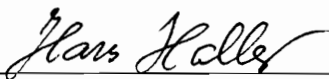


OBJECTIVES AND INCENTIVES IN FINANCIAL MARKETS

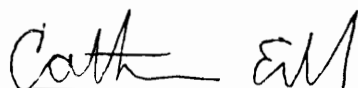
by
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Dissertation submitted to the Faculty of the
Virginia Polytechnic Institute and State University
in partial fulfillment of the requirements for the degree of
DOCTOR OF PHILOSOPHY
in
Economics

APPROVED:



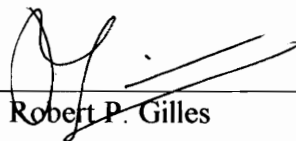
Hans Haller, Chairman



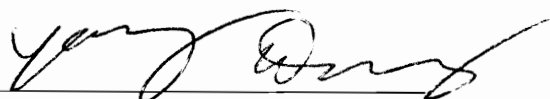
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OBJECTIVES AND INCENTIVES IN FINANCIAL MARKETS

by

Chung-Shu Liu

Committee Chairman: Hans Haller

Economics

(ABSTRACT)

This dissertation is a collection of papers investigating objectives and incentives in financial markets.

The first essay (Chapter 2) deals with the endogenous determination of credit history, credit-worthiness, loans and efforts by borrowers over time. A financial market with adverse selection and moral hazard is analyzed. Facing the adverse selection, lenders are not able to offer separate contracts to different types of borrowers. However, knowing borrowers' credit histories, lenders are able to assign different credit worthiness to borrowers that have different credit histories, and offer different contracts to different groups. It is shown that if borrowers' credit rating is too low, they make low effort to repay their debts. As a borrower acquires a good credit history and has his credit-rating upgraded above a certain point, it becomes worthwhile for him to choose high effort. A low quality borrower may make high effort in early periods in order to build up a good credit history and obtain better terms in the future contracts then shift back to the low effort even though his project continues to succeed when he approaches the end of his life.

The second essay (Chapter 3) analyzes the effect of exogenous noise on shareholders' unanimous choice in the capital market where investors obtain asymmetric information about future returns. The exogenous noise, which comes with the random exogenous supply of the risky asset, is allowed to grow proportionally or disproportionately with the replica of the economy. We show that initial shareholders of a firm tend to approve the firm's maximization of its net market value asymptotically, when the number of replicas of the economy increases. We have proved that if the exogenous noise grows proportionally with the size of the economy, the asymptotic unanimity property holds. With specific parameters in our model, the unanimity property still holds as the exogenous noise grows disproportionately with the economy.

The third essay (Chapter 4) aims at identifying conditions for inefficient investment in both the capital market and the credit market. In the capital market, using the mean-variance model for capital asset pricing to determine the level of investment, one can obtain an under-investment result. Firms putting too much weight on their own variance of the return leads to inefficiency. If the variance of each firm's return is relatively small compared to its covariance with the market or the market becomes very competitive, then the level of investment approaches the optimal level. However, if each firm's decision is independent of the other firms' decisions, then the optimal investment level is never approximated. In the loan market, due to the dead weight loss of bankruptcy, the optimal level of investment can not be attained if renegotiation is impossible. The socially optimal level of investment can always be attained if renegotiation is allowed.

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Chapter 1. Introduction

This paper deals with the information problem and the efficiency problem in financial markets. Financial markets function as intermediates between those who need funds and those who have excess funds. If any participant has better information than the others he may benefit from his superior information. Basically there are two types of financial markets: the credit market and the equity market. The information and efficiency problems associated with different types of markets may have different characteristics.

In the credit market, lenders are owners of funds and borrowers are users of funds. Thus there exists a principal-agent relationship between lenders and borrowers. Borrowers should act on behalf of lenders' interests. However, lenders and borrowers may conflict in their interests. Like most agency models, the separation of owners and users of funds may cause two problems: first, the characteristic of a borrower may not be known to the lender at the time of contracting. All such models with pre-contractual information are categorized as adverse selection. Secondly, borrowers may take some action which will affect the borrowers' interest but is unobservable by lenders. Models with hidden action are categorized as moral hazard.

Adverse selection occurs in the credit market when lenders and borrowers have asymmetric information about the quality of projects at the time of contracting. The information about the borrowers' managerial ability or the distribution of returns of projects is known to the borrowers but not to the lenders. Including an adverse selection effect in their model, Stiglitz and Weiss (1981) demonstrated that since lenders cannot distinguish between good and bad borrowers, there is a pooling equilibrium with credit rationed. An increase in interest rate has two effects: it increases the lenders' payoff

directly; however it also causes the borrowers with low-risk projects to drop out and reduces lenders expected payoff indirectly. In the beginning, increasing the interest rate will increase lenders' payoff. However, if the interest rate rises beyond a certain point, lenders' expected payoffs begin to drop. Thus, banks may reject some loans in equilibrium even though those borrowers are willing to pay a higher interest rate. Credit rationing occurs due to the inability for banks to acquire information about borrowers' types. If a good borrower is able to convey information about his quality to lenders, he may obtain a better contract. Ross (1977) has shown that if the borrowers are firms, and they are allowed to raise funds from both the credit market and the capital market, then firms can use the financial structure to signal the quality of their projects. In equilibrium separate contracts are offered to different borrowers and there is no credit rationing.

Adverse selection is not the only problem occurring in the credit market; lenders may also face the moral hazard problem. Since borrowers do not take the lenders' interests fully into account in making effort to repay their loans after loans have been granted to them. Credit rationing also occurs in the loan market with moral hazard. An increase in the interest rate induces borrowers to switch from low-risk projects to high-risk projects. Thus profits of banks fall with the increase in interest rates. Stiglitz and Weiss (1981) do not discuss any incentive scheme that can be used to overcome the moral hazard effect. However, in another paper, Stiglitz and Weiss (1983) modeled incentive effects on borrowers' behavior in an intertemporal setup. Termination of the bank-borrower relationship can be used to penalize borrowers whenever they default and elicit the desired action by the borrowers.

Adverse selection and moral hazard may occur simultaneously in credit markets. Diamond (1989) analyzes the joint influence of adverse selection and moral hazard in the credit market. Once a borrower defaults, the information about his type is revealed to

lenders. In addition, reputation can be used as an incentive scheme to eliminate the conflict of interest about the choice of risk in investment between borrowers and lenders.

In chapter two, a financial market with adverse selection and moral hazard is analyzed. Lenders do not know borrowers' quality at the time of initial contracting and are not able to control the actions made by borrowers after contracts are signed. Borrowers' credit histories play an important role at the time of contracting. Although the borrowers' true quality will never be revealed, lenders can extract some information about the borrowers' ability from their credit histories. Knowing borrowers' credit histories, lenders update their belief about borrowers' type, and assign different credit worthiness to borrowers that have different credit histories. Thus different contracts are offered to different groups accordingly.

It is shown that if the fraction of low quality borrowers is very large, all borrowers start financing their project with low credit rating. The repayment charged by banks may be so high that both types of borrowers make low effort to repay their debts. As a borrower's projects succeed over several periods, he may acquire a good credit history and may have his credit-rating upgraded. When a borrower's credit-worthiness is above a certain point, it becomes worthwhile for him to choose high effort. If the population proportion of the low quality borrowers is lower than a certain level, because he considers future borrowing, a low quality borrower may make high effort in early periods in order to build up a good credit history and obtain better terms in the future contracts. However, the low quality borrowers may shift back to the low effort even though his project continues to succeed because with a finite life time, his future rent may become too small when he approaches the end of his life. Only if a borrower's credit-worthiness exceeds a certain value, the borrower provides high effort until the end period.

In the capital market, shareholders are the owners of funds. A manager is delegated to carry out shareholders preferred options. However, since there are many shareholders, interests among them may be different; therefore, they may have different opinions about how the firm should be operated. Thus, two questions can be raised: First, can the initial shareholders reach an unanimous agreement upon the objectives of a firm? Second, if an unanimous agreement can be reached, what does it stipulate?

Unanimous agreement among investors may not be reached in an economy with incomplete markets. If a proposal would alter the state-distributions of returns, the unanimity would not be obtained. However, if a proposal would not alter the set of state-distributions of returns available in the whole economy, or if the shareholders value only the mean and variance of their portfolios, then the shareholders would be unanimous in their preference as asserted by Ekern and Wilson (1974).

In economic analysis, the assumption that firms are to maximize their net market value is usually made. This assumption implies that pursuing net market value maximization, the firm will act on behalf of its shareholders. It is true only under certain circumstances. If an unanimous agreement cannot be reached among shareholders, some shareholders must not prefer their firm to maximize its net market value. Even if there is a unanimity among the shareholders about the objective of the firm, the objective needs not to be net market value maximization. As Jensen and Long (1972) and Stiglitz (1972) demonstrated, the level of investment chosen to maximize the net market value of the firm need not coincide with the unanimous preference of its shareholders.

Bester (1982) has shown that the initial shareholders want to maximize the net market value of the firm as the economy becomes very competitive. Bester includes no information problem in his model. When shareholders have asymmetric information about

the price of the risky asset, Haller (1984) has shown that the initial shareholders want to maximize the net market value of the firm as the economy becomes very competitive.

In the third chapter, the effect of exogenous noise on shareholders' unanimous choice in the capital market is analyzed. The exogenous noise, which comes with the random exogenous supply of the risky asset, is allowed to grow proportionally or disproportionately with the replica of the economy. The rational expectations equilibrium concept is adopted to test if net market maximization is desirable as competition increases. We show that initial shareholders of a firm tend to approve the firm's maximization of its net market value asymptotically, when the number of replicas of the economy increases. It seems no matter how the exogenous noise grows, net market value maximization becomes desirable when the basic economy is replicated many times. We have proved that if the exogenous noise grows proportionally with the size of the economy, the asymptotic unanimity property holds. With specific parameters in our model, the unanimity property still holds as the exogenous noise grows disproportionately with the economy.

In the fourth chapter, optimality in both the capital market and the credit market is discussed. In the capital market, we survey the two most important papers written by Jensen and Long (1972) and Stiglitz (1972) and try to identify the conditions for inefficient investment. Using the mean-variance model for capital asset pricing to determine the level on investment, one can obtain an under-investment result. Although the old project affects the decision of new investment, it is not the key factor that causes under-investment. Firms putting too much weight on their own variance of the return leads to inefficiency. If the variance of each firm's return is relatively small compared to the covariance of the market, then the level of investment approaches the optimal level. In addition, if we let the market become more competitive by increasing the number of firms, the aggregate investment made by firms will approach the socially optimal level. However,

if each firm's decision is independent of the other firms' decisions, then the optimal investment level is never approximated, even though the number of firms goes to infinitive.

In the credit market, due to the dead weight loss of bankruptcy, the optimal level of investment can not be attained if renegotiation is impossible. If borrowers and creditors can renegotiate the repayment in case firms refuse to repay, then creditors will never really take over projects and the dead weight loss can always be prevented. The socially optimal level of investment can always be attained if renegotiation is allowed.

REFERENCES

- Bester, H., 1982, On Shareholder Unanimity in the Mean-Variance Model, *Economics Letters*, 10, 363-367.
- Bester, H, and M. Hellwig, 1987, Moral Hazard and Equilibrium Credit Rationing, in *Agency Theory, Information and Incentives*, Bamberg, K. and K. Spremann, eds. Berlin: Springer-Verlag.
- Diamond, D. W., 1989, Reputation Acquisition in Debt Markets, *Journal of Political Economy*, 97, 828-862.
- Ekern, S. and R. Wilson, 1974, On the Theory of the Firm in an Economy with Incomplete Markets, *The Bell Journal of Economics and Management Science*, 5, 171-184.
- Haller, H., 1984, Asymptotic Net Market Value Maximization, Discussion Paper No 138, Sonderforschungsbereich 138, University of Bonn.
- Jensen, M. C. and J. B. Long, Jr., 1972, Corporate Investment under Uncertainty and Pareto Optimality in the Capital Markets, *The Bell Journal of Economics and Management Science* 3, 154-174.
- Merton, R, and M. Subrahmanyam, 1974, The Optimality of a Competitive Stock Market, *The Bell Journal of Economics and Management Science* 5, 145-170.
- Ross S., 1977, The Determination of Financial Structure: The Incentive Signaling Approach, *The Bell Journal of Economics and Management Science*, 8, 23-40.
- Stiglitz, J., 1972, On the Optimality of the Stock Market Allocation of Investment, *The Quarterly Journal of Economics* 86, 25-60.
- Stiglitz, J., 1981, The Allocation Role of the Stock Market: Pareto Optimality and Competition, *The Journal of Finance* 36, 235-251.

Stiglitz, J. & A. Weiss, 1981, Credit Rationing in Markets with Imperfect Information, *American Economic Review*, 71, 393-410.

Stiglitz, J. & A. Weiss, 1983, Incentive Effects of Termination: Applications to the Credit and Labor Market, *American Economic Review*, 73, 912-927.

Chapter 2. Credit History and Credit Worthiness

1. INTRODUCTION

This paper deals with asymmetric information in the credit markets. In the credit market, lenders are owners of funds and borrowers are users of funds. As in most agency models, the separation of owners and users of funds may cause two problems: first, the characteristic of a borrower may not be known to the lender at the time of contracting. All such models with pre-contractual information are categorized as "adverse selection." Secondly, borrowers may take some action that will affect the borrowers' interest but is unobservable by lenders. Models with hidden action are categorized as "moral hazard."

Adverse selection occurs in the credit market when lenders and borrowers have asymmetric information about the quality of projects at the time of contracting. There are many potential borrowers seeking loans. These potential borrowers are not homogeneous. Different types of borrowers differ in their managerial ability, and/or they own projects with different distributions of returns. The information about the borrowers' managerial ability or the distribution of their returns of projects is known to the borrowers but not to the lenders. After signing debt contracts borrowers choose the level of effort that affects the probability of success of their projects but cannot be monitored by banks. Thus, banks also face moral hazard. In a multi-period setting, an optimal debt contract must take into account both problems in each period.

Unless lenders are able to distinguish good borrowers from bad borrowers, different contracts cannot be offered to different types of borrowers. Usually bad borrowers benefit from the pooling of good borrowers and bad borrowers. Thus, bad borrowers will want to mimic good borrowers and good borrowers will like to convey

information about their type to get better contracts. Like firms in Ross's (1977) paper, they can use the financial structure to signal the quality of their projects, or as in Bester (1985), collateral can be used as a signaling device, then in equilibrium separate contracts can be offered to different borrowers.

If there is no signaling mechanism available for firms to convey information to banks, banks may initiate some actions to access information about a potential borrower's quality. If borrowers are firms, banks can review the potential borrowers' financial condition, repayment histories, profitability, or probability of success of their investments. If borrowers are individuals, banks may want to acquire information about their employment and their credit histories. The phenomenon in credit market for individuals is particularly interesting. Organizations are actually trading on information. In local area, there exists a kind of organization called credit bureau, which collects borrowers' repayment information of their loans and credit cards. Banks can acquire an individual's credit history from these organizations when they receive an loan application from a borrower.

In the following, we are particularly interested in the role of borrowers' credit histories in determining their credit-worthiness and the debt contracts in equilibrium. With the type of borrower unknown to the bank, the amount of repayment is determined according to a borrower's credit-worthiness when a lender negotiates with the borrower on a debt contract. Credit-worthiness is the probability of success that is assigned by banks to a borrower in a particular group with the same credit history. Banks assign the credit-worthiness to a potential borrower according to the information revealed by the borrower's credit history and the belief of success of each type when a certain action is taken.

A borrower's credit history is a sequence of records that show the borrower's ability to repay his debt. A borrower will apply for a new loan for another project at the beginning of each period. If the borrower cannot return his repayment, he is in default. The inability to repay his debt will be recorded in his credit history that can be observed by the public at no cost.

Borrowers' credit histories are important for future borrowing. Since banks do not know the type of borrower, they can infer information about borrowers' type from their credit histories only. At the beginning of the first period, borrowers have no credit histories, they all belong to the same group and banks assign the same credit-worthiness to each of them. After the first period, borrowers may have different credit histories. Borrowers are partitioned into various groups, depending on credit histories. Lenders update their beliefs about borrowers' type according to Bays' rule. Different credit-worthiness is assigned to different groups of borrowers with the same credit history.

When applying for a new loan, a borrower with a good credit history may have his credit-worthiness upgraded and may get a loan more easily and at more favorable terms. On the contrary, a borrower with a bad credit history may have his credit-worthiness downgraded and may have some difficulty in getting a loan or may obtain a loan with less favorable terms.

There are several papers related to credit-worthiness and credit history. Broeker (1990) models an adverse selection situation where banks use independent tests to assess borrowers' credit-worthiness. In his model, information cannot be exchanged by banks. Since Broeker adopts a one period model, a potential borrower has no credit history. A loan is granted according only to each bank's independent test. Thus, borrowers have no incentive to build up good credit history.

Bizer and DeMarzo (1992) model a situation where banks face the moral hazard problem: after loan contracts are signed borrowers choose their effort levels which affect their probabilities of success. Borrowers are allowed to borrow from more than one bank. However, all loans are made in the first period, and the project is undertaken in the second period. Thus, no bad record will be revealed in a borrower's credit history. A borrower's credit history is a sequence of accept-reject records and the amount of loan and debt agreed to by the borrower and each bank. The borrower's ability to repay a loan is not revealed by his credit history. Since borrowers are allowed to borrow sequentially from more than one bank, the probability of repaying prior loans decreases. This results in more borrowing and a higher interest rate than if borrowers can only obtain one loan.

Diamond (1989) models the dynamic relationship in a different way. He allows borrowers to obtain only one loan to finance their production in each period. Borrowers' quality is not known to lenders. However, if a bad record appears in a borrower's credit history, his quality is revealed to lenders, and no one will lend to the borrower again. In each period, borrowers choose to undertake risky projects or projects after loans are made. Although risky projects are preferable to a certain type of borrowers, the threat of cutting off credit in the future may have them choose the safe projects instead. Over time, if a good reputation is acquired by a borrower, he will have a strong incentive to choose a safe project.

In Diamond's (1989) paper, if a borrower fails to repay, lenders will know that this is not a good borrower. With the different set-up in my model, borrowers' quality is never revealed to lenders, thus borrowers' credit will never be cut off by lenders. The following outcomes are possible in equilibrium. Lenders are able to assign different credit worthiness to borrowers that have different credit histories, and offer different contracts to different groups. If the fraction of low-quality borrowers is very large, all borrowers begin financing

their projects with low credit rating. The repayment charged by banks may be so high that both types of borrowers make low effort to repay their debts. If a borrower's projects succeed over several periods, he may acquire a good credit history and may have his credit-rating upgraded. When a borrower's credit-worthiness is above a certain point, it becomes worthwhile for him to choose high effort. However, borrowers may shift back to the low effort even though his project continues to succeed because with a finite life time, his future rent from a good credit history may become too small when he approaches the end of his life. Only if a borrower's credit-worthiness exceeds a certain value will the borrower provide high effort until the final period.

The present paper is organized as follows. Section 2 presents the basic setting of the model. Section 3 derives the equilibrium of the one-period model. Section 4 presents the equilibrium of the multi-period model. Concluding remarks are offered in section 5.

2. BASIC SETTING

We will consider a credit market where borrowers own projects but do not have any wealth to invest. They can get loans only through banks. On the other hand, banks can get an infinite amount of funds from depositors at a fixed interest rate equal to 0, but do not have their own projects. We assume that there is a complete financial market, where banks compete in making loans. Banks are risk neutral, thus banks always make zero expected profit in equilibrium.

Borrowers live for T periods. All projects last for one period, and each project requires the same initial investment of size $k = 1$. At the beginning of each period, borrowers seek funds to make investments. If a project succeeds, it yields high return Y_s ; if it fails, it yields low return Y_f , which is assumed to be 0. We assume $Y_s \geq k \geq Y_f$. At the end of each period, the project's return is realized and is distributed between the borrowers

and the lenders. The borrowers must repay all their debts. If the borrowers cannot repay their debts, they are in default and the lenders seize projects.

There are two types of borrowers. Let Q denote the type of borrower, $Q \in \{A, B\}$. A borrower's type is known to him but not to banks. However, the proportion of type A borrowers, λ_0 , is common knowledge. Types differ in their ability to yield high and low returns, given the same level of effort.

In addition to types, the probability of success is also affected by the level of effort exerted by the borrowers. We assume that borrowers can only choose either high level of effort h or low level of effort l . By choosing effort level e , $e \in \{h, l\}$, a type Q borrower will yield high return Y_s with probability p_e^Q and low return Y_f with probability $1 - p_e^Q$. We assume $p_h^Q \geq p_l^Q$ -- the probability of success is higher when high effort is chosen by a type Q borrower. In addition, the probability of success of type A is higher than that of type B , if both types choose the same level of effort e , i.e., $p_e^A > p_e^B$.

Borrowers must determine how much to borrow and how much to consume in each period. At the beginning of each period, a borrower and a lender negotiate about the debt contract. If the borrower cannot get the loan, he can apply to another bank until he gets a loan or is rejected by all banks. If the borrower gets the loan, he will invest in the project and choose the level of effort.

A debt contract is denoted by the pair (k, D_t) , where k is the money borrowed by the borrower and D_t is the borrower's repayment obligation in period t . Since $k = 1$, a debt contract can be identified by its repayment. Since $Y_s \geq k \geq Y_f$, an acceptable D_t must also satisfy the constraint: $Y_s \geq D_t \geq Y_f$. We assume that all debt contracts are enforceable, e.g., borrowers will never default if they yield a return higher than their repayment obligation. At the end of each period, if a project succeeds, the bank receives D_t and the borrower gets $Y_s - D_t$. If it fails, the bank receives Y_f and the borrower gets 0.

The borrower's utility v_t is a function of monetary payment and effort as follows:

$$v_t(w_t, e_t) = w_t - \phi(e_t), \quad (1)$$

where w_t is the borrower's monetary payment, e_t is the effort chosen by the borrower, and $\phi(e_t)$ is the disutility of the borrower when the borrower chooses effort e in period t . A borrower's utility increases with his monetary payment and decreases with the level of effort chosen. As stated before, a borrower can choose only between high effort and low effort. For simplicity, we let $\phi(l) = \phi_l = 0$, and $\phi(h) = \phi_h$. Thus, after signing a contract (k, D) , the borrower's expected utility is $p_h(Y_s - D) - \phi_h$ for high effort h and $p_l(Y_s - D)$ for low effort l . We assume that lenders are risk neutral, therefore, the lender's expected payoff is $p_h D + (1 - p_h)Y_f$ and $p_l D + (1 - p_l)Y_f$, when the borrower chooses high effort and low effort, respectively.

An important question is how to determine the repayment when a bank negotiates with a borrower. Since banks are risk neutral, they will approve a loan only if their expected gross returns are greater than or equal to their cost, k ; the interest paid to its depositors is assumed to be 0 in this paper. Not knowing borrowers' types, banks are facing an adverse selection problem when signing the contract. In addition, since the borrowers choose the level of effort after contracts are signed, the banks also face a moral hazard problem. In each period, when signing a debt contract, a bank must take into account both problems.

The proportion of type A borrowers is λ_0 , $0 \leq \lambda_0 \leq 1$. Although banks do not know borrowers' types, they know the proportion of each type and the probability of yielding high return for each type when a certain action is taken. Thus, the lenders can calculate the probability of repayment for a particular borrower -- which is called the credit-worthiness of the borrower.

In the multi-period setting, at the end of each period, if a borrower cannot repay his debt, then a bad record will result. A borrower's ability to repay his or her debt is partly revealed by the borrower's previous record of repayment, which is called a credit history and can be observed by the public at no cost. The credit history affects a borrower's current and future credit-worthiness and future borrowing. A borrower with a good credit history may have his or her credit-rating upgraded and may secure a loan more easily and with more favorable terms. On the contrary, a borrower with a bad credit history may have his or her credit-rating downgraded and may have some difficulty securing a loan or receive a loan with less favorable terms. In the next sub-section, we will discuss the effect of credit history on a borrower's credit-worthiness.

Credit History and Credit-Worthiness

Credit history is a sequence of records that shows what debt contracts are signed by a borrower and whether the borrower repays his debts. At the end of each period, if the borrower repays his debt, the success S will be recorded in his credit history. If he fails to repay his debt, the record of failure F will appear. Let H_t denote the borrower's credit history at date t . At date 0, borrowers have no credit history. At date 1, a borrower's project could either be success or failure, i.e., $H_1 = (S_1)$ or (F_1) . At date 2, H_2 could be (S_1, S_2) , (S_1, F_2) , (F_1, S_2) or (F_1, F_2) .

Knowing a borrower's credit history, banks assign credit-worthiness to the borrower. Credit-worthiness is the probability of success of a project that is assigned by banks to borrowers with the same credit history. We assume that banks are able to access a borrower's credit history at no cost. Once observing the borrower's credit history, the bank updates its belief about the probability that the borrower is type A . Then, the bank assigns the credit-worthiness to the potential borrower.

The credit-worthiness of a borrower is calculated by a lender according to the information revealed by a borrower's credit history and the knowledge of the probability of success of each type when certain action is taken. Clearly at the beginning of the first period, since no borrowers have a credit history, they belong to the same group and banks assign the same credit-worthiness to them. Let $\lambda_t(H_{t-1})$ denote the belief that the borrower with credit history H_{t-1} is type A. Let (ea_t, eb_t) denote the pair of choices of type A and type B in period t, respectively. In addition, let ${}_{ea,eb}P$ be the credit-worthiness of the potential borrower and ${}_{ea,eb}D$ be the repayment asked by the potential lender when a pair of efforts (ea, eb) is chosen by type A borrowers and type B borrowers in the second stage. Given λ_0 , the prior belief that the borrower is type A, and the probability of success of each type, the banks are able to determine the credit-worthiness of the borrower. In period 1, $\lambda_1 = \lambda_0$, and if the pair of efforts (ea_1, eb_1) are made, then the borrower's credit-worthiness is

$${}_{ea_1,eb_1}P_1 = \lambda_1 P_{ea_1}^A + (1 - \lambda_1) P_{eb_1}^B,$$

where ea and $eb \in \{h, l\}$, different types can choose different actions.

At the beginning of period 2, borrowers may have different credit histories. Different credit-worthiness is assigned to different groups of borrowers. To determine the credit-worthiness of a borrower when the borrower applies for a loan, first, a bank must determine the probability that the borrower is type A. Suppose the lender expects (ea_1, eb_1) to be taken by borrowers. In equilibrium, expectation will be correct since incentive compatibility constraints are met. Thus, if the borrower's project succeeded in the previous period, the lender's belief will be

$$\lambda_2(S_1) = \frac{\lambda_1 P_{ea_1}^A}{\lambda_1 P_{ea_1}^A + (1 - \lambda_1) P_{eb_1}^B}.$$

Since $\lambda_2 > \lambda_1$, the bank believes the probability that the borrower is type A increases. Thus the credit-worthiness of this group is

$$P_2(S_1) = \lambda_2(S_1)p_{ea_2}^A + (1 - \lambda_2(S_1))p_{eb_2}^B,$$

which is greater than P_1 , if both types of borrower still choose the same effort as in period one. With one period of good credit history, the borrower's credit-worthiness is upgraded.

If the borrower failed in the first period, $H_1 = (F_1)$, the lender's belief is

$$\lambda_2(F_1) = \frac{\lambda_1(1 - p_{ea_1}^A)}{\lambda_1(1 - p_{ea_1}^A) + (1 - \lambda_1)(1 - p_{eb_1}^B)},$$

which is smaller than λ_1 . Thus the credit-worthiness of this group is

$$P_2(F_1) = \lambda_2(F_1)p_{ea_2}^A + (1 - \lambda_2(F_1))p_{eb_2}^B,$$

which is smaller than P_1 , if both types of borrow still choose the same effort as in period one.

At date 2, a borrower's credit history H_2 could be (S_1, S_2) , (S_1, F_2) , (F_1, S_2) or (F_1, F_2) . One can get:

$$\lambda_3(S_1, S_2) = \frac{\lambda_2(S_1)p_{ea_2}^A}{\lambda_2(S_1)p_{ea_2}^A + (1 - \lambda_2(S_1))p_{eb_2}^B}$$

$$\lambda_3(S_1, F_2) = \frac{\lambda_2(S_1)(1 - p_{ea_2}^A)}{\lambda_2(S_1)(1 - p_{ea_2}^A) + (1 - \lambda_2(S_1))(1 - p_{eb_2}^B)}$$

$$\lambda_3(F_1, S_2) = \frac{\lambda_2(F_1)p_{ea_2}^A}{\lambda_2(F_1)p_{ea_2}^A + (1 - \lambda_2(F_1))p_{eb_2}^B}, \text{ and}$$

$$\lambda_3(F_1, F_2) = \frac{\lambda_2(F_1)(1 - p_{ea_2}^A)}{\lambda_2(F_1)(1 - p_{ea_2}^A) + (1 - \lambda_2(F_1))(1 - p_{eb_2}^B)}.$$

At date 3, once the bank updates its belief λ_3 , it can update the credit-worthiness of the borrower:

$$P_3(H_2) = \lambda_3(H_2)p_{ea_3}^A + (1 - \lambda_3(H_2))p_{eb_3}^B.$$

If a borrower's credit history $H_2 = (S_1, S_2)$, then we have $\lambda_3 > \lambda_2$. The lender's belief that a borrower is type A increases as the borrower succeeds in two consecutive periods. If the borrower does not change his effort, the result can be $P_3 > P_2$. A borrower's consecutive success makes banks upgrade the borrower's credit-worthiness further. On the contrary, if a borrower succeeds in the first period and fails in the second period, his credit-worthiness will be downgraded. Comparing P_3 with credit history H_2 being one of the following: (S_1, S_2) , (S_1, F_2) , (F_1, S_2) or (F_1, F_2) , we know P_3 is the largest when $H_2 = (S_1, S_2)$ and P_3 is the smallest when $H_2 = (F_1, F_2)$. A good credit history makes good credit rating and a bad credit history makes bad credit rating.

In period t , lenders know a borrower's credit history H_{t-1} , they update their beliefs

$$\lambda_t(H_{t-2}, S_t) = \frac{\lambda_{t-1}(H_{t-2})p_{ea_{t-1}}^A}{\lambda_{t-1}(H_{t-2})p_{ea_{t-1}}^A + (1 - \lambda_{t-1}(H_{t-2}))p_{eb_{t-1}}^B} \quad (2)$$

$$\lambda_t(H_{t-2}, F_t) = \frac{\lambda_{t-1}(H_{t-2})(1 - p_{ea_{t-1}}^A)}{\lambda_{t-1}(H_{t-2})(1 - p_{ea_{t-1}}^A) + (1 - \lambda_{t-1}(H_{t-2}))(1 - p_{eb_{t-1}}^B)} \quad (3)$$

The credit-worthiness of the borrower is

$$p_t(H_{t-1}) = \lambda_t(H_{t-1})p_{ea_t}^A + (1 - \lambda_t(H_{t-1}))p_{eb_t}^B \quad (4)$$

3. ONE-PERIOD MODEL

Understanding the connection between credit-worthiness and credit-history can help us to find equilibrium in the multi-period model. However, the multi-period model is very complicated. Thus we will study the one-period model first as a benchmark solution.

One Period Model with Complete Information about the Borrower's Type

The timing in the one-period model is as follows: first, a lender and a borrower negotiate about a debt contract. After the contract is signed the borrower chooses the level of effort, which cannot be controlled by the lender. With complete information, the

lender can offer different contracts to different types of borrowers. However, facing moral hazard, the lender must offer an appropriate incentive to the borrower in order to induce the borrower to choose a certain level of effort.

To make the model interesting, we assume that each project is worth investing in regardless what effort chosen by the borrower, i.e.,

$$p_h Y_s + (1 - p_h) Y_f - \phi_h \geq k, \quad (7-A)$$

$$p_l Y_s + (1 - p_l) Y_f \geq k. \quad (7-B)$$

For simplicity, let us assume that $Y_f = 0$ and $k = 1$. Given a debt contract with repayment D , the borrower's expected utility is $p_h(Y_s - D) - \phi_h$ when he chooses h , and $p_l(Y_s - D)$ when he chooses l . A borrower will choose $e = h$ if

$$p_h(Y_s - D) - \phi_h \geq p_l(Y_s - D).$$

If h is chosen by the borrower, the lender knows that the probability of success of the project is p_h , and the credit-worthiness of the borrower would be p_h . The lender's expected payoff would be $p_h D + (1 - p_h) Y_f$. In the financial market with complete information, the

lender will ask repayment

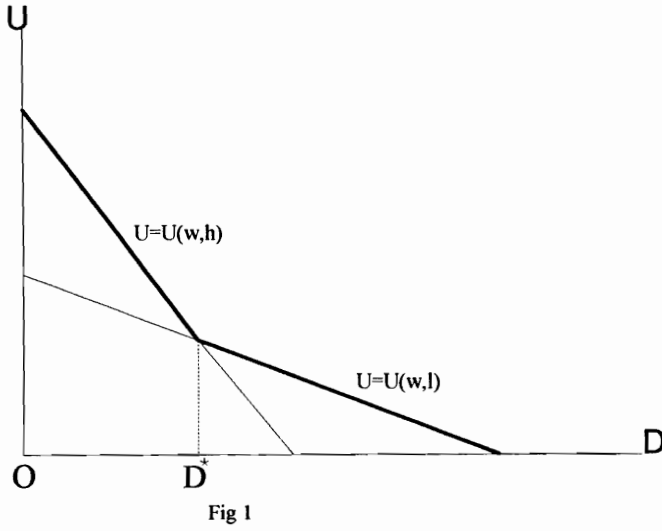
$$D = \frac{k - (1 - p_h) Y_f}{p_h} = \frac{1}{p_h}.$$

A borrower will choose $e = l$ if

$$p_l(Y_s - D) \geq p_h(Y_s - D) - \phi_h.$$

If l is chosen by the borrower, the credit-worthiness of the borrower would be p_l . The lender would then ask repayment

$$D = \frac{k - (1 - p_l) Y_f}{p_l} = \frac{1}{p_l}.$$



As shown in the figure, at $D^* = Y_s - \frac{\phi_h}{p_h - p_l}$, the borrower is indifferent between high effort h and low effort l . If $D > D^*$, the borrower will choose l , and if $D < D^*$, he will choose h .

One Period Model with Incomplete Information about the Borrower's Type

With complete information lenders can offer different contracts to different types of borrowers. However, this is not the case with incomplete information. The model with incomplete information can also be modeled as a two-stage game. In the first stage, a lender negotiates a debt contract with a borrower. In the second stage, after a debt contract (k, D) is signed, the borrower chooses the level of effort to maximize his payoff.

Given a debt contract (k, D) , and effort e , both the bank's payoff $\Pi(D, e)$ and the borrower's utility $u(D, e)$ are functions of repayment and borrowers' effort. In the second stage, given repayment D , if the borrower chooses the optimal level of effort e^* , then $e^*(D) = \text{argmax } U(D, e)$. In the first stage, the borrower determines the optimal debt contract with repayment $D^* = \text{argmax } u(D, e^*(D))$.

Let us assume that in the first stage a debt contract is signed by both parties. The borrower's expected utility of type Q is

$$u_e^Q(D, e) = p_e^Q(Y_s - D) - \phi_e^Q. \quad (8)$$

In order to assure action e^* will be chosen by the borrower, a contract offered to the borrower should provide proper incentives. The borrower of type Q will choose effort e^* , if his expected payoff is higher than if he would choose the other effort e , i.e., if the following condition is satisfied:

$$p_{e^*}^Q(Y_s - D) - \phi_{e^*}^Q \geq p_e^Q(Y_s - D) - \phi_e^Q, \quad (9)$$

where e^* and $e \in \{l, h\}$. Equation (9) is the incentive compatibility constraint of type Q borrowers. Given the repayment D , a borrower will choose high or low level of effort depending entirely on which one yields the higher utility.

Equality in equation (9) means the borrower of type Q is indifferent between choosing high effort and low effort. Let D^{A*} and D^{B*} denote these critical values of repayment for type A and type B , respectively. One gets

$$D^{A*} = Y_s - \frac{\phi_h^A}{p_h^A - p_l^A} \text{ and} \quad (10A)$$

$$D^{B*} = Y_s - \frac{\phi_h^B}{p_h^B - p_e^B}. \quad (10B)$$

With incomplete information, lenders face not only moral hazard, but also the adverse selection problem. In the first stage, since lenders do not know the type of a potential borrower, unlike in the model of complete information, they cannot offer different contracts to different types of borrowers. To negotiate with the borrower, the lenders need to know the probability of getting repaid -- the credit-worthiness of the borrower. As we stated before, the credit-worthiness is the prior belief of success assigned to a potential borrower by banks. When evaluating a potential borrower's credit-

worthiness, lenders must take into account the effort levels chosen by both types of borrowers. The borrower's credit worthiness is

$${}_{ea,eb}P = \lambda p_{ea}^A + (1 - \lambda)p_{eb}^B, \text{ where } e \in \{h, l\}. \quad (11)$$

The expected revenue of banks must be greater than or equal to their cost

$$pD + (1 - p)Y_f \geq k. \quad (12)$$

In the competitive credit market, the bank is offered the lowest such expected revenue.

Thus we get

$${}_{ea,eb}D = \frac{k - (1 - {}_{ea,eb}P)Y_f}{{}_{ea,eb}P} = \frac{I}{{}_{ea,eb}P}. \quad (13)$$

Given D , a borrower's expected utility must be greater than or equal to 0,

$$u^e(D, e) = p_e^e(Y_s - D) - \phi_e^e \geq 0, \quad (14)$$

which is the borrower's participation constraint. If equation (14) is not satisfied a contract will not be signed by the borrower.

From equation (11), we know that P increases with λ . Given all the other parameters, the smaller (larger) the proportion of type A , the smaller (larger) the credit-worthiness is assigned to the borrower. From (12), we know that the lower the borrower's credit-worthiness is, the larger is the repayment charged by the lender. Thus, if λ is too small, the participation constraint may not be satisfied and the borrower may not want to sign the contract.

Summarizing the conditions stated above, a debt contract $(k, {}_{ea^*,eb^*}D)$ will be signed by a borrower and a lender, and the pair of efforts (ea^*, eb^*) will be chosen by borrowers of type A and type B , if:

$$\begin{aligned} p_{ea^*}^A(Y_s - D) - \phi_{ea^*}^A &\geq p_{ea^*}^A(Y_s - D) - \phi_{ea^*}^A \geq 0, \\ p_{eb^*}^B(Y_s - D) - \phi_{eb^*}^B &\geq p_{eb^*}^B(Y_s - D) - \phi_{eb^*}^B \geq 0, \text{ and} \\ {}_{ea^*,eb^*}D &= \frac{I}{{}_{ea^*,eb^*}P}. \end{aligned}$$

Lemma 1.

- (a) If $\frac{\phi^B}{p_h^B - p_l^B} < \frac{\phi^A}{p_h^A - p_l^A}$, then (h, l) cannot be a pair of efforts chosen by borrowers of type A and type B in equilibrium.
- (b) If $\frac{\phi^B}{p_h^B - p_l^B} > \frac{\phi^A}{p_h^A - p_l^A}$, then (l, h) cannot be a pair of efforts chosen by borrowers of type A and type B in equilibrium.

Proof. To have (h, l) as a pair of equilibrium choices, we must find an optimal contract (k, D) such that

$$p_h^A(Y_s - D) - \phi^A \geq p_l^A(Y_s - D) \text{ and}$$

$$p_h^B(Y_s - D) - \phi^B \leq p_l^B(Y_s - D).$$

This means

$$Y_s - \frac{\phi^B}{p_h^B - p_l^B} \leq D \leq Y_s - \frac{\phi^A}{p_h^A - p_l^A}$$

$$\frac{\phi^B}{p_h^B - p_l^B} \geq \frac{\phi^A}{p_h^A - p_l^A}.$$

Thus, If $\frac{\phi^B}{p_h^B - p_l^B} < \frac{\phi^A}{p_h^A - p_l^A}$, then (h, l) cannot be a pair of efforts in equilibrium.

The proof of (b) is similar to the proof of (a).□□

This lemma tells us that if there is a feasible contract such that (h, l) is a pair of optimal choices of borrowers of type A and type B, then there is no feasible contract such that (l, h) is a pair of optimal choices and vice versa.

The equilibrium choices of borrowers in the one-period model can be described by means of four subintervals R1-R4 of the unit interval, some of which may be empty. These intervals cover the unit interval and are pairwise disjoint except for R2 and R3 which may overlap.

Proposition 1. The equilibrium choices of borrowers in the one-period model are as follows:

(a) (h, h) is a pair of optimal choices of borrowers of both types if $\lambda_0 \in R1$.

(b) (h, l) is a pair of optimal choices of borrowers of both types if $\lambda_0 \in R2$.

(c) (l, h) is a pair of optimal choices of borrowers of both types if $\lambda_0 \in R3$.

(d) (l, l) is a pair of optimal choices of borrowers of both types if $\lambda_0 \in R4$.

Proof. See Appendix A.

Example 1:

Let the return of each project be 3 if it succeeds and is 0 if it fails. The probabilities of success for type A are 0.9 if high effort h is chosen, and 0.5 if low effort l is chosen. The probabilities of success for type B are 0.6 if high effort h is chosen, and 0.4 if low effort l is chosen. The disutility of making high effort is 0.3 for type A and 0.3 for type B . The proportion of population of type A is $\lambda = 0.08$.

If lenders have complete information, when a borrower of type A applies for a loan, they offer the contract with $rD^A = 1.111$ to the borrower, and the type A borrower will choose high effort h which is what the lenders expect him to choose. Thus, this is an equilibrium contract. If the lenders offer the debt contract with $rD^A = 2.000$, they expect the borrower choose high effort h . However, the borrower prefers low effort to high effort. Thus this contract cannot be an equilibrium contract.

It is impossible to find a debt contract that offers appropriate incentives to a type B borrower to induce him to choose high effort h and is acceptable to lenders. In equilibrium, lenders will ask for $rD^B = 2.5$, and the borrower of type B chooses low effort l .

Table 1

Information	Type	Debt	Effort	Payoff
Complete Information	A	${}_h D^A = 1.111111$	h	1.4
			l	.9444444
		${}_l D^A = 2.000$	h	.6
			l	.5
	B	${}_h D^B = 1.666667$	h	.500000
			l	.5333334
	${}_l D^B = 2.500$	h	0	
		l	.2	
Incomplete Information	A	${}_{h,h} D = 1.602564$	(h, h)	.9576923
			(l, h)	.698718
		${}_{h,l} D = 2.272727$	(h, l)	.3545454
			(l, l)	.3636364
		${}_{l,h} D = 1.689189$	(h, h)	.8797297
			(l, h)	.6554054
		${}_{l,l} D = 2.45098$	(h, l)	.1941176
			(l, l)	.2745098
	B	${}_{h,h} D = 1.602564$	(h, h)	.5384616
			(h, l)	.5589744
		${}_{h,l} D = 2.272727$	(h, h)	.1363637
			(h, l)	.2909091
	${}_{l,h} D = 1.689189$	(l, h)	.4864865	
		(l, l)	.5243244	
	${}_{l,l} D = 2.45098$	(l, h)	.02.941175	
		(l, l)	.2196078	

If lenders do not know the type of a borrower, there exists no separating equilibrium. However, we can find a pooling equilibrium. When the debt contract ${}_{h,h} D = 1.602564$ is offered by a lender, the lender expects both types of borrowers choose high effort h . Although, a type A borrower prefers high effort h to low effort l , a type B borrower prefers l to h . Thus, this contract cannot be an equilibrium contract. The debt contract with ${}_{l,l} D = 2.45098$ is the only one that the lender's expectation coincides with the choices of both types of borrower. Thus this is the only equilibrium and both types of borrower will choose low effort.

4. MULTI-PERIOD MODEL

From the previous section we know that credit-worthiness is the determinant factor of repayment for a debt contract. The credit-worthiness assigned by banks to a borrower is determined by the prior belief that the borrower is type A , and by the probability of success for each type of borrowers. As long as a debt contract provides enough incentive, i.e., the incentive compatibility constraint is satisfied, certain action will be taken by the borrower. The differences between a multi-period setting and a one period setting are as follows: first, banks can extract some information about the type of a borrower from his credit history. After a project is realized and the credit-history of the borrower is revealed to the public, the beliefs about a borrower's type will be updated. Second, borrowers must take into account the effect of their choices in the current period upon their future payoff.

The expected utility function of a borrower is the summation of the borrower's expected payoff in each period. For simplicity, we assume that the discount factor is 1, thus

$$U_t = p_{e_t}^0(Y_s - D_t) - \phi_{e_t}^0 + p_{e_t}^0(U_{t+1}(S)) + (1 - p_{e_t}^0)(U_{t+1}(F)). \quad (15)$$

$U_{t+1}(S)$ is the expected utility in the future if the borrower's project succeeds in period t ; it should be weighted by the probability of success $p_{e_t}^0$ if the borrower chooses the effort e_t in period t . Similarly, $U_{t+1}(F)$ is the expected utility in the future if the borrower's project fails in period t ; it should be weighted by the probability of failure $1 - p_{e_t}^0$ if the borrower chooses the effort e_t in period t .

The timing of this model in each period is similar to the timing in the one period model. At the beginning of period t , a borrower applies for a new loan to start a new project. Knowing the borrower's credit history, H_{t-1} , the lenders update his belief λ_t about which type of the borrower this one will be. According to this belief, and taking the

borrower's possible action into account, the lender assigns credit-worthiness to the borrower and offers a debt contract to the borrower. After signing the debt contract, the borrower chooses the level of effort to maximize his expected utility from period t on. At the end of the period, the return of the project is realized and distributed between the lender and the borrower. In addition, the failure or success of the project is recorded in the borrower's credit history. Then, the next period comes and the whole procedure is repeated.

In equilibrium, in each period given the repayment D_t , the borrower chooses the optimal level of effort e_t^* to maximize his expected payoff thereafter. An optimal contract can be determined, which minimizes the bank's expected return subject to the borrower's incentive compatibility constraint and participation constraint.

In the first stage, given the choices of different types of borrowers, the lender will offer a debt contract, which yields an expected payoff equaling the borrower's investment k ($=1$). Since in the competitive credit market the bank will yield the lowest expected payoff, we get

$$D_t = \frac{I}{e_{ea^*, eb^*} P_t}, \quad (16)$$

where $e_{ea^*, eb^*} P_t = \lambda_t P_{ea^*}^A + (1 - \lambda_t) P_{eb^*}^B$.

Given the actions taken by both types of borrower, an increase in P_t decreases D_t . Banks will ask more repayment if the borrower's credit-worthiness is low. A good credit history can help the borrower get better terms when he applies for loans.

The borrower's choices in period t not only affect his payoff in this period but will also affect his payoff thereafter. A borrower's choice of effort level affects his payoffs in two ways. First, the choices affect the probability of the success. As stated in our assumption, the probability of success of a project is higher when high effort is chosen

than when low effort is chosen. Second, the choice will affect the level of repayments, and therefore the payoff, in the current period and in the future periods.

By using the two-period model, we can see effects of a borrower's current choice on his current and future payoffs. We know the expected payoff in each period is a function of debt and effort in that period:

$$u_t^Q = p_{e_t}^Q(Y_s - D_t) - \phi_{e_t}^Q. \quad (17)$$

Thus, if a borrower of type Q chooses effort e_1 in the first period, the borrower's expected payoff is

$$U = [p_{e_1}^Q(Y_s - D_1) - \phi_{e_1}^Q] + p_{e_1}^Q[p_{e_2(S_1)}^Q(Y_s - D_2(S_1)) - \phi_{e_2(S_1)}^Q] + (1 - p_{e_1}^Q)[p_{e_2(F)}^Q(Y_s - D_2(F)) - \phi_{e_2(F)}^Q] \quad (18)$$

The term in the first bracket is the borrower's expected payoff in the first period. The term in the second bracket is the borrower's expected payoff in the second period if his project succeeds, which is multiplied by the probability of success in the first period. The term in the third bracket is the borrower's expected payoff in the second period if his project fails which is multiplied by the probability of failure in the first period.

In the first period choosing different level of effort will change the probability of success of the project, $p_{e_1}^Q$, and change the disutility of effort, $\phi_{e_1}^Q$; therefore, the borrower's payoff in each period is changed. In addition, The level of repayment offered by the lender is associated with the level of effort chosen by the borrower. Choosing a different level of effort not only changes the repayment in D_1 but also changes $D_2(S_1)$ and $D_2(F)$, since the level of repayment is determined by the credit-worthiness of the borrower, which is also affected by the borrower's choice.

To get the equilibrium, one can solve the two-period model backward. If the borrower's project succeeds in the first period, in the second period the borrower of type Q will choose the effort level $e_2^*(S)$, such that

$$p_{e_2^*(S_1)}^Q(Y_s - D_2(S_1)) \geq p_{e_2(S_1)}^Q(Y_s - D_2(S_1))$$

$$D_2(S_1) = \frac{I}{e_{a_2, e_{b_2}} \cdot P_2(S_1)},$$

where $e_{a_2(S_1), e_{b_2}(S_1)} P_2(S_1) = \lambda_2(S_1) p_{ea_2(S_1)}^A + (1 - \lambda_2(S_1)) p_{eb_2(S_1)}^B$

and $\lambda_2(S_1) = \frac{\lambda_1 p_{ea_1}^A}{\lambda_1 p_{ea_1}^A + (1 - \lambda_1) p_{eb_1}^B}$

If the borrower's project fails in the first period, in the second period the borrower of type Q will choose the effort level $e_2^*(S)$, such that

$$p_{e_2^*(F_1)}^Q(Y_s - D_2(F_1)) \geq p_{e_2(F_1)}^Q(Y_s - D_2(F_1))$$

$$D_2(F_1) = \frac{I}{e_{a_2^*(F_1), e_{b_2}^*(F_1)} P_2(F_1)}$$

where $e_{a_2(F_1), e_{b_2}(F_1)} P_2(F_1) = \lambda_2(F_1) p_{ea_2(F_1)}^A + (1 - \lambda_2(F_1)) p_{eb_2(F_1)}^B$

$$\lambda_2(F_1) = \frac{\lambda_1 (1 - p_{ea_1}^A)}{\lambda_1 (1 - p_{ea_1}^A) + (1 - \lambda_1) (1 - p_{eb_1}^B)}$$

Lemma 2. If (ea_1, eb_1) is the pair of efforts chosen by a type A borrower and a type B borrower and borrowers' projects succeed in the first period, then, in the second period,

(a) (h_2, h_2) will be chosen if

$$\lambda_0 \geq \frac{dl1 p_{eb_1}^B}{(1 - dl1) p_{ea_1}^A + dl1 p_{eb_1}^B}$$

(b) (h_2, l_2) will be chosen if

$$\frac{dl2 p_{eb_1}^B}{(1 - dl2) p_{ea_1}^A + dl2 p_{eb_1}^B} \leq \lambda_0 \leq \frac{dh2 p_{eb_1}^B}{(1 - dh2) p_{ea_1}^A + Ch2 p_{eb_1}^B}$$

(c) (l_2, h_2) will be chosen if

$$\frac{dl3 p_{eb_1}^B}{(1 - dl3) p_{ea_1}^A + dl3 p_{eb_1}^B} \leq \lambda_0 \leq \frac{dh3 p_{eb_1}^B}{(1 - dh3) p_{ea_1}^A + dh3 p_{eb_1}^B}$$

(d) (l_2, l_2) will be chosen if

$$\lambda_0 \leq \frac{dh4p_{eb_1}^B}{(1-dh4)p_{ea_1}^A + dh4p_{eb_1}^B}$$

Proof. If the pair of efforts (ea_1, eb_1) is chosen and borrowers' projects succeed in the first period, then $\lambda_2 = \frac{\lambda_0 p_{ea_1}^A}{\lambda_0 p_{ea_1}^A + (1-\lambda_0)p_{eb_1}^B}$. From proposition 1-(a), we know that if

$dll \leq \lambda_2$, then (h_2, h_2) is a pair of equilibrium choices in period 2. From

$$\lambda_2 = \frac{\lambda_0 p_{ea_1}^A}{\lambda_0 p_{ea_1}^A + (1-\lambda_0)p_{eb_1}^B} \geq dll, \text{ we get}$$

$$\lambda_0 \geq \frac{dll p_{eb_1}^B}{(1-dll)p_{ea_1}^A + dll p_{eb_1}^B}.$$

The remaining parts (b), (c), and (d) can be proved by using the same argument.

Lemma 3. If (ea_1, eb_1) is the pair of efforts chosen and borrowers' projects succeed in the first period, then, in the second period,

(a) (h_2, h_2) will be chosen if

$$\lambda_0 \geq \frac{dll(1-p_{eb_1}^B)}{(1-dll)(1-p_{ea_1}^A) + dll(1-p_{eb_1}^B)}.$$

(b) (h_2, l_2) will be chosen if

$$\frac{dl2(1-p_{eb_1}^B)}{(1-dl2)(1-p_{ea_1}^A) + dl2(1-p_{eb_1}^B)} \leq \lambda_0 \leq \frac{dh2(1-p_{eb_1}^B)}{(1-dh2)(1-p_{ea_1}^A) + (1-dh2)p_{eb_1}^B}$$

(c) (l_2, h_2) will be chosen if

$$\frac{dl3(1-p_{eb_1}^B)}{(1-dl3)(1-p_{ea_1}^A) + dl3(1-p_{eb_1}^B)} \leq \lambda_0 \leq \frac{dh3(1-p_{eb_1}^B)}{(1-dh3)(1-p_{ea_1}^A) + dh3(1-p_{eb_1}^B)}$$

(d) (l_2, l_2) will be chosen if

$$\lambda_0 \leq \frac{dh4(1-p_{eb_1}^B)}{(1-dh4)(1-p_{ea_1}^A) + dh4(1-p_{eb_1}^B)}$$

The lemmas above tell us about borrowers' choices in the second period. Knowing borrowers' choices in the second period, we will be able to find borrowers' equilibrium

choices in the first period. Assume e_2^* 's are the equilibrium choices in the period 2. In the first period a type Q borrower will choose effort e_1^* if

$$p_{e_1}^Q(Y_s - D_1) - \phi_{e_1}^Q + p_{e_1}^Q[p_{e_2(S_1)}^Q(Y_s - D_2(S_1)) - \phi_{e_2(S_1)}^Q] + (1 - p_{e_1}^Q)[p_{e_2(F_1)}^Q(Y_s - D_2(F_1)) - \phi_{e_2(F_1)}^Q] \\ \geq p_{e_1}^Q(Y_s - D_1) - \phi_{e_1}^Q + p_{e_1}^Q[p_{e_2(S_1)}^Q(Y_s - D_2(S_1)) - \phi_{e_2(S_1)}^Q] + (1 - p_{e_1}^Q)[p_{e_2(F_1)}^Q(F_1)(Y_s - D_2(F_1)) - \phi_{e_2(F_1)}^Q]$$

where $_{ea_1, eb_1} D_1 = \frac{I}{_{ea_1, eb_1} P_1}$

$$_{ea_1, eb_1} P_1 = \lambda_1 P_{ea_1}^A + (1 - \lambda_1) P_{eb_1}^B$$

Given the equilibrium choices in period 2, the equilibrium choices of borrowers in period 1 can be described by means of subintervals of the unit interval.

Proposition 2. Let $(ea_2(S), eb_2(S))$ is the pair of optimal choices in period 2 when the project succeeds, and $(ea_2(F), eb_2(F))$ is the pair of optimal choices in period 2 when the project succeeds in period 1. In case $R3 = \phi$ ($R2 = \phi$) the borrowers' optimal choices in period 1 are as follows:

- (a) (h_1, h_1) is a pair of optimal choices in period one, if $d5 \leq \lambda_0$.
- (b) (h_1, l_1) ((l_1, h_1)) is a pair of optimal choices in period one, if $d6 \leq \lambda_0 \leq d5$.
- (c) (l_1, l_1) is a pair of optimal choices in period one, if $\lambda_0 < d6$.

Proof. See Appendix B.

Lemma 4. If a borrower chooses low effort l in the first period, then he will continue to choose low effort in period two if his project fails in period one.

Lemma 5. If $\lambda_0 \in B1$, then high effort will be chosen by both types in period two if their projects succeed.

Lemma 6. If $\lambda_0 \in B4$, then low effort will be chosen by both types in period two if their projects fail.

Example 2:

This example will use the two-period model to show how the future rents affect a borrower's choice in the current period. The setting is the same as in example 1.

In equilibrium, both types of borrowers will choose high level of effort in period one and the contract with ${}_{h_1, h_1}D_1 = 1.602564$ is an equilibrium contract. If their projects succeed in period one, the pair of effort (h_2, l_2) will be chosen and the contract with ${}_{h_2, l_2}D_2 = 2.429907$ will be signed in period 2. On the other hand, if their projects fail in period one, the pair of effort (l_2, l_2) will be chosen and the contract with ${}_{l_2, l_2}D_2 = 2.486773$ will be signed in the second period.

However, the contract with ${}_{h_1, h_1}D_1 = 1.602564$ cannot be an equilibrium contract if there is only one period. When offering this contract, the bank expects both types of borrowers choose high effort. Nevertheless, the payoff for a type B borrower is 0.5384616 when high effort is chosen, and is 0.5589744 when low effort is chosen. Thus, instead of choosing high effort, the borrower of type B will choose low effort. This results the bank's expected payoff being lower than its loan. As a result, this contract cannot be an equilibrium. As shown in example 1, the pair of effort (l_1, l_1) will be chosen and the contract with ${}_{l_1, l_1}D_1 = 2.45098$ will be signed in equilibrium if there is only one period. The payoffs for the type A borrowers and the type B borrowers will be 0.2745098 and 0.2166078, respectively.

In the two-period model, if the debt contract ${}_{l_1, l_1}D_1 = 2.45098$ is offered and the borrower of type B chooses low effort in the first period, in the second period, the expected payoff for the type B borrower will be 0.3091703 if his project succeeds, and

will be 0.2166113 if his project fails. Weighted these future expected payoffs by the probabilities of success and failure in current period and adding them to the payoff in the first period, we get the total expected payoff for a type *B* borrower equal to 0.4732427. If the type *B* borrower chooses high effort *h* in the first period, he increases the probability of success in the first period, therefore, the expected payoff in the second period is increased. However, he also lowers the expected payoff in the current period to 0.0294118. Adding the expected payoffs in both periods, we get the total payoff for the type *B* borrower equal to 0.3015585, which is lower than the payoff when low effort is chosen by the borrower. Thus, the borrower of type *B* will choose low effort.

Similarly, we can add up the expected pay off for a type *A* borrower in both periods. A type *A* borrower's expected payoffs are 0.577264 and 0.6077098 by choosing high effort *h* and low effort *l*, respectively. To maximize the type *A* borrower's expected payoff, the borrower will choose low effort. Since both types of borrowers will choose low effort *l*, the contract with $l, l, D_l = 2.45098$ is feasible.

Table 2

Q	D_1	e_1	$E(U)$	$E(U_1)$	H_1	λ_2	p_2	D_2	e_2	$E(U_2)^*$
A	1.602564	h_1	1.373606	0.9576923	S	0.1153846	0.4115385	2.429907	h	0.4336133
					F	0.0212766	0.4021277	2.486773	l	0.2566137
		l_1	1.043831	0.698718	S	0.1153846	0.4115385	2.429907	h	0.4336133
					F	0.0212766	0.4021277	2.486773	l	0.2566137
	2.272727	h_1	0.8588582	0.3545454	S	0.1636364	0.4163636	2.401747	h	0.5320755
					F	0.0142857	0.4014286	2.491103	l	0.2544484
		l_1	0.7568983	0.3636364	S	0.1636364	0.4163636	2.401747	h	0.5320755
					F	0.0142857	0.4014286	2.491103	l	0.2544484
	1.689119	h_1	1.162981	0.8797297	S	0.06756756	0.4067568	2.458472	l	0.2707641
					F	0.09803922	0.4098039	2.440191	l	0.3956333
		l_1	0.9886041	0.6554054	S	0.06756756	0.4067568	2.458472	l	0.2707641
					F	0.09803922	0.4098039	2.440191	l	0.3956333
2.45098	h_1	0.577264	0.1941176	S	0.09803922	0.4098039	2.440191	l	0.3956333	
				F	0.06756756	0.4067568	2.458472	l	0.2707641	
	l_1	0.6077098	0.2745098	S	0.09803922	0.4098039	2.440191	l	0.3956333	
				F	0.06756756	0.4067568	2.458472	l	0.2707641	
B	1.602564	h_1	0.8162082	0.5384616	S	0.1153846	0.4115385	2.429907	l	0.3260504
					F	0.0212766	0.4021277	2.486773	l	0.205291
		l_1	0.8125691	0.5589744	S	0.1153846	0.4115385	2.429907	l	0.3260504
					F	0.0212766	0.4021277	2.486773	l	0.205291
	2.272727	h_1	0.439674	0.1363637	S	0.1636364	0.4163636	2.401747	l	0.3698114
					F	0.0142857	0.4014286	2.491103	l	0.2035587
		l_1	0.5609689	0.2909091	S	0.1636364	0.4163636	2.401747	l	0.3698114
					F	0.0142857	0.4014286	2.491103	l	0.2035587
	1.689119	h_1	0.7401214	0.4864865	S	0.06756756	0.4067568	2.458472	l	0.2166113
					F	0.09803922	0.4098039	2.440191	l	0.3091703
		l_1	0.7964711	0.5243244	S	0.06756756	0.4067568	2.458472	l	0.2166113
					F	0.09803922	0.4098039	2.440191	l	0.3091703
2.45098	h_1	0.3015585	0.0294118	S	0.09803921	0.4098039	2.440191	l	0.3091703	
				F	0.06756756	0.4067568	2.458472	l	0.2166113	
	l_1	0.4732427	0.2196078	S	0.09803921	0.4098039	2.440191	l	0.3091703	
				F	0.06756756	0.4067568	2.458472	l	0.2166113	

* The Expected payoff is calculated at the beginning of the 2nd period.

Let us reconsider the case where a debt contract with ${}_{h_1, h_1}D_1 = 1.602564$ is offered. If a type B borrower chooses high effort h_1 , his expected payoff is 0.5384616 in the first period. If his project succeeds, the contract with ${}_{h_2, l_2}D_2 = 2.429907$ will be signed and the type B borrower's expected payoff will be 0.3260504 in the second period. If his project fails, the contract with ${}_{l_2, l_2}D_2 = 2.486773$ will be signed and the type B borrower's expected payoff will be 0.205291 in the second period. Thus, the borrower's total expected payoff will be 0.8162082.

On the other hand, by choosing low effort l in the first period, although the borrower increases the expected payoff in the first period to 0.5589744, he also lowers the possibility of success in the first period, therefore, lowers his expected payoff in the second period. Thus, the total expected payoff for the borrower is 0.8125691, which is smaller than the total value when high effort is chosen. As a result, taking the future rents into account, the borrower will choose high effort instead of low effort in the first period.

For a type A borrower, if the contract with ${}_{h_1, h_1}D_1 = 1.602564$ is offered, by choosing high effort he will get expected payoff equal to 0.9576923 which is already higher than the expected payoff he is able to get by choosing low effort without taking into account of the increase of future expected payoff through the increase of the probability of success. Thus, he will choose high effort in the first period. Since both types of borrowers will choose high effort h , the contract with ${}_{h_1, h_1}D_1 = 1.602564$ is feasible.

Comparing two feasible debt contracts, since the contract with ${}_{h_1, h_1}D_1 = 1.602564$ will give both types of borrowers higher expected payoff than the contract with ${}_{l_1, l_1}D_1 = 2.45098$. Thus, the first contract will be chosen and both types of borrower will choose high effort in the first period.

The effect of future rent on the borrowers' choices in current period has been manipulated in tow-period model above. We can extend the model to more than two

periods. The structure of multi-period model with more than two-periods is a repeated extension of two-period model, but it is too complex to get the equilibrium condition. The following example is some results from numerical simulation of three-period model.

Example 3

In this example we extended the setting in example 2 to three periods. We will change λ_0 to see how this will affect borrowers' choices.

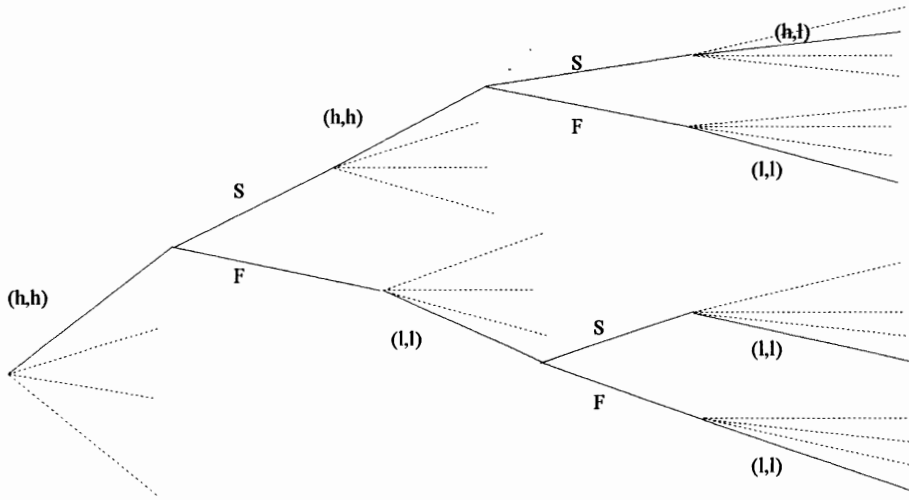
Case 1

If $\lambda_0 = 0.08$ and $T = 3$, we can get the equilibrium choices of both types of borrowers as follows:

t	1	2	3
S_1, S_2	(h, h)	(h, h)	(h, l)
S_1, F_2	(h, h)	(h, h)	(l, l)
F_1, S_2	(h, h)	(l, l)	(l, l)
F_1, F_2	(h, h)	(l, l)	(l, l)

As in example 2, borrowers will not choose high effort if there is only one period. If borrowers live for three periods, the future rent is large enough to let both types of borrowers choose high effort in period 1. In addition, if borrowers' projects succeed in period one, they will also choose high effort in period 2. However, only type *A* borrowers will choose high effort in period 3, if borrowers' projects succeed in both previous periods. The reason for a type *B* borrower shifting from high effort to low effort is that his credit rating is not high enough. Since $\lambda_3 \in R_2$, low effort will be chosen by the borrower.

If borrowers' projects fail in the first period, both types of borrowers will choose low effort in the second period because $\lambda_2 < d5$. Both types of borrowers will continue to choose low effort if their projects fail in period 2 since their credit rating continues to be dropped. If a borrower's project fails in period one, even though his project succeeds in period 2, he will choose low effort in period 3 because their credit rating is too low thus λ_3 fall into set R4.



Case 2

If $\lambda_0 = 0.045$ and $T = 3$, we can get the equilibrium choice of both types of borrowers as follows:

t	1	2	3
S_1, S_2	(h, h)	(h, l)	(h, l)
S_1, F_2	(h, h)	(h, l)	(l, l)
F_1, S_2	(h, h)	(l, l)	(l, l)
F_1, F_2	(h, h)	(l, l)	(l, l)

The choices of both types of borrowers in period three are the same as in case 1. Although the future rent is big enough for both types of borrowers to choose high effort in period one, it is no longer large enough to let the borrowers of type *B* maintain high effort in the second period, if their projects succeed in the first period.

Case 3

If $\lambda_0 = 0.4$ and $T = 3$, we can get the equilibrium choice of both types of borrowers as follows:

t	1	2	3
S_1, S_2	(h, l)	(h, l)	(h, l)
S_1, F_2	(h, l)	(h, l)	(l, l)
F_1, S_2	(h, l)	(l, l)	(l, l)
F_1, F_2	(h, l)	(l, l)	(l, l)

In this case, the choices of both types of borrowers in period two and three are the same as case 2. However, the future rent is no longer big enough for the type *B* borrowers to make high effort even in period 1.

5. CONCLUSION

This paper discusses the effect of borrowers' credit histories on their future borrowing. The connection between borrowers' choices of effort and their credit-worthiness is also explored. From borrowers' credit histories, banks can extract some information about the quality of the borrowers and downgrade or upgrade their credit

rating. Knowing this, borrowers must take into account the effect upon their future payoff when they make choices in each period.

From multi-period simulation we can see the effect of future rents upon their current choices. If the fraction of low-quality borrowers is small and the future rent is large enough to compensate the disutility from making high effort, even the borrowers of low quality will choose high effort. If the fraction of low-quality borrowers is very large, all borrowers start financing their projects with low credit rating. The repayment charged by banks will be so high that both types of borrowers will make low effort to repay their debts. As a borrower's projects succeed over several periods, he may acquire a good credit history and may have his credit-rating upgraded. When a borrower's credit-worthiness is above a certain point, it becomes worthwhile for a high quality borrower to choose high effort. When a borrower acquires good credit rating further, the future rent may be higher enough so that even a low quality borrower begins to choose high effort in some periods. However, the low-quality borrower may shift back to the low effort again even though his project continues to succeed because with a finite life time, his future rent from a good credit rating may become too small when he approaches to the end of his life. Only if a borrower's credit-worthiness exceeds a certain value, will the borrower provide high effort until the final period.

APPENDIX A

The proof of proposition 1 follows.

Let R1 - R4, S1 - S4 be as follows.

$$R1 = S1 \cap S0 = \{dl1 \leq \lambda_0 \leq 1\};$$

$$R2 = (S2 \setminus S1) \cap S0 = \{dl2 \leq \lambda_0 \leq dh2\} \text{ if } \frac{\phi^B}{p_h^B - p_l^B} \geq \frac{\phi^A}{p_h^A - p_l^A};$$

$$R2 = \emptyset \text{ if } \frac{\phi^B}{p_h^B - p_l^B} \leq \frac{\phi^A}{p_h^A - p_l^A};$$

$$R3 = (S3 \setminus S1) \cap S0 = \{dl3 \leq \lambda_0 \leq dh3\} \text{ if } \frac{\phi^B}{p_h^B - p_l^B} \leq \frac{\phi^A}{p_h^A - p_l^A};$$

$$R3 = \emptyset \text{ if } \frac{\phi^B}{p_h^B - p_l^B} \geq \frac{\phi^A}{p_h^A - p_l^A};$$

$$R4 = S4 \setminus (S3 \cup S2 \cup S1) \cap S0 = \{0 \leq \lambda_0 \leq dh4\}.$$

$$S0 = \{0 \leq \lambda_0 \leq 1\}$$

$$S1 = \{c1a \leq \lambda_0 \text{ and } c1b \leq \lambda_0\}$$

$$S2 = \{c2a \leq \lambda_0 \leq c2b\}$$

$$S3 = \{c3a \leq \lambda_0 \text{ and } c3b \leq \lambda_0\}$$

$$S4 = \{\lambda_0 \leq c4a \text{ and } \lambda_0 \leq c4b\}$$

$$c1a = -\frac{p_h^A - p_l^A + \phi^A p_h^B - p_h^A p_h^B Y_s + p_l^A p_h^B Y_s}{(p_h^A - p_h^B)(\phi^A - p_h^A Y_s + p_l^A Y_s)}$$

$$c1b = -\frac{p_h^B - p_l^B + \phi^B p_h^B - (p_h^B)^2 Y_s + p_h^B p_l^B Y_s}{(p_h^A - p_h^B)(\phi^B - p_h^B Y_s + p_l^B Y_s)}$$

$$c2a = -\frac{p_h^A - p_l^A + \phi^A p_l^B - p_h^A p_l^B Y_s + p_l^A p_l^B Y_s}{(p_h^A - p_l^B)(\phi^A - p_h^A Y_s + p_l^A Y_s)}$$

$$c2b = -\frac{p_h^B - p_l^B + \phi^B p_l^B + (p_l^B)^2 Y_s - p_h^B p_l^B Y_s}{(p_h^A - p_l^B)(\phi^B - p_h^B Y_s + p_l^B Y_s)}$$

$$c3a = -\frac{p_h^A - p_l^A + \phi^A p_h^B - p_h^A p_h^B Y_s + p_l^A p_h^B Y_s}{(p_l^A - p_h^B)(\phi^A - p_h^A Y_s + p_l^A Y_s)}$$

$$c3b = -\frac{p_h^B - p_l^B + \phi^B p_h^B - (p_h^B)^2 Y_s + p_h^B p_l^B Y_s}{(p_l^A - p_h^B)(\phi^B - p_h^B Y_s + p_l^B Y_s)}$$

$$c4a = -\frac{p_h^A - p_l^A + \phi^A p_l^B - p_h^A p_l^B Y_s + p_l^A p_l^B Y_s}{(p_l^A - p_l^B)(\phi^A - p_h^A Y_s + p_l^A Y_s)}$$

$$c4b = -\frac{p_h^B - p_l^B + \phi^B p_l^B + (p_l^B)^2 Y_s - p_h^B p_l^B Y_s}{(p_l^A - p_l^B)(\phi^B - p_h^B Y_s + p_l^B Y_s)}$$

To induce both types of borrowers to choose a pair of efforts (h, h) , the incentive constraints must be satisfied.

$$p_h^A(Y_s - {}_{h,h}D) - \phi^A \geq p_l^A(Y_s - {}_{h,h}D).$$

Plug the optimal repayment ${}_{h,h}D = \frac{1}{\lambda_0 p_h^A + (1 - \lambda_0) p_h^B}$ in the above inequality and solve it.

We get $\lambda_0 \geq -\frac{p_h^A - p_l^A + \phi^A p_h^B - p_h^A p_h^B Y_s + p_l^A p_h^B Y_s}{(p_h^A - p_h^B)(\phi^A - p_h^A Y_s + p_l^A Y_s)} = c1a, c1b, c2a, \dots, c4b$ can be solved

the same way.

According to the assumptions $p_h^A \geq p_l^A$ and $p_h^B \geq p_l^B$, we have

$$\lambda_0 p_h^A + (1 - \lambda_0) p_h^B \geq \lambda_0 p_h^A + (1 - \lambda_0) p_l^B \geq \lambda_0 p_l^A + (1 - \lambda_0) p_l^B, \text{ i.e.,}$$

$${}_{h,h}P \geq {}_{h,l}P \geq {}_{l,l}P$$

Thus ${}_{h,h}D \leq {}_{h,l}D \leq {}_{l,l}D$.

From ${}_{h,h}D \leq {}_{h,l}D$, one can get

$$p_h^A(Y_s - {}_{h,h}D) - \phi_h^A \geq p_h^A(Y_s - {}_{h,l}D) - \phi_h^A.$$

If the contracts with ${}_{h,h}D$ and ${}_{h,l}D$ are feasible, the contract with repayment ${}_{h,h}D$ offers higher expected payoff to a type A borrower than the contract with repayment ${}_{h,l}D$.

From ${}_{h,h}D \leq {}_{h,l}D$, one can also get

$$p_l^B(Y_s - {}_{h,h}D) \geq p_l^B(Y_s - {}_{h,l}D).$$

If contracts with repayment ${}_{h,h}D$ are feasible, h is preferable to borrowers of type B , we have

$$p_h^B(Y_s - {}_{h,h}D) - \phi_h^B \geq p_l^B(Y_s - {}_{h,h}D). \text{ Thus we get}$$

$$p_h^B(Y_s - {}_{h,h}D) - \phi_h^B \geq p_l^B(Y_s - {}_{h,l}D).$$

If the contracts with ${}_{h,h}D$ and ${}_{h,t}D$ are feasible, the contract with repayment ${}_{h,h}D$ offers higher expected payoff to a type B borrower than the contract with repayment ${}_{h,t}D$. Since the contract with repayment ${}_{h,h}D$ offers higher expected payoff to both types of borrower than the contract with repayment ${}_{h,t}D$, the contract with repayment ${}_{h,h}D$ will be signed and high effort will be made. The set of equilibrium condition for a type A borrower to choose high effort and a type B borrower to choose low effort should exclude the possibility of (h, h) being an equilibrium. Thus, we get $R2 = S2 \setminus S1 \cap S0$. By the similar argument we can get

$$R3 = S3 \setminus (S2 \cup S1) \cap S0$$

$$R4 = S4 \setminus (S3 \cup S2 \cup S1) \cap S0.$$

APPENDIX B

The proof of proposition 2 follows.

To start with, let $d5$ and $d6$ be defined as follows. $d5 = \text{Max}\{c5a, c5b\}$, where $c5a$ is the lower bound of λ_0 which satisfies the following inequality:

$$\begin{aligned} p_h^A(Y_s - {}_{h,h}D_1) - \phi_h^A + p_h^A(p_{ea_2(S_1)}^A(Y_s - D_2(S_1)) - \phi_{ea_2(S_1)}^A) + (1 - p_h^A)(p_{ea_2(F_1)}^A(Y_s - D_2(F_1))) - \phi_{ea_2(F_1)}^A \geq \\ p_l^A(Y_s - {}_{h,h}D_1) + p_l^A(p_{ea_2(S_1)}^A(Y_s - D_2(S_1)) - \phi_{ea_2(S_1)}^A) + (1 - p_l^A)(p_{ea_2(F_1)}^A(Y_s - D_2(F_1))) - \phi_{ea_2(F_1)}^A \end{aligned} \quad (B1)$$

$c5b$ is the lower bound of λ_0 which satisfies the following inequality:

$$\begin{aligned} p_h^B(Y_s - {}_{h,h}D_1) - \phi_h^B + p_h^B(p_{eb_2(S_1)}^B(Y_s - D_2(S_1)) - \phi_{eb_2(S_1)}^B) + (1 - p_h^B)(p_{eb_2(F_1)}^B(Y_s - D_2(F_1))) - \phi_{eb_2(F_1)}^B \geq \\ p_l^B(Y_s - {}_{h,h}D_1) + p_l^B(p_{eb_2(S_1)}^B(Y_s - D_2(S_1)) - \phi_{eb_2(S_1)}^B) + (1 - p_l^B)(p_{eb_2(F_1)}^B(Y_s - D_2(F_1))) - \phi_{eb_2(F_1)}^B \end{aligned} \quad (B2)$$

$d6 = \text{Min}\{c6a, c6b\}$, where $c6a$ is the upper bound of λ_0 which satisfies the following inequality:

$$\begin{aligned}
& p_h^A(Y_s -_{l_1, l_1} D_1) - \phi_h^A + p_h^A(p_{ea_2(S_1)}^A(Y_s - D_2(S_1)) - \phi_{ea_2(S_1)}^A) + (1 - p_h^A)(p_{ea_2(F_1)}^A(Y_s - D_2(F_1))) - \phi_{ea_2(F_1)}^A \leq \\
& p_l^A(Y_s -_{l_1, l_1} D_1) + p_l^A(p_{ea_2(S_1)}^A(Y_s - D_2(S_1)) - \phi_{ea_2(S_1)}^A) + (1 - p_l^A)(p_{ea_2(F_1)}^A(Y_s - D_2(F_1))) - \phi_{ea_2(F_1)}^A
\end{aligned} \tag{B3}$$

$c6b$ is the upper bound of λ_0 which satisfies the following inequality:

$$\begin{aligned}
& p_h^B(Y_s -_{l_1, l_1} D_1) - \phi_h^B + p_h^B(p_{eb_2(S_1)}^B(Y_s - D_2(S_1)) - \phi_{eb_2(S_1)}^B) + (1 - p_h^B)(p_{eb_2(F_1)}^B(Y_s - D_2(F_1))) - \phi_{eb_2(F_1)}^B \leq \\
& p_l^B(Y_s -_{l_1, l_1} D_1) + p_l^B(p_{eb_2(S_1)}^B(Y_s - D_2(S_1)) - \phi_{eb_2(S_1)}^B) + (1 - p_l^B)(p_{eb_2(F_1)}^B(Y_s - D_2(F_1))) - \phi_{eb_2(F_1)}^B
\end{aligned} \tag{B4}$$

Assume both contracts with $_{h_1, h_1} D_1$, and $_{l_1, l_1} D_1$ are available to borrowers of type B . Since

$$_{h_1, h_1} D_1 \leq_{l_1, l_1} D_1,$$

we have

$$\begin{aligned}
& p_l^B(Y_s -_{h_1, h_1} D_1) + p_l^B(p_{eb_2(S_1)}^B(Y_s - D_2(S_1)) - \phi_{eb_2(S_1)}^B) + (1 - p_l^B)(p_{eb_2(F_1)}^B(Y_s - D_2(F_1))) - \phi_{eb_2(F_1)}^B \\
& \geq \\
& p_l^B(Y_s -_{l_1, l_1} D_1) + p_l^B(p_{eb_2(S_1)}^B(Y_s - D_2(S_1)) - \phi_{eb_2(S_1)}^B) + (1 - p_l^B)(p_{eb_2(F_1)}^B(Y_s - D_2(F_1))) - \phi_{eb_2(F_1)}^B
\end{aligned}$$

This inequality together with B2, we get

$$\begin{aligned}
& p_h^B(Y_s -_{h_1, h_1} D_1) - \phi_h^B + p_h^B(p_{eb_2(S_1)}^B(Y_s - D_2(S_1)) - \phi_{eb_2(S_1)}^B) + (1 - p_h^B)(p_{eb_2(F_1)}^B(Y_s - D_2(F_1))) - \phi_{eb_2(F_1)}^B \geq \\
& p_l^B(Y_s -_{l_1, l_1} D_1) + p_l^B(p_{eb_2(S_1)}^B(Y_s - D_2(S_1)) - \phi_{eb_2(S_1)}^B) + (1 - p_l^B)(p_{eb_2(F_1)}^B(Y_s - D_2(F_1))) - \phi_{eb_2(F_1)}^B
\end{aligned}$$

Thus, if both contracts with $_{h_1, h_1} D_1$, and $_{l_1, l_1} D_1$ are available to borrowers of type B , the contract with $_{h_1, h_1} D_1$ will be chosen by borrowers of type B .

The remaining parts (b) and (c) can be proved by using the same argument.

REFERENCES

- Bester, H. 1985, Screening vs. Rationing in Credit Markets with Imperfect Information, *American Economic Review*, 75, 850-855.
- Bester, H, and M. Hellwig, 1987, Moral Hazard and Equilibrium Credit Rationing, in *Agency Theory, Information and Incentives*, Bamberg, K. and K. Spremann, eds. Berlin: Springer-Verlag.
- Bizer, S.D. & P.M. DeMarzo, 1992, Sequential Banking. *Journal of Political Economy*, 100, 41-61.
- Broecker, T., 1990, Creditor-Worthiness Tests and Interbank Competition. *Econometrica* 58, 429-452.
- Diamond, D.W., 1989, Reputation Acquisition in Debt Markets, *Journal of Political Economy*, 97, 828-862.
- Gale, D. and M. Hellwig, 1985, Incentive Compatible Debt Contracts: The One-Period Problem, *Review of Economic Studies*, 52, 647-663.
- Kreps, D.M. & R. Wilson, 1982, Reputation and Imperfect Information, *Journal of Economic Theory*, 27, 253-279.
- Nakamura, L.I. & S. Shaffer, 1991, Optimal Acceptance Policies for Journals. Working Paper No. 91-15. Federal Reserve Bank of Philadelphia.
- Ross S., 1977, The Determination of Financial Structure: The Incentive Signaling Approach, *The Bell Journal of Economics and Management Science*, 8, 23-40.
- Stiglitz, J. & A. Weiss, 1981, Credit Rationing in Markets with Imperfect Information, *American Economic Review*, 71, 393-410.
- Stiglitz, J. & A. Weiss, 1983, Incentive Effects of Termination: Applications to the Credit and Labor Market, *American Economic Review*, 73, 912-927.

Chapter 3. Asymptotic Unanimity with Exogenous Noise

1. INTRODUCTION

The assumption that firms are to maximize their net market value is usually made in the capital market. This assumption implies that there is a unanimous agreement upon the objectives of a firm among shareholders. Unfortunately, this is not always true. Since there are many shareholders, interests among them may be different; therefore, they may have different opinions about how the firm should be operated. Ekern and Wilson (1974) have shown that if a project would alter the state-distributions of returns, the unanimity would not be obtained.

Since shareholders are the owners of funds, a manager is delegated to carry out shareholders preferred options. If a unanimous agreement cannot be reached among shareholders, some shareholders must not prefer their firm to maximize its net market value. The manager of a firm will not know what to pursue.

There are a lot of papers dealing with the unanimity problem, and an extensive survey can be found in Baron (1979). In the present paper, we are interested in the problem of "asymptotic unanimity." We define asymptotic unanimity as the property that net market maximization of a firm tends to coincide with the interest of all shareholders of the firm as the economy becomes very competitive.

The asymptotic unanimity property has been first introduced by Bester (1982). Using a mean-variance model, Bester has shown that the initial shareholders want to maximize the net market value of the firm as the replica of the economy becomes infinitely large.

Bester includes no information problem in his model. There is a stream of literature tackling the information problem in a competitive stock market without considering the unanimity problem. Grossman (1975) assumes that individuals observe a noisy private signal about the return of the risky assets, but concludes that current risky asset prices reveal enough information to individuals who, therefore, have no reason to acquire private information.

Hellwig (1980) extends Grossman's (1975) model and introduces an additional exogenous noise into the model. In addition to the noise accompanying the private signals, there is a noisy exogenous supply of risky assets in Hellwig's model. He showed the relative importance of information available to an individual depends on the individual's preference.

Adding asymmetric information to Bester's original model, Haller (1984) has shown the asymptotic unanimity property holds without exogenous noise. Following his line, we distinguish several different types of investors. Each investor of a particular type receives noisy private information about the future return of the firm. We also adopt Hellwig's assumption -- there is an exogenous random supply of risky assets. In our model, we will let the exogenous noisy random supply grow proportionally or disproportionately with the replica of the economy. We are interested in whether the asymptotic unanimity property depends on how the noise grows with the replica of the economy.

We show that initial shareholders of a firm tend to approve maximization of net market value of the firm as the replica of the economy becomes larger and larger, in case the noisy supply grows proportionally with the size of the economy. With restrictions of some parameters in our model, the unanimity property still holds when the exogenous noise grows disproportionately with the economy.

The present paper is organized as follows. In section 2, we introduce the basic model of the replica economy. In section 3, we introduce asymmetric information and let the noise grow with the replica of the economy. We will prove that the asymptotic unanimity property holds, when the exogenous noise grows proportionately with the economy. In section 4, we extend the asymptotic unanimity property to a particular case where the exogenous noise grows disproportionately with the economy. Some concluding remarks are made in section 5.

2. BASIC MODEL

Our basic model is a modification of Bester's (1982) two-period model with periods 0 and 2. We also include a random supply in our model as pioneered in Hellwig (1980). In period 0, investors are endowed with initial wealth, which can be used for the production of a commodity¹ in period 2. At date 2, the spot price of this commodity is random, and its distribution is given by the random variable $\tilde{p} \in \mathbb{R}_+$. If wealth is not used for production, it may be saved with interest rate $\delta - 1$.

There exist F different types of firms ($j = 1, \dots, F$). In order to produce an output $x_j \in \mathbb{R}_{++}$ at date 2, firm j has to invest an input of $C_j(x_j)$ units of wealth at date 0. For each j , $C_j(\cdot)$ is assumed to be convex and differentiable. In the l -fold replica of the original economy each type of firms appears l times. The k -th firm of type j will be indexed by " jk ". Thus, x_{jk} is the output of firm jk . In order to produce an output x_{jk} , firm jk needs to invest $C_j(x_{jk})$ units of wealth. The market value of firm jk is V_{jk} .

There are A different types of investors ($a = 1, \dots, A$), whose problem is to allocate their initial wealth between shares of the $F \cdot l$ firms and borrowing or lending of wealth. In

¹The are n commodities in Bester's paper. To simplify our analysis, we let $n = 1$ in this paper.

the l -fold replica, each type of investors appears l times and " ak " indicates the k -th investor of type a . Investor ak has initial endowment $e_{ak} \in \mathbb{R}_+$. We assume all investors of the same type own the same amount of endowment, i.e., $e_{ak} = e_{aq} = e_a$. Let $s_{ak} = (s_{ak,11}, \dots, s_{ak,F1}) \in \mathbb{R}^{F1}$ be the share holding plan of agent ak where $s_{ak,jk}$ is agent ak 's share of firm jk . At date 0, agent ak 's initial share holding of firms is $\bar{s}_{ak} \in \mathbb{R}^{F1}_+$. We assume that the k -th investor of type a originally holds only shares of the k -th firm of each type, i.e., $\bar{s}_{ak,jk} = \bar{s}_{a,j}$ for all a, j, k , and $\bar{s}_{ak,jq} = 0$ for $q \neq k$. Let d_{ak} denote the amount of investor ak 's lending. Then, his or her budget set is given by all portfolios (s_{ak}, d_{ak}) that satisfy

$$s_{ak}V + d_{ak} = e_{ak} + \bar{s}_{ak}(V - C), \quad (1)$$

where $V = (V_{11}, \dots, V_{F1}) \in \mathbb{R}^{F1}$ is the vector of firms' market values and $C = (C_1(x_{11}), \dots, C_1(x_{11})) \in \mathbb{R}^{F1}$ the vector of firms' inputs.

Let the distribution of spot price of the commodity, \bar{p} , be normal with mean $m \in \mathbb{R}_{++}$ and variance $M \in \mathbb{R}_+$. Let $X = (x_{11}, \dots, x_{F1})$ be the vector of firms' production plans. Then the joint distribution of the firms' returns at date 2 is normal with mean $\bar{m} = (mx_{11}, \dots, mx_{F1}) \in \mathbb{R}^{F1}_+$ and variance-covariance matrix

$$\bar{M} = MX'X = M \begin{pmatrix} x_{11}x_{11} & \cdot & \cdot & \cdot & x_{11}x_{F1} \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ \cdot & & & & \cdot \\ x_{F1}x_{11} & \cdot & \cdot & \cdot & x_{F1}x_{F1} \end{pmatrix}.$$

We assume that each consumer's utility of period 1 wealth exhibits constant absolute risk aversion $r_a \in \mathbb{R}_{++}$. Then ak 's preferences over portfolios are represented by the utility index

$$u_{ak}(s_{ak}, d_{ak}) = \bar{m}s_{ak} + \delta d_{ak} - \frac{1}{2}r_a s_{ak}' \bar{M} s_{ak}. \quad (2)$$

Asymmetric Information

The spot price of the risky asset, \tilde{p} , is unknown to investors ex ante. However, investors have some incomplete and asymmetric information about the price. Each investor of type α receives a private signal \tilde{t}_α with noise. Specifically $\tilde{t}_\alpha = \tilde{p} + \tilde{\varepsilon}_\alpha$, and $\tilde{\varepsilon}_\alpha \in \mathbb{R}$ is normally distributed with mean 0 and variance N . $\tilde{p}, \tilde{\varepsilon}_1, \dots, \tilde{\varepsilon}_A$ are assumed mutually independent random variables. N is positive.

Exogenous Noise

There is an exogenous noise in the economy. Following Hellwig (1980), we assume that the economy is subject to a random shock -- an exogenously given supply \tilde{Z}^l of the risky asset \tilde{p} in addition to the endogenous supply x .² The noisy supply is assumed to take the form $\tilde{Z}^l = l^\nu \tilde{z}_*$ in the l -fold replica economy, with a constant $\nu > 0$. Thus the total supply is

$$l^\nu \tilde{z} = l^\nu \tilde{z}_* + x \text{ where } \nu \in \mathbb{R}_+.$$

We get

$$\tilde{z} = \tilde{z}_* + x / l^\nu.$$

\tilde{z}_* is a random variable, which is independent of l . If $\nu = 1$, the noise grows proportionally with the replica of the economy. The noise grows less than proportionally relative to the size of the economy if $\nu < 1$, and the noise grows more than proportionally relative to the size of the economy if $\nu > 1$. The random vector $(\tilde{p}, \tilde{z}_*, \tilde{\varepsilon}_1, \dots, \tilde{\varepsilon}_A)$ has a normal distribution with mean $(m, \bar{z}_*, 0, \dots, 0)$ and non-singular diagonal variance-covariance matrix Γ with diagonal (M, Δ, N, \dots, N) .

²The noisy exogenous supply of risky asset was first adopted by Hellwig (1980).

Let W^l denote the value of one unit of output in the l -fold replica economy. Investors will not only use the private information but also the additional information provided by W^l . W^l is an equilibrium valuation function in a rational expectations equilibrium where investors have taken account of the information revealed by W^l , when forming their demand.

In a rational expectations equilibrium, it can be shown that W^l will be a function of the pooled information $t = (t_1, \dots, t_A)$ and the vector of production plans $X = (x_{I1}, \dots, x_{FI})$. We consider $W^l(t, X)$ of the linear form

$$\tilde{W}^l = \pi_0(x) + \sum_a \pi_a(x) \tilde{t}_a - \gamma(x) \tilde{z}, \quad (3)$$

where $\tilde{t}_a = \tilde{p} + \tilde{\varepsilon}_a$, and $\tilde{z} = \tilde{z}_* + x / l^v$. To show the existence of an equilibrium of type (3), we proceed like Hellwig (1980).

Lemma 1. The system of equations

$$Q_a = \frac{l^{1-v} N (\sum_b Q_b^2 - Q_a Q) + \Delta}{N r_a (\sum_{b \neq a} Q_b^2 + \Delta)}, \quad a = 1, \dots, A; \quad (4a)$$

$$Q = \sum_a Q_a; \quad (4b)$$

has a solution Q_1, \dots, Q_A with $0 < Q_a < \frac{l^{1-v}}{N r_a}$.

Proof. (4) corresponds to a system of equations in Hellwig (1980). Following the procedure corresponding to lemma 3.1 of Hellwig (1980), we obtain the proof.

For Q_1, \dots, Q_A, Q as in lemma 1. In the following, we write π_0 for $\pi_0^l(x)$, γ for $\gamma^l(x)$ and define $W = \tilde{W}^l$.

Lemma 2. Define $\bar{z} = \bar{z}_* + x / l^v$ and $\gamma, \pi_0, \pi_1, \dots, \pi_A$ by

$$\frac{1}{\gamma} \left[1 + l^{1-\nu} \sum_a \frac{1}{r_a} \frac{\sum_{b \neq a} Q_b}{N \sum_{b \neq a} Q_b^2 + \Delta} \right] \quad (5a)$$

$$= \delta^{l^{1-\nu}} \sum_a \frac{(M+N)(N \sum_{b \neq a} Q_b^2 + \Delta) + (Q - Q_a)^2 MN}{M N r_a (N \sum_{b \neq a} Q_b^2 + \Delta)};$$

$$\frac{l}{\gamma} \pi_0 = l^{1-\nu} \sum_a \frac{l}{r_a} \left[\frac{m}{M} - \frac{\sum_{b \neq a} \pi_b (\pi_0 - \gamma z)}{N \sum_{b \neq a} \pi_b^2 + \gamma^2 \Delta} \right]; \quad (5b)$$

$$\pi_a = \gamma Q_a \text{ for } a = 1, \dots, A. \quad (5c)$$

Then $\gamma, \pi_0, \pi_1, \dots, \pi_A$ constitute a linear rational expectations equilibrium of the form (3).

Proof. Suppose (3) is valid. Define $\pi^l(x) = \sum_{a=1}^A \pi_a(x)$. We will write π for $\pi^l(x)$. Define

$$\tilde{t} = t = (t_1, \dots, t_A). \text{ Following the procedure in Grossman (1976), investor } ak \text{ demands}$$

$$s_{ak}(W) = \frac{E(\tilde{p}|t_a, W) - \delta W}{r_a \text{Var}((\tilde{p}|t_a, W))} \quad (6)$$

units of the asset \tilde{p} .

Given (3), the triple $(\tilde{p}, \tilde{t}_a, \tilde{W})$ has normal distribution with mean $(m, m, \pi_0 + \pi m - \gamma \bar{z})$ and variance-covariance matrix

$$\Psi = \begin{pmatrix} M & M & \pi M \\ M & M+N & \pi M + \pi_a N \\ \pi M & \pi M + \pi_a N & \pi^2 M + N \sum \pi_b^2 + \gamma^2 \Delta \end{pmatrix}.$$

Provided $\gamma \neq 0$, then Ψ is non-singular. Conditional on (t_a, W) , \tilde{p} is normally distributed with mean E_a and variance β_a , where

$$E_a = E(\tilde{p}|t_a, W) = m + (M, \pi M) \begin{pmatrix} M+N & \pi M + \pi_a N \\ \pi M + \pi_a N & \pi^2 M + N \sum \pi_b^2 + \gamma^2 \Delta \end{pmatrix}^{-1} \begin{pmatrix} t_a - m \\ W - \pi_0 + \pi m - \gamma \bar{z} \end{pmatrix};$$

$$\beta_a = \text{Var}(\tilde{p}|t_a, W) = M - (M, \pi M) \begin{pmatrix} M+N & \pi M + \pi_a N \\ \pi M + \pi_a N & \pi^2 M + N \sum \pi_b^2 + \gamma^2 \Delta \end{pmatrix}^{-1} \begin{pmatrix} M \\ \pi M \end{pmatrix}.$$

Define

$$D_a \equiv \begin{vmatrix} M+N & \pi M + \pi_a N \\ \pi M + \pi_a N & \pi^2 M + N \sum_{b \neq a} \pi_b^2 + \gamma^2 \Delta \end{vmatrix}$$

$$= (M+N)(N \sum_{b \neq a} \pi_b^2 + \gamma^2 \Delta) + (\pi - \pi_a)^2.$$

We obtain

$$E_a = \alpha_{a0} + \alpha_{a1} t_a + \alpha_{a2} W, \quad (7)$$

$$\beta_a = \frac{MN}{D_a} \left[N \sum_{b \neq a} \pi_b^2 + \gamma^2 \Delta \right], \quad (8)$$

where

$$\alpha_{a0} = \frac{mN}{D_a} \left[N \sum_{b \neq a} \pi_b^2 + \gamma^2 \Delta \right] - \frac{MN}{D_a} \sum_{b \neq a} \pi_b (\pi_0 - \gamma \bar{z})^2; \quad (9)$$

$$\alpha_{a1} = \frac{N}{D_a} \left[N \left(\sum_b \pi_b^2 - \pi_a \pi \right) + \gamma^2 \Delta \right]; \quad (10)$$

$$\alpha_{a2} = \frac{MN}{D_a} \sum_{b \neq a} \pi_b. \quad (11)$$

Let z be the realization of \tilde{z} . The total supply of the risk asset \tilde{p} is $l^v z$, and the total demand for the risk asset is $\sum_{a,k} s_{a,k}(W)$. We get the market clearing condition:

$$l^v z = \sum_{a,k} s_{a,k}(W). \quad (12)$$

Since $\sum_{a,k} s_{a,k}(W) = l \sum_a \frac{E(p/t, W) - \delta W}{r_a \beta_a}$, we obtain

$$z = l^{1-v} \sum_{a,k} s_{a,k}(W)$$

$$= l^{1-v} \sum_a \frac{E(p/t, W) - \delta W}{r_a \beta_a} \quad (13)$$

$$= l^{1-v} \sum_a \frac{\alpha_{a0}}{r_a \beta_a} + l^{1-v} \sum_a \frac{\alpha_{a1}}{r_a \beta_a} t_a + l^{1-v} \sum_a \frac{\alpha_{a2} - \delta W}{r_a \beta_a}.$$

If $\gamma \neq 0$, (3) can be solved for z :

$$z = \frac{l}{\gamma} \pi_0 + \frac{l}{\gamma} \sum_a \pi_a t_a - \frac{W}{\gamma}. \quad (14)$$

Equating coefficients in (13) and (14), we get

$$\frac{l}{\gamma} \pi_0 = l^{1-v} \sum_a \frac{\alpha_{a0}}{r_a \beta_a}, \quad (15)$$

$$\frac{l}{\gamma} \pi_a = l^{l-\nu} \frac{\alpha_{a1}}{r_a \beta_a}, \quad (16)$$

$$-\frac{l}{\gamma} = l^{l-\nu} \sum_a \frac{\alpha_{a2} - \delta}{r_a \beta_a}. \quad (17)$$

Substituting the expressions from (9) - (11) into (15) - (17), we get

$$\frac{l}{\gamma} \pi_0 = l^{l-\nu} \sum_a \frac{l}{r_a} \left[\frac{m}{M} - \frac{\sum_{b \neq a} \pi_b (\pi_0 - \gamma z)}{N \sum_{b \neq a} \pi_b^2 + \gamma^2 \Delta} \right]; \quad (18)$$

$$\frac{l}{\gamma} \pi_a = l^{l-\nu} \frac{N(\sum_b \pi_b^2 - \pi_a \pi) + \gamma^2 \Delta}{N r_a (N \sum_{b \neq a} \pi_b^2 + \gamma^2 \Delta)}; \quad (19)$$

$$-\frac{l}{\gamma} = l^{l-\nu} \left[\sum_a \frac{1}{r_a} \frac{\sum_{b \neq a} \pi_b}{N \sum_{b \neq a} \pi_b^2 + \gamma^2 \Delta} - \frac{(M+N)(N \sum_{b \neq a} \pi_b^2 + \gamma^2 \Delta) + (\pi - \pi_a)^2 MN}{M N r_a (N \sum_{b \neq a} \pi_b^2 + \gamma^2 \Delta)} - \delta \sum_a \frac{1}{r_a} \right]. \quad (20)$$

(4) together with (5) give a solution to (18) - (20). Thus γ , π_0 , π_1, \dots, π_A defined by (5) constitute a linear rational expectations equilibrium.

3. THE NOISY SUPPLY GROWS PROPORTIONALLY WITH THE ECONOMY

We are interested in the effect of noisy exogenous supply on the asymptotic unanimity property in the l -replica economy. We define $(x_1^l, \dots, x_F^l) \in \mathbb{R}_+^F$ to be a profile of most preferred production plans of shareholder jk given l and the rational expectations equilibrium (3) - (5), if $x_{jk} = \hat{x}_j^l$ maximizes $\hat{u}_{ak}^l(\hat{X}^l | x_{jk})$ for $j = 1, \dots, F$ where $\hat{X}^l = (\hat{x}_1^l, \dots, \hat{x}_F^l, \dots, \hat{x}_1^l, \dots, \hat{x}_F^l)$. As stated above, the noise supply may grow proportionally, less than proportionally or more than proportionally relative to the size of the economy, which is determined by the parameter ν in our model. We will prove all initial shareholders tend to approve the firm maximizing its net market value as the market

becomes more and more competitive, i.e., as $l \rightarrow \infty$, if the exogenous noise grows proportionally with the size of the economy.

Proposition 1. Let $\nu = 1$. For some sequence (x_1^l, \dots, x_F^l) of profiles of shareholder ak 's preferred production plans given t and the rational expectations equilibrium (3) - (5), let

$$\bar{x}_j = \lim_{l \rightarrow \infty} \hat{x}_j^l, j = 1, \dots, F.$$

Then for $j = 1, \dots, F$,

$$\bar{x}_j \in \arg \max \bar{s}_{a,j} (\lim_{l \rightarrow \infty} V_{jk}^l (\bar{X} | x_{jk}) - C_j (x_{jk})),$$

where $\bar{X}^l = (\bar{x}_1, \dots, \bar{x}_F, \dots, \bar{x}_1, \dots, \bar{x}_F) \in \mathbb{R}_+^{Fl}$ and V^l belongs to the rational expectations equilibrium (3) - (5).

Proof. From the proof of lemma 2, we can see that $\alpha_{a0}, \alpha_{a1}, \alpha_{a2}, \beta_a, \pi_0, \pi_1, \dots, \pi_A, \gamma$ are independent of l . From (3) we get $W = g_1 + g_2 x / l$ where $g_2 < 0$ and g_1, g_2 are independent of l and x . Since $V_{ij} = W x_{ij}$, we get

$$V_{ij} = g_1 x_{ij} + g_2 \frac{x}{l} x_{ij}. \quad (21)$$

We also obtain

$$\begin{aligned} E_a - W &= \alpha_{a0} + \alpha_{a1} t_a + \alpha_{a2} W - W \\ &= \alpha_{a0} + \alpha_{a1} t_a + (\alpha_{a2} - 1)W \\ &= \alpha_{a0} + \alpha_{a1} t_a + (\alpha_{a2} - 1)(g_1 + g_2 x / l) \\ &= g_3 + g_4 \frac{x}{l}. \end{aligned}$$

From investor ak 's budget constraint

$$s_{ak} W + d_{ak} = e_{ak} + \bar{s}_{ak} (V - C),$$

we get $d_{ak} = -s_{ak} W + e_{ak} + \bar{s}_{ak} (V - C)$.

Investor ak 's expected utility is

$$\begin{aligned}
\hat{u}_{ak}(x) &= \delta d_{ak} + s_{ak} E_a - \frac{1}{2} r_a s_{ak} \beta_a s_{ak} \\
&= -\delta s_{ak} W + \delta e_{ak} + \delta \bar{s}_{ak} (V - C) + s_{ak} E_a - \frac{1}{2} r_a s_{ak} \beta_a s_{ak} \\
&= \delta e_{ak} + \delta \bar{s}_{ak} (V - C) + \frac{(E_a - \delta W)^2}{2r_a \beta_a}.
\end{aligned} \tag{22}$$

The first order condition with respect to x_{jk} for investor ak to maximize his expected utility \hat{u}_{jk} at \hat{X} is

$$\begin{aligned}
L_j^* &\equiv \delta \bar{s}_{a,j} \frac{\partial}{\partial x_{jk}} (V_{jk} - C_j(x_{jk})) + \frac{\partial}{\partial x_{jk}} \left[\delta \sum_{h \neq j} \bar{s}_{a,h} (V_{h,k} - C_h(x_{hk})) + \frac{(E_a - \delta W)}{2r_a \beta_a} \right] \\
&\equiv K_1 + K_2 = 0.
\end{aligned} \tag{23}$$

We need to prove $K_1 = \delta \bar{s}_{a,j} \frac{\partial}{\partial x_{jk}} (V_{jk} - C_j(x_{jk})) = 0$ as $l \rightarrow \infty$. Since $L_j^* = 0$, $K_2 = 0$ as

$l \rightarrow \infty$ will yield the assertion. From (23) and (21), we get

$$\begin{aligned}
K_2 &= \frac{\partial}{\partial x_{jk}} \left[\delta \sum_{h \neq j} \bar{s}_{a,h} (V_{h,k} - C_h(x_{hk})) + \frac{(E_a - \delta W)}{2r_a \beta_a} \right] \\
&= \delta g_2 \sum_{h \neq j} \bar{s}_{a,h} x_{hk} \frac{1}{l} + g_4 \frac{1}{r_a \beta_a} (g_3 + g_4 \frac{x}{l}) \frac{1}{l}.
\end{aligned}$$

Taking limits yields $K_2 = 0$ as $l \rightarrow \infty$. Thus $K_1 = 0$ as $l \rightarrow \infty$.

4. THE NOISY SUPPLY GROWS DISPROPORTIONALLY WITH THE SIZE OF THE ECONOMY

We have proved that the asymptotic unanimity property holds if the noisy supply grows proportionally with the size of the economy. We are wondering if the property still holds as the noisy supply grows disproportionately with the size of the economy. The noise may grow less than proportionally relative to the size of the economy if $\nu < 1$ or more than proportionally relative to the size of the economy if $\nu > 1$.

In the following, we assume $F = 1$, $C(s) = 1/3s^2$, $A=2$, $\bar{s}_1 = \bar{s}_2 = 1/2$, $r_1 = r_2 = 1$, $e_1 = e_2 = 0$, $\delta = 1$, $m = 1$, $M = 1$, $N = 1$, $\Delta = 1$, $\bar{z}_* = 1$, $\tilde{z}_* = 0$, t_1 and $t_2 \in \mathbb{R}$. We will show that the asymptotic unanimity property still holds for several $\nu \neq 1$.

In the l -fold replica economy, let a most preferred production plan of investor ak be defined as before:

$x_{jk} = \hat{x}_j^l$ maximizes $\hat{u}_{ak}^l(\hat{X}^l | x_{jk})$ for $j = 1, \dots, F$ where $\hat{X}^l = (\hat{x}_1^l, \dots, \hat{x}_F^l, \dots, \hat{x}_1^l, \dots, \hat{x}_F^l)$. Let

$$\bar{x}_j = \lim_{l \rightarrow \infty} \hat{x}_j^l, j = 1, \dots, F.$$

We can show

$$\bar{x}_j \in \arg \max_{\bar{x}_{a,j}} (\lim_{l \rightarrow \infty} V_{jk}^l(\bar{X} | x_{jk}) - C_j(x_{jk})) \text{ for } j = 1, \dots, F, \nu = 1/2, 1, 3/2, 2.$$

Proof. From (4), we get

$$Q_a^3 + Q_a - l^{1-\nu} = 0; \tag{24}$$

$$\pi_a = \gamma Q_a. \tag{25}$$

From (18) and (20) we get

$$\pi_0 = \frac{2l^{1-\nu} \gamma (1 + Q_a^2 + Q_a \bar{z})}{1 + 2l^{1-\nu} Q_a + Q_a^2}, \tag{26}$$

$$\gamma = \frac{l^{\nu-1} (1 + Q_a^2) + 2Q_a}{2(2 + 3Q_a^2)}. \tag{27}$$

Investor ak 's expected utility is

$$\begin{aligned} u_{ak}(x) &= \delta e_a + \delta \bar{s}_{ak} (V - C) + \frac{1}{2r_a \beta_a} (E_a - \delta W)^2 \\ &= e_a + \frac{1}{2} (V - C) + \frac{1}{2\beta_a} (E_a - W)^2, \end{aligned} \tag{28}$$

where

$$V_k = Wx_k,$$

$$W = \pi_0 + \sum_a \pi_a t_a - \gamma z,$$

$$D_a = \gamma^2 (3Q_a^2 + 2),$$

$$E_a = \alpha_{a0} + \alpha_{a1} t_a + \alpha_{a2} W,$$

$$\beta_a = \frac{1+Q_a^2}{2+3Q_a^2},$$

$$\alpha_{a0} = \frac{1}{D_a}(\pi_a^2 + \gamma^2 - \pi_a(\pi_0 - \gamma z)),$$

$$\alpha_{a1} = \frac{\gamma^2}{D_a},$$

$$\alpha_{a2} = \frac{Q_a}{\gamma(2+3Q_a^2)},$$

Let $L_k = 0$ be the necessary condition with respect x_k for the maximization of u_{ak} .

$$L_k = \frac{1}{2} \frac{\partial(V_k - C(x_k))}{\partial x_k} + \frac{1}{2} \frac{\partial}{\partial x_k} \frac{(E_a - W)^2}{\beta_a}$$

$$= \frac{1}{2}(L1_k + L2_k) = 0;$$

where

$$L1_k = \frac{\partial(V_k - C(x_k))}{\partial x_k}, \quad (29)$$

$$L2_k = \frac{\partial}{\partial x_k} \frac{(E_a - W)^2}{\beta_a}. \quad (30)$$

To show that a firm's initial shareholders want to maximize the firm's net market value asymptotically, we need to prove $L1_k \rightarrow 0$ as $l \rightarrow \infty$. Since $L_k = 0$, $L2_k \rightarrow 0$ as $l \rightarrow \infty$

yields the assertion. Simplifying (30), we get

$$L2_k = \frac{\partial}{\partial x_k} \frac{(E_a - W)^2}{\beta_a}$$

$$= \frac{t_1(2l - l^v Q_a - l^v Q_a^3) - t_2(l^v Q_a + l^v Q_a^3) + (1 + Q_a^2)lx_k}{2l^2(2 + 3Q_a^2)}. \quad (31)$$

The only positive solution of (24) is

$$Q_a = -\frac{\left(\frac{2}{3}\right)^{\frac{1}{3}} l^{\frac{\nu}{3}}}{(9l + \sqrt{3}\sqrt{27l^2 + 4l^{2\nu}})^{\frac{1}{3}}} + \frac{(9l + \sqrt{3}\sqrt{27l^2 + 4l^{2\nu}})^{\frac{1}{3}}}{(18)^{\frac{1}{3}} l^{\frac{\nu}{3}}}. \quad (32)$$

Plugging Q_a into equation (31), we find $L2_k \rightarrow 0$ as $l \rightarrow \infty$ for $\nu = 1/2, 1, 3/2, 2$. Thus, in this example a firm's initial shareholders want to maximize the firm's net market value asymptotically, no matter how the noise grows with the economy.

5. CONCLUSION

We have extended Bester's (1982) model to an economy where there exists noise coming from a random exogenous supply of the risky asset. In the current paper, the linear rational expectations equilibrium is adopted to test if net market value maximization becomes desirable as competition increases. We have proved, in general, that if the noise grows proportionally with the size of the economy, the asymptotic unanimity property holds as in Bester's paper. If the noise does not grow proportionally with the size of the economy, although we cannot prove the asymptotic unanimity property in general, we did demonstrate the asymptotic unanimity property in particular cases.

It is a wild guess that, in case of linear rational expectations equilibrium, the unanimity property holds no matter how the noise grows with the economy. However, the model is getting too complex for us to obtain the general result if the noise grows disproportionately to the size of the economy.

³ Q_a depend on parameter ν as $l \rightarrow \infty$. As shown in lemma 2, Q_a is independent of l when $\nu = 1$.

However, if $\nu < 1$, $Q_a = \infty$ as $l \rightarrow \infty$, and if $\nu > 1$, $Q_a = 0$ as $l \rightarrow \infty$.

REFERENCES

- Baron, D. P., 1979, Investment Policy, Optimality, and the Mean Variance Model, *The Journal of Finance* 34, 207-232.
- Bester, H., 1982, On Shareholder Unanimity in the Mean-Variance Model, *Economics Letters*, 10, 363-367.
- Black, F., 1986, Noise, *The Journal of Finance* 91, 529-541.
- Ekern, S. and R. Wilson, 1974, On the Theory of the Firm in an Economy with Incomplete Markets, *The Bell Journal of Economics and Management Science*, 5, 171-184.
- Grossman, S., 1976, On the Efficiency of Competitive Stock Markets Where Trades Have Diverse Information, *The Journal of Finance* 31, 573-585.
- Haller, H., 1984, Asymptotic Net Market Value Maximization, Discussion Paper No 138, Sonderforschungsbereich 138, University of Bonn.
- Hellwig, M. F., 1980, On the Aggregation of Information in Competitive Markets, *Journal of Economic Theory* 22, 477-498.

Chapter 4. Identification of Financial Market Conditions for Inefficient Investment

1. INTRODUCTION

This paper discusses optimality in financial markets. The investment level is chosen to evaluate the deviation of firm's investment from the socially optimal level. Since firms are not interested in the social welfare when making investments, usually, the optimality is not reached. The factors that cause deviation from optimality and means to obtain the optimality are discussed in the paper.

In the capital market, we survey the two most important papers, Jensen and Long (1972) and Stiglitz (1972). Jensen and Long use the mean and variance model to analyze a situation where firms, which own old projects, have an opportunity to make a new investment. They conclude that firms will choose a level of investment which is lower than the socially optimal level. Since the decision of new investment is made under the influence of the old projects, one may suspect that the inefficiency may come from the effect of old projects. In addition, the authors assume that firms are not allowed to adjust the scale of their old projects, which is also not convincing. To identify the factor of the inefficiency, one can eliminate the old projects in the model. Then the setup is similar to the Stiglitz model. We conclude that the efficiency occurs because each firm is too concerned about the variance of its return than it should be in the optimality condition. If the variance of each firm's return is relative small compared to the covariance of the market, or the market becomes more competitive by increasing the number of firms, then the level of investment approaches the optimal level.

In the loan market, we use a simple setup to model a situation where the value of a project is worth less when taken over by creditors. Thus, if a firm cannot repay its debt, and creditors take over the project, a debt weight loss will occur. Lenders compete in making loans in the credit market; therefore, they acclaim an the expected payoff that breaks even. Since firms are interested solely in maximizing their expected payoff subject to creditors' break even constraint, social optimality should not be expected. Due to the dead weight loss of bankruptcy, the optimal investment level can not be attained if renegotiation is not allowed. However, if borrowers and creditors can renegotiate the repayments if firms do not repay, then creditors will never really take over projects, and the dead weight loss can always be prevented. Thus socially optimal level of investment can always be attained.

The present paper is organized as follows. In section 2, we survey the most important papers dealing with the optimality in the stock market. In section 3, we analyze the optimality in the loan market. Some conclusion remarks are made in section 4.

2. OPTIMALITY IN THE STOCK MARKET

Following Jensen and Long (1972), we use the mean-variance model for the pricing of capital assets. Assume all investors have constant absolute risk aversion⁴ and homogeneous expectations. Investor i has the preference function $U^i = U^i(e_i, v_i)$, where e_i is his expected cash flow, and v_i is the variance of the cash flow. The equilibrium value of the j th firm is

$$V_j = \frac{1}{r} [\bar{D}_j - \lambda \sigma_{jM}] \text{ for all } j = 1, 2, \dots, n, \quad (1)$$

where

⁴ This assumption assure the market price per unit of risk, λ , is constant. See Stiglitz (1972, 1981).

\bar{D}_j = the expected value of the total end-of period cash flow to the stockholders of firm j ,

$r = 1 + i$, where i is the one period riskless rate of interest at which every consumer can borrow or lend,

$\sigma_{jk} = \text{Cov}(\tilde{D}_j, \tilde{D}_k)$ for all j and k ,

$\sigma_{jM} = \sum_k \sigma_{jk}$ = covariance of the total cash flows of the firm, \tilde{D}_j , with \tilde{D}_M the total cash flows from all firms,

$\sigma_M^2 = \text{Var}(\tilde{D}_M)$,

$\lambda = [\bar{D}_M - rV_M] / \sigma_M^2$ = risk discount factor = market price per unit of risk, and

$V_M = \sum_k V_k$ = total market value of all firms.

Equation (1) tells us that in equilibrium the present value of the j th is the expected payment minus an amount occurred from the risk taken by the firm, which is the correlation of the firm's payment with the market times the risk discount factor⁵.

Consider a new investment opportunity that promises a random return per unit of investment, $\tilde{\rho}$, available to all firms. The distribution of the random return per unit of investment for all firms is identical, and has mean $\bar{\rho}$ and variance σ_ρ^2 , i.e.,

$$\bar{\rho}_i = \bar{\rho}_j = \bar{\rho}$$

and $\sigma_i^2 = \sigma_j^2 = \sigma_\rho^2$, for all i and j .

Constant stochastic returns to scale of investment is assumed as in Jensen and Long's paper⁶. Thus if I_j is the amount of investment made by firm j , then $I_j\tilde{\rho}$ is the random return for the firm. The covariance of returns are

⁵ See Fama, E. and Miller, M. (1972).

$\text{Cov}(\tilde{\rho}_i, \tilde{\rho}_j) = \sigma_{ij} = \beta\sigma_\rho^2$, where $0 \leq \beta \leq 1$ for all $i \neq j$,
and $\text{Cov}(\tilde{\rho}, \tilde{D}_j) = \sigma_{j\rho}$.

The Corporate Investment Criterion

After investments are made, the value of firm j is

$$V_j' = \frac{1}{r} \left[(D_j + I_j \bar{\rho}) - \lambda (\sigma_{jM} + \sigma_{j\rho} \sum_k I_k + I_j \sigma_{M\rho} + \beta \sigma_\rho^2 I_j \sum_{k \neq j} I_k + I_j^2 \sigma_\rho^2) \right]. \quad (2)$$

Define $\Delta V_j = V_j' - V_j$, we get

$$\Delta V_j - I_j = \frac{1}{r} \left[(\bar{\rho} - r) I_j - \lambda (\sigma_{j\rho} \sum_k I_k + I_j \sigma_{M\rho} + \beta \sigma_\rho^2 I_j \sum_{k \neq j} I_k + I_j^2 \sigma_\rho^2) \right] \quad (3)$$

Firm j chooses the level of investment to maximize the net market value of its new investment, $\Delta V_j - I_j$. We get the first order condition

$$\frac{1}{r} \left[(\bar{\rho} - r) - \lambda (\sigma_{j\rho} + \sigma_{M\rho} + \beta \sigma_\rho^2 \sum_{k \neq j} I_k + 2 I_j \sigma_\rho^2) \right] = 0,$$

i.e.,

$$(\bar{\rho} - r) - \lambda (\sigma_{j\rho} + \sigma_{M\rho} + \beta \sigma_\rho^2 \sum_{k \neq j} I_k + 2 I_j \sigma_\rho^2) = 0. \quad (4)$$

Solving (4) for I_j , we get

$$I_j^* = \frac{1}{2\lambda\sigma_\rho^2} [(\bar{\rho} - r) - \lambda (\sigma_{j\rho} + \sigma_{M\rho} + \beta \sigma_\rho^2 \sum_{k \neq j} I_k)]. \quad (5)$$

For simplicity, let us assume that all firms make a positive investment⁷. From (5) we get the aggregate investment

⁶ This assumption simplify the analysis, however, it will not affect the result in general. Let $h(I)$ is the expected return of investment I , and $g^2(I)$ is the variance. All we need is to assume $h'(I) \geq 0$, $h''(I) \leq 0$, and $g'(I) \geq 0$, $g''(I) \geq 0$. See Stiglitz (1972).

⁷ Suppose there k firms that do not make new investment, where $0 \leq k \leq n$. We can rearrange the array, let

$$I^v = \sum_j^n I_j^v = \frac{n}{\lambda \sigma_\rho^2 [\beta(n-1) + 2]} [(\bar{\rho} - r) - \lambda \sigma_{M\rho}] - \frac{\sum_1^n \sigma_{j\rho}}{\lambda \sigma_\rho^2 [\beta(n-1) + 2]}. \quad (6)$$

The Social Welfare Criterion

Consider an investor who holds a fraction δ of the market portfolio in equilibrium.

The rate of exchange of expected wealth for variance of wealth is

$$\frac{\frac{\partial e}{\partial \delta}}{\frac{\partial v}{\partial \delta}} = \frac{D_M - rV_M}{2\delta \sigma_M^2} = \frac{\lambda}{2\delta} \quad (7)$$

Suppose that firm j increases its investment I_j . The rate of exchange of expected wealth for variance of wealth for the investor offered by the new investment is

$$\frac{\frac{\partial e}{\partial I_j}}{\frac{\partial v}{\partial I_j}} = \frac{\delta(\bar{\rho} - r)}{2\delta^2 (\sigma_{M\rho} + \beta \sigma_\rho^2 \sum_{k \neq j} I_k + I_j \sigma_\rho^2)} = \frac{(\bar{\rho} - r)}{2\delta (\sigma_{M\rho} + \beta \sigma_\rho^2 \sum_{k \neq j} I_k + I_j \sigma_\rho^2)} \quad (8)$$

Thus the Pareto optimal level of investment is attained when the rate of exchange of expected return for the variance offered by the new investment is equal to the rate of exchange available for investors in the market. Equating (7) and (8), we get

$$(\bar{\rho} - r) - \lambda (\sigma_{M\rho} + \beta \sigma_\rho^2 \sum_{k \neq j} I_k + I_j \sigma_\rho^2) = 0. \quad (9)$$

Equation (9) is the social welfare optimum criterion. Solving it for I_j , we get the optimal investment for firm j

$$I_j = 0 \text{ for } j = 1, 2, \dots, k-1.$$

The aggregate investment is

$$I^v = \sum_{j=k+1}^n I_j^v = \frac{n-k}{\lambda \sigma_\rho^2 [\beta(n-k-1) + 2]} [(\bar{\rho} - r) - \lambda \sigma_{M\rho}] - \frac{\sum_{k+1}^n \sigma_{j\rho}}{\lambda \sigma_\rho^2 [\beta(n-k-1) + 2]}.$$

$$I_j^o = \frac{1}{\lambda\sigma_\rho^2(1-\beta)}[\bar{\rho} - r - \lambda(\sigma_{M\rho} + I\beta\sigma_\rho^2)] \quad (10)$$

Summing up I_j^o for all j , we have

$$I = \sum_{j=1}^n I_j^o = \frac{n}{\lambda\sigma_\rho^2(1-\beta)}[\bar{\rho} - r - \lambda(\sigma_{M\rho} + I\beta\sigma_\rho^2)]$$

Solving the equation above for I , we get

$$I^o = \frac{n}{\lambda\sigma_\rho^2[1+(n-1)\beta]}[\bar{\rho} - r - \lambda\sigma_{M\rho}] \quad (11)$$

Comparing (6) and (11), we know $I^o \geq I^v$. The total investment made by firms tends to be less than the optimal level from the social welfare point of view.

To identify the cause of this inefficiency, we compare the first order condition for firms' value maximization of equation (4) to the social welfare maximization criteria of equation (9). The covariance $\sigma_{j\rho}$ and the variance $I_j\sigma_\rho^2$ have twice the effect⁸ in firms' value maximization as compared with the social welfare criterion. Firms' choices are affected by the covariance between the new investment and its old project. To see this, let us, following Merton and Subrahmanyam (1974), rewrite the value function (2) as

$$V_j'(I_j; I) \equiv V_j(0; I) + I_j g(I) \\ = \frac{1}{r} \left[\bar{D}_j - \lambda(\sigma_{jM} + \sigma_{j\rho} \sum_k I_k) \right] + I_j \frac{1}{r} \left[\bar{\rho} - \lambda(\sigma_{M\rho} + \beta\sigma_\rho^2 \sum_{k \neq j} I_k + I_j \sigma_\rho^2) \right].$$

We define

$$V_j(0; I) \equiv \frac{1}{r} \left[\bar{D}_j - \lambda(\sigma_{jM} + \sigma_{j\rho} \sum_k I_k) \right], \quad (12)$$

and

$$I_j g(I) \equiv I_j \frac{1}{r} \left[\bar{\rho} - \lambda(\sigma_{M\rho} + \beta\sigma_\rho^2 \sum_{k \neq j} I_k + I_j \sigma_\rho^2) \right]. \quad (13)$$

⁸ One should not forget $\sigma_{j\rho}$ is also in $\sigma_{M\rho}$ since $\sigma_{M\rho} = \sigma_{j\rho} + \sum_{i \neq j} \sigma_{i\rho}$.

$V_j(0; I)$ is the market value of all old assets owned by the j th firm, given that the new aggregate investment in the project is I . The market value of old assets is affected by $\sigma_{\rho j}$, the covariance between the firm's random cash flow \tilde{D}_j and the random cash flow per unit of new project. Without the new investment, the value of the j th firm would be

$$V_j(0; 0) = \frac{1}{r} [\bar{D}_j - \lambda \sigma_{\rho M}].$$

Thus the new investment will change the market value of a firm's old assets. Even though a firm does not make any investment, the value of the firm is still changed due to the aggregate investment of the other firms. Since $\lambda > 0$ and $\sigma_{\rho j} \geq 0$, we have

$$V_j(0; I) \leq V_j(0; 0).$$

The value of the old project of firm j will decrease if there is any new investment made by firms.

$g(I)$ is the value per unit of new investment, and $I_j g(I)$ is the value of firm j 's new investment. The value per unit of new investment is also affected by the covariance between the cash flow per unit of new investment and the total cash flow of old assets.

To isolate the investment decision from being annoyed by old projects, we can assume that $\sigma_{\rho j} = 0$, and hence $\sigma_{\rho M} = 0$, as in Jensen and Long's paper. Or we can just assume there is no old project, thus there is no covariance between the old project and new project; in addition, the covariance between old projects, $\sigma_{\rho M}$ does not exist then, either. This change makes the model very similar to the setup in Stiglitz's (1972) paper. Thus we expect results similar to his paper. Without old projects, the value of firm j becomes

$$V_j = \frac{1}{r} \left[I_j \bar{\rho} - \lambda (\beta \sigma_{\rho}^2 I_j \sum_{k \neq j} I_k + I_j^2 \sigma_{\rho}^2) \right],$$

which can be attained from (2) or (13).

From (4), both setups will give us the same first order condition for value maximization for firm j :

$$(\bar{\rho} - r) - \lambda(\beta\sigma_\rho^2 \sum_{k \neq j} I_k + 2I_j \sigma_\rho^2) = 0.$$

From (9), the optimal social welfare condition is changed to

$$(\bar{\rho} - r) - \lambda(\beta\sigma_\rho^2 \sum_{k \neq j} I_k + I_j \sigma_\rho^2) = 0.$$

We get

$$I^v = \frac{n}{\lambda\sigma_\rho^2[\beta(n-1) + 2]}(\bar{\rho} - r),$$

and

$$I^o = \frac{n}{\lambda\sigma_\rho^2[\beta(n-1) + 1]}(\bar{\rho} - r).$$

Comparing I^v and I^o , we obtain an under-investment result -- the aggregate investment made by firms is lower than the socially optimal level with or without pre-existing investment. However, if the variance of each firm's return is relative small compared to the covariance of the market, then the level of investment approximates the optimal level. In addition, if we let the market become more competitive by increasing the number of firms, we get $I^v \rightarrow I^o$ as $n \rightarrow \infty$.

If each firm's decision is independent of the other firms' decision, i.e., $\beta = 0$, then the investment would be

$$I_j^v = \frac{1}{2\lambda\sigma_\rho^2}(\bar{\rho} - r),$$

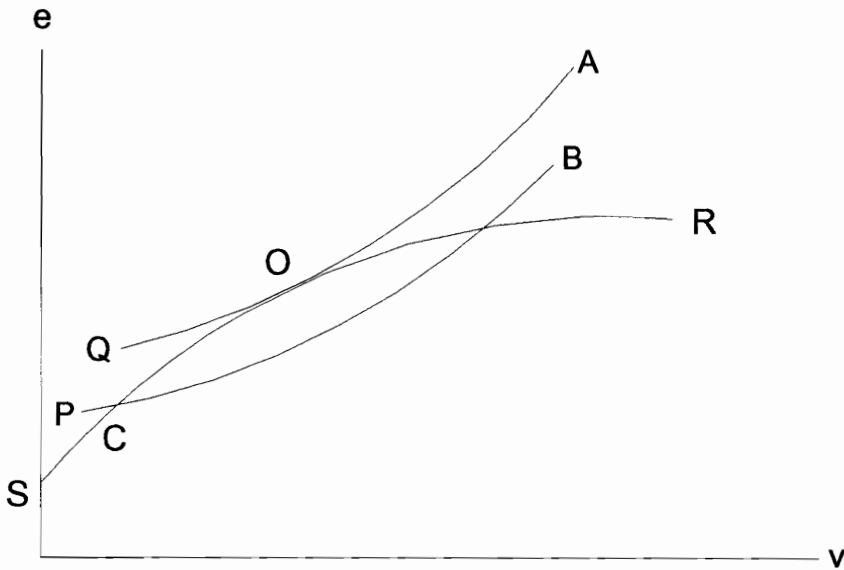
$$I_j^o = \frac{1}{\lambda\sigma_\rho^2}(\bar{\rho} - r).$$

$$I_j^o = 2I_j^v,$$

thus $I^o = 2I^v$.

This result does not depend on the number of firms, n . The optimal investment level is never approximated, even if the number of firms approaches to infinitive.

As shown in Stiglitz (1972), if all investors are identical and all firms in the market are identical, the level of investment for all firms will be identical. The market equilibrium can be illustrated in the following figure.



QA and PB are the indifference curves of a typical investor. SR is the investment opportunity locus. C is the point of value maximization investment and O is the point of optimal social welfare investment. Due to under-investment, the investor will not obtain the highest utility.

Social Wealth Criteria

Summing up V_j for all j , we get the market value of all firms V_M .

The social wealth maximization problem is to maximize

$$\Delta V_M(I) - I^s = \frac{I}{r} \left[(\bar{p} - r) \sum_j I_j - \lambda (2I^s \sigma_{Mp} + \beta \sigma_p^2 \sum_j \sum_{k \neq j} I_j I_k + \sigma_p^2 \sum_j I_j^2) \right] \quad (14)$$

with respect to $\{I_j\}$, subject to: $\sum_j I_j = I^s$. From the first order condition, we get

$I_j = I_k$ for all j and k ⁹.

This implies $I_j = \frac{1}{n}I^s$. We can rewrite (14) as

$$\Delta V_M(I) - I^s = \frac{1}{r} \left[(\bar{\rho} - r)I^s - \lambda \left(2I^s \sigma_{M\rho} + \frac{n-1}{n} I^{s^2} \beta \sigma_\rho^2 + \frac{1}{n} I^{s^2} \sigma_\rho^2 \right) \right] \quad (15)$$

Thus the problem becomes to maximize (15) with respect to I^s , subject to $I^s \geq 0$.

We can derive the first order condition

$$(\bar{\rho} - r) - 2\lambda \left(\sigma_{M\rho} + \frac{n-1}{n} I^s \beta \sigma_\rho^2 + \frac{1}{n} I^s \sigma_\rho^2 \right) = 0 \quad (16)$$

In order to compare to the first order condition of (4), we rewrite (16) as

$$(\bar{\rho} - r) - 2\lambda \left(\sigma_{M\rho} + (I^s - I_j) \beta \sigma_\rho^2 + I_j \sigma_\rho^2 \right) = 0, \quad (17)$$

where $I_j = \frac{1}{n}I^s$. In the first order condition for maximizing individual firm's value, only the variance of the individual firm and the covariance between the firm's new project and old project have double effect on the firm's investment decision. However, in (17), all variance and covariance have the double negative effect on individual firm's investment decision.

This reduces the level of investment. Solving (16) for I^s , we get

$$I^s = \frac{n}{2\lambda \sigma_\rho^2 [1 + \beta(n-1)]} [(\bar{\rho} - r) - 2\lambda \sigma_{M\rho}]. \quad (18)$$

Since $I^s \leq I^v$, there is an over investment for firms.

If there is no pre existing projects, or $\sigma_{\rho j} = 0$ for all j , the level of investment will be

$$I^s = \frac{n}{2\lambda \sigma_\rho^2 [1 + \beta(n-1)]} (\bar{\rho} - r) \quad (19)$$

which is only half of the value of optimal social welfare investment.

If firms make independent choice, i.e., $\beta = 0$, we have

$$I^s = \frac{n}{2\lambda \sigma_\rho^2} (\bar{\rho} - r),$$

⁹ See Jensen and Long (1972).

which is equal to the aggregate investment of individual firm.

3. EFFICIENCY IN THE LOAN MARKET

In this section we set up a model to analyze efficiency in the loan market. Firms have no initial wealth to make investments. Instead of linear stochastic returns to scale, we assume decreasing returns to scale of investment. The stochastic return for an investment I is

$$\tilde{x}f(I), \tag{20}$$

where \tilde{x} is a random variable with distribution function $G(x)$ and density function $g(x)$ which has positive support on $[0, x^u]$; $f(I)$ is a concave function with $f' > 0$ and $f'' < 0$.

Risk neutral creditors have access to liquidity with per unit cost r , and they compete for borrowers in the loan market. Creditors may take over projects if firms do not repay their loans. However, the value of projects is worth less when owned by creditors than when owned by firms. The value of a project is $\alpha\tilde{x}f(I)$ if it is taken over by its creditor, where $0 < \alpha < 1$.

A loan contract can be specified as: $\gamma = (R, I)$, where R is the repayment of the firm. The realization of x can be observed by both parties but cannot be observed by a third party. Thus the contract is not enforceable. After the project is realized, the return, $xf(I)$ is distributed between the firm and the creditor.

Socially Optimal Level of investment

The optimal social investment is obtained by maximizing the expected payoff of a project minus the cost of investment. That is to choose the level of investment to maximize

$$\int_0^{x^u} xf(I)dx - rI. \tag{21}$$

We get the first order condition:

$$f'(I) \int_0^{x^u} x dx - r = 0. \quad (22)$$

Equilibrium without Renegotiation

Since lenders compete for borrowers in the loan market, the optimal contract, $\gamma^* = (R^*, I^*)$, must maximize the borrower's expected payoff, which guaranteeing the lender's break-even point. If $x f(I^*) < R^*$, the firm cannot repay its debt, the creditor takes over the project. If $x f(I^*) \geq R^*$ and the firm decides not to repay, then the creditors will take over the project; therefore, the firm will repay R^* . We define

$$\hat{x} \equiv \frac{R}{f(I^*)}. \quad (23)$$

Thus the expected payoff for the creditor and the borrower are:

$$V(\gamma) = [1 - G(\hat{x})]R + \int_0^{\hat{x}} \alpha x f(I) g(x) dx - rI, \quad (24)$$

and

$$U(\gamma) = \int_{\hat{x}}^{x^u} x f(I) - R g(x) dx. \quad (25)$$

The problem is to maximize the expected payoff of the borrower subject to the lender's participation constraint, $V(\gamma) \geq 0$.

Let

$$\mathcal{L} = \int_{\hat{x}}^{x^u} x f(I) - R g(x) dx + \lambda \left[(1 - G(\hat{x})) + \int_0^{\hat{x}} \alpha x f(I) g(x) dx - rI \right]. \quad (26)$$

We obtain the first order condition:

$$-[1 - G(\hat{x})] + \lambda [1 - G(\hat{x}) - (1 - \alpha) \hat{x} g(\hat{x})] = 0; \quad (27A)$$

$$\int_{\hat{x}}^{x^u} x f'(I) g(x) dx + \lambda \left[\int_0^{\hat{x}} \alpha x f'(I) g(x) dx + (1 - \alpha) g(\hat{x}) \hat{x} R \frac{f'(I)}{f(I)} - r \right] = 0; \quad (27B)$$

$$[1 - G(\hat{x})] + \int_0^{\hat{x}} \alpha x f(I) g(x) dx - rI = 0. \quad (27C)$$

If the level of investment without negotiation is social optimal, solving (27A) - (27C) for I , we would also be able to get the same level of investment as in equation (22).

It is similar as to solve three variables R , I and λ from four equations (22) and (27A) - (27C). We are strongly doubt that we can get the results. However, it is difficult to solve it directly from the equations without further restriction. In the following cases, we will use uniform distribution of return and a special investment function to show that the level of investment without renegotiation is not social optimal.

Case 1

Let $f(I) = \sqrt{I}$ and $g(x) = 1$ for $x \in [0, 1]$, $x'' = 1$.

From (22), we get

$$\int_0^1 \frac{1}{2} \sqrt{I} = r.$$

Solving it for I , we have the socially optimal level of investment

$$I = \frac{1}{16r^2}. \quad (28)$$

Without renegotiation, from (27A) - (27C), we get

$$-(1 - \hat{x}) + \lambda(1 - 2\hat{x} + \alpha\hat{x}) = 0 \quad (29A)$$

$$\left(\frac{1}{4\sqrt{I}} \right) (1 - \hat{x}^2) + \lambda \left[\frac{1}{2\sqrt{I}} \left(\frac{1}{2} \alpha\hat{x}^2 + (1 - \alpha)\hat{x}R \right) - r \right] = 0 \quad (29B)$$

$$\frac{1}{2} \sqrt{I} \alpha\hat{x}^2 + (1 - \hat{x})R - rI = 0 \quad (29C)$$

Plugging the socially optimal investment in (29A) - (29C) we find that $I = \frac{1}{16r^2}$ cannot be a solution of (29A) - (29C). Thus the investment without renegotiation does not reach socially optimal level.

Equilibrium with Renegotiation

In this subsection we assume that renegotiation is allowed after the return of the project is realized. Suppose that the new payment R^n is agreed on by both parties. The borrower gets

$$xf(I^*) - R^n,$$

and the creditor's payoff is

$$R^n - rI^*.$$

R^n is the Nash bargaining solution with bankruptcy payoff $\alpha \tilde{x}f(I)$ as the creditor's threat point to take over the project. We assume that both parties have the same bargain power, thus R^n maximizes

$$(R^n - \alpha \tilde{x}f(I^*))^{1/2} (xf(I^*) - R^n)^{1/2}. \quad (30)$$

Solving the equation, we get

$$R^n = \frac{1+\alpha}{2} xf(I^*). \quad (31)$$

A borrower will repay, if

$$xf(I^*) - R^* \geq xf(I^*) - R^n,$$

i.e., $R^n \geq R^*$. Thus we have

$$\frac{1+\alpha}{2} xf(I^*) \geq R^*.$$

$$x \geq \frac{2R^*}{f(I^*)1+\alpha}.$$

We define

$$\hat{x} \equiv \frac{2R^*}{f(I^*)1+\alpha}.$$

The expected payoffs for the borrower and the investor are

$$U(\gamma) = \int_{\hat{x}}^{x^*} xf(I)g(x)dx - [1 - G(\hat{x})]R + \int_0^{\hat{x}} \frac{1-\alpha}{2} xf(I)g(x)dx, \quad (32)$$

and

$$V(\gamma) = [1 - G(\hat{x})]R + \int_0^{\hat{x}} \frac{1+\alpha}{2} xf(I)g(x)dx - rI. \quad (33)$$

The problem becomes to maximizing the borrower's expected payoff subjected to $V(\gamma) \geq 0$.

Let

$$\begin{aligned} \ell = & \int_{\hat{x}}^{x^u} xf(I)g(x)dx - [1 - G(\hat{x})]R + \int_0^{\hat{x}} \frac{1-\alpha}{2} xf(I)g(x)dx \\ & + \lambda \left[(1 - G(\hat{x}))R + \int_0^{\hat{x}} \frac{1+\alpha}{2} xf(I)g(x)dx - rI \right]. \end{aligned} \quad (34)$$

One yields the first order conditions:

$$-[1 - G(\hat{x})]R + \lambda[1 - G(\hat{x})]R = 0; \quad (35A)$$

$$\int_{\hat{x}}^{x^u} xf'(I)g(x)dx + \int_0^{\hat{x}} \frac{1-\alpha}{2} xf'(I)g(x)dx + \lambda \left[\int_0^{\hat{x}} \frac{1+\alpha}{2} xf'(I)g(x)dx - r \right] = 0;$$

$$(35B)$$

$$[1 - G(\hat{x})]R + \int_0^{\hat{x}} \frac{1+\alpha}{2} xf(I)g(x)dx - rI = 0$$

$$(35C)$$

From (35A) we get

$$\lambda = 1.$$

Plugging it into (35B) and rearranging the equation, we obtain

$$f'(I) \int_0^{x^u} xg(x)dx - r = 0$$

Thus, if renegotiation is allowed, the equilibrium level of investment will reach the socially optimal investment. The reason is, through renegotiation, the dead weight loss can always be prevented -- the creditor will not really take over the project, he only threatens to take it over. The aggregate expected payoff of the borrower and the lender is

$$U(\gamma) + V(\gamma) = \int_0^{x^u} xf(I)g(x)dx - rI,$$

which is exactly equal to the social welfare function.

We get the same first order condition:

$$f'(I) \int_0^{x^u} xg(x)dx - r = 0$$

4. CONCLUSION

Using the mean-variance model for capital asset pricing to determine the level of investment, one can obtain an under-investment result -- the aggregate investment made by firms is lower than the socially optimal level of investment.

Although the old project affects the decision of new investment, it is not the key factor that causes under investment. Firms putting too much weight on their own variance of the return leads to inefficiency. If the variance of each firm's return is relative small compared to the covariance of the market, then the level of investment approaches the optimal level. In addition, if we let the market become more competitive by increasing the number of firms, the aggregate investment made by firms will approach the socially optimal level. However, if each firm's decision is independent of the other firms' decision, then the optimal investment level is never approximated, even though the number of firms goes to infinitive.

In the loan market, due to the dead weight loss of bankruptcy, the optimal level of investment can not be attained if renegotiation is impossible. If borrowers and creditors can renegotiate the repayment in case firms refuse to repay, then creditors will never really take over projects and the dead weight loss can always be prevented. The socially optimal level of investment can always be attained if renegotiation is allowed.

REFERENCES

- Fama, E. and Miller, M. 1972, *The Theory of Finance*, New York: Holt, Rinehart & Winston.
- Jensen, M. C. and J. B. Long, Jr., 1972, Corporate Investment under Uncertainty and Pareto Optimality in the Capital Markets, *The Bell Journal of Economics and Management Science* 3, 154-174.
- Merton, R, and M. Subrahmanyam, 1974, The Optimality of a Competitive Stock Market, *The Bell Journal of Economics and Management Science* 5, 145-170.
- Stiglitz, J., 1972, On the Optimality of the Stock Market Allocation of Investment, *The Quarterly Journal of Economics* 86, 25-60.
- Stiglitz, J., 1981, The Allocation Role of the Stock Market: Pareto Optimality and Competition, *The Journal of Finance* 36, 235-251.

vita

Chung-Shu Liu was born in Ilan, Taiwan, on February 28, 1958. He received his B.S. in industrial design from National Cheng Kung University in 1980, his first M.A. in economics from National Chung Hsing University in 1986. In 1987, he married to Chen-Min Chang. Then he entered Virginia Polytechnic Institute and State University in August 1988, received an M.A. in 1990 and completed his Ph.D. in April 1994.

A handwritten signature in black ink, appearing to read 'C. S. Liu', with a long horizontal flourish extending to the right.