REFUTATION TEXT OFFERS MIXED EFFECTS ON NIGERIAN PRE-SERVICE TEACHERS' USE OF THE REPRESENTATIVENESS HEURISTIC

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This study enlisted a refutation text to foster conceptual change among 71 preservice teachers of mathematics education in situations that might elicit the use of the representativeness heuristic. Statistically significant differences were found between respondents' pre- and post-tests, with more than half exhibiting normative reasoning after prior use of the heuristic. Analysis of their written explanations revealed differential patterns in their reasoning across two effect categories. Implications for misconception research and practice are discussed.

Keywords: Conceptual Change, Probability, Refutation Text, Representativeness.

Probabilistic misconceptions have been documented to differ in the extent to which they are amenable to change (Jones et al., 1997), with the representativeness heuristic being a prominent one. As the name suggests, it involves quantifying the occurrence of a random phenomenon "by the degree to which it: (i) is similar in essential characteristics to its parent population; and (ii) reflects the salient features of the process by which it is generated." (Kahneman & Tversky, 1972, p. 430). A variety of studies have been conducted to ascertain the prevalence of the heuristic among different populations, especially preservice teachers (PST). In Wilkins (2007), 11 out of 15 PSTs relied on it. The rate was 17.5% in Kustos and Zelkowski (2013).

Several efforts have been made for the jettisoning of misconceptions for normative ideas. They have mostly included programs involving predictions and data analyses through engaging in experiments and simulations (Fischbein & Gazit, 1984; Jones et al., 1997). Although these interventions have returned modest gains, the fruitfulness of conceptual change (CC) strategies in overcoming mathematical misconceptions (Lem et al., 2017) may offer a commensurate path to forging acceptable conceptions of probability. Consequently, this study adopts one CC approach—refutation text—as a probable mechanism to overcome the misconception involving the use of the representativeness heuristic among PSTs.

Theoretical Framework

Refutation texts: offering a switch from S1 to S2

Misconceptions are known to be resistant to change because they portray beliefs about chance phenomena. And since beliefs are personal, subjective truths, the feeling of agency conditions the holder to stick to them irrespective of their source. With an alternative conception offering valid explanations in certain contexts (Savard, 2014), the automaticity and effortlessness (see properties of S1, Kahneman, 2003) characterizing alternative conceptions purports to be valid across contexts. Consequently, getting the holder toward CC will involve a radical shift from existing to new conceptions, abled due to at least three reasons: dissatisfaction, intelligibility, and plausibility (Posner et al., 1982).

A refutation text (RT) offers an avenue for enabling this switch. By exposing the misconception, a cognitive conflict is created that might cause a person to attend to salient aspects of the problem they would otherwise not have. Based on the strengths of the belief, this encounter with such explicit information prepares the ground for alternative explanations that might help resolve the conflict, setting the stage for an activation of a cognitive ecology that decontextualizes the problem, a feature of S2 thinking (Kahneman, 2003).

Commonly used RTs are characterized by three properties: (1) description of a misconception, (2) refutation of the misconception, and (3) subsequent provision of a normative conception (Chi, 2008). Refuting a misconception comes with reasons why such beliefs are not viable, generalizable, or consistent, to—as Leron and Hazzan argue—bring "S2 to intervene in its role as critic of S1" (2006, p. 109). Consequently, it may be valuable to enlist a RT to answer the following questions: (1) What are the effects of a RT on PSTs' use of the representativeness heuristic? (2) What qualitative affordances exist within and across the pattern of effects?

Methods

The study involved 71 PSTs of mathematics education at a university in south-western Nigeria. Following informed consent, they were issued a pre-test containing a question to elicit their use of the representativeness heuristic which was immediately followed with an issuance of the RT (without being told about a post-test which was administered after 2 hours).

Table 1: Pre-test and Post-test on the Representativeness Heuristic

Pre-test	Post-test
An experiment requires you to flip a penny 100 times and record whether the penny comes up heads or tails. On the first 10 flips the penny comes up heads. After flipping the penny 90 more times, how many heads would you expect to get out of the total 100 flips? Answer………..… (Please explain your answer below).	Suppose you toss a fair coin six times, recording the result of each toss. For instance, if you toss a head and then five tails in a row, you would write H T T T T T. Which is the *least likely* result? (a) H T H T H T (b) H H T H T T (c) H H H T T T (d) T T T H T T (e) All are equally likely. (Please explain your answer below).

Following the plurality of indicators of the representativeness heuristic (see Tversky & Kahneman, 1974), I adopted the hermeneutical position of content analysis (Mayring, 2015) to develop a RT—that largely attended to the misconception of chance indicator of the heuristic typified by less runs and more switches—with a goal of influencing S1 thinkers to interrogate their reasoning to fit into S2. The RT was content-validated by a professor of mathematics education who teaches a graduate-level probability methods' course. The content of the RT is as shown in Table 2.

Table 2: Refutation Text on the Representativeness Heuristic

Refutation Text

In a game of Ludo[3] between Ghufran and Imran, Ghufran threw three double sixes and a 3,4 (i.e., 6-6, 6-6, 6-6, 3-4). When it was Imran's turn, he threw a 4-5. After counting his scores, Ghufran again threw almost the exact same sequence (i.e., 6-6, 6-6, 6-6, 1-5). In protest, Imran said the game was rigged, threatening to opt out of it. While it may be difficult to convince him otherwise, the scenario does not in any way suggest cheating but a valid random process. Specifically, underlying such thinking is a misconception called *representativeness*. Because the throw of the pair of dice represents a random process, individuals often expect the outcomes to appear in a strictly random manner however short or long the sequence. Had Ghufran thrown a sequence like 6-6, 5-4, 3,2, 1-3, 4-4, 1-6, 1-1, 4-2 rather than 6-6, 6-6, 6-6, 3-4 and 6-6, 6-6, 6-6, 1-5 on his first eight throws, Imran might not have alleged rigging. But both sets of outcomes represent valid random processes. As such, they have the same chance of occurring.

To analyze the data, I coded the keys 1—for both pre- (*55*) and posttests (option *e*)—and other options 0, making the data suitable for analysis using the McNemar (Binomial) Test. To answer RQ1, the test was used to determine if an intervention effect existed as measured by changes either in the positive or negative direction. To answer RQ2, two respondents' written explanations were described across positive and negative effects.

Results and Discussion

Quantitative

The RT had a major influence on 48 PSTs (see Table 3). Specifically, 40 respondents switched from the heuristic use to normative conception. In contrast, a fifth of those that were positively influenced were impacted negatively by the intervention. The RT also had no effect on 18 respondents. 5 respondents were consistent in their exhibition of the normative conception at the pre and posttests (reinforcing effect). Collapsing the rows (pretest) and columns (posttest) shows the prevalence of this misconception before (81.7%) and after (36.6%) the intervention. Similarly, 18.3% and 63.4% were without the misconception before and after. Overall, I documented a statistically significant change in PSTs' responses (N=71, p<.001) associated with pre and post-test, suggesting that the RT had an effect in getting PSTs to undergo CC.

Table 3: Prevalence and Changes in the use of the Representativeness Heuristic Before and After Intervention

	All		
	0	1	T
0	18	40	58
1	8	5	13
T	26	45	71

[3] Ludo is a boardgame played by 2 or 4 people. A turn is initiated when an opponent fails to throw a double six on a pair of dice.

Qualitative Differences

In this section I provide excerpts from two respondents who made switches between S1 and S2 thinking as inferred from results of the quantitative analysis.

Table 4: Excerpts Across Positive and Negative Effects

Effects	Excerpts	
	Pre	Post
Positive	Since each flip of a fair coin is independent, the probability of getting heads remains 0.5. Therefore, you would expect to get heads approximately 50 times out of 100 flips, regardless of the outcomes of the first 10 flips.	They are equally likely to have the same result because the probability of getting head or tail in each outcome is equal.
Negative	The chances of getting heads and tails are equal because you cannot determine how many heads will come up after the total experiment but since we have 10 flips already and the chances of the remaining 90 times flipping is equal, you can say $10+½(90) = 55$.	The least likely result of the tossing of a fair coin six times is getting three heads and three tails in a row as a possible outcome of an event.

The explanations from the respondent (Opebe) who made a positive switch show an understanding of randomness at the pre and post-tests. Therefore, evidence for the influence of the RT in causing Opebe to engage in the slow, systematic thinking of S2 at the posttest lies in his jettisoning of the need to balance out the outcome, causing him to conceive of all the options as equally likely. In contrast, a scrutiny of Moladun's excerpt presents unclear results even as I classified its effect negative. While she avoided the heuristic at the pretest by applying the base-rate frequency, she initially claimed that the probability was indeterminate. It is possible her answer at the post test was more of an outcome approach (see Konold, 1989) than a desire to map the sample to population. For this reason, it is not clear if the RT really had a negative effect on her because a similar reasoning that masks misconception of chance (see Tversky & Kahneman, 1974) undergirds her explanations both before and after she engaged with the RT.

Conclusions

This study set out to ascertain the effects of a RT on PSTs use of the representativeness heuristic. Based on quantitative analysis, the intervention promoted CC although posing differential effects on the respondents. Qualitative analysis showed that the intervention was mostly effective when the RT matched with the specific representativeness indicator. Owing to the demonstrated potential of the intervention, it may be valuable to enlist CC strategies into the design of instructional programs for fostering normative probabilistic ideas. Therefore, textbook authors may find the structure of RTs useful as a complement, or alternative to the existing writing style that largely involves mere presentation of normative ideas. Overall, this study offers preliminary evidence for an approach other than experiments and simulations in building probabilistic conceptions, one that may especially be worthwhile in countries where students have little access to materials to conduct such experiments.

Limitation and Future Research

Restricting the RT to a single indicator of representativeness (misconception of chance) is a major limitation of the study. For this reason, the RT better attends to the post test than the pretest whose normative reasoning owes to attending to the base-rate frequency. Future research might incorporate more indicators that would lend the RT to a broad array of chance situations.

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