DECISIONS UNDER UNCERTAINTY

BY STATE FISH AND WILDLIFE AGENCIES

by

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CHAPTER I

INTRODUCTION

> "So far we have been managing game by the 'seat-of-our-pants,' and fortunately getting away with it. We must develop game management from the art that it has been to a precise science."[1]

With increasing demand on federal and state money, governmental agencies are coming under ever-increasing scrutiny. The legislators and the public are becoming increasingly efficiency-minded. They want to see results from their investments in the various agencies. The days when it was enough to be protecting motherhood and producing apple pie are fast drawing to an end. State fish and wildlife agencies must be able to allocate their time, monies, and resources efficiently and show resulting benefits, or funds for such activities will continue to decline.

Background

> "The purpose of the Game Division's program is to manage Michigan's wildlife species in a manner which will . . . provide maximum recreational opportunities for . . . hunters and . . . other general wildland recreationists with a specific interest in wildlife."[2]

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[1]Verme, L. J., "Summary," White-Tailed Deer in the Midwest, a symposium presented at the 30th Midwest Fish and Wildlife Conference, Columbus, Ohio, December 9, 1968, p. 34.

[2]Miller, J. E., Interoffice Communication, Michigan Department of Natural Resources, November 24, 1969.

Within state governmental structures, an agency, a department, or a commission exists which is responsible for managing the state's fish and wildlife populations. In Virginia this function is performed by the Virginia Commission of Game and Inland Fisheries; in Georgia, the Game and Fish Commission; in South Carolina, the Wildlife Resources Department; in Ohio, the Division of Wildlife; in Michigan, the Department of Natural Resources; in Texas, the Parks and Wildlife Department.

Usually a primary goal of the agency is to maximize recreation opportunities that result from the utilization of the state's fish and wildlife populations. The agency pursues this goal by managing those factors which influence the rate of growth, replacement, and distribution of the fish and wildlife populations, as well as the rates, levels, and methods of harvesting these fish and wildlife populations.

Many management procedures are available to an agency. Populations can be manipulated by stocking, by disease and predator control, by habitat management, or the like, to influence animal size, and to influence population size and distribution. The harvesting of fish and wildlife populations, and thus the population size, can be influenced by regulating the length of the hunting and fishing season, the bag or creel limit, the method of take, by policing, or by improving access to hunting and fishing areas. Actions such as these are choices which must be made if these populations are to be regulated by more than the natural controls of the population and the environment.

Statement of Problem

A state fish and wildlife agency has three basic resources at its disposal to apply in an effort to achieve its goal or objective. These basic resources are the agency's budget, the natural resources to be managed, and the skills, judgment, and experience of those persons within the agency.

No state fish and wildlife agency operates with an unrestricted budget; that is, there are never enough funds to undertake all desirable activities. Thus the problem becomes: how can the decision makers of an agency, employing their skills, judgment, and experience, best make the necessary decisions as to the uses of these funds and natural resources?

This problem is complicated by the fact that the decision maker is generally uncertain of the outcome of an action which he would like to consider. For example, suppose a manager is contemplating stocking a stream that had not been stocked before. He does not want to stock it with more fish than necessary, but how many are necessary depends on the stream capacity and on how many people will fish the stream if it is stocked. Thus he does not know if, say, 1,000 fish are stocked whether or not he has stocked too many fish. That is, the "best" action he should take depends upon which of a set of future possible conditions actually occurs. Decision making under such circumstances is referred to as decision making under uncertainty[3] if the probability of occurrence of each one of the set of future possible conditions is not known with certainty. Traditional decision methods generally have not been structured so as to consider such uncertainty.

To summarize and simplify the problem: how can a state fish and wildlife decision maker, employing his skills, judgment, and experience, best make decisions, under uncertainty, as to the uses of agency funds and natural resources?

Objectives

The objectives of the study are to:

(1) Demonstrate that there is a need for procedures for decision making under uncertainty in fish-and-wildlife-agency decision problems.

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[3]Another possible source of uncertainty is that the outcome may simply be unknown, regardless of the future conditions. One might argue that a rational decision maker would not consider an action if he did not know the outcome. But consider this: "Forage clearings have been used in deer and turkey management since 1935 and by 1965 over 30,000 acres in 22 states were devoted to this practice. The estimated replacement value of these clearings is $3.7 million and it is estimated that $4 million has been spent in maintenance.

We have no sound evidence of the effect of clearings on wildlife production, movements, and harvest. To my knowledge no 'before and after' data are available to support the common assumption that clearings increase harvest and hunter success." (Larson, J. S., "Agricultural Clearing as Sources of Supplemental Food and Habitat Diversity for White-Tailed Deer," White-Tailed Deer in the Southern Forest Habitat, proceedings of a symposium, Nacogdoches, Texas, March 25-26, 1969, pp. 46-47.) For this study the above argument is accepted and such an action alternative will not be considered as relevant.

(2) Demonstrate how decision theory can be applied to decision making under uncertainty with emphasis toward fish-and-wildlife-agency decision problems.

Procedure

Following a brief review of literature, the study will proceed in the following manner:

(1) Several hypothetical decision problems covering a wide range of fish-and-wildlife-agency decision problems will be introduced and various sources of uncertainty will be suggested.

(2) Decision theory will be introduced and its advantages will be discussed.

(3) Following a general discussion on how to formulate decision problems under uncertainty so as to facilitate the application of decision theory, a hypothetical fish-and-wildlife-agency decision problem will be so formulated.

(4) The application of decision theory to fish-and-wildlife-agency decision problems in general and to the hypothetical problem in (3) specifically will be made.

(5) Several hypothetical fish-and-wildlife-agency decision problems, including those in (1), will be formulated in accordance with the method described in (3).

Review of Literature

The original theoretical groundwork for what has become known as decision making under uncertainty is generally credited to Bayes. To quote E. C. Molina[4] in "Bayes' Theorem: An Expository Presentation," "Bayes' theorem made its appearance . . . in an essay which occupies pages 370 to 418 of the Philosophical Transactions, Vol. 53, for 1763, by Richard Price." Molina quotes Price, "I now send you an essay which I have found amongst the papers of our deceased friend, Mr. Bayes, and which, in my opinion, has great merit, and well deserves to be preserved.'" It was to take some 170 years before the theorists were to "discover" what Bayes in his genius had deduced and another two decades and a world war[5] before the value of Bayes' work, as foreseen by Price, would be realized.

There was much development of the technical aspects of the theory behind decision making under uncertainty during the 1940's. But perhaps the best known attempt to "decode" this decision theory in a form more readily understood was by L. J. Savage.[6] In his article entitled

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[4]Price, Richard, "Bayes' Theorem: An Expository Presentation," The Annals of Mathematical Statistics, Vol. II, 1931.

[5]To quote from Luce, R. D., and Raiffa, H., Games and Decisions, John Wiley & Sons, Inc., New York, 1957, p. 3: "During that period [World War II] considerable activity developed in scientific, or at least systematic, approaches to problems that had been previously considered the exclusive province of men of 'experience.' These include such topics as logistics, submarine search, air defense, etc."

[6]Savage, L. J., "The Theory of Statistical Decision," Journal of the American Statistical Association, Vol. 46, 1951, pp. 55-67.

"The Theory of Statistical Decision," he set forth an "informal exposition" of Abraham Wald's developments in decision theory, presented by Wald in Statistical Decision Functions[7]. Savage went on to expand this into a book entitled The Foundations of Statistics[8] in 1954.

Much has been written on decision theory and decision making under uncertainty. Luce and Raiffa[9], while concentrating on game theory, devoted a chapter to "Individual Decision Making under Uncertainty" in 1957. Chernoff and Moses[10], in 1959, coauthored Elementary Decision Theory in 1959, a result of nine years of study in statistics and the decision making process. To quote from the Preface, "In recent years, statistics has been formulated as the science of decision making under uncertainty. This formulation represents the culmination of many years of development and, for the first time, furnishes a simple and straightforward method of exhibiting the fundamental aspects of a [decision] problem." In 1964, Pratt, Raiffa, and Schlaifer[11] presented

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[7]Wald, Abraham, Statistical Decision Functions, John Wiley & Sons, New York, 1950.

[8]Savage, L. J., The Foundations of Statistics, John Wiley & Sons, New York, and Chapman Hall, London, 1954.

[9]Luce, R. D., and Raiffa, H., Games and Decisions, John Wiley & Sons, London, 1957.

[10]Chernoff, H., and Moses, L. E., Elementary Decision Theory, John Wiley & Sons, New York, N. Y., 1959.

[11]Pratt, J. W., Raiffa, H., and Schlaifer, R., "The Foundations of Decision under Uncertainty: An Elementary Exposition," Journal of the American Statistical Association, Vol. 59, No. 306, June, 1964, pp. 353-375.

"The Foundations of Decision under Uncertainty: An Elementary Exposition" which presents a mathematically oriented discussion of the various aspects of decision making under uncertainty. More recently R. L. Winkler[12] has written several articles on various methods for determining subjective probabilities.

There have been some applications of decision theory in the field of agricultural economics and natural resources. For example, in 1960, Dillon and Heady[13] published a bulletin entitled Theories of Choice in Relation to Farmer Decisions, in which they examined farmers' decisions in the decision theory context. Various choice criteria were examined. In 1967 Halter and Dean[14] published a manuscript in limited quantity, Decisions Under Uncertainty: Application to Agriculture, in which they examined in depth several applications which had been made. Their conceptualization of the decision making process followed that of Chernoff and Moses in Elementary Decision Theory, cited earlier. In 1968,

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[12]Winkler, R. L., "The Assessment of Prior Distributions in Bayesian Analysis," Journal of the American Statistical Association, Vol. 62, No. 318, September, 1967, pp. 776-800; "The Quantification of Judgment: Some Methodological Suggestions," Journal of the American Statistical Association, Vol. 62, No. 320, December, 1967, pp. 1105-1120; "The Consensus of Subjective Probability Distributions," Management Science, Vol. 15, No. 2, October, 1968, pp. B:61-B:75.

[13]Dillon, J. L., and Heady, E. O., Theories of Choice in Relation to Farmer Decisions, Iowa State University Press, Bulletin 485, Ames, Iowa, October, 1960.

[14]Halter, A. N., and Dean, G. W., Decisions Under Uncertainty: Applications to Agriculture, Oregon State University, Corvallis, Oregon, 1967.

E. F. Thompson[15] wrote "The Theory of Decision under Uncertainty and Possible Applications in Forest Management" in which he pointed out that "uncertainty is present in most forest management decision-making situations."

[15]Thompson, E. F., "The Theory of Decision under Uncertainty and Possible Applications in Forest Management," Forest Science, Vol. 14, No. 2, June, 1968, pp. 156-163.

CHAPTER II

DECISION THEORY AND FISH AND WILDLIFE MANAGEMENT DECISION PROBLEMS

Suppose that you, as a wildlife manager, are faced with this problem: you are in charge of managing a deer population on a state-owned area in such a manner as to satisfy the wants of the deer hunters in the region. Suppose further that you want to know how much money you should allocate to deer habitat improvement so as to maintain a deer herd of the size necessary to satisfy the hunter wants over time without wasting money. You know that more intensive habitat improvements result in more deer which will satisfy more hunters. The problem is that the allocation decision must be made before the future hunter wants are known. In the next chapter it will be demonstrated how such a problem might be handled. But first, some background is necessary.

Classification of Knowledge Situations

In the following section the types of fish and wildlife decision problems for which decision theory might provide a solution are discussed. However, wide variations occur in the terminology used to describe the various knowledge situations which a decision maker might face. To avoid confusion, it is desirable to set down the classification of knowledge situations, under which a decision problem occurs, which will be used for the remainder of this study.

The gamut of knowledge situations which the decision maker may face will be divided or classified into three classes: certainty, risk, and uncertainty.

When the decision maker must decide between two actions, he is under one of the following knowledge situations:

(1) Certainty, "if each action is known to lead invariably to a specific outcome."[16]

(2) Risk, which will be divided into:

(a) objective risk, if each action leads to one of a set of possible outcomes, each outcome occurring with a known probability[17]; or

(b) subjective risk, if each action leads to one of a set of possible outcomes, each outcome occurring with a subjective probability which the decision maker believes is "sufficient to provide acceptable chance of success and he is willing to decide and act,"[18] or is "assumed as meaningful by the decision maker."[19]

(3) Uncertainty, which will be divided into:

(a) partial uncertainty, if each action leads to one of a set of possible outcomes, each outcome occurring with

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[16]Luce and Raiffa, op. cit., p. 13.

[17]Luce and Raiffa, op. cit., p. 15.

[18]Hedges, T. R., Farm Management Decisions, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1963, p. 578.

[19]Luce and Raiffa, op. cit., pp. 277, 300.

a subjective probability which is not assumed as sufficient by the decision maker at a particular moment in time, and steps can be taken "to acquire the necessary additional knowledge so that he can act,"[20] i.e., move to subjective risk; or

(b) complete uncertainty, if each action leads to one of a set of possible outcomes, each outcome occurring with a definite but completely unknown probability.

The bulk of formal theory in economics, psychology, and management sciences is based on the assumptions of certainty.[21] Typically, decision making under certainty takes the form: given a set of possible acts, the decision maker chooses the one which maximizes or minimizes some given index such as profit, cost, or benefit-cost ratio, or equates two given indices such as supply versus demand, or marginal cost versus marginal return.

However, this assumption of certainty is more often than not unrealistic. Frequently, decision makers must act under conditions of uncertainty, or at very best, risk.

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[20]Hedges, T. R., _op. cit._, p. 579.

[21]Luce and Raiffa, _op. cit._, p. 15.

Selected Examples of Fish and Wildlife Management Decision Problems

Uncertainty is present in many fish and wildlife management decision problems, though it is often assumed away. A complete listing of the factors included in all fish and wildlife decision problems that may be subject to uncertainty would be a very formidable task.

The following are some examples of problems[22] which a fish or wildlife decision maker might face and some of the factors, or future action environments[23], of the problem which may be subject to uncertainty:

1. Problem: How to determine the length and time of year for a fishing or hunting season to achieve a desired harvest.

   Possible sources of uncertainty:

   (a) public wants;

   (b) number of animals and their distribution;

   (c) weather conditions during the season.

2. Problem: How to determine the number of wardens to effectively enforce the hunting or fishing regulations.

   Possible sources of uncertainty:

   (a) number of habitual violators;

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[22]Personal communication with Dr. R. H. Giles, Division of Forestry and Wildlife Sciences, Virginia Polytechnic Institute and State University, Blacksburg, Virginia, 1971.

[23]The action environment is the total of the factors under which an action, which was necessarily decided on beforehand, is taken.

(b) number of dissatisfied hunters or fishermen;

(c) ease of surveilance.

3. Problem: How to decide whether to overhaul old equipment or to buy new equipment.

Possible sources of uncertainty:

(a) amount of use in the future;

(b) cost of repairs per 100 hours of use if old equipment is kept;

(c) cost of repairs per 100 hours of use if new equipment is bought.

4. Problem: How to determine how much money to allocate to acquire a certain acreage of land. If more money is allocated than used, the unused amount is returned to the state treasury.

Possible sources of uncertainty:

(a) availability of suitable land;

(b) cost of land.

5. Problem: How to determine how much money should be spent on dikes to assure a specific number of ducks.

Possible sources of uncertainty:

(a) rainfall, which determines water area;

(b) available food supply;

(c) number of ducks in area.

6. Problem: How to decide on number of fish to stock in a stream to assure a certain catch.

Possible sources of uncertainty:

(a) water flow;

(b) native supply;

(c) food supply;

(d) ratio of fish caught to total number of fish.

In the following chapter (Chapter III), the decision problem, discussed at the start of this chapter, will be used as an example to demonstrate the application of decision theory under uncertainty. In Chapter IV, the other problems described in this section will be formulated within the decision theory framework. In preparation for Chapter III, the decision theory framework will now be reviewed.

Review of Decision Theory

Decision theory provides a systematic approach to decision making under knowledge conditions of risk and uncertainty. In fish and wildlife management decision problems, risk or uncertainty is often present, and thus it would seem that decision theory is a potentially valuable tool for fish and wildlife decision makers.

The use of decision theory does not insure perfect decision making. Just as with any decision tool, the results of the technique can be no better than the information used. However, decision theory does provide a distinct benefit; it allows the decision maker to take risk and uncertainty into account. Since many fish and wildlife management problems contain risk or uncertainty, this means that the decision maker's analysis will be closer to the real-world situation, and his decision is likely to be improved.

A decision problem exists if and only if there is a choice of alternative Actions $(A, A_2, \ldots A_k)$. Decision theory is applicable to the problem if:

(1) each action leads to one of a set of possible outcomes, or Consequences;

(2) the set of possible consequences can be associated with a set of distinct future events, or States of Nature $(S_1, S_2, \ldots S_j)$, which make up an uncertain factor of the action environment[24];

(3) some one of the states of nature is bound to occur and no two states of nature have overlapping definitions, i.e., the states are mutually exclusive and collectively exhaustive.[25] Thus, the probabilities of occurrence $(P(S_1), P(S_2), \ldots, P(S_j))$ (whether they are known, subjective, or unknown) of all the states of nature sum to 1.

Formulation of the Problem

The first step in solving a decision problem is to formulate the problem, i.e., develop a well-defined statement of the problem. If the decision is to be the best possible, the formulation must be precise

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[24]Chernoff and Moses, op. cit., p. 2.

[25]Schlaifer, Robert, Introduction to Statistics for Business Decisions, McGraw-Hill Book Company, Inc., New York, 1961, pp. 7-8.

and detailed. The quality of the decision is directly related to the formulation of the problem. The following outline will be used:

1. Objective
2. Constraints on the objective and on the actions
3. Alternative actions
4. Assumptions
5. States of nature
6. Unit of the consequences
7. Consequences

The hypothetical problem described at the beginning of this chapter, which will be referred to as the deer problem, will be formulated in preparation for Chapter III. The problem will intentionally be kept simple since the primary purpose of Chapter III will be to demonstrate methodology and not to solve a specific problem.

1. Objective:

   The objective for the deer decision problem is to determine the best action to take to achieve a sustainable deer harvest that will satisfy public wants.

2. Constraints on the objective and actions:

   The decision maker may place constraints or restrictions on the objective. For example, the decision maker may want to minimize the amount of money spent, or minimize the risk of not satisfying the public want, or equate harvest to public want over time, or some other constraint.

He may also place constraints on the actions. For example, he may want to set a maximum and/or minimum on the amount that can be spent, restrict the initial investment to a maximum amount, use only presently employed labor force, or some other constraint, or some combination of constraints.

3. Alternative actions:

The alternative actions are the relevant actions available to the decision maker to influence the size of the deer herd so as to achieve a sustainable deer harvest. For this application it will be assumed that three actions (A_1, A_2, A_3) are available, where A_1, A_2, A_3 are levels of increasing habitat improvement on a state-owned area. A_1 represents an allocation of $18,000 to deer habitat improvement; A_2, $22,000; A_3, $26,000.

4. Assumptions as to the knowledge situation:

It is important for the decision maker to realize the assumptions he makes. His ultimate decision is "My decision is . . . , _assuming_ . . . ." For this application, it will be assumed that the only source of uncertainty (state-of-nature set) is the extent of public wants.

This assumption implies that the basic technological relationships are known. That is, it is known that if X dollars are spent for habitat improvement, Y deer can be harvested each year. Such an assumption, while "removing" some of the uncertainty, is probably far from the real-world situation.

5. States of nature:

The states of nature must be defined in such a manner that one is bound to occur (collectively exhaustive) and no two have overlapping definitions (mutually exclusive). For this application, it will be assumed that there are three collectively exhaustive and mutually exclusive states of nature (S_1, S_2, S_3) where S_1 represents a hunter want for 1,000 deer; S_2, 1,500 deer; S_3, 2,000 deer.

6. Unit of the consequence:

Since it is desired to maintain a sustainable harvest and to balance harvest with public want, the obvious unit of consequence might be the difference between the number of deer which can be harvested while maintaining a sustainable harvest and the number of deer the public wants. However, this implies that the consequence of not meeting the public want by 150 deer is exactly equal to the consequence of having an excess of 150 deer. This is unlikely to be the case.

This inconsistency can be overcome by the use of the concept of utility which refers to the satisfaction obtained by an action. The satisfaction the decision maker receives from an action then can be defined in units of utility.[26]

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[26]For a detailed discussion of utility, see Leftwich, R. H., The Price System and Resource Allocation, The Dryden Press, Inc., Minsdale, Illinois, 1970, Chapter 4; Chernoff and Moses, op. cit., Chapter 4.

For this application it will be assumed that the decision maker's satisfaction for equating harvest with public want is 1,000 units of utility. The utility loss for not meeting public want is one unit per deer. The utility loss for exceeding the number of deer needed and thus wasting time and money that could be spent elsewhere is one unit per two deer. In other words, it will be assumed that the satisfaction lost by not meeting public want by 100 deer is twice that lost by exceeding public want by 100 deer.

If the concept of utility is used, it must be used with extreme care. Utility is a personal evaluation and may differ not only among decision makers but also with changes of scale.

7. Consequences:

The consequences of each action must be known though the accuracy may or may not be critical. (This aspect is discussed in the footnote on page 49.)

The available actions, states of nature, and consequences for this application can be summarized in the following manner as the <u>decision matrix</u>:

States of nature \ Actions	A_1	A_2	A_3
S_1	C_{11}*	C_{12}	C_{13}
S_2	C_{21}	C_{22}	C_{23}
S_3	C_{31}	C_{32}	C_{33}

* C_{11} represents the expected consequence if Action A_1, is taken and State S_1 occurs.

To facilitate the application, values will be assigned to the above decision matrix. First, the consequences will be in the form of the difference between the number of deer that can be harvested and the number of deer the public wants; or

$C_{jk} = A_k$ (in number of deer) - S_j (in number of deer)

such that:

States of nature \ Actions	A_1 (1,200 deer) \$18,000	A_2 (1,700 deer) \$22,000	A_3 (2,000 deer) \$26,000
	Consequences in number of deer		
S_1 (1,000 deer)	+200*	+700	+1,000
S_2 (1,500 deer)	-300**	+200	+ 500
S_3 (2,000 deer)	-800	-300	0

* +200 indicates an excess of 200 deer.

** -300 indicates a deficiency of 300 deer.

These numbers are entirely hypothetical and are being used only to facilitate the application of the theory.

Next, these excesses and deficiencies in deer numbers can be converted to units of utility, as described previously, to provide the following decision matrix:

Actions: States of nature	A_1 \$18,000	A_2 \$22,000	A_3 \$26,000
	Consequences in units of utility		
S_1	900*	650	500
S_2	700**	900	750
S_3	200	700	1,000

* C_{11} = 1,000 units - (+200 deer) (.5 deer/unit) = 900 units.

** C_{21} = 1,000 units - (300 deer) (1 deer/unit) = 700 units.

The deer problem is now in a form suitable for the application of decision theory to the decision problem.

CHAPTER III

DECISION THEORY APPLICATION

In this chapter decision theory will be applied to the deer problem. The application will begin by assuming a knowledge condition of complete uncertainty and will continue through to the condition defined as subjective risk. Emphasis will be placed on:

(1) How the decision maker can improve his knowledge condition;

(2) The various options open to a decision maker if he must make a decision based on the available knowledge; and

(3) The mechanics and reasoning behind such options.

Application of the Theory to the Deer Decision Problem

A. Assuming Complete Uncertainty:

Under complete uncertainty, the decision maker has no knowledge as to the probability of occurrence of the states of nature. Yet, due to some circumstance, he nonetheless considers it necessary to make a choice. Hopeless as the situation appears, some decision making procedures or criteria[27] are available.

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[27]These criteria are discussed further in Luce and Raiffa, *op. cit.*, Chapter 13; Chernoff and Moses, *op. cit.*, Chapter 5; and Halter and Dean, *op. cit.*

1. The maximin criterion specifies that the decision maker should choose the action with the best worst consequence, i.e., maximize the minimum outcome. Applying this principle to the deer decision problem, the best choice would be A_2 since 650 units of utility represent the best worst consequence (as compared to 500 units for A_3 and 200 units for A_1). Maximin has the merit that it provides an extremely conservative choice for problems in which conservation might make good sense.

A possible objection to the maximin criterion is that it does not possess the property of "row linearity." To illustrate, suppose the decision maker with the deer problem decides that he would gain an additional 300 units of utility in the event that S_1 occurred because he would have less worry. The action matrix would be:

States of nature \ Actions	A_1	A_2	A_3
S_1'	900 +300 = 1,200	650 +300 = 950	500 +300 = 800
S_2	700	900	750
S_3	200	700	1,000

and the optimum act would be A_3 rather than A_2. "It can be argued in this case that since the constant had the same effect on each action that it should not affect the decision maker's action."[28]

2. The minimax regret criterion (suggested by Savage[29]). The action matrix, if in terms of positive consequences, must be transformed to an action regret matrix. Looking at the deer problem, if S_1 occurred, there would be no regret[30] if A_1 were taken (900-900), a regret of 250 if A_2 were taken (900-650), and a regret of 400 if A_3 were taken (900-500). The action regret matrix would be:

States of nature		Actions: A_1	A_2	A_3
	Maximum gain	Regret in regret units		
S_1	900	900 -900 = 0	900 -650 = 250	900 -500 = 400
S_2	900	900 -700 = 200	900 -900 = 0	900 -750 = 150
S_3	1,000	1,000 - 200 = 800	1,000 - 700 = 300	1,000 -1,000 = 0

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[28]Halter and Dean, op. cit., p. 60.

[29]Savage, L. J., "The Theory of Statistical Decision," Journal of the American Statistical Association, 1951, Vol. 46, pp. 55-67.

[30]To quote Halter and Dean, op. cit., p. 60: "If the decision maker takes an action and the state of nature occurs for which the gain is largest for his act, then he will have no regret. However, if he takes an action for which the gain is not the largest, and the same state of nature occurs, then he will have a regret of the difference between the largest gain and that which he receives."

The criterion prescribes the choice of the action with the <u>min</u>imum and <u>max</u>imum regret, A_2[31].

Minimax regret does possess the property of "row linearity." However, it does not possess the property of "independence of irrelevant alternatives." Suppose that in the deer problem, for some reason A_1 was found to be unworkable and was dropped. The new action regret matrix becomes:

States of nature \ Actions	A_2	A_3
S_1	0	150
S_2	0	150
S_3	300	0

The criterion would now prescribe the choice of A_3 over A_2 even though A_2 and A_3 are independent of A_1.

Another objection, only when the consequences are measured in utility, comes from Chernoff[32]: " . . . it has never been clearly demonstrated that differences in utility do in fact measure . . . regret. In other words it is not clear that the 'regret' of going from a state of utility 5 to a state of

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[31]The results of this criterion and the maximin criterion may differ. If (A_2, S_3) were 550 units of utility, the solution using maximin would still be A_2, but with a new regret of 450 regret units for (A_2, S_3), the solution using minimax regret would be A_3.

[32]Chernoff, H., "Rational Selection of Decision Functions," <u>Econometrica</u>, Vol. 22, 1954, pp. 422-443.

utility 3 is equivalent in some sense to that of going from a state of utility 11 to one of utility 9."

3. The <u>pessimism-optimism index criterion</u> (Hurwicz index). The Hurwicz index provides a means of making a decision related to the decision maker's degree of pessimism (or optimism). The maximin and minimax regret criteria both assume that the decision maker is ultraconservative (completely pessimistic). The Hurwicz index considers both the best and the worst outcomes for each action.

First, the decision maker must decide on a value of B_1 as a fixed number between, or inclusive of, 0.0 and 1.0, which is indicative of his pessimism. Thus, 0.0 indicates no pessimism, while 1.0 indicates complete pessimism (no optimism). The action prescribed will be the one which maximizes the index: $(B)\min_k C_{jk} + (1-B)\max_k A_{jk}$.

For the deer problem, assume that the decision maker is neither optimistic nor pessimistic; that is, B = 0.5. Then:

	Consequence				
Action	Minimum	Maximum	(B) (Min)	(1-B) (Max)	Sum
A_1	200	900	(.5)(200) = 100	(.5)(900) = 450	550
A_2	650	900	325	450	775
A_3	500	1,000	250	500	750

Thus, the Hurwicz criterion prescribes that A_2 be chosen.

Notice that nothing was said concerning the states of nature. The Hurwicz index deals only with the decision maker's attitude, whether pessimistic or optimistic, toward the consequences of the actions. The maximin and minimax regret criteria are, in effect, the Hurwicz index criterion when B = 1.0, i.e., is completely pessimistic. The Hurwicz index criterion does not possess the property of "row linearity" nor the property of "independence of irrelevant alternatives."

4. The minimin criterion. The minimin criterion is, in effect, the Hurwicz index criterion where B = 0.0, i.e., no pessimism or completely optimistic. It also does not possess the property of "row linearity." Based on this criterion, the decision maker with the deer problem would choose S_3.
5. The "principle of insufficient reason" (LaPlace) criterion. This criterion assumes that, since there is no information as to the occurrence probabilities of the states of nature, the decision maker should assume that each state has an equal probability of being the true state.

Assuming the necessary condition that the states are collectively exhaustive and mutually exclusive, the application of LaPlace to the deer problem follows:

States of nature	$P(\theta_j)$ based on LaPlace criterion	Weighted expected utility A_1	A_2	A_3
S_1	1/3 or .33	300	217	167
S_2	1/3 or .33	233	300	250
S_3	1/3 or .33	67	233	333
	Expected utility for each action	600	750	750

This criterion prescribes the choice of either A_2 or A_3.

W. T. Morris states, "The very statement of this principle raises strong feelings of doubt. Its highly gratuitous character leaves little to be said for it, except it is simple and gives definite selections among the alternatives."[33] While it may be argued that the states should be listed in such a way as to make them equally likely, to do so assumes that some knowledge is available concerning the occurrence probabilities of the states. The criterion does have the properties of "row linearity" and "independence of irrelevant alternatives."

There are no arguments based on deductive proofs which can be used to defend the use of one criterion over the others. The arguments for or against are based either on intuition or failure to

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[33] Morris, W. T., The Analysis of Management Decisions, Richard D. Irwin, Inc., Homewood, Illinois, 1964, p. 387.

possess some property. While Milnor[34] and Luce and Raiffa[35] produced lists of axioms, desirable properties which a criterion should have, none of the criteria that have been discussed have all of the properties. To quote Morris[36] again: "It seems . . . more reasonable to simply conclude that the choice of a principle must remain a matter of reflecting a decision maker's attitude toward (complete) uncertainty in a given decision-making context."

B. Assuming Partial Uncertainty:

"A common criticism of (the previous) criteria . . . is that they are rationalized on some notion of complete ignorance [uncertainty]. In practice, however, the decision maker usually has some vague partial information concerning the true state. No matter how vague it is, he may not wish to endorse any characterization of complete ignorance [uncertainty], and so the heart is cut out of criteria based on this notion."[37]

1. Personalistic probabilities. While certainty is an unrealistic assumption for many decision problems in fish or wildlife management, the fish or wildlife decision maker may possess at least some knowledge about the occurrence probabilities (prior)

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[34] Milnor, J. W., "Games Against Nature," Thrall, R. M.; Coombs, C. H.; and Davis, R. L., (eds.), Decision Processes, John Wiley & Sons, Inc., New York, 1954.

[35] Luce and Raiffa, op. cit., pp. 287-296.

[36] Morris, W. T., op. cit., p. 395.

[37] Luce and Raiffa, op. cit., p. 299.

probabilities) of the states of nature as a result of his judgment and experience. The question is, how can he use his judgment and experience (and/or the judgment and experience of others) to the best advantage in his decision making?

The decision maker's judgments, guesses, or feelings about the likelihood of a future event may be very roughly classified as personalistic probabilities.[38] "The personalistic theory of probability prescribes that a person should use personal probability assessments in decision making and that these assessments, though not necessarily correct, should correspond with his judgments. Since the judgments exist solely in the assessor's mind, there is no way to prove whether or not this requirements is satisfied. All self-consistent . . . assessments are admissible as long as the individual feels they correspond with his judgments."[39]

Obviously, the closer to "correct" the personalistic probabilities are, the "better" the decision will be. Several methods have been devised to improve the personalistic probability. These methods include:

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[38]Morris. W. T., op. cit., p. 232.

[39]Winkler, R. L., "The Quantification of Judgment: Some Methodological Suggestions," Journal of the American Statistical Association, December, 1967, Vol. 62, p. 1105.

(a) The Savage hypothetical acts method.[40] The Savage method involves considering a series of simple hypothetical decision problems under uncertainty, using the states from the real problem, and resolving them. These solutions are then used to infer a prior probability.

To illustrate, consider the states from the deer problem and two hypothetical acts, H_1 and H_2:

States of nature \ Actions	H_1	H_2
S_1 (1,000 deer)	\$ 0	X
S_2 (1,500 deer)	\$ 0	X
S_3 (2,000 deer)	\$100	X

with the indicated monetary equivalents. A value for X must be chosen so that the decision maker is indifferent to choosing H_1 or H_2. That is, at what minimum amount (X) would the decision maker be indifferent between receiving X for certain and having a chance of receiving \$100 if S_3 does turn out to be the true state? Suppose that the decision maker with the deer problem feels that there is not a very good chance that S_3 will occur and be

[40]Luce and Raiffa, op. cit., pp. 300-302.

indifferent between receiving \$20 for certain and taking the chance on receiving \$100. The decision maker has, in effect, assigned a subjective probability to S_3 of $\frac{20}{100}$ or .20.

Now, the procedure must be repeated with:

States of nature \ Actions	H_1	H_2
S_1	\$ 0	Y
S_2	\$100	Y
S_3	\$ 0	Y

Suppose, for this situation, that the decision maker is just as willing to receive \$85 certain as to take a chance on receiving \$100 if S_2 does turn out to be the true state. Thus, Y = \$85 and his subjective probability for S_2 becomes $\frac{85}{100}$ or .85.[41]

The procedure must be repeated again with:

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[41]His subjective probability for S_3 is .85 assuming that, if his utility of the \$100 were 1 unit, the utility of the \$20 is .2 unit. According to Luce and Raiffa (*op. cit.*, pp. 300-301), the Savage school suggests that the dollar values be changed to reflect the decision maker's utility returns for X given that his utility for \$100 is 1 unit. Thus, if the decision maker's utility from receiving \$20 was .3 unit, then his subjective probability for S_3 would be .3, not .2.

States of nature \ Actions	H_1	H_2
S_1	\$100	Z
S_2	\$ 0	Z
S_3	\$ 0	Z

Suppose he decides on a value of \$30 for Z; thus his subjective probability of S_1 is .30.

The decision maker has arrived at his subjective probabilities for the states in the deer problem:

States of nature	Subjective probability
S_1	.30
S_2	.85
S_3	.20

However, his judgment is <u>inconsistent</u> because the sum of his subjective probabilities does not equal 1 (.30 + .85 + .20 = 1.35). Then he must repeat the entire process until the sum of his subjective probabilities is equal to one.

(b) The modified hypothetical acts method.[42] Two modifications are suggested to the Savage method. The first involves a

[42]As modified by the author.

switch from the use of \$100 to the average expected utility of the various acts for the state for which the subjective probability is being determined.

Thus, for the deer problem, the three simple hypothetical decision problems would be:

Actions / States of nature	H_1	H_2
S_1	0	X
S_2	0	X
S_3	633*	X

* (200 + 700 + 1,000) / 3 = 633.

Actions / States of nature	H_1	H_2
S_1	0	Y
S_2	783*	Y
S_3	0	Y

* (700 + 900 + 750) / 3 = 783.

and

States of nature \ Actions	H_1	H_2
S_1	683*	Z
S_2	0	Z
S_3	0	Z

* (900 + 650 + 500) / 3 = 683.

This first modification has a disadvantage in that the decision maker must think in terms of units of utility rather than the more common unit of dollars. However, it has the advantage that it puts the decision problem of deciding on the subjective probabilities within the same context of the decision problem in question, rather than using \$100 and modifying X, Y, Z to reflect the utility gained from receiving an amount less than \$100. Thus, the assumption is <u>not</u> made that the decision maker's utility distribution from \$0 to \$100 is the same as his utility distribution for the decision problem consequences.

The second modification has to do with the problem of inconsistency. The Savage method implies that the inconsistency stems from errors in judgment. The method prescribes that the process be repeated until the subjective probabilities sum to 1, i.e., to force modification of judgment to establish consistency. The second modification avoids this by assuming that, while the judgments are made rationally, the decision maker <u>unconsciously</u> biases his judgment because of the type of decision process used. This bias is assumed to be a result of his aversion to risk or his tendency to gamble when confronted with a betting situation; i.e., when is he indifferent between receiving X and taking a chance on receiving Y? Thus, if he had an aversion to risk, he would require a smaller X rather than gamble for Y; if he had a tendency to gamble, he would require a higher X rather than gamble for Y. It is finally assumed that he is consistent in his bias. The modification will be illustrated by the deer problem.

It was assumed that the decision maker with the deer problem developed the following probabilities on his first attempt:

States of nature	Subjective probability
S_1	.30
S_2	.85
S_3	.20
$\Sigma P(S)$	1.35

The modification assumes that the decision maker had a tendency to gamble and that he was consistent in this tendency. Thus, the probabilities should be adjusted downward by multiplying each one by $\frac{1.00}{\Sigma P(S)}$, i.e., $\frac{1.00}{1.35}$ or .74, to negate this tendency so that his subjective probabilities will be consistent; i.e.:

States of nature	Subjective probability
S_1	.22
S_2	.63
S_3	.15
$\Sigma P(S)$	1.00

Thus, rather than force the decision maker to modify his judgment to be consistent, it is assumed that he is subconsciously biased consistently.

The following methods are designed for use when the decision maker is able to consult one or more persons whom he considers reasonably expert with regard to the states of nature. The question then becomes how to combine two or more different subjective probabilities in arriving at a subjective probability upon which to base a decision.

(c) The weighted-average method.[43] The most obvious method of combining subjective probabilities is by weighting the

[43]Winkler, R. L., "The Consensus of Subjective Probability Distributions," Management Science, Vol. 15, No. 2, October, 1968, pp. 61-64.

probabilities from each judgment and combining into a "pooled"[44] set of subject probabilities. The only restriction is that the new probabilities sum to 1, i.e., that they are consistent.

(1) Equal weights. Under the concept of equal weights, the decision maker makes a subjective decision that there is no difference in quality between the judgments. Looking at the deer problem, suppose the decision maker gathered the following information from persons who he felt were reasonably expert with regard to hunter wants and familiar with the decision problem situation:

States of nature \ Source of probability	The decision maker	Expert No. 1	Expert No. 2
S_1	.22	.34	.30
S_2	.63	.57	.50
S_3	.15	.09	.20
$\Sigma P(S)$	1.00	1.00	1.00

Applying the concept of equal weights, the decision maker would multiply the subjective probability for

[44]Winkler, R. L., op. cit., p. 61.

each judgment for each state by 1/3 or .33 and sum the results. Thus:

States of nature	Consensus of probabilities
S_1	.29*
S_2	.57
S_3	.15
$\Sigma P(S)$	1.01**

* (.22)(.33) + (.34)(.33) + (.30)(.33) = .29 or (.22 + .34 + .30) ÷ 3 = .29.

** 1.01 ≠ 1.00 due to rounding error.

(2) Weights proportional to a ranking. Now suppose that the decision maker with the deer problem felt that his assessment of the states was equal to that of Expert No. 1 but better than that of Expert No. 2. Then he might assign a rank of 1 for Expert No. 2, and a rank of 2 for himself and Expert No. 2. Thus he would weight his probabilities and those of Expert No. 1 by $\frac{2}{1 + 2 + 2}$ or .4. Expert No. 2's probabilities would be weighted by $\frac{1}{1 + 2 + 2}$ or .2. Thus:

States of nature	Consensus of probabilities
S_1	.28*
S_2	.58
S_3	.14
$\Sigma P(S)$	1.00

* (.22)(.4) + (.34) (.4) + (.30)(.2) = .28.

Suppose, however that a study was recently completed, based on responses to a questionnaire which estimated the following probabilities:

States of nature	Study No. 1
S_1	.27
S_2	.48
S_3	.25
$\Sigma P(S)$	1.00

Now suppose that the decision maker with the deer problem thought it was a very good study, but he did not want to ignore his judgment and the judgment of the two experts. He might then decide to rank the results of the study, his opinion, and the judgments of Experts No. 1 and No. 2 as 10, 5, 4, and 1, respectively.

The weights become $\frac{10}{20}$, $\frac{5}{20}$, $\frac{4}{20}$, and $\frac{1}{20}$, respectively.

The result becomes:

States of nature	Consensus of probabilities
S_1	.27*
S_2	.54
S_3	.19
$\Sigma P(S)$	1.00

* (.27)(.5) + (.22)(.25) + (.34)(.20) + (.30)(.05) = .27.

(3) Weights proportional to a self-rating. This concept involves having each assessor "rate himself on a scale from 1 to c, where c is the highest rating and 1 is the lowest."[45] Each assessor is then ranked in accordance with his self-rating. Looking at the deer problem, suppose the decision maker felt he could not make a judgment concerning the probabilities, but as a result of a questionnaire to three experts, he received the following (c set at 5):

[45]Winkler, R. L., op. cit., p. 64.

States of nature \ Source of probability	Expert No. 1	Expert No. 2	Expert No. 3
S_1	.34	.30	.28
S_2	.57	.50	.67
S_3	.09	.20	.05
$\Sigma P(S)$	1.00	1.00	1.00
Self-rating	4	2	3
Ranking by decision maker	2	–	1
Weight	.66		.33

If the decision maker decides that a self-rating of less than 3 would mean that the expert's assessment would be dropped, the ranking would be based on self-rating minus 2. The consensus would thus be:

States of nature	Consensus of probabilities
S_1	.32
S_2	.62
S_3	.08
$\Sigma P(S)$	1.00

(d) Feedback and reassessment.[46] One possible method of improving the prior subjective probabilities, before applying a weighted-average method to the judgment of two or more persons, is to provide each person with all of the judgments and to give him an opportunity to reconsider his judgment. These reassessed probabilities could be analyzed as above or "recycled" again. If, by recycling, these judgments converge into a common judgment, then a weighted-average method would be unnecessary.

(e) Groups reassessment.[47] After the initial judgments, the experts might be brought together and, as a group, attempt to arrive at a single consensus or merely discuss the various factors which influenced their judgment. In this way, factors which might not have been considered can be brought out.

A disadvantage of requiring that a consensus be reached is that the consensus may be slanted because of "natural leaders" and individual tactics. If a consensus is not required, then each expert can be given an opportunity afterwards to reassess his subjective probabilities, and a consensus can be reached by a weighted-average method.

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[46]Winkler, R. L., <u>op</u>. <u>cit</u>., pp. 69-70.

[47]Winkler, R. L., <u>op</u>. <u>cit</u>., pp. 70-71.

Which of the previous methods will provide the "best" prior subjective probabilities? Since the "correct" (objective) probabilities have been assumed to be unknown, and in the great majority of fish and wildlife management decision problems is unknown, it is impossible to select a method on the basis that it is more nearly "correct" than the other methods.[48] The method or methods that are used depends upon the situation, how reasonable the method(s) appears to the decision maker, and how sensitive the decision outcome is to changes in the probabilities. Sensitivity will be discussed below, but it is necessary first to explain briefly the classical approach to decision making under risk.

Once the decision maker obtains subjective probabilities concerning the occurrence (prior probabilities or "likelihood" of occurrence [posterior probabilities[49]] of the states of nature) which he feels are sufficient to make a decision, the following method can be used to arrive at a decision. First, the average expected consequence for each act is the sum of the potential outcomes, each of which is "weighted" by the occurrence probability for each particular state of nature. To illustrate this procedure, the example problem will be used.

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[48]Winkler, R. L., op. cit., p. 74.

[49]Posterior probabilities will be discussed later (see page 51).

Suppose the decision maker with the deer problem decides to make a decision based on the following subjective probabilities:

States of nature	Subjective probabilities
S_1	.32
S_2	.60
S_3	.08
$\Sigma P(S)$	1.00

The average[50] expected consequences, in this case utility, for the actions based on the subjective probabilities and the decision matrix (p. 22) becomes:

States of nature	P(S)	Weighted expected utility		
		A_1	A_2	A_3
S_1	.32	(900)(.32) = 288	208	160
S_2	.60	(700)(.60) = 420	540	450
S_3	.08	(200)(.08) = 16	56	80
Average expected utility of each action		Σ = 724	Σ = 804	Σ = 690

[50]The average expected consequence of an action is <u>not</u> the consequence the decision maker can expect if the action is taken, but rather the average expected consequence if the action were taken repeatedly.

The maximizing action then would be A_2 with an average expected utility of 804 units, as opposed to 724 average expected units for A_1 and 690 average expected units for A_3.

The sensitivity of the decision to probability changes indicates the effect of a change in the subjective probabilities on the action choice prescribed by the decision. Coming back to the original question concerning what methods should be employed, an examination of sensitivity using the deer problem would be helpful.

In the deer problem, the subjective probabilities arrived at using the modified hypothetical actions methods were .22, .63, and .15 for S_1, S_2, and S_3, respectively. Using these probabilities, the average expected utilities for the actions would be 669 units for A_1, 815 units for A_2, and 732.5 units for A_3. The maximizing action would be A_2. Now, how sensitive the decision is to changes in the subjective probabilities can be explored. This must be determined by trial and error although examination can aid. For example, it would seem that, for the deer problem, since there is a large spread between the average expected utilities for S_1 and S_2, the subjective probability of S_1 could be increased and that of S_2 decreased. Suppose the decision maker tried .32, .53, and .15 for S_1, S_2, and S_3. The average expected utility for A_1, A_2, and A_3 would be 689 units, 790 units, and 707 units. A_2 still is the maximizing action. Thus after one trial, it is known that the solution will not change if the subjective probabilities are in the following ranges:

States of nature	Range of probabilities
S_1	.22 to .32
S_2	.53 to .63
S_3	.15
$\Sigma P(S)$	must = 1.00

Several trials indicate that the solution will not change for the following ranges:

States of nature	Range of probabilities
S_1	.00 to .40
S_2	.40 to 1.00
S_3	.00 to .30
$\Sigma P(S)$	must = 1.00

In this case, the decision is <u>insensitive</u> within a rather wide range of probability changes, and the decision maker's subjective probabilities (.22, .63, and .15 for S_1, S_2, and S_3, respectively) are well within this range. Thus, while the decision maker might not have placed a great deal of confidence in his probabilities originally, after determining the sensitivity of the decision, he might be quite willing to make a decision solely on his original subjective probabilities determined by the modified hypothetical acts method.[51]

Thus, the decision maker's decision to accept his subjective probabilities and make a decision is a matter of confidence. If he is not confident of his subjective probabilities, it might be worth his while to try another method. However, suppose that after arriving at a set of subjective probabilities he is still not satisfied; or, even though satisfied, he would like to improve them. It may be that additional information is available or can be obtained which cannot be used to develop subjective or objective probabilities but which might be used in the decision process by use of the Bayesian strategy to revise his subjective probabilities.

2. Revised subjective probabilities. This section will be introduced by way of illustration using the example problem. The

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[51]On page 20 it was stated that "the consequences of each action must be known though the accuracy may or may not be critical." The criticalness of the accuracy of the consequences can be determined in a manner similar to the method described for determining the sensitivity of the decision--i.e., vary the consequences of the actions while holding the subjective probabilities constant. For example, in the deer problem, if the expected outcome of A_1 had been 1,000 deer rather than 1,200 deer, with the resulting consequences of 1,000 units of utility if S_1 occurs, 500 units if S_2 occurs, and 0.0 units if S_3 occurs; would the decision change, given the decision maker's subjective probabilities (.22, .63, and .15 for S_1, S_2, and S_3, respectively)? Thus, by trial and error the criticalness of the accuracy of the consequences can be determined.

decision maker with the deer problem, using his judgment and the modified hypothetical acts method, arrived at the subjective prior probabilities of:

States of nature	Subjective prior probabilities
S_1	.22
S_2	.63
S_3	.15
$\Sigma P(S)$	1.00

Now, suppose someone had developed a "predicting" formula (experiment) which estimates hunter wants for future seasons based on license sales for past seasons. When applied to the decision maker's problem situation, the experiment "predicted" that the harvest demand would be 1,000 deer, i.e., $\hat{S}_1$ (observation). However, it has not proven to be a perfect "predictor." Application to similar situations in the past has demonstrated that when S_1 did occur, $\hat{S}_1$ had been predicted 3/4, or .75, of the time; when S_2 occurred, $\hat{S}_1$ had been predicted 1/3, or .33, of the time; when S_3 occurred, $\hat{S}_1$ had been predicted 1/4, or .25, of the time.

At first glance, these data would seem inconsistent since .75 + .33 + .25 do not sum to 1.00. Notice, however, that the figures are not the probabilities that S_1, S_2, and S_3, respectively, will occur given that $\hat{S}_1$ was predicted--they are not

$P(S_1|\hat{S}_1)$, $P(S_2|\hat{S}_1)$, $P(S_3|\hat{S}_1)$, where $\hat{S}_1$ represents the predicted state. Rather, the figures are $P(\hat{S}_1|S_1)$, $P(\hat{S}_1|S_2)$, and $P(\hat{S}_1|S_3)$, respectively. These probabilities will be referred to as conditional probabilities, i.e., the probability of the observation (in this case the prediction $\hat{S}_1$) is conditional on which state is assumed as having occurred.

The revised prior probability for the deer problem (the posterior $P(S_1)$, given that S_1 was predicted), i.e., $P(S_1|\hat{S}_1)$, can be calculated[52] in the following manner:

$$P(S_1|\hat{S}_1) = \frac{P(S_1)P(\hat{S}_1|S_1)}{P(S_1)P(\hat{S}_1|S_1)+P(S_2)P(\hat{S}_1|S_2)+P(S_3)P(\hat{S}_1|S_3)}$$

$$P(S_1|\hat{S}_1) = \frac{(.22)(.75)}{(.22)(.75)+(.63)(.33)+(.15)(.25)} = .40$$

The other two conditional probabilities become:

$$P(S_2|\hat{S}_1) = \frac{(.63)(.33)}{.41} = .51$$

and:

$$P(S_3|\hat{S}_1) = \frac{(.15)(.25)}{.41} = .09$$

Thus, given that the study (experiment) predicted S_1:

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[52]The formula that is used in calculating the conditional probabilities is known as Bayes formula:

$$L(S_i|\hat{S}_j)^* = \frac{P(S_i)P(\hat{S}_j|S_i)}{P(S_1)P(\hat{S}_j|S_i)+P(S_2)P(\hat{S}_j|S_2)+\ .\ .\ .\ + P(S_{in})P(\hat{S}_j|S_{in})}$$

where: i = a specific state
in = total number of states
j = the specific observation from the experiment.

* Reads the conditional probability of state i given S_j. For background see: Luce and Raiffa, op. cit., pp. 309-313; Halter and Dean, op. cit., pp. 170-182; Hillier, F. S., Lieberman, G. J., Introduction to Operations Research, Holden-Day, Inc., San Francisco, 1967, pp. 82-89; and Morris, W. T., op. cit., pp. 440-442.

States of nature	Posterior probabilities $P(S \mid \hat{S}_1)$
S_1	.40
S_2	.51
S_3	.09
$\Sigma P(S \mid \hat{S}_1)$	= 1.00

When the posterior probabilities (if $\hat{S}_1$, prediction of S1, is observed) are compared with the prior probabilities, it is obvious that the use of an experiment may change the prior probabilities considerably. The posterior probabilities will still sum to one. As before, the decision is made between the available actions by finding the expected value of each act, using the posterior probabilities, and selecting the largest [most favorable].[53] In the deer problem, the expected values would be:

[53]Halter and Dean, op. cit., p. 85.

States of nature	P(S)	Weighted expected utility A_1	A_2	A_3
S_1	.40	(900)(.40) = 360	260	200
S_2	.51	(700)(.51) = 357	459	383
S_3	.09	(200)(.09) = 18	63	90
Average expected utility of each action		$\Sigma = 735$	$\Sigma = 782$	$\Sigma = 673$

Thus, the decision was not changed (A_2 is still the maximizing action) when $\hat{S}_1$ (prediction of S_1) was observed.

The obvious question is, of what value is the experiment? To answer this, it is necessary, first, to find the posterior probabilities and expected values for the other possible observations, namely $\hat{S}_2$ and $\hat{S}_3$. The computations for the deer problem are shown in Table I.[54] To make the notation easier: prediction of S_1 will be Observation Z_1; a prediction of S_2 will be Observation Z_2; and a prediction of S_3 will be Observation Z_3.

[54]This tabular form is from Chernoff and Moses, op. cit., p. 179.

Table I. Computation of Posterior Probabilities and Resultant Expected Consequences of the Actions for the Deer Problem

States of nature S_i	Conditional probabilities $P(Z_1 \mid S_i)$* Z_1	Z_2	Z_3	Prior probabilities $P(S_i)$		Experiment observations Z_1	Z_2	Z_3
S_1	.75	.15	.10	.22	$P(S_1)P(Z_j \mid S_1)$	.165	.033	.022
S_2	.33	.47	.20	.63	$P(S_2)P(Z_j \mid S_2)$	.208	.296	.126
S_3	.25	.35	.40	.15	$P(S_3)P(Z_j \mid S_3)$	.037	.053	.060
					$\Sigma P(Z_j)$	.410	.382	.208
					Posterior probabilities			
					$P(S_1 \mid Z_j)$	$\frac{.165}{.410}=.402$	$\frac{.033}{.382}=.086$	$\frac{.022}{.208}=.106$
					$P(S_2 \mid Z_j)$	$\frac{.208}{.410}=.402$	$\frac{.296}{.382}=.775$	$\frac{.126}{.208}=.606$
					$P(S_3 \mid Z_j)$	$\frac{.037}{.410}=.091$	$\frac{.053}{.382}=.139$	$\frac{.060}{.208}=.288$
					$\Sigma P(S_i \mid Z_j)$	1.00	1.00	1.00
						Consequences in utility units		
					$P(S_i \mid Z_j)$ times consequences = expected consequences	$a_1=734.9$	$a_1=647.7$	$a_1=577.2$
						$a_2=781.3$	$a_2=850.7$	$a_2=815.9$
						$a_3=672.2$	$a_3=763.2$	$a_3=795.5$
					Maximizing action	a_2	a_2	a_2
					Maximum expected consequence	$a_2=781.3$	$a_2=850.7$	$a_2=815.9$

Weighted average utility = $P(Z_j)+P(S_i \mid Z_j)$
(.410)(781.3)+(.382)(850.7)+(.208)(815.9)= 815.0

States of nature \ Actions	A_1	A_2	A_3
	Consequences in utility units		
S_1	900	650	500
S_2	700	900	700
S_3	200	700	1000

* Where i = total number of states and j = total number of observations.

Examination of the expected consequences based on the posterior probabilities indicates that the Bayesian strategy to maximize utility units would be A_2 no matter which of the three observations were observed. This type of strategy is known as a pure strategy. The weighted average utility is the same as if no observation were taken, i.e., (.22)(650) + (.63)(900) + (.15)(1,000) = 815 units. Thus there is no economic gain from taking an observation. It also might be pointed out that this cannot be determined until the conditional probabilities are known but can be determined before an observation is taken.

This does not mean that the effort in obtaining the conditional probabilities is wasted. The fact that Bayesian strategy indicates a pure strategy implies that the additional information that went into the experiment was consistent with the decision maker's prior subjective probabilities. It does not mean that his probabilities are "correct" or that another experiment would not provide an economic gain. It does give the decision maker some basis for more confidence in his prior subjective probabilities and thus may aid him in the decision problem of whether or not he is willing to base a decision on his subjective probabilities.

For the sake of further discussion, assume that in the deer problem, the expected consequence for a_3 if Z_3 is observed (presently it is 795.5 units of utility) as 825 units.

The weighted average utility then would be (.410)(781.3) + (.382)(850.7) + (.208)(825) = 816.9, 1.9 units above the weighted average using only the prior probabilities. Thus the utility gain from the experiment would be 1.9 units, on the <u>average</u> and <u>over time</u>. The Bayesian strategy would dictate a mixed strategy, that is, if Z_1 or Z_2 is observed, Action 2 should be taken. If Z_3 is observed, Action 3 should be taken.

The question then is whether the 1.9 units of expected gain and added confidence are worth the cost and effort to obtain it. Or, what if the gain were 50 units? One limit is known for sure--the gain resulting from a perfect predictor given the prior subjective probabilities. The perfect predictor for the deer problem would have the following conditional probabilities:

State of nature	Experiment observations		
	Z_1	Z_2	Z_3
	Conditional probabilities		
S_1	1	0	0
S_2	0	1	0
S_3	0	0	1

where Z_1 was a prediction of S_1, and so on. The Bayesian strategy then would be: if Z_1 is observed, choose Action A_1; if Z_2 is observed, choose Action A_2; and if Z_3 is observed, choose Action A_3. The weighted average expected consequence would be:

(.22)(900) + (.63)(900) + (.15)(1,000) = 915 units. The expected gain would be 100 units. Thus this is the maximum, given the prior subjective probabilities. Again the decision maker must use his judgment. Obvious experience and judgment, or availability to such, are invaluable in decision making under uncertainty.

Only one example, the deer problem, has been used to demonstrate the use of an experiment and the Bayesian formula to utilize additional information not used[55] in arriving at a prior subjective probability. Any additional data which acts as a "predictor" to some extent can be used if it can be set up in such a way as to be able to correlate the observations with the states as conditional probabilities. Thus, a study which "predicts" probabilities should be incorporated into the decision maker's, and/or some expert's, subjective probabilities as indicated previously; or, if the decision maker considers the predicted probabilities to be "good enough," they could be used to arrive at posterior probabilities for the decision.

Other possibilities for an experiment for the example problem might be:

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[55] Using some of the same data to generate the posterior probabilities "has the distinct disadvantage that it incorporates the prior probability given by the same data" into the posterior probability. Halter and Dean, op. cit., p. 178.

(a) Determine the ratio of rifle sales one season to the wants the following season. This ratio would be only a partial "predictor" since rifles are used for other purposes. Regression analysis might be used with this type of experiment.

(b) Employ computer simulation using historical data to predict the hunter wants based on the previous two years and other factors (predict wants, not probabilities). The problem may arise here that information fed into the computer may have been used, at least subconsciously, to develop the prior subjective probabilities.

(c) Use some measure of public acceptance of hunting, say on a scale from 1 to 10, to predict the wants the following year. For example, the probability that public acceptance was 3, given that the harvest wants (S) was 1,500 deer for the following year, would be a conditional probability.

Summary

Some concluding remarks are in order at this point. In this chapter, decision making by a state fish or wildlife official under a condition of uncertainty was discussed. Various methods were demonstrated when the decision maker finds himself under complete uncertainty but nevertheless must make a decision. It was pointed out that each of the methods has strong objectionable characteristics. Thus, if it is at all possible to develop a set of personalistic probabilities, this would be advisable in many cases.

Under personalistic probabilities several methods were described as feasible for obtaining a personalistic probability. This might be done by the decision maker alone, in conjunction with one or more experts and/or studies, or solely by use of experts and/or studies. It was pointed out that a decision as to whether to accept some set of subjective probabilities and make a decision or try to improve the set must be based on the decision maker's, and/or other's, judgment and experience. The question of sensitivity of the solution to changes in the probabilities was discussed from the point of view of aiding in the decision as to whether or not to try to improve the prior subjective probabilities. If the decision maker decided that he would make a decision based on his subjective probability, he immediately changed a knowledge condition to that of subjective risk. Under subjective risk, the decision maker arrived at a decision based on the sum of the consequences of each action weighted by the prior subjective probabilities. The decision then became the action which maximized his gain or utility or, less commonly, minimized his loss or regret.

The Bayesian strategy was then introduced as a method for improving on a prior subjective probability. The Bayesian formula was introduced by way of the example problem, and the significance of various possible outcomes was discussed. The method of determining the value of the experiment was illustrated as well as the value of the "perfect" predictor. It was pointed out again that the ultimate decision to make a decision must be based, to some extent at least, on judgment and experience. Once a set of posterior probabilities are determined and the decision maker decides to make a decision, his decision would be based on the strategy and the observation from the experiment.

Thus it becomes obvious that decision making under uncertainty is in itself uncertain! There are many guidelines but few rules. Intuition, judgment, experience, and experimentation can improve the knowledge situation and, in doing so, can improve the decisions.

CHAPTER IV

VARIOUS MANAGEMENT PROBLEMS FORMULATED WITHIN THE DECISION THEORY FRAMEWORK

Several general management problems were briefly discussed in Chapter II as examples of the type of problems in which uncertainty might be encountered. These will now be formulated within the decision theory framework.

Formulation of Six Management Problems

Problem 1: How to determine the length and time of year for a fishing or hunting season to achieve a desired harvest.

The deer problem for the previous chapter was to determine the monies necessary to provide a sustainable harvest to meet public wants. Suppose that the decision maker chose A_2, and spent $22,000 for habitat improvement. Suppose further that for a particular year the state of nature turned out to be S_3, a public want for 2,000 deer. Since A_2 was taken, only 1,700 deer could be harvested without reducing the number that could be harvested the following year below the 1,700-deer level.

The decision maker's problem now is how to limit the harvest to 1,700 deer. He could change the regulations or shorten the season. A change in regulations would be met with public disapproval. The public would negate the effects of a shorter season by intensifying their efforts if it were announced beforehand. To cut off the season when 1,700 deer had been harvested would also be met with disapproval. However, the decision maker can effectively "shorten" the season by scheduling it in such a manner that there are more unsuitable hunting days--i.e., rainy days--than during the usual season.

The decision maker's problem and action matrix might be something like this:

Action	Description
A_1	September 15 to October 15
A_2	October 1 to November 1
A_3	October 15 to November 15
A_4	November 1 to December 1
A_5	November 15 to December 15

States of nature	Description
S_1	20% below normal precipitation
S_2	10% below normal precipitation
S_3	Normal precipitation*
S_4	10% above normal precipitation
S_5	20% above normal precipitation

* Normal being average for the specific time period based on historical data.

Actions: States of nature	A_1	A_2	A_3	A_4	A_5
S_1	Consequences (in units of utility)				
S_2	= utility if 1,700 deer are har-				
S_3	vested minus utility lost if				
S_4	less than or more than 1,700				
S_5	deer are harvested				

Once the consequences are determined, the decision maker would be ready to solve the problem using decision theory under uncertainty. The prior probabilities might be obtained from historical data or a weather bureau prediction. A possible experiment might be some method of prediction based on the weather for the previous year or on the weather for that year up to the point that the decision must be made.

Problem 2: How to determine the number of wardens to effectively enforce the hunting or fishing regulations.

Suppose a manager is responsible for seeing that a trout stream is policed "effectively." The decision maker's first problem is to define "effectively" in such a way as to be meaningful and measurable. Suppose the decision maker decided that "effectively" would be the maximum difference between the savings in stocking costs from controlling violations and the cost of policing. Assuming that the number of fish needed to meet public wants is known, the states become the percent of the public which are potential violators. Thus:

States of nature	Description
	% which are potential violators
S_1	5%
S_2	10%
S_3	15%

Action	Description
	No. of wardens
A_1	0*
A_2	1
A_3	2
A_4	3
A_5	4

* A_1 allows the decision maker a choice of doing nothing; oftentimes such a choice is not considered but it is generally a real possibility.

States of nature \ Actions	A_1	A_2	A_3	A_4
S_1	Consequences in utility units			
S_2	or dollars, positive			
S_3	or negative			

The decision maker's choice becomes the action with the highest dollar return (saving).

Problem 3: How to decide whether to overhaul old equipment or to buy new equipment.

For this hypothetical problem, suppose that the decision maker is uncertain of the amount of use that a piece of equipment will provide during the next year. Suppose also that he is uncertain of repair costs per 100 hours of use if the old equipment is kept. He does know the price of replacing the equipment and the expected life, in hours, for the new equipment.

States of nature	Description
	Hours of use
S_1	100
S_2	200
	Cost of repairs per 100 hr.
S_3	25
S_4	50

Action	Description
A_1	Use old equipment
A_2	Buy new equipment

Cost of new equipment = $350

Expected life of new equipment = 10 years

Cost per year (depreciation) and assuming no repair cost the first year = $35.

States of nature \ Actions	A_1	A_2
S_1 and S_3	\$ 25	\$ 35
S_1 and S_4	\$ 50	\$ 35
S_2 and S_3	\$ 50	\$ 70
S_2 and S_4	\$100	\$ 70

The criterion for making a decision would be minimization of costs.

An additional operation is necessary to arrive at a prior subjective probability:

States of nature	Subjective probabilities
S_1 and S_3	$P(S_1)P(S_3)$
S_1 and S_4	$P(S_1)P(S_4)$
S_2 and S_3	$P(S_2)P(S_3)$
S_2 and S_4	$P(S_2)P(S_4)$
$\Sigma P(S_1,S_2)$	1.00
$\Sigma P(S_3,S_4)$	1.00
$\Sigma(P(S_1,S_2)P(S_3,S_4))$	1.00

Now, say an experiment was possible for S_3 and S_4. The posterior probabilities, assuming two observations, Z_1 and Z_2, would be:

States of nature	Observation Z_1	Z_2
S_1 and S_3	$P(S_1)P(S_3 \mid Z_1)$	$P(S_1)P(S_3 \mid Z_2)$
S_1 and S_4	$P(S_1)P(S_4 \mid Z_1)$	$P(S_1)P(S_4 \mid Z_2)$
S_2 and S_3	$P(S_2)P(S_3 \mid Z_1)$	$P(S_2)P(S_3 \mid Z_2)$
S_2 and S_4	$P(S_2)P(S_4 \mid Z_1)$	$P(S_2)P(S_4 \mid Z_2)$

Problem 4: How much money to allocate to acquire a certain acreage of land. If more money is allocated than used, the unused amount is returned to the state treasury.

Assume in this case that the two sources of uncertainty are the availability of suitable land and the cost of land, and that the decision maker wants to acquire 1,000 acres. In this case, however, if the amount of land that is available is dependent upon the amount the agency is willing to pay, the states of uncertainty can be reduced to the cost per acre needed to acquire the desired acreage.

States of nature	Description
	Cost per acre
S_1	\$100
S_2	\$150
S_3	\$200
S_4	\$250

Action	Description
	Amount to be allocated
A_1	\$100,000
A_2	\$150,000
A_3	\$200,000
A_4	\$250,000

The action matrix, in terms of number of acres which would be purchased and amount of excess funds, becomes:

Actions / States of nature	A_1 \$100,000	A_2 \$150,000	A_3 \$200,000	A_4 \$250,000
	Acres that would be purchased (Amount of excess funds)			
S_1 (\$100/acre)	1,000 (\$ 0)	1,000 (\$ 50,000)	1,000 (\$100,000)	2,500 (\$150,000)
S_2 (\$150/acre)	667 (\$ 0)	1,000 (\$ 0)	1,000 (\$ 50,000)	1,000 (\$100,000)
S_3 (\$200/acre)	500 (\$ 0)	750 (\$ 0)	1,000 (\$ 0)	1,000 (\$ 50,000)
S_4 (\$250/acre)	400 (\$ 0)	600 (\$ 0)	800 (\$ 0)	1,000 (\$ 0)

Since only 1,000 acres are needed, the decision maker maximizes his utility if he can purchase 1,000 acres and have no funds left over. Suppose that one unit of utility is gained for each acre which can be purchased up to 1,000 acres and one unit of utility is lost for each \$200 of excess funds. In terms of utility the action matrix becomes:

Actions / States of nature	A_1	A_2	A_3	A_4
	Units of utility			
S_1	1,000	750	500	250
S_2	667	1,000	750	500
S_3	500	750	1,000	750
S_4	400	600	800	1,000

The decision theory problem is to choose the action which maximizes the decision maker's utility.

Problem 5: How to determine how much money should be spent on dikes to insure a specific number of ducks.

The number of ducks which can be attracted to a specific area depends on the number of ducks in the vicinity, as well as other factors such as water area, food supply, and remoteness. For this example, only the number of ducks in the vicinity will be considered uncertain. The water area is determined by the amount of diking, i.e., the actions. Thus the problem becomes:

States of nature	Description
	Ducks in the vicinity
S_1	8,000
S_2	12,000
S_3	16,000

Action	Description
	The amount to be spent on dikes
A_1	\$10,000
A_2	\$15,000
A_3	\$20,000

States of nature \ Actions	A_1	A_2	A_3
S_1	Consequences in units of utility gained by attracting a certain number of ducks minus the units loss from the cost of building dikes		
S_2			
S_3			

The optimum choice becomes the action which maximizes the decision maker's utility.

Problem 6: How to decide on the number of fish to stock in a stream to insure a certain catch.

The sources of uncertainty that might be considered are rate of water flow, native supply, and ratio of fish caught to total number of fish. The decision maker wants to satisfy a known public want without overstocking. The problem becomes:

States of nature	Description
	Rate of water flow
S_1	1,000 g.p.m.*
S_2	1,500 g.p.m.
S_3	2,000 g.p.m.
	Native supply
S_4	2,000 fish
S_5	3,000 fish
	Ratio of fish caught to total number**
S_6	.6
S_7	.8

* g.p.m. = gallons per minute.

** The stocked fish will be easier to catch, because of tameness acquired at the fishery, than will the native fish.

Action	Description
	In number of fish to be stocked
A_1	5,000
A_2	7,500
A_3	10,000

States of nature \ Actions	A_1	A_2	A_3
S_1, S_4, S_6			
S_1, S_4, S_7	Consequences might be developed		
S_1, S_5, S_6			
S_1, S_5, S_7	similar to the method used in		
S_2, S_4, S_6			
S_2, S_4, S_7	the deer problem (in the		
S_2, S_5, S_6			
S_2, S_5, S_7	last section of		
S_3, S_4, S_6			
S_3, S_4, S_7	Chapter II)		
S_3, S_5, S_6			
S_3, S_5, S_7			

Any experimental evidence would be channeled in the same fashion as suggested for Problem 3. The decision maker would then choose the action which maximized this utility.

Combining Several Related Decision Problems within a Decision Theory Framework

The preceding example problems cover a wide range of problems. However, they all involved "single action choices," i.e., either A_1 or A_2 or A_3. There are a wide range of problems in fish and wildlife management which require coordinating two or more actions. For example, consider a "multiple action choices" problem such as the following.

A decision maker in deer management wants to determine: the level of deer habitat improvement (deer problem) to maintain a sustainable harvest to meet public want; the time of year for the hunting season (Problem 1) to prevent over-harvest; and the number of wardens to minimize the illegal taking of deer (Problem 2) to prevent higher than expected harvests; and do this at minimum cost. Thus the problem becomes:

States of nature	Description
	Public want in number of deer (one deer per hunter)
S_1	1,000
S_2	2,000
	Precipitation in percent below normal (-), normal, or above normal (+)
S_3	-20%
S_4	-10%
S_5	Normal
S_6	+10%
S_7	+20%
	Percent of hunters who are potential violators (take two deer)
S_8	5%
S_9	10%
S_{10}	15%

Action	Description
	Amount to be spent on habitat improvement
A_1	\$18,000
A_2	\$22,000
A_3	\$26,000
	Dates of season
A_4	September 15 to October 15
A_5	October 1 to November 1
A_6	October 15 to November 15
A_7	November 1 to December 1
A_8	November 15 to December 15
	Number of wardens
A_9	0
A_{10}	1
A_{11}	2
A_{12}	3
A_{13}	4
A_{14}	5

The action matrix becomes:

States of nature combinations \ Action combinations	A_1 A_4 A_9	A_1 A_4 A_{10}	A_1 A_4 A_{11}	A_1 A_4 A_{12}	A_1 A_4 A_{13}	A_1 A_4 A_{14}	A_1 A_4 A_{10}	...	A_3 A_8 A_{14}
S_1, S_3, S_8 S_9 S_{10}									
S_1, S_4, S_8 S_9 S_{10}									
S_1, S_5, S_8 S_9 S_{10}	Consequences* in units of utility summed for the three action groups (habitat improvement, season dates, and warden numbers**)								
S_1, S_6, S_8 S_9 S_{10}									
S_1, S_7, S_8 S_9 S_{10}									
. . . S_2, S_7, S_{10}									

* Care must be taken when combining the actions that the units of the consequences are expressed in similar units, or can be converted to a common unit.

** Losses and/or gains in number-of-wardens action matrix were demonstrated with dollar values; the dollars must be converted to a common consequence unit such as units of utility.

The action which maximizes the decision maker's utility becomes a set of actions such as (A_2, A_6, A_{11}) which prescribes \$22,000 to be spent on habitat improvement; set the hunting season for October 15 to November 15; and employ two wardens.

Suppose now that the cost of two wardens is \$10,000 and the average expected saving in costs for habitat improvement is \$3,000, thus the cost of the combined actions would be \$19,000 plus \$10,000, or \$29,000 to have the highest <u>likelihood</u> of meeting public deer wants, without over-harvesting.

Balancing a Budget within a Decision Theory Framework

Suppose that there is a state fish and wildlife agency with an anticipated budget of \$100,000 for next year. Suppose further that the objective of this agency is the maximization of recreation man-days, and that within this agency there are two divisions: wetland wildlife management, and deer management. Thus, the decision maker's problem is to propose an allocation of the \$100,000 between the two divisions in pursuit of the objective of the agency. Assume that he does want to maximize recreation man-days but, because of hardcore demand for both deer and wetland wildlife recreation, he does not want to eliminate either subprogram. Since the budget is \$100,000 and assuming no negative returns, the allocations to the two subprograms should sum to \$100,000 to maximize man-days.

At increments of $1.00, there are 99,999 possible combinations or possible actions.[56] To simplify the problem, the director decided that $25,000 increments will be considered, and thus the possible actions are:

Action	Budgeted expenditures for: Wetland wildlife management	Deer management
A_1	$25,000	$75,000
A_2	$50,000	$50,000
A_3	$75,000	$25,000

This "increments" method is advantageous in that wide ranges of change can be considered. However, if there were several divisions and a smaller increment were used, a great deal of information would be necessary to solve the problem. An alternative is to consider changes in expenditures with the sum of the changes being equal to the change between the budget for the current year and the budget for the following year. For example, if the current budget was $90,000 with $42,000 going to wetland wildlife management and $48,000 going to deer management, then five possible actions might be:

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[56] $100,000.00 plus $.00 and $.00 plus $100,000.00 are possible combinations but not possible actions since neither subprogram is to be eliminated.

Action	Changes in budget allocations for: Wetland wildlife management	Deer management
A_1	- 5,000	+15,000
A_2	0	+10,000
A_3	+ 5,000	+ 5,000
A_4	+10,000	0
A_5	+15,000	- 5,000

The objective then would be to maximize the increase in recreation man-days.

Consider another problem: suppose a decision maker has to choose between two completely unrelated actions, each of which would cost the same amount. For example, suppose a director has to choose between stocking a stream (A_1), and managing a game area (A_2), and, further, that he wants to maximize recreation man-days. He is concerned with the following states of nature:

States of nature	Description
S_1	Low want for fish
S_2	High want for fish
S_3	Low want for deer
S_4	High want for deer

The director anticipates the following consequences:

States of nature \ Actions	A_1	A_2
	No. of expected man-days	
S_1	1,000	
S_2	1,200	
S_3		925
S_4		1,300

However the states must be combined since, as they are, more than one state will occur at the same time--i.e., S_1 or S_2 <u>and</u> S_3 or S_4. The combinations would be: S_1 and S_3; S_1 and S_4; S_2 and S_3; and S_2 and S_4. The action matrix becomes:

States of nature \ Actions	A_1	A_2
	No. of man-days	
S_1, S_3	1,000	925
S_1, S_4	1,000	1,300
S_2, S_3	1,200	925
S_2, S_4	1,200	1,300

The subjective probabilities would be handled in a manner similar to the method explained on page 66.

Inclusion of Secondary Positive Consequences, Negative Consequences, and Indirect Production Costs within a Decision Theory Framework

Decision theory is advantageous in that it allows the decision maker to consider uncertainty as to the state of nature in making decisions. Problems arise, however, from the fact that only one input class and only one output or consequence class can be used. Thus, inputs must be expressed in common units such as utility units, as dollars, man-days, animal units, or some other unit.

A. Negative Consequences and Secondary Positive Consequences:[57]

Looking at the deer management problem, if the consequences were measured in units of utility, how could such negative consequences as deer and hunter damage losses to farms and public and commercial forests be taken into consideration? If consequences were measured in man-days, how could the consequences of protecting a bird species from extinction be compared to other activities? One solution to problems of this type would be to measure all consequences in utility units or dollars. This is often a rather difficult, if not impossible, task. What is the utility gained or value of an animal which cannot be hunted and is rarely seen? What is the monetary value of preventing the extinction of a species?

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[57]The term, secondary, is used to mean secondary to the primary goal such as maximization of man-days. Such secondary consequences of an action may be of more importance than the primary consequence.

B. Indirect Production Costs:

Indirect production costs have been defined as costs, either monetary or in resources, to another agency or to the private sector which results from the implementation of a management practice. For example, if an agricultural clearing for deer habitat improvement is opened, the timber production for that area is lost. If a maintenance method involving equipment such as tractor mowers, or plows, or planters, is considered, timber loss from such a road should be included. In the case of clearings and roads, such an indirect cost would most likely be low since the area in relation to total forest area is not likely to be significant and the land utilized is not likely to be highly productive for timber. Thus, if the loss is small and similar to the indirect costs of the alternatives, the decision as to the 'optimum' method of maintenance and number and size of the clearings is not likely to be greatly affected by ignoring such costs. If such indirect production costs are high, they should be considered. This presents a problem, since such costs do not affect the budget of fish and game agency and since the consequences of the action are not measured in dollars. The use of utility units often proves less than satisfactory, also.

C. Suggestion for Handling Indirect Production Costs, Secondary Positive Consequences, and Negative Consequences:

The consequences of k actions and j states of nature may be represented by a k by j matrix of scalar values when set up in tabular form such as:[58]

$$\begin{array}{cc} & \text{Actions} \\ \text{States of nature} & \begin{bmatrix} C_{11} & C_{12} & \cdots & C_{1k} \\ C_{21} & C_{22} & \cdots & C_{2k} \\ \vdots & \vdots & & \vdots \\ C_{j1} & C_{j2} & \cdots & C_{jk} \end{bmatrix} \end{array}$$

Each entry or element of this matrix is a scalar quantity which is the potential consequence of a particular action and state of nature. Indirect production costs, secondary positive consequences, and negative consequences could be included by considering each element to be a column matrix of order x by 1, where x notates the various consequences of a particular action and state of nature. Thus C_{11} would become:

$$\begin{bmatrix} C \\ C \\ \vdots \\ C_x \end{bmatrix}_{11} = [C]_{11}$$

For example, if the potential consequences of a particular action and state of nature were 30,000 deer ('primary' consequence), 30

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[58]For more detail, refer to page 92.

acres of lost timber growth to a state forest (indirect production cost), \$15,000 in hunter-related profits to state businesses ('secondary' positive consequence), \$12,000 in damages to farms and privately owned forests[59] (negative consequence), and the equivalent of a loss of 40 acres of timber due to deer and hunter damage (negative consequence), the column matrix then would be:

$$\begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix}_{jk} = \begin{bmatrix} +30{,}000 \\ +15{,}000 \\ -12{,}000 \\ +\quad 70 \end{bmatrix}_{jk}$$

where: C_1 = number of deer

C_2 = state hunter-related business profits in dollars

C_3 = state agricultural gains in dollars

C_4 = state forest loss in acres

Using this method, the action matrix for some S_1 might be:

[59] An alternative would be to subtract the \$12,000 from \$15,000, yielding a positive \$3,000 in secondary consequences. However, such an alternative would not indicate the income redistribution effect of the activity.

States of nature \ Actions	Agricultural clearings A_1 \$0.00	A_2 \$15,000	A_3 \$20,000	A_4 \$25,000	Explanation of consequence
S_1	5,000	22,000	30,000	33,000	Number of deer
	+3,000	+13,000	+15,000	+16,000	State business economic gain
	-1,000	- 7,000	-12,000	-20,000	Agricultural economic gain
	+ 3	+ 50	+ 70	+ 110	State forest acreage loss
⋮					
S_j					

Where possible extinction of a species is involved, a probability might be assigned such that a zero probability would mean extinction and 1 would indicate survival of the species.

If the decision maker wants to maximize the number of deer, he would choose the action with the maximum expected C_1.[60] In addition, he would know the expected values of C_2, C_3, and C_4 for the maximizing action. By examining the next best alternative the decision maker may find that a small drop in C_1 might result in a favorable shift in C_2, C_3, and/or C_4. By examining various alternative actions, and by using his judgment and experience, the decision maker might ultimately decide to take some action other than the one which maximizes or minimizes the primary consequence.

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[60]For the basic mathematical operations needed for the use of such column matrices in decision theory, see Appendix A.

CHAPTER V

SUMMARY AND CONCLUSION

It has been demonstrated in this thesis that decision theory can be applied to a wide range of fish and wildlife management decision problems.

Summary

Decision making under uncertainty may be summarized in the following potential situation and action sequences:

1. Facing a decision problem which contains uncertainty and which can be formulated in such a manner that decision theory can be applied, the decision maker can make a decision assuming complete uncertainty; or
2. If he has some knowledge as to the occurrence of the states of nature, i.e., partial uncertainty, and assuming that he can arrive at subjective probabilities (prior or posterior) which he is satisfied with, i.e., he is willing to make a decision based on the probabilities; then
3. He can use the probabilities to make a decision, i.e., he has improved his knowledge condition from that of uncertainty to that of subjective risk; or

4. If he is unwilling to make a decision based on the probabilities, he must make his decision based on the assumption of complete uncertainty.

Application of decision theory under uncertainty by fish and wildlife agency decision makers was illustrated by a hypothetical deer management problem. An assumed budget allocation problem was described and the various knowledge situations (certainty, risk, uncertainty) were discussed. By the use of other examples, the wide range of fish and wildlife decision problems under uncertainty were demonstrated. A brief review of decision theory followed and the necessary factors (actions, consequences, states of nature) of any decision problem under uncertainty were discussed. The various steps in formulating a decision problem within the decision theory framework were introduced and applied to the deer problem. The application of decision theory to fish and wildlife management problems was demonstrated and illustrated by application to the example problem. To begin with, complete uncertainty was assumed and several decision criteria (maximin, minimax regret, minimin, "principle of insufficient reason") were discussed. It was pointed out that no one criterion was "best."

Partial uncertainty was then assumed. It was pointed out that complete uncertainty rarely occurs since fish and wildlife managers generally possess some knowledge concerning the problem situation. The use of personalistic probabilities as an expression of this knowledge was discussed and demonstrated. Methods of obtaining such subjective probabilities (Savage's hypothetical acts method, modified hypothetical acts method, weighted-average method, feedback and reassessment, group reassessment) were introduced. It was explained that the "best" method was dependent upon several factors which were discussed. However, it was pointed out that, if additional information in the form of an experiment ("predictor") was available or could be obtained, the subjective probabilities could be modified to form posterior probabilities.

The examples that were used to demonstrate the wide range of fish and wildlife decision problems under uncertainty were formulated within the decision theory framework. A method for combining several related decision problems into a single problem was demonstrated. This was followed by a suggestion for balancing a budget within the decision theory framework.

Conclusion

> "It is difficult for us (wildlife managers) to sit back and go through the processes of analysis and planning when so much needs to be done. It is easier to get at the job and leave the detailed planning until later. That is where expediency overcomes our better judgment."[61]

Fish and wildlife managers face a multitude of decision problems. Some are sufficiently unimportant that they can be made without detailed analysis. However, these managers are also faced with many decision problems that are of sufficient magnitude to warrant the careful

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[61]Longwood, F. R., "The Selection, Design, and Analysis of Wildlife Habitat Studies," paper read at the Northeast Fish and Wildlife Conference, 1962.

weighing of alternatives and consequences. Until recently, the analytical methods for decision making available to these managers have been based on traditional economic theory. "Such theory is built primarily around deterministic models based on assumption of complete knowledge and certainty." Unfortunately, these "decision makers usually face incomplete knowledge and a large element of uncertainty."[62]

Thus, fish and wildlife management would seem to be an especially relevant area for the application of decision theory. Not only does decision theory provide for the systematic consideration of uncertainty, but it also enables the decision maker to bring his experience and expertise into the decision process.

Decision theory is by no means an Utopian process which will guarantee a "perfect" decision. It can provide a method for improved decision making for many decision problems which the fish and wildlife manager may face. Nor is it expected that this study will become a "handbook for decision making" for such managers. Rather it is hoped that it will provide the impetus for further study and consideration of the application of decision theory to various fish and wildlife management decision problems.

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[62]Lundgren, Allen L., "Introductory Remarks," presented at the 15th World Congress, International Union of Forestry Research Organizations, Forest Economics Section, Gainesville, Florida, March 19, 1971, p. 2.

"So far we have been managing game by the 'seat-of-our-pants,' and fortunately getting away with it. We must develop game management from the art that it has been to a precise science."[63] [64]

[63] Verme, L. J., *op. cit.*, p. 34.

[64] Decision making under uncertainty is more precise than decision making under certainty, *if* certainty was *assumed* when, in fact, the knowledge condition was one of uncertainty.

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APPENDIX A

BASIC MATHEMATICAL OPERATIONS NECESSARY FOR THE USE OF COLUMN MATRICES WITHIN AN ACTION MATRIX

As long as the consequences in a column matrix are scalar quantaties (numbers), then there is no problem with the mathematics of decision theory. A column matrix can be multiplied by a scalar by multiplying each element in the matrix by the scalar[65], as follows:

$$\begin{bmatrix} 30{,}000 \\ 15{,}000 \\ -12{,}000 \\ +\quad 70 \end{bmatrix}_{21} \quad \text{multiplied by .2 becomes} \quad \begin{bmatrix} +6{,}000 \\ +3{,}000 \\ -2{,}400 \\ +\quad 14 \end{bmatrix}$$

[65]Campbell, H. G., An Introduction to Matrices, Vectors, and Linear Programming, Appleton-Century-Crofts, Division of Meredith Corporation: New York, 1965, p. 8.

Column matrices can be summed if they are conformable.[66] That is:

$$\begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix}_{11} + \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix}_{21} = \begin{bmatrix} C_{111} \\ C_{211} \\ C_{311} \end{bmatrix} \begin{matrix} + \\ + \\ + \end{matrix} \begin{bmatrix} C_{121} \\ C_{221} \\ C_{321} \end{bmatrix}$$

while:

$$\begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix}_{11} + \begin{bmatrix} C_1 \\ C_4 \\ C_5 \end{bmatrix}_{21}$$

cannot be summed. However, in this case where C_4 and C_5 do not exist in $[C_{11}]$, and C_2 and C_3 do not exist in $[C_{21}]$, if $[C]_{11}$ and $[C]_{21}$ are assumed to be:

$$\begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{bmatrix}_{11} \quad \text{and} \quad \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \end{bmatrix}_{21}$$

where C_4 and C_5 are equal to zero in $[C]_{11}$, and where C_2 and C_3 are equal to zero in $[C]_{21}$, then they can be summed.

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[66] Campbell, H. G., *op. cit.*, p. 7.

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Thesis

DECISIONS UNDER UNCERTAINTY BY
STATE FISH AND GAME AGENCIES

by

Edward Franklin Morgan

Abstract

Decision makers within state fish and wildlife agencies face a multitude of decision problems. These decision makers are faced with many decision problems that are of sufficient magnitude to warrant the careful weighing of alternatives and consequences. Until recently, the few analytical methods for decision making available to these decision makers have been based on traditional economic theory with assumptions of complete knowledge and certainty.

This study investigates several hypothetical problems which such decision makers may encounter and it is concluded that these decision makers often face incomplete knowledge and uncertainty. Based on that conclusion, the applicability of decision theory to fish and wildlife management problems is explored. Emphasis is placed on how the decision maker can improve his knowledge condition, the various options open to a decision maker if he must make a decision based on the available knowledge, and the mechanics and reasoning behind such options.

The study concludes that fish and wildlife management problems are a relevant area for the application of decision theory. Not only does decision theory provide for the systematic consideration of uncertainty,

but it also enables the decision maker to bring his experience and expertise into the decision process. While decision theory is by no means an Utopian process which will guarantee a "perfect" decision, it can provide a method for improved decision making for many decision problems which the fish and wildlife manager may face.