

An analytic model of the
food consumption behavior of health-conscious individuals

by

S. Murthy Kambhampaty

Dissertation submitted to the Faculty of the
Virginia Polytechnic Institute and State University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

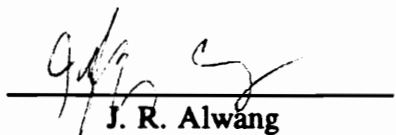
in

Agricultural and Applied Economics

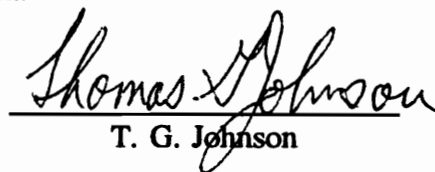
APPROVED:



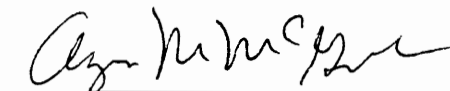
P. J. Driscoll, Chair



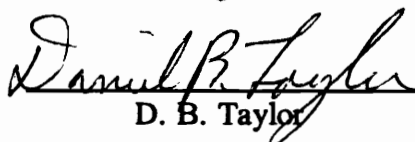
J. R. Alwang



T. G. Johnson



A. M. McGuirk



D. B. Taylor

1994

Blacksburg, Virginia

AN ANALYTIC MODEL OF THE
FOOD CONSUMPTION BEHAVIOR OF HEALTH-CONSCIOUS INDIVIDUALS

by

S. Murthy Kambhampaty

Paul J. Driscoll, Chair

Agricultural and Applied Economics

(ABSTRACT)

Evidence of changing patterns of food consumption behavior is presented.

Previous attempts at explaining these changes are critically reviewed and the need for an alternate approach is identified. A model of consumer behavior in which utility from food consumption is maximized subject to outlay for foods and limits on the consumption of fat, cholesterol, sodium, and/or other food components is proposed. This model yields a system of demands that are functions of prices and outlay as well as the composition of food and limits on the consumption of these components.

The structure of this model is examined and restrictions on consumer food demands are derived. The derivation of individual demands based on the proposed model is demonstrated using a specific indirect utility function. Tests of the joint hypotheses that fat or cholesterol consumption determines food demand are defined. The computation of aggregate food demand elasticities with respect to changes in prices and changes in attributes such as fat or cholesterol consumption is

demonstrated. Data necessary for estimating the parameters of the model and testing hypotheses are identified.

The model proposed in this study allows tests of the hypothesis that food demands are not affected by food composition as well as measurement of these effects.

Keywords: household behavior, food demand, diet-health awareness, rationing, data collection, aggregation, translog model, economic welfare

To
the late Smt. Durvasula Suryakanthamma
and
the late Kum. Kambhampaty Venkata Sitalakshmi

Acknowledgements

I would like to thank Paul Driscoll for his able guidance of my research. I should thank him also for having read the many drafts of my dissertation and the paper we wrote, for his prompt attention to my requests, and above all for showing me how to finish a research endeavor such as this. I would also like to thank Tom Johnson for financial support during the early part of my graduate studies, Dan Taylor for his constant encouragement, and Anya McGuirk and Jeff Alwang for many stimulating discussions. Finally, there are those of my family and friends who made it possible, bearable, and even fun.

Contents

Acknowledgements	v
I Consumer health awareness in food consumption	1
I. 1 Current research on food demand with health concerns	4
I. 1a The household production approach	5
I. 1b Evidence of taste change resulting from health awareness ..	7
I. 2 Modeling health concerns as auxiliary consumption constraints ..	9
I. 3 Organization of the study	11
II Alternatives for modeling food demands in the presence of diet-health awareness	13
II. 1 Models of "taste change"	13
II. 2 Health concerns as auxiliary constraints	24
II. 2a Consumer demand with health-related constraints	24
II. 2b Consumer theory with "rationing"	28
III Consumer demands with health-related constraints	32
III. 1 Consumer theory	33
III. 2 Utility maximization with auxiliary constraints	35
III. 3 Comparative statics of the proposed model	38
III. 3a The Slutsky equation and symmetry	39
III. 3b The Le Chatelier principle	42

III. 3c	The indirect utility function and the expenditure function	45
III. 3d	Commodity groups, separability and decentralization . . .	47
III. 4	Constrained maximization with mixed constraints	49
IV	Implementation of the model	51
IV. 1	The derivation of individual demands	51
IV. 1a	Testing for additional constraints on food consumption . .	55
IV. 2	Market demands with health concerns	58
IV. 2a	Computation of market demand elasticities	60
IV. 3	Data needs	60
V	Conclusions	62
	References	65
	Vita	71

I Consumer health awareness in food consumption

Writing for the United States Department of Agriculture's *Food Review*, Frazao and Cleveland state that

Public health efforts to increase consumer awareness of diet-disease relationships aim to make consumers understand that what they eat may affect their health (p. 16).

Federal efforts in this regard include publication of *Dietary Guidelines for Americans* and requirements that food packagers label products for their content of different vitamins, minerals, protein, fat, and cholesterol; also, the popular press has publicized medical research on the effects of food on various aspects of health. Surveys of consumer attitudes toward food consumption suggest that this information has affected consumer behavior. Capps and Schmitz report:

In a recent Food Marketing Institute Survey, 93% of respondents indicated concerns about the nutritional content of foods. Almost two-thirds of those surveyed in a U.S. Department of Agriculture (USDA) study reported they had adjusted household diets in the previous three years for health and nutrition reasons (p. 21, cited from Jones and Weimer).

If health awareness has indeed affected food consumption behavior, then food producers and marketers will necessarily be interested in the magnitude and direction of these impacts. Health concerns potentially affect both the price and quantity demanded of various foods, and competitive producers must know the direction and magnitude of these effects in order to adjust product lines and optimize levels of production.

Frazao and Cleveland (F-C) report that changes in consumers' diets, while having changed their spending patterns on foods have not necessarily lowered their intakes of fat. Based on analysis of the 1989 and 1990 *Diet and Health Knowledge Survey* (DHKS) and the 1989 and 1990 *Continuing Survey of Food Intake by Individuals* (CSFII), F-C report that "... meal planners aware of diet-disease relationships were not more likely than others to meet the recommended intakes for fat, saturated fat, or cholesterol (p. 18)." Their analysis finds that

. . . meal planners tended to underestimate the amount of fat and saturated fat in their diets, but overestimate the amount of cholesterol. Whereas 41 percent of the meal planners thought the level of fat in their diets was "about right," and 49 percent thought their diets were "about right" for saturated fat, only 25 percent of the meal planners actually met the recommendations for fat and saturated fat. Conversely, slightly more than half of the meal planners thought their diets were "about right" for cholesterol, although nearly three-fourths met the recommendation for cholesterol (p. 18).

These findings suggest that while consumers might seek to limit their fat and/or cholesterol intake, it is their *perceptions* about the content of fat and cholesterol in the foods they consume, and hence in their diets, that influences the outcome as reflected in expenditure patterns on food. Putler and Frazao find that women with a higher probability for diet-health awareness show no greater reduction in fat intake over others in USDA surveys of food consumption from 1985-1988. Their conclusion, consistent with Frazao and Cleveland, is that "[C]onsumers may be having difficulties making effective food substitutions in their diets, perhaps due to insufficient knowledge about the relative fat content of different food groups (p. 18)." The model proposed here postulates a process in which consumers make food choices, and hence substitutions, based on relative prices as well as relative *perceived* content of fat,

cholesterol, and possibly other nutrients, effectively modeling the patterns observed in previous studies of food consumption patterns.

Blaylock and Smallwood declare a need for nontraditional ways of incorporating "subjective influences" into the analysis of food demand. In their own words:

The fact remains that people buy foods that contain substances: some good, some harmful if eaten in excess. Economists have studied food choices for many years using classical demand theory as a guide. This theory and its extensions basically postulate that food choices are determined by prices, income, and the socio-economic characteristics of individuals. Clearly, other, more subjective influences, such as knowledge and attitudes, are at work—but usually ignored. Society, in turn, will bear enormous opportunity costs if we do not break the chains that bind us to traditional types of analyses (Blaylock and Smallwood, p. 1).

The model proposed here addresses this concern by allowing food demands to be affected by consumers' perceptions of the level to which they should limit their intake of selected nutrients and their perceptions of the composition of these nutrients in food, and by allowing the measurement of these effects on food demand. Nonetheless, the model remains solidly grounded on the theory of consumer behavior, and is shown to have features consistent with the standard model.

In summary, this study seeks the specification of a model of food demand behavior in the presence of consumer diet-health awareness¹; a model which specifically relates consumer food demands to the market price and income variables *as well as* to measures of food composition (or perceptions thereof). The rest of this chapter is dedicated to presenting the background for the study and introducing the

¹The term "diet-health awareness" as used throughout this study is borrowed from Frazao and Cleveland. It is not attributed at every occurrence in the interest of brevity, but the above should be borne in mind.

proposed model. In the following section, current research on food demand in the presence of health concerns is first discussed. The need for an alternative approach is pointed out in critical evaluation of the currently available models/approaches. The proposed model is introduced in the subsequent section. In a final section, the content of the rest of the dissertation by chapter is presented.

I. 1 Current research on food demand with health concerns

Current research on health awareness in food consumption varies widely in objectives and methods used. It is useful to organize this literature into broad categories for our evaluation along the lines of Capps and Schmitz (C-S). One category of studies includes those that examine the impact of social, economic, and demographic indicators on dietary quality. Since these studies do not yield models of demand that are functions of food characteristics, the evaluation in C-S is not expanded upon. A different category includes those studies that follow the production function approach of Stigler and Becker and of Gorman-Lancaster (Gorman [1956] 1980, 1976; Lancaster). Included here are hedonic price models, since "hedonic price functions" are derived from a particular form of household production function (attributable to Gorman-Lancaster) in the consumer's utility maximization problem. The general version of the household production model is discussed at some length to show that despite its analytical elegance, it is not applicable in the current context.

A final category includes those studies which seek evidence of "taste change" or "structural change" in the demand for food. The premise of these studies is that changes in the parameters of demand functions (systems) evidence changes in tastes (possibly due to health-related factors). This literature is also evaluated, with summary of previous criticisms (Alston and Chalfant 1991a, 1991b; Chalfant and Alston; Eales and Unnevehr; Kirman; Stoker), and with additional observations about "taste change" research. Finally, an alternative behavioral model is proposed that permits demands to change due to changing perceptions about fat and cholesterol content of foods as well as due to prices and income, while maintaining stable preferences or "tastes."

I. 1a The household production approach

Stigler and Becker (S-B) (see also Becker (1965, [1975] 1983)) propose a model in which households combine goods purchased in the market with household labor, human capital, and non-human capital to produce the "primary" commodities (such as musical appreciation, euphoria, etc.) which are choice variables in the utility function. Utility from "home-produced goods" is maximized subject to technological constraints, and to the full income constraint. S-B demonstrate that *this model can be used to explain a wide variety of behavior previously considered to be characteristic of changing tastes, while maintaining stable preferences.* The model is widely

recognized (Deaton and Muellbauer 1980) for its usefulness in modeling consumer (household) behavior when non-market goods are being consumed.

Discussion of applications of the household production model can be found in Pollak and Wachter, Deaton and Muellbauer (1980), and Barnett (1977, 1981). The treatment here follows that of Deaton and Muellbauer (1980). The household is represented as maximizing a utility index $U(\cdot)$ the arguments of which are "primary" choice commodities (in the notation of S-B and Barnett (1981)) $Z_i, i \in \{1, 2, \dots, r\}$; these commodities are produced using market goods, household labor, household human and non-human capital, and given technology, so that total costs $C(\mathbf{p}, \boldsymbol{\omega}, \mathbf{Z}; \mathbf{k})$ are less than or equal to full household income, where $C(\cdot)$ represents the minimum cost, given technology, of producing the utility maximizing vector of "home-produced" goods \mathbf{Z} out of market goods \mathbf{x} at prices \mathbf{p} , the labor supply l_0 of household members to household production at wages $\boldsymbol{\omega}$, and a fixed vector of human and non-human capital \mathbf{k} . Deaton and Muellbauer (1980) suggest that the household's problem can be viewed as a two stage optimization in which the household first minimizes short-run costs given \mathbf{Z} and \mathbf{k} , and then chooses the level of \mathbf{Z} which maximizes utility subject to the full household income constraint \mathbf{Y} ; hence the system of (structural) demands resulting from the model can be estimated "... using methods (such as instrumental variables) to overcome simultaneity bias (p. 249)."

While this approach accounts for changing demands while retaining stable preferences (Stigler and Becker), we find it inapplicable in the present context. The

main difficulty in applying this approach here is that fat and cholesterol cannot be regarded as "primary" commodities of the consumer's choice problem, and thus the model does not give demands for food commodities such as beef, pork, and chicken as functions of market prices, expenditures, and fat and cholesterol composition. Were household behavior modeled with "health" as the primary commodity, market goods other than help would necessarily be included in the problem, and again the model would not give food demands as functions of food prices, expenditures, and the components of food. Nonetheless, as Deaton and Muellbauer (1980) point out, the model should give pause to the economist looking at data from food consumption surveys and considering the argument that "tastes have changed."

I. 1b Evidence of taste change resulting from health awareness

As opposed to the approaches discussed above, which hold preferences stable, there has been considerable research effort expended in seeking evidence of changes in tastes resulting from health concerns (Capps and Schmitz, Chalfant and Alston, Eales and Unnevehr). A selection of these studies is briefly reviewed below².

Capps and Schmitz report rejection of the null hypothesis that cholesterol information does not shift the intercept of per capita demands of the Rotterdam type. Choi and Sosin model information as contributing to multiplicative "technical change"

²The next chapter presents a lengthier critique.

in the structure of preferences; they find evidence for a logistic evolution of the parameters of the translog indirect utility function over the time period 1953-1984.

Eales and Unnevehr test for structural change in the per capita demand for meats using an AIDS model and find that meat demands have shifted away from time consuming, low quality meat products towards higher quality ones. They conclude that structural change has been induced by a need for convenience in food preparation as the percentage of women in the labor force has increased and question the validity of the "taste change" argument.

Alston and Chalfant critically review the "taste change" literature and conclude that there is not enough evidence in the data to support the hypothesis of structural change, and that the results are extremely sensitive to the choice of functional form which has no sound basis (Alston and Chalfant 1991a, 1991b). Based on non-parametric tests seeking violations of the Weak Axiom of Revealed Preference, WARP, and of Varian's (1982, 1983) Generalized Axiom of Revealed Preference, GARP (Chalfant and Alston), they conclude that relative prices and income alone explain all of the variation in aggregate meat consumption data. Wohlgenant uses a flexible functional form based on the Fourier series expansion in (relative) meat prices to model beef demand and comes to the conclusion that aggregate beef consumption data are completely explained by relative prices of meats and real personal consumption expenditures.

An additional criticism, to those of Chalfant and Alston, of modeling health concerns solely in terms of changing preferences is identified here: the logical extension of the taste change argument is that, after the complete diffusion of knowledge about the health effects of food consumption, food demands would no longer be a function of the nutritional composition of food but merely of the standard price and expenditure variables³. However, even without taste change, i.e. with constant preferences, *demands will always be influenced by the nutritional composition of food* if health concerns place nutrition related constraints on the food consumption of individuals.

I. 2 Modeling health concerns as auxiliary consumption constraints

Deaton and Muellbauer (1980) distinguish between constraints on consumption and restrictions on preferences as follows:

One of our chief concerns in this book is to show how appropriate assumptions on preferences and the constraints households face are required in different contexts ... to gain insights into particular kinds of behavior, and to justify particular empirical approaches. ... The borderline between whether restrictions are to be placed on preferences or on the constraints faced by the household is sometimes a subtle one (p. 244).

They suggest that the simplicity of the "explanation" should count towards the decision. The argument here is that the way in which consumers have altered their food consumption patterns as a result of diet-health awareness is best reflected in a

³If, on the other hand, taste change is modeled as persistent over the long run, its effect on demands may far outweigh that of prices and income.

model of consumer utility maximization subject to income and auxiliary constraints on food consumption.

As was shown above, the problem with the household production approach is that it is not easily applied in the current context. Further, if Frazao and Cleveland's findings are any indication, it would seem that consumers apply rather limited decision rules in planning their diets given diet-health awareness. For example they may adopt the dietary guideline to consume less than 300 mg of cholesterol a day from all foods, and less than 65g of fat per day. Thus augmenting the standard version of the consumer model with constraints to represent these decision rules will be the most fruitful strategy for modeling consumer food demand behavior in the presence of health concerns. This augmented model still permits incorporating other types of heterogeneity by the introduction of demographic and other factors (either in the utility function or as constraints).

The model postulated is as follows: each household/consumer maximizes a utility function subject to a budget constraint and auxiliary constraints which bind the consumer's intake of (saturated and/or total) fat and/or cholesterol to a maximum level attained by linear combinations of the foods they consume. The model is easily extended to constraints on the consumption of sodium and other components of foods. Further, the weights given different foods and the constraint bounds are based on individual information/ perceptions rather than on universally true quantities. This aspect distinguishes the model proposed here from "rationing" models, and the

distinction is elaborated upon in the next chapter. In the context of the proposed model, it is entirely possible for health information to leave consumer indifference maps unaltered and to influence the demands for different foods just as do relative prices and income.

The following chapters are devoted to presenting the proposed model, to examining the structure of this model and comparing it to the standard model, and to deriving the market demands when some consumers are quantity constrained by their health concerns while others are not. The implementation of the model and its implications for data collection are also discussed. Since the data necessary to implement the proposed model are not available, the contribution of this dissertation is mainly to extend the theory of consumer behavior, and to argue for the collection of data which would allow estimation and testing of the theory.

I. 3 Organization of the study

The rest of the dissertation is organized as follows: specific studies representing the "taste change" approach presented above are critically evaluated in the first part of Chapter II. The discussion aims to give a detailed account of the theoretical and econometric issues raised by these studies. The proposed model is presented formally in the final section of the chapter, where it is compared to the literature on rationing. Chapter III examines the structure of the proposed model, comparing it to the standard model of the utility maximizing consumer. This chapter establishes that consumer

food demands in light of health concerns can be analyzed using the conventional model of the consumer with auxiliary constraints, without resorting to "taste change" arguments. Chapter IV presents the derivation of market demands implied by the proposed model, and demonstrates the implementation of the current model. Data needs for its implementation are discussed. A strong implication of the proposed model is that aggregation over individuals must be constructed first over groups and then as the sum of the group-wise aggregated demands. The final chapter presents some concluding remarks.

II Alternatives for modeling food demands in the presence of diet-health awareness

In this chapter, "taste change" models are evaluated in some detail and the proposed model is formally presented. In the first section, models of "taste change" in light of health concerns are discussed in detail. Some theoretical and empirical problems of these models are presented. The reader familiar with this literature should skip ahead to the following section where the proposed model, which avoids many of these problems, is formally presented. This later section also draws a distinction between the proposed model and "rationing" models.

II. 1 Models of "taste change"

In this section, a selection of studies which follow the "taste change" approach to analyzing the effect of health concerns on food demand are reviewed. Also, results which refute the hypothesis of "taste change" are discussed, and previous criticism of the approach is presented. It should be pointed out at the outset that in many studies which fall into this category, the author(s) tests for structural change but does not specifically test whether the structural change is fully explained by *health* concerns. The implication appears to be that any change in preferences away from beef must be

seen as being caused by awareness of a cholesterol-disease relationship. Some authors have identified other possible causes for changes in tastes; see, for example, Eales and Unnevehr reviewed below.

Capps and Schmitz (C-S) test whether meat demands are modified by cholesterol awareness. They use the "cholesterol awareness index" of Brown and Schrader along with aggregate quantity data for 1966–1988 from *Food Consumption, Prices, and Expenditures* and price data from *Livestock and Poultry Outlook*; demands are specified according as the Rotterdam model. The demand for each of beef, pork, poultry, and fish are specified as being affected, through the intercept, by cholesterol awareness. The null hypothesis is that there is no structural change; the alternate being that cholesterol exogenously shifts demand schedules for these four meat aggregates. The restrictions of homogeneity, symmetry, and adding up are imposed; weak separability of the meats group from all other goods in the consumers choice set is implicitly assumed. One of the equations in the system is dropped and the iterative Zellner procedure is used to estimate the parameters (Capps and Schmitz). C-S find that the cholesterol information index lagged a half-year is significant in the equations for pork, poultry, and fish, concluding that there does exist evidence for structural change in the demand for beef.

Having thus found "evidence of structural change," C-S do not test whether *all* structural change is explained by the cholesterol information index. It is thus possible that the cholesterol information index is found significant simply because it is

correlated with other, possibly more important, factors such as demographic change, for example. Also, criticism of the use of per capita data and the implied

"representative agent" model by Kirman and Stoker applies here. For example,

Taken at face value, representative agent models have the same value as traditional, ad hoc macroeconomic equations; namely they provide only statistical descriptions of aggregate data patterns, albeit descriptions that are straightjacketed by the capricious enforcement of restrictions of optimizing behavior by a single individual (Stoker, p. 1829).

Stoker goes on to argue that

... it is entirely possible for there to exist substantial heterogeneity in individual responses together with a simple, possibly linear relationship between the associated aggregates. While this setting immediately raises doubts as to the interpretation of the aggregate relationship, one could ask whether the aggregate equation could serve as a good tool for prediction. *This amounts to renouncing any possible behavioral interpretation of such an equation*, and justifying such aggregate equations through the need for parsimony in a larger modeling context (pp. 1857-1858).

(Emphasis added.)

Thus a simple aggregate equation may be an adequate statistical description of the data, but cannot be given any behavioral interpretation. This implies, in turn, that estimating a Rotterdam model with per capita data does not permit the inference of changes in consumer preferences. This is also the conclusion to be drawn from Barnett (1979, 1981) who proves that where the Rotterdam system of demand equations is integrable to a community utility function, that utility function is Cobb-Douglas, which is too restrictive; where it is not integrable, it can be regarded as a statistical approximation ". . . designed to forecast value share equations (Barnett, 1981, p. 316)."

Even if one did find evidence for changing consumer preferences, however, it is the logical extension of this argument that, as information about the health effects of

various foods becomes universally available, the corresponding "preference shifts" will eventually cease. Thus, if cholesterol awareness completely explains changing preferences, the effect of this information should eventually be fully reflected in prices and income, and therefore demands should be explained again by relative prices and income alone. Further, since preferences are fixed within time periods, cross-sectional demand analyses will not identify the effect of food composition on demands.

Choi and Sosin estimate a system of demands for red meat, poultry, and other foods presuming structural change similar to Solow's "disembodied technical change."

The consumer's problem at time t is transformed from

$$(II. 1a) \quad \begin{aligned} & \max_{\{X_t\}} U_t(X_t) \\ & s.t. \quad P_t X_t = Y_t \end{aligned}$$

to

$$(II. 1b) \quad \begin{aligned} & \max_{\{z_t\}} U_t(z_t) \\ & s.t. \quad \Phi_t z_t = Y_t \end{aligned}$$

where $\phi_{it} = p_{it} / a_i(t)$ and $z_{it} = a_i(t)x_{it}$. The z_t s represent the information augmented quantities consumed of the respective X_t s. Ch-S specify a translog indirect utility function and consumption of information augmented meats, $z_{it} = a_i(t)x_{it}$, where $i \in \{\text{beef, poultry, other foods}\}$. Roy's identity is then applied to derive the expenditure share equations

$$\begin{aligned}
m_i^F &= \frac{p_i^F X_i^F}{Y^F} \\
&= \frac{c^*_i + \sum_j c_{ij}(\log(p_j^F/m^F))}{\sum_k c^*_k + \sum_k \sum_j c_{jk}(\log(p_j^F/m^F))} \\
\text{(II. 2)} \quad &\forall i \in F \triangleq \{\text{red meat, poultry, other foods}\}
\end{aligned}$$

where

$$c^*_i = c_i + \sum_j c_{ij} \log(b_j(t))$$

and the system (II. 2) is estimated for step and logistic evolutions of the $b_j(t)$ s. Each system is then tested for the null hypothesis that there is no structural change.

Aggregate (per capita) time-series data were used for the years 1953–1984 from *Food Consumption, Prices, and Expenditure* and *U.S. Food Consumption Sources of Data and Trends*.

Ch-S find the null hypothesis of no structural change rejected, and further that the smooth logistic evolution in the parameters for beef only and for beef as well as for poultry fares best in terms of the percentage of observations violating monotonicity or quasi-convexity and also in terms of the likelihood ratio test of the model with multiplicative structural change against the model without. They conclude that over the period from 1975-1985 there was a smooth evolution of preferences away from beef and towards poultry and other foods. While Choi and Sosin point to evidence for structural change in the demand for beef and poultry, they do not specify a model which allows them to test for the cause of the change. Further, the implicit

assumption in using aggregate data (per capita) is that market commodities are augmented by information at the same rate for every consumer, or that the aggregate data behave as though there existed a "representative consumer" whose behavior corresponds to that of the per capita aggregates. The former assumption is unrealistic and the problems of the latter assumption are discussed in detail by Kirman. Finally, the functional form specified gives an approximation for individual preferences, and is not appropriate for use with per capita data. (See Lewbel (1987) for a discussion of the translog model in individual form and the (approximate) aggregations over individuals. Stoker and Lewbel 1988, 1989 present a similar discussion of the individual and aggregate versions of demand equations generated by an AIDS specification.)

Eales and Unnevehr (E-U) use the AIDS model for aggregate data to first test for the separability of meat demands and then test structural change across separable meat groups. Aggregate data used are variously from *Food Consumption, Prices, and Expenditures, Livestock Slaughter, and Poultry Slaughter* for the years 1965-1985. Tests for weak separability are based on the result that "... the off-diagonal term in the Slutsky substitution matrix is proportional to the income derivatives of the two separable goods." The tested hypotheses are derived, as restrictions on the coefficients of the appropriately specified AIDS model, from the main hypothesis that

$$(II. 3) \quad \frac{S_{ik}}{\partial X_i / \partial Y} = \frac{S_{jk}}{\partial X_j / \partial Y}$$

for all i, j in each weakly separable group and k outside the separable group. From their tests for separability they conclude that

Tests for weak separability among meat products . . . suggest that tests for structural change in the aggregate meats may be biased. . . . Two types of significant shifts in meat demand were identified in meat products: an exogenous constant annual 6.4% growth in demand for chicken parts/processed from 1965 to 1985 and a 3.5% decline in demand for beef table cuts after 1974. . . . Increased demand for convenience is a plausible explanation for the growth in chicken parts demand because the value of time for the principal meal preparer has increased during the last twenty-five years (pp. 530–531).

Eales and Unnevehr argue that the change in structure is not due to a cholesterol "scare" but rather due to an increased demand for convenience in preparation resulting, in turn, from the increased participation of women in the laborforce. This article represents a thorough effort to specify a demand system consistent with aggregating individual behavior and testing for structural change within the context of such a model. However, as argued previously, consumer preferences, and demands, should become stable as households stabilize their working patterns. Also, E-U suggest that "[I]n contrast to separability, the concept of structural change does not have a strong theoretical foundation. Evidence of change in parameters may not indicate an actual change in preferences but simply a misspecification of the model (p. 524)." This latter conclusion also applies to (parametric) separability results since the hypothesis in equation (II. 3) must be tested in the context of some specified functional form (in this case that of the AIDS model). Finally, the specification of structural change chosen simply postulates a one-time shift in demands; E-U argue that the way in which the

shift is modeled is suggestive of the causes, but this explanation is ad hoc.

Accordingly, they find evidence for a one-time shift in preferences away from beef table cuts and towards chicken parts.

Chalfant and Alston⁴ conduct non-parametric tests to examine whether changes in the demand for meat cannot actually be explained by relative prices and income alone, and come to the unfashionable conclusion that there is no evidence for a change of preferences in the per capita data from the mid-sixties through early eighties for both Australia and the U.S. They conduct nonparametric tests of the axioms of revealed preference looking for preference reversals over food consumption bundles for the period of their study. The weak axiom of revealed preferences (WARP) states that

$$(II. 4) \quad p_j X_i \geq p_i X_j \Rightarrow X_i \mathbf{R} X_j \text{ and } X_j \text{ not } \mathbf{R} X_i \text{ for } i \neq j$$

Letting $\Phi_{ij} = p_i X_j$, Chalfant and Alston look for violations of the weak axiom of revealed preference by looking for situations such that $\Phi_{ii} > \Phi_{ij}$ and $\Phi_{jj} > \Phi_{ij}$. Further, the strong axiom of revealed preference (SARP, transitivity of preferences) states that

$$(II. 5) \quad X_i \mathbf{R} X_j \text{ and } X_j \mathbf{R} X_k \Rightarrow X_i \mathbf{R} X_k \text{ and } X_k \text{ not } \mathbf{R} X_i \text{ for } i \neq j$$

⁴See also Alston and Chalfant 1991a, 1991b. The earlier study especially includes an extensive critique of the "taste change" literature which calls into question the conclusion of changing preferences.

which is violated if, for some X_i , X_j , and X_k , $\Phi_{ii} > \Phi_{ij}$, $\Phi_{jj} > \Phi_{jk}$, and $\Phi_{ik} > \Phi_{ii}$.

Varian's (1985) procedure for testing for violations of SARP within confidence limits is used.

Finding no violations of either WARP or SARP using the data above, they conclude that in fact preferences have been stable over time; what they should conclude is that there exists a utility function consistent with the data and which exhibits stable preferences, which is not the same as their conclusion. Afriat's theorem (Varian 1982, 1983) states that the data satisfy "cyclical consistency" if and only if "there exists a nonsatiated utility function that rationalizes the data (Varian 1982, p. 946)." Varian (1982) shows that if the data satisfy SARP, they satisfy cyclical consistency. Thus, if the data satisfy SARP, there exists a nonsatiated utility function which is consistent with the data, but this criterion does not test whether the data are, in fact, generated by such a utility function. This is, of course, generally true of all nonparametric "tests."

Chalfant and Alston acknowledge the problems of aggregating across consumers, but suggest that since they do not find preference reversals over the period, the aggregate data could be consistent with theory. Finally, they suggest that studies which test for structural change in the data in the context of models that impose a functional form on consumer preferences—the parametric versus the nonparametric approach—may find evidence for structural change due to "... inappropriate

aggregation of individual data, the exclusion of relevant variables such as the opportunity cost of time, or some other specification error (p. 392)."

Wohlgenant specifies a Fourier flexible functional form to model the demand for beef. Based on goodness of fit tests, Wohlgenant finds that the relative prices of competing meats explain all of the variation in aggregate beef consumption data. The Fourier flexible functional form is found to have "higher explanatory power" than a log-log form with some interaction terms included. The high R^2 value for the estimated Fourier flexible form is interpreted as evidence in favor of the hypothesis that beef consumption data are fully explained by income and the relative prices of beef and competing meats. The null hypothesis that the sinusoidal terms in the Fourier flexible specification are jointly insignificant is rejected. Also, the quadratic specification with parameter structural change modeled by a dummy variable ($D=1$ if year is 1973 or later) is rejected in favor of the Fourier flexible form ". . . using the J-test for nonnested regression models (p. 326)." This study points to the conclusions that the beef consumption data may be generated by consumers with stable preferences, beef consumption being determined by the relative prices of beef, competing meats, and income. High value of the adjusted R^2 is not evidence in favor of the model, as McGuirk and Driscoll have shown.

In the survey above, the following problems were identified with the "taste change" approach: First, tests of taste change assume the functional form chosen is correct. When it is not, "evidence" for taste change may simply be an artifact of a

misspecified model. In instances (Chalfant and Alston; Wohlgenant) where nonparametric or semi(non)parametric methods were used, the hypothesis of stable preferences was not rejected. Further, the models specified in investigating the data for structural change often do not allow the finding of taste change to be identified as being due to health concerns; even in the case of Capps and Schmitz who find evidence for taste change due to cholesterol information, they do not test whether this indicator completely explains preference shifts. Note that the chosen indicator of cholesterol awareness, based as it is on articles published in medical journals does not actually specify how this information is related to household/individual decisions regarding food consumption. Further, "taste change" models should account for demographic changes in the population which might cause the hypothesis of stable preferences to be rejected when using aggregate data even though individual household preferences remain stable. Finally, any "preference shifts" caused by health concerns would eventually become reflected in relative prices and expenditures and hence the implication of the approach is that health and nutrition indicators should not be arguments in the demands for food commodities, except as one-time shifters of demand or during the period of adjustment. This dissertation seeks a model in which some consumers' demands will always be affected by the composition of food, and which reflects differences across individual consumers.

II. 2 Health concerns as auxiliary constraints

In this section, the proposed model is first discussed in detail and then formally presented. It is then compared with "rationing" models which are similar in structure; it will be argued that there are important differences in the two specifications which leave the rationing literature wanting in the current context. The discussion will show that, while the rationing literature anticipates the approach suggested in this dissertation, it does not give demand (share) equations as functions of prices, outlay, (perceived) proportions of fat, cholesterol, and other food components, and limits on the consumption of these components. Thus this study seeks to contribute to the literature on applied consumer theory, and provides a sound basis for collecting data necessary to study the food demands of health-conscious consumers.

II. 2a Consumer demand with health-related constraints

If health concerns are specified as imposing additional constraints on food consumption, consumers' demands for foods in the presence of health concerns can be modeled without many of the problems identified above. In this section, the main features of this approach *vis a vis* the taste change approach are discussed. The model is then formally defined.

It was pointed out in the introduction that if health concerns do impact food demands then food producers would like to know the magnitude and direction of these

impacts in order to adjust their product mix and output levels. Demands which are a function of the composition of food would provide the necessary information. A model which yields such demands also could meet the challenge posed by Blaylock and Smallwood. In many of the taste change models, food demands are tested for structural change but the models do not provide information on the behavior of demands with respect to health indicators or nutritional factors. Further, as was argued previously, the logical extension of the taste change argument is that after preferences have been fully modified with respect to the new information, food demands will not be affected by food composition, only relative prices and outlay. The alternative approach proposed here gives the derivation of food demands as functions of prices and income as well as proportions of components, such as fat and cholesterol for example, in foods along with bounds on the consumption of the components, fat and cholesterol in the example. Further, these are permanent features of the demand system since, for individuals who are effectively constrained, demands for food commodities will always be a function of health and nutrition factors.

An implicit assumption of the proposed model is, further, that consumer *preferences* are not necessarily affected by health concerns but rather that health concerns affect the quantities consumed of different goods (similarly as does the budget constraint). For instance, one may like to eat bacon and eggs for breakfast but realize that bacon and eggs contain large amounts of fat and cholesterol. Thus, a steady diet of bacon and eggs is avoided in following guidelines for consuming low

quantities of cholesterol and fat or saturated fat. This is the direct implication of expressing health concerns as constraints on the consumption of fat or cholesterol, or sodium in the consumer's utility maximization problem. It is the maintained hypothesis of this study that preferences are stable and well-behaved, avoiding questions of how tastes come about and how they are modified which are better left to psychologists, sociologists or somebody else (Silberberg; Stigler and Becker). It should be pointed out that this assumption, standard in the economics literature and supported by studies such as those of Chalfant and Alston, allows the specification of flexible forms which approximate the utility function and its derivatives. The demands under the alternate model below can then be derived based on the specified form of the direct or indirect utility function or the expenditure function, as can hypotheses for testing the structure of consumer demands. Further, the proposed model allows the specification of changing preferences and of demographically induced differences in tastes, retaining all of the flexibility of the standard model.

The proposed model is formally stated below, using the problem of utility maximization from meat consumption, assuming that the meats group is weakly separable from other foods. While this example is used here and in the discussion in Chapter IV to facilitate exposition, it should be noted that the proposed model applies to studying demands for all foods. The model can also be applied in other context where the consumption of some component of purchased commodities (commodity) is

restricted without altering preferences. In said example, the proposed model of the health-conscious consumer's behavior in meat consumption is:

$$\begin{aligned}
 & \max_{\{X^M\}} U^M = U^M(X^M) \\
 \text{(II. 6)} \quad & \text{s.t. } p^M \cdot X^M = Y^M \\
 & f^M \cdot X^M = F^M
 \end{aligned}$$

i.e. the consumer maximizes her or his utility from meats, represented by a stable, well-behaved utility function, subject to outlay for meats and auxiliary constraints which place limits on the consumption of fat and cholesterol from meats. It is assumed that the content of fat and cholesterol in meats are in fixed proportions, and that these proportions vary over time. Meat demands derived from this model are of the form

$$\text{(II. 7)} \quad X^M = X^M(p, Y, f^M, F^M)$$

The utility function allows for variations in preferences across households due to demographic and other differences (Gorman 1976; Pollak and Wales 1981, 1992; Lewbel 1985). Further, by appropriate choice of utility function form, the hypothesis that consumer demands for meats are not affected by the fat and cholesterol content of meats can be tested through restrictions on the parameters of the estimated demand system, which is familiar econometric practice.

The model is developed further in the next chapter, and some of the standard results from consumer theory such as Roy's Identity and the Slutsky equation are

shown to derive from this "augmented" problem equivalently to the standard model. Empirical issues involved in implementing the model are discussed in the subsequent chapter along with an example of deriving individual demand (share) equations. This chapter is concluded with a discussion of rationing models.

II. 2b *Consumer theory with "rationing"*

To the extent that both approaches impose additional quantity restrictions on the consumer, the proposed model and rationing models are similar in structure. However, the two specifications differ in that rationing is an *institutional* constraint, while the additional constraints introduced in the proposed model arise from a particular specification of *individual* behavior. Thus, in the rationing model component proportions f and the constraints F are usually all determined by a legislative or an administrative order and are held fixed (or may be allowed to vary, for example, in fixed proportion to household income); and while the consumer's demands can be derived as functions of (p, Y, f, F) , they cannot be estimated as such due to the lack of variation in (f, F) . In this model, the component proportions and constraints are based on individual perceptions, and thus vary over time for the same individual as perceptions change (due to education and media exposure), and vary across households as well.

The original comparative statics result on household demands with rationing constraints, the Le Chatelier result, was derived by Samuelson (1947) and elaborated upon by Tobin and Houthakker. Studies by Graaff and, later, by Neary and Roberts have sought to extend the model by relating the rationed and unrationed demands through the expenditure, or cost, function. Neary and Roberts demonstrate that restricted and unrestricted demands can be related through "virtual prices"—the prices at which the freely chosen levels of consumption will be identical to those chosen when the ration is imposed—and by the fact that the unrestricted and restricted minimum costs will be equal when the price of the rationed good equals the virtual price.

Deaton (1981) and Deaton and Muellbauer (D-M, 1981) have extended the analysis of Neary and Roberts into areas where households face restrictions on their consumption of individual goods, i.e. "strict rationing," and where there may be some variation in the ration across households, thus allowing the estimation of these models. Deaton and Muellbauer give functional forms for estimating consumer demands with "involuntary" unemployment, while Deaton estimates a model of household commodity demands where the consumption of the non-durable good, housing, is treated as "strictly" rationed. However, this ". . . method of generating rationed from unrationed demands (Deaton 1981, p. 71)" severely limits the generality of the functional forms chosen. Deaton (1981) suggests that ". . . analysis of rationed behavior can proceed from the specification of a suitable function" for the restricted

cost function. He gives the example of a "rationed AIDS (or RAIDS)" model and the expenditure share equations with a single ration on housing consumption.

Jackson seeks to extend the method of Neary and Robert to "points rationing"—where the sum of goods consumptions weighted by their "points prices" must equate to a "points ration"—and also to more general functional constraints, but suggests that

The application of rationing theory to specific preferences is hampered by algebraic intractability. . . . A partial remedy is to reverse the usual practice, defining preferences immediately in a rationed version, without corresponding unrationed functions (Deaton, 1981). Unless this simplifying procedure is adopted, there is little prospect of using rationing theory in empirical work (pp. 338–339).

Thus, Jackson surrenders to "algebraic intractability," failing to provide empirically testable models for "points rationing," which might have served the purpose of the current study. In this study, demand share equations are derived as functions of prices, expenditures, and exogenous factors related to food composition, as also are hypotheses which allow the testing of the standard theory as restrictions on the parameters of the extended model.

While the literature on rationing theory anticipates this study, it does not provide the results, as this study does, which allow the analysis of consumer food demands with the health concerns identified in the introductory chapter. The current study further differs from the literature reviewed above in focussing on household level decisions, and by deriving the demand share equations by specifying an indirect utility function rather than an expenditure or cost function. Finally, since in most cases

consumers who account for the components of foods in planning their diets will always do so, this study does not explore the relationship between rationed and unrationed demands, nor does it suffer for this omission.

III Consumer demands with health-related constraints

This study proposes a model of consumers who each maximize utility from food consumption subject to budget and additional constraints on the consumption of various components of foods. Individuals are postulated to differ in whether or not they enforce constraints on their consumption of, for example, fat and of cholesterol, and in their perceptions of the content of fat and cholesterol in various foods. Thus aggregate demands must be computed by adding the demands of heterogeneous consumer groups, and the demands of consumers within each group must also be estimated separately since they will have different forms depending on the effective constraints upon the consumer. Further, for those consumers who are effectively constrained in the consumption of fat for example, the limit of fat consumption and the proportions of fat (or their perceptions) in various food commodities will always be arguments in their food demands. In effect, the number of constraints enforced determines the arguments in the consumer's demand functions. Thus, consumers who face different constraints have demand functions with different sets of arguments. This rules out aggregate demands as functions of a single set of arguments.

In this chapter, the individual consumer demands are derived for the augmented model proposed. It will be shown that the constraint coefficients and bounds will be

arguments of the consumer's demand functions. Standard comparative statics results from consumer theory are derived. The dual structure of the model is examined. It is shown that weak separability of the food group allows the problem analyzed to be restricted to maximization of utility from consumption of food commodities subject to the optimal allocation of expenditures on foods. Finally, the model is compared to the mathematical programming version of an optimization problem with mixed constraints, and an important distinction is drawn.

III. 1 Consumer theory

The standard version of the problem of the consumer is to choose a consumption bundle so as to

$$(III. 1) \quad \max_{\{X\}} \{U(X) \mid p \cdot X = Y \text{ and } X \in R_+^n\}$$

Note that the budget constraint is binding (expenditures on all goods add up to total disposable income) if the consumer's behavior exhibits local nonsatiation, i.e. $U_i(X) > 0$ for at least one i . The above can be expressed using the Lagrangian function as

$$(III. 2) \quad \max_{\{X\}} \mathcal{L} = U(X) + \mu(Y - p \cdot X)$$

The first order (necessary) conditions for this problem to have a solution are

$$(III. 3a) \quad \frac{\partial \mathcal{L}}{\partial X_i} = 0 \Rightarrow U_i - \mu p_i = 0 \quad \forall i \in \{1, 2, \dots, n\}$$

$$(III. 3b) \quad \frac{\partial \mathcal{L}}{\partial \mu} = 0 \Rightarrow \mathbf{p} \cdot \mathbf{X} = Y$$

while the necessary second order conditions are met if the utility function satisfies the regularity conditions of single valuedness, quasi-concavity, twice continuous differentiability, and monotonicity. The first order conditions in (III.3) can be solved for the Marshallian (ordinary) demands $X(\mathbf{p}, Y)$ and the Lagrange multiplier $\mu(\mathbf{p}, Y)$.

Note that the ordinary demands must satisfy the budget constraint given prices and income, and that since an equiproportionate change in \mathbf{p} and Y does not affect the budget constraint (see (III. 1)), the solution must remain unaltered for such changes. Thus $X(\mathbf{p}, Y)$ and $\mu(\mathbf{p}, Y)$ must be homogeneous of degree zero in prices and income.

The model proposed augments the standard problem with additional constraints. It is maintained that adding constraints on the consumption of cholesterol and fat by consumers is the theoretically and empirically preferred way of modeling the effect of health concerns of individuals or households on food demands. It is also maintained that health concerns relate to specific components of food and are implemented as binding (or effective) *implicit* constraints on the consumption of that particular

component of food. In the following section, the structure of the model with added constraints is demonstrated.

III. 2 Utility maximization with auxiliary constraints

Consider the addition of a single additional binding constraint to the standard consumer demand problem⁵: this constraint might represent the upper bound on the amount of (saturated) fat the consumer's diet allows or the amount of cholesterol s/he is constrained to consume. The consumer's problem is now to choose the consumption bundle to

$$(III. 4) \quad \max_{\{X\}} \{U(X) \mid p \cdot X = Y \text{ and } f \cdot X = F \text{ and } X \in \mathbf{R}_+^n\}$$

(Note that X is a column n -vector and p and f are row n -vectors.) This is the consumer's problem in standard form augmented by a constraint on the consumption of some characteristic F , where the elements of the row-vector f are the proportions of that characteristic in the various goods consumed. In the context of this model, the vector f varies across consumers when a) consumers have different perceptions about the fat content of foods, or b) goods like hamburger are treated as homogeneous regardless of fat content, or c) consumers' perceptions change over time. Of course,

⁵Non-binding constraints are considered below.

when goods like "beef" are in fact aggregated goods, the associated f_i will also differ across consumers.

The Lagrangian version of the problem (III. 4) is

$$(III. 5) \quad \max_{\{X\}} \mathcal{L} = U(X) + \mu(Y - p \cdot X) + \lambda(F - f \cdot X)$$

while the first order conditions are

$$(III. 6a) \quad \frac{\partial \mathcal{L}}{\partial X_j} = 0 \Rightarrow U_j - \mu p_j - \lambda f_j = 0 \quad \text{for all } j \in \{1, 2, \dots, n\}$$

$$(III. 6b) \quad \frac{\partial \mathcal{L}}{\partial \mu} = 0 \Rightarrow p \cdot X = Y$$

$$(III. 6c) \quad \frac{\partial \mathcal{L}}{\partial \lambda} = 0 \Rightarrow f \cdot X = F$$

The system of equations (III.6) includes $n+1+1$ equations in $n+2$ unknowns (X, μ, λ) with the parameters of the system being (p, Y, f, F) . Provided that this system of equations is independent, one can solve for the unknowns (X, μ, λ) in terms of the parameters (p, Y, f, F) ; thus the solution to the above system of first order conditions is $X(p, Y, f, F)$, $\mu(p, Y, f, F)$, $\lambda(p, Y, f, F)$. Again, since the constraints (and hence the problem)

are invariant to multiplication of either (p, Y) or (f, F) by any factor, $X(p, Y, f, F)$, $\mu(p, Y, f, F)$, $\lambda(p, Y, f, F)$ must be homogeneous of degree 0 in (p, Y) and in (f, F) .

This model yields demands that are functions of the perceived fat content of foods consumed and the constraint on fat consumption, as well as of prices and expenditures. In general, all demands X_j will be functions of (p, Y, f, F) regardless of whether or not X_j contains fat. For some separable structures, some demands may be independent of (f, F) .

Additional constraints modify the model in a straightforward way. Let the system of m auxiliary constraints be written as

$$(III. 7) \quad \mathbf{f}_k \cdot \mathbf{X} = F_k \quad \forall k \in \{1, 2, \dots, m\}$$

where \mathbf{f}_k is the k^{th} row of a matrix which represents the perceived proportions of characteristics (such as fat and cholesterol) of foods in the vector \mathbf{X} and F_k is the k^{th} element of a (column) vector which gives the totals of each characteristic for all foods in the bundle \mathbf{X} .

The Lagrangian version of this problem is

$$(III. 8) \quad \max_{\{\mathbf{X}\}} \mathcal{L} = U(\mathbf{X}) + \mu(Y - \mathbf{p} \cdot \mathbf{X}) + \sum_{k=1}^m \lambda^k (F_k - \mathbf{f}_k \cdot \mathbf{X})$$

with the resulting system of first order conditions

$$(III. 9a) \quad \frac{\partial \mathcal{L}}{\partial X_j} = 0 \Rightarrow U_j - \mu p_j - \sum_{k=1}^m \lambda^k f_{kj} = 0 \quad \forall k \in \{1, 2, \dots, m\}$$

$$(III. 9b) \quad \frac{\partial \mathcal{L}}{\partial \mu} = 0 \Rightarrow \mathbf{p} \cdot \mathbf{X} = Y$$

$$(III. 9c) \quad \frac{\partial \mathcal{L}}{\partial \lambda^k} = 0 \Rightarrow \sum_{i=1}^n f_{ki} X_i = F_k \quad \forall k \in \{1, 2, \dots, m\}$$

This is a system of $n+1+m$ equations in the unknowns $(X_{n \times 1}, \mu_{1 \times 1}, \lambda_{m \times 1})$ with parameters $(\mathbf{p}_{1 \times n}, Y_{1 \times 1}, \mathbf{f}_{m \times n}, \mathbf{F}_{m \times 1})$; its solution is given by $X(\mathbf{p}, Y, \mathbf{f}, \mathbf{F}), \mu(\mathbf{p}, Y, \mathbf{f}, \mathbf{F}), \lambda(\mathbf{p}, Y, \mathbf{f}, \mathbf{F})$. Again, since X must satisfy all of the constraints which in this case are linear, the X are homogenous of degree 0 in (\mathbf{p}, Y) and in (\mathbf{f}_k, F_k) for all k in $\{1, 2, \dots, m\}$.

III. 3 Comparative statics of the proposed model

This section is devoted to establishing some of the familiar comparative statics results from consumer theory which provide useful restrictions in empirical analysis. The Slutsky equation, symmetry, and the "law of demand" for compensated price

responses are derived. The Le Chatelier result is also proved. The dual structure of the model, indirect utility and cost functions, is demonstrated.

III. 3a *The Slutsky equation and symmetry*

Taking the total derivatives of the first order conditions (III.9), we get

$$(III. 10a) \quad \sum_{j=1}^n U_{ij} dx_j - d\mu p_i - \sum_{k=1}^m d\lambda^k f_{ki} = \mu^1 dp_i + \sum_{k=1}^m \mu^{k+1} df_{ki}$$

$$(III. 10b) \quad \sum_{i=1}^n p_i dX_i = dY - \sum_{j=1}^n X_j dp_j$$

$$(III. 10c) \quad \sum_{i=1}^n f_{ki} dX_i = dF_k - \sum_{j=1}^n X_j df_{kj}$$

The system of equations (III.10) can be written in partitioned matrix form, letting

$$H = \partial^2 U / \partial X \partial X', \text{ as}$$

$$(III. 10') \quad \begin{bmatrix} \mathbf{H} & : & \mathbf{p}' & : & \mathbf{f}' \\ \dots & \dots & \dots & \dots & \dots \\ \mathbf{p} & : & \mathbf{0} & : & \mathbf{0} \\ \dots & \dots & \dots & \dots & \dots \\ \mathbf{f} & : & \mathbf{0} & : & \mathbf{0} \end{bmatrix} \begin{bmatrix} d\mathbf{X} \\ \dots \\ -d\mu \\ \dots \\ -d\lambda^1 \\ \vdots \\ -d\lambda^m \end{bmatrix} = \begin{bmatrix} \mu d\mathbf{p}' + \sum_{k=1}^m \lambda^k d\mathbf{f}'_k \\ \dots \\ dY - \mathbf{X}' d\mathbf{p}' \\ \dots \\ dF_1 - \mathbf{X}' d\mathbf{f}'_1 \\ \vdots \\ dF_m - \mathbf{X}' d\mathbf{f}'_m \end{bmatrix}$$

where k is again the row index. This system can now be solved for the dX_i and $d\mu^k$ terms. For example, applying Cramer's rule for the solution of systems of linear equations,

$$(III. 11) \quad dX_i = \frac{1}{B} \left[\sum_{j=1}^n \left(\mu dp_j + \sum_{k=1}^m \lambda^k df_{kj} \right) B_{ji} + (dY - \mathbf{X}' d\mathbf{p}') B_{n+1,i} + \sum_{k=1}^m (dF_k - \mathbf{X}' d\mathbf{f}'_k) B_{n+1+k,i} \right]$$

where any B_{ij} is the ij^{th} cofactor of the "bordered Hessian" matrix B . The response of demand for good X_i with respect to a change in price p_j can now be derived as $\partial X_i / \partial p_j$, which from (III. 11) is

$$(III. 12) \quad \frac{\partial X_i}{\partial p_j} = \frac{\mu B_{ji} - X_j B_{n+1,i}}{B}$$

Also

$$(III. 13) \quad \frac{\partial X_i}{\partial Y} = \frac{B_{n+1,i}}{B}$$

$$(III. 14) \quad \frac{\partial X_i}{\partial f_{kj}} = \frac{\lambda^k B_{ji} - X_j B_{n+1+k,i}}{B}$$

$$(III. 15) \quad \frac{\partial X_i}{\partial F_k} = \frac{B_{n+1+k,i}}{B} \quad \forall k \in \{1,2,\dots,m\}$$

If we call the first term on the right hand side of (III. 12) s_{ij} , the "compensated" price response, we have

$$(III. 16) \quad s_{ij} = \mu \frac{B_{ji}}{B}$$

and $s_{ii} \leq 0$ since the "marginal utility of income" is always positive, since the signing term on the co-factor of a diagonal element is positive, and since B is negative semi-definite. This is the "law of demand" (Deaton and Muellbauer 1980, p. 45). Further, since B is symmetric, $s_{ij} = s_{ji}$. Finally, from (III. 12), (III. 13), and (III. 16) we can write the "compensated price response" as

$$(III. 17) \quad s_{ij} = \frac{\partial X_i}{\partial p_j} + X_j \frac{\partial X_i}{\partial Y}$$

which is the Slutsky equation. Note that the above derivations constitute proof of some fundamental propositions of consumer theory—the "law of demand," symmetry, and the Slutsky equation—without resort to duality. In later (sub-)sections, the above are derived from a "dual" representation of preferences in the form of an indirect utility function and an expenditure function, but first an important consequence of adding constraints to the consumer's utility maximization problem is discussed, and proven.

III. 3b The Le Chatelier principle

Samuelson ([1947] 1983) states the Le Chatelier principle as follows:

if in a given position of equilibrium a (compensated) change in price is made, the resulting change in amount demanded of that good will be greater if the individual is not subjected to the extra constraints of rationing than if he is subjected to such constraints; furthermore, the introduction of each new constraint will make demand more inelastic (pp. 168-169).

In the context of the proposed model, consider the addition of a constraint (such as on saturated fat consumption) whose bound is large enough to just admit the optimal (without the added constraint) bundle. The Le Chatelier principle implies that the compensated own-price responses will be smaller in size (more inelastic) with the added constraint than without it. This result is proved here for composite constraints

of the form proposed. (Samuelson ([1947] 1983, pp. 100-3) demonstrates the method, which is adapted here.)

In the interests of generality, let the consumer originally face m constraints, so that with the added one, s/he faces $(m+1)$ constraints. Let the added constraint, imposed at X^0 , be of the form

$$(III. 18) \quad f_{m+1} \cdot X^0 = F_{m+1}$$

where f_{m+1} is a row n -vector. Further, let us assume that the bound F_{m+1} is such that the constraint is just binding, i.e. the chosen optimal consumption vector X^0 without the added constraint satisfies it with equality. Now, let the compensated own-price responses for the augmented system be denoted ${}^{m+1}s_{ii}$, while the compensated price response without the added constraint in (III. 18) is denoted ${}^m s_{ii}$. We have

$$(III. 19) \quad \begin{aligned} & {}^{m+1}s_{ii} - {}^m s_{ii} \\ &= \mu^1 \left(\frac{{}^{m+1}\mathbf{B}_{ii}}{{}^{m+1}\mathbf{B}} - \frac{{}^m\mathbf{B}_{ii}}{{}^m\mathbf{B}} \right) \\ &= \mu^1 \left(\frac{{}^{m+1}\mathbf{B}_{ii}}{{}^{m+1}\mathbf{B}} - \frac{{}^{m+1}\mathbf{B}_{n+m+1, n+m+1 \cdot ii}}{{}^{m+1}\mathbf{B}_{n+m+1, n+m+1}} \right) \\ &= \mu^1 \left(\frac{{}^{m+1}\mathbf{B}_{ii} \cdot {}^{m+1}\mathbf{B}_{n+m+1, n+m+1} - {}^{m+1}\mathbf{B} \cdot {}^{m+1}\mathbf{B}_{n+1+m, n+1+m \cdot ii}}{{}^{m+1}\mathbf{B} \cdot {}^{m+1}\mathbf{B}_{n+m+1, n+m+1}} \right) \end{aligned}$$

We can apply the following theorem of Jacobi (Samuelson, 1960)

$$(III. 20) \quad \Delta \Delta_{ii \cdot jj} = \Delta_{ii} \Delta_{jj} - \Delta_{ij} \Delta_{ji}$$

where Δ is the determinant of a matrix and Δ_{ij} is the jj^{th} co-factor of the ii^{th} co-factor of the matrix. Equation (III. 19) can then be written as

$$\begin{aligned}
 & {}^{m+1}S_{ii} - {}^mS_{ii} \\
 \text{(III. 21)} \quad & = \mu^1 \left(\frac{{}^{m+1}B_{ii} {}^{m+1}B_{n+m+1, n+m+1} - {}^{m+1}B_{ii} {}^{m+1}B_{n+m+1, n+m+1} + {}^{m+1}B_{n+m+1, i} {}^{m+1}B_{i, n+m+1}}{{}^{m+1}B {}^{m+1}B_{n+m+1, n+m+1}} \right) \\
 & = \mu^1 \frac{({}^{m+1}B_{n+m+1, i})^2}{{}^{m+1}B {}^{m+1}B_{n+m+1, n+m+1}}
 \end{aligned}$$

since the "bordered Hessian" matrix B is symmetric. Since the second-order condition for utility maximization with a total of r constraints is (Silberberg, p. 177)

$$\text{(III. 22)} \quad \text{Sign}(B_m) = (-1)^{m-r} \text{ where } m = 1+2r, \dots, n+r$$

the sign of the two terms in the denominator of (III. 21) is identical ($= (-1)^n$), and its sign must be positive. Further, since $\mu^1 \geq 0$ and since the numerator is always positive, we get the general result

$$\text{(III. 23)} \quad {}^mS_{ii} \leq {}^{m-1}S_{ii} \leq \dots \leq {}^0S_{ii} \leq 0$$

This is the result stated by the Le Chatelier principle (Samuelson [1947] 1983).

III. 3c *The indirect utility function and the expenditure function*

The usefulness of the indirect utility function and of Roy's identity are pointed out by Pollak and Wales (1992): "... functional form specification for a demand system usually begins with the assumption about the form of the indirect utility function and uses Roy's identity to derive the ordinary demand functions (p. 10)." In this section, the indirect utility function and Roy's identity for the proposed model are derived.

The indirect utility function can be derived by substituting the ordinary demand functions into the utility function to get the envelope of the utility function. Thus,

$$(III. 24) \quad V(\mathbf{p}, Y, \mathbf{f}, F) = U(X(\mathbf{p}, Y, \mathbf{f}, F))$$

The *expenditure function* is defined as the inverse of the indirect utility function at any level of utility, or

$$(III. 25) \quad V(\mathbf{p}, e(\mathbf{p}, U^0, \mathbf{f}, F), \mathbf{f}, F) \equiv U^0$$

and by Shephard's Lemma, compensated demands, s_j , are given by

$$(III. 26) \quad s_j \equiv \frac{\partial e(\mathbf{p}, U^0, \mathbf{f}, F)}{\partial p_j}$$

and since the expenditure function is concave (Varian 1984, p. 123) $s_{ii} \leq 0$ (the law of demand). Also, Young's Theorem states that partial derivatives—here of the

expenditure function—are invariant to the order of differentiation and hence we must have $s_{jk} = s_{kj}$ for all $\{j,k\}$ which gives symmetry.

Compensated demands are identically equal to Marshallian demands X_j

$$(III. 27) \quad s_j(\mathbf{p}, U^0, \mathbf{f}^0, \mathbf{F}^0) \equiv X_j(\mathbf{p}, e(\mathbf{p}, U^0, \mathbf{f}^0, \mathbf{F}^0), \mathbf{f}^0, \mathbf{F}^0)$$

Differentiating (III. 25) results in

$$(III. 28) \quad \begin{aligned} dU|_{U^0} &\equiv \sum_{i=1}^n \frac{\partial V}{\partial p_i} dp_i + \frac{\partial V}{\partial Y} dY + \sum_{i=1}^n \sum_{k=1}^m \frac{\partial V}{\partial f_{ki}} df_{ki} + \sum_{k=1}^m \frac{\partial V}{\partial F_k} dF_k \\ &\equiv \sum_{i=1}^n \frac{\partial V}{\partial p_i} dp_i + \frac{\partial V}{\partial Y} de(\mathbf{p}, U^0, \mathbf{f}, \mathbf{F}) + \sum_{i=1}^n \sum_{k=1}^m \frac{\partial V}{\partial f_{ki}} df_{ki} + \sum_{k=1}^m \frac{\partial V}{\partial F_k} dF_k \end{aligned}$$

A change in a single price p_j which leaves U^0 unaltered gives

$$(III. 29) \quad \begin{aligned} 0 &\equiv \frac{\partial V(\mathbf{p}, Y, \mathbf{f}^0, \mathbf{F}^0)}{\partial p_j} + \frac{\partial V(\mathbf{p}, Y, \mathbf{f}^0, \mathbf{F}^0)}{\partial Y} \frac{\partial e(\mathbf{p}, U^0, \mathbf{f}^0, \mathbf{F}^0)}{\partial p_j} \\ &\Rightarrow \frac{\partial V(\mathbf{p}, Y, \mathbf{f}^0, \mathbf{F}^0)/\partial p_j}{\partial V(\mathbf{p}, Y, \mathbf{f}^0, \mathbf{F}^0)/\partial Y} \equiv s_j(\mathbf{p}, U^0, \mathbf{f}^0, \mathbf{F}^0) \equiv X_j(\mathbf{p}, Y, \mathbf{f}^0, \mathbf{F}^0) \end{aligned}$$

The second identity in the result in (III. 29) is Roy's identity.

Taking partial differentials with respect to p_k on both sides of (III. 27) results in

$$(III. 30) \quad \begin{aligned} \frac{\partial s_j(\mathbf{p}, U^0, \mathbf{f}^0, \mathbf{F}^0)}{\partial p_k} &\equiv \frac{\partial X_j(\mathbf{p}, Y, \mathbf{f}^0, \mathbf{F}^0)}{\partial p_k} + \frac{\partial X_j(\mathbf{p}, Y, \mathbf{f}^0, \mathbf{F}^0)}{\partial Y} \frac{\partial e(\mathbf{p}, U^0, \mathbf{f}^0, \mathbf{F}^0)}{\partial p_k} \\ &\Rightarrow \frac{\partial X_j}{\partial p_k} \equiv \frac{\partial s_j}{\partial p_k} - X_j^0 \frac{\partial X_j}{\partial Y} \end{aligned}$$

which is the Slutsky equation.

By extending the results of the previous section, which were derived using the "primal" approach, to dual specifications, the applicability of the analysis is widened. In the next chapter, the convenience of the dual approach is demonstrated when individual demands are derived from a specific indirect utility functional form which has well-defined approximation properties.

III. 3d *Commodity groups, separability and decentralization*

Blackorby, Primont, and Russell (B-P-R) define strong decentralizability as follows: "If it is possible for the consumer to optimally allocate the r th category expenditure, y_r , knowing only intracategory prices, we say that the problem [utility maximization over all goods subject to income] ... is characterized by *strong decentralizability* with respect to sector r (p. 177)." They go on to prove (Thm 5.3) that as long as the utility function can be written as $U \equiv \hat{U}(U^1(X^1), X^2, \dots)$, then the consumers utility maximization problem is strongly decentralizable in the first partition. Thus the consumer can treat the problem as being to maximize consumption on goods in the first partition subject to a budget constraint for in-group expenditures to be bound by the optimal outlay on the group.

This result can be shown to hold when the standard problem is augmented with additional constraints on the consumption of fat and cholesterol, so that we may write

$$\begin{aligned}
& \max_{\{X^1\}} U^1(X^1) \\
\text{(III. 32)} \quad & \text{s.t. } p^1 \cdot X^1 = Y - \sum_{j \in I^1} p_j X_j^0 \equiv Y^1(p, Y) \\
& f_k^1 \cdot X^1 = F_k - \sum_{j \in I^1} f_{k,j} X_j^0 \equiv F_k^1(f, F) \quad \forall k \in 1, \dots, m
\end{aligned}$$

where X_j^0 is the optimal level of consumption of all X_j not in the r^h partition (See Pollak; B-P-R, pp.184-189, Thms. 5.1, 5.2, and 5.3). The procedure for the proof is to show that the first order conditions for the maximization of the total utility function U subject to budget and constraints on the consumption of cholesterol and fat imply the first order conditions for the problem of maximizing utility from the weakly separable group, given the optimal outlay on the weakly separable group and the additional constraints. This holds if the constraints involve goods in the weakly separable group only. The proof is not presented here since decentralization is implied by the separable structure of the *utility* function and hence is invariant to the number of constraints facing the consumer. Finally, (III. 32) can be written as

$$\begin{aligned}
& V^1(p^1, Y^1, f^1, F^1) \equiv \max_{\{X^1\}} U^1(X^1) \\
\text{(III. 33)} \quad & \text{s.t. } p^1 \cdot X^1 = Y - \sum_{j \in I^1} p_j X_j^0 \equiv Y^1(p, Y) \\
& f_k^1 \cdot X^1 = F_k - \sum_{j \in I^1} f_{k,j} X_j^0 \equiv F_k^1(f, F) \quad \forall k \in 1, \dots, m
\end{aligned}$$

which allows the use of results derived in the previous section, on the indirect utility function, to derive theoretically plausible demand systems and the restrictions on their parameters for the decentralized problem.

III. 4 Constrained maximization with mixed constraints

Up to this point, only binding constraints have been studied. Now consider the problem with inequality constraints. Takayama (1985, pp. 102-104) shows that if an objective function (a utility function in our case) is to be maximized given some equality (binding) constraints and some inequality (not necessarily binding) constraints, the Karush-Kuhn-Tucker or Arrow-Hurwicz-Uzawa first order conditions state that if a constraint is binding, its shadow price will be positive, and if it is non-binding, its shadow price will be zero. Note that, to test these first order conditions, one needs actual physical values of the constraints and a function which describes the production of the constrained commodity by various consumption vectors.

Given that it is impossible to obtain data on *physical* constraints through direct questioning, the practical alternative is to indirectly determine whether consumers enforce any constraints. For example, consumers could be asked the following three questions: 1) "Are you concerned about consumption of fat?" 2) "Would your food purchases differ if you were not concerned about fat?" 3) "What is the percentage of fat in various foods purchased?" The responses to these questions could then be used

to determine whether consumers enforce an *implicit* fat constraint, the perceived proportions of fat in various foods consumed, and the bound on the implicit constraint.

In this dissertation, it is assumed that the fat and cholesterol constraints, in the meat demand example chosen, are non-redundant to the budget constraint over the "relevant" ranges of (X, p, Y, f, F) . The population is argued to be made up of consumers (households) who maximize utility subject to budget and other constraints, and consumers differ, along with other possible forms of "heterogeneity" (Stoker 1993), on whether or not they constrain fat and/or cholesterol consumption.

Data necessary for estimating demands in the context of the proposed model are discussed in the following chapter, using the example of meat demands of a consumer who restricts her/his fat intake. The procedure for deriving individual demands is demonstrated. The aggregation of individual demands to obtain market demands when consumers enforce different constraints is also discussed, and formulae for estimating price elasticities are provided.

IV Implementation of the model

This chapter examines the implementation of the proposed model of individuals whose food consumption behavior can be represented by the standard utility maximization model with additional constraints on, for example, the consumption of fat and/or cholesterol. It is argued that not all consumers face the same (number of) constraints; thus the population is assumed to be comprised of groups of consumers where consumers in each group face no constraints at all or face a constraint(s) on the consumption of fat and/or cholesterol. In this chapter, the derivation of individual demands from a specified indirect utility function is demonstrated. Non-parametric tests of constraints are discussed. The method of computing (mean or per capita) aggregate demands and of computing demand elasticities is also demonstrated. Finally, the data necessary for estimating and testing the system of demands derived from the proposed model are discussed.

IV. 1 The derivation of individual demands

Roy's identity for an indirect utility function with additional arguments relating to the perceived constraints faced by consumers was derived in the previous section. Here that general result is translated into an example with a specific indirect utility

function. It should first be pointed out that due to the fact that consumers/households will be successful in enforcing different constraints, demand functions cannot be estimated using aggregated data. While this increases the complexity of the analysis, it also frees the economist from the restrictive assumption of quasi-homotheticity of the (indirect) utility function (as, for example, with the AIDS model). The functional form chosen for the individual indirect utility function is the indirect translog utility function (Christensen, Jorgnesen, and Lau; Lewbel 1987). Since the indirect translog utility function can be considered to be a second-order Taylor series approximation to an unknown indirect utility function, its first derivatives can be considered to be first-order Taylor series approximations to the first derivatives of the unknown utility function. Further, as Driscoll has shown, the domain over which the translog approximation converges to the unknown function includes the entire space of real numbers, and hence is valid over the range of the data (which economic theory postulates as restricted to the positive orthant).

Assume that consumer's preferences for beef, pork, and poultry are separable from all other goods, and further that the components of the aggregates "beef," "pork," and "poultry" are separable from their complements. The "augmented" indirect translog utility function (of second-order) can be written, for consumers who face a constraint on fat consumption only, as

$$\begin{aligned}
\ln V = & \alpha_0 + \sum_{i=1}^3 \alpha_i \ln \frac{p_i}{Y} + \frac{1}{2} \sum_{i=1}^3 \sum_{j=1}^3 \beta_{ij} \ln \frac{p_i}{Y} \ln \frac{p_j}{Y} \\
& + \sum_{i=1}^3 \gamma_i \ln \frac{f_i}{F} + \frac{1}{2} \sum_{i=1}^3 \sum_{j=1}^3 \delta_{ij} \ln \frac{f_i}{F} \ln \frac{f_j}{F} \\
& + \frac{1}{2} \sum_{i=1}^3 \sum_{j=1}^3 \eta_{ij} \ln \frac{f_i}{F} \ln \frac{p_j}{Y}
\end{aligned}
\tag{IV. 1}$$

where the f_i, f_j , represent perceived proportions of fat in the three foods while F is the level to which fat consumption is constrained. Symmetry, $\beta_{ij} = \beta_{ji}$, $\delta_{ij} = \delta_{ji}$, should be imposed for data analysis. Note that the above functional specification satisfies the homogeneity and symmetry restrictions of consumer theory derived in the previous section. The budget shares for each commodity j generated by this functional specification can be obtained by applying Roy's Identity in logarithms

$$M_j = - \frac{\partial \ln V / \partial \ln p_j}{\partial \ln V / \partial \ln Y}. \text{ Thus}$$

$$\begin{aligned}
M_j = & \frac{\alpha_j + \sum_{i=1}^3 \beta_{ji} \ln \frac{p_i}{Y} + \sum_{i=1}^3 \eta_{ij} \ln \frac{f_i}{F}}{\sum_{i=1}^3 \alpha_i + \sum_{i=1}^3 \sum_{j=1}^3 \beta_{ij} \ln \frac{p_i}{Y} + \sum_{i=1}^3 \sum_{j=1}^3 \eta_{ij} \ln \frac{f_i}{F}}
\end{aligned}
\tag{IV. 2}$$

which can be estimated using non-linear estimation procedures⁶ (Amemiya; Judge, et. al.).

⁶Since the γ_i and δ_{ij} parameters cannot be identified in the current context, along with α_0 , this model can be used for examining welfare effects of changing prices using the Compensating and Equivalent Variations, but cannot be used to analyze the effects of changing f_i .

The null hypothesis that consumer demands for food commodities are completely explained by prices and expenditures alone can be tested using this model. The joint hypothesis

$$(IV. 3) \quad H_0 : \eta_{ij} = 0 \quad \forall i, j \in \{1, 2, 3\}$$

can be tested using a χ^2 -test for parameter restrictions on a system of equations such as the "generalized Wald" statistic (Amemiya, p. 145; also Gallant and Jorgenson). Thus, using this model one can test, using household level data and parametric restrictions, the hypothesis suggested by the previous results of Chalfant and Alston, and Wohlgenant.

Estimation and testing of individual demands could well depend on the data available. At this point, no known data set reports all of the information necessary for estimating and testing the above model; one of the objectives of this study has been to argue for the collection of such data. Either cross-section or long panel data could be used, but due to the formidable expense of collecting long panels (Bobst, et. al.), it is more realistic to anticipate the availability of data for the cross-section. In order to estimate equation (IV. 2) with cross-section data the assumption that all consumers (at least those who successfully enforce the same number of constraints) have identical preferences must be added; a long panel would obviate the need for this assumption by allowing the estimation of individual demands for each household. However, the latter strategy will be derailed if there does not exist sufficient variability in (f_k, F_k) ,

which is entirely possible for consumers who are on strict diets for medical or other reasons. Nonetheless, data from a panel could be pooled for consumers in the same group (the defining characteristic of a group being that its members enforce the same number of constraints), under the assumption that all members of the group have the same preferences. Systematic differences in preferences across consumers in the group can, of course, be modeled using demographic *scaling* and/or *translating* (Gorman 1976; Pollak and Wales 1981), or some other form of modifying function (Lewbel 1985). The availability of a long panel also facilitates nonparametric tests of the hypothesis above. The strategy for nonparametric tests is discussed below.

IV. 1a Testing for additional constraints on food consumption

The nonparametric approach to testing consumer behavior, suggested by the work of S. N. Afriat and implemented in different contexts by Afriat (1967, 1976), Diewert and Parkan, Chalfant and Alston, and Varian (1982, 1983), seeks contradictions to the axioms of revealed preference in the data; failure to find contradictions is interpreted as meaning that the data "rationalize" demand theory. In this section, the nonparametric approach is elaborated upon. A generalized axiom of revealed preference for consumers maximizing utility subject to outlay, and also to fat and/or cholesterol constraints, is derived. The procedure for implementing non-parametric tests based on this (modified) axiom is then outlined as in Varian (1982).

Varian (1983) proposes nonparametric tests for various hypotheses about consumer behavior. Included are tests for rationing behavior; which are adapted to the proposed model. Varian's exposition is demonstrated in this sub-section.

First, some definitions are necessary. Varian (1982) defines the following:

Given an observation x^i and a bundle x :

- (1) x^i is *directly revealed preferred* to x , written $x^i R^0 x$, if $p^i x^i \geq p^i x$.
- (2) x^i is *strictly directly revealed preferred* to x , written $x^i P^0 x$, if $p^i x^i > p^i x$.
- (3) x^i is *revealed preferred* to x , written $x^i R x$, if $p^i x^i \geq p^i x^j$, $p^j x^j \geq p^j x^k$, ..., $p^m x^m \geq p^m x$ for some sequence of observations $(x^i, x^j, x^k, \dots, x^m)$. In this case we say the relation R is the transitive closure of the relation R^0 .
- (4) x^i is *strictly revealed preferred* to x , written $x^i P x$, if there exist observations x^j and x^k such that $x^j R x^i$, $x^j P^0 x^k$, $x^k R x$. [$x R x$ by convention.] (p. 947)

Now, consider the theorem of Afriat (in Varian 1982).

The following conditions are equivalent:

- (1) There exists a non-satiated utility function that rationalizes the data;
- (2) The data satisfies "cyclical consistency"; that is,
 $p^r x^r \geq p^r x^s$, $p^s x^s \geq p^s x^1$, ..., $p^q x^q \geq p^q x^r$,
implies
 $p^r x^r = p^r x^s$, $p^s x^s = p^s x^1$, ..., $p^q x^q = p^q x^r$.
- (3) There exist numbers U^i , $\lambda^i > 0$, $i = 1, \dots, n$, such that
 $U^i \leq U^j + \lambda^j p^j (x^i - x^j)$ for $i, j = 1, \dots, n$.
- (4) There exists a nonsatiated, continuous, concave, monotonic utility function that rationalizes the data (p. 946).

Varian (1983) modifies the theorem of Afriat by defining the following

"Generalized Axiom of Revealed Preference (GARP):"

The data satisfies GARP if $x^i R x^j$ implies $p^j x^j \leq p^j x^i$ (p. 100).

Given the above definition Afriat's theorem is modified by replacing his condition (2) with

- (2) the data satisfies GARP (Varian, 1983, p. 100).

Varian then gives the algorithm for testing his condition (2) above.

Varian (1983) also gives non-parametric tests for rationing which can be used in the current context. The rationing model is specified as

$$(IV. 4) \quad \begin{aligned} & \max U(X) \\ & s.t. \quad p^i X \leq Y^i, \\ & \quad \quad a^i X \leq b^i, \quad i=1, \dots, n. \end{aligned}$$

where in this case the index i refers to the observations of the data. Using the fact that the solution to this problem must satisfy the Karush-Kuhn-Tucker conditions, Varian (1983) proves the following theorem:

The following conditions are equivalent:

- (1) The data can be rationalized by a continuous, concave, monotonic, non-satiated utility function;
- (2) There exist numbers $u^i, \lambda^i \geq 0, \mu^i \geq 0$ with $\mu^i = 0$ if $a^i X^i < b^i$ and $\lambda^i = 0$ if $p^i X^i < Y^i$ such that:
 $U^i \leq U^j + (\lambda^j p^j + \mu^j a^j)(x^i - x^j)$ (p. 108).

The theorem suggests the following combinatorial rule which is analogous to Varian's GARP:

$$\text{If } x^j R x^i \text{ then } p^j x^j \leq p^i x^i \text{ and } a^j x^j \leq a^i x^i.$$

A modified version of Warshall's algorithm presented in Varian (1982) must be used for testing the data for violations of this rule. First the modified rule is tested to see if the consumer enforces all of the constraints being considered. If the rule is violated, then one of the constraints is dropped and the rule tested again, and so on until some combination of constraints, including the budget constraint is not violated. This procedure can be used to test for implicit constraints enforced by consumers, and also for putting them in various categories based on the number and type of implicit constraints they enforce. The functional specification of the indirect utility function

will then vary based on the category into which the consumer falls. Essentially, the specification should be such that it accounts for all of the constraints being enforced.

IV. 2 Market demands with health concerns

One of the aims stated at the outset was to develop a model of food demand that would address the information needs of food producers and marketers. These agents will be interested in aggregate or market (consumer) demands for food commodities. Thus it is necessary to develop a measure of aggregate demands and their elasticities.

Given the heterogeneity of consumer demands across groups, no "aggregate demand" function exists under the model proposed. Thus, aggregate demands must be derived by summing individual demands over the population. Let there be N households in the population with N_1 consumers facing a budget constraint only, N_2 consumers facing a cholesterol consumption constraint only, N_3 consumers facing a fat consumption constraint only, and N_4 consumers facing both a cholesterol and a fat consumption constraint; let the demands of consumers in each of these groups be represented respectively as ${}_1X(p_{r1}, Y_{r1})$, ${}_2X(p_{r2}, Y_{r2}, f_{r2}, F_{r2})$, ${}_3X(p_{r3}, Y_{r3}, f_{r3}, F_{r3})$, and ${}_4X(p_{r4}, Y_{r4}, f_{r4}, F_{r4})$. The aggregate demand for good j in the meat group can now be expressed as

$$\begin{aligned}
(IV. 5) \quad X_j &= \sum_{r=1}^N X_{j,r} \\
&= \sum_{r=1}^{N_1} {}_1X_{j,r1} + \sum_{r=1}^{N_2} {}_2X_{j,r2} + \sum_{r=1}^{N_3} {}_3X_{j,r3} + \sum_{r=1}^{N_4} {}_4X_{j,r4} \\
&\quad \text{where } N_1 + N_2 + N_3 + N_4 = N
\end{aligned}$$

and hence the mean, or per capita, demand for good j is

$$\begin{aligned}
(IV. 6) \quad \mu_{X_j} &= \frac{N_1}{N} \mu_{{}_1X_j} + \frac{N_2}{N} \mu_{{}_2X_j} + \frac{N_3}{N} \mu_{{}_3X_j} + \frac{N_4}{N} \mu_{{}_4X_j} \\
\text{where } \mu_{X_j} &= \frac{X_j}{N}; \mu_{{}_sX_j} = \frac{{}_sX_j}{N}, s \in \{1, 2, 3, 4\}
\end{aligned}$$

The conditional means in (IV. 6) can be estimated using data from a representative sample, by estimating individual demand functions and then computing the sample averages within groups. Some important points should be noted here. The sample average demand for each good j in each group is calculated as the average of individual demands for members of the (sample) group, *not* by evaluating the estimated function at the average of the data. Total demand X_j is estimated by \hat{X}_j obtained by scaling sample mean demand up by population size. Since estimated demands obtained from the translog model are consistent but not necessarily unbiased due to nonlinearity (Amemiya, p. 257), the estimator \hat{X}_j is only consistent.

IV. 2a *Computation of market demand elasticities*

The elasticity of good j with respect to price k can be calculated using the result in equation (IV. 6). Let the k^{th} price change from p_k^0 to $p_k^1 = p_k^0 + 0.01 \times p_k^0$. Equation (IV. 6) can be used to calculate the new level of mean aggregate demand for good j at the old and new prices. The price elasticity of good j can now be calculated using the (discrete form of) the definition (Varian 1987, p. 107), whereby

$$(IV. 7) \quad \hat{\epsilon}_{jk} = \frac{\frac{\hat{X}_j^1 - \hat{X}_j^0}{\hat{X}_j^0}}{\frac{1.01 p_k^0 - p_k^0}{p_k^0}}$$

and, in turn, the Allen elasticity is given by

$$(IV. 8) \quad \hat{\sigma}_{jk} = \frac{\hat{\epsilon}_{jk}}{\hat{M}_k}$$

Consistency of \hat{X}_j implies that as sample $n \rightarrow \infty$, $\hat{X}_j \rightarrow X_j$, and therefore $\hat{\epsilon}_{jk} \rightarrow \epsilon_{jk}$; $\hat{\epsilon}_{jk}$ is consistent.

IV. 3 **Data needs**

In order to test the model, one would need to estimate consumer/household individual demand functions of the type derived above. Consumers are (potentially) differentiated in a) their preferences, b) their incomes, c) the commodities whose

consumption they effectively constrain, and d) their perceptions of the proportion of various foods comprised by these commodities. Thus data necessary are household data, if possible over several periods of time; the data should report the quantities consumed of various market foods, the prices at which these foods were purchased, which commodities, if any, were being constrained, and the consumer's (meal planner's) perceptions of the content of fat and cholesterol in each serving of the various foods consumed. Individual consumer demands can then be estimated for each household. If the model specified allows for some of the coefficients to be common across individuals (e.g., by specifying that preferences are identical across individuals), panel data methods (see, for example, Hsiao) can be used to simplify the estimations, and to increase the efficiency of the estimators. Further, if it is believed that consumers do not have precise and/or accurate information about the proportions of fat and cholesterol in various foods, or of the recommended limits on consumption of fat and cholesterol, and if a long panel of data such as the above is available, non-parametric tests based on the axioms of revealed preference (Afriat 1967, 1976; Alston and Chalfant 1991a, 1991b; Chalfant and Alston; Diewert and Parkan; Varian 1982, 1983, 1985) can be used to examine which consumers effectively enforce which constraints.

The following, final, chapter summarizes and concludes the dissertation.

V Conclusions

It was argued that consumers' health awareness in food consumption is reflected in behavior through the imposition of additional implicit constraints on consumption. Existing approaches considered were the household production approach and the "taste change" approach. The household production approach while empirically useful in other contexts was judged to be inapplicable in the context of this study. The "taste change" approach was found to be wanting theoretically and also the conclusion from logical extension of the argument that "tastes have changed" is that tastes will finally become stable again at which point demand behavior is explained by prices and income alone. A further implication of this approach is that prices and expenditures explain all of the cross-sectional variation in food consumption data, whereas it is maintained here that food composition accounts for some of the differences in consumer expenditures for food; this assumption is justified by the results presented in the introductory chapter. The proposed model yields consumer demands for foods which are determined by prices, outlay, the composition of foods, and implicit constraints on consumption of food components such as fat, cholesterol, and other components of food. The model also permits incorporation of demographic or other

systematic differences in preferences. As an extension of food demand analysis, the model meets the challenge posed by Blaylock and Smallwood.

Standard restrictions on demands implied by consumer theory were derived for the proposed model. The procedure for deriving demand share equations from a specified indirect utility function was demonstrated using the translog indirect utility function. Elasticity formulae were computed for individual and (per capita or mean) aggregate demands. Data necessary for estimating the model proposed were identified as cross-section or, if possible, long panel data with information about prices and quantities of various foods purchased, income, and perceptions of, in the example chosen, fat and cholesterol in foods consumed. Such data would enable tests of whether food consumption patterns are fully explained by prices and income alone, and their collection is strongly encouraged. (No such data set is currently available.) If the null hypothesis is rejected, i.e. if consumers effect a desire for good health by limiting fat and/or cholesterol consumption as modeled here, then models of taste change are misspecified since they imply that, in the cross-section, prices and expenditures will completely explain all variation in the data.

The main implications of the study are that *perceived* proportions of fat, cholesterol, and other relevant components in foods help determine consumer demands for foods, along with prices and outlay. Thus, efforts to improve the accuracy of perceptions about the composition of foods and the recommended intakes of various components of food are crucial to ensuring that consumers will be able to make well

informed choices about food consumption given health concerns. Additionally, if perceptions are more or less accurate, changes in the actual fat and cholesterol contents of foods will have predictable effects on demands. These effects may be determined through the suggested model.

Finally, this dissertation focussed on meat demand analysis due to a need, identified in the introductory and second chapters, for a theoretically sound and empirically elegant approach. In the discussion, it was argued that health concerns of consumers affect their spending patterns but do not necessarily alter their preferences. The proposed model can easily be extended to the many areas of economic analysis which manifest this property, such as consumer behavior in the presence of concerns about food safety, sustainable economic development, and environmental destruction. Such extensions would allow tests of whether these concerns have affected consumer preferences, or whether they have changed consumer spending patterns simply to seek to reduce the use of pesticides, inorganic fertilizers, and fluorocarbons among the possible examples.

References

- Afriat, S. N. 1967. The construction of utility functions from expenditure data. *International Economic Review* 8: 67–77.
- . 1976. *The combinatorial theory of demand*. London: Input-Output Publishing Co.
- Alston, J. M., and J. A. Chalfant. 1991a. Can we take the con out of meat demand studies? *Western Journal of Agricultural Economics* 16: 36–48.
- . 1991b. Unstable models and incorrect forms. *American Journal of Agricultural Economics* 73: 1171–81.
- Amemiya, T. 1985. *Advanced econometrics*. Cambridge: Harvard University Press.
- Barnett, W. A. 1977. Pollack and Wachter on the household production function approach. *Journal of Political Economy* 85: 1073–82.
- . 1979. Theoretical foundations for the Rotterdam model. *Review of Economic Studies* 46: 109–30.
- . 1981. *Consumer demand and labor supply*. New York: North-Holland.
- Becker, G. S. 1965. A theory of the allocation of time. *Economic Journal* 75: 493–517.
- . [1975] 1983. *Human capital: a theoretical and empirical analysis, with special reference to education*. 2d ed. Midway reprint. Chicago: University of Chicago Press.
- Blackorby, C., D. Primont, and R. R. Russell. 1978. *Duality, separability, and functional structure: theory and economic applications*. New York: Elsevier North-Holland.

- Blaylock, J. and D. Smallwood. In this issue. *Journal of Agricultural Economics Research* 45, no. 1: 1–2.
- Bobst, B. W., R. E. Branson, R. C. Haidacher, E. E. Jacobs, R. Raunika, B. J. Senauer, D. Smallwood, D. S. Tilley, and L. R. de Zapata. 1987. Data sources for demand analysis. In *Food demand analysis: problems, issues, and empirical evidence*. Ed. R. Raunika and C. Huang. Ames, Iowa: Iowa State University Press.
- Braschler, C. 1983. The changing demand structure for pork and beef in the 1970s: implications for the 1980s. *Southern Journal of Agricultural Economics* 15: 105–10.
- Brown, D. J., and L. F. Schrader. 1990. Cholesterol information and shell egg consumption. *American Journal of Agricultural Economics* 72: 548–55.
- Capps, O., Jr., and J. D. Schmitz. 1991. A recognition of health and nutrition factors in food demand analysis. *Western Journal of Agricultural Economics* 16: 21–35.
- Chalfant, J. A., and J. M. Alston. 1988. Accounting for changes in tastes. *Journal of Political Economy* 96: 391–410.
- Chang, H. S., and H. Kinnucan. 1991. Advertising, information, and product quality: the case of butter. *American Journal of Agricultural Economics* 73: 1195–203.
- Chavas, J.-P. 1983. Structural change in the demand for meat. *American Journal of Agricultural Economics* 65: 148–53.
- Choi, S., and K. Sosin. 1990. Testing for structural change: the demand for meat. *American Journal of Agricultural Economics* 72: 227–36.
- Christensen, L. R., D. W. Jorgenson, and L. J. Lau. 1975. Transcendental logarithmic utility functions. *American Economic Review* 65: 367–83.
- Deaton, A. S. 1981. Theoretical and empirical approaches to consumer demand under rationing. In *Essays in the theory and measurement of consumer behavior*. Ed. A. S. Deaton. Cambridge: Cambridge University Press.
- . 1992. *Understanding consumption*. London: Clarendon Press.

- , and J. Muellbauer. 1980. *Economics and consumer behavior*. Cambridge: Cambridge University Press.
- . 1981. Functional forms for labor supply and commodity demands with and without quantity restrictions. *Econometrica* 49: 1521–32.
- Diewert, E., and C. Parkan. 1978. Test for consistency of consumer data and nonparametric index numbers. Working Paper 78–27, University of British Columbia, Canada.
- Driscoll, P. J. 1994. When flexible forms are asked to flex too much. *Journal of Agricultural and Resource Economics* 19: 183–196.
- Eales, J. S., and L. J. Unnevehr. 1988. Demand for beef and chicken products: separability and structural change. *American Journal of Agricultural Economics* 70: 521–32.
- Fraza, Betsy, and Linda Cleveland. 1994. Diet-health awareness about fat and cholesterol—only a start. *Food Review* (January–April): 15–22.
- Gallant, A. R., and D. W. Jorgenson. 1979. Statistical inference for a system of simultaneous, nonlinear, implicit equations in the context of instrumental variable estimation. *Journal of Econometrics* 11: 275–302.
- Gorman, W. M. [1956] 1980. A possible structure for analyzing quality differentials in the egg market. Reprinted. *Review of Economic Studies* 47(1980): 843–56.
- . 1976. Tricks with utility functions. In *Essays in economic analysis*. Ed. M. J. Artis and A. R. Nobay. Cambridge: Cambridge University Press.
- Graaff, J. de V. 1948. Rothbarth's 'virtual price system' and the Slutsky equation. *Review of Economic Studies* 16: 91–5.
- Hsiao, C. 1986. *The analysis of panel data*. Cambridge: Cambridge University Press.
- Jackson, W. A. 1991. Generalized rationing theory. *Scottish Journal of Political Economy* 38, no. 4: 335–42.
- Jones, J., and J. Weimer. 1980. A survey of health-related food choice. *National Food Review* (Fall): 16–8.

- Jorgenson, D. W., L. J. Lau, and T. Stoker. 1982. The transcendental logarithmic model of aggregate consumer behavior. In *Advances in econometrics*, Volume 1. Ed. R. Basman and G. Rhodes. Greenwich, Connecticut: JAI Press.
- Kirman, A. P. 1992. Whom or what does the representative individual represent? *Journal of Economic Perspectives* 6: 117–36.
- Lancaster, K. J. 1966. A new approach to consumer theory. *Journal of Political Economy* 74: 132–57.
- Lewbel, A. 1989. Exact aggregation and a representative consumer. *Quarterly Journal of Economics* 104: 622–33.
- . 1988. Exact aggregation, distribution parameterizations, and a nonlinear representative consumer. In *Advances in econometrics*, Volume 7. Ed. G. F. Rhodes and T. B. Fomby. Greenwich, Connecticut: JAI Press.
- . 1987. Fractional demand systems. *Journal of Econometrics* 36: 311–37.
- . 1985. A unified approach to incorporating demographic or other effects into demand systems. *Review of Economic Studies* 52: 1-18.
- McGuirk, A. M., and P. J. Driscoll. 1993. R²: don't be turned on by a tight fit. Staff Paper SP-93-10. Blacksburg, Virginia: Department of Agricultural and Applied Economics, Virginia Polytechnic Institute and State University.
- Neary, J. P., and K. W. S. Roberts. 1980. The theory of household behavior under rationing. *European Economic Review* 13: 25-42.
- Pollack, R. A. 1971. Conditional demand functions and the implications of separable utility. *Southern Economic Journal* 37: 423-433.
- , and M. L. Wachter. 1975. The relevance of the household production function and its implications for the allocation of time. *Journal of Political Economy* 83: 255–77.
- Pollack, R. A., and T. W. Wales. 1981. Demographic variables in demand analysis. *Econometrica* 49: 1533-1551.
- . 1992. *Demand systems specification and estimation*. New York: Oxford University Press.

- Putler, D. S., and E. Frazao. Consumer awareness of diet-disease relationships and dietary behavior: the case of dietary fat. *Journal of Agricultural Economics Research* 45, no. 1: 3–17.
- Rothbarth, E. 1941. The measurement of changes in real income under conditions of rationing. *Review of Economic Studies* 8: 100–107. (Cited in Tobin and Graaff.)
- Samuelson, P. A. [1947] 1983. *Foundations of Economic Analysis*. Enl. ed. Cambridge, MA: Harvard University Press.
- . 1960. An extension of the Le Chatelier principle. *Econometrica* 28: 368–79. (Also available as Ch. 42 in *The collected scientific papers of Paul A. Samuelson*. Ed. J. E. Stiglitz. Cambridge: M. I. T. Press, 1966.)
- Silberberg, E. 1990. *The structure of economics*. 2d ed. New York: McGraw Hill.
- Stoker, T. M. 1993. Empirical approaches to the problem of aggregation over individuals. *Journal of Economic Literature* 31: 1827–74.
- Stigler, G. J., and G. S. Becker. 1977. De gustibus non est disputandum. *American Economic Review* 67: 76–90.
- Takayama, A. 1985. *Mathematical economics*. New York: Cambridge University Press.
- Tobin, J. 1952. A survey of the theory of rationing. *Econometrica* 20: 512–53.
- , and Houthakker, H. 1951. The effects of rationing on demand elasticities. *Review of Economic Studies* 18: 140–53.
- Varian, H. R. 1982. The nonparametric approach to demand analysis. *Econometrica* 50: 945–73.
- . 1983. Nonparametric tests of consumer behavior. *Review of Economic Studies* 50: 99–110.
- . 1984. *Microeconomic analysis*. 2d ed. New York: W. W. Norton.
- . 1985. Nonparametric analysis of optimizing behavior with measurement error. *Journal of Econometrics* 30: 445–58.

———. 1987. *Intermediate microeconomics*. New York: W. W. Norton.

Wohlgenant, M. K. 1985. Estimating cross elasticities of demand for beef. *Western Journal of Agricultural Economics* 10: 322–29.

Vita

Srinivasa Srirama Satyanarayana Murthy Kambhampaty was born in Visakhapatnam, India on 28 October 1964. He received the Bachelor of Engineering (Electrical and Electronics Engineering) degree from Andhra University in his hometown in May 1985. His interest in public policy led him to advanced studies for which he received the Master of Urban Affairs (August 1990), the Master of Arts in Economics (May 1994) and, with the completion of this dissertation, the Doctor of Philosophy in Agricultural and Applied Economics (December 1994) all from Virginia Polytechnic Institute and State University.

A handwritten signature in black ink, reading "Srinivasa Srirama Satyanarayana Murthy Kambhampaty". The signature is written in a cursive style with a large, decorative flourish at the end.