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PREDICTIVE EQUATIONS FOR BOLTED CONNECTIONS

by

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(ABSTRACT)

A FORTRAN computer program applying the ultimate strength/instantaneous center of rotation method was written to generate the eccentricity coefficients used for the study of this project.

For single line bolted connections, the value of the eccentricity coefficient is determined by several independent variables: NR (number of rows in the bolted connection), B (distance between two adjacent bolts in a vertical column), X_o (horizontal distance from centroid to applied load), and θ (the load angle). From the relationships between the eccentricity coefficient and the independent variables, it was observed that a mathematical model of the eccentricity coefficient with respect to the independent variables is hard to determine. Hence, statistical equations for predicting the eccentricity coefficients were developed by using the Buckingham's PI-Theorem and regression analysis. The precision of the statistical equations is discussed, and several ways to improve the precision are presented in this paper.

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Chapter 1

INTRODUCTION

1.1 GENERAL

When the line of action of an applied load on a connection does not exactly pass through the centroid of a bolted connection group, the connection is called an eccentrically loaded connection. Some examples, such as a bracket connection, a standard beam connection, and a girder web splice are shown in Figure 1.

The effect of an eccentric load on the connection can be treated as a combination of direct shear and rotational moment. The capacity of the connection can be assumed to be the summation of the resistance of each connection element, which carries an equal share of the shear load and a distributed portion of the moment. The strength of a connection group, in terms of P_u can be determined by the following relationship

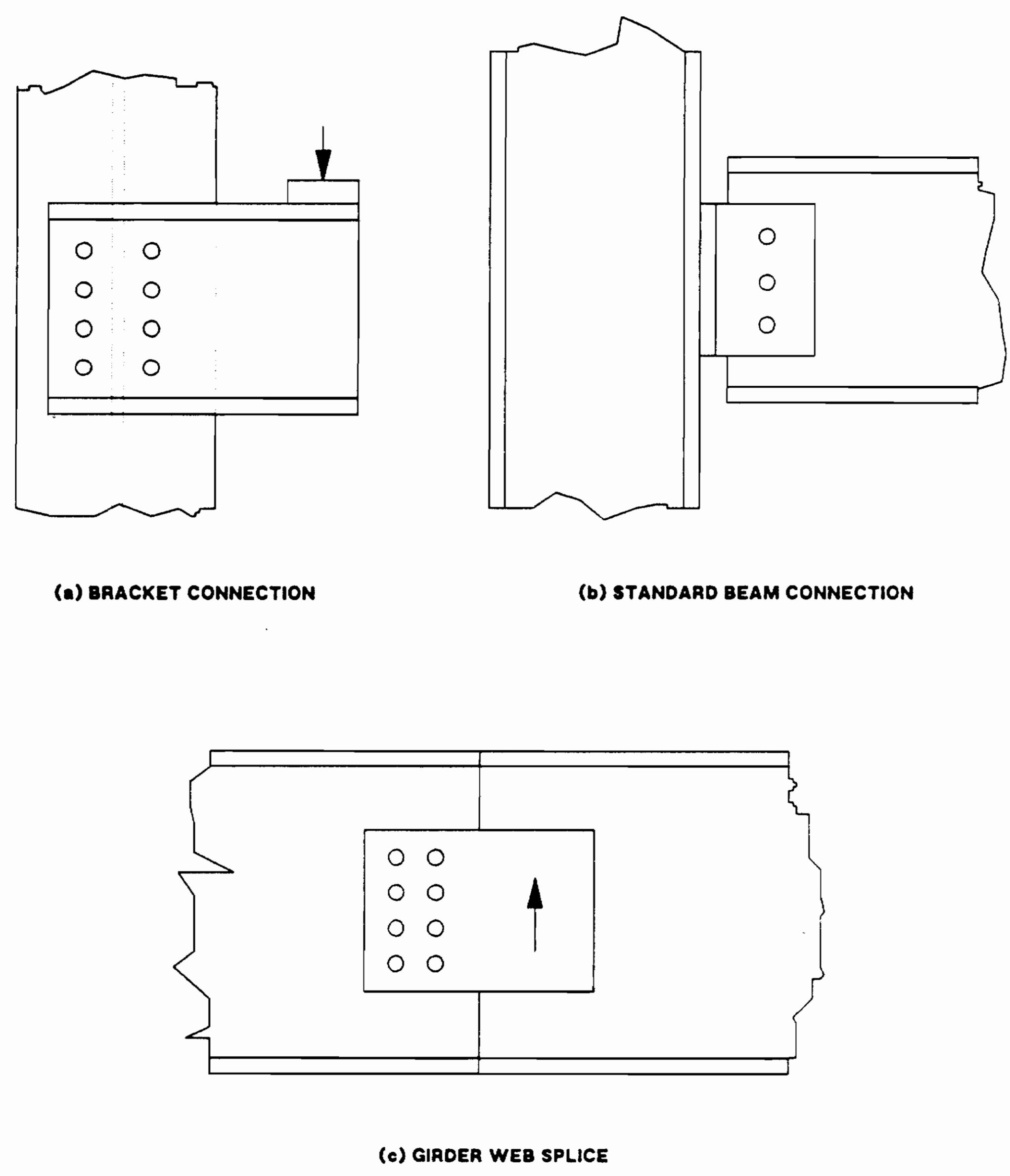

Figure 1. Examples of Eccentrically Loaded Connections

$$P_u = C \times r \tag{1.1}$$

where

P_u = allowable eccentric load

C = coefficient of the eccentrically loaded connection
(eccentricity coefficient)

r = nominal strength of one fastener

Because the eccentric load could be inclined and the exact center of rotation is unknown, the eccentricity coefficient is hard to determine. Prior to the 8th Edition of the American Institute of Steel Construction Manual (1980) (hereafter referred to as the 8th Ed. Manual), elastic analysis was utilized to determine the eccentricity coefficients. However, the elastic method usually provides inconsistent factors as well as answers that are too conservative. Therefore, an ultimate strength/instantaneous center of rotation method, which is the most accurate method in predicting the capacity of eccentrically loaded connection, was introduced in the 8th Ed. Manual. This method is also described in the 1st Edition of the Load and Resistance Factor Design (LRFD) Manual (1986). Unfortunately, this method is iterative and therefore only practically applicable if a computer is available for use.

Alternate methods were developed and are used in the new 9th Edition AISC ASD Manual (1989) (hereafter referred to as 9th Ed. Manual). These two methods take the inclined eccentric loads into account, however, they either give results that are too conservative or need tabulated eccentricity coefficients for vertical load as base data. Since the elastic method does not render acceptable results and the other methods are not very convenient to apply, it was desired to develop an alternate method. A method using simple statistical equations for determining the eccentricity coefficients is developed in this paper.

1.2 LITERATURE REVIEW

In the 7th Edition AISC Manual (1973), elastic analysis is applied to calculate the eccentricity coefficients. It is simply assumed that each fastener supports an equal share of vertical load and an additional load due to moment, which is caused by the eccentricity. The additional load due to moment is proportional to the distance between each fastener and the centroid of the connection group. Since elastic analysis does not consider the inelastic behavior of the fasteners, it generally provides excessively conservative results. It is also restricted to vertically applied loads.

An ultimate strength/instantaneous center of rotation method appears in the 8th Ed. Manual. The ultimate strength method for eccentrically loaded bolted connections was developed by Crawford and Kulak (1971), and the ultimate strength method for welded connections was then presented by Butler et al. (1972). The ultimate strength method is the best predictor for the eccentricity coefficient. However, this method is iterative and therefore is only suitable for practical use with the help of a computer. The 8th Ed. Manual has listed the coefficients for 0°(vertical) eccentric loads, and the 1st Edition AISC LRFD Manual (hereafter referred to as the LRFD Manual) added the coefficients for 45° and 75° eccentric loads. When the load angle is other than 0°, 45°, 75° , or 90°, the coefficient is still unknown. Applying linear interpolation between load angles could be significantly unconservative, especially for large load angles, and is therefore not recommended.

Tide (1980) discussed the development of the design tables for eccentrically loaded welded connections in the 8th Ed. Manual.

Brandt (1982a) proposed a rapid calculation procedure to determine the ultimate strength eccentricity coefficients of bolted connections. He applied an algorithm to improve the solution of the eccentricity coefficient from each successive trial iteration and utilized an h-square extrapolation to approximate the final solution. Brandt (1982b) also extended the calculation procedure to welded connections.

Avigdor Rutenberg (1984) developed an approximate method for the nonlinear analysis of the eccentrically loaded bolted connections. This method provides conservative solutions. For some larger connection groups, it requires the help of a computer because hand computations become tedious.

Iwankiw (1987) introduced several methods for the design of eccentric and inclined loads on bolted and welded connections. In addition to the ultimate strength method, he described Rotation of Inclined Load to Vertical Method, Full Plastic Analysis Method, Algebraic Addition of Direct Horizontal Shear and Eccentric Moment Resistances Method, and an extended Elastic Method.

The Rotation of Inclined Load to Vertical Method assumes that any inclined load can be rotated to the vertical position. That is, no matter what the load angle is, the applied load is assumed to be vertical, and the eccentricity coefficients can be directly obtained from the tables of vertical load in the AISC Manual. This method is simple but it can produce unacceptable solutions when the values of θ and X_o are both large.

The Full Plastic Analysis applies the concept of plastic steel design theory. It assumes that every fastener in the connection group has reached its ultimate resistive force R_{ult}. Hence, this method yields unconservative answers, since the fasteners do not behave in this manner.

The Algebraic Addition of Direct Horizontal Shear and Eccentric Moment Resistances Method first calculates the vertical and horizontal components of an inclined load, then evaluates the load effects of the vertical and horizontal components respectively, and finally adds the two load effects algebraically. It provides eccentricity coefficients between the elastic analysis solutions and the ultimate strength solutions. This method was further modified by Iwankiw (1988) and adopted in the new 9th Ed. Manual.

The extended Elastic Method includes horizontal load components of inclined loads. Although it renders quick answers, the results tend to be rather conservative.

The latest 9th Ed. Manual introduces two alternate methods to determine the eccentricity coefficients. Alternate method one is an extended elastic method which adds the effect of the horizontal component of inclined loads. However, this method still gives overly conservative results in some cases and yields inconsistent factor of safety. Alternate method two uses a trigonometric equation to approximate the coefficients for inclined loads. Although this method permits one to obtain the coefficients for any load angle, it is also conservative and requires the vertical load coefficients tabulated in the AISC Manuals. However, this is the most functional of all the methods and presently the method of choice for designers, if the ultimate strength/instantaneous center method is not used.

1.3 ULTIMATE STRENGTH METHOD

The most accurate of the predictive methods is the ultimate strength (instantaneous center) approach. It is used as a basis for the statistical development of this paper, therefore it is reviewed in detail.

The resistance of a connection group can be assumed to act perpendicularly to the radius of rotation (see Figure 2). It is obtained by summing the individual fastener resistance force, which is determined from the load-deformation relationship in Figure 3. The deformation of a fastener can be computed by assuming it to be linearly proportional to its distance from the instantaneous center, and can be expressed as follow:

$$\Delta = \frac{r}{r_{max}} \Delta_{max} \tag{1.2}$$

where

Δ = total deformation of a fastener

Δ_{max} = total deformation of the fastener most remote from the instantaneous center

r = fastener's radius of rotation from the instantaneous center

r_{max} = r of the fastener most remote from the instantaneous center

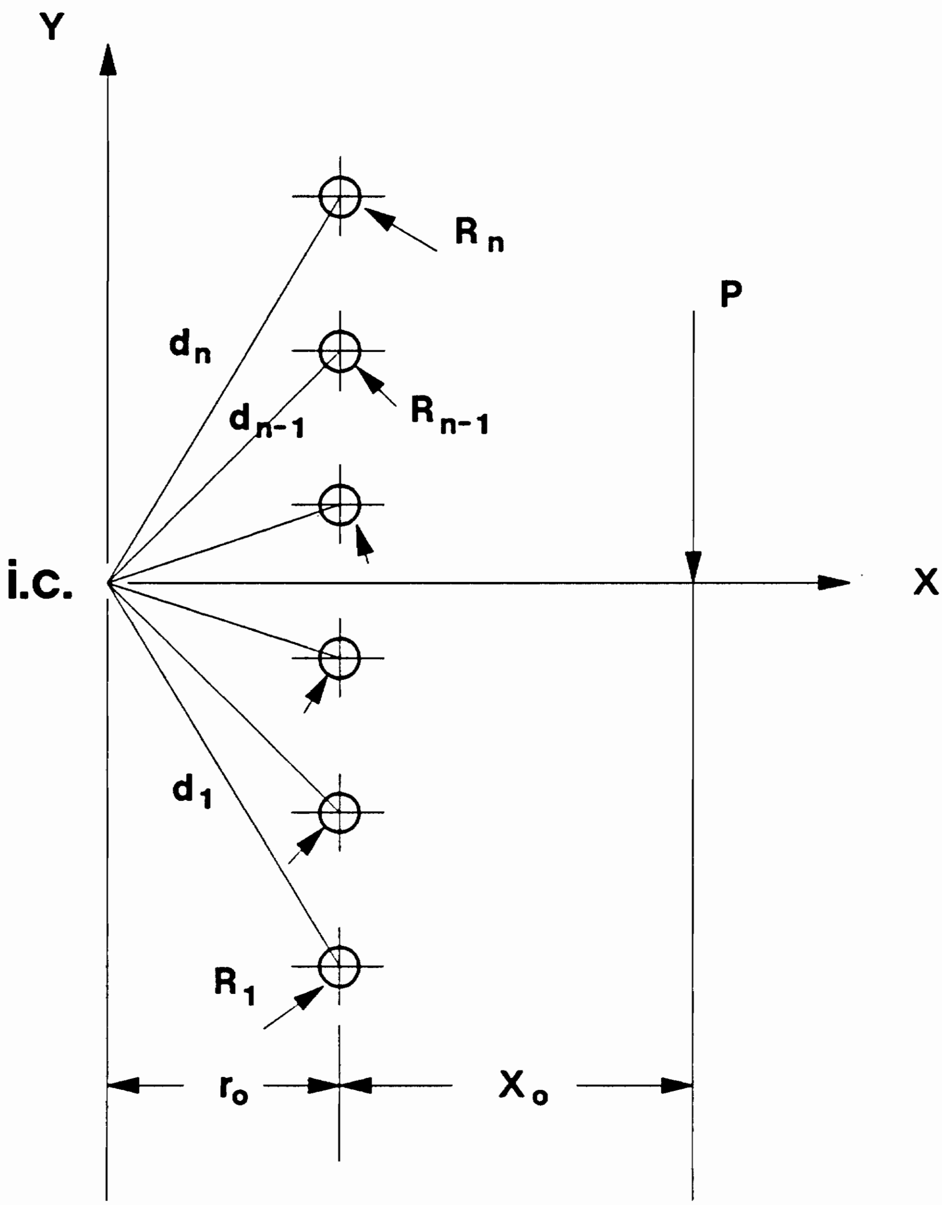

Figure 2. Illustration of the Instantaneous Center of Rotation

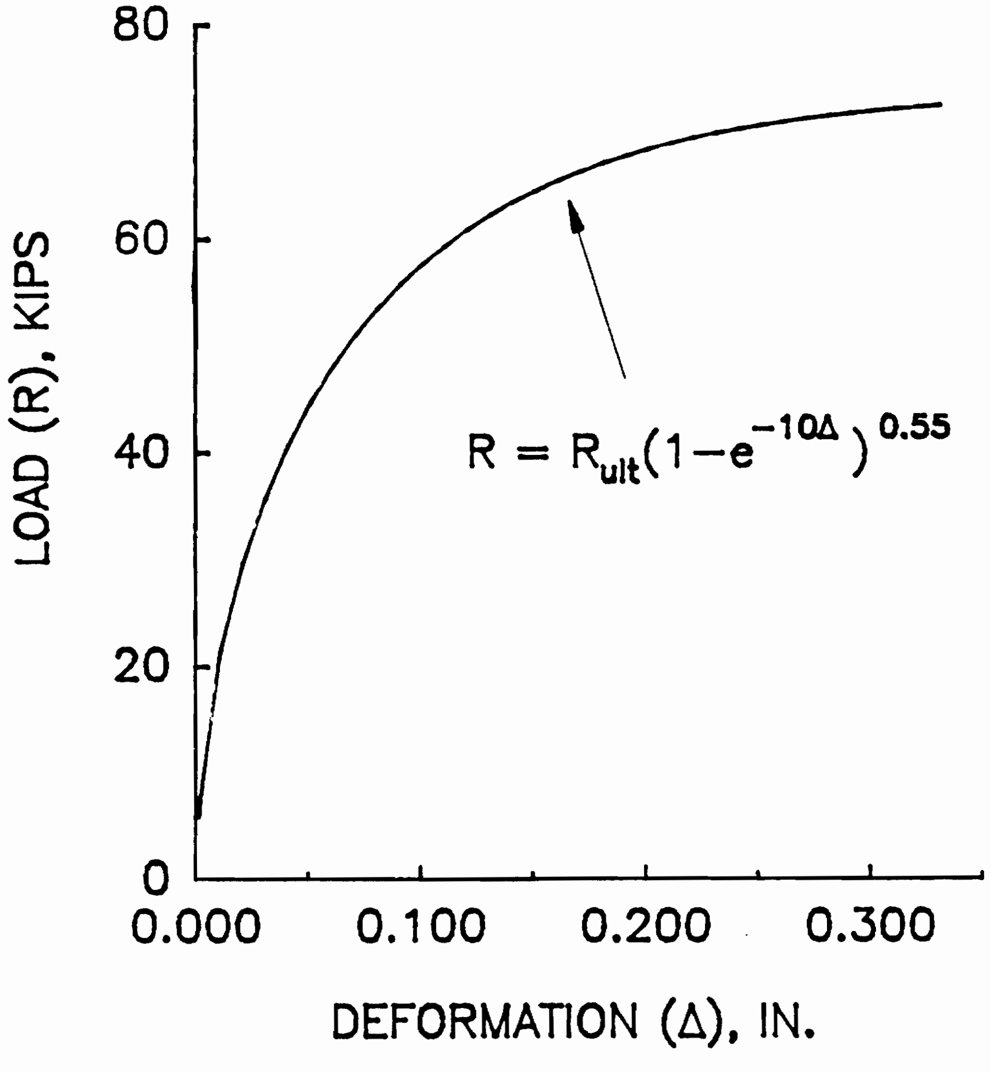

Figure 3. Load-Deformation Relationship

Referring to Crawford and Kulak (1971), the load-deformation relationship of a single fastener is expressed as:

$$R = R_{ult}\left(1 - e^{-10\Delta}\right)^{0.55} \tag{1.3}$$

where

R = shear force in a single bolt

R_{ult} = ultimate shear load of a single bolt

e = base of natural logarithm = 2.71828....

According to the LRFD Manual, "The load-deformation relationship data is based upon the following values obtained experimentally for 3/4 in. dia. ASTM A325 bolts:

R_{ult} = 74 kips

Δ_{max} = 0.34 in."

When the correct location of the instantaneous center has been located, the three equilibrium Equation (1.4) will equal to zero.

$$F_x = P_x + \sum R_x \tag{1.4a}$$

$$F_y = P_y + \sum R_y \tag{1.4b}$$

$$M = M_p + \sum R_M \qquad (1.4c)$$

where

F_x = x (horizontal) component of unbalanced force

P_x = x (horizontal) component of applied load P_u

R_x = x (horizontal) component of the resistance force in one fastener

F_y = y (vertical) component of unbalanced force

P_y = y (vertical) component of applied load P_u

R_y = y (vertical) component of the resistance force in one fastener

M = unbalanced moment

M_p = moment of applied load P_u about the instantaneous center

R_M = resistance moment in one fastener

The non-dimensional coefficient C is obtained by dividing the ultimate load P_u by R_{ult} and can be expressed as:

$$C = \frac{P_u}{R_{ult}} \qquad (1.5)$$

In order to find the correct location of the instantaneous center, trial iterations must be employed. This makes the ultimate strength method hard to apply without the use of a computer, since the number of iterations may be large and the calculation procedure is somewhat tedious.

1.4 ORGANIZATION

A condensed outline of the design problem in eccentrically loaded connection is presented in Chapter One. Chapter One also includes the literature review which discusses the current methods for determining the eccentricity coefficient. Chapter Two describes the computer program which employs the ultimate strength/instantaneous center of rotation method to generate the eccentricity coefficient. Statistical equations for the coefficient are presented in Chapter Three, and their precision is also discussed. Finally Chapter Four includes the summary and conclusions of the work and suggestions for future research.

Chapter 2

COMPUTER PROGRAM

2.1 INTRODUCTION

A FORTRAN program was written using the ultimate strength method described in Chapter One. In the design aids of the LRFD Manual, the upper limits of the bolted connection are twelve horizontal rows and four vertical columns. This limit is also used as the limit of connections considered in this study. The coefficient of any bolted connection within this dimension can be obtained from the program. If the coefficient of a bolt connection having different dimensions needs to be determined, the dimension statement in the program can be changed. Both in the LRFD Manual and in the program, the applied load P_u is considered to act on the horizontal line extending from the centroid of the bolt connection, so that there is only a horizontal distance X_o from the centroid to P_u and the vertical distance between the load and the centroid is considered to be zero (see Figure 4).

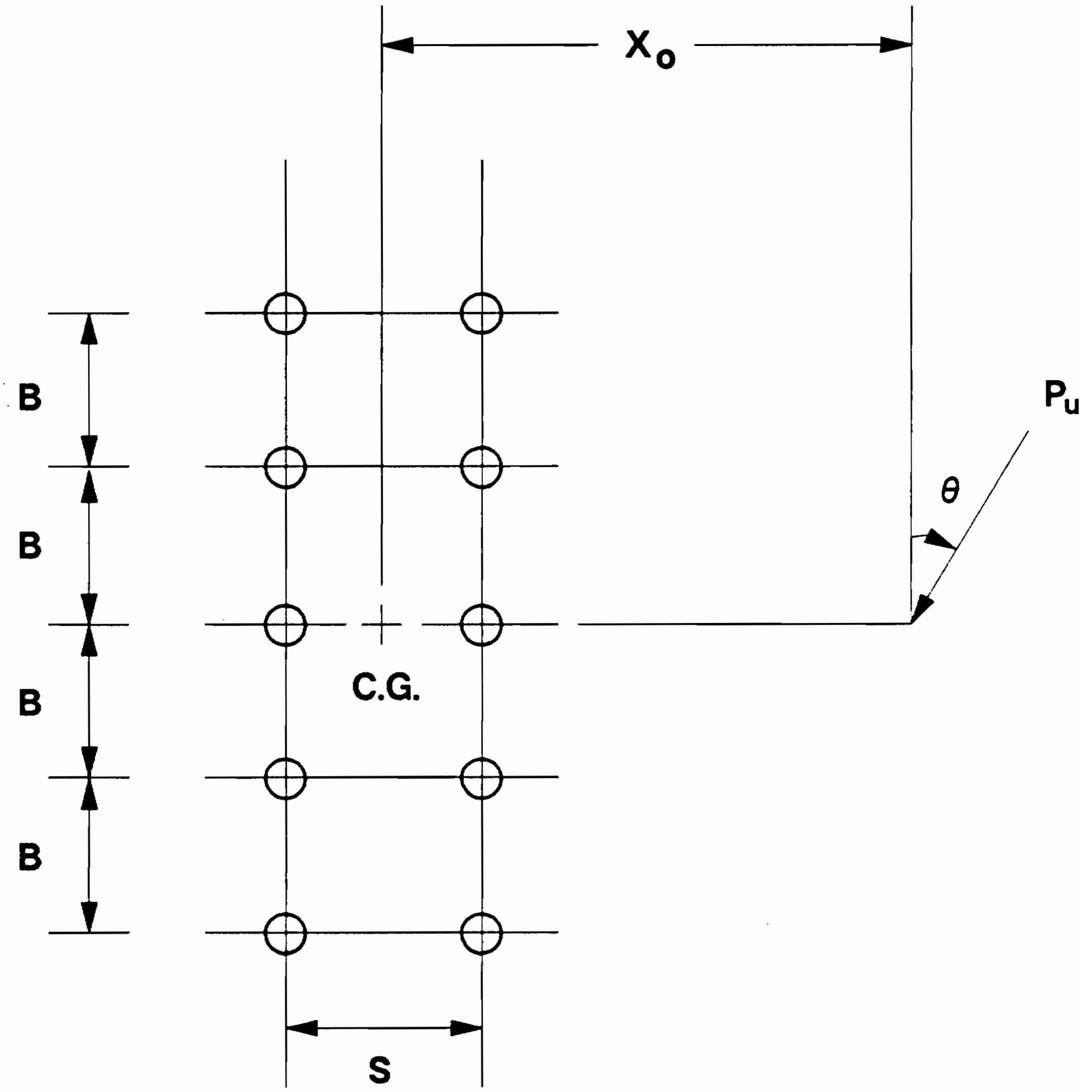

Figure 4. Illustration of A Typical Bolted Connection Subject to An Eccentric Load

The loading angle θ is measured clockwise from the vertical direction (see Figure 4), so that when the loading angle equals to 90°, the moment due to P_u about the centroid is zero and the coefficient for the connection is simply equal to the number of bolts.

In the program, the load P_u is normalized to have a unit magnitude. The distance between any two adjacent bolts in a vertical column is assumed to be constant. Similarly, the distance between any two adjacent bolts in a horizontal row is assumed to be constant.

2.2 *INPUT DATA*

The input data of the program are:

NC = number of columns in the bolted connection

NR = number of rows in the bolted connection

B = distance between two adjacent bolts in a vertical column

S = distance between two adjacent bolts in a horizontal row; (spacing)

X_o = horizontal distance from centroid to applied load P_u

θ = the load angle

With the above input data, this program automatically generates the coordinates of every bolt and computes the eccentricity coefficient.

2.3 CALCULATION PROCEDURE

The calculation procedure of this program followed the procedure described by Brandt (1982), and is summarized in the following six steps:

1. Generate the coordinates of every bolt

2. Assume the instantaneous center of rotation to be at the centroid of the bolted connection and calculate the moment of P_u about centroid

3. Use the elastic concept to locate the elastic instantaneous center of rotation

4. Calculate the deformation Δ by Equation 1.2 and the resistance force R by Equation 1.3 in each bolt

5. Find the unbalanced forces F_x, F_y, and the unbalanced moment M in Equation 1.4

6. Locate the new instantaneous center of rotation and iterate until Equation 1.4 is satisfied or certain convergence criteria are reached.

2.4 CONVERGENCE CRITERIA

Brandt (1982) applied an h-square extrapolation equation to approximate the final value of the coefficient. However, the coefficients obtained by this extrapolation equation are different from the tabulated coefficients in the LRFD Manual. The convergence criterion in the LRFD Manual was applied in this program. That is, the program keeps iterating to locate the instantaneous center of rotation and compute the coefficients until a convergence criterion of 1% is satisfied. In other words, when the difference between two successive coefficients divided by the coefficient generated from the last iteration is less than 1%, the final solution is considered to have been found. This convergence criterion can be expressed as:

$$\frac{|C_{n-1} - C_n|}{C_{n-1}} < 0.01 \tag{2.1}$$

where

C_n = the last generated coefficient

C_{n-1} = the 2nd from the last generated coefficient

After running the program, it was discovered that Equation 2.1 could never be satisfied in some situations (when NR is less than four, Equation 2.1 occasionally cannot be reached). Hence an extra convergence criterion was applied in the program as follows: if Equation 2.1 is not reached within ten iterations, the program terminates and the coefficient from the last iteration is chosen as the final solution.

Chapter 3

PARAMETRIC STUDY AND EQUATION DEVELOPMENT

3.1 INTRODUCTION

The objective of the project is to develop a predictive strength equation for a connection with one vertical row of bolts. The FORTRAN computer program stated in Chapter Two was used to generate the eccentricity coefficients for the predictive strength equation. From the relationships between the eccentricity coefficient and the independent variables, it was observed that a mathematical model of the coefficient with respect to the independent variables is hard to determine. Hence, by applying Buckingham's PI-Theorem and regression analysis, statistically based equations are developed to fit the data generated by the computer program.

3.2 *INDEPENDENT AND DEPENDENT VARIABLES*

For a one vertical column bolt connection, the input data NC (number of columns) and S (spacing) are constants and equal to 1 and 0, respectively. Thus, there are a total of four independent variables: NR (number of rows), θ (the load angle), X_o (horizontal distance from centroid to applied load), and B (distance between two adjacent bolts in a vertical column) in the equations. Referring to the LRFD Manual, the ranges of the four independent variables are as follows:

NR = 2 through 12

θ = 0° through 90°, or in radian, 0 through $\frac{\pi}{2}$

X_o = 2 in. through 36 in.

B = 3 in. and 6 in.

It is apparent that the eccentricity coefficient C is the only dependent variable. With the above independent variables, C can be expressed as a function of NR, θ, X_o, and B.

$$C=f(NR,\ \theta,\ X_o,\ B) \tag{3.1}$$

3.3 RELATIONSHIP BETWEEN INDEPENDENT VARIABLES AND THE COEFFICIENT

The relationships between each individual independent variable and the eccentricity coefficient are shown in the following figures. Figures B.1 through B.6 (see Appendix B) present the load angle θ with respect to the eccentricity coefficient C when B is equal to three inches in Figures B.1 through B.3 and B is equal to six inches in Figures B.4 through B.6. It was found that when X_o is small, the relationship between θ and C is almost a straight line parallel to the θ -axis. In other words, θ has very little influence on C and it can be assumed that the relationship between θ and C is constant when X_o is less than four inches. But as X_o increases, C is more sensitive to θ , especially when θ is greater than 65°, the slope begins to increase sharply. When X_o is close to thirty-six inches (greater than twenty-four inches), θ has considerable influence on C. It was also found that if the other variables are kept constant, the eccentricity coefficient when B equals six inches is larger than the coefficients when B equals three inches. This is because as B increases, the resistance arm of every bolt increases, and therefore so does the capacity of the connection group.

Figures C.1 through C.6 (see Appendix C) show C plotted against X_o. It was found that X_o is inversely proportional to C; however, when θ is close to 90°, the relationship between X_o and C is almost a straight line parallel to the X_o-axis. Hence it can be concluded that when θ approaches 90° there is a constant relationship between X_o and C, since there is no eccentricity moment due to the applied load P_u, and C is equivalent to the number of bolts, no matter what the values of X_o and B are. It was also observed that as θ increases beyond 45°, the relationship between X_o and C is almost linear, especially when B is equal to six inches.

Figures D.1 through D.9 (see Appendix D) the variation in C with respect to B. It was found that the relationship between B and C can always be regarded as linear. When X_o is small, it was observed that C is almost independent of B and θ. This is because the eccentricity moment is insignificant compared to the shear force. In other words, when the effect of the eccentricity moment is much smaller than the effect of the shear force, increasing the value of B does not significantly increase the capacity of the bolted connection.

Figures E.1 through E.6 (see Appendix E) show the variation in C with respect to NR. Generally, the relationship between NR and C can be regarded as linear, unless X_o is large, or θ is close to 0°. In other words, when the eccentricity moment is small, NR and C have a linear relationship. Because the figures for B equals six inches are similar to the figures for B equals three inches, only six figures were presented for NR with respect to C.

It was noticed that there are irregularities in some curves of Figures B.1, C.3, C.6, E.1, E.2, and E.6. Theoretically, the final answer of the eccentricity coefficient is obtained when the unbalanced force and unbalanced moment equal to zero. However, the two convergence criteria stated in section 2.4 are applied to determine the final solution; therefore, the computer program might occasionally generate coefficients which make the curves in the above figures discontinuous.

3.4 DIMENSIONAL ANALYSIS

The units of most physical quantities can be expressed by mass(M), length(L), and time(T). Hence, in dimensional analysis, these three units are usually chosen as the reference units, sometimes called fundamental or basic units. Maxwell (1871) gave the definition of dimension of a unit as follows: "When a given unit varies as the nth power of one of these fundamental units, it is said to be of n dimensions as regards that unit." Isaacson (1975) further explained the meaning of dimension as follows: "If a quantity, q, has the dimensions of a, b, and c respectively with regard to mass, length, and time, we say briefly that its dimensional representation is given by

$$q = M^a L^b T^c \text{''}$$

When the dimensions of a physical quantity are reduced to zero, the quantity is called dimensionless. Since the eccentricity coefficient is dimensionless, all of the independent variables in the statistical equation should also be dimensionless. Among the four independent variables, NR and θ are already dimensionless, however, X_o and B both have a reference unit of length. In order to make X_o and B dimensionless, Buckingham's PI-Theorem, the most important theorem in dimensional analysis, was applied. The indepentent variable B was chosen as the normalizing variable and the four independent variables were reduced to three dimensionless pi-terms

$$\pi_1 = NR \qquad \pi_2 = \theta \qquad \pi_3 = \frac{X}{B} \tag{3.2}$$

Then the eccentricity coefficient C can be expressed as a function of π_i :

$$C = f(\pi_1, \pi_2, \pi_3) \tag{3.3}$$

And the form of the predictive equation for C might be

$$C = e^{A}\, \pi_1^{n_1}\, \pi_2^{n_2}\, \pi_3^{n_3} \tag{3.4}$$

or

$$C = A + n_1\, \pi_1 + n_2\, \pi_2 + n_3\, \pi_3 \tag{3.5}$$

Thus, by applying Buckingham's PI-Theorem, the unknown function between the eccentricity coefficient and the independent variables was simplified to some undetermined constants, which can be found by using the regression analysis.

Before applying the regression analysis, a data file which includes the values of the eccentricity coefficient and the values of the independent variables was generated by the computer program stated in Chapter Two. The "REG" (regression) procedure in SAS, a general purpose statistical program, was then applied to fit linear regression equations to the data by the least-square method. A linear regression equation with multiple independent variables generally has the form

$$Y = A + n_1\, X_1 + n_2\, X_2 + \cdots + n_i\, X_i \tag{3.6}$$

where

Y = dependent variable

A = interception

X_i = independent variables

n_i = constants of the independent variables

The "REG" procedure generates the constants A and n_i of the regression equation. The precision of the regression equation is determined by the R^2 value. R^2, the square of the coefficient of correlation, indicates how well the regression line fits for given data. The R^2 value is always between zero and one. If the R^2 value is close to one, it means that the regression line conforms to the given data very well, but if R^2 is close to zero, then there is a poor fit between the data and the regression line.

3.5 PREDICTING EQUATIONS

The ranges of the independent variables were chosen as

NR = 2, 4, 6, 8, 10, 12

$\theta = 0, 0.1745, 0.3491, 0.5236, 0.6981, 0.8727, 1.0472, 1.2217, 1.3963, 1.5533$ (*radian*)

(0°, 10°, 20°, 30°, 40°, 50°, 60°, 70°, 80°, 89°)

X_o = 4, 8, 12, 16, 20, 24, 28, 32, 36 in.

B = 3, 4.5, 6 in.

Because θ = 1.5708 (90°) is a special case, θ = 1.5533 (89°) was selected instead. With the ranges presented above, there are 1620 eccentricity coefficients associated with 1620 combinations of independent variables. Since the regression procedure in SAS always generates a regression equation like Equation 3.6, for a predicting equation like Equation 3.4, the logarithms of the eccentricity coefficients and all the pi-terms in Equation 3.2 should be taken first. Because the logarithm of zero is infinity, instead of using the value of $\theta = 0$, θ was assumed to be 0.00001 when applying the regression analysis for Equation 3.4. The linear model regression analysis was then applied to obtain the constants A and n_i. The predicted eccentricity coefficient C was determined by exponentiating the predicted value calculated from the regression equation. Thus, Equation 3.4 becomes

$$C = e^{-1.1944} (NR)^{1.6346} (\theta)^{0.0461} \left(\frac{X_o}{B} \right)^{-0.5295} \tag{3.7}$$

$(R^2 = 0.8704)$

For a predicting equation like Equation 3.5, directly applying the linear model regression analysis yields all the constants. And the predicting equation is

$$C = -0.9238 + 0.7692(NR) + 1.9638(\theta) - 0.3840\left(\frac{X_o}{B}\right) \tag{3.8}$$

$(R^2 = 0.8746)$

A plot of the coefficients from Equation 3.7 versus the coefficients from the computer program is shown in Figure 5. Figure 6 presents the coefficients from Equation 3.8 versus the coefficients from the computer program. Two lines indicating the 25% error limit were drawn in each figure.

It was found in Figure 6 that Equation 3.8 might generate negative eccentricity coefficients which are not reasonable. Several special data groups were discovered in both figures. In Figure 5, the six special data groups above the top 25% error limit line described C when

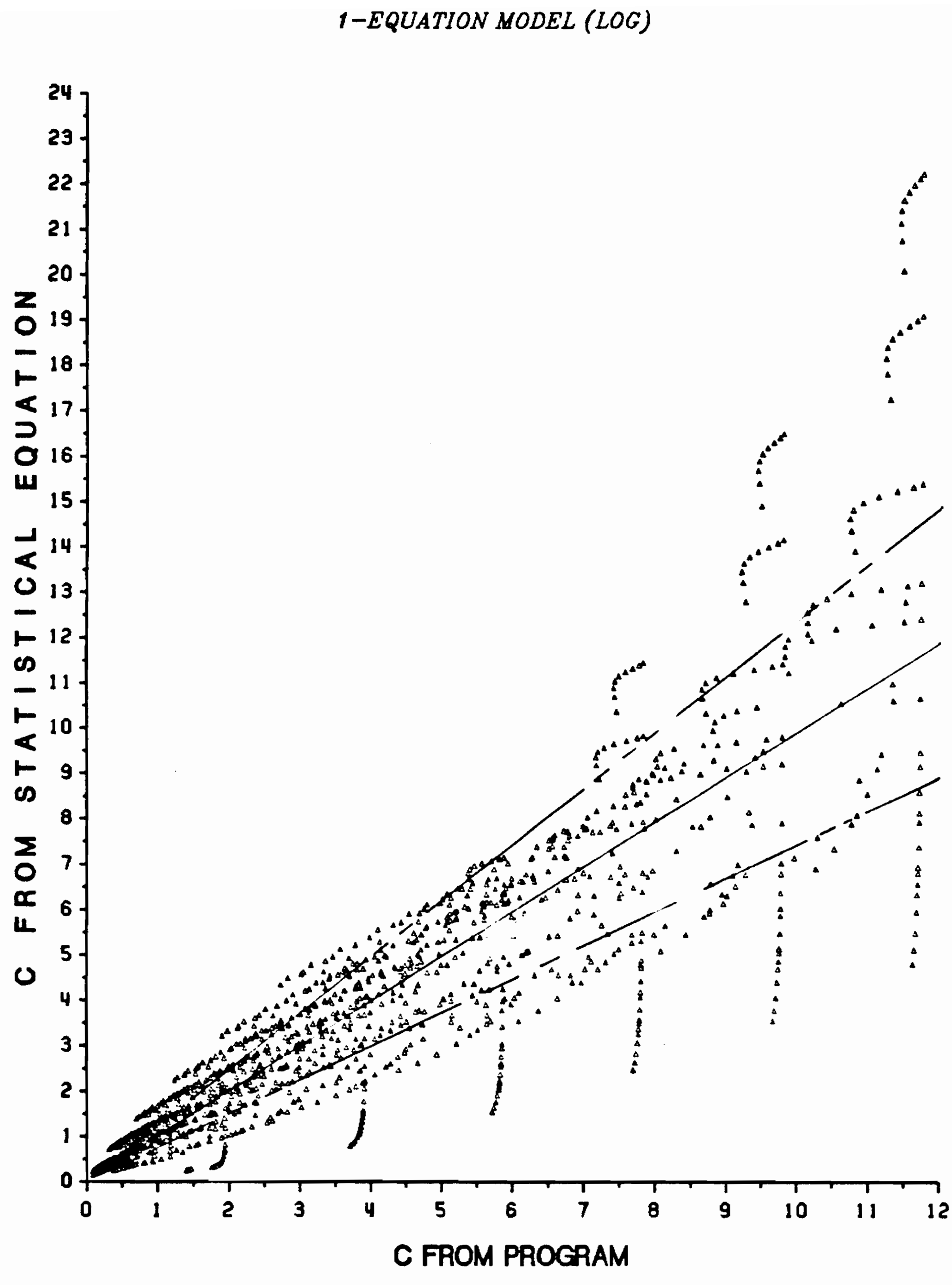

Figure 5. The Predicted Coefficients from Eq.(3.7) vs the Coefficients from Computer Program

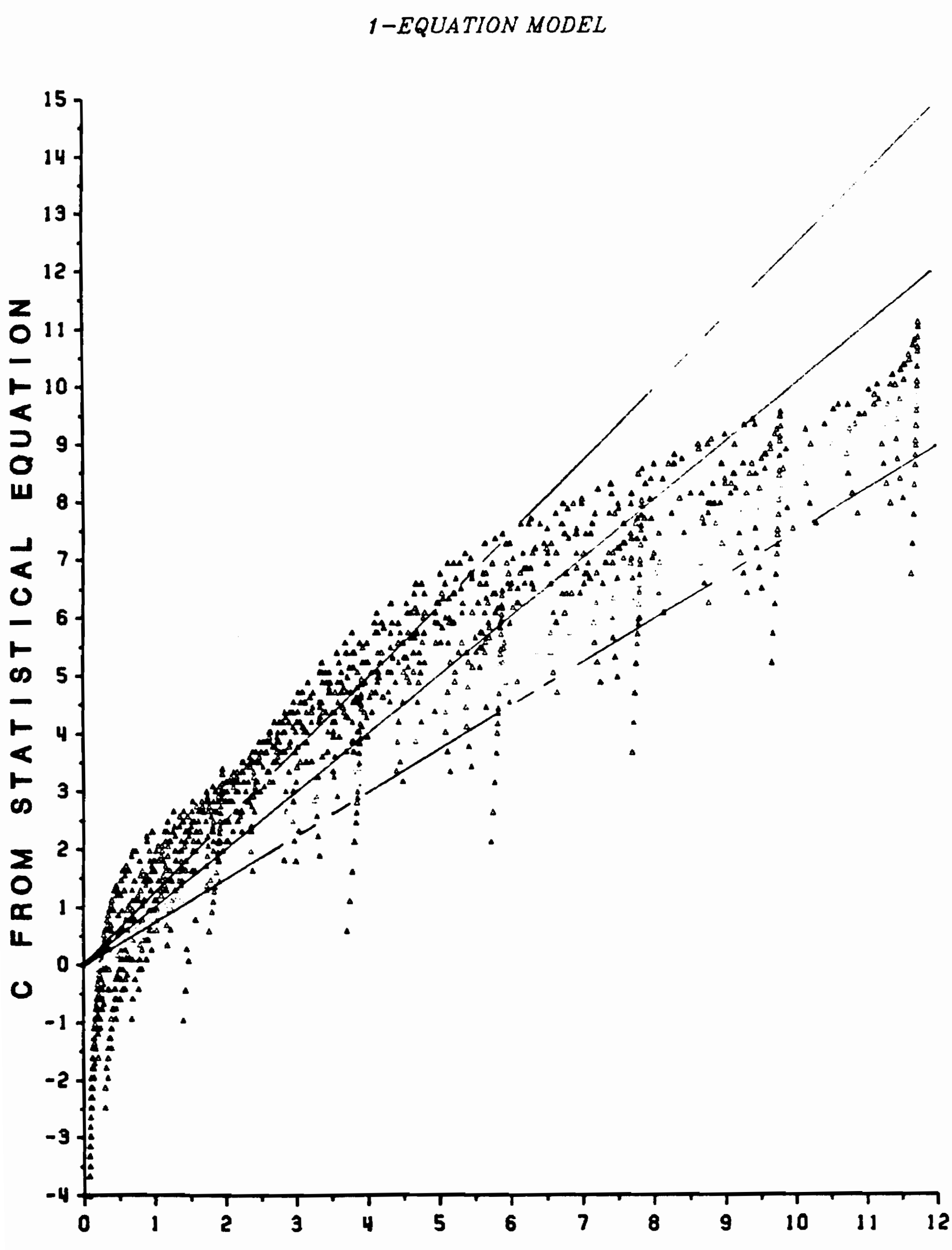

Figure 6. The Predicted Coefficients from Eq.(3.8) vs the Coefficients from Computer Program

X_o = 4 in. and 8 in.

B = 3 in. through 6 in.

θ = 10° through 89°

NR = 8 through 12

Figures B.1, B.4, D.1, D.2, and D.3 demonstrate that θ and B has insignificant influence on C when X_o is equal to two inches. If Equation 3.7 is used, including θ causes large errors when X_o is small.

In both Figures 5 and 6, the six data groups below the bottom 25% error limit line were identified describing C when

θ = 89°

X_o = 12 in. through 36 in.

B = 3 in. through 6 in.

NR = 2 through 12

In this case, it can be seen from Figures C.3, C.6, D.3, D.6, and D.9 that C is almost independent of X_o and B. Only independent variable NR has sufficient influence upon C to be required in the equation, and both X_o and B can be ignored. Hence, Equations 3.7 and 3.8 give large errors when θ is 89°.

Neither Equation 3.7 nor Equation 3.8 provide satisfactory results because the R^2 values are not close enough to one and because there are so many data points beyonds the 25% error limit lines in both Figures.

After numerous trials, it was found that there are two ways to improve the precision of the predicting statistical equations. These are presented in the following section.

3.6 METHODS TO IMPROVE THE PRECISION OF THE PREDICTING EQUATIONS

3.6.1 Divide the Ranges of the Independent Variables

If the 1620 combinations of variables are divided into several parts, and more than one equation is used, the precision of the predicted coefficient can be improved. As stated above, when X_o is small and θ is close to 90°, Equations 3.7 and 3.8 yield large errors. Hence, the two sets of data when X_o is small (less than eight inches) and θ is close to 90° (greater than 80°) were separated from the other data

1. X_o < 8 in. (the range of the other independent variables were not separated)

2. θ > 80° (the range of the other independent variables were not separated)

3. data not included in 1. and 2.

Instead of one statistical equation, three equations were used to predict the eccentricity coefficients. The three sets of A and n_i of Equations 3.4 and 3.5 are presented in Tables 1 and 2, and the results are illustrated in Figures 7 and 8 respectively. From Figures 7 and 8, it was discovered that those special data groups still exist; however, their precision has been improved and most of the errors have been restricted within the 25% error limit lines.

In addition to one-equation and three-equation models, a nine-equation model was tried, and the 1620 combinations were divided into nine parts by separating the ranges of the independent variables θ and X_o into three sections (the ranges of NR and B were not separated) (see Table 3). The nine sets of constants A and n_i of Equations 3.4 and 3.5 are shown in Tables 4 and 5 respectively, and the results are presented in Figures 9 and 10. There are still six special data groups in both Figures 9 and 10. These data groups were identified as the second case (θ = 89°, X_o = 12 in. through 36 in., B = 3 in. through 6 in., NR = 2 through 12) in section 3.5. It was observed that the precision has been remarkably improved. Only a few data points exceed the 25% error limit lines. An 18-equation model for Equations 3.4 and 3.5 were also tested by further dividing the independent variable NR into two parts (NR_1 = 2,4,6 and NR_2 = 8,10,12). The constants are listed in Tables 6 through 9, and the results are shown in Figures 11 and 12. Both Figures show more improved precision than Figures 9 and 10.

3.6.2 Increase The Number of PI-Terms

Three independent pi-terms NR, θ , and $\frac{X}{B}$ have been used. However, if arbitrarily combining the three pi-terms by multiplication or division, more pi-terms can be ob-

Table 1. The constants of Eq.(3.4) (3-equation model)

	A	n_1	n_2	n_3	R^2
1.	-0.7685	1.3066	0.0076	-0.2264	0.9531
2.	-1.6812	1.2241	3.2845	-0.1493	0.9689
3.	-1.2764	1.7491	0.0247	-0.6750	0.9345

Table 2. The constants of Eq.(3.5) (3-equation model)

	A	n_1	n_2	n_3	R^2
1.	-0.4942	1.0113	0.6079	-0.8291	0.9926
2.	-12.6939	0.9970	8.5776	-0.1585	0.9899
3.	-0.5207	0.7097	1.5466	-0.3724	0.8929

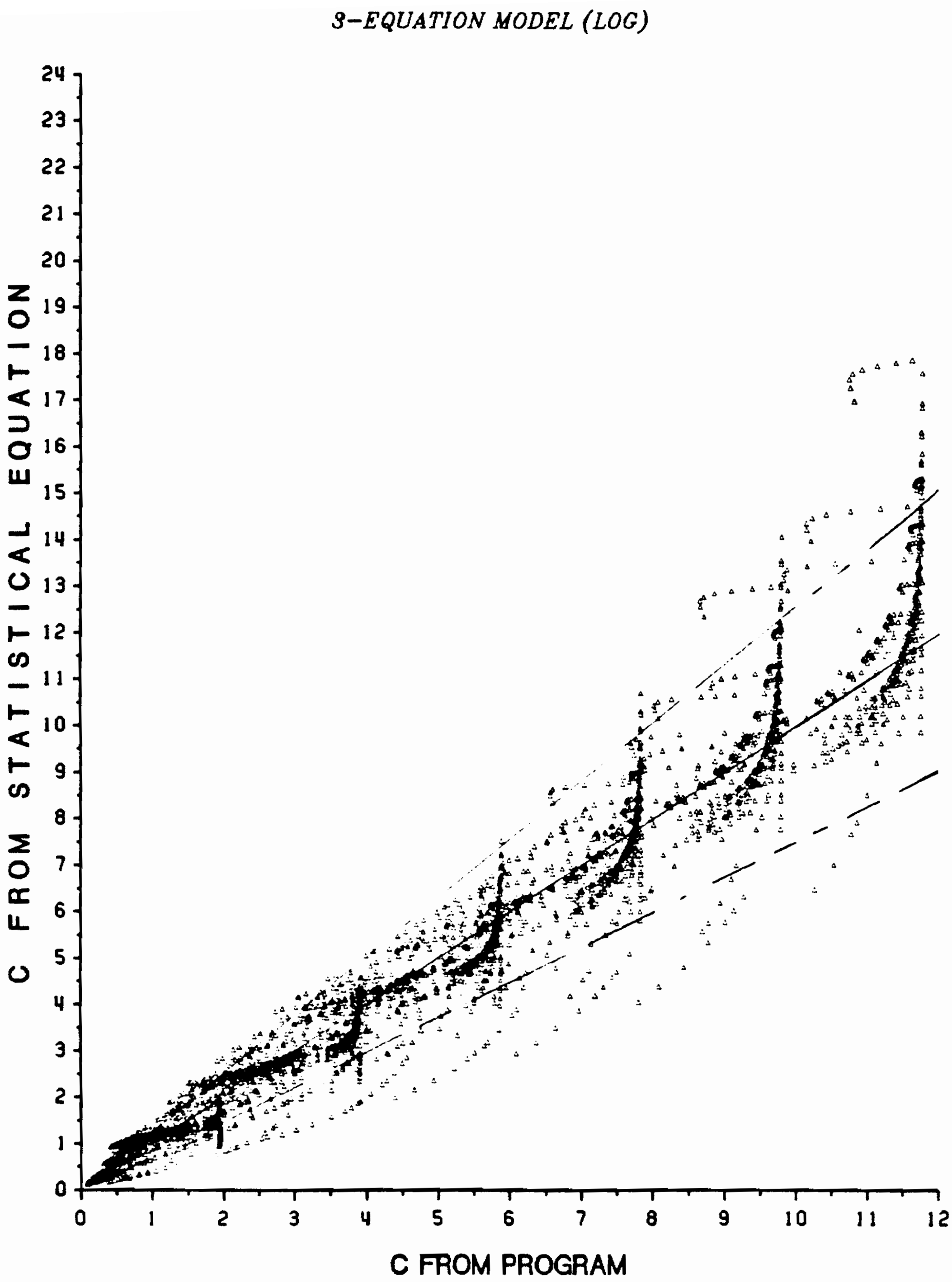

Figure 7. The Predicted Coefficients from Eq.(3.4) vs The Coefficients from the Computer Program (3-equation model)

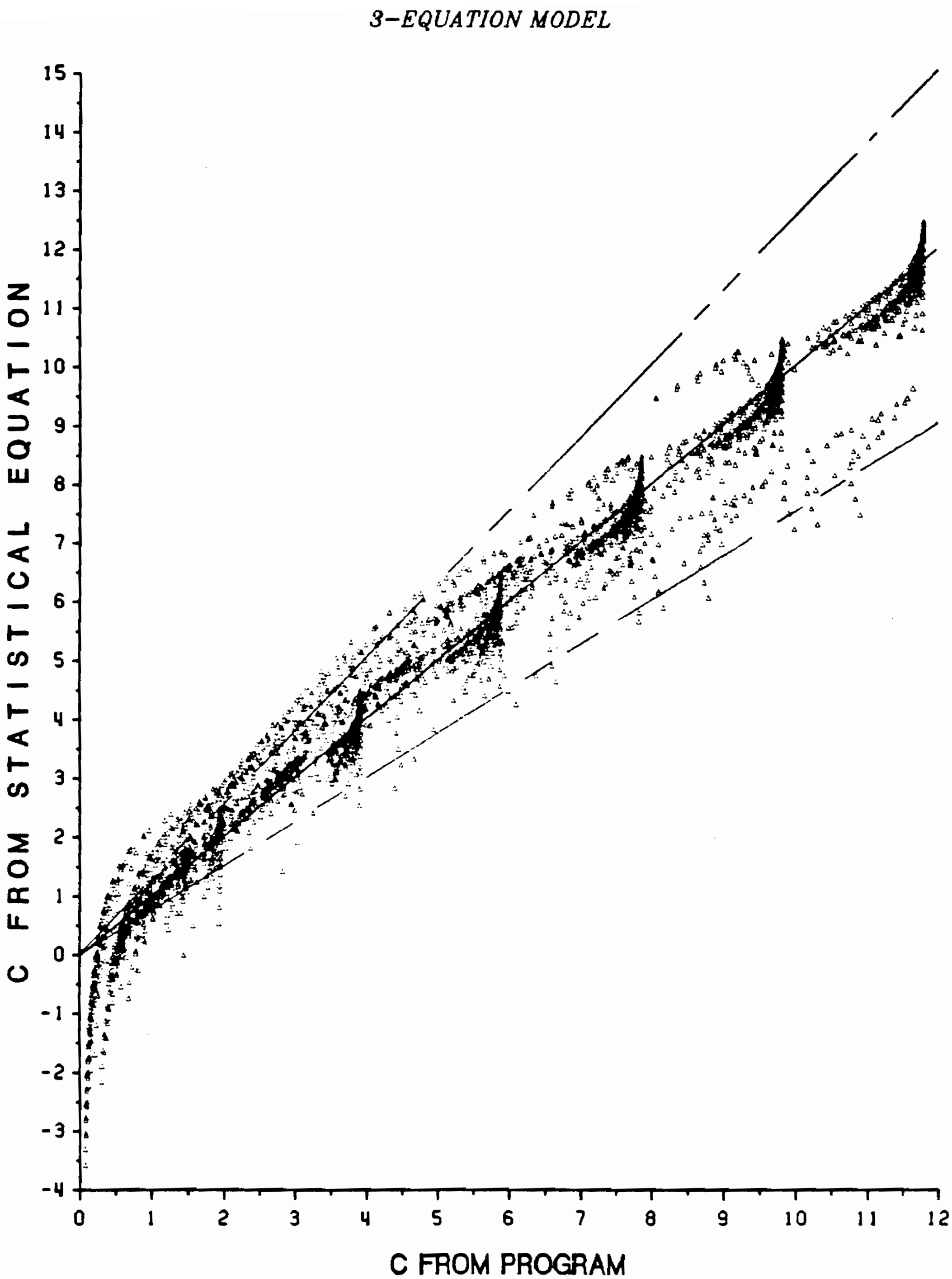

Figure 8. The Predicted Coefficients from Eq.(3.5) vs The Coefficients from the Computer Program (3-equation model)

Table 3. The Ranges of θ and X_0 for 9-equation Model

θ_1	θ_2	θ_3
0° 10° 20° 30° (0,0.1745,0.3491,0.5236)	40° 50° 60° (0.6981, 0.8727, 1.0472) ( ): in radian	70° 80° 89° (1.2217, 1.3963,1.5533)

(a)

X_{o1}	X_{o2}	X_{o3}
4 in. 8 in. 12 in.	16 in. 20 in. 24 in.	28 in. 32 in. 36 in.

(b)

Table 4. The constants of Eq.(3.4) (9-equation model)

x_o \ θ	θ_1	θ_2	θ_3
x_{o1}	A = -1.3756 n_1 = 1.6093 n_2 = 0.0002 n_3 = -0.5113 R^2 = 0.9815	A = -1.1323 n_1 = 1.5180 n_2 = 0.2848 n_3 = -0.4037 R^2 = 0.9827	A = -0.7546 n_1 = 1.1827 n_2 = 1.0925 n_3 = -0.1454 R^2 = 0.9685
x_{o2}	A = -1.4661 n_1 = 1.8736 n_2 = 0.0037 n_3 = -0.8577 R^2 = 0.9984	A = -1.2287 n_1 = 1.7725 n_2 = 0.5890 n_3 = -0.6925 R^2 = 0.9954	A = -1.3861 n_1 = 1.3543 n_2 = 2.6332 n_3 = -0.3071 R^2 = 0.9584
x_{o3}	A = -1.4106 n_1 = 1.9249 n_2 = 0.0055 n_3 = -0.9363 R^2 = 0.9990	A = -1.1395 n_1 = 1.8547 n_2 = 0.7499 n_3 = -0.8105 R^2 = 0.9982	A = -1.7336 n_1 = 1.4531 n_2 = 3.6223 n_3 = -0.3909 R^2 = 0.9554

Table 5. The constants of Eq.(3.5) (9-equation model)

x_o \ θ	θ_1	θ_2	θ_3
x_{o1}	A = 0.3273 n_1 = 0.9431 n_2 = -0.0375 n_3 = -0.9856 R^2 = 0.9815	A = -0.7285 n_1 = 0.9549 n_2 = 1.0729 n_3 = -0.8363 R^2 = 0.9877	A = -4.0432 n_1 = 0.9999 n_2 = 2.9122 n_3 = -0.3147 R^2 = 0.9925
x_{o2}	A = 0.5363 n_1 = 0.6343 n_2 = 0.2446 n_3 = -0.4404 R^2 = 0.9312	A = -1.1503 n_1 = 0.7113 n_2 = 1.8064 n_3 = -0.4015 R^2 = 0.9624	A = -9.9845 n_1 = 0.9300 n_2 = 7.3155 n_3 = -0.2373 R^2 = 0.9813
x_{o3}	A = 0.4391 n_1 = 0.4399 n_2 = 0.3113 n_3 = -0.2125 R^2 = 0.9132	A = -1.1921 n_1 = 0.5480 n_2 = 1.9157 n_3 = -0.2287 R^2 = 0.9404	A = -13.1874 n_1 = 0.8575 n_2 = 9.6278 n_3 = -0.1581 R^2 = 0.9649

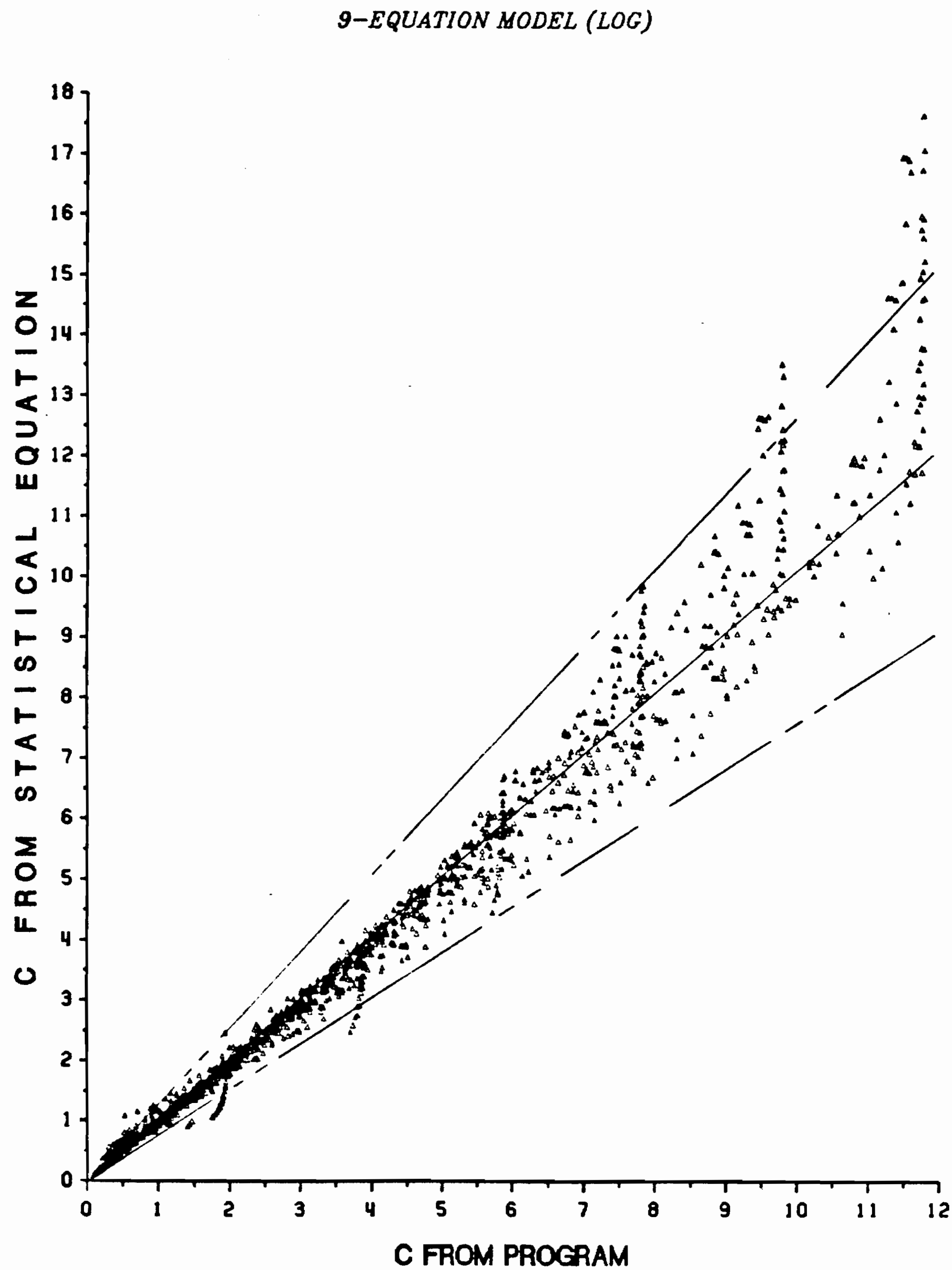

Figure 9. The Predicted Coefficients from Eq.(3.4) vs The Coefficients from the Computer Program (9-equation model)

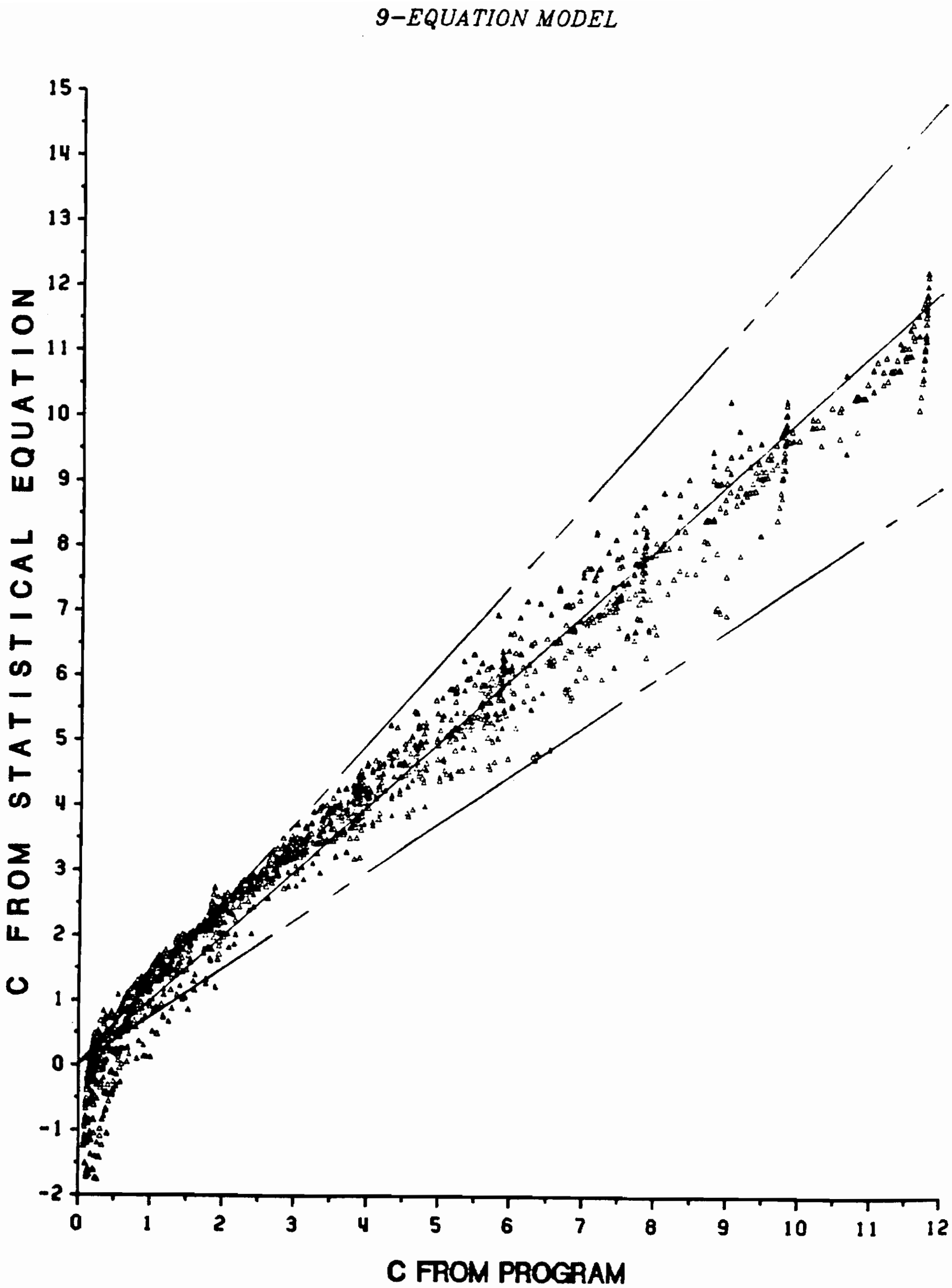

Figure 10. **The Predicted Coefficients from Eq.(3.5) vs The Coefficients from the Computer Program (9-equation model)**

Table 6. The constants of Eq.(3.4) (18-equation model, NR = 2,4,6)

X_o \ θ	θ_1	θ_2	θ_3
X_{o1}	A = -1.4198 n_1 = 1.7350 n_2 = 0.0012 n_3 = -0.6955 R^2 = 0.9892	A = -1.2020 n_1 = 1.6627 n_2 = 0.4155 n_3 = -0.5471 R^2 = 0.9869	A = -0.9904 n_1 = 1.2391 n_2 = 1.7446 n_3 = -0.2225 R^2 = 0.9491
X_{o2}	A = -1.3766 n_1 = 1.9062 n_2 = 0.0053 n_3 = -0.9380 R^2 = 0.9986	A = -1.1381 n_1 = 1.8730 n_2 = 0.7462 n_3 = -0.8122 R^2 = 0.9983	A = -1.7215 n_1 = 1.4456 n_2 = 3.7348 n_3 = -0.3897 R^2 = 0.9453
X_{o3}	A = -1.3369 n_1 = 1.9281 n_2 = 0.0065 n_3 = -0.9735 R^2 = 0.9983	A = -1.0221 n_1 = 1.9053 n_2 = 0.8579 n_3 = -0.8907 R^2 = 0.9991	A = -2.0619 n_1 = 1.5663 n_2 = 4.9159 n_3 = -0.5007 R^2 = 0.9492

Table 7. The constants of Eq.(3.4) (18-equation model, NR = 8,10,12)

x_o \ θ	θ_1	θ_2	θ_3
x_{o1}	A = -0.9091 n_1 = 1.3539 n_2 = -0.0007 n_3 = -0.3271 R^2 = 0.9410	A = -0.6905 n_1 = 1.2738 n_2 = 0.1541 n_3 = -0.2603 R^2 = 0.9577	A = -0.3511 n_1 = 1.0777 n_2 = 0.4405 n_3 = -0.0683 R^2 = 0.9395
x_{o2}	A = -1.3451 n_1 = 1.7632 n_2 = 0.0021 n_3 = -0.7774 R^2 = 0.9926	A = -0.9724 n_1 = 1.5644 n_2 = 0.4318 n_3 = -0.5727 R^2 = 0.9855	A = -0.8351 n_1 = 1.2087 n_2 = 1.5315 n_3 = -0.2245 R^2 = 0.9476
x_{o3}	A = -1.4000 n_1 = 1.8862 n_2 = 0.0044 n_3 = -0.8990 R^2 = 0.9943	A = -1.0071 n_1 = 1.7170 n_2 = 0.6419 n_3 = -0.7304 R^2 = 0.9919	A = -1.1614 n_1 = 1.2828 n_2 = 2.3287 n_3 = -0.2810 R^2 = 0.9459

Table 8. The constants of Eq.(3.5) (18-equation model, NR = 2,4,6)

x_o \ θ	θ_1	θ_2	θ_3
x_{o1}	A = 0.1984 n_1 = 0.8099 n_2 = 0.0491 n_3 = -0.6652 R^2 = 0.9373	A = -0.6784 n_1 = 0.8631 n_2 = 0.8427 n_3 = -0.5750 R^2 = 0.9626	A = -4.2942 n_1 = 0.9788 n_2 = 3.1197 n_3 = -0.2942 R^2 = 0.9774
x_{o2}	A = 0.1406 n_1 = 0.4155 n_2 = 0.1651 n_3 = -0.1750 R^2 = 0.9103	A = -0.7922 n_1 = 0.5332 n_2 = 1.0574 n_3 = -0.1955 R^2 = 0.9360	A = -8.2411 n_1 = 0.8562 n_2 = 5.9380 n_3 = -0.1421 R^2 = 0.9606
x_{o3}	A = 0.1000 n_1 = 0.2674 n_2 = 0.1366 n_3 = -0.0736 R^2 = 0.9131	A = -0.6278 n_1 = 0.3714 n_2 = 0.8783 n_3 = -0.0943 R^2 = 0.9176	A = -9.8394 n_1 = 0.7799 n_2 = 7.1012 n_3 = -0.0968 R^2 = 0.9262

Table 9. The constants of Eq.(3.5) (18-equation model, NR = 8,10,12)

x_o \ θ	θ_1	θ_2	θ_3
x_{o1}	A = 0.1540 n_1 = 1.0266 n_2 = -0.1241 n_3 = -1.3061 R^2 = 0.9970	A = -1.0018 n_1 = 1.0139 n_2 = 1.3031 n_3 = -1.0977 R^2 = 0.9938	A = -3.7472 n_1 = 1.0039 n_2 = 2.7046 n_3 = -0.3351 R^2 = 0.9636
x_{o2}	A = -0.0717 n_1 = 0.8221 n_2 = 0.3242 n_3 = -0.7058 R^2 = 0.9605	A = -2.2058 n_1 = 0.8523 n_2 = 2.5554 n_3 = -0.6074 R^2 = 0.9813	A = -11.9724 n_1 = 0.9840 n_2 = 8.6929 n_3 = -0.3325 R^2 = 0.9597
x_{o3}	A = -0.1431 n_1 = 0.6011 n_2 = 0.4860 n_3 = -0.3513 R^2 = 0.9483	A = -2.5521 n_1 = 0.6982 n_2 = 2.9530 n_3 = -0.3631 R^2 = 0.9724	A = -16.9312 n_1 = 0.9281 n_2 = 12.1544 n_3 = -0.2195 R^2 = 0.9556

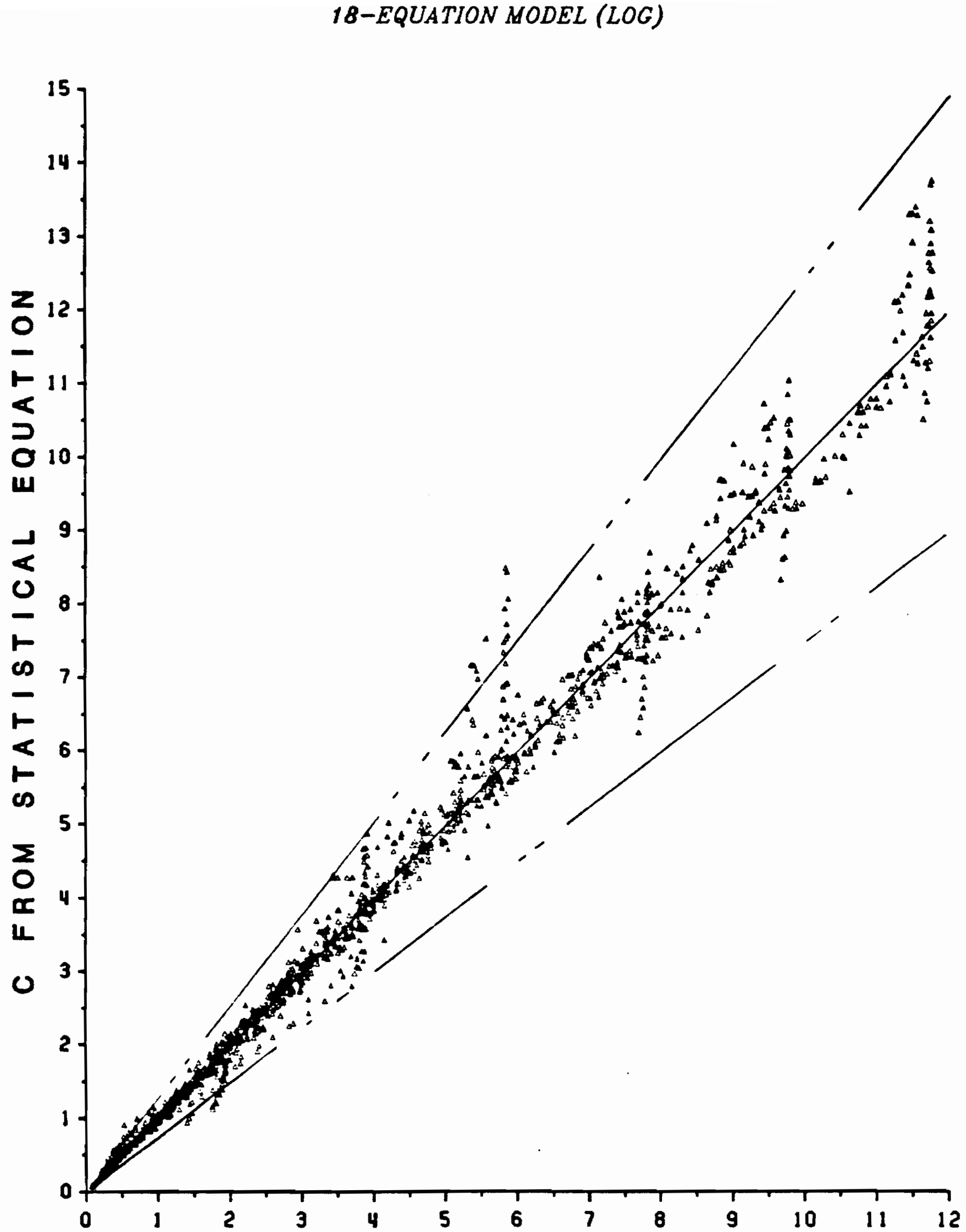

Figure 11. The Predicted Coefficients from Eq.(3.4) vs The Coefficients from the Computer Program (18-equation model)

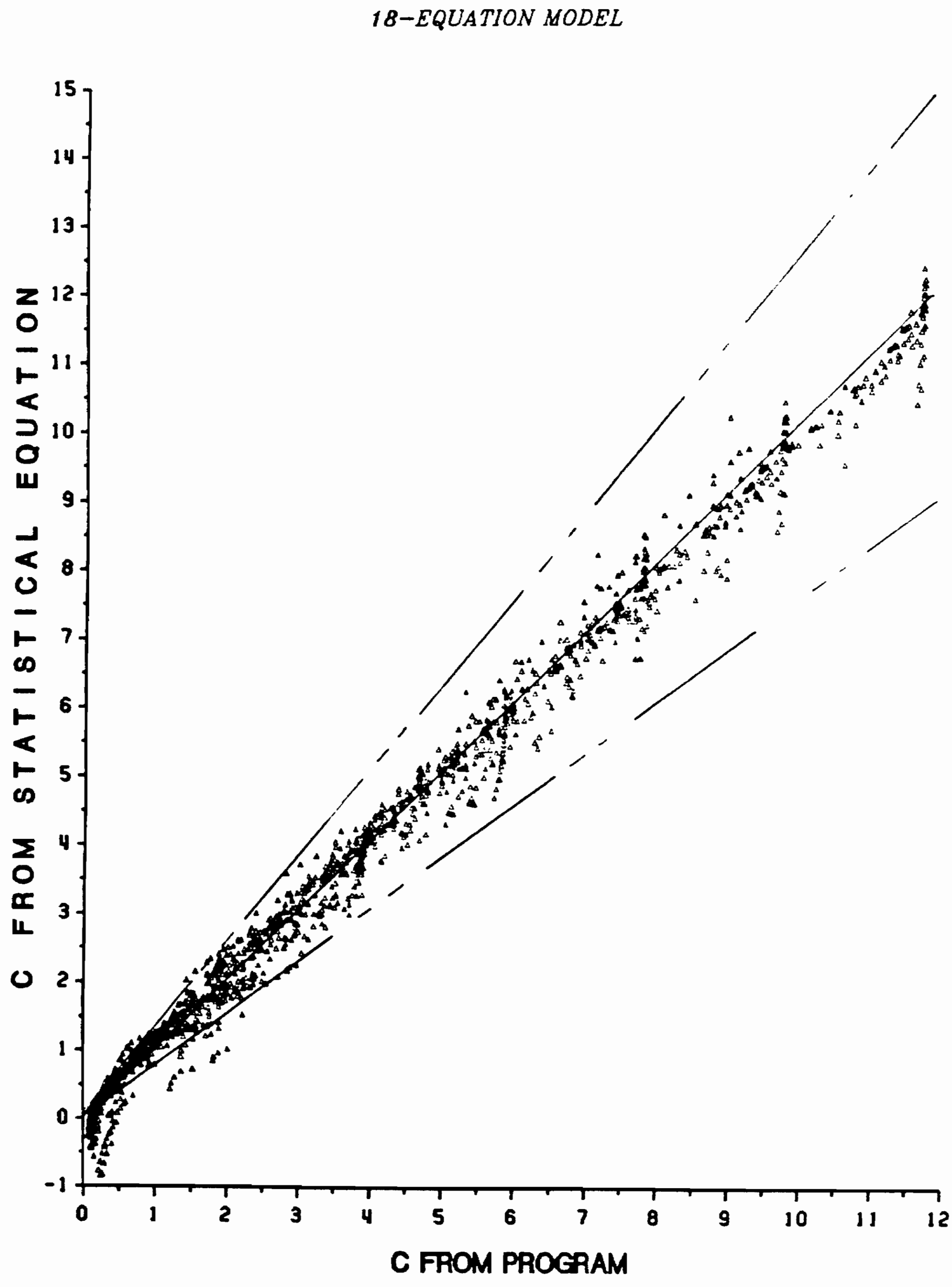

Figure 12. The Predicted Coefficients from Eq.(3.5) vs The Coefficients from the Computer Program (18-equation model)

tained. It was found that the precision of the predicting equations is proportional to the number of the pi-terms. Equations with eight pi-terms equations were tested. The R^2 values of Equations 3.7 and 3.8 with eight pi-terms are 0.9710 and 0.9318 respectively. Although the R^2 values can be improved remarkably, eight pi-terms equations are not applicable because they provide incorrect equation trends.

3.7 EQUATION TREND

The eccentricity coefficient is inversely proportional to the value of X_o , but proportional to the values of NR, θ, and B. Hence, any predicting equations should give higher eccentricity coefficient when the value of NR, θ, or B increases, and should yield lower eccentricity coefficient when X_o increases. In other words, constants n_1 and n_2 in both Equations 3.4 and 3.5 should be positive, and constant n_3 should be negative. The constants in Equations 3.7, 3.8, and in Tables 1,2, and 4 through 7 all show the right sign except the constant n_2 in row one, column one (X_{o1}, θ_1) in Tables 5, 7, and 9. As stated in Section 3.5, when X_o is small, θ can be omitted from the predicting equation. Thus, if θ is included in the predicting statistical equations, the sign of constant n_2 could be wrong.

Chapter 4

SUMMARY, CONCLUSIONS, AND FUTURE RESEARCH

4.1 SUMMARY

A computer program using the ultimate strength method was written to determine the coefficient of eccentrically loaded bolted connection. By comparing the eccentricity coefficients tabulated in the LRFD Manual and the eccentricity coefficients generated by this FORTRAN program, confidence in this program was established.

From Section 3.2, it was found that the exact function of the eccentricity coefficient with respect to all independent variables is hard to determine. This is because each independent variable has considerable influence on the eccentricity coefficient and the relationship between individual independent variable and the eccentricity coefficient is greatly influenced by the other independent variables.

Statistical equations for predicting eccentricity coefficients were developed. Figures 5 through 12 show the precision of these equations. If the eccentricity coefficients for the case when X_o is smaller than or equal to eight inches and θ is 89° (or close to 90°) are not included in these figures, most errors of the predicted coefficients are less than 25%. It is recognized that using nine or eighteen equations to predict the eccentricity coefficient for single line bolted connections is not practical for design application. However, it is worth attempting if nine equations can predict the eccentricity coefficients for connections with multiple lines of bolts.

For connections with multiple lines of bolts, there are two more independent variables, NC (number of columns) and S (spacing) that would have to be included in the equations. It is stated in Section 3.6.2 that the precision of the predicting equations is proportional to the number of pi-terms, so if NC and S are included in the equations, the precision of the predicted coefficients are expected to be improved.

In this paper, a nine-equation model was obtained by simply separating the ranges of both independent variables θ and X_o into three sections (see Table 3). The division of the independent variables can be more specific, for example, the eccentricity coefficients when X_o is less than or equal to eight inches and when θ is close to 90° can be separated from the other eccentricity coefficients.

4.2 CONCLUSIONS

Although one statistical equation does not yield satisfactory results, a nine-equation model provides acceptable precision and restricts most of the errors within 25% error range. If the division of the independent variables are more specified, it is believed that the precision of the statistical equations could be improved and the number of the equations could be reduced.

For connections with multiple lines of bolts, there are five pi-terms, NR, NC, θ, $\frac{X}{B}$, and $\frac{S}{B}$ in the statistical equation. Because the precision of the regression equation is proportional to the number of pi-terms, it is believed that statistical equations can provide acceptable predicted eccentricity coefficients for multiple lines bolted connections.

Using statistical equations to predict the eccentricity coefficient provides the benefit of saving space. Unlike tabulated eccentricity coefficients, statistical equations require a small space in the Manual. Statistical equations can also be easily programmed in computer for computer aided design.

4.3 FUTURE RESEARCH

Further studies could include developing the predicting statistical equations for multiple lines bolted connections. Statistical equations for welded connections can also be developed by the same procedure. There are several method for determining the eccentricity coefficients, future study can also include comparing all methods in detail.

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Appendix A

LIST OF THE FORTRAN COMPUTER PROGRAM

```
      PROGRAM ELBC
***********************************************************************
*                                                                     *
*      The program applies the ultimate strength, instantaneous center *
*      of rotation method to generate the coefficients of eccen-      *
*      trically loaded bolted connentions.                            *
*                                                                     *
***********************************************************************
*                                                                     *
*                          INDEX NOTATION                             *
*                                                                     *
***********************************************************************
*                                                                     *
*      NB : Quantity of bolts                                         *
*      ANGLE : The angle of the applied eccentric load                *
*      XP : X-distance from applied force to centroid                 *
*      YP : Y-distance from applied force to centroid                 *
*      XB(I) : X-Coordinate of bolt I                                 *
*      YB(I) : Y-Coordinate of bolt I                                 *
*      P : Applied eccentric load                                     *
*      XO : X-Coordinate of centroid from P                           *
*      YO : Y-Coordinate of centroid from P                           *
*      PJ : Polar moment of inertia                                   *
*      PX : Horizontal component of normalized applied force P ( of   *
*           unit magnitude )                                          *
*      PY : Vertical component of normalized applied force P          *
*      MO : Moment of P about centroid                                *
*      D(I) : Distance of bolt from instantaneous center              *
*      D2 : Square of D                                               *
*      SD2 : Summation of D2                                          *
*      DMAX : largest magnitude of D                                  *
*      DX(I),DXX(I) : X-distance of bolt I from instantaneous center  *
*      DY(I),DYY(I) : Y-distance of bolt I from instantaneous center  *
*      AX : X-Component of distance from centroid of bolt group       *
*           to instantaneous center                                   *
*      AY : Y-Component of distance from centroid of bolt group       *
*           to instantaneous center                                   *
*      MP : Moment about instantaneous center of normalized P         *
*      M(I) : Moment about instantaneous center of bolt force         *
*             divided by RULT                                         *
*      SM : Summation of M(I)                                         *
*      RULT : Ultimate bolt force capacity                            *
*      RF : Ratio of force on each bolt ( R ) to the RULT             *
*      DELTA(I) : Deformation of bolt I                               *
*      RX(I) : X-Component of bolt force                              *
*      RY(I) : Y-Component of bolt force                              *
*      SRX : Summation of RX(I)                                       *
*      SRY : Summation of RY(I)                                       *
*      FX : X-Component of unbalanced force                           *
*      FY : Y-Component of unbalanced force                           *
*      CU,CUT(I) : Bolt coefficient for ultimate strength solution    *
*                                                                     *
***********************************************************************
C----
      REAL MO, MP, M(48)
      DIMENSION XB(48), YB(48), D(48), DELTA(48), RX(48), RY(48),
     +          DX(48), DY(48), RF(48), XB2(12,4), YB2(12,4),
     +          CUT(18,12), DXX(48), DYY(48), XO(18), B(2)
      INTEGER FLAG
      DATA YO/ 0./
```

```
C-----------------------------------
C       Read data and print headings.
C-----------------------------------
        OPEN(1,FILE='ELBC.DAT')
        OPEN(3,FILE='ELBC.OUT')
C---------------------------------------------------------------
C       Referring to 1st AISC LRFD Manual,
C       the input X0 are 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 14, 16, 18,
C                        20, 24, 28, 32, 36 in.
C       the input B are 3 and 6 in.
C---------------------------------------------------------------
        READ(1,*)(X0(I),I=1,18)
        READ(1,*)(B(I),I=1,2)
        WRITE(*,'(1X)')
        WRITE(*,3)
 3      FORMAT(1X,'Units: in, kips, degree ( clockwise : positive )')
        WRITE(*,'(1X)')
        WRITE(*,4)
 4      FORMAT(1X,'Input NC, NC:The number of fasteners ',
     +            'in a horizontal row.')
        READ(*,*) NC
        WRITE(*,5)
 5      FORMAT(1X,'Input NRI, NRI:The least number of fasteners ',
     +            'in a vertical column.')
        READ(*,*) NRI
        WRITE(*,6)
 6      FORMAT(1X,'Input S, S:The distance between two adjacent ',
     +            'columns of fasteners.')
        READ(*,*) S
        WRITE(*,7)
 7      FORMAT(1X,'Input ANGLE, ANGLE:The load angle')
        READ(*,*) ANGLE
        WRITE(3,8) ANGLE
 8      FORMAT(1X,'ANGLE =',F4.1,' DEGREE')
C-----------------------------------------------------------------------------
C           Calculate the X-component and Y-component of the applied force P.
C           Assume that P equals to the unit downward force, that is, P = -1.
C-----------------------------------------------------------------------------
            PX = -1 * SIN(ANGLE/180.*3.141593)
            PY = -1 * COS(ANGLE/180.*3.141593)
C--------------------------------------------------------------
C           Start the loop of pitch. Pitches = 3 in. and 6 in.
C--------------------------------------------------------------
            DO 1100 N=1,2
C------------------------
C           Print headings.
C------------------------
               WRITE(3,'(1X)')
               WRITE(3,10) B(N)
 10            FORMAT(1X,'B =',F3.1,'in')
               WRITE(3,'(1X)')
               WRITE(3,12)
 12            FORMAT(3X,'X0',34X,'NR')
               WRITE(3,13)
 13            FORMAT(1X,'---------------------------------------------',
     +            '------------------------------')
               WRITE(3,14)
 14            FORMAT(3X,'in','    1     2     3     4     5     6     7',
     +         '     8     9    10    11    12')
               WRITE(3,13)
```

```
C-------------------------------------------------------------------
C             Start the loop of X0. X0 is horizontal distance from the
C             bolt centroid to the applied force P. The values of X0 are
C             listed in the 1st LRFD Manu.
C-------------------------------------------------------------------
              IF(ANGLE.EQ.90) THEN
                DO 2000 NX0=1,18
                   DO 2000 NR=NRI,12
                      CUT(NX0,NR)=NR
2000            CONTINUE
              ELSE
              DO 1200 NX0=1,18
C-------------------------------------------------------------
C                Start the loop of NR. NR are number of rows, from
C                NRI( the least number of fasteners in a vertical
C                column ) to 12.
C--------------------------------------------------------------
                 DO 1050 NR=NRI,12
                    NB=NC*NR
C--------------------------------------------------
C                   Generate the coordinates of bolts.
C--------------------------------------------------
                    X1=(NC-1)*S/2.
                    Y1=(NR-1)*B(N)/2.
                    DO 105 I=1,NR
                       DO 105 J=1,NC
                          XB2(I,J)=X1-S*(J-1)
                          YB2(I,J)=Y1-B(N)*(I-1)
 105                CONTINUE
C----
                    K=1
                    DO 110 I=1,NC
                       DO 110 J=1,NR
                          XB(K)=XB2(J,I)
                          YB(K)=YB2(J,I)
                          K=K+1
 110                CONTINUE
C----
                    DO 111 I=1,NB
                       DXX(I)=XB(I)
                       DYY(I)=YB(I)
 111                CONTINUE
C---------------------------------------------------------
C                   Calculate the moment of P about centroid.
C---------------------------------------------------------
                    M0 = PY*X0(NX0)-PX*Y0
C--------------------------------------------------------
C                   Calculate the polar moment of inertia J.
C--------------------------------------------------------
                    PJ=0.
                    DO 115 I=1,NB
                       PJ=PJ+(XB(I)**2)+(YB(I)**2)
 115                CONTINUE
C----
                    FX=PX
                    FY=PY
                    X0T=X0(NX0)
                    Y0T=Y0
C----------------------------------------------------------------
C                   Locate the instantaneous center and check the
```

```
C                 1% convergence criterion. If the 1% convergence
C                 criterion is not satisfied in 10 loops, the 10th
C                 generated coefficient is chosen as the final
C                 answer.
C-----------------------------------------------------------------
                  CUP=-1.0
                  FLAG=1
C-----------------------------------------------------------------
C                   Locate the instantaneous center. AX and AY are
C                   X and Y component of distance from centroid
C                   of bolt group to instantaneous center.
C-----------------------------------------------------------------
1111                AX=-(FY*PJ)/(NB*MO)
                    AY=(FX*PJ)/(NB*MO)
                    X=XOT-AX
                    Y=YOT-AY
C----------------------------------------------------------------
C                   Calculate MP, the moment about instnataneous
C                   center of applied force P.
C----------------------------------------------------------------
                    MP=PY*X-PX*Y
C----------------------------------------------------------------
C                   Calculate DX and DY, the X and Y distance of
C                   bolt I from instantaneous center, also DMAX,
C                   the largest distance of bolt from instant-
C                   aneous center.
C----------------------------------------------------------------
                    DMAX=0.
                    SD2=0.
                    DO 120 I=1,NB
                      DX(I)=DXX(I)-AX
                      DY(I)=DYY(I)-AY
                      D2=DX(I)**2+DY(I)**2
                      D(I)=SQRT(D2)
                      SD2=SD2+D2
                      DXX(I)=DX(I)
                      DYY(I)=DY(I)
                      IF(DMAX.LT.D(I)) THEN
                        DMAX=D(I)
                      ENDIF
 120                CONTINUE
C------------------------------------------------------------------
C                   Calculate DELTA(I), RF(I), M(I), and SM(I).
C                   DELTA(I): the deformations of bolts
C                   RF(I): ratio of force on each bolt to the RULT
C                   RULT: ultimate bolt force capacity
C                   M(I): the moment about instantaneous center of
C                         bolt force divided by RULT
C                   SM: Summation of M(I)
C------------------------------------------------------------------
                    SM=0.
                    DO 125 I=1,NB
                      DELTA(I)=0.34*D(I)/DMAX
                      RF(I)=(1.-EXP((-10)*DELTA(I)))**0.55
                      M(I)=RF(I)*D(I)
                      SM=SM+M(I)
 125                CONTINUE
C----------------------------------------------------------------
C                   Calculate CU, bolt coefficient for ultimate
C                   strength solution, and RULT.
```

```
C                       CUD: The difference between two successive
C                            coefficients.
C-----------------------------------------------------------------
                        CU=ABS(SM/MP)
                        CUD=ABS((CUP-CU)/CUP)
                        IF(CUD.GE.0.01.AND.FLAG.LE.10) THEN
                        FLAG=FLAG+1
                        CUP=CU
                        RULT=(-MP)/SM
C----------------------------------------------------------------------
C                       Calculate RX, RY: X and Y component of bolt force,
C                       and their summation, SRX, SRY.
C----------------------------------------------------------------------
                        SRX=0.
                        SRY=0.
                        DO 130 I=1,NB
C---------------------------------------------------------------
C                          In case that D(I) equals to 0, skip the
C                          calculations of RX(I) and RY(I). Since
C                          this will make them indefinite.
C---------------------------------------------------------------
                           IF(D(I).NE.0) THEN
                             RX(I)=(-DY(I))/D(I)*RF(I)*RULT
                             RY(I)=DX(I)/D(I)*RF(I)*RULT
                           ENDIF
                           SRX=RX(I)+SRX
                           SRY=RY(I)+SRY
 130                    CONTINUE
C--------------------------------------------------------------------
C                       Calculate FX: the X-component of the unbalance
C                                     force F
C                                 FY: the Y-component of the unbalance
C                                     force F
C--------------------------------------------------------------------
                        FX=PX+SRX
                        FY=PY+SRY
                        XOT=X
                        YOT=Y
                        GO TO 1111
                        ENDIF
                        CUT(NXO,NR)=CU
 1050              CONTINUE
 1200           CONTINUE
                ENDIF
C-----------------------------------
C                  Print the results.
C-----------------------------------
                DO 2001 NN=1,18
                   WRITE(3,20) XO(NN),(CUT(NN,J),J=1,12)
 20                FORMAT(1X,F4.1,12(1X,F5.2))
 2001           CONTINUE
C----
 1100     CONTINUE
C----
          CLOSE(1)
          WRITE(*,'(1X)')
          STOP
          END
```

Appendix B

THE RELATIONSHIP BETWEEN THE ECCENTRICITY COEFFICIENT AND THE LOAD ANGLE

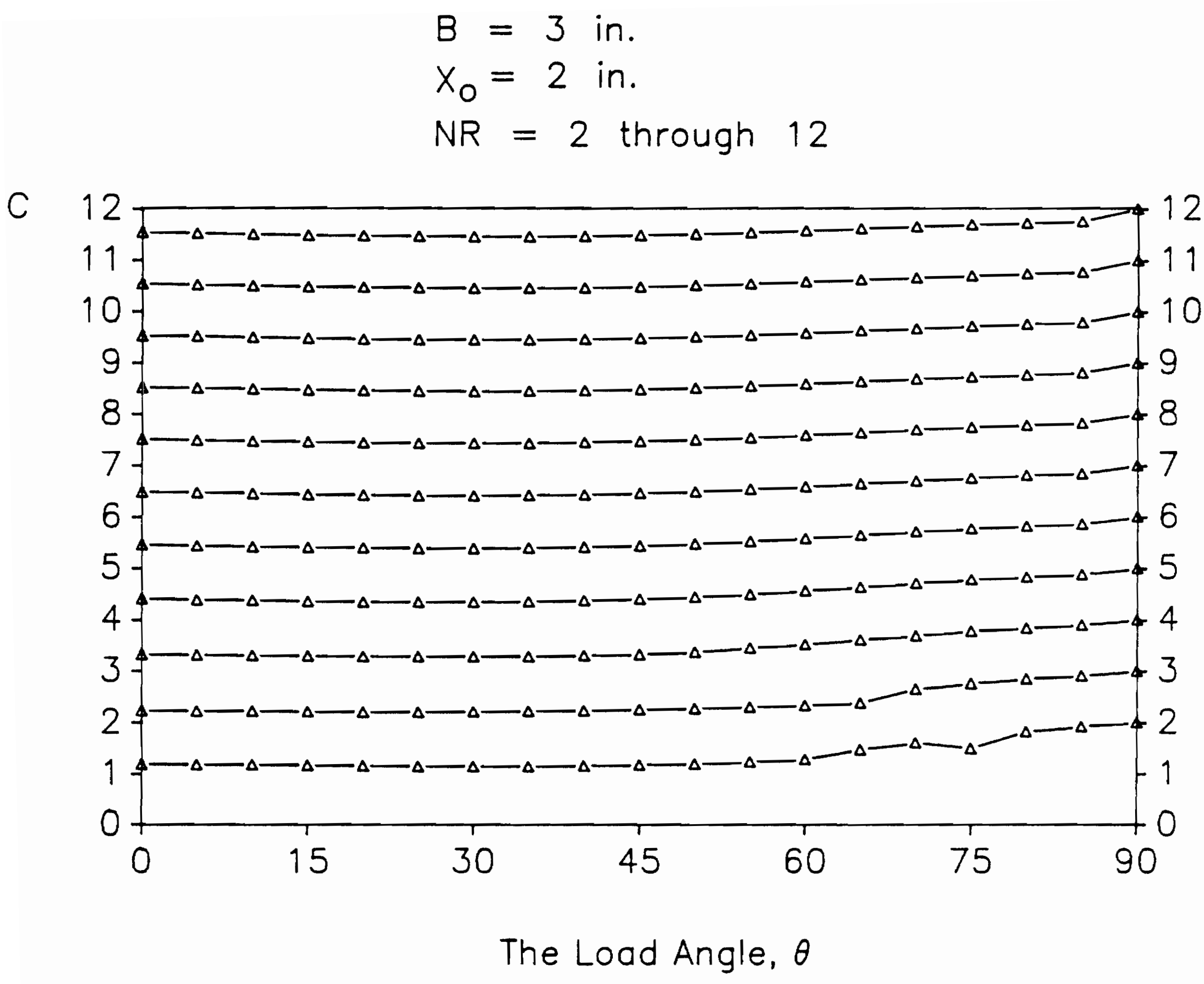

Figure B.1. The Eccentricity Coefficient vs The Load Angle (B = 3 in., X_o = 2 in., NR = 2 through 12)

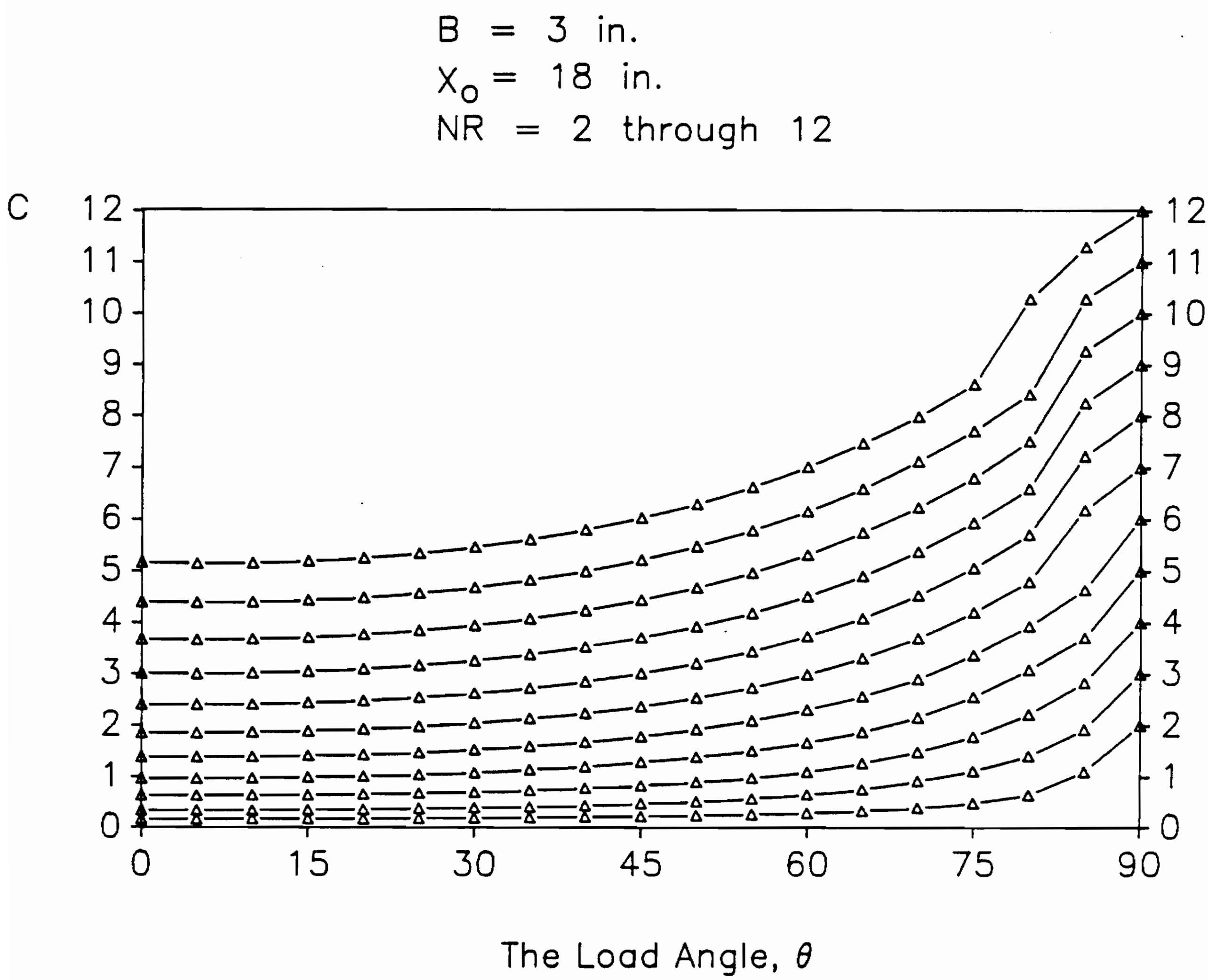

Figure B.2. The Eccentricity Coefficient vs The Load Angle (B = 3 in., X_o = 18 in., NR = 2 through 12)

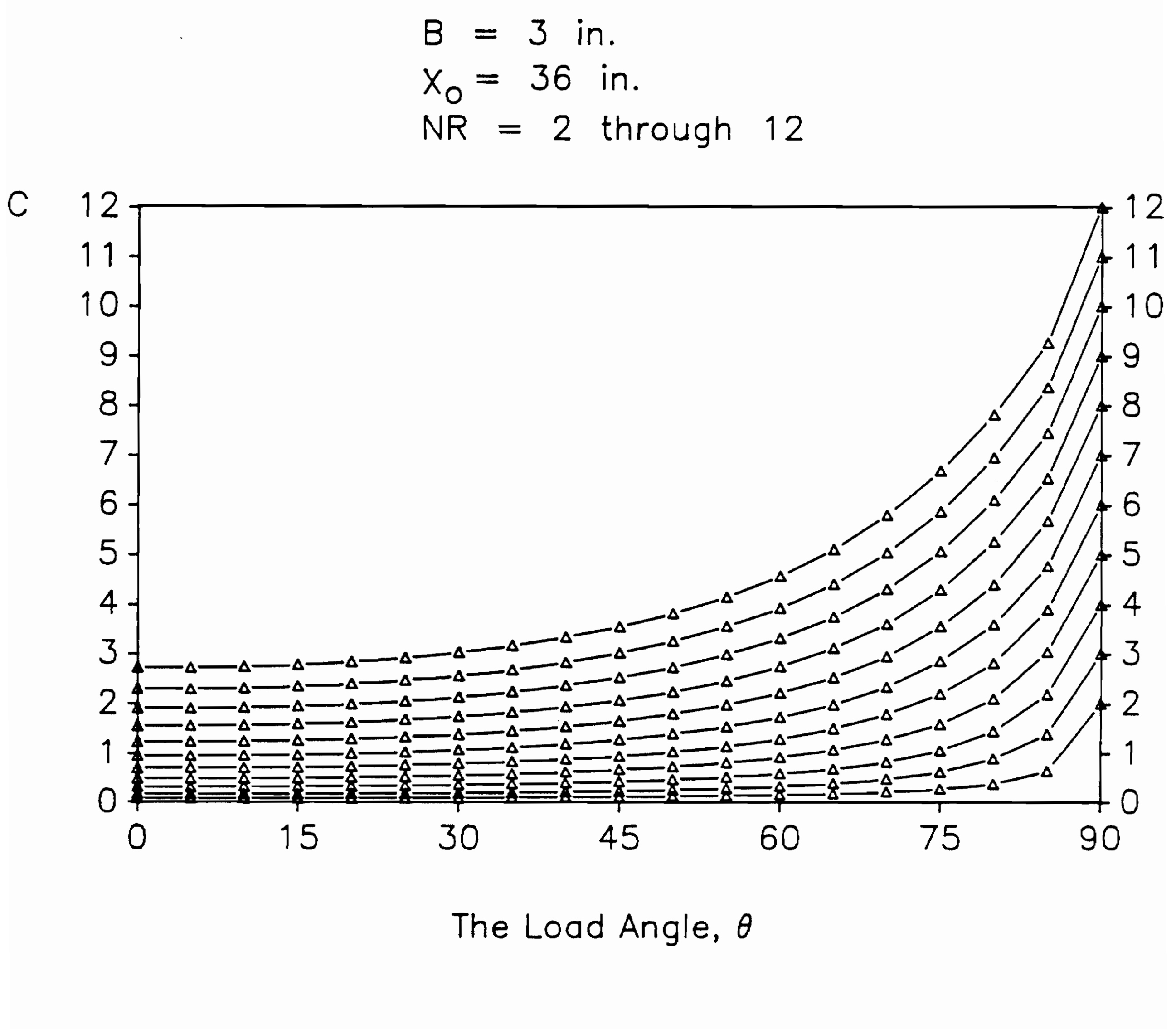

Figure B.3. The Eccentricity Coefficient vs The Load Angle (B = 3 in., X_o = 36 in., NR = 2 through 12)

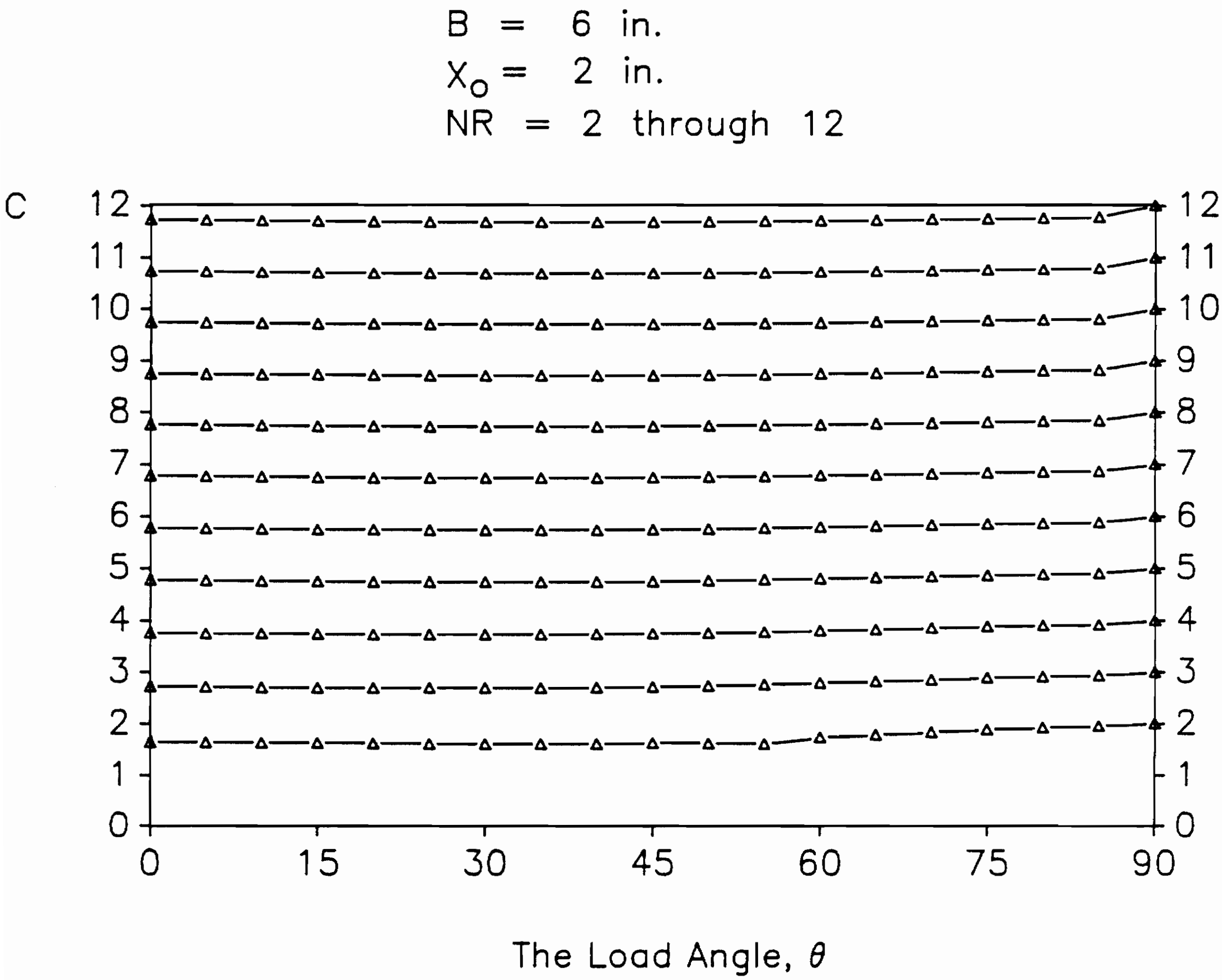

Figure B.4. The Eccentricity Coefficient vs The Load Angle (B = 6 in., X_o = 2 in., NR = 2 through 12)

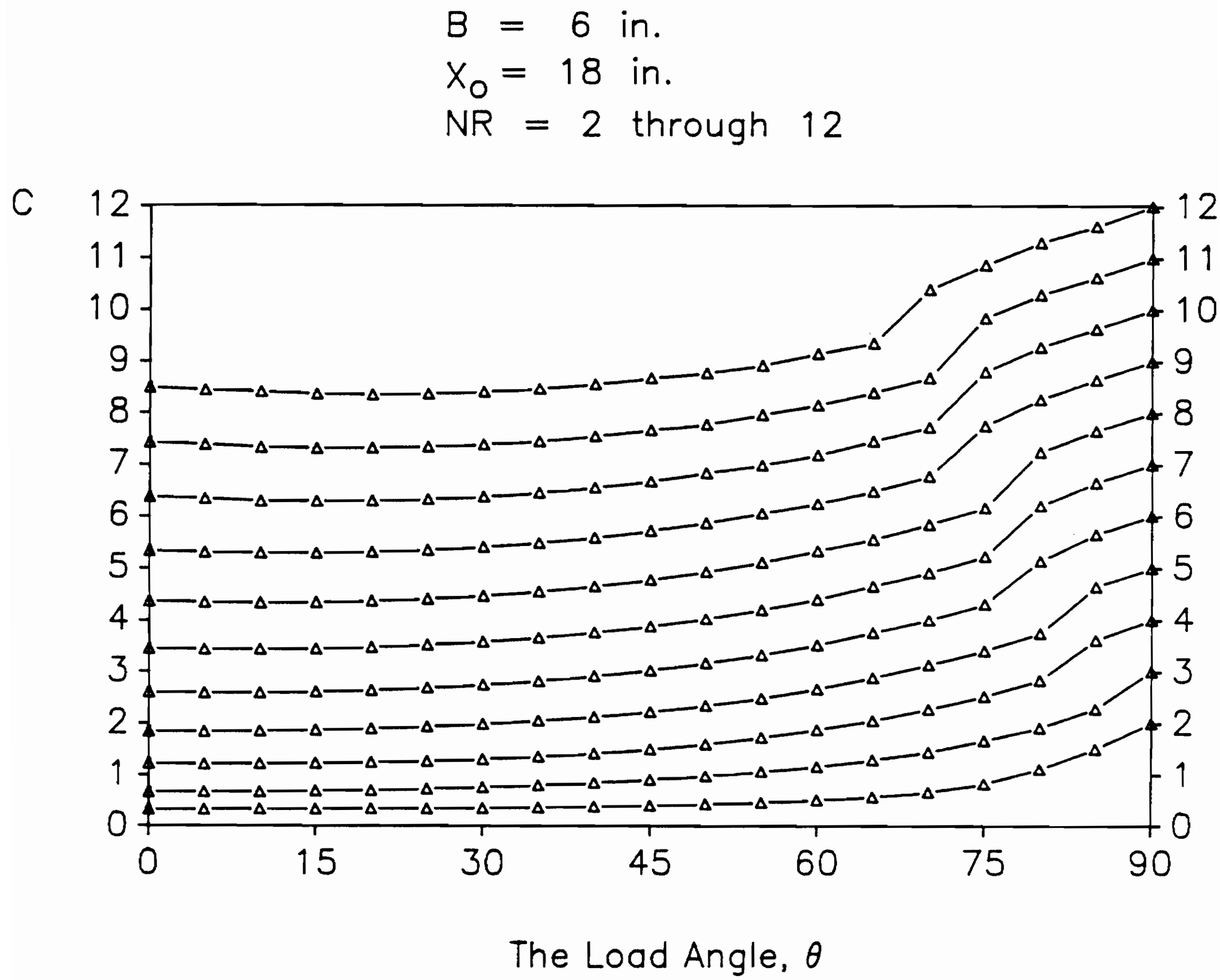

Figure B.5. The Eccentricity Coefficient vs The Load Angle (B = 6 in., X_0 = 18 in., NR = 2 through 12)

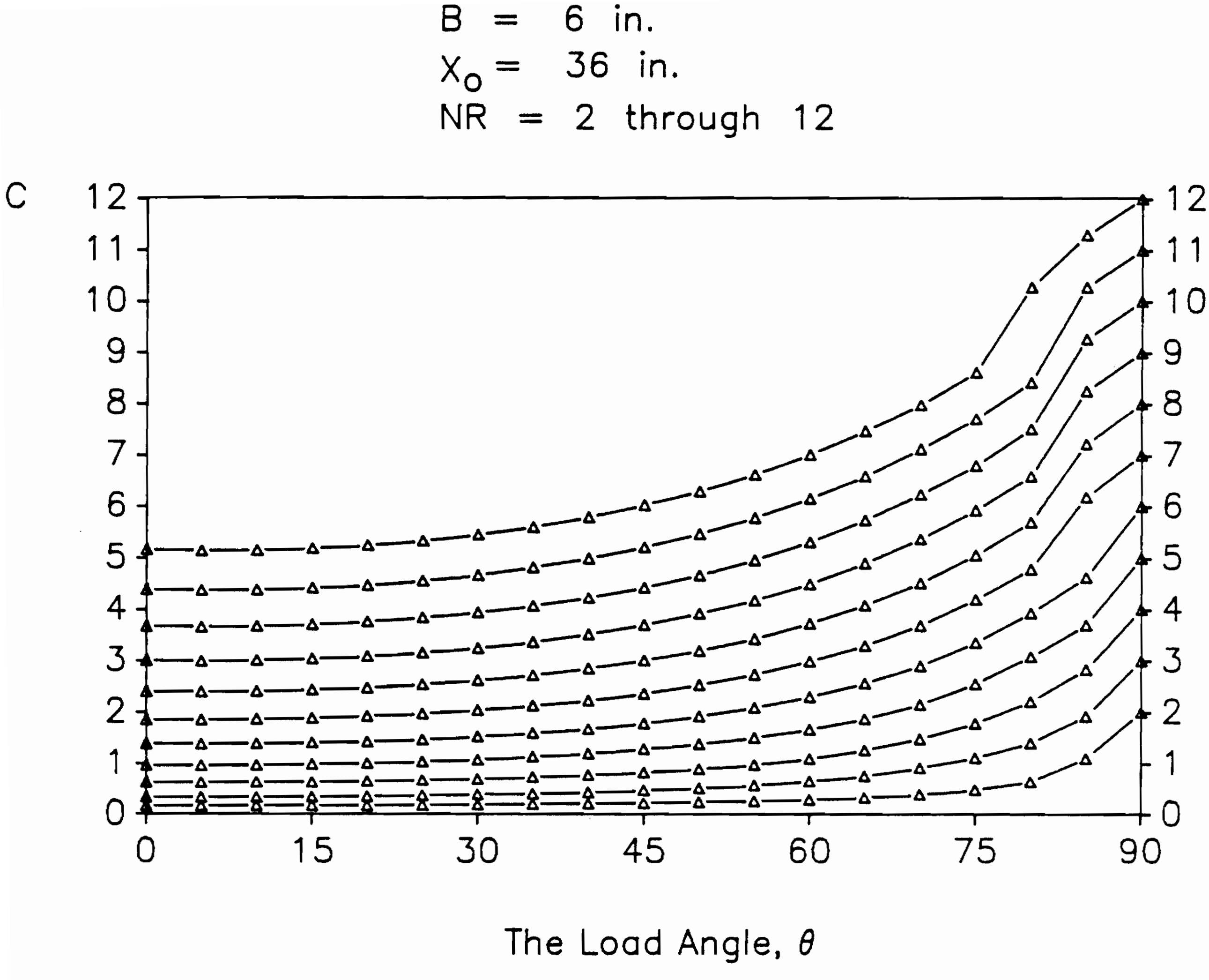

Figure B.6. The Eccentricity Coefficient vs The Load Angle (B = 6 in., X_o = 36 in., NR = 2 through 12)

Appendix C

THE RELATIONSHIP BETWEEN THE ECCENTRICITY COEFFICIENT AND X_o

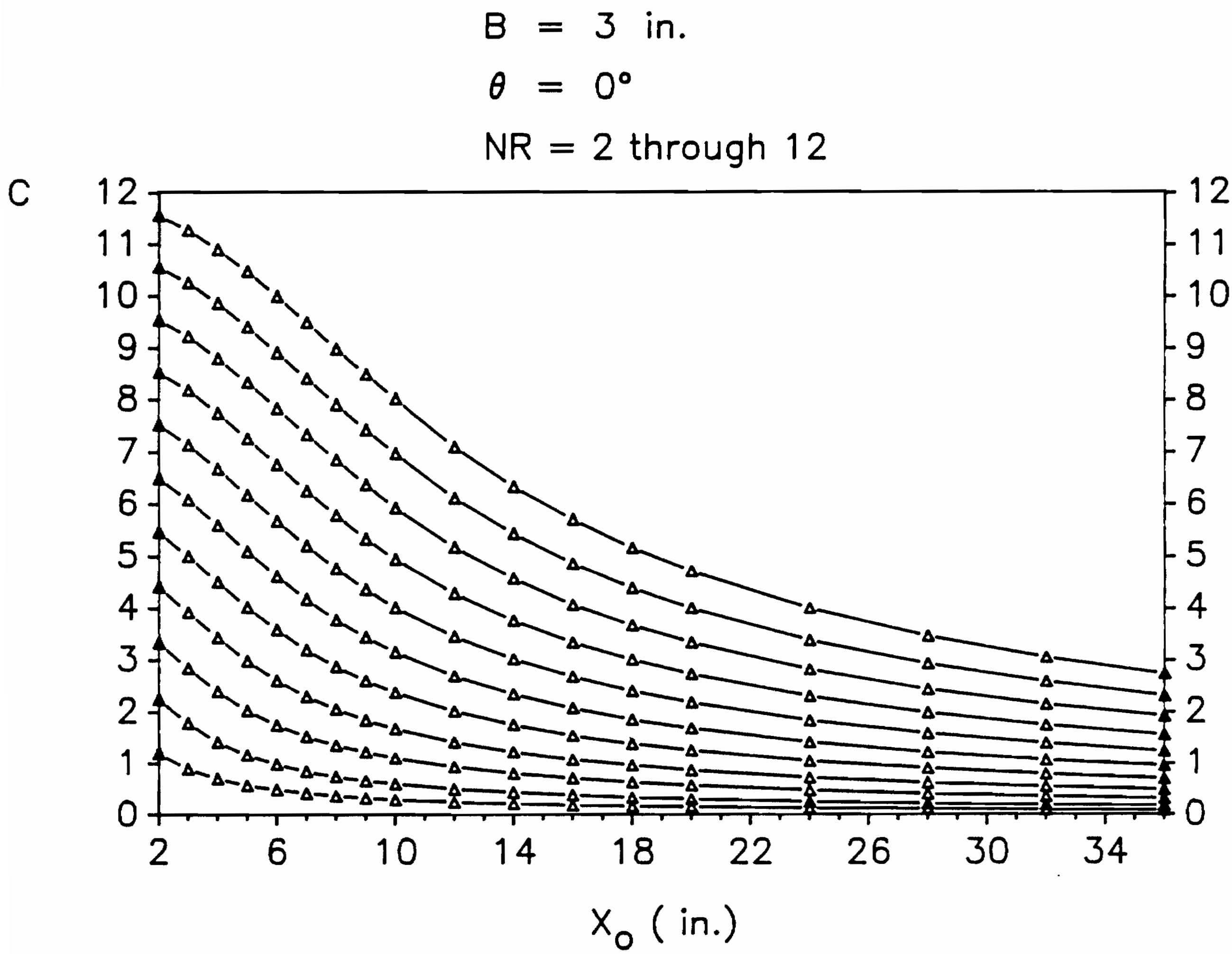

Figure C.1. The Eccentricity Coefficient vs X_o (B = 3 in., θ = 0°, NR = 2 through 12)

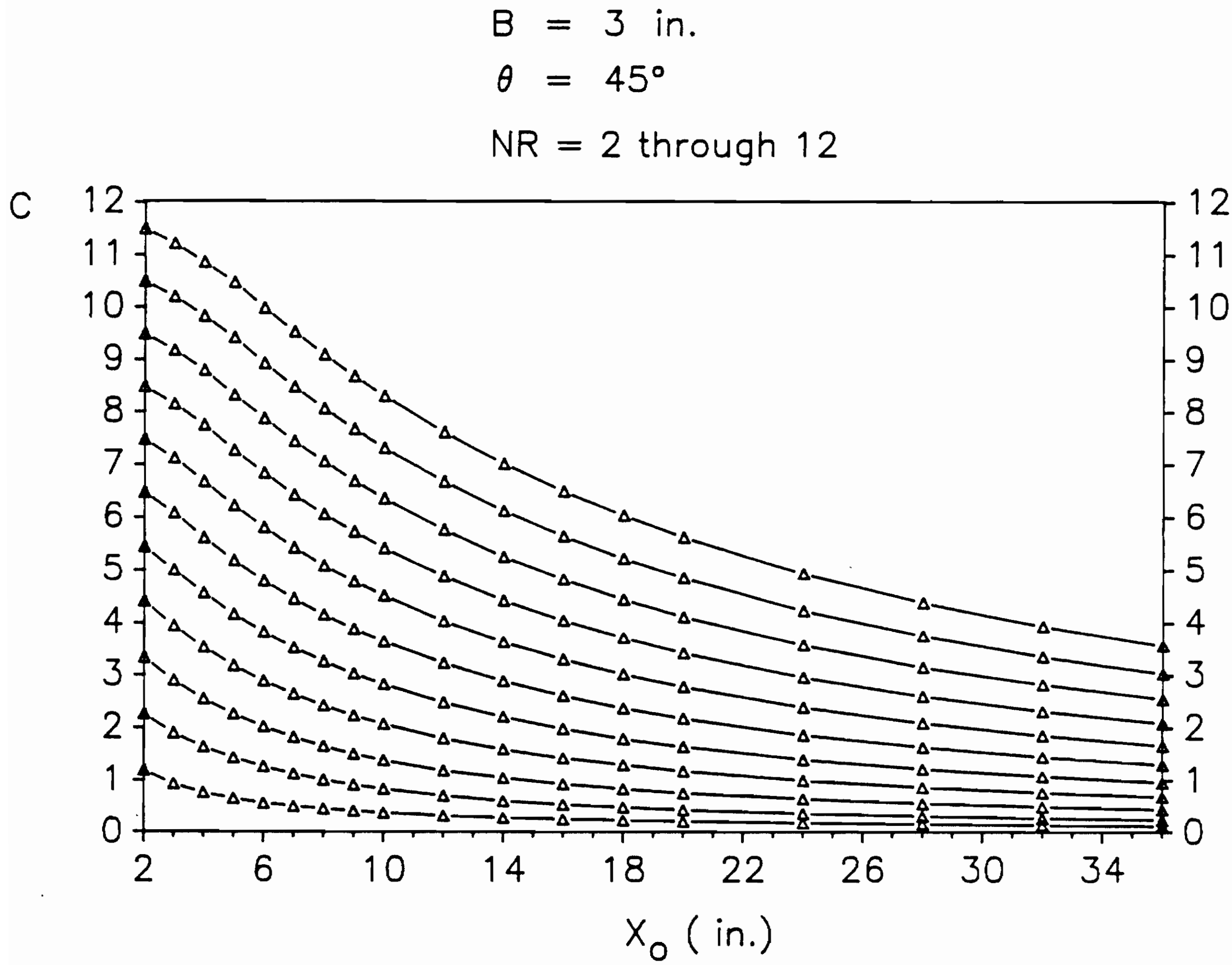

Figure C.2. The Eccentricity Coefficient vs X_o (B = 3 in., θ = 45°, NR = 2 through 12)

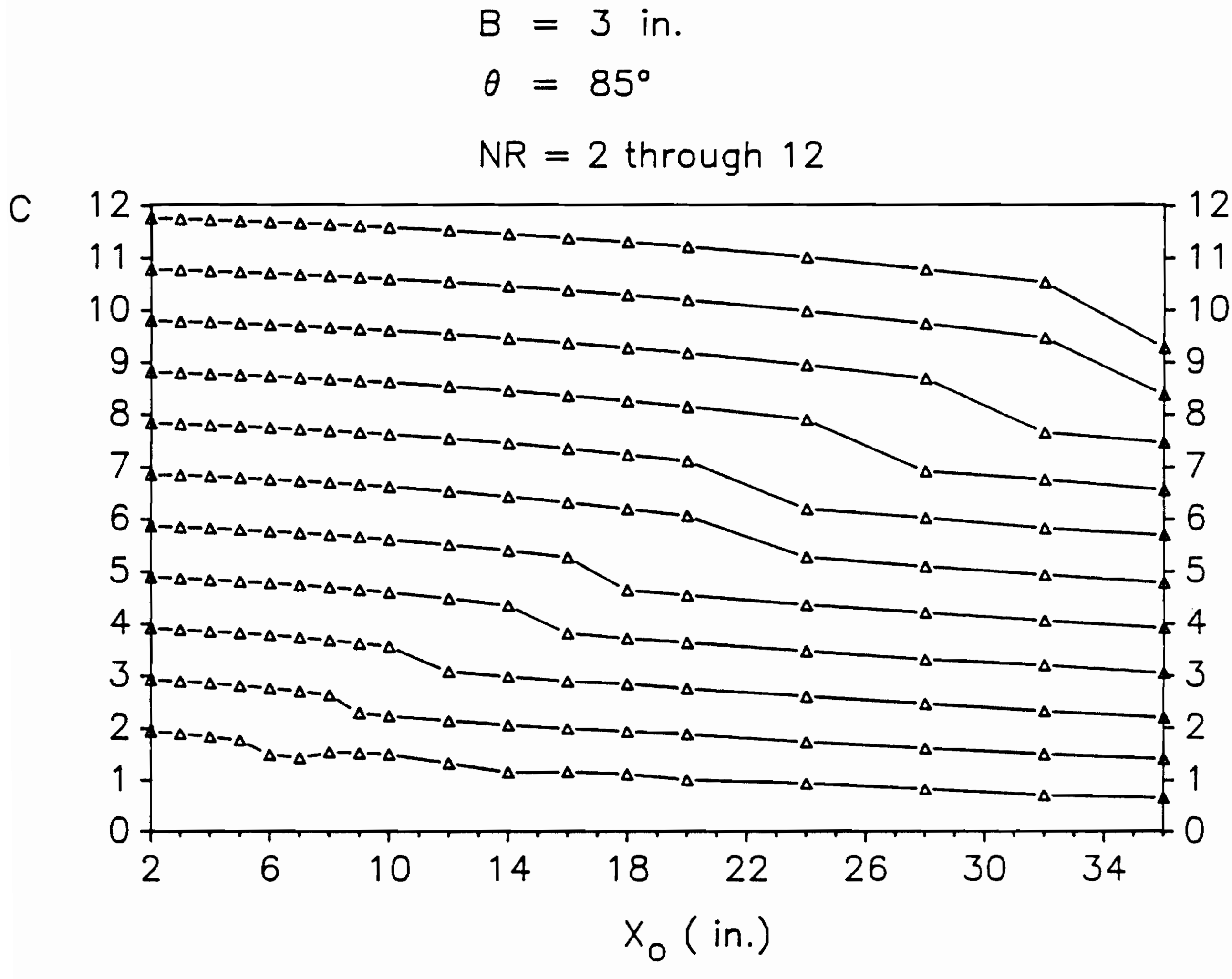

Figure C.3. The Eccentricity Coefficient vs X_0 (B = 3 in., θ = 85°, NR = 2 through 12)

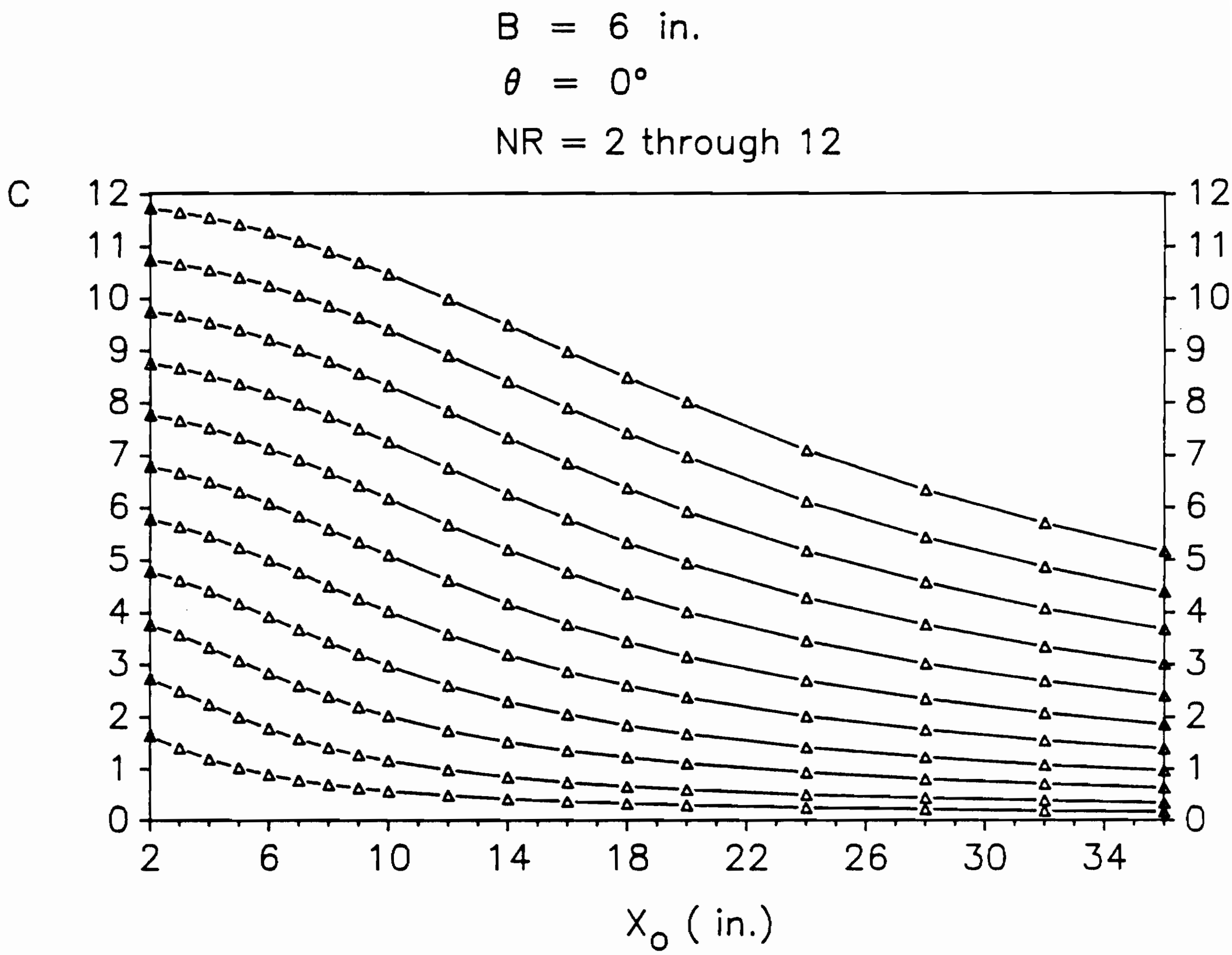

Figure C.4. The Eccentricity Coefficient vs X_o (B = 6 in., θ = 0°, NR = 2 through 12)

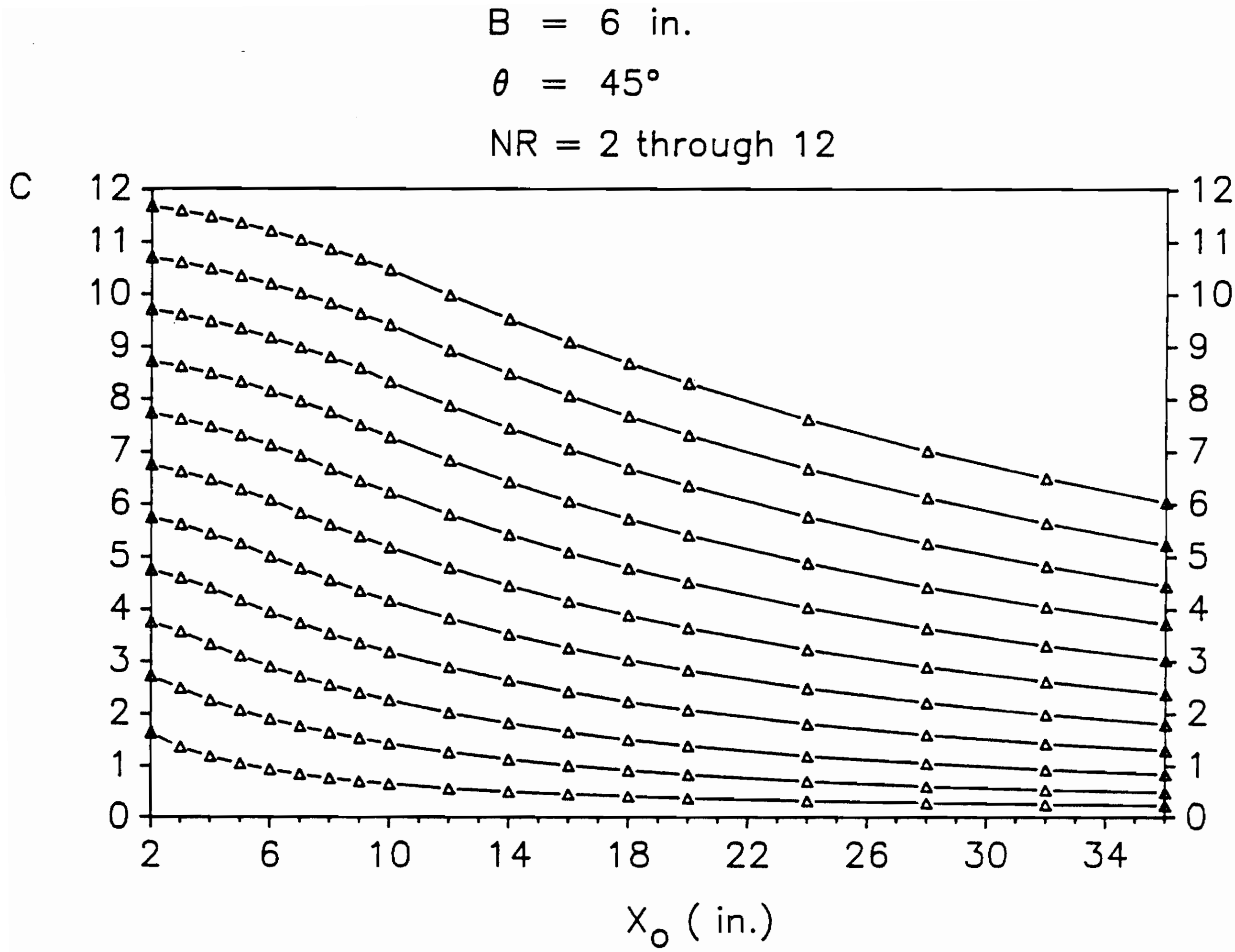

Figure C.5. The Eccentricity Coefficient vs X_o (B = 6 in., θ = 45°, NR = 2 through 12)

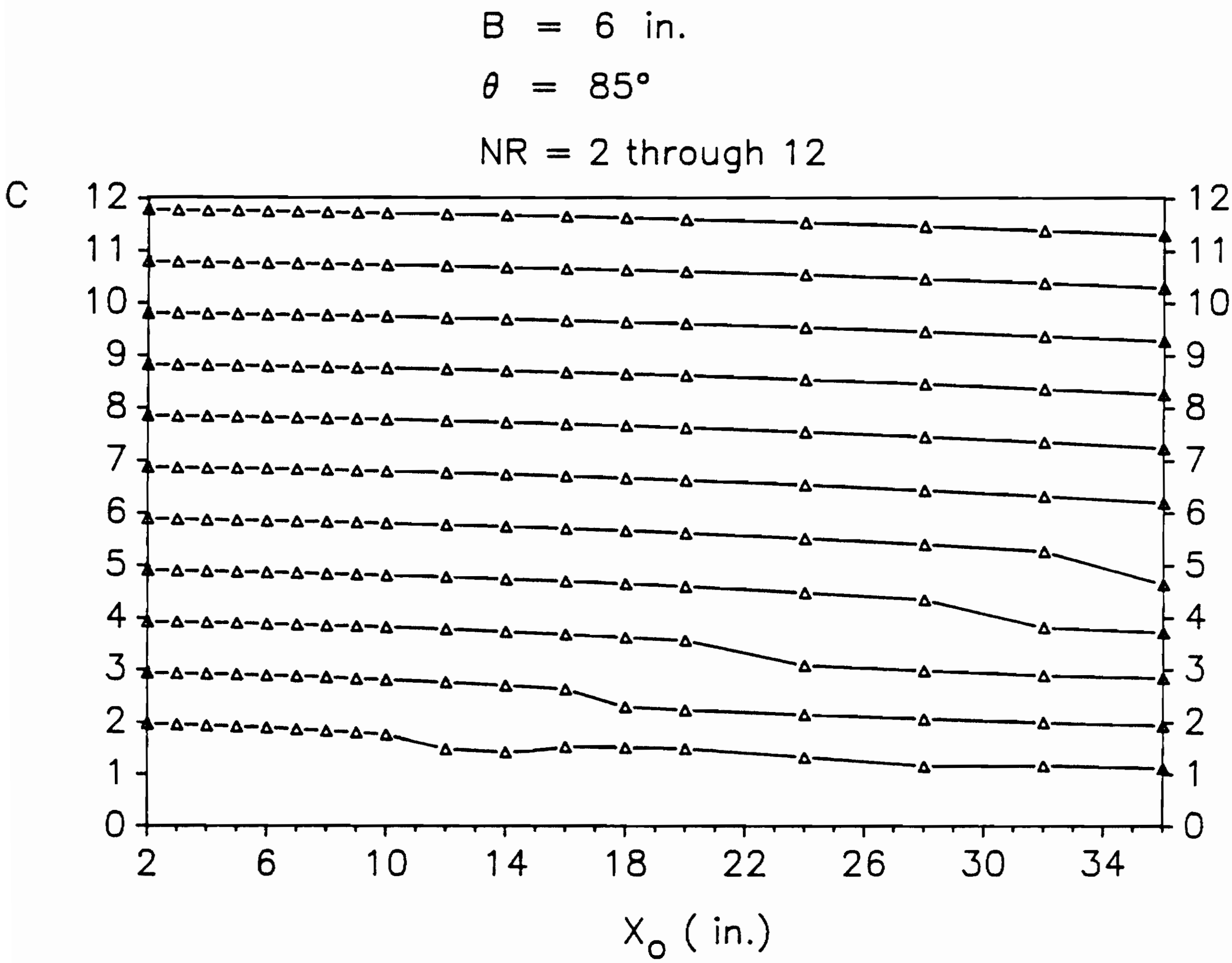

Figure C.6. The Eccentricity Coefficient vs X_o (B = 6 in., θ = 85°, NR = 2 through 12)

Appendix D

THE RELATIONSHIP BETWEEN THE ECCENTRICITY COEFFICIENT AND B

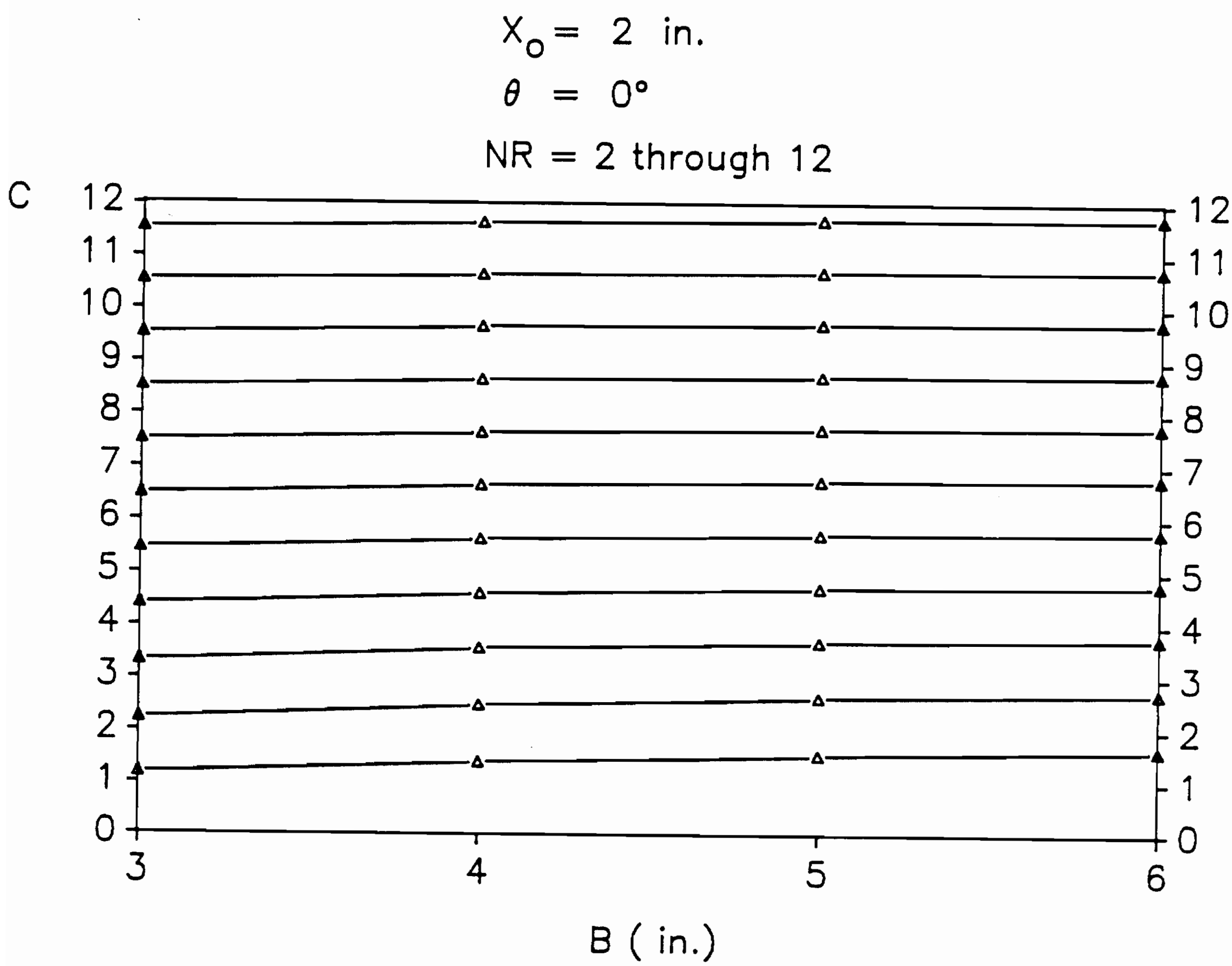

Figure D.1. The Eccentricity Coefficient vs B (X_o = 2 in., θ = 0°, NR = 2 through 12)

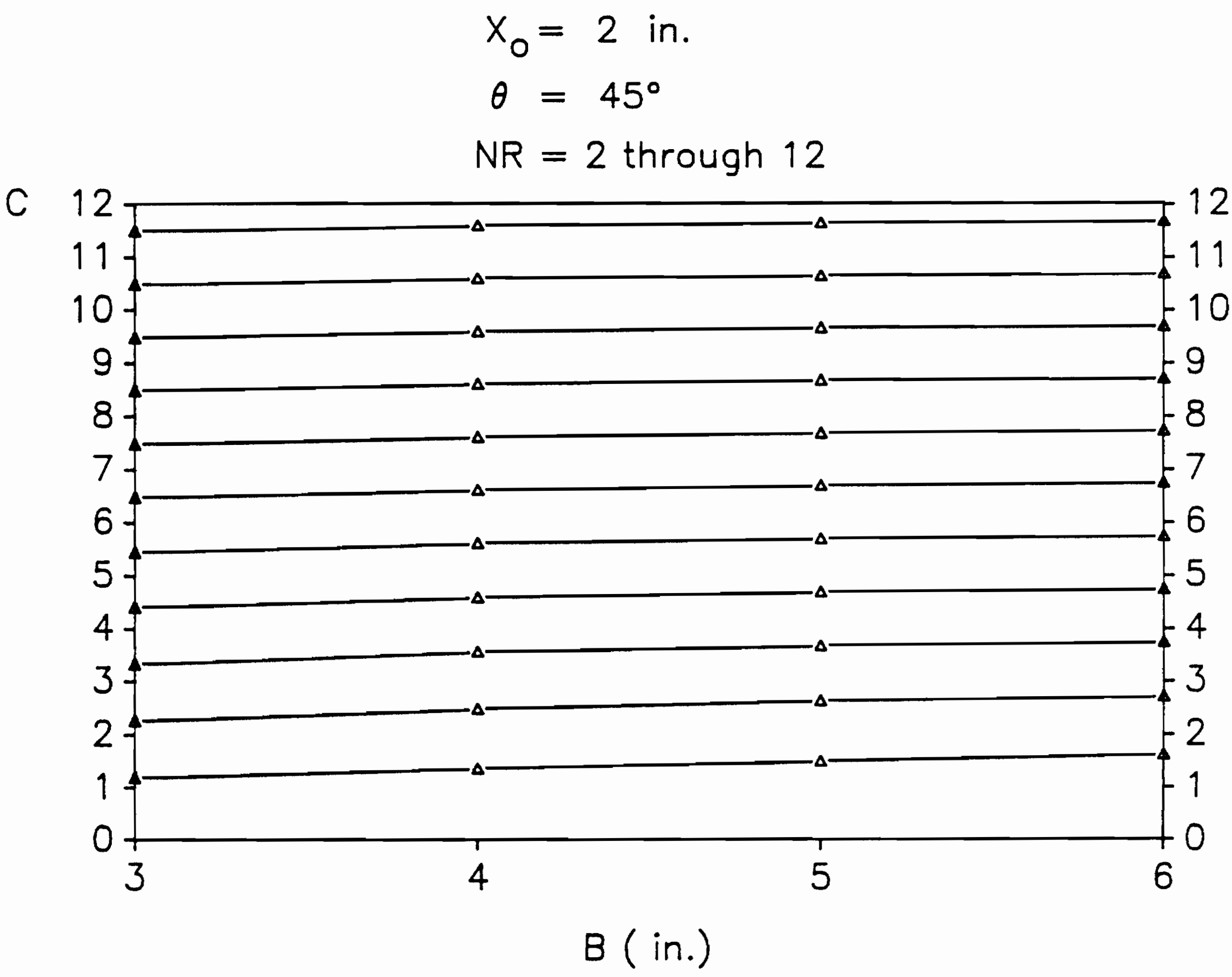

Figure D.2. The Eccentricity Coefficient vs B (X_o = 2 in., θ = 45°, NR = 2 through 12)

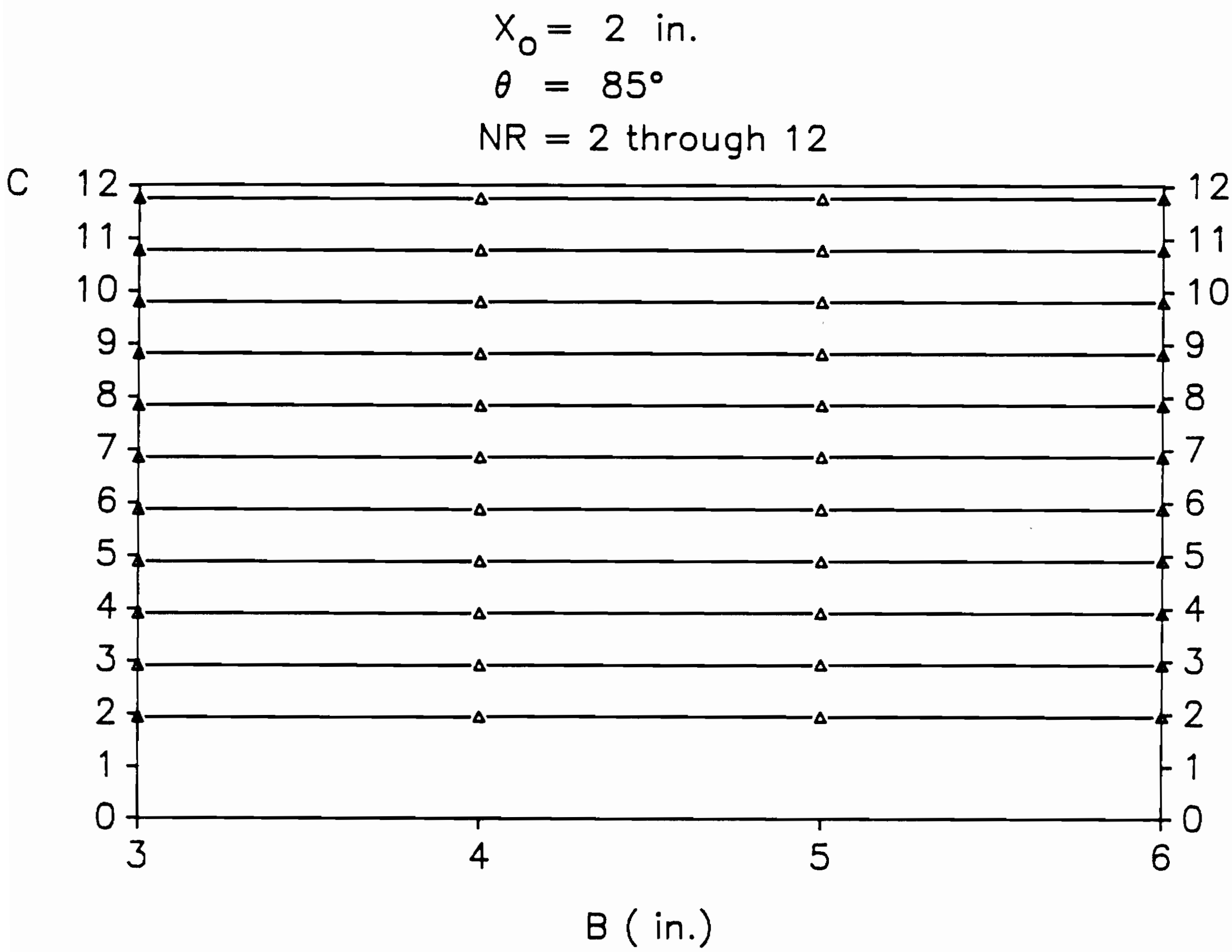

Figure D.3. The Eccentricity Coefficient vs B (X_o = 2 in., θ = 85°, NR = 2 through 12)

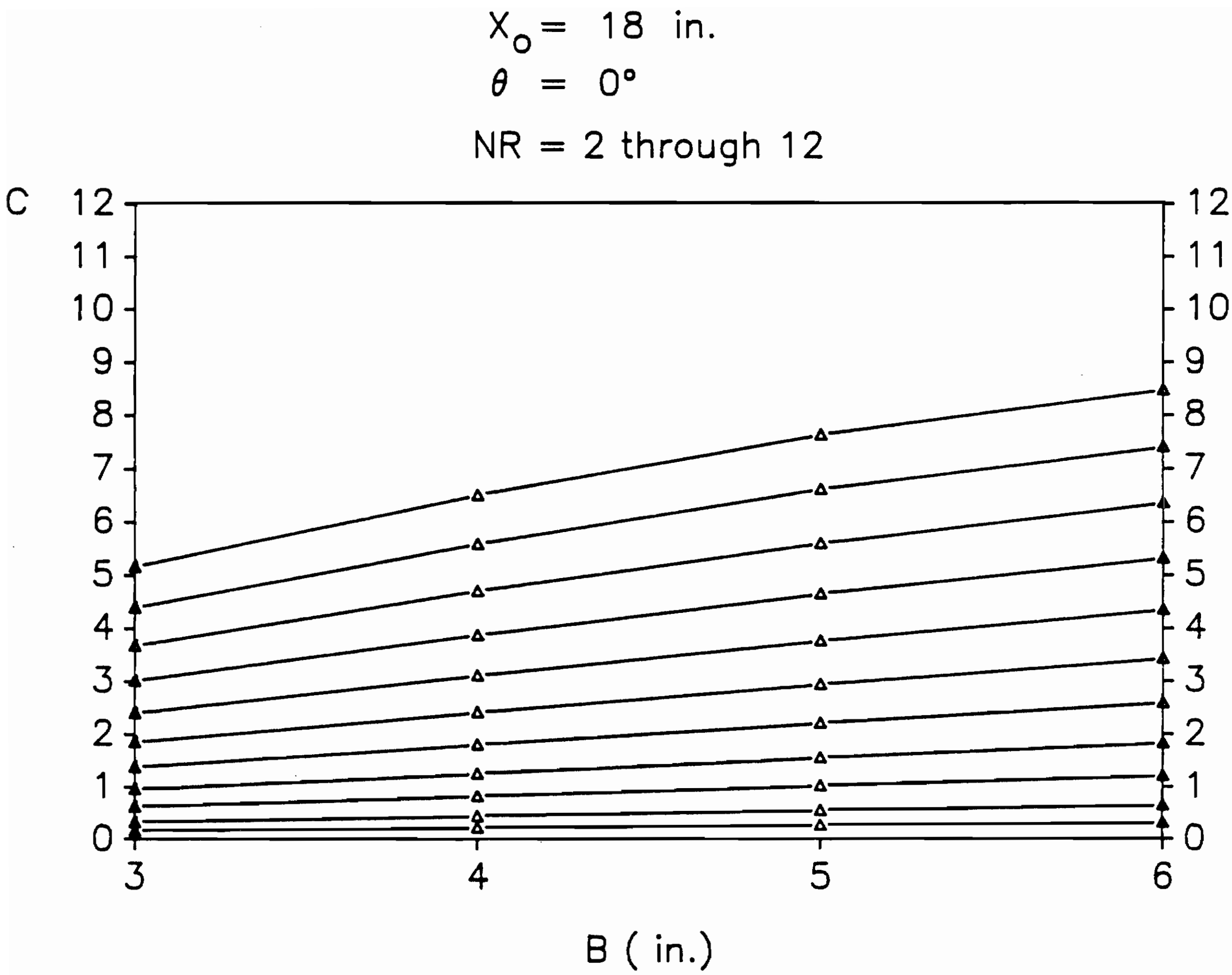

Figure D.4. The Eccentricity Coefficient vs B (X_o = 18 in., θ = 0°, NR = 2 through 12)

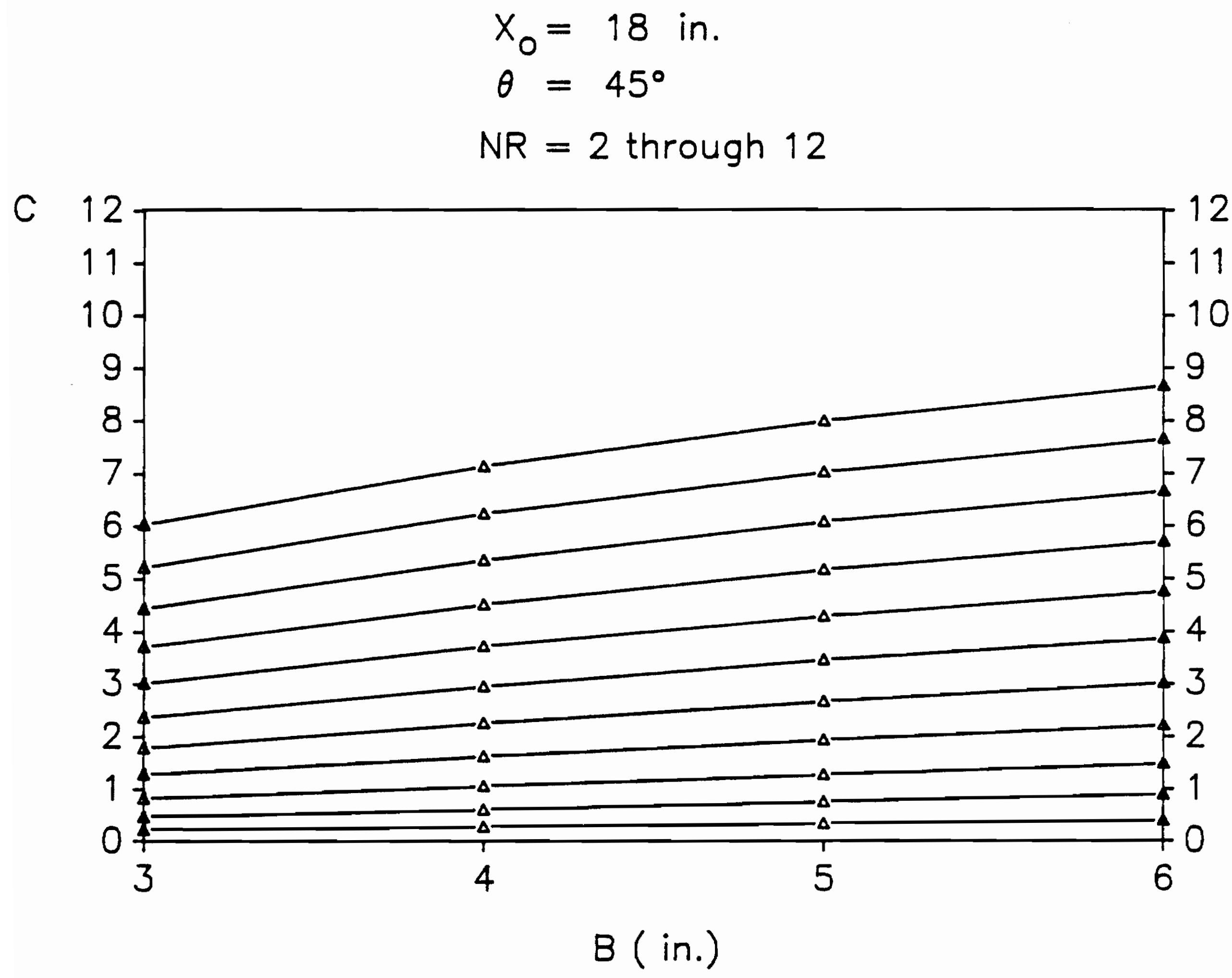

Figure D.5. The Eccentricity Coefficient vs B (X_o = 18 in., θ = 45°, NR = 2 through 12)

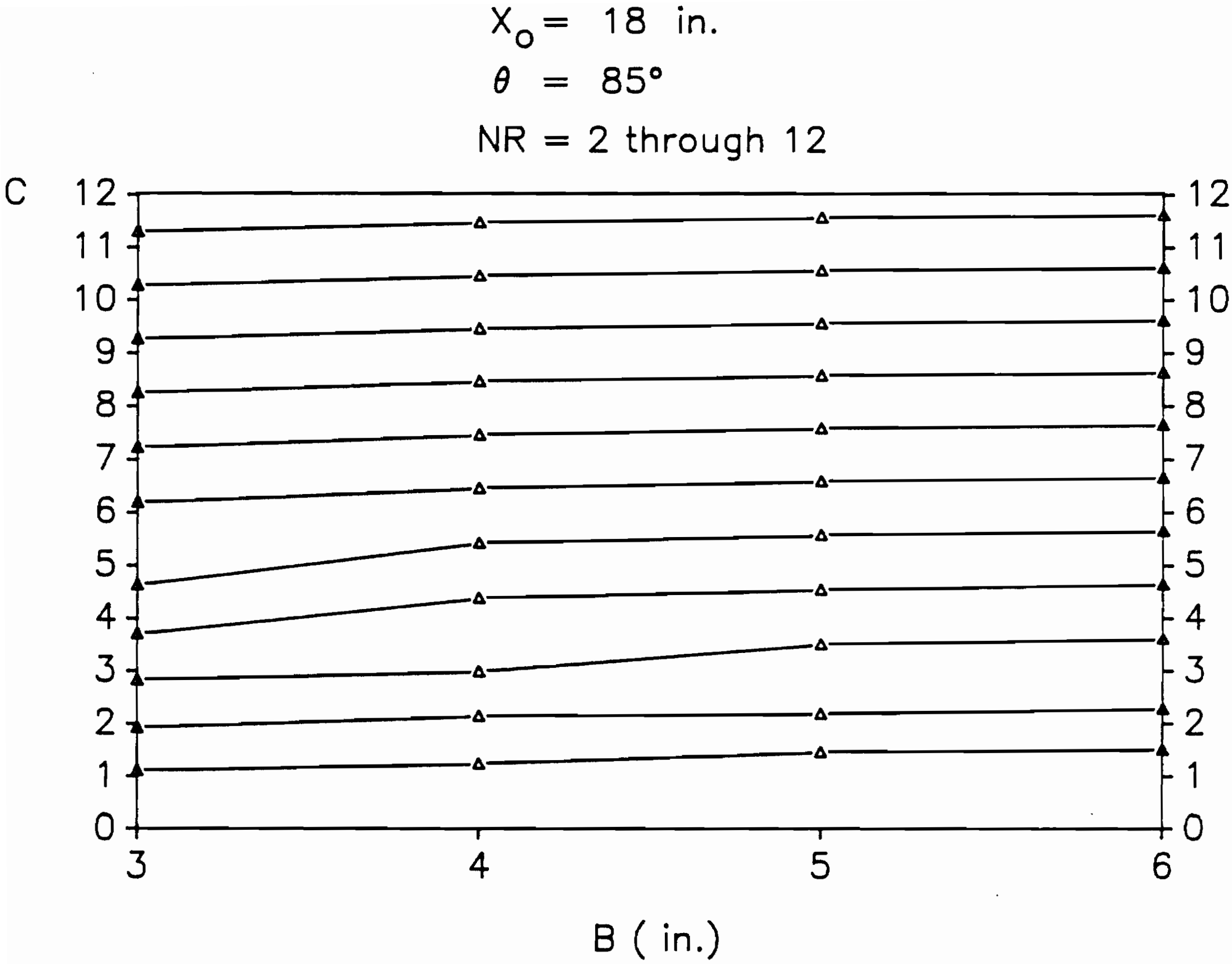

Figure D.6. The Eccentricity Coefficient vs B (X_o = 18 in., θ = 85°, NR = 2 through 12)

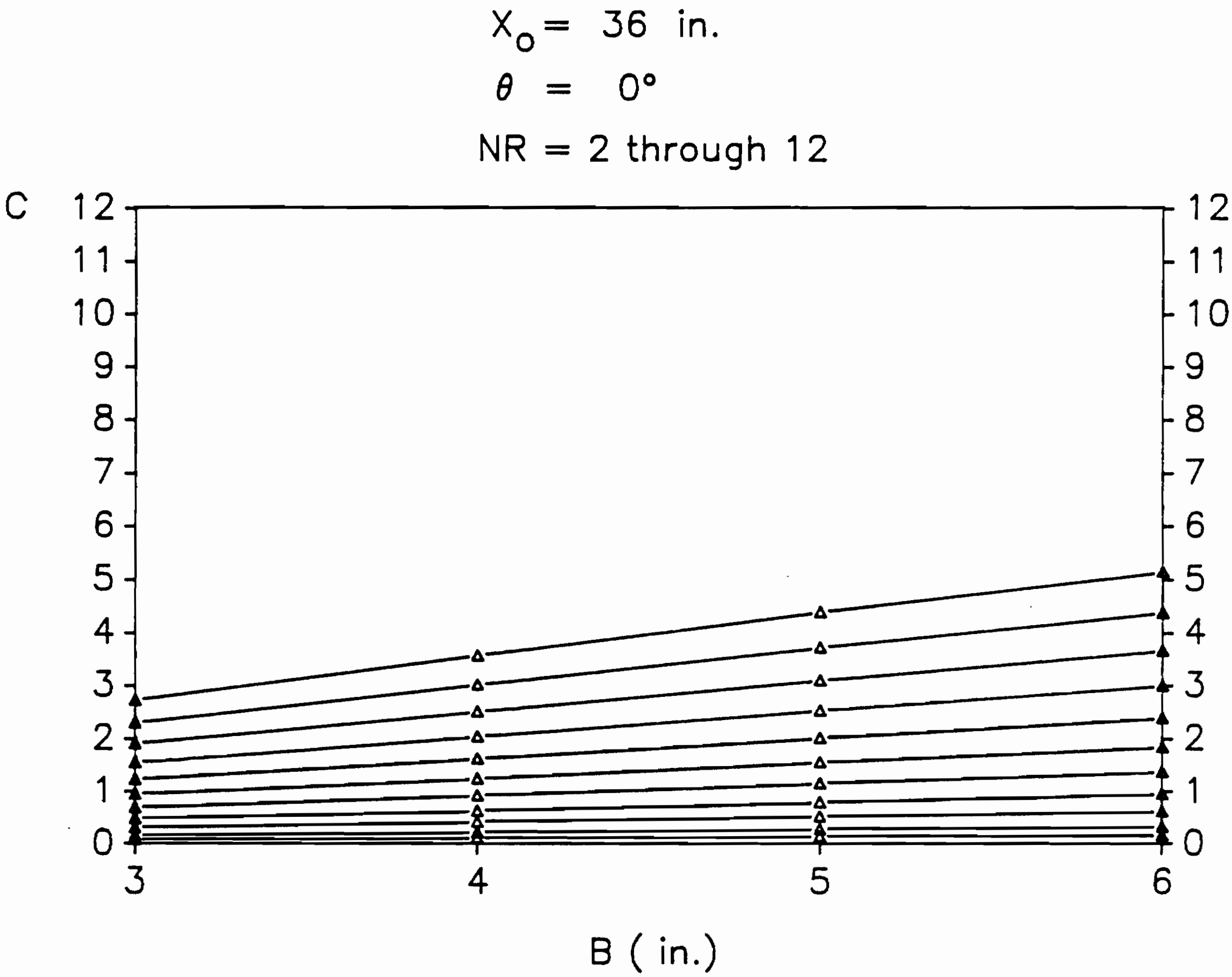

Figure D.7. The Eccentricity Coefficient vs B (X_o = 36 in., θ = 0°, NR = 2 through 12)

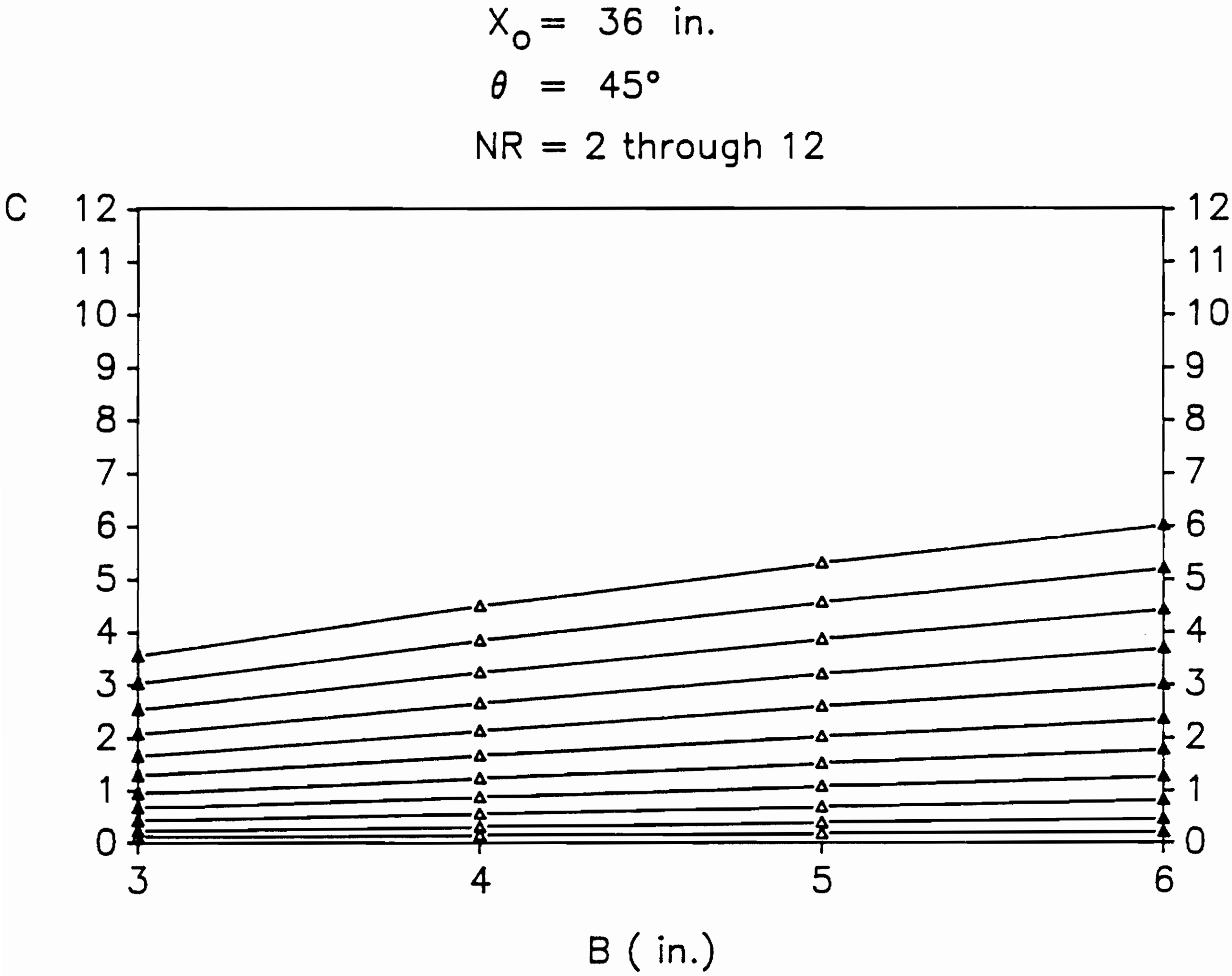

Figure D.8. The Eccentricity Coefficient vs B (X_o = 36 in., θ = 45°, NR = 2 through 12)

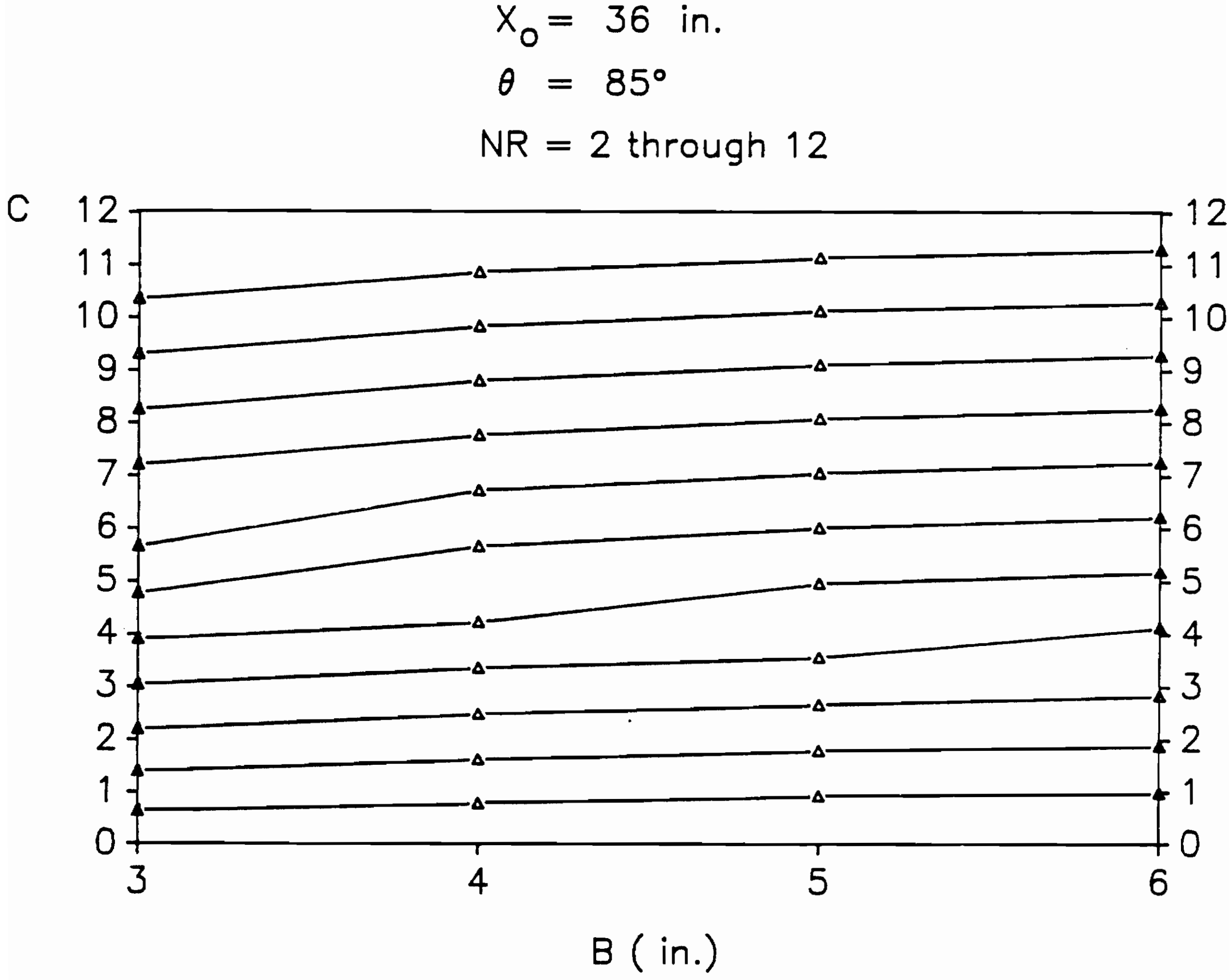

Figure D.9. The Eccentricity Coefficient vs B (X_o = 36 in., θ = 85°, NR = 2 through 12)

Appendix E

THE RELATIONSHIP BETWEEN THE ECCENTRICITY COEFFICIENT AND NR

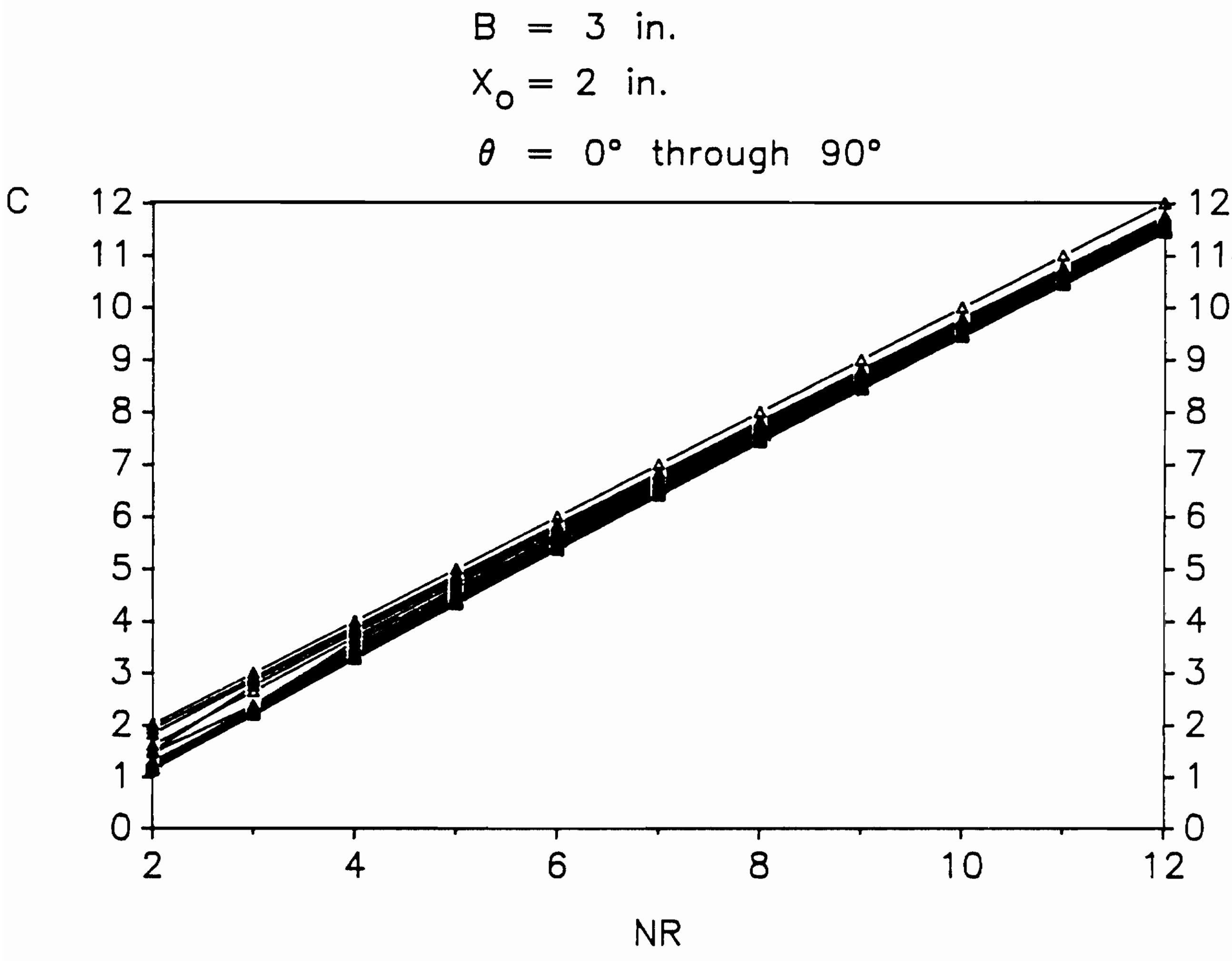

Figure E.1. The Eccentricity Coefficient vs NR (B = 3 in., X_o = 2 in., θ = 0° through 90°)

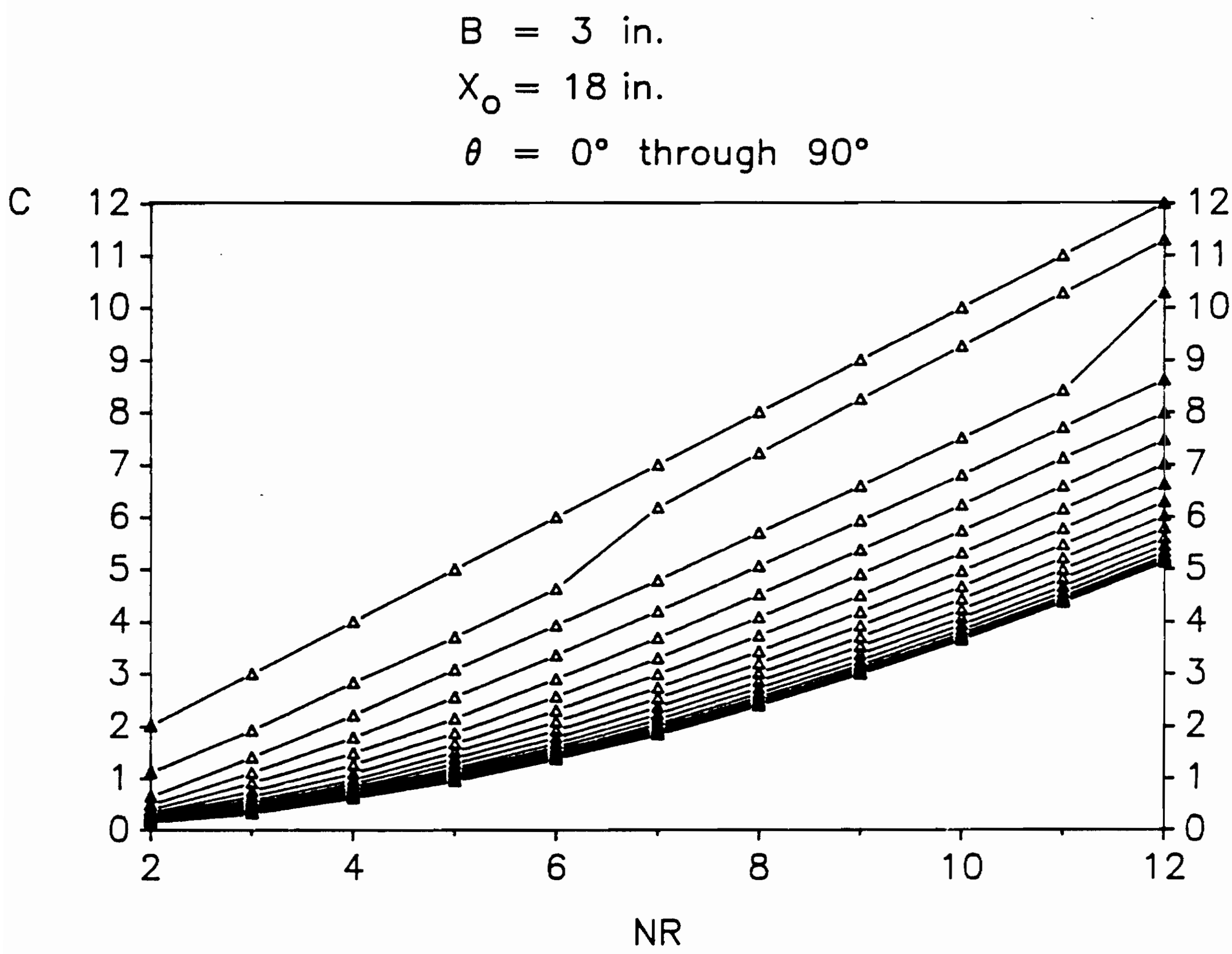

Figure E.2. The Eccentricity Coefficient vs NR (B = 3 in., X_o = 18 in., θ = 0° through 90°)

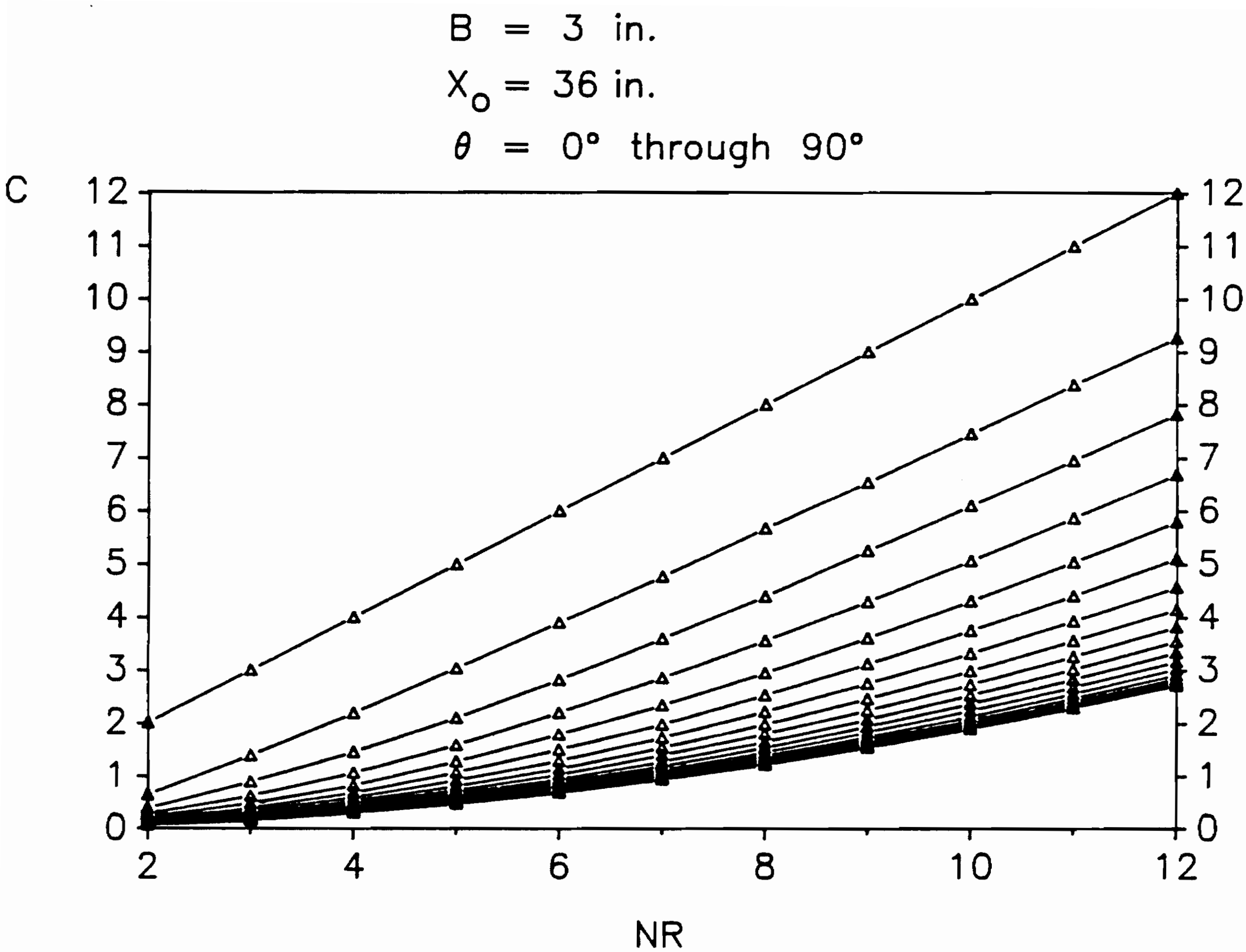

Figure E.3. The Eccentricity Coefficient vs NR (B = 3 in., X_o = 36 in., θ = 0° through 90°)

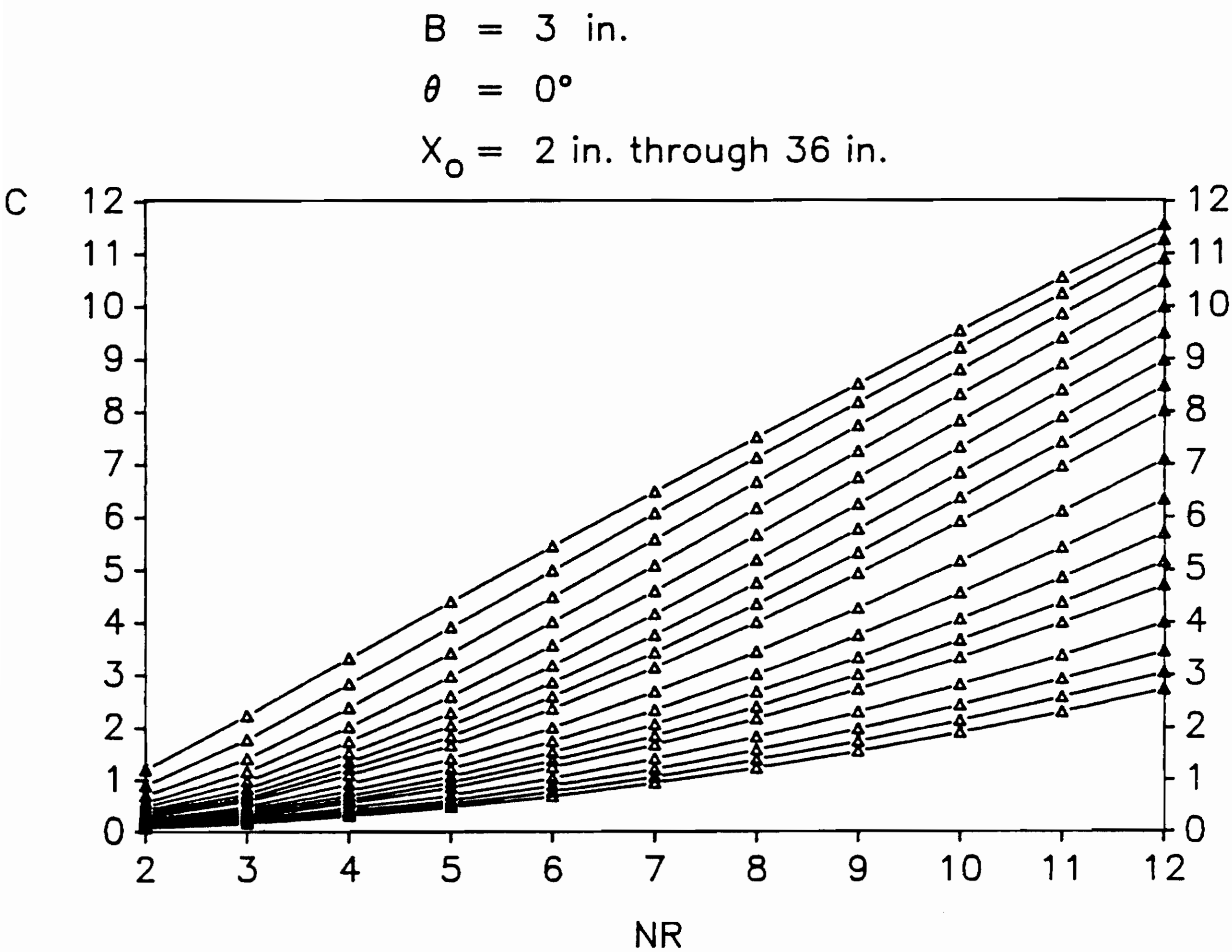

Figure E.4. The Eccentricity Coefficient vs NR (B = 3 in. θ = 0°, X_o = 2 in. through 36 in.)

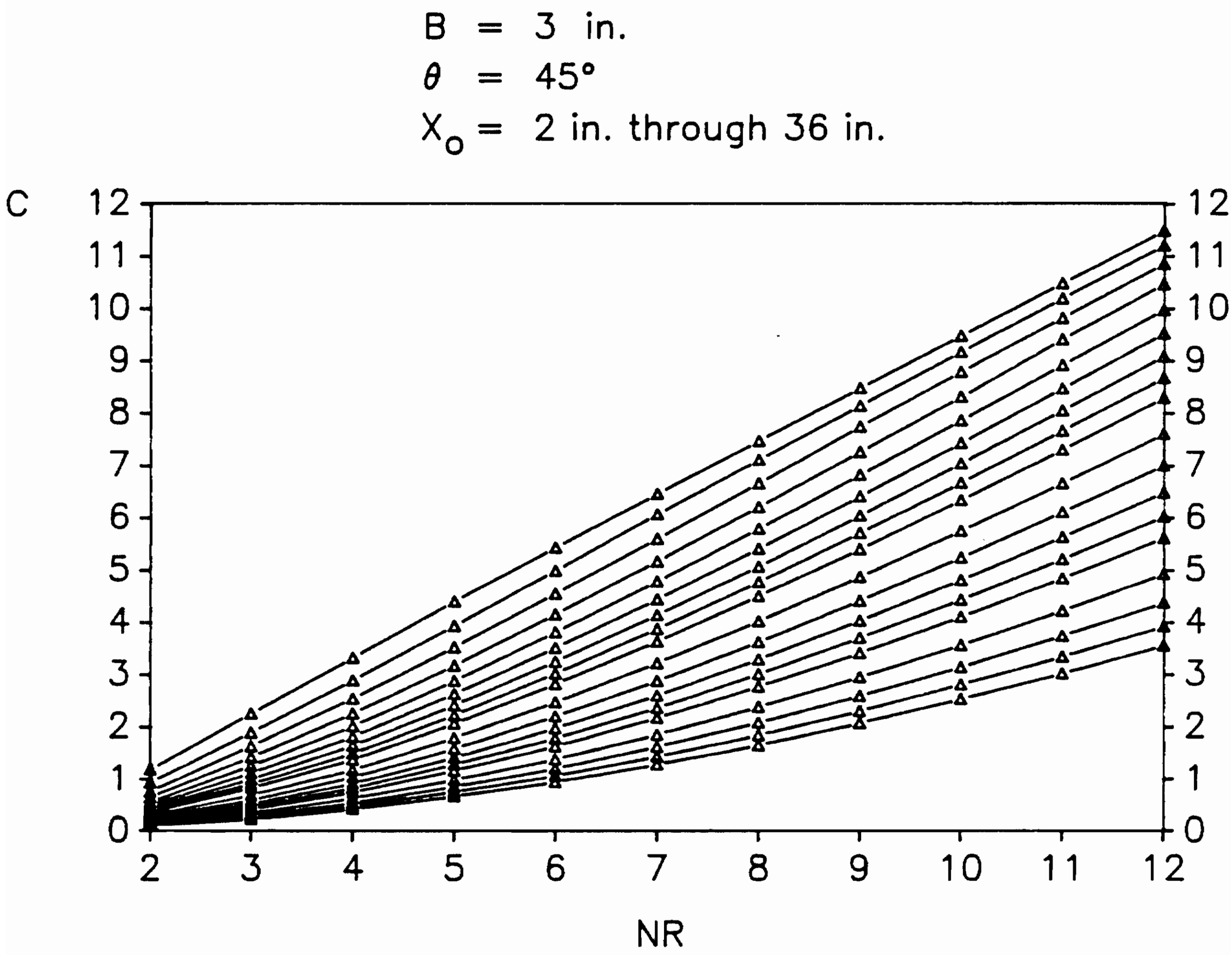

Figure E.5. The Eccentricity Coefficient vs NR (B = 3 in. θ = 45°, X_o = 2 in. through 36 in.)

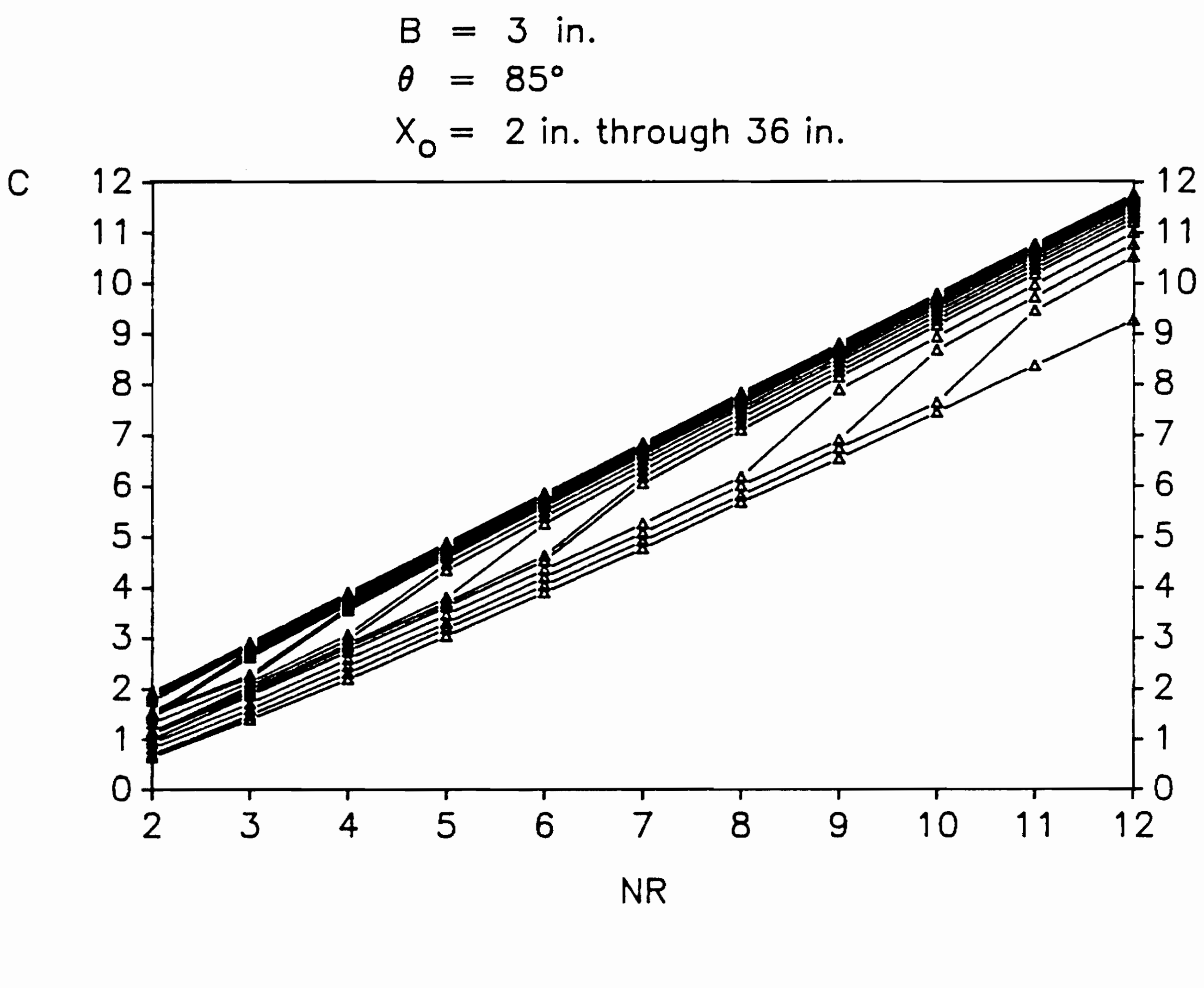

Figure E.6. The Eccentricity Coefficient vs NR (B = 3 in. θ = 85°, X_0 = 2 in. through 36 in.)

Appendix F

DESIGN EXAMPLE

Using 7/8-in. A325-N bolts, find the eccentricity coefficient when
NR = 10, θ = 60 , X_o= 20 in., B = 3 in., NC = 1, S = 0 in.
(connection with one vertical row of bolts)

C_u = 5.00 from the computer program

1. LRFD From Table X, angle = 45°
   C = 4.10 (18.0% conservative compared to C_u)
   Using ϕr_v = 21.1 kips, P_{max} = 86.51 kips

2. ASD From 9th Ed. Manu. C_o= 3.33
   Use Alternate Method 2:

C_{max} = 10

$$A = \frac{10}{3.33} = 3.00$$

$$\frac{C_a}{C_o} = \frac{3.00}{(\sin 60° + 3.00\cos 60°)} = 1.268 > 1.0 \quad \text{o.k.}$$

C_a = 1.268(3.33) = 4.22 (15.4% conservative compared to C_u)

Using r_v= 12.6 kips, P_{max} = 4.22(12.6) = 53.2 kips

3. Coefficients from Eq.3.4 in this paper

1-Eq. Model

$$C = e^{-1.1944}(NR)^{1.6346}(\theta)^{0.0461}\left(\frac{X_o}{B}\right)^{-0.5295}$$

$$C = e^{-1.1944}(10)^{1.6346}\left(\frac{\pi}{3}\right)^{0.0461}\left(\frac{20}{3}\right)^{-0.5295}$$

= 4.79 (4.2% conservative compared to the C_u)

3-Eq. Model (see Table 1.)

$$C = e^{-1.2764}(10)^{1.7491}\left(\frac{\pi}{3}\right)^{0.0247}\left(\frac{20}{3}\right)^{-0.6750}$$

= 4.36 (12.8% conservative compared to C_u)

9-Eq. Model (see Table 4.)

$$C = e^{-1.2287}(10)^{1.7725}\left(\frac{\pi}{3}\right)^{0.5890}\left(\frac{20}{3}\right)^{-0.6925}$$

= 4.79 (4.2% conservative compared to C_u)

Vita

The author was born on November 21, 1962 in Changhua, Taiwan, the Republic of China. He graduated from Changhua High School in June 1981. In 1983, after passing the college entrance test, he entered the National Cheng Kung University and graduated with a Bachelor of Science Degree in Civil Engineering in the June of 1986. After working two years, he came to Virginia Polytechnic Institute and State University in the fall semester of 1988 for his graduate study. Upon completion of this project, the author will go back to his home country to work as a civil engineer in civil engineering consulting company.