THE CHARGE DEPENDENCE OF THE NUCLEAR FORCES AND THE A = 14 ISOBARIC TRIPLET

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I. INTRODUCTION

It has long been recognized that the main contribution to the specifically nuclear portion (i.e., short-range part) of the nucleon-nucleon interaction is charge independent. Evidence for this conclusion comes from several sources, e. g., the binding energies for the nuclear energy levels in isobaric nuclei, two-body scattering data, and gamma decay selection rules.

The specifically electromagnetic portion of the nucleon-nucleon interaction has two primary sources. The first, and by far the more important, is the Coulomb interaction between the protons in the nucleus. The second is the coupling of the intrinsic and orbital magnetic moments of the nucleons to the static electric field produced by the protons in the nucleus. This leads to a small spin-orbit interaction. (See Appendix VIII.)

The Coulomb interaction, especially in heavier nuclei, will shift the nuclear energy levels rather substantially, up to 1000 Mev or so for the heaviest nuclear systems. The spin-orbit effect arising from the magnetic moments is substantially smaller, on the order of one Mev.

That there should be a specifically nuclear charge-dependent interaction can be seen by a study of the source of the nuclear interaction, the pi-mesons. There are three of these, the $\pi^{\pm}$ and the π^{o}.

In first order, a neutron-proton system can interact via the exchange of any one of the three pions, while a neutron-neutron or proton-proton system can interact only through the interchange of a neutral pion. In first order, the difference in mass between the charged and neutral pions then leads, in the static limit, to a charge symmetric (but not charge independent) nucleon-nucleon potential of the form [7, 8, 18, 52]

$$V(r)= \frac{1}{3}\left(\frac{g}{M}\right)^2 \vec{\sigma}^1\cdot\vec{\sigma}^2\left\{\vec{\tau}^1\cdot\vec{\tau}^2\mu^2\frac{e}{r}^{-\mu r}+\eta\tau_z^1\tau_z^2\left[\mu e^{-\mu r}-2\frac{e}{r}^{-\mu r}\right]\right\} \qquad (1)$$

where μ is the mass of the charged pions, $\eta = (M_{\pi^{\pm}} - M_{\pi^{o}})/M_{\pi^{\pm}}$, g in the pion-nucleon coupling constant, and M is the nucleon mass. The second term of this potential causes a few per-cent violation of charge independence. (The notation $\tau_z = +\frac{1}{2}$ for a neutron, and $-\frac{1}{2}$ for a proton, is used throughout this paper.)

Inclusion of the radiative corrections to these first order calculations destroys even the charge symmetry that is present in (1). [48] This breakdown is of the order of a percent.

It is clear, therefore, that any investigation into the charge dependence of the specifically nuclear interaction must use refined methods to peer behind the masking effects of the electromagnetic contribution to the nucleon-nucleon interaction.

As an aid in determining the choice of nuclei to be studied it is evident that, the lighter the nucleus, the less the screening due to the explicitly electromagnetic effects. Also, the calculations for the lighter nuclei are considerably less burdensome due to the fewer number of particles in each shell.

As an aid in the study of the charge dependence Blin-Stoyle and LeTourneux [9, 10] suggested the use of a phenomenological two-nucleon potential that would be suitable for model calculations (see Appendix I). It is of the form:

$$V_{12}=[(\tau_z^1+\tau_z^2)p + \tau_z^1\tau_z^2 q + \vec{\sigma}^1\cdot\vec{\sigma}^2\{(\tau_z^1+\tau_z^2)r + \tau_z^1\tau_z^2 s\}]V(r_{12}) \qquad (2)$$

Here p, q, r, and s measure the departure of the nuclear forces from charge independence, and are constants to be determined from the experimental data. $V(r_{12})$ is assumed to contain all of the radial dependence.

It is the aim of this study to obtain improved estimates for the values of p, q, r, and s by a study of the mass 14 system, Nitrogen, Carbon, and Oxygen. To do this, we will use a Harmonic Oscillator Shell Model Calculation.

In order to carry out this program, improved estimates of the Shell Model wave functions for the isotriplet and the isosinglet states are obtained. This is done by setting up the Hamiltonian matrix using data from effective interaction calculations and comparing the three triplet matrices so obtained to determine the mixing of the two-particle s-d states into the desired nuclear wave functions.

Up to now the only states that had been considered in the study of the mass 14 system were the p shell states. Our estimates show that the amplitude of mixing of the s-d shell state is large enough to yield non-negligible corrections to previous calculations.

These wave functions are then used to obtain estimates of the explicitly electromagnetic effects. Upon comparing the energy levels of the isobaric triplet states it is found that there remain small residual shifts that are then attributed to the charge dependent potential. Equations are then developed for p, q, r, and s.

The wave functions are checked by studying their prediction of various nuclear characteristics. Good results are obtained for the ground state magnetic dipole and electric quadripole moments. In addition the M1 transition lifetime from the first excited state in N^{14} to the ground state is also predicted well.

II. EVIDENCE FOR THE CHARGE DEPENDENCE OF THE NUCLEAR FORCES

The effect of adding a charge-dependent part to the nuclear Hamiltonian

Suppose the charge independent part of the nuclear Hamiltonian is H_o and that the charge dependent portion is V, and can be treated as a small perturbation on H_o. The eigenstates, ψ_n of H_o are simultaneously eigenstates of the square of the total isotopic spin operator, T^2 and its z-component, T_z. More explicitly:

$$\begin{aligned} H_o\psi_n(T) &= E_n\psi_n(T) \\ T^2\psi_n(T) &= T(T+1)\psi_n(T) \\ T_z\psi_n(T) &= \frac{N-Z}{2}\psi_n(T) \end{aligned} \qquad (3)$$

Upon adding the charge dependent perturbation, V, to H_o, we have for the eigenstates, ψ, of the total nuclear Hamiltonian:

$$H\psi=(H_o+V)\psi=E\psi \qquad (4)$$

where,

$$\psi=\psi_o(T) + \sum_{T',n\neq 0} \alpha_T(T')\psi_n(T') \tag{5}$$

and, if V is "small" compared to H_o, we have:

$$\alpha_T(T')= \frac{\langle\psi_n(T')|V|\psi_o(T)\rangle}{E_o(T)-E_n(T')} \tag{6}$$

this being the result of first-order perturbation theory.

The α_T (T') are a measure of the isotopic spin impurities with isotopic spin T' in a state which is predominantly of isotopic spin T. It is expected, at least for the low-lying states of light nuclei, that the α_T (T') are quite small for $T \neq T'$, i.e., these states are relatively pure isotopic spin states since the Coulomb contribution is small. More recently it has been found, by direct reaction studies, that even for heavier nuclei, the isobaric spin impurities for low-lying levels are quite low.[3, 4] This was explained by Lane and Soper [35] who showed that, actually, the isobaric spin purity, near the stability line, increases with increasing mass number. They argued that, in actual nuclei the surplus neutrons (which by themselves have pure isotopic spin $T = \frac{N - Z}{2}$) dilute the isobaric

spin impurities of the core to produce a very pure total isotopic spin for the whole nucleus.

These statements are true only for states of low excitation lying near the stability line. In general, the higher the excitation, or the farther from the stability line, the higher will be the isotopic spin impurity of a given state. This impurity can become quite large, up to 0.5, even for states of the lighter nuclides. [71]

The earlier work

Studies of α_T (T') can yield a large amount of information about the strength of the charge-dependent portion of the nucleon-nucleon interaction. Early studies of this number were carried out quite vigorously by Wilkinson and co-workers [14, 31, 57, 62-72] and others.[5, 6]

The main tool that was used is the electric dipole transition in self conjugate nuclei ($T_z = 0$). Since such systems occur only for relatively light nuclei, their results pertain mostly to nuclei with $A \leq 20$.

In these self-conjugate nuclei a selection rule on the electric dipole transition of $\Delta T = \pm 1$ holds. [72] Observation of the validity of this rule gives a rough estimate of the

mixture parameters α_T (T'). For example, suppose that there is a transition from a T ≠ 0 excited state to the T = 0 ground state of a self conjugate nucleus. Then the transition is assumed to take place between the admixture of a T = 1 state into the T = 0 excited state (the ground state is assumed to be relatively pure). Thus, the dipole transition width, or lifetime, will be determined by the parameter $\alpha_0(1)$ of the excited state. Clearly, the less the admixture the slower the transition, or the longer the lifetime. Such transitions should, thus, seem to be greatly retarded with respect to similar transitions from a T = 1 to a T = 0 state.

In addition to these dipole transitions, reactions like x (α,γ)y were used to study the isospin purity of some nuclear states. It is assumed that the alpha particle has very little isotopic spin impurity, and has T = 0. It is thus possible to study states in the system y with the same isospin as the ground state of the system x.

Also, suppose a given nuclear state is unstable to particle emission, i.e., there may be a (γ, n) or (γ, p) reaction. Then the probabilities for decay from this

excited state via the n or p mode to different members of the same isotopic spin multiplet (at least the main, or basic, state is the same) should differ only by Clebsh-Gordon coefficients in the isotopic spin. For example, "if a state is nominally a member of an isospin triplet (T = 1) and emits neutrons or protons to residual T = 1/2 states then the T = 0 contamination can do this and the relative reduced widths depart from the charge independent expectation by a factor of order" [72]

$$\frac{1 + \alpha_1(0)}{1 - \alpha_1(0)} \quad . \qquad (7)$$

A further and more complete test of charge independence (the above methods really test only charge symmetry) was made by a study of the energy separation of isobaric triplet levels. These are a group of three levels, all with the same mass number, A, and with $(N-Z)/2 = \pm 1, 0$. For most nuclear systems they will consist of the ground states of the $|T_z| = 1$ nuclei and one of the very low-lying excited states of the self-conjugate nucleus. The explicitly electro-magnetic effects such as the Coulomb interaction

and the neutron-proton mass difference are first subtracted and the levels are then compared. If the specifically nuclear interaction were charge independent then the resulting levels would be degenerate.

The result of all of the above estimates was that the nucleon-nucleon interaction is charge independent. That is the n-n and the p-p and the n-p forces all have the same magnitude. These results were estimated to be accurate to within a few percent. This is just what was expected on the basis of the meson arguments mentioned above.

Results from the study of Beta Decay [13]

With the advent of the Conserved Vector Current theory of Beta decay, a new tool for the study of the impurity coefficients $\alpha_T(T')$, was made available. This is the allowed Fermi matrix element.

If the Conserved Vector Current theory is correct, or, if mesonic effects can be neglected, then the allowed Fermi matrix element is just:

$$M_f = \langle f | T^{\pm} | i \rangle, \qquad (8)$$

where $T^{\pm}$ is just the total isotopic spin raising or lowering operator. This matrix element has the isotopic spin selection rules $\Delta T_z = \pm 1, \Delta T = 0$ and has the value $\sqrt{(T \mp T_z)(T \mp T_z + 1)}$ + for electron decay and - for positron decay.

The extraction of $|M_f|$ from the experimental data can be accomplished in two ways. The first is by a study of $J = 0 \rightarrow J = 0$ transition rates. In this case the ft value for the transition is proportional to the reciprocal of the Fermi matrix element: [33]

$$1/(ft) = K|C_f M_f|^2 \qquad (9)$$

The constant K may be evaluated from superallowed transitions ($J = 0 \rightarrow J = 0$, $\Delta T = 0$) and is found to be: [24]

$$1/K = 6090 \pm 30 \text{ sec.} \qquad (10)$$

In a transition $J = 0 \rightarrow J = 0$ with $\Delta T = 1$, it is assumed that the Fermi part of the transition occurs via the admixture into the state of the lower $|T_z|$, the analogue of the state with higher $|T_z|$. (The state with the lower absolute value of T_z is observed to be the daughter in

electron decay and is usually the parent in positron decay. [13]) Calling the isospin of the state with the lower absolute value of T_z, T_1, and the higher T_2 ($T_2 = T_1 + 1$), then the ft value for the transition becomes:

$$ft = \frac{(ft)_{sa}}{[\alpha_{T_2}(T_1)]^2} \tag{11}$$

where $(ft)_{sa}$ is just the ft value for the analogous superallowed transition.

For the transitions which are not $J^{\pi} = 0 \rightarrow J = 0$, it can be shown that: [13]

$$\alpha_{T_2}(T_1) = \frac{\rho^2(6090\pm30)}{(ft)(1+\rho^2)(2T_2)} \tag{12}$$

where $\rho = \left| \frac{C_f M_f}{C_{gt} M_{gt}} \right|$ and C_f and C_{gt} are the Fermi and Gamow-Teller coupling constants, M_f and M_{gt} are the analogous nuclear matrix elements. Determination of the α's requires, in addition to the measurement of the ft value of the transition, a measurement of the quantity ρ^2. This is usually done either by an aligned parent decay experiment or by a $\beta\gamma$ circular polarization correlation experiment, both of which are extremely difficult to execute.

A collection of Fermi matrix elements found in this way, together with the ratio M_f/M_{gt} is given by Daniel and Schmitt. [16]

Analysis of all of these results shows that the isotopic spin mixing parameters so found cannot be explained solely by the specifically electromagnetic interactions, but that there is a small and persistent deviation which is attributed to the charge dependence of the specifically nuclear interaction.

Evaluation of the Charge-Dependent Parameters

Attempts to fit the parameters of the charge-dependent potential (2) to the experimental beta decay data have been carried out by several investigators. [19, 10, 12, 16, 37-40] These workers concentrated on fitting the parameters $\alpha_T(T')$, after removal of explicit electromagnetic interaction effects. The procedure is to compute all of the charge-dependent effects, including the potential (2), and equate the resulting expression containing the parameters p, q, r, and s to the energy differences of the isobaric states.

Also the energy separation of the A = 6 [39] and the A = 14 [1, 37, 39] and the A = 24 [40] triplets have been investigated by a similar procedure. The theoretical energy differences are calculated parametrically in terms of the charge-dependent parameters p, q, r, s and equated to the experimental shifts. The resulting equations are then solved for p, q, r, and s.

The final analyses all show that p, q, r, and s are on the order of 10^{-3}, but all of the uncertainties are quite large; e.g., we quote: [16]

$$\begin{aligned} p &= 0.00 \pm 0.03 \qquad & q &= -0.15 \pm 0.12 \\ r &= -0.01 \pm 0.02 \qquad & s &= -0.08 \pm 0.07 \end{aligned} \tag{13}$$

III. CALCULATIONS IN THE A = 14 SYSTEM

The Wave Functions

The calculations that must be performed in estimating the charge dependent effects require a detailed knowledge of the nuclear wave functions. Visscher and Ferrell [59] obtained the necessary wave functions for the mass 14 system.

Their semiphenomenological calculation was performed by taking as many of the nuclear force parameters as possible from the low-energy nuclear data, and then calculating a model Hamiltonian matrix which was then diagonalized to obtain the nuclear energy level eigenfunctions and eigenvalues. During this process they fixed the remaining parameters by requiring that their wave functions describe the anomalously long lifetime for the beta transition from the ground state of C^{14} to the ground state of N^{14}.

The Shell Model basis functions considered in their calculation restrict the nucleons to move only in the 1s and 1p shells. This greatly limits the burden of the calculation, and, in their framework, makes the calculation possible. But, as we shall show below, this procedure

ignores contributions to the wave functions from other configurations which are not negligible.

The wave functions obtained by Visscher and Ferrell are subject to another criticism. They do not reproduce the M1 transition rate between the first excited state and the ground state of N^{14}. [53]

Visscher and Ferrell did their intermediate coupling calculation using L-S coupled basis states. This means that, with the restriction to basis states describing particles occupying a filled 1s core and ten other particles in the 1p shell, the isotriplet levels, $(J,T) = (0,1)$, are described by the basis states $^1S_0$, and $^3P_0$, while isosinglet levels, $(J,T) = (1,0)$, are described by the basis states $^3S_1$, $^1P_1$, and $^3D_1$. In this framework the wave functions for the ground state of N^{14} (isosinglet), and the ground states of C^{14}, and O^{14}, and the first excited state of N^{14} (isotriplet) are, respectively:

$$\begin{aligned} \psi(1,0) &= [C_S\psi(^3S_1) + C_P\psi(^1P_1) + C_D\psi(^3D_1)]\chi^0{}_0 \\ \psi(0,1) &= [C_S'\psi(^1S_0) + C_P'\psi(^3P_0)]\chi^1{}_{M_1} \end{aligned} \qquad (14)$$

where $\chi^T_{M_T}$ is the isobaric part of the nuclear wave functions.

Visscher and Ferrell obtained $(C_S, C_P, C_D) = (0.173, 0.355, 0.920)$ and $(C_S', C_P') = (0.764, 0.646)$.

Detailed Calculations

Detailed calculations of the charge-dependent effects using these wave functions, have been carried out by Altman and MacDonald. [1] In their calculation they performed an analysis of the energy separation of the first isobaric triplet of levels in the A = 14 system. This yields two equations for the unknown parameters p, q, r, and s:

$$\begin{aligned} -27.84q + 58.88\ s &= 0.118 \text{ Mev}; \\ -250.96p + 247.1\ r &= -0.018 \text{ Mev}. \end{aligned} \qquad (15)$$

In order to obtain another equation involving these four parameters they analyzed the difference in ft values between the O^{14} and C^{14} beta decays to the ground state of N^{14}. A fourth equation was obtained through an analysis of the difference between the n-p and p-p scattering lengths.

They assumed that the $O^{14} - C^{14}$ ft value difference arises because of the admixture, into the ground states of these two nuclei, of other states of isotopic spin 1. These admixtures are assumed to be caused by the charge-

dependent potential (2), and have different amplitudes for the two nuclei because of the different numbers of protons in them.

One is not able to obtain unambiguous equations for the parameters in this way because, experimentally, all that can be measured in a beta lifetime experiment is the absolute square of a matrix element, and not the matrix element itself. Thus, two sets of results are quoted, one for the matrix elements positive, and the other for the matrix elements negative.

The final results of their work are:

$$p = -2.165 \times 10^{-3} \quad \text{or} \quad 8.85 \times 10^{-3}$$
$$r = 2.126 \times 10^{-3} \quad \text{or} \quad 9.06 \times 10^{-3} \qquad (16)$$
$$q = 1.5 \times 10^{-2} \quad \text{and} \quad s = -5.0 \times 10^{-3}$$

and are roughly an order of magnitude smaller than those obtained by Daniel and Schmitt (13).

A Priori Calculation

A slightly different approach was taken by Lovitch. [37, 38] In this work a potential similar to (1) was computed. This potential takes into account the one and two pion exchange graphs, the charged and uncharged pion mass

difference, and a difference in the charged and uncharged pion-nucleon coupling constants.

With this potential and the other specifically electromagnetic interactions Lovitch was able to calculate the "expected" energy shifts for the first isobaric triplet in the A = 14 system, using the wave functions of Visscher and Ferrell. Calculations were made both with and without the specifically nuclear charge dependent term.

The results of these calculations showed that distinctly better agreement to the isotriplet energy differences was obtained with the charge dependent nuclear force than without it. The percentage error was decreased from approximately 3% to 0.3%.

However, the use of this meson potential has no distinctly greater validity over inserting a gaussian factor in (2), since, when nucleons are in a complex nucleus, their interactions may be different than when they are relatively free, such as in a scattering experiment.

Also, it should be noted that both of the above calculations were executed using the Harmonic Oscillator Shell Model. Both Lovitch and Altman and MacDonald estimated

the model dependency of their calculations and found them to be small.

Objection to These Calculations

Blin-Styole and Nair [11] have pointed out that both of the above calculations may be in danger. The Coulomb energies used in obtaining the energy shifts and the estimates of the charge dependent effects are sensitive to correlation effects in the nucleus. This means that higher configurations may become important. This sensitivity is not large compared to the total Coulomb energy, but upon taking differences, and second differences (which is done in solving the equations for p, q, r, and s) they can become highly significant, and therefore "can have an important effect on any conclusions which may be drawn about the charge dependence of the specifically nuclear forces."

Blin-Stoyle and Nair calculated the effect of mixing in configurations higher than those considered by Visscher and Ferrell utilizing Serber and Rosenfeld mixtures for the nuclear potential. The configurations that were considered were those obtained by exciting one particle through two oscillator levels, or two particles through

one oscillator level. The equations from the isotriplet energy shifts that they obtained are (Altman and MacDonald's results are included for comparison):

$$\begin{aligned} -p + r &= 1.4 \times 10^{-4} \\ -q + 2.1s &= 4.2 \times 10^{-3} \end{aligned} \qquad \text{Altman \& MacDonald}$$

$$\begin{aligned} -p + 0.9r &= -0.8 \times 10^{-3} \\ -q + 2.6x &= 4.5 \times 10^{-3} \end{aligned} \qquad \text{Rosenfeld mixture} \qquad (17)$$

$$\begin{aligned} -p + 1.3r &= 3.7 \times 10^{-3} \\ -q + 4.2s &= 8.0 \times 10^{-3} \end{aligned} \qquad \text{Serber mixture}$$

It is clear from these results that the inclusion of higher configurations in the calculation can change the estimates for p, q, r, and s by an order of magnitude. Thus, better estimates of the nuclear wave functions, which include at least the most important part of the excited configurations mentioned above, are necessary for estimating the parameters p, q, r and s that represent the effective charge dependent interaction.

IV. ESTIMATES OF THE A = 14 WAVE FUNCTIONS

Results from Effective Interaction Calculations

The effective interaction method which is to be described below deals with a method for finding the matrix elements of some "effective" Hamiltonian. This "effective" Hamiltonian is to be distinguished from the "real" Hamiltonian and any "model" Hamiltonian operator.

The effective Hamiltonian is a matrix which yields some specific experimental results when used in conjunction with shell model calculations. These results are usually taken to be energy eigenvalues. One does not usually require that the "effective" Hamiltonian matrix reproduce all of the experimental results; e.g., that its eigenvectors reproduce experimental quadripole and dipole moments, etc. This distinguishes it from the matrix of the real Hamiltonian operator. Of course, it is hoped that as the calculations are improved, agreement with experiment on all fronts is obtained.

Also, the effective Hamiltonian operator is not a "model" Hamiltonian. Model Hamiltonians are usually given

some specific form. This form is deduced from arguments relating to invariance principles, e. g., rotational invariance. These model Hamiltonians then contain unknown constants which are fixed by requiring results, calculated by using these "model" operators to agree with experimental data.

No pre-assumed form is forced on the effective interaction Hamiltonian, except that in accordance with the general shell model assumptions it is conceived to be written in two parts; H_0, a single-particle energy portion, and V, a weak, two-particle interaction.

The first step in the computation of nuclear wave functions in any shell model calculations is the assumption of a basis. For instance, some authors have attempted to describe the ground state of N^{14} via a closed 1s shell and ten other particles moving in the p shell coupled to J = 1, T = 0. (See the previous chapter.)

Then, normally, the matrix of some model Hamiltonian operator is computed in this assumed basis. For the case described above, this is a 3 x 3 matrix. This Hamiltonian matrix is then diagonalized. The diagonalization procedure

yields both the eigenfunctions and the eigenvalues of the assumed Hamiltonian.

The effective interaction method dodges the problem of assuming a model óperator. The procedure outlined below was described by Glaudemans, et. al. [25] Their calculation pertained to nuclei with mass numbers between Si^{29} and Ca^{40}. A similar calculation was done by Amit and Katz [2] in the mass range from He^{5} to O^{16}. Still a third calculation was carried out by Cohen and Kurath [15] in the same mass range.

There is one point on which the whole structure lies: If one assumes two-particle residual interactions, then all that is necessary for the computation of matrix elements between complicated many-particle states is a knowledge of two-particle matrix elements. This is so because the more complicated matrix elements of a two-particle operator can be expressed as a well-defined superposition of two-particle matrix elements. The details of this reduction are outlined in Appendix II.

The object, then, is to obtain the single-particle matrix elements of the single-particle portion of the effec-

tive Hamiltonian, H_o, and necessary two-particle matrix elements of the residual two-particle portion V. This collection of matrix elements, both single-particle and two-particle, will be denoted by h_r in the following. The number of parameters h_r is always smaller than the amount of input data.

The procedure goes as follows: Select a set of shell model basis functions $\{|\psi_i>;\ i = 1,k\}$ which will be used to describe some of the known energy levels of nuclei in a predetermined mass range. An "effective" Hamiltonian matrix "p" is then formally constructed for each set of energy levels of given (J, T) of interest in each nucleus. The matrix elements of the total effective Hamiltonian operator, H_e, are expressed in terms of the unknown single-particle energies, and the unknown two-particle interaction matrix elements that arise, as seen above, from the many-particle matrix elements. Thus:

$$<\psi_i|H_e|\psi_j>^p = \sum_r \alpha_r(i,j,p)h_r \qquad (18)$$

where the h_r are the unknown single-particle and two-particle matrix elements. The $\alpha_r(i, j, p)$ are known

expansion parameters which arise, in part, from the decomposition of the many-particle matrix elements into two-particle matrix elements by the methods of Appendix II. The letters "i" and "j" refer to specific states of the basis. "p" carries all other information such as the specific nucleus to which the matrix refers, the spin and parity of the levels which are described by this matrix, etc.

The eigenvectors satisfying

$$\sum_i a_{ik} \langle \psi_j | H_e | \psi_i \rangle = E^k a_{jk} \tag{19}$$

form an orthogonal matrix which diagonalizes the matrices described by (18):

$$\sum_{i,j} a_{jk} \langle \psi_j | H_e | \psi_i \rangle a_{il} = E^k \delta_{l,k} \tag{20}$$

The process of solving (18) and (19) for the quantities h_r and a_{jk} is a complex task since the resulting equations are not linear. Therefore, a linearizing iteration procedure is set up. This procedure is as follows: The parameters h_r are first estimated, and the matrix elements $\langle \psi_i | H_e | \psi_j \rangle^p$ are determined. From these the matrices

diagonalizing the energy matrices "p" are determined. The eigenvalues of the energy matrices are also found. These eigenvalues are then compared with the experimental values (after Coulomb effects have been eliminated). Those which fit best are then used in (20) and the parameters h_r are redetermined. This process is then repeated, using more and more experimental energy values at each stage, until the results become stationary. This procedure is found to converge rapidly.

The effective interaction calculation provides a powerful tool for determining nuclear wave functions without recourse to assumptions about the detailed structure of the "best" model Hamiltonian to be used in shell model calculations. Also, it provides a tool for studying model nuclear Hamiltonians in the shell model. A two-particle matrix element found by the effective interaction method can be compared to the same matrix element calculated on the basis of an assumed model Hamiltonian and the two results compared. In this way various nuclear force parameters might be obtained. This procedure was studied in more detail by Cohen and Kurath. [15] The two-particle

interaction matrix elements of Amit and Katz [2] are used in the following. Their calculation restricts particles to move in the filled 1s shell and in the 1p shell. Their results for the pertinent two-particle matrix elements are presented in Table I.

States of Interest and their Principal Configurations

The first four $J^{\pi} = 1^{+}$ isobaric singlet and the first three $J^{\pi} = 0^{+}$ isobaric triplet states in A = 14 are investigated in the present work. (See the Figure.) Of these, all but the most highly excited isotriplet state are well known. That is, their spin, parity, and isospin assignments are quite firm. (See the Nuclear Data Sheets. [42].)

The excitation energy of the last isotriplet level may be inferred indirectly. Kasky, et. al. [32] and Latore and Armstrong [36] both indicate the presence of a possible $J^{\pi} = 0^{+}$ level in the region 10-11 mev. Both groups of experimenters find indications of the presence of a broad resonance in this region.

In addition, a theoretical investigation by True [58] in which the presence of s-d effects (without core excitation) are taken into account yields a $(J^{\pi}, T) = (0^{+}, 1)$ level

at 10.93 mev. All three of these bits of information then indicate a level somewhere in the region of 10 - 11 mev.

The composition of the well-known states in N^{14} was investigated by Warburton and Pinkston. [60] They analyzed the data arising from electromagnetic transition widths, reduced widths, and inelastic scattering cross sections in order to obtain shell model assignments for many of the lower lying levels of the Nitrogen nucleus.

Their analysis considered the mass 14 system to be composed of an inert 1s core and ten other particles to be distributed among the $p_{1/2}$, $p_{3/2}$, and s-d shells. Their pertinent results are given in Table II.

The s-d states considered by Warburton and Pinkston are of the same type as those discussed by Blin-Stoyle and Nair. (See Chapter III.) That is, they promote two nucleons through "one" oscillator level.

The s-d level consists of three closely spaced shell model levels, the $d_{5/2}$, $2s_{1/2}$ and $d_{3/2}$ levels. This close spacing (they are all within four mev. of each other) then leads one to speak of their "near degeneracy." Any

calculation which attempts to include the effects of nucleons moving in one of these levels should also include the effects of nucleons moving in the other two. This is so because any energy denominators that appear in a pertubation approach will all have approximately the same value because of the near degeneracy of these levels.

Configurations Used in the Present Analysis

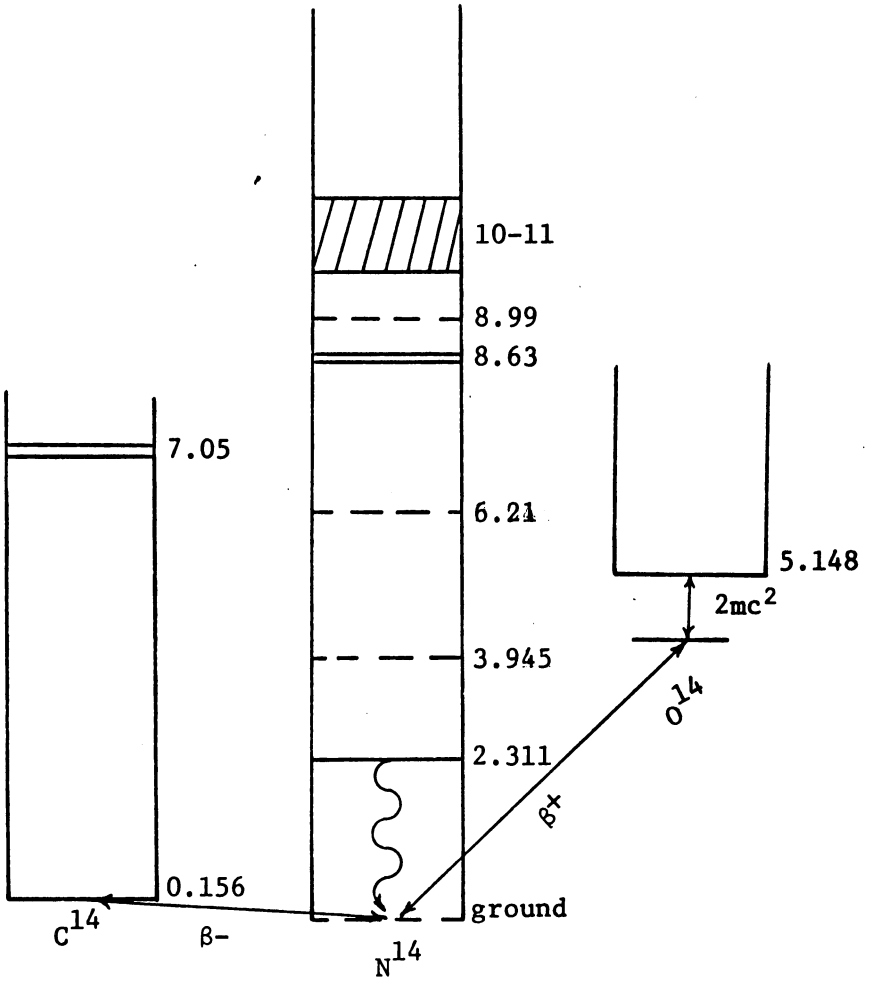
In view of the above work it is evident that improvement on the wave functions of Visscher and Ferrell can be accomplished by considering, in addition to the configurations with ten nucleons moving in the 1p shell, other configurations which describe eight nucleons moving in the 1p shell, and two other nucleons moving in the s-d shell. One nucleon alone cannot be removed from the 1p shell to the s-d orbit, since the parity of a "p" particle is negative, while that of the states of interest is positive. The s-d shell is considered because, as pointed out by Warburton and Pinkston, the pairing energy is large enough so that the s-d configurations should lie below the 2p and 1f configurations.

TABLE I

$\langle\psi_1(J,T)\|H_e\|\psi_2(J,T)\rangle$				
States			(J,T)	
$\|\psi_1\rangle$	$\|\psi_2\rangle$	(0,1)	(1,0)	(1,1)
$(1/2)^2$	$(1/2)^2$	2.40	3.92	- - -
$(3/2)^2$	$(3/2)^2$	4.25	5.63	- - -
(1/2,3/2)	(1/2,3/2)	- - -	8.95	-2.55
(1/2,3/2)	$(1/2)^2$	- - -	1.89	- - -
(1/2,3/2)	$(3/2)^2$	- - -	0.00*	- - -
$(1/2)^2$	$(3/2)^2$	-2.01	0.00*	- - -

Some two-body matrix elements for particles in the 1p shell, in the j-j coupling scheme from Amit and Katz. [2] All energies are in Mev.

*These matrix elements were found to have no significance in their calculation and were therefore arbitrarily set equal to zero.



FIGURE

The energy levels and transitions studied in the isobaric triplet A = 14. The dashed lines have T = 0, single solid lines have T = 1 and indicate the known members of the first isobaric triplet. The double solid lines have T = 1 and indicate the two known members of the second isotriplet. The cross-hatched level is the third T = 1 state whose energy is uncertain. All energies are in Mev. and are referred to the ground state of N^{14}.

TABLE II

N^{14} Excitation Energy (Mev.)	(J^{π},T)	Shell Model Assignment	N^{14} Energies Relative to the O^{16} core (Mev.)
0.00	(1+,0)	$p_{1/2}^{-2}$	-22.959
2.311	(0+,1)	$p_{1/2}^{-2}$	-25.370
3.945	(1+,0)	$p_{3/2}^{-1}p_{\frac{1}{2}}^{-1}$	-26.904
6.21	(1+,0)	s-d	-29.169
8.63	(0+,1)	s-d	-31.58
8.99	(1+,0)	?	-31.95
~10.5 mev.	(0+,1)	?	-33.4

Shell Model assignments for the energy levels of N^{14}. [60]

In the following it will be convenient to refer all of our calculations to the ground state of O^{16}. This is a particularly stable nuclide and we assume that it consists of closed 1s and 1p shells. That is, it is assumed that this nucleus is in a pure configuration $(1s^4\ 1p^{12})$. The mass 14 system in this case will be described as holes in the O^{16} core. For example, C^{14} is described as some superposition of at least two proton holes in the O^{16} core. Also the j-j coupling scheme will be used throughout.

The basis states that will be considered then are the following:

a) two holes in the $p_{1/2}$ shell

b) two holes in the $p_{3/2}$ shell

c) four holes in the $p_{1/2}$ shell and two particles in the s-d shell

d) one hole in the $p_{1/2}$ shell and one hole in the $p_{3/2}$ shell.

Labeling these we have:

$$
\begin{aligned}
|\psi_1\ (J,T)> &= |p_{1/2}^{-2}\ \psi_c:(J,T)> \\
|\psi_2\ (J,T)> &= |p_{3/2}^{-2}\ \psi_c:(J,T)> \\
|\psi_3\ (J,T)> &= |p_{1/2}^{-4}\ (s\text{-}d)^2\psi_c:(J,T)> \\
|\psi_4\ (J,T)> &= |p_{1/2}^{-1}\ p_{3/2}^{-1}\psi_c:(J,T)>
\end{aligned}
\qquad (21)
$$

where the symbol $(s\text{-}d)^2$ stands for a two-particle s-d wave function and ψ_c represents the O^{16} core.

The first problem that must be solved is the determination of the s-d state. For the case $(J, T) = (0, 1)$ only one choice which will preserve the antisymmetry of the two-particle state is possible, namely

$$|(s\text{-}d)^2(J,T)> = \alpha|d^2_{5/2}(J,T)> + \beta|d^2_{3/2}(J,T)> + \gamma|2s^2_{1/2}(J,T)> \qquad (22)$$

where the mixing amplitudes, α, β and γ are as yet unknown.

For $(J, T) = (1, 0)$, in addition to the three components in (22) two others are possible, those with single particles in the $1d_{3/2}$ orbit and the other particle in the $1d_{5/2}$ orbit or in the $2s_{1/2}$ orbit. It will be assumed in the following that the mixing of these two states is negligible and they will be neglected.

The procedure outlined in the following will be adopted in order to determine the coefficients α, β, and γ in (22).

The energy matrices for the $(J, T) = (0, 1)$ isotriplet are computed for each isobaric component. These energy matrices differ only by an additive constant along the

diagonal. These additive constants represent the shift in energy levels that is expected because of the neutron-proton mass difference and the coulomb interaction. The parameters α, β, and γ are then determined by requiring the matrices to be identical, after subtraction of these diagonal constants. This yields two equations for the three parameters. The normalization requirement supplies the third.

Computation of the Matrices - General

In order to evaluate the Hamiltonian matrices for the nuclear levels, the binding energies to the core for the various particles and holes must be known. The method adopted here to estimate these quantities follows very closely that of Amit and Katz who found that after their iteration procedure was complete, their first estimates of the single-particle energies differed very little from the final stationary values.

Suppose M(N, Z) is the mass of a particular state in a nucleus, excited or ground, which has N neutrons and Z protons. Then:

$$M(N,Z)c^2 = (ZM_H + NM_N)c^2 + E_0(N,Z) \qquad (23)$$

where M_H is the mass of a Hydrogen atom, M_N is the mass of the neutron and $E_0(N,Z)$ is the interaction energy of the nucleons in this state. Define E_Z and E_n for our purposes by the equations:

$$E_0(N,Z\pm1) = E_0(N,Z) + E_Z$$

and (24)

$$E_0(N\pm1,Z) = E_0(N,Z) + E_n$$

where we take E_n and E_Z to be the interaction energies of a single neutron and proton (+) or a single neutron or proton hole (-) with the core (N, Z).

For a proton hole, states of N^{15} are used, while for a neutron hole, states of O^{15} are used to compute E. For the values of the interaction energies with the core for the s-d particles, states of the nuclides O^{17} and F^{17} are used. All of the data for these calculations are taken from the nuclear data sheets. [42] The results are presented in Tables III and IV.

Two-particle and two-hole Coulomb energies are also needed. In the C^{14} nucleus there are two proton holes, and in the O^{14} nucleus there are two protons in the s-d shell. These are found in Appendix IV using a

Harmonic Oscillator Shell Model calculation. The results are given in Table V for an oscillator spring constant of $0.5997\ fm^{-1}$. [37]

The energy eigenvalues that will be associated with the matrices are known with respect to the ground states of N^{14}, O^{14}, and C^{14}. Since this calculation is refering to the energy levels as holes in the ground state of O^{16}, the energy eigenvalues of the matrices must be referred to this state as well. Using the results of Everling, et. al. [20] we find:

$$\begin{aligned} E(O^{14}) &= -E_O - 28.018\text{Mev} = B - M(O^{14}) \\ E(N^{14}) &= -E_N - 22.959\text{Mev} = B - M(N^{14}) \\ E(C^{14}) &= -E_C - 23.115\text{Mev} = B - M(C^{14}) \end{aligned} \tag{25}$$

where

$$B = M(O^{16}) - M_H - M_N$$

and the E's are the experimental excitation energies of the nuclear levels. The quantities on the left-hand side of (25) are the energy eigenvalues that are to be used in the calculations that follow and are given in Table II.

TABLE III

(lj)	neutron	proton
p 1/2	-15.67	-12.13
p 3/2	-21.83	-18.46

Interaction energies of neutron and proton holes with the O^{16} core. All energies are in Mev.

TABLE IV

(lj)	neutron	proton
1d	-4.142	-0.597
2s1/2	-3.271	-0.097
1d	0.938	4.503

Interaction energies of s-d particles with the O^{16} core. All energies are in Mev.

The Isotriplet Matrix and the s-d State

With the basis states (21) the matrices for the (J, T) = (0, 1) triplet are:

$$\langle\psi_1|H|\psi_1\rangle = E_c - m_1 - m_2 + I^1_{1/2} + I^2_{1/2} + \langle p^2_{1/2}|H_2|p^2_{1/2}\rangle$$

$$\langle\psi_2|H|\psi_2\rangle = E_c - m_1 - m_2 + I^1_{3/2} + I^2_{3/2} + \langle p^2_{3/2}|H_2|p^2_{3/2}\rangle$$

$$\langle\psi_3|H|\psi_3\rangle = E_c - m_1 - m_2 + 2I^p_{1/2} + 2I^n_{1/2} + I^1_{sd} + I^2_{sd} + y^* + \langle p^4_{1/2}|H_4|p^4_{1/2}\rangle + \langle(s\text{-}d)^2|H_2|(s\text{-}d)^2\rangle$$

$$\langle\psi_1|H|\psi_2\rangle = \langle p^2_{1/2}|H_2|p^2_{3/2}\rangle \qquad (26)$$

$$\langle\psi_1|H|\psi_3\rangle = x^*$$

$$\langle\psi_2|H|\psi_3\rangle = 0$$

In these equations E_c is the rest mass energy of the core O^{16}, M_1 is the rest mass energy of the first particle removed, and M_2 is the rest mass energy of the second particle removed from the core. The I's are the interaction energies of the particles and holes with the core, and are given in Tables III and IV. The subscripts on the interaction Hamiltonian are to indicate on how many particles these operators act. The indicated interaction Hamiltonians are the total interactions, including

Coulombic effects. Y* is the interaction of the s-d particles with the four $p_{1/2}$ holes which is unknown. All of the two-particle matrix elements are coupled to (J, T) = (O, 1). The matrix element $\langle\psi_1|H|\psi_3\rangle$ is also unknown and is called x*.

The derivation of equations (26) is not too complicated, and only one of them will be indicated here, the first. The total energy of interaction in the state $|\psi_1\rangle$ including all of the rest mass energies is just the rest mass energy of the core less the rest mass energy of the two particles removed, plus the interaction of the holes with the core. To this must be added the interaction of the holes with each other. This is the last term in each of the indicated matrix elements. This hole-hole interaction was shown by Racah [44] to be the same as that for the corresponding particles. Thus the last two-hole matrix element is equal to the corresponding two-particle matrix element.

With these equations and with the data in Tables I and III to V and the results of Appendix IV on the four-particle matrix element the matrices for the isotriplet states become:

TABLE V

j \ i	1	2	3	4	5
1	0.5167	0.0812	-0.1489	-0.0497	-0.0335
2		0.5742	-0.0727	-0.1828	-0.0474
3			0.4857	0.1702	0.0385
4				0.5104	0.0461
5					0.3744

Coulomb Energies of the State (A4-1). These results are computed from Table A4-3 with $e^2 \sqrt{\nu/2\pi} = 0.3445$ Mev.

For N^{14}

$$B + \begin{bmatrix} -25.43 & -2.01 & x^* \\ -2.01 & -36.08 & 0 \\ x^* & 0 & -35.97 + y^* + f_1(\alpha,\beta,\gamma) \end{bmatrix} \tag{27a}$$

For C^{14}

$$B + \begin{bmatrix} -20.36 & -2.01 & x^* \\ -2.01 & -31.06 & 0 \\ x^* & 0 & -35.19 + y^* + f_2(\alpha,\beta,\gamma) \end{bmatrix} \tag{27b}$$

For O^{14}

$$\begin{bmatrix} -29.78 & -2.01 & x^* \\ -2.01 & -40.29 & 0 \\ x^* & 0 & -36.75 + y^* + f_3(\alpha,\beta,\gamma) \end{bmatrix} \tag{27c}$$

where the functions used in the 3-3 elements are:

$$f_1(\alpha,\beta,\gamma) = -4.739\alpha^2 + 5.343\beta^2 - 3.368\gamma^2 \tag{28a}$$

$$f_2(\alpha,\beta,\gamma) = -1.194\alpha^2 + 9.006\beta^2 - 0.149\gamma^2 + 0.7137\alpha^2 + 0.6792\beta^2 + 0.5235\gamma^2 + 0.4760\alpha\beta + 0.1336\alpha\gamma + 0.1091\beta\gamma \tag{28b}$$

$$f_3(\alpha,\beta,\gamma) = -8.284\alpha^2 + 1.680\beta^2 - 6.542\gamma^2 \tag{28c}$$

Comparison of the 1-1 and 2-2 elements of the matrices (27) shows that:

$$O^{14} = C^{14} - 4.28\text{Mev}$$
$$C^{14} = N^{14} + 5.05\text{Mev} \qquad (29)$$

This implies, upon equating the 3-3 elements:

$$3.545\alpha^2 + 3.663\beta^2 + 3.174\gamma^2 = 3.497$$

and (30)

$$0.7137\alpha^2 + 0.6792\beta^2 + 0.5235\gamma^2 + 0.4760\alpha\beta + 0.1336\alpha\gamma + 0.1091\beta\gamma = 0.7878$$

With the normalization condition,

$$\alpha^2 + \beta^2 + \gamma^2 = 1 \qquad (31)$$

there are three equations that may be solved for the coefficients α, β, and γ. Two solutions are found to exist:

Alpha	Beta	Gamma
0.9250	0.1103	0.3636
0.2150	0.7914	0.5723

(32)

The first of these is used in the following calculations. In the Shell Model the $d_{5/2}$ level lies below the $d_{3/2}$ level, and is liable to be mixed in with lower configurations more readily than the $d_{3/2}$ level.

With this choice for the s-d state the matrices for the (J, T) = (0, 1) levels in the nuclei C^{14}, N^{14}, and O^{14} may now be determined. These matrices differ only by an additive constant along the diagonal, and therefore any unitary transformation that diagonalizes one of them does the same to the other two. Thus in the following studies we need deal only with one of them, say that for N^{14}, which is:

$$B + \begin{bmatrix} -25.43 & -2.01 & x^* \\ -2.01 & -36.08 & 0 \\ x^* & 0 & -37.84 + y^* \end{bmatrix} \qquad (33)$$

Indeed, these last statements are just a restatement of the isotriplet nature of these levels.

The matrix elements in (33) are not yet fixed. In addition to the complete lack of knowledge of x* and y* there are small uncertainties in the rest of the matrix elements (except those which are zero because of the

orthogonality of the states). These uncertainties are intrinsic to the effective interaction method for finding the various two-particle interaction matrix elements.

In this process the "best fit" to the observed energy levels of a number of parameters is found. This fitting procedure yields statistical estimates (with specified errors) for the parameters.

In the following we will further analyze this matrix with reference to the experimental data, the energy eigenvalues, the Gamow-Teller beta transition to the ground state of N^{14} and the M1 transition to this same ground state. But before proceeding further, the state vector describing the ground state of N^{14} must be obtained. Hence we turn our attention to the T = 0 states and the associated isosinglet matrix.

The Isosinglet Matrix

The isosinglet matrix is derived in a way very similar to that used for the isotriplet matrix. In this case, however, there is a four-dimensional basis. As is shown by the previous work of Visscher and Ferrell and Amit and Katz, all three of the p shell states contribute

significantly. In the present calculation we add to this set of basis states the s-d shell state as the fourth vector of the basis. We take the amplitudes of the various configurations in the s-d state to be the same as those found in the analysis of the isotriplet levels.

We are thus led to a consideration of the description of the isosinglet levels in terms of all of the basis functions (21). The matrix is:

$$\langle\psi_1|H|\psi_1\rangle = E_c - m_n - m_p + I^p_{1/2} + I^n_{1/2} + \langle p^2_{1/2}|H_2|p^2_{1/2}\rangle$$

$$\langle\psi_2|H|\psi_2\rangle = E_c - m_n - m_p + I^p_{3/2} + I^n_{3/2} + \langle p^2_{3/2}|H_2|p^2_{3/2}\rangle$$

$$\langle\psi_3|H|\psi_3\rangle = E_c - m_n - m_p + 2I^p_{1/2} + 2I^n_{1/2} + I^p_{sd} + I^n_{sd} + \langle p^4_{1/2}|H_4|p^4_{1/2}\rangle + y$$

$$\langle\psi_4|H|\psi_4\rangle = E_c - m_n - m_p + \tfrac{1}{2}(I^n_{1/2} + I^p_{1/2}) + \tfrac{1}{2}(I^n_{3/2} + I^p_{3/2}) + \langle p_{1/2}p_{3/2}|H_2|p_{1/2}p_{3/2}\rangle \qquad (34)$$

$$\langle\psi_1|H|\psi_2\rangle = \langle p^2_{1/2}|H_2|p^2_{3/2}\rangle$$

$$\langle\psi_1|H|\psi_3\rangle = x$$

$$\langle\psi_1|H|\psi_4\rangle = \langle p^2_{1/2}|H_2|p_{1/2}p_{3/2}\rangle$$

$$\langle\psi_2|H|\psi_3\rangle = 0$$

$$\langle\psi_2|H|\psi_4\rangle = \langle p^2_{3/2}|H_2|p_{1/2}p_{3/2}\rangle$$

$$\langle\psi_3|H|\psi_4\rangle = 0$$

With these expressions, and with the data in Tables I and III - V, and the results of Appendix IV, the matrix for the N^{14} J = 1, T = 0 levels becomes

$$B + \begin{bmatrix} -23.88 & 0 & x & 1.89 \\ 0 & -34.66 & 0 & 0 \\ x & 0 & -40.55 + y & 0 \\ 1.89 & 0 & 0 & -25.09 \end{bmatrix} \qquad (35)$$

It is evident that this matrix is already partially diagonalized. But (see Table VI) the eigenvalue appearing at the (2,2) matrix element does not correspond to any of the experimental values. This is because Amit and Katz were unable, by their calculation, to fix the (1,2) and (4,2) matrix elements. Thus, for the present work these matrix elements must be treated as unknowns.

Also, all of the other matrix elements (except those that are zero by the orthagonality conditions) are subject to small uncertainties, as described above in the discussion of the isotriplet matrix.

Further Analysis of the Matrices

In order to be able to fix the free parameters in the isosinglet and isotriplet matrices, comparison must be made to the experimental data.

This data consists of the energies of the observed levels, the magnetic dipole and electric quadripole moments of the ground state, and the β and M_1 transition lifetimes from the first isotriplet state to the ground state of N^{14}. Expressions for these moments and transition rates are calculated in Appendices V, VI.

The first matrix that must be analyzed is the isosinglet matrix. This is so because the ground state of N^{14} must be known before any of the transition rates may be computed.

The parameter y appearing in (35) is first estimated by requiring that the trace of the matrix be the sum of the first four pertinent eigenvalues in Table II. These are all states with $(J^{\pi}, T) = (1^+, 0)$. This gives: $y = 13.12$ Mev.

Next the off diagonal elements were studied. This is for the two-fold reason that x is as yet completely unknown, and that the observed quantities (energies, etc.)

will depend most sensitively on the impurities of the configurations used to describe the known energy levels

The procedure adopted for this analysis was to vary each off diagonal element separately until a minimum of some function, called the error function, was found. Each element was varied sequentially at one pass until a minimum of the error function for that pass was found The process was then repeated for several more passes in view of the fact that changing one element might affect the behavior of another. The method was found to converge quite rapidly, no more than six passes being required for the results to become stationary.

The error function used at this stage was one involving the energies, being the sum of the squares of the differences between the observed and calculated energies. It was not expected that the results would be satisfactory at this stage and, indeed, they were not.

Next the full matrix was subjected to the same procedure. (Elements (3 2) and (3,4) were always kept zero, since for a two-particle interaction they are automatically so.) The error function still involved only the energies.

The process again converged, yielding very good results for the energy eigenvalues as is to be expected. In addition, ground-state magnetic dipole and electric quadripole moments were obtained which agreed very well with experiment. The resulting matrix is given in (37)

$$\begin{bmatrix} -25.765 & -1.350 & 2.413 & 2.044 \\ -1.240 & -31.486 & 0 & 0.108 \\ 2.413 & 0 & -26.535 & 0 \\ 2.044 & 0.1080 & 0 & -27.192 \end{bmatrix} \tag{37}$$

The changes induced on the matrix by the above procedures can be seen to be small, no more than 10% for any known element.

In order to test the uniqueness of the result so obtained, other calculations were performed in an attempt to find other minima of the error function. The only one found was with the phase of the (1, 3) element changed.

This is to be expected because the structure of the matrix (35) is such that all of the energy eigenvalues depend on x quadratically. In addition, both of the ground state moments depend on the admixture of the s-d state into the ground state quadratically. Thus, by this procedure it is impossible to fix the phase of this element.

TABLE VI

Energy (Mev.)	$(p_{1/2})^2$	$(p_{3/2})^{-2}$	$p_{1/2}^{-1}\ p_{3/2}^{-1}$	$(s\text{-}d)^2$	μ (Nucl. Magnet)	Q $\times 10^{-26}$ cm^2
-22.95	0.9258	-0.1195	0.3617	--	0.351	1.06
-22.95	0.7655	-0.1165	0.3663	0.5160	0.3949	1.275
-26.90	-0.1034	0.0132	-0.7304	0.6750	0.5777	1.224
-29.17	-0.5560	0.3500	0.5557	0.5094	0.5314	-0.610
-31.95	0.3068	0.9294	-0.1529	-0.1368	0.6704	-0.938

Amplitudes

Wave functions for the J = 1, T = 0 levels in N^{14}. The first level is that quoted by Visscher and Ferrell for the ground state. The other four levels are those obtained by our analysis.

TABLE VII

Energy (Mev.)	$(p_{1/2})^2$	$(p_{3/2})^2$	$p_{1/2}^{-1}\ p_{3/2}^{-1}$	μ (Nucl. Mag.)	Q $X10^{-26}cm^2$
-22.95	0.9258	-0.1195	0.3617	0.351	1.06
-22.95	0.9162	0.0697	0.3946	0.321	1.10
-29.15	-0.3969	0.2930	0.8699	0.620	-0.76
-31.89	0.0550	0.9536	-0.2961	0.749	-0.76

Amplitudes

Wave functions for the J = 1, T = 0 levels in N^{14}, with the s-d contribution turned off. The first line is Fisscher and Ferrell's results.

The states so obtained are presented in Table VI, along with their calculated moments and energies.

In order to check the compatibility of these calculations with those done by Visscher and Ferrell, the s-d contribution in the matrix (35) was turned off. This was done by fixing the (3,1) element as zero, leaving the (3,3) element unchanged from that obtained by the trace condition as described above. The above procedure was then repeated. First the off diagonal and then the entire matrix was studied. These results compare quite favorably with those of Visscher and Ferrell. (See Table VII.)

As mentioned above, the ground state moments obtained by the present calculations agree very well with experimental data. The experimental moments are: [40]

$$\mu = 0.404 \text{ nuclear magnetons}$$
$$Q \simeq 1. \times 10^{-26} \text{cm}^2 \qquad (36)$$

It is to be emphasized that this calculation of the ground state did not partake, in any way, of fitting anything other than the experimental energy levels. This is in contrast to the calculations of Visscher and Ferrell who forced the experimental beta decay on their results.

The isotriplet matrix was then analyzed in a fashion similar to that used for the isosinglet matrix.

First the parameter y* was estimated using the trace condition. The appropriate energy eigenvalues are listed in Table II. It was found that y* = 9.04 Mev.

Next the off diagonal elements were studied. The error function again pertained only to the energies. After the minimum of the error function was found by varying the off diagonal elements, all of the matrix elements were varied (except those that should be zero because of orthogonality). The resulting matrix is:

$$\begin{bmatrix} -26.240 & 0.102 & 02.154 \\ 0.102 & -34.502 & 0 \\ -2.154 & 0 & -30.712 \end{bmatrix} \qquad (38)$$

The effects of the experimental uncertainty of the third eigenvalue were studied by using several neighboring values for this number. The states so obtained differed but little from each other, on the order of a percent change in each amplitude.

The eigenvectors of (38) are given in Table VIII.

Experimental M_1 [53] and Beta [42] reduced matrix elements for the transitions from the first isobaric triplet of levels to the ground state of N^{14} are compared with those calculated using the wave functions obtained by the present analysis in (39). It should be noted that there are two experimental beta transitions, one from the ground state of O^{14} and the other from the ground state of C^{14}.

		Exp.	Ours	
M1		.5	-0.5104	
Beta	O^{14}	1.06×10^{-2}	8.3×10^{-2}	(39)
	C^{14}	2.02×10^{-3}		

The reason for the two different experimental reduced matrix elements is that isospin is, strictly, not a good quantum number; that is, the forces between nucleons are charge dependent (e.g., the Coulomb force). These charge-dependent forces mix into the states we are considering other configurations of higher isotopic spin which do not contribute to the beta transition. Thus the reduced beta matrix elements depart from the charge independent value.

The fact that the reduced matrix elements should differ from each other, even neglecting the charge

dependence of the nuclear forces is that the Coulomb forces are different in the ground states of C^{14} and O^{14}. This would produce slightly different mixing in of other states and so lead to different transition rates.

The present outlook is that of initially assuming charge independence (after corrections for Coulomb and n-p mass difference effects). This can be seen in the method of determining the s-d state. This was done by comparing the isotriplet matrices to each other and requiring that they be identical (aside from a multiple of the unit matrix). This is the assumption of charge independence.

The number we have found for the Beta transition rate is, then, the charge independent value. Corrections to this value have to be made in order to obtain the rates that are influenced by the admixture of other isospin states. These corrections are influenced by the charge dependent interaction (2).

That reasonable values for the parameters p, q, r, and s can be found by making these corrections is shown by the work of Altman and MacDonald [1]. These authors used the wave functions of Visscher and Ferrell to find

TABLE VIII

Energy (Mev.)	Amplitudes $(p_{1/2})^2$	$(p_{3/2})^2$	$(s\text{-}d)^2$
-25.37	0.9274	0.0104	-0.3739
-31.58	0.3738	0.0131	0.9274
-34.497	-0.0145	0.9999	-0.0083

Wave functions for the J = 0, T = 1 levels in C^{14}, N^{14} and O^{14} obtained by the present analysis.

values for p, q, r, and s by comparing the ft-value difference between the O^{14} and C^{14} decays. Their results are given in (16).

The calculated M_1 transition rate in (39) compares quite favorably to the experimental value. Previous estimates of this number were off by about 30%. It is thus seen that the wave functions obtained in this work describe the N^{14} nucleus well.

In addition, the wave functions obtained by the present work may be compared with those found by True. [58] As mentioned above, True studied the spectrum of the N^{14} system by using a model Hamiltonian operator. His basis states considered the N^{14} system to be composed of a closed 1s and $1p_{3/2}$ core and two other particles moving in the $p_{1/2}$ or s-d shells. He was able to obtain qualitative agreement with the observed energy spectrum. The wave functions obtained for the first excited state (J = O, T = 1) by True and the present analysis are compared in (40).

2.311 Mev. State	$p_{1/2}^2$	$p_{3/2}^{-2}$	$d_{3/2}^2$	$d_{5/2}^2$	$2S_{1/2}^2$
Present	0.9274	0.01040	-0.04124	-0.34590	-0.13595
True's	-0.9501	-	0.1139	0.2635	0.1219

(40)

It is seen that the two wave functions compare well, aside from an overall phase factor. Comparison of the ground state results is not so fruitful because there are four basis states that are different for the two calculations. True considered, in addition to the s-d state (22) the additional two basis vectors that are possible, (see the beginning of this chapter) while he omitted the core excited states $p_{3/2}^{-2}$ and $p_{3/2}^{-1}p_{1/2}^{-1}$. He found that the mixing amplitudes for the two additional s-d states were small.

It is concluded that the wave functions that have been obtained by the present work are the "best" that the method will allow. We may now proceed to the calculations to determine the equations for the charge-dependent parameters.

V. RESULTS AND SUMMARY

Charge-Dependent Results

This section deals with the process of analyzing the energy shifts of the first isobaric triplet of levels in A=14. The energy shifts that must be explained are: [38]

$$\text{and} \quad \begin{aligned} O^{14} - N^{14*} &= 3.6159 \pm 0.0015 \text{ Mev.} \\ C^{14} - N^{14*} &= -2.9375 \pm 0.0012 \text{ Mev.} \end{aligned} \tag{41}$$

These values are already corrected for the n-p mass difference.

The largest contribution to these shifts comes from the Coulomb forces between the protons. The effect of the Coulomb force can be calculated by the methods and results of Appendix VII. With the value $e^2 \sqrt{\nu/2\pi} = 0.3445$. Mev. [38, 59] and the wave functions of Table VIII, we find

$$\text{Coulomb} \quad \begin{aligned} O^{14} - N^{14*} &= 3.525 \text{ Mev.} \\ C^{14} - N^{14*} &= -2.939 \text{ Mev.} \end{aligned} \tag{42}$$

The spin-orbit interaction generated by the motion of the nucleons (See Appendix VIII) contributes a small shift. We find:

$$\text{spin orbit} \quad \begin{cases} O^{14} - N^{14*} = 0.0145 \text{ Mev.} \\ C^{14} - N^{14*} = -0.0145 \text{ Mev.} \end{cases} \tag{43}$$

Combining all of these results we find that there is a residual shift

$$\text{residual} \quad \begin{cases} O^{14} - N^{14*} = 91 \text{ Kev.} \\ C^{14} - N^{14*} = 13 \text{ Kev.} \end{cases} \tag{44}$$

It is this shift that we attribute to the charge dependent portion of the specifically nuclear interaction. We see, then, that the n-p force is stronger than the n-n or p-p force. This agrees qualitatively with Altman and Macdonald. [1]

We assume a radial dependence in (2) of $V(r) = V_o \times \exp(-r^2/r_o^2)$, with a well depth of 50 Mev. and a range such that $r_o \mu^{1/2} = 1$. [38, 59] The results of the charge-dependent potential (2) may be computed, using the methods of Appendix VII. We find:

$$O^{14} - N^{14*} = -56.66p - 3.407q + 66.06r + 6.724s$$

nuclear: (45)

$$C^{14} - N^{14*} = 56.66p - 3.407q - 66.06r + 6.724s$$

Upon equating the residual shifts (44), to the nuclear shifts (45) we obtain:

$$-6.814q + 13.446s = 0.104 \text{ Mev.}$$
$$-113.32p + 132.12r = 0.078 \text{ Mev.} \quad (46)$$

Casting these into the form quoted by Blin-Stoyle and Nair [11] we find:

$$-q + 1.97s = 1.5 \times 10^{-2}$$
$$-p + 1.17r = 6.9 \times 10^{-4} \quad (47)$$

Care must be taken when comparing this result to those quoted in (17). Different conventions were assumed about the form of (2) and the z component of the isotopic spin for the neutron and proton. Blin-Stoyle and Nair used the Pauli (iso)spin operators rather than the (iso)spin operators in (2). This yields a difference of a factor of two in p and r and a factor of four in q and s. Because of the difference in the isospin assignments, the signs of p

and r are reversed. Taking all of these things into account our results are:

$$-q' + 1.97s' = 3.82 \times 10^{-3}$$

and (48)

$$-p' + 1.17\, r' = -3.4 \times 10^{-4}$$

which are comparable to the Rosenfeld mixture results of Blin-Stoyle and Nair (17).

It is now necessary to obtain two other equations for the parameters p, q, r, and s in order to find what their actual values would be. Within the context of the present analysis this could be done by computing the effect of the charge-dependent potential (2) on the beta decay rates of C^{14} and O^{14}, and comparing this to the deviation of these results from the charge-independent value found in the last chapter.

Summary

In this work we have obtained wave functions for the first few low-lying states in A=14. These wave functions take into account the effects of particles moving within the s-d shell. Good agreement with experimental data are obtained for the ground state properties of N^{14} as well as

the M1 transition rate from the first excited state of N^{14} to its ground state. Moreover, this analysis has led to a more symmetric interpretation of the beta transitions from the ground state's of C^{14} and O^{14} to the N^{14} ground state.

The objection of Blin-Stoyle and Nair with respect to the previous charge-dependent calculations was answered by including the effects of particles moving within the s-d shell. The results obtained were found to be comparable, but not equal, with their calculation using a Rosenfeld mixture for the charge-independent portion of the nucleon-nucleon force.

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APPENDIX I

THE CHARGE DEPENDENT TWO-BODY POTENTIAL

The general form of the two-body, static, charge dependent interaction between nucleons may be found in the following way: [9, 10]

Let H_n be the general nuclear Hamiltonian:

$$H_n = H_o + V, \tag{A1-1}$$

where H_o is the charge independent part of H_n and V is the charge dependent part, composed of two-body interactions, i.e.,

$$V = \sum_{i<j}^{A} V_{ij} \quad , \tag{A1-2}$$

where A is the number of nucleons in the system being considered.

Since charge must be conserved, we require the charge operator, Q, to commute with H_n, i.e.,

$$[Q, H_n] = 0. \tag{A1-3}$$

This may be cast into a more useform form if we use the

Isotopic Spin formalism to express Q:

$$Q = \frac{e}{2}\,[A - T_z] \tag{A1-4}$$

where T_z is the Z-component of the total Isotopic Spin operator

$$T_z = \sum_{i=1}^{A} \tau_z^i \tag{A1-5}$$

Since H_o is, by definition, an Isotopic Spin scalar (A1-4) reduces to

$$[T_z, V] = 0 \tag{A1-6}$$

Inserting (A1-2) and (A1-5) into (A1-6) and realizing that each commutator that results must vanish independently, we find

$$[\tau_z^i + \tau_z^j, V_{ij}] = 0 \tag{A1-7}$$

Let us consider how to construct the most general possible form of V_{ij} in its dependence on the operators $\vec{\tau}_i$ and $\vec{\tau}_j$. Clearly, since these are a representation of the rotation group of spin 1/2, we can build, at most, a first rank tensor, i. e., a product of two matrices, one of

which operates on the ith particle and the other on the jth particle. We have then, using the sum rules on repeated indices,

$$V'_{ij} = a_k \tau_k^i + b_k \tau_k^j + c_{kl} \tau_k^i \tau_l^j \quad . \tag{A1-8}$$

We may invoke the fact that the two particles are indistinguishable in the Isotopic Spin formalism. We, then, have the following results:

$$\begin{aligned} a_k &= b_k \\ & \qquad (\ell,k = z,+,-) \\ c_{k\ell} &= c_{\ell k} \end{aligned} \tag{A1-9}$$

from which we find

$$V_{ij} = a_k(\tau_k^i + \tau_k^j) + \tfrac{1}{2} c_{k\ell}(\tau_k^i \tau_\ell^j + \tau_\ell^i \tau_k^j) \tag{A1-10}$$

Now applying (A1-7), we find that only a few of the terms in (A1-10) result in a vanishing commutator; these are

$$\begin{aligned} & \tau_z^i + \tau_z^j \\ & \tau_z^i \tau_z^j \\ & \tau_+^i \tau_-^j + \tau_-^i \tau_+^j \end{aligned} \tag{A1-11}$$

But, since $c_{+,-} = c_{-,+}$, the last two terms appear together

as the sum

$$\tau_+^i\tau_-^j + \tau_-^i\tau_+^j = -\vec{\tau}^i\cdot\vec{\tau}^j + \tau_z^i\tau_z^j \ . \qquad \text{(A1-12)}$$

Dropping the scalar term since it is a charge independent term, we find

$$V_{ij} = \alpha(\tau_z^i + \tau_z^j) + \beta\tau_z^i\tau_z^j \qquad \text{(A1-13)}$$

where α and β contain the radial and spin dependences, etc. If we restrict ourself to a static potential, i.e., one with no momentum dependent terms, we have for the general form of the two-body static charge dependent potential:

$$V_{ij} = V_1(r_{ij})[p(\tau_z^i + \tau_z^j) + q\tau_z^i\tau_z^j] + V_2(r_{ij})[r(\tau_z^i + \tau_z^j) + s\tau_z^i\tau_z^j]\vec{\sigma}_i\cdot\vec{\sigma}_j \qquad \text{(A1-14)}$$

where V_1 and V_2 contain all of the radial dependence and the σ's are the Pauli spin operators. Summing and assuming that V_1 equals V_2, yields

$$V = \sum_{i<j} V(r_{ij})[p(\tau_z^i + \tau_z^j) + q\tau_z^i\tau_z^j + \{r(\tau_z^i + \tau_z^j) + s\tau_z^i\tau_z^j\}\vec{\sigma}_i\cdot\vec{\sigma}_j] \qquad \text{(A1-15)}$$

where p, q, r, and s measure the departure of the interaction from strict charge independence.

An example of this last statement is that if both p and r are zero, the neutron-neutron and proton-proton potentials are the same, and the nuclear forces would be charge symmetric.

APPENDIX II

SHELL MODEL CALCULATIONS

One of the basic assumptions of the Shell Model is that a nuclear system can be described by a wave function which is a superposition of products of a reasonably small number of single-particle wave functions, or states. If the isotopic spin formalism is used, in which neutrons and protons are considered as different states of a single-particle called the nucleon, then this requires wave functions which are completely antisymmetric under the interchange of any two of these nucleons.

Another Shell Model assumption, that the nucleons move in an averaged central field on which is superimposed a weak (residual) nucleon-nucleon force as a correction, leads to the concept of nuclear shells. A shell is defined as a set of degenerate single-particle energy eigenvalues of the unperturbed Hamiltonian. It is assumed in the simplest model that these shells are filled successively, starting from the lowest and proceeding upward as each shell is filled, bearing in mind always that the Pauli Exclusion Principle must be fulfilled.

One is led naturally to treat the wave function of a nuclear system in terms of these nuclear shells. Antisymmetric wave functions are constructed for all the nucleons in a given shell and then this set of wave functions is explicitly antisymmetrized. In practice this program is not always carried out completely. One then deals with just a few unfilled shells and ignores the filled shells, calling them inert.

To carry out this program, the various wave functions for different numbers of nucleons in different shells have been studied and classified by various authors. [17-20, 22, 23, 27-30] The approach has been through the methods of group theory, and some of the quantum numbers used in this classification are strictly of a group theoretic nature. For instance, the seniority, s, and the reduced isotopic spin, t, are two such quantum numbers. Two other, more familiar, quantum numbers used in the classification are the total angular momentum J and the total isotopic spin T; likewise, their z-projections are also used, but for compactness of notation will not be indicated. All other quantum numbers necessary for the detailed

classification of a nuclear shell state are grouped under a common symbol, say α. For convenience, the seniority and the reduced isotopic spin will not be indicated and may be thought to be subsumed in α.

Often in the course of this discussion vector coupled wave functions involving several groups of particles will appear. To avoid writing numerous vector coupling coefficients an appropriate notation is necessary. A set of vector coupled wave functions will be enclosed in brackets. Commas will separate each term, and semicolons will indicate intermediate coupling. Colons will indicate the final result. Thus

$$[\psi_1(J_1T_1\alpha_1),\psi_2(J_2T_2\alpha_2);J'T',\psi_3(J_3T_3\alpha_3):JT] \qquad \text{(A2-1)}$$

indicates functions 1, 2 coupled to J', T', and this coupled with 3 to give the final result J, T.

For the purpose of constructing the fully antisymmetric wave functions in some definite and manageable way, and also because of the simplicity it lends to the calculations, the method of fractional parentages was developed. This was first done for the analogous atomic case by Racah [43-46] and then applied by him and others [17-20, 22, 23

27-30, 51] to the nuclear case. The idea goes as follows: Suppose one knows the properly antisymmetric wave functions for N-1 nucleons in a given shell. Call these $\psi(j^{N-1}(JT\alpha))$ and supposé $\psi(j)$ is the wave function for a single particle. (Clearly the total angular momentum for one particle is just j, and the isotopic spin is 1/2. Thus, these quantities are not explicitly listed.) The set of wave functions

$$[\psi(j^{N-1}(JT\alpha),\psi(j):J'T'] \qquad \text{(A2-2)}$$

form a set of functions which are antisymmetric in the first N-1 nucleons and have no special symmetry for the Nth nucleon. These form a complete set of functions for N nucleons. The projection of this set onto the subset of completely antisymmetrized functions is the method of fractional parentages. Let $\psi(j^n(J'T'\alpha'))$ be the fully antisymmetric wave function for n nucleons. Then by the general expansion theorems of quantum mechanics

$$\psi(j^n(J'T'\alpha')) = \sum_{JT\alpha} [j^n(J'T'\alpha')\{\}j^{n-1}(JT\alpha),j:J'T']x \qquad \text{(A2-3)}$$

$$[\psi(j^{n-1}(JT\alpha)),\psi(j):JT]$$

The coefficients in this expansion are the fractional parentage coefficients (cfp). The symbol $|\}$ is used to denote that this is not a unitary transformation, but rather a projection.

The wave functions can be coupled in any scheme desired. L-S and j-j couplings are commonly used. The cfp are different for these different couplings, of course. The cfp have been calculated and tabulated for a limited number of cases. [17-19, 22, 23, 25, 27-30]

A nuclear system need not be in a pure configuration. (A configuration here means that there are N_1 particles in the first shell, characterized by a set of eigenvalues $\{\alpha_1\}$, N_2 particles in the second shell . . ., etc.). It may be described by a mixed configuration, i.e., a superposition of several different pure configurations. In this more complicated case the above scheme is still useful, since as is shown below the evaluation of matrix elements is still straightforward.

Keeping these things in mind, we shall now discuss the evaluation of matrix elements for one- and two-particle operators. Suppose one wishes to evaluate the matrix element for a single-particle operator F where

$$F = \sum_{i=1}^{N} f_i \qquad \text{(A2-4)}$$

between two states $\psi(j^n(JT\alpha))$ and $\psi(j^n(J'T'\alpha'))$ where all the particles are equivalent, i.e., belong to the same shell. Then using (A2-3) there results:

$$\langle\psi(j^N(JT\alpha))|F|\psi(j^N(J'T'\alpha'))\rangle = \sum_{\substack{\alpha''J''T'' \\ \alpha'''J'''T'''}} [j^N(JT\alpha)||j^{N-1}(J''T''\alpha''),j:JT] \times$$

$$[j^{N-1}(J'''T'''\alpha'''),j:J'T'||j^N(J'T'\alpha')] \times$$

$$\langle j^{N-1}(J'''T'''\alpha'''),j:J'T'|F|j^{N-1}(J''T''\alpha''),j:JT\rangle \qquad \text{(A2-5)}$$

Since all of the nucleons are treated as identical, the matrix element for F is just N times that for a single particle, say that for the Nth. Then (A2-5) reduces to:

$$\langle\psi(j^N(JT\alpha))|F|\psi(j^N(J'T'\alpha'))\rangle = N\sum_{\alpha''J''T''} [j^N(JT\alpha)||j^{N-1}(J''T''\alpha''),j:JT] \times$$

$$[j^{N-1}(J''T''\alpha''),j:J'T'||j^N(J'T'\alpha')]\langle\psi(j)|f|\psi(j)\rangle \qquad \text{(A2-6)}$$

The evaluation of matrix elements for two-body interactions poses a more difficult problem. This is so, because, in order to evaluate these matrix elements, two particles have to be separated from the many-particle wave functions, with all of the attendant proliferation of vector coupling coefficients and multiplication of cfp. To get around this problem the N→N-2 cfp are defined. It is through these that the auxiliary quantum numbers "s" and "t" receive their proper definition, and their meaning can be studied (see, _e.g._, Reference 55). Suppose $\psi(j^n(JT\alpha))$ is an antisymmetric wave function for N nucleons. Then following a chain of reasoning similar to that for the N → N - 1 cfp one gets:

$$\psi(j^N(JT\alpha)) = \sum_{\substack{\alpha' J' T' \\ J'' T''}} [j^{N-2}(J'T'\alpha'), j^2(J''T'') : JT | \} j^N JT\alpha] \times [\psi(j^{N-2}(J'T'\alpha')), \psi(j^2(J''T'')) : JT] \qquad \text{(A2-7)}$$

With this definition it will be possible to calculate two-body matrix elements of operators such as:

$$V = \sum_{i<j} V_{ij} \qquad \text{(A2-8)}$$

As might be expected, there is a close connection between the $N \rightarrow N-2$ and the $N \rightarrow N-1$ cfp. This can be found by using (A2-3) twice, rearranging the vector couplings, and then comparing this result to the definition for the $N \rightarrow N-2$ cfp. The result is

$$[j^{N-2}(J_1T_1\alpha_1),j^2(J_2T_2):JT|\}j^N(JT\alpha)] = \sum_{J'T'\alpha'}(-)^{J_1+T_1+J+T} \times$$

$$\sqrt{[J'][J_2][T'][T_2]}\begin{Bmatrix} j & j & J_2 \\ J_1 & J & J' \end{Bmatrix}\begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & T_2 \\ T_1 & T & T' \end{Bmatrix} \times \qquad \text{(A2-9)}$$

$$[j^{N-1}(J'T'\alpha'),j:JT|\}j^N(JT\alpha)][j^{N-2}(J_1T_1\alpha_1),j:J'T'|\}j^{N-1}(J'T'\alpha')]$$

where $\begin{Bmatrix} A & B & C \\ D & E & F \end{Bmatrix}$ is a 6-j symbol as defined by Rotenberg, et. al. [50] and $[J] = 2J + 1$.

If a nuclear state is described by a mixed configuration, then one is confronted with the problem of calculating two-body matrix elements when some of the particles are in one shell and some are in another. The method of fractional parentages reduces this to a relatively straightforward task. Suppose $\psi_1(j_1^{n_1}(J_1T_1\alpha_1))$ and $\psi_2(j_2^{n_2}(J_2T_2\alpha_2))$

are the antisymmetrized wave functions for n_1 particles in the j_1 shell and n_2 particles in the j_2 shell, respectively, and that a non-antisymmetrized wave function coupled to given (J, T) is:

$$\psi = [\psi_1(j_1^{n_1}(J_1T_1\alpha_1)), \psi_2(j_2^{n_2}(J_2T_2\alpha_2) : JT] \qquad \text{(A2-10)}$$

then calculations with this wave function will yield the same results as the properly antisymmetrized wave functions would, provided antisymmetric two-body states are used in taking the two-body matrix elements that arise.

When taking matrix elements of a two-particle operator between states like (A2-10), first write it in the form:

$$V = \sum_{i<j}^{n_1} V_{ij} + \sum_{n_1<i<j}^{n_2} V_{ij} + \sum_{i=1}^{n_1} \sum_{n_1<j}^{n_2} V_{ij} \qquad \text{(A2-11)}$$

These matrix elements clearly reduce to three terms. Two consist of matrix elements for particles belonging to the same shell, while the third term is for particles belonging to two different shells.

The evaluation of these last matrix elements, when particles are in different shells, requires some added techniques. For this problem, the generalized cfp are constructed. These generalized cfp allow one to extract the wave function of the nth nucleon from a fully anti-symmetrized state consisting of nucleons moving in two different shells. (Clearly this is all that is necessary when dealing with two-nucleon interactions.) Amit and Katz [2] find:

$$\psi(j^{a}j'^{,b}\alpha JT) = \sum_{\alpha_1 J_1 T_1} (j^{a-1}j'^{,b-1}(\alpha_1 J_1 T_1)j\alpha JT)(j^{a-1}j'^{,b}(\alpha_1 J_1 T_1)j\{|j^{a}j'^{,b}\alpha JT)$$

$$+ \sum_{\alpha_1 J_1 T_1} \psi(j^{a}j'^{,b-1}(\alpha_1 J_1 T_1)j'\alpha JT)(j^{a}j'^{,b-1}(\alpha_1 J_1 T_1)j'\{|j^{a}j'^{,b}\alpha JT), \tag{A2-12}$$

$$(j^{a-1}j'^{,b}(\alpha_1 J_1 T_1)j\{|j^{a}j'^{,b}\alpha JT) = \sqrt{\frac{a}{n}}\,(j^{a-1}(\alpha_2 J_2 T_2)j\{|j^{a}\alpha'J'T')$$

$$\sqrt{(2J'+1)(2T'+1)(2J_1+1)(2T_1+1)}\quad xW(J'J''jJ_1;JJ_2)W(T'T''\tfrac{1}{2}T_1;TT_2), \tag{A2-13}$$

and

$$(j^{a}j'^{,b-1}(\alpha_1 J_1 T_1)j'\{|j^{a}j'^{,b}\alpha JT) = \sqrt{\frac{b}{n}}\,(-1)^{k}(j'^{,b-1}(\alpha_2 J_2 T_2)j'\{|j^{b}\alpha''J''T'')$$

$$x\sqrt{(2J''+1)(2T''+1)(2J_1+1)(2T_1+1)}\ W(J''J'j'J_1;JJ_2)W(T''T'\tfrac{1}{2}T_1;TT_2), \tag{A2-14}$$

and

$$K = 2T' + T'' - T + T_2 - T_1 + 2J' + J'' - J + J_2 - J_1.$$

where the W functions are the Racah functions as defined by, e.g., Rose. [49] These results may then be used to perform calculations in the ways indicated above.

It is evident from the above discussion that the whole paraphenalia of Shell Model calculations is reduced, via the powerful techniques of fractional parentages, to the evaluation of one-body and two-body matrix elements.

There are other methods that are associated with Shell Model calculations. Among these is the idea of particles and holes. There are general theorems which allow one to reduce the labor of calculating some many-particle matrix elements by replacing them with few-particle matrix elements. In special cases, namely when near closed shells, this is convenient.

APPENDIX III

THE TALMI INTEGRAL DECOMPOSITION

The use of harmonic oscillator wave functions in Shell Model calculations allows the explicit evaluation of the various radial integrals (the Slater Integrals) that develop. This method was introduced by Talmi [54] and the technique expanded by Kronheimer [34]. This analysis summarizes their work. (See also Thieberger [56].)

The whole point of the method is to reduce the evaluation of any given Slater Integral to a sum over Talmi integrals, the latter being denoted by I and defined by:

$$I_\ell(\nu/2) = \frac{\int_\theta^\infty r^{2\ell+2} e^{-\nu r2/2} V(r)dr}{\int_0^\infty r^{2\ell+2} e^{-\nu r2/2} dr} \qquad \text{(A3-1)}$$

where the ν refers to the harmonic oscillator spring constant.

The three-dimensional harmonic oscillator Hamiltonian in spherical coordinates is given by:

$$H = \frac{1}{2m} (p^2 + m^2\omega^2 r^2) \qquad \text{(A3-2)}$$

and its eigenfunctions are:

$$\psi_{n\ell}(r,\theta,\phi) = \frac{R_{n\ell}(r)}{r} Y_{\ell}^{m}(\theta,\phi) \tag{A3-3}$$

where the Y_{ℓ}^{m} are the ordinary spherical harmonics and the radial part $R_{n\ell}$ is

$$R_{n\ell}(r) = N_{n\ell} \exp(-\nu r^2/2) L_{n+\ell+1/2}^{\ell+1/2}(\nu r^2) \tag{A3-4}$$

where $\nu = m\omega/\hbar$ and the $L_{n+\ell+1/2}^{\ell+1/2}$ are the associated Laguerre polynomials and are defined by

$$L_{n+\ell+1/2}^{\ell+1/2}(\nu r^2) = \sum_{k=0}^{n} (-)^k (2\nu)^k \binom{n}{k} \frac{(2\ell+1)!!}{(2\ell+2k+1)!!} \tag{A3-5}$$

and $N_{n\ell}$ is the normalization constant for the associated Laguerre polynomials and is given by:

$$N_{n\ell}^2 = \frac{2^{\ell-n+2}(2\ell+2n+1)!!\ \nu^{\ell+3/2}}{\sqrt{\pi}\ n!\ \{(2\ell+1)!!\}^2} \tag{A3-6}$$

and the double factorial notation has its usual meaning, except $(-1)!!$ is defined to be 1, for later purposes.

Also the notation $\binom{n}{\ell}$ has been used for the binomial coefficient.

A product of two radial functions is given by:

$$R_{n\ell}(r)\, R'_{n'\ell'}(r) = N_{n\ell}N_{n'\ell'}e^{-\nu r^2}\, r^{\ell+\ell'+2}\sum_{s=0}^{n+n'} A_s\, r^{2s} \qquad \text{(A3-7)}$$

where,

$$A_s=(-)^s(2\nu)^s\sum_k \binom{n}{k}\binom{n'}{s-k}\frac{(2\ell+1)!!(2\ell'+1)!!}{(2\ell+2s+2k+1)!!(2\ell'+2s-2k+1)!!} \qquad \text{(A3-8)}$$

and the sum is taken over all terms for which the factorials are defined.

The definitions of the direct (F^k), and the Exchange (G^k) Slater integrals are:

$$F^k(n,\ell;n',\ell') = \int_0^\infty \int_0^\infty R_{n\ell}{}^2(r_1)R_{n'\ell'}^2(r_2)V_k(r_1,r_2)dr_1dr_2 \qquad \text{(A3-9)}$$

and

$$G^k(n,\ell;\, n',\ell') = \int_0^\infty \int_0^\infty R_{n\ell}(r_1)R_{n\ell}(r_2)R_{n'\ell'}(r_1)R_{n'\ell'}(r_2)V_k(r_1r_2)dr_1dr_2$$

where V_k is found from the expansion of the two-body potential $V(r_{12})$ in terms of the Legendre polynomials:

$$V(\vec{r}_{12}) = \sum_k P_k(\text{Cos}\ \omega_{12})V_k(r_1r_2) \qquad \text{(A3-10)}$$

where ω_{12} is the angle between $\vec{r}_1$ and $\vec{r}_2$.

Inserting the Harmonic Oscillator wave functions (A3-3) in the definition of the Slater integrals (A3-9) there results:

$$F_k(n,\ell;n',\ell') = N^2_{n\ell}N^2_{n'\ell'} \sum_{s=0}^{2n}\sum_{t=0}^{2n'} \frac{A_s A_t}{\nu^{(\ell+s+3/2)}\nu^{(\ell'+t+3/2)}} \quad x$$

$$\sum_{m=0}^{s+t+\ell+\ell'} \phi_m(\ell+\ell'+2s,\ell+\ell'+2t,k) I_m(\gamma/2) \qquad \text{(A3-11a)}$$

$$G_k(n,\ell;n',\ell') = N^2_{n\ell}N^2_{n'\ell'} \sum_{s,t=0}^{n+n'} \frac{A_s A_t}{\nu^{(\ell+s+3/2)}\nu^{(\ell'+t+3/2)}} \quad x$$

$$\sum_{m=0}^{s+t+\ell+\ell'} \phi_m(\ell+\ell'+2s,\ell+\ell'+2t,k) I_m(\nu/2) \qquad \text{(A3-11b)}$$

where the results of Kronheimer have been used and

$$\phi(L_1,L_2,k) = \frac{\pi(2k+1)}{2^{2L+4}} \sum_{g=0}^{[k,\ell]} (-)^g \frac{(2\ell+1)!!}{(2\ell+2g+1)!!} \sum_{p=g}^{[k,L-\ell+g]} \binom{k+p}{k}$$

$$\binom{k}{p}\binom{p}{g} \sum_{m=p+g}^{L-1} C_{A,B}\ (2g-2p+2m)!\,(2L+1-2g-2m)!!\,(2m-1)!! \qquad \text{(A3-12)}$$

and where $A = L_1 + 1-p$, $B = L_2 + 1-p$, $C_{A,B}(N)$ is the coefficient of t^N in the expansion of $(1 + t)^A x(1-t)^B$, $[A, B]$ denotes the lesser of A or B, and $L = (L_1 + L_2)/2$.

The Talmi integrals for the Coulomb and short-range Gaussian potentials can be worked out directly. They are given in Table (A3-1).

TABLE (A3-1)

Nuclear and Coulomb Talmi Integrals

$V(r)$	$I_\ell(\nu/2)$
$V_0 \exp(-r^2/r_0^2)$	$V_0[\nu r_0^2/(2+\nu r_0^2)]^{\ell+3/2}$
e^2/r	$e^2\sqrt{\nu/2\pi}\,\frac{2^{\ell+1}\,\ell!}{(2\ell+1)!!}$

APPENDIX IV

COULOMB AND FOUR-PARTICLE MATRIX ELEMENTS

Coulomb Matrix Elements

We are interested in Coulomb matrix elements between the two-particle states

$$|\psi_1\rangle = |p^2_{1/2}\rangle,\ |\psi_2\rangle = |p^{-2}_{3/2}\rangle,\ |\psi_3\rangle = |d^2_{3/2}\rangle,$$
$$|\psi_4\rangle = |d^2_{5/2}\rangle,\ |\psi_5\rangle = |2s^2_{1/2}\rangle \qquad \text{(A4-1)}$$

where the letters p, d, s, indicate the values of the orbital angular momentum and the subscripts 1/2 etc., indicate the j value for each particle. All of these states are coupled to J = O, T = 1 which is not indicated explicitly.

The conversion of all of these states to the L-S coupling scheme may be carried out using the transformation coefficients

$$\alpha = \langle (s_1,s_2)S;(\ell_1,\ell_2)L:JM | (s_1,\ell_1)j_1;(s_2,\ell_2)j_2:JM\rangle$$
$$= \sqrt{[S][L][j_1][j_2]} \begin{Bmatrix} S & \frac{1}{2} & \frac{1}{2} \\ L & \ell_1 & \ell_2 \\ J & j_1 & j_2 \end{Bmatrix} \qquad \text{(A4-2)}$$

where the bracket is a 9-j symbol as defined in [50].

TABLE A4-1

L-S	$d^2_{5/2}$	$d^2_{3/2}$	$p^2_{1/2}$	$p^2_{3/2}$	$2s^2_{1/2}$
0	$\sqrt{3/5}$	$\sqrt{2/5}$	$\sqrt{1/3}$	$\sqrt{2/3}$	1
1	$-\sqrt{2/5}$	$\sqrt{3/5}$	$\sqrt{2/3}$	$-\sqrt{1/3}$	0

Table of transformation coefficients between the j-j and L-S Coupling Scheme for J = 0, T = 1.

For J = O this reduces to

$$\alpha = (-)^{j_1+S+\ell_1+1/2}\,[L][j_1]\begin{Bmatrix} 1/2 & 1/2 & S \\ \ell_2 & \ell_1 & j_1 \end{Bmatrix}\delta_{j_1,j_2}\delta_{L,S} \qquad \text{(A4-3)}$$

where the bracket is a 6-j symbol.

The Coulomb interaction is diagonal in the L-S coupling scheme. Hence, each of the matrix elements

$$M_{ij} = \langle\psi_i|V_c(r)|\psi_j\rangle \qquad \text{(A4-4)}$$

where $V_c(r)$ is the Coulomb interaction, can be written as a sum over diagonal L-S coupled matrix elements.

$$M_{ij} = \sum_{L=0} a_{ij}^{L}\langle(n_i\ell_i)^2L=S|V_c(r)|(n_j\ell_j)^2L=S\rangle \qquad \text{(A4-5)}$$

where n_i and n_j indicate the quantum numbers of the single-particle radial wave functions.

Each of these L-S coupled matrix elements can be further decomposed into Slater integrals. There are two cases:

$(n_i\ell_i) = (n_j\ell_j)$

$$\langle(n_i\ell_i)^2L|V_c(r)|(n_j\ell_j)^2L\rangle = (-)^L[\ell_i]^2\sum_k\begin{pmatrix}\ell_i & \ell_i & k\\ 0 & 0 & 0\end{pmatrix}^2 \times \begin{Bmatrix}\ell_i & \ell_i & L\\ \ell_i & \ell_i & k\end{Bmatrix} F^k(n_i,\ell_i;n_j,\ell_j) \qquad \text{(A4-6)}$$

and

$$(n_i \ell_i) \neq (n_j \ell_j)$$

$$\langle (n_i \ell_i)^2 L | V_c(r) | (n_j \ell_j)^2 L \rangle = (-)^L [\ell_i]^2 \sum_k \begin{pmatrix} \ell_i & \ell_j & k \\ 0 & 0 & 0 \end{pmatrix}^2 \times \begin{Bmatrix} \ell_i & \ell_i & L \\ \ell_j & \ell_j & k \end{Bmatrix} G^k(n_i, \ell_i; n_j, \ell_j) \qquad \text{(A4-7)}$$

where the F^k and the G^k are the direct and exchange Slater integrals (A3-9) and the $\begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix}$ are the 3-j symbols as defined by Rotenberg, et. al. [50]

The decomposition of these Slater integrals to Talmi integrals by the use of oscillator wave functions has been carried out by Thieberger. [56] Those results that are pertinent are given in Table A7-4.

For the Coulomb interaction the Talmi integrals are given in Table A3-1. Inserting these values for the I_p into equations (A4-8) and (A4-7) and then inserting these in turn into (A4-5) yields the matrix M_{ij} that is tabulated in Table A4-3.

Four-Particle Matrix Element Calculation

Four particles in the $p_{1/2}$ shell can only couple to J = O and T = O. Using the results of Appendix II on the Shell Model techniques we have, for a charge independent interaction:

TABLE A4-2

a^k_{ij}	k=0					k=1				
i \ j	1	2	3	4	5	1	2	3	4	5
1	1/3	$\sqrt{2}/3$	$\sqrt{2/15}$	$1/\sqrt{5}$	$1/\sqrt{3}$	2/3	$-\sqrt{2}/3$	$\sqrt{2/5}$	$-2/\sqrt{15}$	0
2	$\sqrt{2}/3$	2/3	$2/\sqrt{15}$	$\sqrt{2/5}$	$\sqrt{2/3}$	$-\sqrt{2}/3$	1/3	$-1/\sqrt{5}$	$\sqrt{2/15}$	0
3	$\sqrt{2/15}$	$2/\sqrt{15}$	2/5	$\sqrt{6}/5$	$\sqrt{2/5}$	$\sqrt{2/5}$	$-1/\sqrt{5}$	3/5	$-\sqrt{6}/5$	0
4	$1/\sqrt{5}$	$\sqrt{2/5}$	$\sqrt{6}/5$	3/5	$\sqrt{3/5}$	$-2/\sqrt{15}$	$\sqrt{2/15}$	$-\sqrt{6}/5$	2/5	0
5	$1/\sqrt{3}$	$\sqrt{2/3}$	$\sqrt{2/5}$	$\sqrt{3/5}$	1	0	0	0	0	0

Coefficients a^k_{kj} as defined in (A4-5)

TABLE A4-3

j \ i	1	2	3	4	5
1	3/2	2/6	$-11\sqrt{2}/36$	$-\sqrt{3}/12$	$-7/72$
2	$\sqrt{2}/6$	5/3	$-19/90$	$-13\sqrt{6}/60$	$-7\sqrt{2}/72$
3	$-11\sqrt{2}/36$	$-19/90$	$141/100$	$121\sqrt{6}/600$	$19\sqrt{2}/240$
4	$-\sqrt{3}/12$	$-13\sqrt{6}/60$	$121\sqrt{6}/600$	$889/600$	$19\sqrt{3}/240$
5	$-7/72$	$-7\sqrt{2}/72$	$19\sqrt{2}/240$	$19\sqrt{3}/240$	$313/288$

This table gives the Coulomb matrix elements between the states (A4-1) in units of $e^2\sqrt{\nu/2\pi}$ where e is the charge on the electron, and ν is the oscillator constant.

$$\langle p^4_{1/2}|H_{int4}|p^4_{1/2}\rangle = 6\sum_{\substack{J'T'\\J''T''}} \langle (1/2)^2 J'T'|H_{int2}|(1/2)^2 J'T'\rangle \times$$

$$[(1/2)^4 J=0T=0|\}(1/2)^2(J''T'');(1/2)^2(J'T'):J=0T=0]^2 \qquad \text{(A4-8)}$$

Using equation (13) of Appendix II the $N \to N-2$ cfp become

$$[(1/2)^4 J=0T=0|\}(1/2)^2(J'T');(1/2)^2(J''T''):J=0T=0] = \frac{(-)^J}{\sqrt{2}}\delta_{J',J''}\delta_{T',T''} \qquad \text{(A4-9)}$$

Inserting (A4-9) into (A4-8) one obtains:

$$\langle p^4_{1/2}|H_{int4}|p^4_{1/2}\rangle = 3\{\langle (1/2)^2 J=0T=1|H_{int2}|(1/2)^2 J=0T=1\rangle + \langle (1/2)^2 J=1T=0|H_{int2}|(1/2)^2 J=1T=0\rangle\} \qquad \text{(A4-10)}$$

For the Coulomb interaction

$$V_c(r) = \frac{e^2}{r}(1/2 - \tau^1_z)(1/2 - \tau^2_z) \qquad \text{(A4-11)}$$

so

$$\langle JTT_z|V_c(r)|JTT_z\rangle = \frac{T_z(T_z - 1)}{2}\langle JTT_z|e^2/r|JTT_z\rangle \qquad \text{(A4-12)}$$

using the results of Appendix II again, we obtain

$$\langle (p_{1/2})^4|V_{c4}|(p_{1/2})^4\rangle = \langle (1/2)^2 J=0T=1|e^2/r|(1/2)^2 J=0T=1\rangle \qquad \text{(A4-13)}$$

For the total interaction Hamiltonian:

$$H_4 = H_{int4} + V_{c4} \tag{A4-14}$$

one obtains

$$\langle p^4_{1/2}|H_4|p^4_{1/2}\rangle = \langle p^4_{1/2}|H_{int4}|p^4_{1/2}\rangle + \langle p^4_{1/2}|V_{c4}|p^4_{1/2}\rangle \tag{A4-15}$$

Using the previous results for the two-particle Coulomb interaction and the results of Amit and Katz [2] the four-particle matrix element is just

$$\langle p^4_{1/2}|H_4|p^4_{1/2}\rangle = 19.476\text{Mev}. \tag{A4-16}$$

APPENDIX V

TRANSITIONS FROM THE (J = 0, T = 1) STATES IN A = 14 TO THE (J = 1, T = 0) GROUND STATE OF N^{14}

The M1 and Gamow-Teller transitions from the isovector states in the mass 14 system to the ground state isosinglet of N^{14} can be computed in parallel ways. This is because the operators inducing these transitions are very similar, in part, and their reduced matrix elements (reduced, that is, in isotopic spin) are essentially the same.

M1 Lifetime

The M1 operator is the more complicated of the two operators involved in these transitions and will be considered first.

Weidenmuller [61] gives the expression for the M1 lifetime τ

$$\frac{1}{\tau} = \frac{1}{3}\ \frac{e^2}{\hbar c}\ \omega\ \left[\frac{E}{Mc^2}\right]^2 \sum_{m_i m_f} \frac{|\langle J_i m_i|0|J_f m_f\rangle|^2}{(2J_i+1)} \qquad \text{(A5-1)}$$

where

$$0 = \frac{1}{2} \sum_i^A [\ell_\mu^i + (\mu_p + \mu_n)\sigma_\mu^i] + \sum_i^A [\ell_\mu^i + (\mu_p - \mu_n)\sigma_\mu^i]\ \tau_Z^i \qquad \text{(A5-2)}$$

and $\omega = kc$. E is the energy of the transition and M is the mass of a proton.

The sum over the magnetic quantum numbers in (A5-1) can be evaluated, in terms of the angular momentum reduced matrix element, by application of the Wigner Eckhart theorem. One then obtains:

$$\frac{1}{\tau} = \frac{1}{3}\,\frac{e^2}{\hbar c}\,\omega\,\left[\frac{E}{Mc^2}\right]^2\,\frac{|\langle J_i T_i M_{ti}||0||J_f T_f M_{tf}\rangle|^2}{(2J_i+1)}\,\varepsilon \qquad \text{(A5-3)}$$

Here ε is 1 if $(J_i, J_f, 1)$ satisfy the triangle rules for the addition of angular momenta, and is zero otherwise. The reduced matrix element is that defined by Talmi and de Shalit. [55]

For a transition between an isotriplet and isosinglet level, such as one here, the isoscalar portion of the operator (A5-2) makes no contribution. This is so because a scalar operator has non-vanishing matrix elements only between states with the same vector quantum numbers. In the following, therefore, we need deal only with the isovector, the second, term in this equation. This operator will be called O_{eff}.

Using the wave functions (21), of the text, the only non-vanishing matrix elements of O_{eff} are $\langle\psi_1|0|\psi_1'\rangle$, $\langle\psi_2|0|\psi_2'\rangle$, $\langle\psi_3|0|\psi_3'\rangle$, $\langle\psi_4|0|\psi_1'\rangle$, and $\langle\psi_4|0|\psi_2'\rangle$.

Primes denote the excited state functions [(J, T) = (0, 1)] and unprimed vectors denote the ground state functions [(J, T) = (1, 0)]. All of the other matrix elements that would arise in computing the transition probability vanish because a single particle operator such as (A5-2) can couple at most one particle in different orbitals.

Of these matrix elements, the first and third are essentially two-particle matrix elements. The second reduces to a two-particle one as can be seen by the method of fractional parentages. Using the cfp of Glaudemans, et. al. [25] we obtain:

$$\langle p_{3/2}^{\,6}\,(1,0)|O_6|p_{3/2}^{\,6}(0,1)\rangle = \langle p_{3/2}^{\,2}(1,0)|O_1|p_{3/2}^{\,2}(0,1)\rangle \qquad \text{(A5-4)}$$

It is a relatively straightforward task to compute the reduced matrix element:

$$\langle (\ell j)^2\ J = 0\ ||\ \ell^1 + (\mu_p - \mu_n)\sigma^1||(\ell j)^2\ J = 1\rangle = -\frac{1}{\sqrt{j(j+1)}}\ x$$

$$\{\tfrac{1}{2}\,[\ell(\ell+1) + j(j+1) - 3/4] + (\mu_p - \mu_n)[3/4 + j\,(j+1) - \ell(\ell+1)]\} \qquad \text{(A5-5)}$$

The matrix element for the isotopic spin portion is ($M_t = 0$):

$$\langle (\tfrac{1}{2})^2 \; T=1 \,||\, \tau_Z^1 \,||\, (\tfrac{1}{2})^2 \; T=0 \rangle = \frac{1}{2} \qquad \text{(A5-6)}$$

The reduced matrix element of the two-particle operator O_{eff} is just twice the product of these last two equations, i.e., the right-hand side of (A5-4).

The ten-particle matrix elements $\langle \psi_4 | O_{eff} | \psi_1' \rangle$ and $\langle \psi_4 | O_{eff} | \psi_2' \rangle$ may be evaluated using the generalized cfp as developed by Amit and Katz [2], and quoted in Appendix II, and using the cfp of Glaudemans, et al.

These turn out to be:

$$\langle \psi_4 || O_{eff} || \psi_1^1 \rangle = \frac{1}{\sqrt{3}} (1-2(\mu_p-\mu_n))$$

$$\langle \psi_4 || O_{eff} || \psi_2^1 \rangle = -\frac{1}{\sqrt{6}} (1-2(\mu_p-\mu_n)) \qquad \text{(A5-7)}$$

Beta Lifetime

The beta decay from the ($J = 0$, $T = 1$) states in the mass 14 system (these states are the ground states of C^{14} and O^{14}) to the ground state ($J = 1$, $T = 0$) of N^{14} proceeds via the allowed Gamow-Teller mode.

The transition rate for this process is described (in the usual β decay notation) by:

$$\frac{K}{ft} = \left| \frac{C_V}{C_A} \right|^2 \quad \frac{|\langle J_i||\sigma||J_f\rangle|^2}{(2J_i + 1)} \qquad \text{(A5-8)}$$

The reduced matrix element is that defined by Talmi and de Shalit and still contains isotopic spin dependence, which can be factored out. This factor is:

$$\langle T=1, m_T | \tau_I^1 | T=0, m_T'=0 \rangle = \mp \frac{1}{\sqrt{2}} \delta_{m_T,1} \qquad \text{(A5-9)}$$

The results for the M1 lifetime may now be used by extracting from them that portion due to the operator σ. This is just the coefficient of $\mu_p - \mu_n$. This term is then to be multiplied by $\mp\sqrt{2}$ which is the ratio of the matrix elements for τ_I and τ_Z.

The final results are quoted in Table (A5-1).

	STATES		$\langle\psi_1\|\|O\|\|\psi_2'\rangle$	
	$\|\psi_1\rangle$	$\|\psi_2'\rangle$	M_1	BETA
(1)	$p_{1/2}^{2}$	$p_{1/2}^{2}$	$-\frac{1}{\sqrt{3}}(2-\frac{g_p-g_n}{2})$	$\mp\sqrt{2/3}$
(2)	$p_{3/2}^{6}$	$p_{3/2}^{6}$	$-\sqrt{\frac{5}{3}}(1+\frac{g_p-g_n}{2})$	$\pm\sqrt{10/3}$
(3)	$p_{1/2}^{-1}\, p_{3/2}^{-1}$	$p_{1/2}^{2}$	$\frac{1}{\sqrt{3}}(1-2\frac{g_p-g_n}{2})$	$\pm 2\sqrt{2/3}$
(4)	$p_{1/2}^{-1} p_{3/2}^{-1}$	$p_{3/2}$	$-\frac{1}{\sqrt{6}}(1-2\frac{g_p-g_n}{2})$	$\mp 2\sqrt{1/3}$
(5)	$d_{3/2}^{2}$	$d_{3/2}^{2}$	$-\sqrt{\frac{7}{5}}(2+\frac{g_p-g_n}{2})$	$\pm\sqrt{14/5}$
(6)	$d_{5/2}^{2}$	$d_{5/2}^{2}$	$-\sqrt{\frac{3}{5}}(3-\frac{g_p-g_n}{2})$	$\mp\sqrt{6/5}$
(7)	$2s_{1/2}^{2}$	$2s_{1/2}^{2}$	$-\sqrt{3}(\frac{g_p-g_n}{2})$	$\pm\sqrt{6}$

TABLE (A5-1)

Reduced matrix elements necessary to compute the M1 and β lifetimes for transitions from the iso-triplet (J = O, T = 1) the ground state of N^{14}.

APPENDIX VI

PROPERTIES OF THE GROUND STATE OF N^{14}

Quadripole Moment

The quadripole moment operator Q_q^2 is [55]

$$Q_q^2 = e \sum_i^A (\tfrac{1}{2} - \tau_z^i)\, r_i^2 Y_q^2(i) \qquad \text{(A6-1)}$$

and the quadripole moment is defined as the expectation value of the zeroth component of this operator in the state of highest weight multiplied by a normalizing factor.

$$Q = \sqrt{\frac{16\pi}{5}}\, \langle \psi(J,M = J) | Q_o^2 | \psi(J,M = J) \rangle \qquad \text{(A6-2)}$$

For N^{14} the ground state has T = O. Hence the isovector portion of the operator (A6-1) does not contribute to the matrix element (A6-2). In the following this term is ignored. The resultant operator is called O_{eff}.

Using the wave functions (21) for the ground state, the only nonvanishing contributions to the quadripole moment come from the diagonal elements and $\langle \psi_1 | O_{eff} | \psi_4 \rangle$, $\langle \psi_2 | O_{eff} | \psi_4 \rangle$.

This is so because a single particle operator such as (A6-1) can couple at most a single particle in different orbitals.

All of the diagonal elements can be expressed as two-particle elements. This is trivially true for $\langle\psi_1|O_{eff}|\psi_1\rangle$ and $\langle\psi_3|O_{eff}|\psi_3\rangle$. Using the methods of fractional parentage one finds:

$$\langle\psi_2|O_{eff}|\psi_2\rangle = -\langle p^2_{3/2}{}^{J=1,T=0}|O_{eff2}|p^2_{3/2}{}^{J=1,T=0}\rangle$$

and; (A6-3)

$$\langle\psi_4|O_{eff}|\psi_4\rangle = \frac{5}{8}\langle p^2_{3/2}{}^{J=1,T=0}|O_{eff2}|p^2_{3/2}{}^{J=1,T=0}\rangle$$

These contain only 3/2 results since a 1/2 orbital cannot have a quadripole moment.

We obtain, after some computation,

$$\langle(\ell j)^2 J,M=J|O_{eff}|(\ell j)^2 J,M=J\rangle = (-)^{j+1/2-J}[J][j]\langle r^2\rangle_{n,\ell} \times$$

$$\begin{pmatrix} j & 2 & j \\ -\frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}\begin{pmatrix} J & J & 2 \\ -J & J & 0 \end{pmatrix}\begin{Bmatrix} J & J & 2 \\ j & j & j \end{Bmatrix} \qquad \text{(A6-4)}$$

where the symbols are the 3-j and 6-j symbols as defined by Rotenberg, et al. [50]

The off-diagonal elements can be similarly computed utilizing the generalized coefficients of fractional parentages. (See Appendix II.) The results are:

$$\langle\psi_4|O'_{eff}|\psi_1\rangle = -\sqrt{\frac{5}{16\pi}}\,\frac{1}{5}\,e\langle r^2\rangle_{n,1}$$
$$\langle\psi_4|O_{eff}|\psi_2\rangle = -\sqrt{\frac{5}{16\pi}}\,\frac{1}{5\sqrt{10}}\,e\langle r^2\rangle_{n,1} \qquad \text{(A6-5)}$$

If one assumes harmonic Oscillator Wave functions, then the radial integrals are evaluable. We obtain

$$\langle r^2\rangle_{1p} = \frac{5}{2\nu}$$
$$\langle r^2\rangle_{1d} = \frac{7}{2\nu} \qquad \text{(A6-6)}$$

Inserting the various values for n, l, j in equation (A6-4) yields the results in which we are interested. These are tabulated in Table (A6-3).

Magnetic Moments

The magnetic moment of a nuclear system is defined as

$$\mu = \langle\psi J, M=J|M_z|\psi J, M=J\rangle \qquad \text{(A6-7)}$$

where the operator M is defined as:

$$\vec{M} = \frac{\mu_0}{2}(\vec{L} + (g_p + g_n)\vec{S}) - \mu_0 \sum_i^A (\vec{\ell}_i + \frac{(g_p - g_n)}{2} \vec{s}_i)\tau_z^i \quad \text{(A6-8)}$$

In taking matrix elements of this operator between T = 0 states, the second, the isovector, portion of this operator makes no contribution. The remaining term will be called M_{eff}.

The present calculations are greatly simplified, if, instead of performing the calculations in the j-j coupling scheme they are transformed to the L-S scheme. The transformation is straightforward and the matrix is given in Table (A6-1) for p shell states.

If we define

$$\mu_{L,S} = \langle L,S|M_{eff}|L,S\rangle \quad \text{(A6-9)}$$

where the states are coupled to J = 1, J_z = 1.

Then

$$\mu_{0,1} = \frac{g_p + g_n}{2} \mu_0$$

$$\mu_{1,0} = \frac{1}{2}\mu_0 \quad \text{(A6-10)}$$

$$\mu_{2,1} = (3/4 - (g_p + g_n)/4)\ \mu_0$$

These results may be transformed to the j-j coupling scheme. The results of this process are given in Table (A6-3).

TABLE (A6-1)

Transformation Matrix between j-j and
L-S Coupling for the p Shell States
J = 1, T = 0

	(L,S)		
j,j	(0,1)	(1,0)	(2,1)
$p^2_{1/2}$	$-\sqrt{3/9}$	$\sqrt{2/3}$	$2\sqrt{15/9}$
$p_{1/2}p^7_{3/2}$	$+4\sqrt{3/9}$	$-\sqrt{2/3}$	$\sqrt{15/9}$
$p^6_{3/2}$	$-\sqrt{30/9}$	$-\sqrt{5/3}$	$\sqrt{6/9}$

TABLE (A6-2)

Transformation Coefficients between j-j and
L-S Coupling for the d Shell Matrix Elements
J = 1, T = 0

	(L,S)		
j,j	(1,0)	(0,1)	(2,1)
$d^2_{3/2}$	9/25	2/25	14/25
$d^2_{5/2}$	14/25	7/25	4/25
$d_{5/2}d_{3/2}$	2/25	16/25	7/25

TABLE (A6-3)

Magnetic moment and quadripole moment matrix elements of interest for the study of N^{14}. The off-diagonal terms are multiplied by two.

		$\langle\psi_1\|O\|\psi_2\rangle$	
$\|\psi_1\rangle$	$\|\psi_2\rangle$	Magnetic	Quadripole
$p^2_{1/2}$	$p^2_{1/2}$	$\frac{1}{3}\left(2 - \frac{g_p + g_n}{2}\right)$	0
$p^6_{3/2}$	$p^6_{3/2}$	$\frac{1}{3}\left(1 + \frac{g_p + g_n}{2}\right)$	$-\frac{4}{25}\, e\langle r^2\rangle_{1,1}$
$p^{-1}_{1/2}p^{-1}_{3/2}$	$p^{-1}_{1/2}p^{-1}_{3/2}$	$\frac{1}{2}\left(\frac{1}{2} + \frac{g_p + g_n}{2}\right)$	$\frac{1}{10}\, e\langle r^2\rangle_{1,1}$
$p^{-1}_{1/2}p^{-1}_{3/2}$	$p^2_{1/2}$	$\frac{2}{3}\left(\frac{1}{2} - \frac{g_p + g_n}{2}\right)$	$\frac{2}{5}\, e\langle r^2\rangle_{1,1}$
$p^{-1}_{1/2}p^{-1}_{3/2}$	$p^2_{3/2}$	$\frac{\sqrt{10}}{3}\left(\frac{1}{2}\, \frac{g_p + g_n}{2}\right)$	$-\frac{1}{5}\frac{\sqrt{2}}{5}\, e\langle r^2\rangle_{1,1}$
$d^2_{3/2}$	$d_{3/2}$	$\frac{1}{5}\left(3 - \frac{g_p + g_n}{2}\right)$	$\frac{4}{25}\, e\langle r^2\rangle_{1,2}$
$d^2_{5/2}$	$d^2_{5/2}$	$\frac{1}{5}\left(2 + \frac{g_p + g_n}{2}\right)$	$\frac{32}{175}\, e\langle r^2\rangle_{1,2}$
$2s^2_{1/2}$	$2s^2_{1/2}$	$\frac{g_p + g_n}{2}$	0

APPENDIX VII

ENERGY SHIFT DUE TO A CHARGE-DEPENDENT TWO-NUCLEON INTERACTION

The states that are used to describe the isobaric triplet of levels in this calculation are:

$$
\begin{aligned}
|p^2_{1/2}, p^8_{3/2}, 1s^4_{1/2}\rangle &= |\psi_1\rangle \\
|p^4_{1/2}, p^6_{3/2}, 1s^4_{1/2}\rangle &= |\psi_2\rangle \\
|d^2_{5/2}, p^8_{3/2}, 1s^4_{1/2}\rangle &= |\psi_3\rangle \qquad \text{(A7-1)} \\
|d^2_{3/2}, p^8_{3/2}, 1s^4_{1/2}\rangle &= |\psi_4\rangle \\
|2s^2_{1/2}, p^8_{3/2}, 1s^4_{1/2}\rangle &= |\psi_5\rangle
\end{aligned}
$$

The charge-dependent potential

$$V = \sum_{i<j} [(\tau_z^i+\tau_z^j)p + \tau_z^i\tau_z^j q + \vec{\sigma}^i\cdot\vec{\sigma}^j\{(\tau_z^i+\tau_z^j)r + \tau_z^i\tau_z^j s\}]V(r_{ij})$$

will shift the observed energy levels by an amount which can be described by E shift $= \alpha M_T + \beta M_T^2 + \gamma$. In the present work we compare the observed levels to each other. That is, the energies are subtracted. For this case the constant, γ, will have no significance. It will, therefore, be ignored.

Reduction of the many-particle matrix elements to two-particle elements

The matrix elements $\langle\psi_2|V|\psi_3\rangle$, $\langle\psi_2|V|\psi_4\rangle$ and $\langle\psi_2|V|\psi_5\rangle$ will be zero since a two-particle operator such as V can connect at most two-particles in different orbitals.

The diagonal elements with two-particles in the unfilled shell constitute a class of matrix elements that can be reduced very easily. These matrix elements will be equal to matrix elements taken with wave functions that are antisymmetrized only according to shells, provided the resulting two-particle matrix elements are taken with antisymmetric two-particle wave functions. We have then

$$\langle(\ell j)^2(\ell' j')^f(\ell'' j'')^f(J,T)|V|(\ell j)^2(\ell' j')^f(\ell''' j'')^f(J,T)\rangle =$$

$$\sum_{(\ell_1,j_1)=(\ell',j')}^{(\ell'',j'')} \langle(\ell j)^2(\ell_1,j_1)^f(J,T)|V_{2,f}|(\ell j)^2(\ell_1 j_1)^f(J,T)\rangle +$$

$$\langle(\ell j)^2(J,T)|V_2|(\ell j)^2(J,T)\rangle \qquad \text{(A7-2)}$$

where f indicates that the shell is filled and

$$V_{2,f} = \sum_{i=1}^{2}\sum_{j=3}^{f+2} V_{i,j} \qquad \text{(A7-3)}$$

The many-particle matrix elements appearing in (A7-2) can be evaluated by the method of fractional parentages (in this case the cfp are all 1). The procedure is straightforward though tedious. The result is:

$$\langle(\ell j)^2(\ell' j')^f(J,T)|V_{2,f}|(\ell j)^2(\ell' j')^f(J,T)\rangle \doteq \frac{2M_T}{[j]}\sum_J [J]\langle jj'J|(p+\vec{\sigma}_1\cdot\vec{\sigma}_2 r)V(r_{12})|jj'J\rangle_a \tag{A7-4}$$

The symbol $\doteq$ means that the term independent of M_T has been ignored.

The remaining diagonal element can be treated in the same way. One obtains six-particle functions, however, instead of the two-particle functions that appear in (A7-2). These may be evaluated via the cfp:

$$\langle p^6_{3/2}{}^{J=0,T=1}(\ell j)^f|V_{6,f}|p^6_{3/2}{}^{J=0,T=1}(\ell j)^f\rangle = \frac{2M_T}{[j']}\sum_J [J]\langle p_{3/2}(\ell j)J|V_2|p_{3/2}(\ell j)J\rangle \tag{A7-5}$$

and

$$\langle p^6_{3/2}{}^{J=0T=1}|V_6|p^6_{3/2}{}^{J=0T=1}\rangle = 5\langle p^2_{3/2}{}^{J=2}|(p+\vec{\sigma}_1\cdot\vec{\sigma}_2 r)V(r_{12})|p^2_{3/2}{}^{J=2}\rangle M_T + \frac{1}{2}\langle p^2_{3/2}{}^{J=0}|(q+\vec{\sigma}_1\cdot\vec{\sigma}_2 s)V(r_{12})|p^2_{3/2}{}^{J=0}\rangle \tag{A7-6}$$

The results for matrix elements which are off-diagonal but all of which have two particles in the unfilled shells is even more straightforward than the above. The only components of such matrix elements are the two-particle elements taken between the two-particle states which comprise the excess nucleons outside of the core. All of the other components vanish because they contain scalar products in which particles are in different orbitals.

The remaining off-diagonal element may be computed by the method of cfp. The result is

$$\langle\psi_1(M_T)|V|\psi_2(M_T)\rangle = \langle p^2_{3/2}(-M_T)|V_2|p^2_{1/2}(-M_T)\rangle \qquad \text{(A7-7)}$$

The next step in the analysis of the matrix elements is to reduce these further to Slater integrals.

There are two basically different operators whose matrix elements must be computed. These are:

$$V(r_{12}) = V(|\vec{r}_1 - \vec{r}_2|)$$

and (A7-8)

$$\sigma_1 \cdot \sigma_2 V(r_{12})$$

Reduction to Slater Integrals

In order to reduce the j-j coupled matrix elements of these operators to Slater integrals they will be expanded into scalar products of irreducible tensor operators. The matrix elements of these scalar products can then be computed in a straightforward way. Thus:

$$V(r_{12}) = \sum_k V_k(r_1,r_2)(C^k(1)\cdot C^k(2)) \qquad \text{(A7-9)}$$

where,

$$C^k_m = \sqrt{4\pi/(2k+1)}\; Y^k_m$$

Likewise:

$$\vec{\sigma}_1\cdot\vec{\sigma}_2 V(r_{12}) = \sum_{k,r} (-)^{k+r+1}(T^{(1k)r}(1)\cdot T^{(1k)r}(2)) \qquad \text{(A7-10)}$$

where,

$$T^{(1k)r}_m = [\vec{\sigma}\times C^k]^r_m$$

The matrix element of a scalar product between two j-j coupled states that are not antisymmetrized is:

$$\langle j_1 j_2 J|T^k(1)\cdot T^k(2)|j_1' j_2' J\rangle = (-)^{j_2+J+j_1'} \begin{Bmatrix} j_1 & j_2 & J \\ j_2' & j_1' & k \end{Bmatrix} \times$$

$$\langle j_1||T^k||j_1'\rangle\langle j_2||T^k||j_2'\rangle \qquad \text{(A7-11)}$$

For the two types of single-particle reduced matrix elements, one can obtain:

$$\langle j||C^k||j'\rangle = (-)^{j-1/2}\sqrt{[j][j']}\begin{pmatrix} j & j & j' \\ -1/2 & 0 & 1/2 \end{pmatrix}\frac{1}{2}[1+(-)^{k+\ell+\ell'}]$$

and (A7-12)

$$\langle j||T^{(1k)r}||j'\rangle = \sqrt{[j][j'][r]}\begin{Bmatrix} 1/2 & \ell & j \\ 1/2 & \ell' & j \\ 1 & k & r \end{Bmatrix} \times$$

$$6(-)^{\ell}\sqrt{[\ell][\ell']}\begin{pmatrix} \ell & k & \ell' \\ 0 & 0 & 0 \end{pmatrix}$$

With these formulae one can evaluate all of the j-j coupled matrix elements of interest here. For instance, let

$$A'((\ell_1 j_1),(\ell_2 j_2)) = \sum_J [J]\frac{\langle j_1 j_2 J|(p+\vec{\sigma}_1\cdot\vec{\sigma}_2 r)V(r_{12})|j_1 j_2 J\rangle_a}{[j_1][j_2]} \qquad \text{(A7-13)}$$

where the subscript "a" means the wave functions are antisymmetrized between the two particles Then one can obtain

$$A'((\ell_1 j_1)(\ell_2 j_2)) = p[F^0 - \sum_k \begin{pmatrix} j_1 & k & j_2 \\ -1/2 & 0 & 1/2 \end{pmatrix} \frac{1}{2}[1+(-)^{\ell_1+\ell_2+k}]G^k] - r\sum_k [2 - [\ell_1][\ell_2] \begin{Bmatrix} 1/2 & j_1 & \ell_1 \\ k & \ell_2 & j_2 \end{Bmatrix}^2] \begin{pmatrix} \ell_1 & k & \ell_2 \\ 0 & 0 & 0 \end{pmatrix}^2 G^k \quad \text{(A7-14)}$$

The F and G's are direct and exchange Slater integrals respectively.

Table A7-1 gives the results for the cases of interest in this calculation.

The diagonal elements coupled to J = O may be evaluated in general. We obtain

$$\langle j^2 J=0 | V(r_{12}) | j^2 J=0 \rangle = [j] \sum_k \begin{pmatrix} j & k & j \\ -1/2 & 0 & 1/2 \end{pmatrix}^2 F^k \quad \text{(k even)}$$

and (A7-15)

$$\langle j^2 J=0 | \vec{\sigma}_1 \cdot \vec{\sigma}_2 V(r_{12}) | j^2 J=0 \rangle = -[j] \sum_k [2 - [\ell] \begin{Bmatrix} 1/2 & j & \ell \\ k & j & \ell \end{Bmatrix}^2] \begin{pmatrix} \ell & k & \ell \\ 0 & 0 & 0 \end{pmatrix}^2 F^k$$

Results using these last two formulae are given in Table A7-2.

The remaining diagonal elements are:

$$\langle p_{3/2}^{\,2},\ J=2|\ V(r_{12})|\ p_{3/2}^{\,2}\ J=2\rangle = F^{o} - \frac{3}{25}F^{2}$$

and (A7-16)

$$\langle p_{3/2}^{\,2}|J=2\ \ \vec{\sigma}_1\cdot\vec{\sigma}_2\ V(r_{12})|\ p_{3/2}^{\,2}\ J=2) = -\frac{1}{3}F^{o} - \frac{13}{75}F^{2}$$

All of the off-diagonal elements are between states consisting of two particles coupled to p 1/2 and the other state coupled to $(\ell j)^2$ J = 0.

One may obtain for these:

$$\langle p_{1/2}^{2}J=0|V(r_{12})|(\ell j)^{2}J=0\rangle = (-)^{\ell+1}\sum_{k}\sqrt{2[j]}\begin{pmatrix}1/2 & k & j\\ -1/2 & 0 & 1/2\end{pmatrix}^{2}G^{k} \qquad (\ell+k \text{ odd}) \qquad \text{(A7-17)}$$

and

$$\langle p_{1/2}^{2}J=0|\vec{\sigma}_1\cdot\vec{\sigma}_2V(r_{12})|(\ell j)^{2}J=0\rangle = (-)^{\ell}\sum_{k}\sqrt{2[j]}\ (2-3[\ell]\ x \begin{Bmatrix}1/2 & 1/2 & 1\\ k & \ell & j\end{Bmatrix}^{2}\begin{pmatrix}1 & k & \ell\\ 0 & 0 & 0\end{pmatrix}^{2}G^{k} \qquad \text{(A7-18)}$$

The Slater integrals may be reduced to Talmi integrals, if the Harmonic Oscillator Shell Model is used. These transformations are given in Table (A7-4).

TABLE (A7-1)

Reduction of j-j Coupled Matrix Element Sums to Slater Integrals

(ℓ_1,j_1)	(ℓ_2,j_2)	$A^1((\ell_1 j_1),(\ell_2,j_2))$
$p_{1/2}$	$p_{3/2}$	$p[F^o - \frac{F^2}{10}] - r\,[\frac{2F^6}{3} + \frac{F^2}{6}]$
$p_{1/2}$	$1s_{1/2}$	$p[F^o - \frac{G^1}{6}] - \frac{rG^1}{6}$
$p_{3/2}$	$1s_{1/2}$	$p[F^o - \frac{6^1}{6}] - \frac{r6^1}{6}$
$d_{3/2}$	$p_{3/2}$	$p[F^o - \frac{G^1}{60} - \frac{9G^3}{140}] - r\,[\frac{G^1}{4} + \frac{3G^3}{28}]$
$d_{3/2}$	$1s_{/12}$	$p[F^o - \frac{G^2}{10}] - \frac{3rG^2}{10}$
$d_{5/2}$	$p_{3/2}$	$p[F^o - \frac{G^1}{10} - \frac{G^3}{14}] - r[\frac{G^1}{6} + \frac{G^3}{7}]$
$d_{5/2}$	$1s_{1/2}$	$p[F^o - \frac{G^2}{10}] - \frac{3rG^2}{10}$
$2s_{1/2}$	$1p_{3/2}$	$p[F^o - \frac{G^1}{6}] - \frac{rG^1}{2}$
$2s_{1/2}$	$1s_{1/2}$	$p[F^o - \frac{G^o}{2}] - \frac{3rG^o}{2}$

TABLE (A7-2)

Results for Diagonal Matrix Elements

$(\ell j)^2$	$\langle(\ell j)^2 \, V(r_1 2)\,\vert\,(\ell j)^2 J = 0\rangle$	$\langle(\ell j)^2 J = 0\,\vert\,\sigma_1\cdot\sigma_2 V(r_1 2)\,\vert\,(\ell j)^2\rangle$
$p_{1/2}{}^2$	F^0	$-\left(\frac{1}{3}F^0 + \frac{8}{15}F^2\right)$
$p_{3/2}{}^2$	$F^0 + \frac{F^2}{5}$	$-\left(\frac{5}{3}F^0 + \frac{13}{15}F^2\right)$
$d_{3/2}{}^2$	$F^0 + \frac{F^2}{5}$	$-\left(\frac{3}{5}F^0 + \frac{9}{35}F^2 + \frac{16}{35}F^4\right)$
$d_{5/2}{}^2$	$F^0 + \frac{8}{35}F^2 + \frac{2}{21}F^4$	$-\left(\frac{7}{5}F^0 + \frac{16}{35}F^2 + \frac{26}{105}F^4\right)$
$2S_{1/2}{}^2$	F^0	$-3F^0$

TABLE (A7-3)

Off-Diagonal Matrix Elements

(ℓj)	$\langle (P_{1/2})^2 J = 0 \lvert V(r_{12}) \rvert (\ell j)^2 J = 0 \rangle$	$\langle p_{1/2}{}^2 J = 0 \lvert \sigma_1 \cdot \sigma_2 V(r_{12}) \rvert (\ell j)^2 J = 0 \rangle$
$(p_{3/2})^2$	$\frac{\sqrt{2}}{3} F^2$	$-\frac{\sqrt{2}}{3} (2F^0 + F^2)$
$d_{3/2}{}^2$	$-\frac{\sqrt{2}}{3} G^1$	$\frac{\sqrt{2}}{5} (G^1 + \frac{12}{7} G^3)$
$d_{5/2}{}^2$	$-\frac{\sqrt{3}}{7} G^1$	$\frac{8\sqrt{3}}{15} (G^1 + \frac{1}{4} G^3)$
$2S_{1/2}{}^2$	$-\frac{1}{3} G^1$	G^1

TABLE (A7-4)

Decomposition of some Slater Integrals to Talmi Integrals.

(From Talmi and de Shalit) [55]

n_1l_1	n_2l_2	SLATER INTEGRALS
1s	1s	$F^{o'} = I_o$
1s	1p	$F^o = \frac{1}{2}(I_o + I_1)$
		$G^1 = \frac{3}{2}(I_o - I_1)$
1s	1d	$F^o = \frac{1}{4}[(I_o + 1_2) + 2I_1]$
		$G^2 = \frac{5}{4}[(I_o + I_2) - 2I_1]$
1s	2s	$F^o = \frac{1}{8}[5(I_o + I_2) - 2I_1]$
		$G^o = \frac{5}{8}[(I_o + I_2) - 2I_1]$
1p	1p	$F^o = \frac{1}{12}[5(I_o + I_2) + 2I_1]$
		$F^2 = \frac{25}{12}[(I_o + I_2) - 2I_1]$
1p	1d	$F^o = \frac{1}{24}[7(I_o + I_3] + 5(I_1 + I_2)]$
		$F^2 = \frac{35}{24}[(I_o + I_3) - (I_1 + I_2)]$
		$G^1 = \frac{1}{8}[7(I_o - I_3) - (I_1 - I_2)]$
		$G^3 = \frac{49}{24}[(I_o - I_3) - 3(I_1 - I_2)]$
1p	2s	$F^o = \frac{1}{48}[11I_o + 37I_1 - 35I_2 + 35I_3]$
		$G^1 = \frac{1}{16}[11I_o - 41I_1 + 65I_2 - 35I_3]$
1d	1d	$F^o = \frac{1}{240}[63(I_o + I_4) + 28(I_1 + I_3) + 58I_2]$
		$F^2 = \frac{7}{48}[9(I_o + I_4) - 8(I_1 + I_3) - 2I_2]$
		$F^4 = \frac{189}{86}[(I_o + I_4) - 4(I_1 + I_3) + 6I_2]$
1d	2s	$F^o = \frac{1}{96}[15I_o + 40I_1 - 34I_2 - 56I_3 + 63I_4]$
		$G^2 = \frac{5}{96}[15I_o - 44I_1 + 106I_2 - 140\pm_3 + 63I_4]$
2s	2s	$F^o = \frac{41}{64}I_o - \frac{79}{48}I_1 + \frac{385}{96}I_2 - \frac{175}{48}I_3 + \frac{105}{64}I_4$

APPENDIX VIII

SPIN ORBIT INTERACTION GENERATED BY THE MOTION OF NUCLEONS

Suppose a neutron, with a magnetic moment $\vec{\mu}$, where

$$\vec{\mu} = \frac{e}{mc} \mu_n \vec{s} \tag{A8-1}$$

is moving in a radial electric field

$$\vec{E} = \frac{\vec{r}}{r} \frac{dV}{dr} \tag{A8-2}$$

Then if the momentum of the neutron is $\vec{p}$ it sees, in its reference frame, a magnetic field

$$\vec{B} = - \frac{\vec{p}}{mc} \times \vec{E} \tag{A4-3}$$

The interaction energy, H, of this neutron with the electric field is

$$\begin{aligned} H &= -\vec{\mu} \cdot \vec{B} \\ &= \frac{e}{(mc)^2} \mu_n \vec{\ell} \cdot \vec{s} \frac{1}{r} \frac{dV}{dr} \end{aligned} \tag{A4-4}$$

Here μ_n is the magnetic moment of the neutron in nuclear magnetons, M is the mass of a neutron and $\vec{\ell}$ and $\vec{s}$ are the orbital and spin angular momentum operators, respectively.

The analogous result for a proton is:

$$H = (\mu_p - 1/2) \frac{e}{(mc)^2} \vec{\ell}\cdot\vec{s} \frac{1}{r} \frac{dV}{dr} \qquad \text{(A8-5)}$$

Using the isotopic spin formalism we may combine (A8-5) and (A8-4).

There results:

$$H_{\ell,s} = \frac{e}{2(mc)^2} [(1/2 - \tau_z)(2\mu_p - 1) + (1/2 + \tau_z)2\mu_n] \frac{\vec{\ell}\cdot\vec{s}}{r} \frac{dV}{dr} \qquad \text{(A8-6)}$$

The wave function describing the J = 0, T = 1 isobaric triplet in the Mass 14 system is a superposition of states having two holes in the $p_{1/2}$ shell, $p_{3/2}$ shell, four holes in the $p_{1/2}$ shell and two particles in the s-d shell (see equation (22) of the text).

For this system we treat the 14 particle wave functions as two-hole wave functions or four-hole and two-particle wave functions. The portion of the s-d wave function referring to the four holes will not contribute

to the charge dependent results. As before, in Appendix VII, terms containing no M_T dependence will be omitted. The symbol $\doteq$ will imply this.

For a two-particle state and a two-particle operator $H_{\ell,s}$ there results:

$$<(\ell j)^2 J=0,T=1,M_T|H_{\ell,s}|(\ell j)^2 J=0,T=1,M_T> \doteq \frac{e}{2(mc)^2}(2\mu_n - 2\mu_p + 1)(j(j+1) - \ell(\ell+1) - 3/4)M(\ell)\, M_T \tag{A8-7}$$

where $M(\ell)$ is the radial integral that occurs in taking the matrix elements:

$$M(\ell) = \int_0^\infty R_\ell^2(r)\frac{dV}{dr}\,rdr \tag{A8-8}$$

Where $R_\ell(r)$ are the radial wave functions for a particle with angular momentum ℓ. For the Harmonic Oscillator functions

$$R_1 = N_1 \exp(-\nu r^2/2),\quad R_2 = N_2\, r^2 \exp(-\nu r^2/2) \tag{A8-9}$$

The charge distribution ρ giving rise to the potential $V(r)$ is taken to be:

$$\rho = \frac{e}{4\pi}(2N_0^2 + 6N_1^2 r^2)\exp(-\nu r^2) \tag{A8-10}$$

with

$$N_0^2 = \frac{4\nu^{3/2}}{\sqrt{\pi}} \ , \ N_1^2 = \frac{8\nu^{5/2}}{3\sqrt{\pi}} \ , \ N_2^2 = \frac{16\nu^{7/2}}{15\sqrt{\pi}} \ .$$

This charge distribution produces a potential

$$V(r) = 4\pi\int_r^\infty \rho(r')r'dr' + \frac{4\pi}{r}\int_0^r \rho(r')r'^2dr' \qquad \text{(A8-11)}$$

So,

$$\frac{dV}{dr} = -\frac{4\pi}{r^2}\int_0^r \rho(r')r'^2dr' \qquad \text{(A8-12)}$$

Integrating (A8-11) by parts, using (A8-12) for the derivative of the potential there results

$$M(\ell) = 4\pi\int_0^\infty f(\ell)\rho(r)r^2\exp(-\nu r^2)dr \qquad \text{(A8-13)}$$

where

$$f(\ell)\exp(-\nu r^2) = \int \frac{R_\ell^2(r)}{r}\,dr \qquad \text{(A8-14)}$$

Table (A8-1) lists the results for $f(\ell)$ and $M(\ell)$ for 1p and 1d nucleons.

TABLE (A8-1)

Results for the Functions $f(\ell)$ and $M(\ell)$ of the Text

Nucleons	$F(\ell)$	$M_1(\ell) = M(\ell)/e\nu\sqrt{\frac{\nu}{2\pi}}$
1p	$-\frac{N_1^2}{2\nu}$	$-\frac{10}{3}$
1d	$-\frac{4_2^2}{2\nu^2}(1 + \nu r^2)$	$-\frac{41}{15}$

One obtains then:

$$\langle(\ell j)^2 J=0, T=1, M_T | H_{\ell,s} | (\ell j)^2 J=0, T=1, M_T \rangle \doteq$$

$$\frac{1}{8\pi}\left[\frac{\pi}{\alpha E_m}\right]^2 E_0{}^3 M_1(\ell)(j(j+1) \mp \ell(\ell+1) - 3/4)(2\mu_n - 2\mu_p + 1)M_T \qquad \text{(A8-15)}$$

where α is the fine structure constant, E_m the energy equivalent of a nucleon and E_o the oscillator energy.

$$E_0 = e^2\sqrt{\nu/2\pi} \qquad \text{(A8-16)}$$

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THE CHARGE DEPENDENCE OF THE NUCLEAR FORCES AND THE A = 14 ISOBARIC TRIPLET

by
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ABSTRACT

The aim of the present work is to obtain two equations for the four parameters specifying the charge-dependent two-nucleon potential that was suggested by Blin-Stoyle and Le Tourneux. [1]

To do this, new wave functions for the first few low-lying levels of the A = 14 isobaric triplet O^{14}, N^{14}, and C^{14} are found. These wave functions contain contributions from the 2s-1d shell as well as the normal 1p shell components. The method used is a variant of the effective interaction procedure. [2] The wave functions, so obtained, describe the ground state magnetic dipole and electric quadripole moments as well as the M1 gamma transition from the first excited state of N^{14} to its ground state.

The energy separation of the first isobaric triplet of levels is then computed using these wave functions for the charge-dependent Coulomb and spin-orbit forces. The residual shifts are attributed to the charge-dependent nuclear force, and two equations for the four parameters are obtained.

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