Field dispersion measurements - a swept frequency technique" *Technical Digest of Symposium on Optical Fibre Measurements* Boulder, USA, 1984.

23. K. Daikoku and A. Sugimura "Direct measurement of wavelength dispersion in optical fibres - difference method" *Electronics Letters* 14, 149 (1978).

24. A. Sugimura and K. Daikoku "Wavelength dispersion of optical fibers directly measured by 'difference method' in the 0.8-1.6 μm range" *Review of Scientific Instruments* 50, 343 (1979).

25. B. Costa, D. Mazzoni, M. Puleo and E. Vezzoni "Phase shift technique for the measurement of chromatic dispersion in optical fibers using LEDs" *IEEE Journal of Quantum Electronics* QE-18, 1509 (1982).

26. B. Costa, M. Puleo and E. Vezzoni "Phase-shift technique for the measurement of chromatic dispersion in single-mode optical fibers using LEDs" *Electronics Letters* 19, 1074 (1983).

27. P. J. Vella, P. M. Garel-Jones, and R. S. Lowe "Measurement of chromatic dispersion of long spans of single-mode fibre: a factory and field test method" *Electronics Letters* 20, 167 (1984).

28. F. M. Sears, L. G. Cohen and J. Stone "Interferometric measurements of dispersion-spectra variations in a single-mode fiber" *Journal of Lightwave Technology* LT-2, 181 (1984).

29. H. T. Shang "Chromatic dispersion measurement by white-light interferometry on metre-length single-mode optical fibers" *Electronics Letters* 17, 603 (1981).

30. M. Tateda, N. Shibata and S. Seikai "Interferometric method for chromatic dispersion measurement in a single-mode optical fiber" *IEEE Journal of Quantum Electronics* QE-17, 404 (1981).

31. L. G. Cohen and J. Stone "Interferometric measurements of minimum dispersion spectra in short lengths of single-mode fibre" *Electronics Letters* 18, 564 (1982).

32. J. Stone and L. G. Cohen "Minimum-dispersion spectra of single-mode fibres measured with subpicosecond resolution by white-light crosscorrelation" *Electronics Letters* 18, 716 (1982).

33. M. J. Saunders and W. B. Gardner "Precision interferometric measurement of dispersion in short single mode fibers" *Technical Digest of Symposium on Optical Fibre Measurements* Boulder, USA, 1984.

34. L. Oksanen and S. J. Halme "Interferometric dispersion measurement in single-mode fibers with a numerical method to extract the group delays from the measured visibility curves" *Technical Digest of Symposium on Optical Fibre Measurements* Boulder, USA, 1984.

35. H. M. Presby "Optical dispersion of unclad fibers over limited wavelengths" *Applied Optics* 13, 465 (1974).

36. W. D. Bomberger, and J. J. Burke "Interferometric measurement of dispersion of a single-mode optical fibre" *Electronics Letters* 17, 495 (1981).

Appendix A. Numerical Differentiation

We are interested in approximating a second derivative given only a number of empirically determined points. The derivation of the differentiation formula is based on finding a linear combination of Taylor series expansions in which as many unwanted terms as possible cancel. For a second derivative at least three data points must be included for acceptable results; by including more points, it is possible to cancel more unwanted terms.

The derivation which follows is for a five-point differentiation formula. We begin by considering five Taylor series expansions, all truncated at seven terms:

$$f(a+2h) = f(a) + (2h)f'(a) + \frac{(2h)^2}{2!}f''(a) + \frac{(2h)^3}{3!}f'''(a) + \frac{(2h)^4}{4!}f^{iv}(a) + \frac{(2h)^5}{5!}f^{v}(a) + \frac{(2h)^6}{6!}f^{vi}(a)$$

$$f(a+h) = f(a) + hf'(a) + \frac{h^2}{2!}f''(a) + \frac{h^3}{3!}f'''(a) + \frac{h^4}{4!}f^{iv}(a) + \frac{h^5}{5!}f^{v}(a) + \frac{h^6}{6!}f^{vi}(a)$$

$$f(a) = f(a)$$

$$f(a-h) = f(a) - hf'(a) + \frac{h^2}{2!}f''(a) - \frac{h^3}{3!}f'''(a) + \frac{h^4}{4!}f^{iv}(a) - \frac{h^5}{5!}f^{v}(a) + \frac{h^6}{6!}f^{vi}(a)$$

$$f(a-2h) = f(a) - (2h)f'(a) + \frac{(2h)^2}{2!}f''(a) - \frac{(2h)^3}{3!}f'''(a) + \frac{(2h)^4}{4!}f^{iv}(a) - \frac{(2h)^5}{5!}f^{v}(a) + \frac{(2h)^6}{6!}f^{vi}(a)$$

We seek a linear combination of these expansions in which all but the second derivative term will cancel. The best we can do is to leave an error term that is sixth order in h:

$$f(a + 2h) - 16f(a + h) + 30f(a) - 16f(a - h) + f(a - 2h) = -12h^2 f''(a) + 96h^6/720$$

Subtracting the error term from both sides and dividing by the unwanted coefficient simplifies the equation to yeild an explicit expression for $f''(a)$:

$$f''(a) = \frac{-f(a + 2h) + 16f(a + h) - 30f(a) + 16f(a - h) - f(a - 2h)}{12h^2} + \frac{h^4}{90}$$

Note that now the error term is only fourth order in h. We drop this term (along with the higher-order terms truncated at the beginning) to get the approximate differentiation formula:

$$f''(a) \cong \frac{-f(a + 2h) + 16f(a + h) - 30f(a) + 16f(a - h) - f(a - 2h)}{12h^2}$$

Appendix B. Dispersion Calculation Program

```
Program DispersionCalculator;
{
      Calculates numerical value of dispersion
        from experimental data taken at 5 points

      Language: TURBO Pascal
            (MS/DOS version 2.0)

      Data file format:      Wavelength  (nm)
                             Fiber Length  (m)
                             5 phase shifts at 2 MHz intervals
                                   (highest to lowest),
                                    expressed in multiples of pi
                              (seven lines total)
                                                    }


Const
  Npoints                  = 5;
  MHz                      = 1.0E+06;
  c                        = 299792458.;
  DeltaFreq                = 2.0E+06;

Type
  Filename                 = string[8];
  Extension                = string[3];

Var
  FileDrive                : char;
  Infile                   : text;
  Datafile                 : Filename;
  Ext                      : Extension;
  Lambda,FiberLength       : real;
```

```
  Freq                          : real;
  DeltaTheta                    : array [1..Npoints] of real;
  DeltaBeta                     : array [1..Npoints] of real;
  d2BetadFreq2                  : real;
  I                             : integer;
  Dispersion                    : real;


Procedure Initialize;
begin
  ClrScr;
  Write('Drive for data files:');
  Readln(FileDrive);
end;

Procedure GetFileInfo;
begin
  Write('Filename:');
  Readln(Datafile);
  Write('Extension:');
  Readln(Ext);
end;

Procedure OpenInputDataFile;
begin
  Assign(Infile,FileDrive + ':' + Datafile + '.' + Ext);
  Reset(Infile);
end;

Procedure ReadData;
begin
  Readln(Infile,Lambda);
  Readln(Infile,FiberLength);
  For I := 1 to Npoints do
  Readln(Infile,DeltaTheta[I]);
end;

Procedure DisplayHeader;
begin
  ClrScr;
  Writeln('Analysis of ',FiberLength:5:0,' m  fiber sample');
  Writeln('          at ',Lambda:5:1,' nm  wavelength');
end;

Procedure DisplayData;
begin
  GoToXY(1,6);
  Writeln(' Delta Thetas');
  For I := 1 to Npoints do
  Writeln(DeltaTheta[I]:8:3);
end;

Procedure DisplayResult;
begin
  GoToXY(1,15);
```

```
  Writeln('Calculated Dispersion:',Dispersion:10:3,' ns/nm/km');
end;

Procedure CalculateBeta;
begin
  For I := 1 to Npoints do
  DeltaBeta[I] := Pi*DeltaTheta[I]/(FiberLength/1000);
end;

Procedure SecondDerivative;
                        {Calculates second derivative of beta vs. freq.}
begin
  d2BetadFreq2:= (-DeltaBeta[5]+ 16*DeltaBeta[4]-30*DeltaBeta[3]
              + 16*DeltaBeta[2]-DeltaBeta[1])/(12.0*sqr(DeltaFreq));
end;

Procedure CalculateResults;
begin
  Freq := c/(Lambda*1.0E-09);
  CalculateBeta;
  SecondDerivative;
  Dispersion := (-sqr(Freq)*d2BetadFreq2/(2*pi*c));
end;


Begin
  Initialize;
  GetFileInfo;
  OpenInputDataFile;
  ReadData;
  CalculateResults;
  DisplayHeader;
  DisplayData;
  DisplayResult;
End.
```

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