CHAPTER ONE
INTRODUCTION

Over the past decade, much research in mathematics education has focused on reform efforts largely driven by reaction to informative documents developed by the National Council of Teachers of Mathematics [NCTM]. The two major documents, The Curriculum and Evaluation Standards for School Mathematics (1989) and The Professional Standards for Teaching Mathematics (1991), have influenced many discussions about mathematics teaching and learning among classroom teachers, administrators, researchers and other mathematics educators. One driving question is: What types of classroom environments stimulate students to become more mathematically powerful learners? The type of classroom environment described in the Professional Standards is a mathematical community in which conjecturing, investigating, reasoning, and problem solving are key components. These activities are intended to engage students in the development of mathematical ideas as they attempt to make sense of their environment. My study aims to provide information about one secondary mathematics teacher's experiences in reforming his classroom environment to be consistent with NCTM's recommendations.

One tenet of current mathematics reform recommendations focuses on communication in the mathematics classroom. Communication that compels students to interact as a part of the community of the mathematics classroom is a goal. Because we develop understandings throughout our lives as we interact with others, a classroom that enhances communicative interactions seems profitable. Imagine a mathematics classroom in which students' ideas are solicited and valued as important contributions to developing an understanding of concepts and problems. In this classroom, the teacher becomes a collaborating member and "the learning environment evolves as a result of interaction among the teacher and students as they engage in the mathematical content" (Simon, 1995, p. 133). Although in most traditional mathematics classrooms students are asked to individually solve exercises in a teacher-dominated environment, research across disciplines has shown the benefits of some type of cooperative arrangement within the classroom (Augustine, Gruber & Hanson, 1990; Lyman & Foyle, 1990; Slavin, 1991). An analysis of over 122 studies of different types of cooperative learning in mathematics "found cooperation considerably more effective than interpersonal competition or individualistic efforts in promoting achievement and productivity" (Davidson & Kroll, 1991, p. 362). These studies often focused on measuring achievement gains in terms of individual testing or examined comparisons of different cooperative methods.

An extensive body of research has focused on specific aspects of learning within groups in mathematics classes. Cohen's (1994) review of several studies examines elaborated discussion, task-related talk, the structure of groups and the effects on achievement, group roles, and different tasks. The cooperative learning literature has shown many positive effects of grouping using cooperative learning structures such as the jigsaw method, numbered heads together, think-pair-share, team learning, and co-ops (Johnson & Johnson, 1987; Graves & Graves, 1989; Kagan, 1988; Slavin, 1985). Results suggest that "healthy social development, with an overall balance favoring trust rather than distrust of other people, the ability to view situations and problems from a variety of perspectives, a meaningful sense of direction and purpose in life, and an awareness of mutual interdependence with others" develops through cooperative interaction (Johnson, 1991, p. 34). Indeed, the NCTM Curriculum Standards and Teaching Standards advocate classroom environments that actively involve students in discussion of mathematical concepts in small-groups, as well as in whole-class discussions, to further their understanding of mathematical concepts.

Utilizing cooperative groups is one way to generate discussion in a classroom. Much of the research, however, does not provide a holistic picture of interactions among students and teachers in an environment that encourages not only cooperation, but discussion of mathematical thinking. In the current environment of reform in mathematics classrooms, it is important to examine such interactions. This study, in particular, focuses on one teacher's efforts to involve students in discussions of mathematics concepts.

Rationale for the Study

Mathematics teachers at all levels are being asked to reform their instruction by creating learning environments that rely heavily on collaborative communications among class members. The NCTM documents provide a vision of classrooms that are communities in which mathematical reasoning is shared to aid in problem-solving and connecting of mathematics concepts. The challenge for teachers is to develop and maintain strategies to effectively create such environments (Ball, 1993; Silver and Smith, 1996). Focusing on the verbal interactions among members of a secondary mathematics class can extend research concerning sharing of discussion by illustrating the possibilities and challenges encountered by a teacher when attempting to enhance collaboration through discussion. This study attempts to contribute by asking: What are the patterns of discussion evident in a secondary classroom in which the teacher embraces reform-oriented ideas? What challenges does the teacher encounter in attempting to change the patterns of discussion?

Recent research results, completed in elementary classrooms informed by constructivist ideas, have provided a view of patterns of interactions in classrooms that offered varied opportunities for student discussion of ideas (Bauersfeld, 1992; Pirie & Kieran, 1992; Steffe & Wiegel, 1992; Wood, Cobb, Yackel & Dillon, 1993). The goals of these studies were "to simply come to a better understanding of what might have happened in that classroom in particular and, on the other hand, to develop a conceptual framework that might allow us to cope with the complexity of classroom life in a more general way" (Cobb et al., 1992, p. 100). These goals should also be examined in the context of the secondary classroom where social norms are largely traditional and students are often confronted with specialized mathematics content. When examining secondary mathematics classrooms in the current system, many seem to be introducing more non-routine problems and activities, but there are few widespread examples of ones that are engaging students in student-generated discussions of solutions to problems. Many still are largely teacher-directed following traditional patterns of discussion (Schratz and Mehan, 1993). Researchers have called for further examination of classrooms in which teachers are exploring current reform recommendations. The importance of observing a classroom setting lies in examining teachers' actual daily efforts of engaging students in mathematical discussions in classrooms. As Simon (1995) notes, research completed within the daily conditions of the classroom such as "pressure to teach from a preset curriculum" and "to cover particular mathematical content" (p. 142) can inform us of the context dependence of teachers' efforts.

What happens when mathematics teachers attempt to change their strategies to foster effective interaction within their classrooms? Orton (1994) suggests that any attempt to become a more effective teacher and provide a more constructive classroom environment will be hard work. Other recent studies of mathematics teachers involved with reform have reflected this idea by revealing the struggles inherent in implementing previously untried strategies (Gregg, 1995; Haimes, 1996; Lloyd, 1996; Wilson & Lloyd, 1997). The complexity of classroom environments and teachers' sense of efficacy when relying on traditional, tried practices appear to compound the difficulty of reform.

The Emerging Research Questions

When planning this study, I drew on my experiences as a high school teacher and supervisor of student teachers. Throughout my teaching and study in the graduate program, I considered questions about mathematics teaching and learning. I reflected on my former high school students and wondered about the learning experiences I had tried to provide for them. Although my mathematics classroom teaching began in a fairly traditional manner, it was not long before I realized that my explanations of concepts were neither necessary nor sufficient. The publication of the NCTM Standards documents lent some validity to my feelings that students must somehow be more actively involved in the development of mathematics concepts and understandings. Through much reading and reflection, the ideas that are the foundation of NCTM's recommended reforms began to transform my own conceptions of what it meant to have effective mathematics classroom instruction. The image that dwelled in my mind was a classroom in which students wholeheartedly discussed their ideas as they came to terms with emerging mathematical concepts. In order for this picture to develop, collaboration had to become central in the classroom. That is, as teachers we would need to structure tasks and discussions of concepts in a manner that encouraged and required students to collaborate. Students would need to feel that their ideas and questions were valued, so that sharing their reasoning would become a classroom norm (Cobb, Yackel, Wood; 1993).

To address these issues, I conducted a qualitative case study with procedures adapted from those of Lincoln and Guba (1985), Taylor and Bogdan (1984), and Stake (1988) that examined the experience of a secondary mathematics teacher during his efforts to facilitate discussions in a secondary Algebra class. Mr. Lester (a pseudonym) was a veteran mathematics teacher who had established an alignment with Standards-like reforms by participating in the local mathematics education community. In particular, Mr. Lester expressed the desire to create a classroom environment that actively involved students in the discussion of ideas. One of his Algebra classes was observed during the Spring semester of the 1995-96 school year; and he participated in interviews during the Spring and Summer of 1995-96 and the Fall of 1997.

The purpose of this study was to document and analyze whole-class discussion patterns in Mr. Lester's class, as well as to investigate his attempts to implement current reform recommendations. The following questions guided the study:

1) What patterns of verbal interactions concerning the construction of mathematical knowledge exist in class discussions?

2) What is the emerging role of the teacher in attempting to facilitate mathematical discussions in a secondary algebra class?

3) What issues does the teacher confront in attempting to implement mathematical discussions in the classroom?

As the study progressed, the struggles that Mr. Lester encountered in structuring the classroom environment and revising his role, became a focus in the study.

Outline of the Dissertation

A review of the related literature is presented in Chapter Two. The review focuses on the envisioned mathematics classroom environment, constructivist influences on mathematics education reform, and challenges for teachers involved in reform. In Chapter Three I provide a description of the methodology that guided the study, as well as an account of my journey through the research process and how this study evolved and was shaped by my observations in Mr. Lester's classroom. Results related to the previously stated research questions are presented respectively in Chapters Four, Five, and Six. Finally, Chapter Seven gives a short summary of results along with implications of this study.

CHAPTER TWO
REVIEW OF THE RELATED LITERATURE

This chapter presents a review of literature that motivates and addresses the main questions of this study. Specifically, I focus on three main topics: reform-oriented mathematics classroom environments, constructivist influences evident in mathematics education reform, and the challenges facing teachers who involve themselves in enacting reform. First, I provide a short general discussion of the envisioned classroom according to reform recommendations. Second, I provide an overview of several constructivist epistemologies to form a basis for the challenge of current reforms. Third, I examine patterns of discussion in traditional classrooms as compared to those being developed in non-traditional settings and the rationale for changing discussion. Fourth, I consider the changing mathematics classroom environment by examining shifts in the roles of teachers and students and the challenges for mathematics teachers, focusing on research results from similar studies. Finally, the general research questions offered in the first chapter are examined within the framework of this review.

General Discussion of Reform in Mathematics Classrooms

Implementing and examining reform in mathematics classrooms is a continual process. Educators involved in reform efforts often attempt to or are encouraged to shift curriculum and instruction in ways that propose to enhance student learning with increased mathematical understanding. Traditionally, practitioners have viewed reform efforts which are largely based on theoretical frameworks, as dictated by researchers who lack practical experience. As a result, many teachers have either rejected or found little support for these efforts within their classrooms. Eisenhart (1991) suggests that there is a tendency for theory-based research to produce scholarly discourse that only functions well within the academic discipline and the "conclusions produced by the logic of theoretical discourse about educational practice, for example, are often neither practical nor helpful in day-to-day practice"(p. 206). New visions for reform often are discussed in a detached manner at the university level, emphasize policies in a step-by-step procedural manner, and appear to rely on one-time professional development opportunities (McDonald, 1996).

The NCTM Standards, a basis for much current reform discussion in mathematics education circles, provide a basis for reform efforts that can potentially generate collaboration between educators at all levels. An overarching theme in the Standards is the shift from a traditional mathematics curriculum inundated by memorization of isolated facts and procedures to one that emphasizes conceptual understanding of major ideas that connect throughout the development of mathematical knowledge and understandings. At the heart of this vision of a quality mathematics program is the conviction that:

> If students are exposed to the kinds of experiences outlined in the Standards, they will gain mathematical power. This term denotes an individual's ability to explore, conjecture, and reason logically, as well as the ability to use a variety of methods effectively to solve non-routine problems. (NCTM, 1989, p. 5)

This statement implies that mathematics classrooms must be places where non-routine problems are posed regularly and instructors immerse students in a classroom environment in which knowledge is viewed as a product of students' constructions as they interact with each other within the classroom. The type of classroom envisioned in the Professional Standards is one that shifts toward the development of mathematical communities, the use of logic and evidence as verification, the sharing of mathematical reasoning, the encouragement of conjecturing, investigating, and problem solving, and connecting mathematics at all levels. These shifts imply that communication in the classroom become based on student development of statements, questions, and ideas.

Since the widespread dissemination of the Standards documents in the early 90's, many seminars, conferences, and in-service activities have been aimed at aiding classroom teachers in producing this type of environment. There is still much to be accomplished. In particular, more concrete strategies for developing this environment are needed for the enhancement of secondary school classes. The Standards' recommendations for secondary classrooms suggest an underlying assumption that students entering grade nine will have experienced math in a K - 8 Standards context (1989). Currently, it is unlikely that most high school students have experienced previous mathematics classrooms that have shifted towards a Standards environment. Although interest in communication in classrooms has grown, many students are products of traditional procedure-driven mathematics instruction.

The challenge to revise classroom environments as presented in reform-oriented recommendations is largely based on ideas drawn from constructivist principles of teaching and learning. The next section outlines three main ideas that form a basis for challenging the mathematics education community to impact how we teach mathematics.

Constructivist Influences on Mathematics Education Reform

The ideas supporting the individual and social construction of knowledge in the learning of mathematics are rooted in the theories of cognitive constructivism. For mathematics learning this "implies that the child must actively construct its own foundation and its own mathematics" and that "the activity of coming to know the world is fundamentally creative and occurs repeatedly for each child" (Richards & von Glasersfeld, 1980, p.29). This is contradictory to the behaviorist position that observable behaviors constitute what a child learned. Bruer (1993) outlines that behaviorist learning theory as applied to education emphasized the following arrangement of the classroom environment:

> Teachers would present lessons in small, manageable pieces (stimuli), ask students to give answers (responses), and then dispense reinforcement (preferably positive rather than negative) until their students became conditioned to give the right answers. Behaviorism is a simple, elegant scientific theory that has both methodological and intuitive appeal. But humans are more complicated than behaviorism allows. (p. 8)

In contrast, a constructivist view implies that simply responding with correct answers is insufficient to develop meaningful mathematics knowledge. This view, however, does not assert that there exists no mathematical knowledge independent of each learner. As Sinclair (1990) notes, "Our children do not have to reinvent the wheel - they can begin to conceptualize the intricate properties of wheels as they exist in our society" (p. 23).

Although students can develop conceptions that allow them to construct understanding, they are not demanded to reinvent already existing knowledge for no purpose. There is purposeful preexisting knowledge; that is, prior knowledge that can be a basis for new understandings. Ideas, thoughts and skills previously acquired will impact new knowledge that can be detected or processed. Previous knowledge will provide for activating the generation of new knowledge (Flavell, Miller, & Miller, 1985). The information-processing model in cognitive development represents knowledge as information that enters the mind and is processed for either memory or further use. Flavell describes an episode of information processing as "retrieving or assembling a complex plan or strategy for solving a problem, attempting to execute that plan or strategy, revising it if it proves inadequate, and so forth" (p. 9). Information processing in this view is consistent with some of the stances on mathematics learning evidenced in the Standards. That is, learning mathematics should be a process in which students are involved in solving problems through acting on information, creating procedures, and revising their thinking as they explore various possibilities.

In the learning and teaching of mathematics, what does it mean for people to "do" mathematics? In the constructivist view of learning mathematics, the "doing" is termed construction. The NCTM Professional Standards (1991) states:

> Learning is an active, dynamic, and continuous process that is both an individual and a social experience. Children are naturally inquisitive and have a desire to learn. Their early experiences reflect the excitement of discovery. In school, however, limitations of time, place, and perceptions often constrain what is natural as children encounter environments that are not responsive to them as learners. (p. 144)

Since the proliferation of the NCTM Standards documents among mathematics educators, reform-oriented practices based on constructivist principles have gained support. Educators and, to a greater extent, textbook publishers have staked their claims to constructivist, Standards based classrooms. Noddings (1990) notes that constructivists in mathematics education contend that "acceptance of constructivist premises about knowledge and knowers implies a way of teaching that acknowledges learners as active knowers" (p. 10). She also outlines that, even though there are differences in current constructivist views, constructivists basically agree on the following precepts: All knowledge is constructed; There exist cognitive structures that are activated in the processes of construction; Cognitive structures are under continual development; and, Acknowledgment of constructivism as a cognitive position leads to the adoption of methodological and pedagogical constructivism. These ideas form a basis for continued efforts that challenge educators to reform mathematics classrooms.

Ideas Grounding Reform

Traditional mathematics classrooms often follow the pattern of direct instruction as follows: the teacher reviews previous introduced topics, the teacher introduces new topics, the students practice exercises relating to the concept, and the students complete seatwork. Confrey (1990) suggests that there is "an increasing amount of evidence that direct instruction may not provide an adequate base for students' development and for student use of higher cognitive skills" (p. 107). Direct instruction is aligned with views of learning and teaching that are centered on response and knowledge acquisition as opposed to knowledge construction. Within this view the student is seen as passive and the environment is controlled by rewards for correct responses and reprimands for incorrect ones (Mayer, 1992). In knowledge acquisition, "the goal of instruction is to increase the amount of knowledge in the learner's repertoire so that learning outcomes can be evaluated by measuring the amount of knowledge acquired" (p. 407).

In contrast, theories of teaching and learning based on constructivist principles view students as active learners, mathematical learning as the active manipulation of meanings, and recommend learning environments that challenge students' thinking (Davis, Maher, & Noddings, 1990). Among those scholars often cited in the literature are Piaget, Vygotsky, and Bruner. The contributions of each to the underlying constructivist principles inherent in many mathematics education reforms is described below.

Active Production of Knowledge

Piaget's account of learning provides for the active production of knowledge by individual learners. Nesher & Kilpatrick (1990) state that Piaget's contributions are essentially built on the following basic ideas:

> Knowledge derives from the adaptation of the individual to the environment.
> Knowledge can be traced to the individual's way of acting with objects and dealing with situations.

When acting on objects, individuals develop different kinds of knowledge, depending on the kind of abstraction they make.

These positions largely concentrate on the interaction between an individual and an object in its environment. In classroom instruction, the goal of the teacher is to provide experiences through which students can gain purposeful knowledge that can later be applied in other situations. This thought complements a skeletal outline of Piaget's conceptions of assimilation and accommodation through which conceptual learning occurs. Learners organize their experiences into schemata and patterns which are available for application to new situations. In any environment then, "when existing schemata are adequate to the new environment, the [learner] assimilates the new experience to the existing schemata; when existing patterns are inadequate, the [learner] may adapt the schemata and thus accommodate the new experience" (Sainsbury, 1992, p. 110). These schemata and patterns are those constructed by the learner in an attempt to develop knowledge.

The evidence of these ideas in mathematics education reform arise in the call for classrooms that provide opportunities for students to actively participate in the environment. One explanation of this constructivist approach suggests the following: Information does not simply happen to a person, it requires self-generated mental activity by the person to make sense of his environment. Constructivism implies that learning is an interpretive activity and that rather abrupt changes in behavior might be expected when a person achieves insight (Zimmerman, 1981). When students actively participate, social interaction also plays a role in the classroom.

Social Interaction

Within the classroom, social interactions among all participants play a role in the learning that occurs. This view can be seen in Vygotskian ideas in which interactions play a major role. Blanck (1990) states that basic assertions of this theory are: "Mental activity is uniquely human. It is the result of social learning, of the interiorization of social signs, and of the internalization of culture and of social relationships" (p. 44). The assumptions in this theory are sometimes viewed as offering a perspective different from the Piagetian view of individual construction of knowledge. Whereas the Piagetian view deals largely with the individual, the Vygotskian view holds that interpersonal interactions aid in generating individual knowledge. This perspective can be seen in an interpretation by van Oers (cited in Steffe & Tzur, 1994):

> One of the basic tenets of the Vygotskian approach to education is the assumption that individual learning is dependent on social interaction. However, it should be clear from the outset that this is not merely a statement of correlation between individual learning and social context. This thesis should be interpreted in its strongest possible form, proposing that the qualities of thinking are actually generated by the organizational features of the social interaction. (p. 29)

This interpretation implies that the social organization of mathematics classrooms plays a major role in the learning of students.

The social interactions suggested for classrooms based in recent reforms give rise to the application of Vygotsky's ideas concerning assisted performance. Through interaction with a more capable peer or adult, a student "can perform at a developmentally more advanced level when assisted than when acting alone, and this difference in level of performance suggests that a learner has a range of potential rather than some fixed state of ability" (Smagorinsky, 1995, p. 195). The connections between individual knowledge and knowledge that is socially constructed becomes important. Driver and Scott (1995) remind us that a main issue of importance "is that children reorganize and reconstruct experiences of their physical and social environment" [leading to a similarity in the Vygotskian and Pigetian perspective in] "recognizing that the child cannot be a passive recipient of knowledge" (p. 28). Students in mathematics classrooms, instead of relying solely on individually constructed knowledge, should be compelled to share their constructions with others. Bruner's work lends support to these notions.

Shared Meanings

The work of Bruner, spanning decades, provides another perspective from which to view current reforms in mathematics education. As we attempt to provide students with meaningful learning environments, these words of Bruner's seem appropriate to consider: "Our culturally adapted way of life depends upon shared meanings and shared modes of discourse for negotiating differences in meaning and interpretation" (1990, p. 13). The NCTM Standards documents support this statement in its emphasis on classroom discourse and the sharing of mathematical ideas. Learning in this context is collaborative and necessarily relies on the sharing of knowledge among members of the class. This context is recommended as children first begin to learn about mathematical ideas in school with the notion that shared mathematical discourse will continue into the higher grades. Bruner (1986) proposes this foreshadowing of reform:

> I think that its central technical concern will be how to create in the young an appreciation of the fact that many worlds are possible, that meaning and reality are created and not discovered, that negotiation is the art of constructing new meanings by which individuals can regulate their relations with each other. It will not, I think, be an image of human development that locates all of the sources of changes inside the individual, the solo child. (p. 149)

The implication for learning in mathematics classrooms is that created meanings by individuals must be negotiated with others to enhance knowledge within the community of learners. The idea of negotiation is also stated in the Professional Standards (1991) as follows, "Students' learning of mathematics is enhanced in a learning environment that is built as a community of people collaborating to make sense of mathematical ideas" (p .58). Some educators may agree philosophically, but express that in practice these ideas often do not come to fruition. Bruner's much quoted discovery learning has been touted as unjustifiable by some due to misunderstandings in how it applies to classroom instruction. Its emphasis on the "active recreation of knowledge" and objective of "generating a deeper understanding" of concepts makes it possible that a classroom operating under constructivist beliefs "looks like discovery learning all over again" (Orton, 1994, p. 43). Whether the collection of beliefs labeled constructivism are considered as intertwined with Bruner's conceptions or not, either should lead to purposeful activity in our mathematics classrooms. Orton goes on to say that "a closer look at classroom practice implied by the acceptance of constructivist views is now relevant" (p. 43).

Classroom practice that is in the process of change within the current environment of mathematics reform implies that classroom patterns of interaction and the rationale for changing discussion patterns also be altered. These are discussed in the next section.

The Envisioned Mathematics Classroom

Changing the mathematics classroom towards an environment as envisioned in the Standards will require that patterns of interactions within the classroom be reformed. That is, the ways in which teachers and students act and react towards each other need to be examined. The envisioned classroom deviates from traditional patterns of interaction in a way that challenges teachers and students to become engaged in joint mathematical activity.

Classroom Patterns of Interaction

When examining the daily happenings in classrooms, a regularity in the interactions that occur is often noticeable. Traditionally, verbal interactions in many classrooms are restricted between the teacher and one student at a time. The setting is dominated by teacher talk (Flanders, 1973) and teachers often disseminate knowledge in classes that structurally discourage interaction (Brooks & Brooks, 1993). This phenomenon is especially true in mathematics classrooms in which the school mathematics tradition (Cobb, Yackel, & Wood, 1993) perpetuates the view that concepts to be learned are accepted by all members of the class as given by the teacher with little discussion.

Research results indicate that teachers monopolize communication in the classroom, dominate classroom discussion, and maintain a basic structure in classrooms that heavily rely on teacher-student recitation (Good & Brophy, 1987). If mathematics educators want to shift from the traditional mathematics classroom environment, then they must rely upon interactions among students and teachers to a greater extent. The patterns that occur may look different than those traditionally determined. How can examining the patterns of interaction in mathematics classrooms aid us in assessing current reform recommendations? Patterns in classrooms are studied with the interest of making classroom processes understandable. Voight (1995) comments that "we cannot improve the classroom microculture in the same way that we can change the mathematical curriculum" since each classroom culture depends on particular conventions and norms that may be difficult to change. Instead, we may think of the change as an evolution and thus "it is helpful to understand the regularities and dynamics of the processes within the classroom life" (p. 164). The network of routines and obligations within the classroom are described as patterns of interaction and are considered as regularities interactively constituted rather than as fixed rules (Voight, 1994). That is, each classroom is made up of contributing members who jointly construct the emerging patterns of interaction.

Mathematics classrooms in the United States have exhibited well-established patterns over the years. The term traditional will be used to describe mathematics classrooms that exhibit these well-established patterns. Barnes (1992) cites a summary of American classrooms, one as early as 1860, that shows the predominance of question-and-answer methods that are teacher-controlled. The traditional classroom is anchored in this "typical adult-child dyadic view of teaching and learning [that] has de-emphasized student-student relationships in the classroom" and has relied on a system of instruction that emphasizes teacher lectures and seatwork done by students individualistically (Johnson, 1991, p. 31). Through the lack of interaction or discussion in such a classroom, even when students are given the chance to participate in answering questions, the teacher's perspective often continues to dominate. The teacher's dominance is seen in a view presented by Hargreaves (cited in Sainsbury, 1992):

> The teacher defines the situation with reference to her goals and role and this defines appropriate roles for the pupils. In general, they are expected to conform to her definitions in such matters as acceptable style and content of talk and are evaluated according as they succeed in so conforming ... it is difficult to discover the pupils' perspective on all this, as responses to questions will generally mirror the teacher-defined perspective. (p. 95)

The teacher's perspective becomes a major determining factor in the resulting classroom environment. What teachers believe about classroom discourse influences the types of discussion that occur within their classrooms (Cohen, 1990; Thompson, 1994; Wilson & Lloyd, 1997). Does the teacher rely heavily on her interpretations of tasks and solutions? Or, do students actively participate in the reasoning when discovering solution paths?

A widely used description of the traditional pattern in which the teacher's interpretations prevail, is the initiation-reply-evaluation sequence. In this pattern, students respond to ideas and questions that are mostly teacher-generated. The use of this sequence in questioning and presenting material is sometimes preferred in a lecture-only environment. However, Schratz and Mehan (1993) report that in using this technique, elicitations are often in the form of known information questions. That is, teachers use a turn-allocation procedure to "elicit information from students about topics for which they already know the answer" (p. 247). This interaction between a teacher and a student is also referred to as the funnel pattern in which an inappropriate response by the student results in either the teacher telling the student the answer or directing the student in a step-by-step manner to the desired answer (Wood, 1995, p. 213).

Numerous studies have shown that the widely utilized explain-practice instruction of the traditional classroom has failed to foster mathematics achievement (Lo, Wheatley, & Smith, 1994). Because mathematics educators are often interested in increasing students' abilities to explain their reasoning, the ubiquitous appearance of traditional environments in mathematics classrooms at all levels seems contradictory. On the other hand, it would be difficult to prescribe mandatory practices that would provide for non-traditional environments geared towards current reform recommendations being endorsed by many in the mathematics education community. The difficulty in effectively motivating students to participate in ways that enhance their mathematical reasoning raises questions about the appropriateness of some traditional methods and strategies of instruction in many mathematics classrooms. Many teaching methods can be utilized in one classroom in order to meet the needs of each student at any particular time; however, "constructivism would seem to argue against the widespread use of traditional methods because these are less successful in helping individual children" (Orton, 1994, p. 48). Schratz and Mehan (1993) feel that teachers can take a stand for alternatives as they comment, "Different pedagogical purposes require different interactional patterns" (p. 248). What are some different interactional patterns?

In conceptually-oriented classrooms, teachers act to focus students towards rich conceptions of situations, ideas, and relationships among ideas (Thompson et. al., 1994). Reasoning becomes shared in a community of discussion. This type of classroom environment would exhibit interactions in contrast to the traditional pattern. Voight's (1995) examinations of a reform-oriented second grade classroom describes a non-traditional interactional pattern in whole-class settings. He uses the term discussion pattern as a descriptor of the following:

> The students have solved the problem at hand during small-group work. Now, the teacher asks a student to report.
> The student declares a solution to the problem and explains it.
> The teacher contributes to the student's explanation by further questions, hints, reformulations, or judgments, so that a joint explanation or solution emerges and is taken as valid.
> The teacher asks other students for different ways of solving. The first phase starts again. (p. 181)

A noticeable contrast from the traditional elicitation pattern in whole-class settings is the initiation of solution paths and explanations by students rather than solely by the teacher. Although this does not imply that the teacher has no evaluative input into proposed solutions, the main thrust of meaning given to the problem situation is student generated. In addition, the teacher does not evaluate in the traditional sense of being the determiner of the correctness of the solution. Using student-initiated strategies and explanations in concert with teacher input gives way to discussion that is a whole-class negotiation in which the teacher jointly validates solution processes.

This non-traditional pattern is evident in Wood, Cobb and Yackel's (1993) description of the nature of whole-class discussion in their inquiry mathematics classroom project. Through episodes of classroom discussion involving the teacher and students, they illustrate patterns of interaction that are mutually constituted in two contexts. One context exemplifies the form of discourse when the discussion centers on different methods for doing problems, and the second context illustrates the nature of discourse that focuses on interpretations of the given task (p. 60). Following Voight's terminology in describing patterns, several possibilities to describe these illustrations are: teacher initiation-student response-class negotiation pattern, student initiation-presentation-negotiation pattern, exchange of viewpoints pattern, and teacher reduction of ambiguity pattern. Other studies of whole-class interactions have also suggested that non-traditional environments in which the teacher relinquishes a central role are more conducive to increasing student interaction. For example, Edwards and Westgate (1994) summarize the findings of several British studies of whole-class interactions in traditional and "progressive" classrooms. In the traditional setting, teachers talked the most, asked large numbers of questions eliciting brief answers, and most communication was limited in ways that made it relatively safe for those not wishing to participate officially in the lesson. Those classrooms that moved away from the frontal teaching pattern saw the teacher interacting more often with individual students, more pupil-pupil interaction, and more pupil-initiated interaction with the teacher; although the social interaction between students was often limited. In another study, Orton (1994) describes an example of whole-class teaching, called "conflict teaching" as an example of a successful experiment in constructivist approaches to teaching mathematics (p. 45). He outlines its characteristics as follows: The process begins with the exposure of difficulties and misunderstandings perhaps through diagnostic questioning. Then, small-group discussion occurs in which groups come to an agreed view that can be shared with the whole-class for further discussion. This is in many ways congruent to Voight's description of the discussion pattern of interaction.

What classroom patterns of interaction appear to promote the reform envisioned mathematics classroom environment? In Gregg's (1995) study of classroom interactions in an inquiry type mathematics class, the terms given to the observed patterns were: initiation-response-evaluation-echo pattern and response-evaluation-echo pattern. In this study the teacher began by posing a task with no pre-given instructions. A student then volunteered a solution. From this point, the whole-class became "a community of validators" by calling out agreement or disagreement. The teacher then helped in clarifying the solution by "echoing" the volunteer student's solution to facilitate the communication among class members. Discussion would ensue until the teacher reassumed the role as monitor of turn-taking and gave the floor to another student. Although the teacher in this environment maintains a central role, the teacher's authority is diminished in the sense that student ideas drive the direction of discussion. More evidence of teachers attempting to create similar environments may contribute to teachers' confidence in attempting to change the typical traditional pattern found in most mathematics classrooms.

In the context of this study, the focus on communication in classroom reform is foremost. The challenge of using whole-class discussion that involves students in developing understandings is considered in the next section.

Utilizing Discussion in Mathematics Classrooms

Why would mathematics teachers accept the challenge of utilizing reform-oriented strategies in their classrooms? It is important to consider possible reasons for creating a discussion-based mathematics classroom; that is, a classroom in which discussion follows non-traditional patterns. Traditional patterns of discussion can be found in mathematics classrooms that emphasize a procedural view of mathematics learning and teaching. The pattern of discussion in traditional classes exhibit the elicitation pattern that can be described in three phases (Voight, 1994):

(1) The teacher sets up an ambiguous task, the students offer different answers which the teacher evaluates preliminarily.
(2) If the students' contributions are too divergent the teacher guides the student towards one definite argument, solution, etc[1] Believing to help the students, the teacher asks small questions to elicit bits of knowledge.
(3) The teacher and the students reflect and evaluate what has been done (p. 287).

One main ingredient of this traditional sequence is the initiation and confirmation of meanings by the teacher. In this case, students often do not have the chance to direct their own thinking in the classroom. Many times, the third phase in this description is never reached.

Lo, Wheatley and Smith (1994) provide some possible reasons for considering the use of non-traditional discussion: Students can profit from explaining solutions to classmates in a whole-class setting; Students can profit from opportunities to make sense of other students' explanations; Students can profit from being challenged when explaining their methods; The negotiation of certain social norms and beliefs plays an important role in fostering mathematics learning; The need to communicate mathematical ideas can promote meaningful learning; Opportunities to communicate mathematics may foster positive attitudes about mathematics learning; and, Class discussion can provide opportunities for individual students to connect and integrate their mathematical knowledge. These strategies offer many alternatives to the traditional classroom environment. The use of whole-class discussions that provide opportunities for students to communicate their questions, ideas, and reasonings is a viable strategy for providing an environment in which students consistently interact in discussions aimed at developing negotiated mathematical meanings. The bulk of previous research examining such discussions has been completed at the elementary level (Cobb, Yackel & Wood, 1993; Gregg, 1995; Kamii, 1985; Lo, Wheatly & Smith, 1994).

A major research project (Cobb, Yackel, & Wood, 1993) completed in a second-grade elementary mathematics classroom provides a basis for examining the features of a discussion-based mathematics class. The general structure of the classroom was to first present a task to the class. Then, students worked in small groups (pairs in this case) to discuss possible solution paths. After this, the teacher orchestrated whole-class discussions of interpretations and solutions. One radical difference from most traditional mathematics classrooms is that there was no individual paper and pencil seatwork and no grading of written work. As a result of whole-class discussions, two levels of conversation can be noticed in the environment. The two levels were termed "talking about and doing mathematics" and "talking about talking about mathematics" (p. 24). Wood (1993) in examining the teacher's role writes that the "teacher's intention as she led the class discussion was to encourage children to verbalize their solution attempts" (p. 17). Thus, the nature of the discussion occurring in this class was also instrumental in establishing norms necessary for developing a setting in which the students felt safe to express their mathematical thinking.

The challenge for teachers in utilizing discussion of this type is to engage students on a regular basis in whole-class discussions that can be characterized "as engaging in joint mathematical activity" (Cobb et al., 1992, p. 104). That is, students working together to reach a conclusion seem to understand each other's intents and anticipated solution paths. This also helps in making all members of the class feel like part of the community. A discussion-based environment also is more promising in developing mathematical interpreting skills of the students involved.

In a ten-month study of a third grade classroom (Lo, Wheatley & Smith, 1994), a challenge to the teacher's central role was for students to participate verbally in discussion as presenters of ideas and solutions. The teacher normally chose the first presenter based upon observations during small-group work. Then, "presenters were expected to be prepared when they raised their hands and to give clear explanations of their solution methods for a particular task". For subsequent presenters the teacher "would choose those students who seemed to have listened to the first presenter and who she believed had different methods; however, students could still participate in the discussion in other ways, for example, asking for clarification, offering an alternative explanation, or connecting on another student's method" (p. 35). Utilizing discussion in this way illustrated the possibilities for creating meaningful student involvement.

Creating a discussion-based mathematics classroom will rely on several factors. Communication in the classroom should aid students in formulating their own mathematical ideas (Pimm, 1996); close attention should be paid to the oral communication of students through active listening (Pirie, 1996); and, teachers should understand the challenges and dilemmas involved in the journey (Silver & Smith, 1996). What role do teachers and students play in creating a discussion-based mathematics classroom? The next section outlines recommended shifts from the traditional roles.

Changing the Mathematics Classroom Environment

To attach one all-encompassing descriptor to any classroom is nearly impossible. Classrooms tend to develop specific characteristics that are routine yet susceptible to change at any time. The development of shared understandings and norms that structure the classroom environment are generated by both the teacher and the students. In a classroom environment described as traditional, the teacher is perceived as possessing control of the development of understandings and norms. This pattern can be altered if teachers and students are willing to shift their perceptions and expectations of traditional mathematics classroom environments.

Shifts in the Roles of Teachers

Teachers who are responsive to or already embrace reform-oriented recommendations underlying the envisioned classroom, find themselves reflecting on their previous and current instructional practices. In what ways can their instruction become compatible with the principles that support reform? Students in non-traditional settings also face environments that may be unfamiliar. This section examines the recommended shifts in instruction as teachers attempt to reform the structure of their mathematics classrooms and the roles that students should adopt in reform-oriented classrooms.

Shifting instruction away from the traditional teacher-led approach, towards an approach in which students are encouraged to search rather than follow, is a challenging task (Ball, 1992; Lloyd, 1996; Wilson & Ball, 1991; Wilson & Lloyd, 1995, 1997). Teachers encounter difficulties in shifting their pedagogical practices in ways consistent with current reform ideas for several reasons: most teachers were not educated in settings that emphasize student involvement in developing concepts and understandings, most teachers were not trained to teach in non-authoritative ways, and there is little incentive to shift if current practice is perceived as successful (Brooks & Brooks, 1993). Indeed, the efficacy of clear and accurate telling is often desired as opposed to the undermined efficacy of de-emphasized telling (Smith, 1996).

In terms of teachers' roles, the concept of teaching in more non-authoritative ways does not imply that the teacher's authority is completely discounted. Rather, it may be possible for teachers to emphasize student involvement by decreasing their traditional role as "teller." Several studies reviewed by Noddings (1989) suggest that teacher-led explanations and continued telling, even from those who are seen as successful mathematics teachers, are less imperative than some believe. One study of small-group learning (that emphasized student centeredness) versus traditional whole-class instruction (with an "expert" teacher imparting knowledge) found no significant differences in the two settings. Students learned as well in the less teacher-centered environment. Another study in a middle school environment found that small-group communication was effective in promoting problem solving and understanding of certain mathematical structures. As students are encouraged to collaborate in small-groups or whole-class discussions, it is important that the teacher skillfully directs discourse to contribute worthwhile information to the negotiation. In this way, the teacher contributes his expert mathematics knowledge to the on-going discussion as a participant rather than a dictator. The allowance and encouragement of student-led discussions and conversations does not infer mass confusion or chaos. The teacher is responsible for initiating and orchestrating discourse while analyzing patterns of inequality, dominance, and student-felt low expectations that may lead to non-participation (NCTM, 1991). The teacher provides a framework in the classroom within which discussion is encouraged and valued.

When allowing student ideas to generate discussion, teachers also need to be willing to accept explanations that may seem divergent. That is, although student explanations may initially appear to be misleading, the sharing of various reasonings can be directed towards deeper understandings. For example, in Schoenfeld's (1989) description of his problem-solving class he states:

> I remain engaged as a member of the community, making sure that the appropriate mathematical values are respected (Are we really sure? Is there a counterexample? Is the argument alright?). I refrain, however from pronouncing what is right and what is wrong; I pose the issues, and leave it (for as long as possible) for the class to resolve them. (p. 82)

His role becomes that of a questioner who elicits student ideas in a way that compels them to validate their own reasonings. Student contributions and understandings are an integral part of mathematical explanations. Again, the teacher as the content expert is not expected to non-evaluatively support the ideas of students without question. Rather, Eisenhart (1988) suggests that a view of mathematical learning should include the process of acculturation into the mathematical meanings and practices of society. Thus, students' constructions may or may not be potentially powerful for future learning. In considering this, Cobb et. al (1992) remark that one facet of the teacher's role, then, is to facilitate mathematical discussions between students while acting as a participant in the discussion. As a result, "the teacher still acts to constrain students' mathematical interpretations and solutions, but does so in a communicative context that involves the explicit negotiation of mathematical meanings" (p. 102).

Shifts in instruction that are geared towards creating a more student-centered classroom environment are a widely accepted goal of the mathematics education community. However, when individual teachers must contend with executing these shifts within their individual mathematics classrooms they often feel unsupported by colleagues and supervisors within their school and face conflict from students when attempting to create a community in which critical analysis of ideas is accepted (Silver & Smith, 1996). The attitudes and intentions of individual teachers then play an important part in these shifts. Sainsbury (1992) provides the following identification of a desirable attitude for a teacher whose aim is to bring about the type of classroom interactions advised in documents such as the Standards:

> Instead of regarding herself as the custodian of pure specialised knowledge, she has to see herself as a partner, albeit a more knowledgeable partner, in a conversation. Instead of seeing pupils as empty vessels to be filled with desired information, she must see them as active appliers of meaning, using their present understanding to grapple with new material and bring it within their grasp. She must acknowledge and respect this understanding and these attempts, rather that regarding it as irrelevant interference in her own projects. The emphasis must shift from the adult as teacher to the child as learner, and the teacher must redefine her position accordingly. (p. 124)

Identifying with this attitude suggests that shifts in instruction necessarily correspond with the underlying beliefs of teachers. Then, teacher's attitudes about and stance concerning her role impact instruction. Through Barnes' (1992) work with communication in the classroom he concludes that:

> It is possible to show that the way in which teachers think about what constitutes nowledge is often linked to what they think learning and teaching are. That is, a view of knowledge is likely to carry with it a view of classroom communication and of the roles of teacher and pupil in formulating knowledge(p. 139).

The decisions concerning communication and the establishment of roles in the classroom will characterize the instructional setting. For example, a year long study completed in a third grade mathematics class caused the teacher to challenge her views concerning the creation of a constructivist-based classroom (Peterson, 1992). The teacher's revised thinking included changes in her views about the ways in which children learn mathematics. Her reply concerning the change in her views can be seen as an emerging description of teacher change:

> Before everything was more or less programmed [or scripted in CSMP], and it's not that programmed right now, if that makes sense. Because when I teach the lesson, I have an idea what I want them [the children] to understand. And however they arrive at that is okay. I don't know why it's different, but it is different. I'm just not getting the responses [from students] that I had in the CSMP book, because I'm not anticipating or saying to myself, "Well, this is what they should say." Rather, I'm taking what they're giving me and building upon that. I know a lot of time in CSMP, when you [the teacher] have the anticipated response there, if you don't get that, then you would somehow rephrase it and tell it to them. With this [the way I am teaching now] I don't do that; everything comes from them [the students]. And we can build upon what they bring to the lesson, and I think that's really exciting. (p. 171)

The changes in this teacher's views concerning the learning of her students suggests that a teacher's actions, implicitly derived from beliefs, have much to do with renewed efficacy in promoting student collaboration in the class environment.

As teachers shift their roles, students will also be confronted with developing new conceptions of the mathematics classroom environment. Shifts in the role of students in reform-oriented classrooms is discussed next.

Shifts in the Roles of Students

Traditionally, the role of students in a mathematics classroom is one of passive acceptance of a large body of information provided by the teacher (Silver, 1996). As classroom teachers shift their instructional practices, students generally find themselves placed in roles that likewise follow non-traditional patterns. This shift in the classroom constitution may cause dissension in the classroom or confusion about the roles of both teacher and students. Shifting the role of students in the envisioned mathematics classroom will require that traditional interactions change.

Students, who find themselves in a mathematics classroom in which the teacher is changing her instruction, are in many ways forced to accept new roles. Cobb (1990) reminds us that "a student's goal in the classroom is not to learn mathematics per se. Instead, it is to complete tasks in ways that are acceptable with respect to the classroom situation" (p. 205). That is, students will act in ways that are accepted by the teacher, because their goal is to effectively fit into the classroom environment. The expected student roles are largely dependent upon the instructional goals evidenced within the classroom environment. In a classroom that is shifting towards a community of learners, students may be expected to actively contribute to discussions in small-group and whole-class settings, take responsibility for negotiating meanings within the classroom, and participate in problem solving situations in which they will be encouraged "to explore, formulate and test conjectures, prove generalizations, and discuss and apply the results of their investigations" (NCTM, 1989, p. 128). One notable shift in many of these descriptions is that the student becomes a major contributor to developing discussions.

Yackel, Cobb and Wood (1993) express their agreement with this role by saying, "The learning opportunities that arise from children's attempts to communicate with each other include those that arise not only as they attempt to resolve conflicts but also as they verbalize their thoughts in the course of a dialogue and as they attempt to interpret and make sense of their partner's verbalizations" (p. 35). When students become involved in resolving conflicts through verbalizing their conceptions, the role as a passive receptacle of information is precluded. The importance of discussion in the classroom is cited by Gibbs and Orton (1994) as follows:

> There is no guarantee that ideas presented by a teacher will be received by pupils in their intended form. Some form of discussion to allow negotiation of meaning seems essential. A pupil struggling to understand a teacher's utterance may resolve the difficulty by trying to pose a question or by disputing a comment made by a fellow pupil. (p. 104)

This idea is consonant with suggestions that collaborative discussion in the classroom can enhance concept development and problem solving abilities (Lester et. al., 1994; Cobb, Yackel & Wood, 1993).

Damon and Phelps (cited in Lumpe, 1995) outline three types of peer interaction utilized in classrooms: peer tutoring, cooperative learning, and peer collaboration. Each requires students to work together and participate in discussions to some extent. Peer collaboration was found to be the method that is "the most effective peer group technique for enhancing conceptual-change learning and problem solving because of its high level of mutuality and equality between students" (p.307). The reform-oriented moves towards classroom environments that encourage discussion among all members require that the role of students be re-examined.

A reform-oriented environment or non-traditional mathematics class can be described as one in which the environment suggests contrasting norms to the traditional environment. Generally, a non-traditional mathematics classroom environment is characterized by facets of current reform-oriented recommendations largely based on ideas expressed in the NCTM Standards. The challenge to teachers is to reform their classroom practices in ways that are aligned with reform recommendations. This is not expected to be an easily accomplished task. Rather, the complexity of classrooms demands that teachers continually reflect on their practice when accommodating new conceptions and strategies. This challenge is examined next.

Reforming Mathematics Classroom Practice

Teachers intending to reform the environment of their mathematics classrooms along the lines of current reform-oriented recommendations are faced with the realities of resigning comfortable habits that can be difficult to change. When focusing on the structure of classroom discussions, the traditional pattern in which the teacher maintains authority for transmitting knowledge, validating information, and expecting students to silently learn (Silver & Smith, 1996), presents an obstacle to change for many veteran teachers. Lampert (1991) suggests that in order to change the mathematics classroom, teachers must create environments that reject the cultural assumption about mathematics, often shaped by school experience, that "mathematical truth is determined when the answer is ratified by the teacher" (p. 124). Responsibility for creating and validating meanings becomes shared among students and teachers in a reform-oriented environment.

The NCTM Professional Standards emphasize important strategies to be considered by teachers aiming to create mathematical communities in their classrooms. These are: (1) Setting goals and selecting or creating mathematical tasks to help students achieve these goals; (2) Stimulating and managing classroom discourse so that both the students and the teacher are clearer about what is being learned; (3) Creating a classroom environment to support teaching and learning mathematics; and, (4) Analyzing student learning, the mathematical tasks, and the environment in order to make ongoing instructional decisions (p. 5). Many instructors embrace these strategies in theory (Minstrell & Simpson, 1996) but find difficulties when they attempt to integrate these wholly into their classrooms (Cohen, 1990; Gregg, 1995; Haimes, 1996; Wilson & Lloyd, 1995, 1996, 1997). In terms of generating more student participation in discussion of mathematics concepts, teachers may feel that control of the environment is relinquished to students. As a result, teachers may retain many of their traditional strategies (Pirie, 1996; Silver & Smith, 1996).

The practices in a traditional mathematics classroom environment in which teachers check answers to homework, provide explanations, introduce new material, work examples, and assign seatwork is often referred to as the school mathematics tradition (Cobb, Wood, Yackel, & McNeal, 1992; Gregg, 1995). The familiarity of this routine provides a feeling of efficacy for many mathematics teachers. As Smith (1996) writes:

> Telling, irrespective of its pedagogical strengths and weaknesses, provides a clear model for teachers of mathematics to develop a sense of efficacy. Though good telling cannot guarantee that students will learn, it narrows the scope of the content to manageable proportions, clearly defines what the central acts of teaching are and what counts as evidence of student learning, and provides structure for daily classroom life. (p. 393).

That is, until teachers feel comfortable and confident that a non-traditional environment can also realistically represent success in their view, it is unlikely that their practice will noticeably change.

Fullan and Miles (1992) propose orientations that should be incorporated into the thinking and actions of those involved in change efforts to enact successful change. The first three particularly apply to individual classroom teachers by providing advice in the context of change:

> Even in cases where reform eventually succeeds, things will often go wrong before they go right;
> Even the development of a shared vision that is central to reform is better thought of as a journey in which people's sense of purpose is identified, considered, and continuously shaped and reshaped;
> Too often, change-related problems are ignored, denied, or treated as an occasion for blame and defense. (p. 749-750)

The adoption of these views accepts and realizes that change is on-going and difficulties that often occur in a changing environment can be more productively seen as areas for reflection: "Only by tracking problems can we understand what we need to do next to get what we want" (p. 750). The NCTM Standards documents present a similar view through implying the position that teachers are not being asked to remedy or rectify any past or current perceived failures. Rather, the professed intent of the Standards are to provide guidelines for the context of envisioned changes in mathematics classrooms that will enable students to become more productive learners of mathematics. Wilson and Ball (1991) suggest that a constructivist view of learning necessarily allows that teachers also construct understandings of the teaching and learning process on their own terms.

Ball (1992) outlines three concerns when addressing the implementation of reform-oriented strategies: (1) This kind of teaching is hard, and no one is going to produce a system, a formula, or a program that can produce it; (2) Constructing ways to teach mathematics that take these ideas seriously will require new learning, support, and resources; and (3) Teachers are being urged to make their work more uncertain, even as they are simultaneously being asked to produce, more reliably, a set of ambitious outcomes. Indeed, teachers are often expected to "implement" practices seemingly handed down by well-meaning reformers. For example, Cohen

(1990) reports an account of an elementary teacher who, when compelled to revise her mathematics teaching strategies through state efforts, shaped a practice consisting of a mixture of her traditional and reform-oriented instruction. Cohen writes that "the changes in Mrs O's teaching that seemed paradoxical to me seemed immense to her" and that even though mixed practice with confusion seems inevitable, "it often goes unnoticed by those who promote change in teaching" (p. 339). This example points to the need for mathematics teacher educators to examine the changing mathematics classroom with a sense of understanding of the challenges inherent in such change.

Mathematics teachers (like Mr. Lester in my study) who are intent on realizing the vision of the Standards in their classrooms, often find the transition filled with uncertainty. Teachers and researchers who have aimed at realizing the intent of reform recommendations have been faced with challenges. The results of their experiences offer both instructive struggles and useful strategies for others interested in changing their practice. At the elementary level, many studies of mathematics teaching support the evidence that reform is complex and challenges the traditional pedagogical views held by teachers (Cobb, Yackel, & Wood, 1993; Cohen, 1990; Prawat, 1992; Wilson, 1990).

Similar research results at the secondary level are growing. Several studies reveal the struggles encountered within the context of reform experiences (Briscoe, 1991; Gregg, 1995; Haimes, 1996; Lloyd, 1996, 1997). Briscoe (1991) in the study of a high school science teacher suggests that "teachers must examine their beliefs, judgments, and thoughts regarding what they do and how they do it" (p. 197). The teacher with whom she worked believed that his practice needed reform, but had trouble envisioning his role to support the new context. Lloyd (1996) found that a secondary mathematics teacher who implemented and was supported by a reform-oriented curriculum in one class continued to struggle to substantially change his and his students' activities in his other more traditional classes. Haimes (1996) studied a veteran algebra teacher's implementation of a reform-oriented curriculum. His results suggest that the impact of the intended curriculum appeared to be minimal. As a result, he suggests further examination of the difficulties facing teachers in reforming traditional practices. A significant challenge raised by Gregg (1995) is that "encouraging teachers to view their current practice as problematic may not be sufficient to promote reform". The mathematics teachers (in the school in which he completed a study) knew that aspects of their practice were problematic, "but they had developed explanations to account for these problems" (p. 464). These results raise questions concerning how teachers can effectively meet the challenges of reform and teacher educators' roles in fostering environments that support teacher change. It is also evident that unless teachers' conceptions of teaching and learning substantially change, it is doubtful that reform will be extensive or lasting.

Research Questions Revisited

In the current environment of mathematics education reform, teachers are being asked to substantially alter their views of teaching and learning mathematics. While many teachers embrace reform-oriented views, their prior teaching and learning experiences can weigh heavily on attempts to change. When attempting to incorporate reform-oriented recommendations, teachers may often rely on previously tried traditional strategies or selectively mix reform-oriented practices during the change process.

For this study, these issues are addressed in particular as related to the use of class discussions to develop mathematical knowledge. The guiding questions are: What class discussion patterns existed in Mr. Lester's class? What was Mr. Lester's role in facilitating class discussions? What reform-related struggles did Mr. Lester encounter? These questions can be further examined in light of the related literature.

When examining class discussion patterns and the teacher's role, the literature suggests that mathematics teachers traditionally rely on patterns that emphasize teacher initiation and confirmation of meanings. The traditional initiation-reply-evaluation or funnel pattern prevails in many classrooms and the teacher's role tends to reinforce teacher-centered environments. What will characterize class discussions if the teacher has stated intentions and ample knowledge to facilitate reform recommendations? Do class discussion patterns remain traditional in a class in which the teacher wholly embraces reform-oriented views? In attempting to facilitate class discussions, does the teacher's role incorporate reform-oriented strategies?

Implementing reform in the classroom is a challenge that is complex and uncertain. Teachers who may or may not embrace reform-oriented views, but are supported by particular curriculums experience struggles in the context of reform. For example, teachers may struggle with changing their views of their own and their students' roles. Are similar struggles encountered when a teacher wants to revise his classroom environment based on his reform-oriented views as opposed to following an intended curriculum? The next chapter presents the methodological approaches used to investigate these questions.

CHAPTER THREE
RESEARCH METHODS

The purpose of this study was to examine the experience of a secondary mathematics teacher during his efforts to facilitate mathematical discussions in a secondary algebra class. Whole-class discussions and interviews were documented and analyzed to investigate the pattern of discussion, the teacher's role in facilitating discussion, and the struggles encountered by the teacher through his attempt to enact reform-oriented strategies. The investigation focused on the teacher's vision for and actual practice in utilizing reform-oriented strategies for discussion in his Algebra class.

As motivated by the issues outlined in Chapter Two, the following questions guided the study: (1) What patterns of verbal interactions concerning the construction of mathematical knowledge exist in class discussions? (2) What is the role of the teacher in attempting to facilitate mathematical discussions in a secondary Algebra class? (3) What struggles does the teacher confront in attempting to implement mathematical discussions in the classroom? To investigate these questions, I completed a qualitative case study with procedures adapted from those of Lincoln and Guba (1985), Taylor and Bogdan (1984), and Spradley (1980) that examined the experience of a secondary teacher, Mr. Lester, during his efforts to facilitate class discussions in an Algebra class. Data were collected during the Spring and Summer of 1995-96 and the Fall of 1996.

This chapter outlines the methodology of the study, the processes used in completing data collection and data analysis, and my journey through the research process. My journey as a researcher through this case study was at times uncertain. Because there were changes in the focus of the case study as I collected data, I provide in this chapter a reflective account (based on personal log entries) of the eventual direction of the study. The chapter is organized as follows: research design, data collection, data analysis, planning for trustworthiness, limitations and ethical considerations, and my reflections on the research process.

Research Design

Based on the guiding research questions and purpose of this study, I chose a qualitative research methodology. Silverman (1993) defines methodology as "a general approach to studying a research topic [that] establishes how one will go about studying any phenomenon" (p. 2). Because this study proposed to document and analyze whole-class discussion patterns in Mr. Lester's class, as well as to investigate his implementation of current reform recommendations, a qualitative methodology in which classroom interactions were observed in naturally occurring situations and examined in the context of the actual classroom, was appropriate. The objective of qualitative research is "understanding, rather than the ability to generalize or [identify] causes and effects" (Whitt, 1991, p. 407). That is, the focus was on examining whole-class discussions, the role of the teacher, and the experiences of the teacher as they naturally occurred. The goal was to add to the understanding of how veteran mathematics teachers facilitate reform-oriented strategies in their classes.

The objective of this study was to understand and analyze the happenings in Mr. Lester's classroom. To accomplish the objective, I chose to base my research on interpretive fieldwork methods outlined by Erickson (1986), such as: long-term observation in the setting; writing fieldnotes, memos, examples; audiotaping; analytic reflection; and, reporting by description and direct quotes. The research was completed both inductively and deductively as described by Erickson:

> It is true that specific categories for observation are not determined in advance of entering the field setting as a participant observer. It is also true that the researcher always identifies conceptual issues of research interest before entering the field setting. In fieldwork, induction and deduction are in constant dialogue. As a result, the researcher pursues deliberate lines of inquiry while in the field, even though specific terms of inquiry may change in response to the distinctive character of events in the field setting. The specific terms of inquiry may also be reconstrued in response to changes in the fieldworker's perceptions and understandings of events and their organization during the time spent in the field. (p. 121)

Before entering the field setting, the conceptual issues of research interest that I identified were consistent with my experiences as a mathematics teacher and student of mathematics education issues. As I began to study and read more, the notions of a classroom in which students were involved in discussing, justifying, and reasoning with mathematical ideas became foremost in my interests. The basic principles evident in the Standards documents became a basis for my pursuant research. As Patton (1991) writes, the decision about an approach to research rests on a number of ideas, one of which is the "researcher's own beliefs about the origin and nature of human behavior and the appropriate way to study behavior" (p. 392). Thus, my interest in the workings of a mathematics classroom in a non-traditional environment fueled my research.

Position of the Researcher

Through my experience as a mathematics teacher at different levels and the opportunities I've had to observe in many mathematics classrooms as a supervisor of student teachers, my background and knowledge provides a basis for completing research in the classroom. Because the classroom was a familiar setting, I also needed to be conscious of my tactics during observations. As Becker is quoted in Hammersly and Atkinson (1983):

> It is not just the survey method of educational testing or any of those things that keeps people from seeing what is going on. I think, instead, that it is first and foremost a matter of it all being so familiar that it becomes impossible to single out events that occur in the classroom as things that have occurred, even when they happen right in front of you. (p. 92)

Realizing my familiarity with a secondary mathematics classroom and Mr. Lester, this comment suggested that observations of events in the classroom be focused to aid in "seeing" what is happening. In the naturalistic design used in this study, this position can be viewed as an enhancing factor rather than an obstacle. Knowing that as a teacher it was virtually impossible for me to be involved in a classroom setting from a purely observational standpoint, the possibility of investigator-investigated interaction was conceded. Lincoln and Guba (1985) address this aspect of research through the posture of the researcher. By accepting that in a natural setting the interactions that occur cannot be eliminated, the researcher can look at her presence as an opportunity to be exploited. Meaningful research with human subjects "is impossible without the full understanding and cooperation of the respondents" (p. 105).

Applying this framework, the setting was viewed as a community in which the actions and interactions were the primary focus and the researcher "maintain[ed] a balance between being an insider and outsider, between participation and observation" (Spradley, 1980, p. 160). In this study, I most often assumed the role of an observer, although I was not treated as an outsider. I became a familiar member of the class environment. The teacher and students were free to talk to me at any time, although they knew that my purpose was to observe, audio-record, and take notes. Any incidental interactions or discussions were recorded in a personal log. As Lofland and Lofland (1984) commented, "Be neither discouraged nor overconfident about your relationship to the setting. Whatever the relationship, it is simultaneously an advantage and a drawback" (p. 16).

A Qualitative Case Study

In completing a case study, a single-case design was chosen because the case is unique in some way or has the potential to give revelatory results within the domain of inquiry (Yin, 1994). A qualitative case study methodology "focuses on a bounded system, whether a single actor, a single classroom, a single institution, or a single enterprise - usually under natural conditions - so as to understand it in its own habitat" (Stake, 1988). Stake emphasizes that, "the case is something deemed worthy of close watch. It has character, it has totality, it has boundaries....it is a complex, dynamic system" (p. 256). In this study, Mr. Lester was in a unique position to enact significant reform in his mathematics classroom. He was not only a veteran mathematics teacher, but also had participated widely in reform efforts. In particular, Mr. Lester was a teacher whose background, experience, and intentions in reforming his classroom, made him an ideal candidate for more intense examination.

The underlying principles of the research design of this study were found in the concepts of a naturalistic framework which operates from the view that realities are often dependent upon the respondent and circumstances, the knower and what becomes known both shape the research context, and inquiry is bounded within the confines of the researcher and the setting (Ely, 1991). A naturalistic design is appropriate to the focus of this study because the setting involves interactions that are constructed by the participants according to the social reality and experiences of each. As Lincoln and Guba (1985) suggest, "inquiry must be carried out in a 'natural' setting because phenomena of study, whether they may be - physical, chemical, biological, social, psychological - take their meaning as much from their contexts as they do from themselves" (p. 189). The setting of a naturally occurring classroom has the potential for producing meaningful data precisely because it is natural and not controlled. In this study, there was no attempt to intervene in the natural occurring situations of the classroom. An outline of the naturalist framework considers that: realities are multiple, constructed, and holistic; knower and known are interactive; hypotheses are time and context bound; causes and effects are not distinguishable in the traditional sense since entities are mutually shaping; and inquiry is value-bound (Lincoln & Guba, 1985). That is, the outcomes of this study represent the hypotheses constructed through my interaction with the data that emerged from classroom observations and conversations with Mr. Lester.

The next section describes the setting for this study in terms of the school in which the classrooms existed, the teacher, the classroom, and the students.

The Setting and Participants

The setting that was the focus of this study was a secondary mathematics classroom in a high school in Virginia. Perry High School (a pseudonym) can be described as a rural high school. The enrollment in the 1995-96 school year was approximately 950 students, with a racial composition of approximately 870 white and 80 non-white students. The class was chosen for several reasons: (a) the teacher, Mr. Lester, was known to me as one who was reform-minded; (b) the class was a typical secondary level Algebra One class in that it contained mostly 9th and 10th grade students taking Algebra for the first time; and (c) through my work as a supervisor of student teachers, gaining access to the site was relatively uncomplicated. The mathematics department at Perry High had several teachers who were interested in reform-oriented ideas. Mr. Lester and one other teacher, in particular, participated widely in local and state mathematics education opportunities.

The interest in this particular classroom arose as a result of my professional dealings with the teacher, Mr. Neil Lester. Because the teacher was one with whom I had worked with through my position as a supervisor of mathematics student teachers, I had some knowledge of his vision of reform in the mathematics classroom. In particular, Mr. Lester's professional experiences (teaching in the public schools and working with student teachers at the university level) placed him in a unique and promising position to significantly impact his teaching practice based on his visions of reform. His vision of teaching, learning mathematics, and the role of students in a mathematics classroom was based on a setting that was largely non-traditional, based heavily on NCTM recommendations.

Mr. Lester's Algebra classroom setting would be a familiar sight to many mathematics teachers. There were two wall-length chalkboards on the front and left side walls and a desk at the front of the room. Some mathematics posters and former student projects were dispersed throughout the room. Two bookshelves lined the back of one side wall. Mr. Lester was a "traveling teacher" (he moved from one room to another during the school day), so the physical structure of this classroom was not his design. He used a rolling cart to transport his materials, books, and student papers from class to class. Because he had a long, developed working relationship with the teacher whose classroom he used for two periods, there appeared to be little disruption with this aspect of his daily routine. The class in this study was Mr. Lester's 4th block Algebra class. There were 25 students in this class; 12 were female and 13 were male.

Data Collection

In completing a qualitative case study by applying an ethnographic way of looking (Wolcott, 1988), it is important that the data come from an in-depth examination through multiple sources. In this study, data were collected through direct observation, interviews with the teacher, audiotapes of class discussions, and personal log notes written by the researcher. A few samples of teacher developed worksheets were collected, although these were of minor relevance to the focus of the study. Collecting data from multiple sources helped to provide for triangulation in the study as "a way of guarding against researcher bias" and to aid in gaining "a deeper and clearer understanding of the setting and people being studied" (Taylor & Bogdan, 1984, p. 68). Ethnographic data collection can be used in an attempt to holistically examine processes within a classroom with a devaluation of the effectiveness of these processes. Patton

(1991) writes:

> Data collection methods in the qualitative tradition permit a detailed description and analysis of what it is persons know and accomplish as they interact with each other in the occasions they themselves create and manage. This means that qualitative researchers use materials collected from ongoing, naturally occurring occasions of social interaction, usually by means of field and interview notes or with the help of audio- or videotaped recordings and transcripts. (p. 392)

Chilcott (1987) mentions that a salient feature of school ethnography "is that it can raise the consciousness level of professional educators regarding what is occurring around them" (p. 210).

Data were collected during the Spring semester of the 1995-96 school year, the Summer of 1996, and the beginning of the Fall semester of 1996-97. An informal interview was completed with Mr. Lester at the beginning of the Spring semester (January 1996) to talk about his intentions for facilitating discussion in his Algebra class based on his vision for reform (see Chapter Four for more detail). Because the school was on semester blocks, the class was not yet familiar with Mr. Lester. I allowed the first six weeks for the students to become acclimated and for Mr. Lester to develop his routine. During this time I visited the class periodically to informally observe and explain my role in conducting the study. The data, collected in two subsequent phases, consisted of approximately eleven weeks of observation, twenty-two audiotaped class blocks (90 minutes each), and six interviews with Mr. Lester.

Phase One Data Collection

Phase One data collection took place during the Spring semester of the 1995-96 school year during two units of study chosen mutually by Mr. Lester and the researcher. Because Perry High School had recently switched to a semester block schedule, Mr. Lester had to complete a pacing guide for the semester that coincided with the State of Virginia Standards of Learning (SOL's) for Algebra. The pacing guide was a general outline of Algebra topics and corresponding SOL's broken into each six weeks grading period. For example, the first six weeks contained the topics of formulas and graphs (SOL's:1), variable expressions (SOL's: 2), algebraic properties and relationships (SOL's: 3), linear equations (SOL's: 1,2,3), and matrices

(SOL's: 4). We looked at his pacing guide for the second and third six weeks and chose units that were conveniently timed as well as relevant to most Algebra teachers. Each unit consisted of a topic typically covered in an Algebra One class, as well as some supplemental study of a transformational graphing packet used in Perry County. The first unit was on systems of equations; the second, factoring polynomials (with introduction to the quadratic formula).

Whole-class discussions (22 total) were audio-taped and transcribed (the first unit was 12 class blocks, the second unit was 10 class blocks). Observational notes were written during each of these blocks. Observations at the beginning of the study aided in developing a sense of the classroom environment being facilitated by Mr. Lester. For example, it was evident early in the first unit that Mr. Lester put forth a great deal of effort in developing materials that he considered to be worthwhile and potentially beneficial to developing discussion of mathematics concepts. He often spent a lot of class time in attempting to get students to think about possible strategies for solving problems. It also became evident that although he envisioned using small-groups to generate discussion, the lack of his development of effective structure to the groups appeared to prohibit this idea. Later, as Mr. Lester's struggles in facilitating whole-class discussions achieved a larger focus, I observed more closely the strategies that he used to attempt to involve students. In one instance (4/23/96) when solving systems of equations, the class had just completed finding a value for "y" in a system. Mr. Lester posed several questions in the subsequent discussion of the solution such as: Can you think of another method we might use? Would it be easier to use substitution? Why would graphing seem to be a possibility? The use of these types of questions were intended to elicit more student responses.

During Phase One data collection, Mr. Lester was interviewed three times; at the beginning of the first unit, between the first and second unit, and at the end of the second unit. The interviews were approximately one hour in length and were based on general questions such as: What will be happening during this unit? What will you be doing? What will the students be doing? What are your goals for the unit? Are there any specific results you are hoping for? These interviews were used to gather data on Mr. LesterÌs plans and subsequent reflections concerning actual classroom discussions.

Phase Two Data Collection

Phase Two data collection consisted of Mr. LesterÌs coding of class discussion transcripts using a coding sheet constructed by the researcher and two more extensive interviews with Mr. Lester in which he reflected on his professional experience and his experience with the Algebra class during the semester. Each of these were audiotaped and transcribed. During the summer of 1996, Mr. Lester read class transcripts and color-coded them to aid in developing categories for describing the patterns of verbal interactions occurring in the Algebra class. The coding sheet used was constructed by using information in the NCTM Professional Standards as a guide for verbal interactions in class discussions (see Appendix A). For example, some possible phrases concerning teacher dialogue were: promoting (teacher was trying to promote discussion of a topic), posing (teacher was trying to elicit students' thinking with a question), and clarifying (teacher was trying to ask a student to clarify their idea). Some phrases concerning student dialogue were: generalizing (student makes a generalization), responding (student simply responds to a prompt), and presenting (student presents solution to a problem). Because Mr. Lester would also be subsequently interviewed about his experience over the semester, his reading of class discussion transcripts also aided in his recall of the content and patterns of discussion.

In the Fall of 1996-97, the two interviews (approximately two hours each) were completed with Mr. Lester. In the first interview, data about his former experience and background was collected to build a case for his vision of reform. For example, we talked about what he did during his beginning years of teaching that led him to become more involved with professional development programs (such as working closely with university sponsored programs) and complete his master's degree. The second interview was a follow-up conversation based on previous transcript data and observations, relating to Mr. LesterÌs visions of reform filtered through his experience during the study. Questions were formed from my reading of previous transcripts such as:

> Several times you mentioned that "in the past" the connections between graphing and transformations was "pretty clean", but not in this class. What is your reaction to this?
> Do you think the content-coverage issue was part of your frustrations with the class?
> Have your expectations changed in terms of your approach to planning for the class?

What were your overall reactions to student responses to discussions initiated by you? During the interview, the basic questions were used as a guide, not as a rigid set. That is, the responses given by Mr. Lester during the interview contributed to the direction of the conversation.

Observation Notes and Personal Log

While data collection was on-going, classroom observation notes were written. These notes supplemented the audio-recordings of class discussion and provided paraphrases, remarks, and sketches of any visual items (e.g. graphs) that are not found on audio-tape. Hammersly and Atkinson (1983) point out that the main purpose of notes is to aid in identifying and developing appropriate theoretical categories. A personal log was used for writing my own thoughts concerning the study. I brought this with me each time I observed and wrote comments about the research process and reflections of my progress. I also wrote any emerging thoughts concerning possible themes or categories as I read related literature. Ely (1991) provides a description of the personal log as "the place where each qualitative researcher faces the self as instrument through a personal dialogue about moments of victory and disheartenment, hunches, feelings, insights, assumptions, biases, and ongoing ideas about method" (p.69).

These written notes formed a basis for beginning analyses of the data. The next section describes in further detail the procedures used in data analysis.

Data Analysis

In the analysis of qualitative data the main purpose is to derive meaning through the words of participants in a way that allows those who examine it to interact with the information. In Ely's words, "to analyze is to find some way or ways to tease out what we consider to be essential meaning in the raw data; to reduce and reorganize and combine so that readers share the researcher's findings in the most economical, interesting fashion" (p. 140). With this in mind, the genesis of my analysis of the raw data (observational notes, incidental notes, interview transcripts, and, whole-class discussion transcripts) was completed as I read related literature and completed data collection. That is, data analysis began during observations and continued throughout the study.

To organize the data, I drew upon the concept of the audit trail. This concept is outlined in Lincoln and Guba (1985) as a technique to gain confirmability, which is the extent to which the findings are objective and representative of the situation. Files were developed based upon the six audit trail categories: raw data, data reduction and analysis products, data reconstruction and synthesis products, process notes, and instrument development information (p. 319). For the purposes of this study, I used the following categories for my files: observational notes, transcriptions, fieldnotes, analytic memos, personal log, and instruments.

The file labeled *Observational Notes* contained two notebooks of notes (one for each observed unit) written while completing daily observations in the classroom. These included notes about the setting and jottings of actual class discussion. The file labeled *Transcriptions* contained typed transcriptions of interviews. These were read, re-read, coded, and re-coded as themes emerged. The file labeled *Fieldnotes* contained compilations of observational notes and whole-class discussion transcriptions to form a synthesis of the patterns of discussion and the class environment. The file labeled *Analytic Memos* contained notes concerning emerging themes, questions, and developing propositions. Analytic memos were included as an aid in moving the analysis forward. Hammersly and Atkinson (1983) describe these as "not fully developed working papers, but periodic written notes whereby progress is assessed, emergent ideas are identified, research strategy is sketched out, and so on" (p. 164). The *Personal Log* file contained incidental notes, thoughts concerning the research process itself, ideas generated from the literature, and the developing story line. The personal log was important as it allowed for continual reflection of my research process and discounting of the data. Finally, the *Instruments* file contained copies of permission letters granted through the Human Subjects Committee and the school system, consent forms from participants, calendars, interview guides, and any other logistical information.

The data were the empirical facts that led to an understanding of the phenomena under study. Lincoln and Guba (1985) note that, "Data are so to speak, the constructions offered by or in the sources; data analysis leads to a reconstruction of those constructions" (p. 332). The initial data was read in the context of related literature. Then, specific incidents in the data were continually compared as the researcher refined hypotheses, identified properties of the incidents, explored relationships to one another, and integrated them into a construction of understanding. My data analysis was informed by analysis strategies found in Taylor & Bogdan (1984), Lincoln and Guba (1985), Spradley (1979), and Erickson (1986) as described in the remainder of this section. In seeking to demonstrate plausible evidence for the interpretations of the study, my analysis was guided by three stages found in Taylor & Bogdan (1984): discovery, coding, and discounting.

The discovery stage entailed identifying themes and propositions following the given steps: a) read and reread the data; b) keep track of themes, hunches, interpretations, and ideas; c) look for emerging themes; d) develop concepts and propositions; e) read and reread the literature; and, f) develop a story line. During the discovery stage, in looking for emerging themes and developing concepts and propositions, I borrowed from analysis techniques outlined in Lincoln & Guba (1985). For example, the technique of unitizing the data was used as I searched the data to acquire an understanding of Mr. Lester's vision of reform. Mr. Lester indicated his attitude towards teaching mathematics in the following sentences from an interview (10/22/96): "You know, going back to the idea of order of operations. Well, we do it this way so everybody gets the same answer. Well, that sounds like it's sort of fabricated." This comment reveals some of his laments concerning the traditional perspective given by teachers on the concept of order of operations. A unit of information like this aided in developing concepts that suggested direction for further analysis. Also during the discovery stage, interpretation of the data began. For example, as I read transcript data, a concept that arose was authority. Mr. Lester's wording of responses and reactions in class discussions several times captured the meaning of teacher authority as in the following: "At least if I give them the idea or the method first, they latch onto it as being valid because it came from me as opposed to [a student's] method" and "If it comes from the teacher's mouth, it must be correct" (interview 10/22/96). These subsequently led to developing propositions that were grounded in the data.

The coding stage entailed coding the data and analyzing negative cases to deepen my understanding of the setting. In the initial coding, I read transcripts and noted themes by writing ideas as marginal notes. For example, I noted in the first interview that Mr. Lester mentioned "past experience" with students several times as he talked about his vision of mathematics education reform. In another example, I used the theme "professional knowledge" to indicate places where Mr. Lester either explicitly talked about his professional experiences or alluded to his knowledge of current mathematics reform issues. Categories were developed and refined as the whole data set was coded. As the data was re-read, codes were developed to fit the data. For example, the category "Standards" fit the data that included both Mr. Lester's talk about professional knowledge and his use of language in class discussions that implied his knowledge of Standards-like reform.

The final stage of discounting the data entailed interpreting data in the context in which they were collected. My personal log was valuable for this purpose. The notes I took aided me in an intuitive comparison of Mr. Lester's interview responses and how he talked spontaneously about the class. I also was able to interpret the influence of my perceptions on the setting.

Although the entire data set influenced a holistic understanding of the class environment, patterns of discussion, and Mr. Lester's struggles; the data could be divided into two sections in response to the research questions. These are outlined in the next two sections.

Class Discussion Data

Whole-class discussions during two units of study were audiotaped, transcribed, and observed to provide a source of information relating specifically to two research questions: (1) What patterns of verbal interactions concerning the construction of mathematical knowledge exist in class discussions? (2) What is the role of the teacher in attempting to facilitate mathematical discussions in a secondary Algebra class?

In the discovery stage of analyzing class discussion data, I searched for emerging patterns of discussion based on audiotaped data and my observations. As a preliminary tool I drew from Spradley's (1979) concept of domain analysis by writing general domains in the form of "X is a kind of Y" to provide directions such as: "... is a kind of questioning," "... is a kind of explanation," "... is a kind of response," and "... is a kind of solution." This preliminary analysis prompted me to analyze class transcripts by focusing on decision points of the teacher during discussion. For example, I identified places where the teacher provided explanations for a student's response, attempted to have students remember a procedure, or asked a question that aimed at having students explore an idea related to discussion. In looking at student-talk, I focused on responses and questions. I noted that most student responses or questions could be typified as basic, procedural, and one-thought. With these beginning ideas I read through class transcripts making general comments. From my reading of the transcripts and developing ideas, I coded my copies of the transcripts with the following descriptions of the teachers role: reexplain, recall, explore, expand, exhibit, redirect, authority, Standards-intent, and struggles. The first three codes focused on discussion strategies Mr. Lester used during discussion and the second three focused on discussion strategies he could have used (but did not) to facilitate discussion. The last three codes indicated reform-oriented issues.

At the end of the two class discussion units, after transcribing classroom discussion tapes, Mr. Lester was given copies of both units to read and color code. By having Mr. Lester participate at this stage, his contributions could serve two purposes: to offer his perspective on the emerging patterns of discussion as a participant in analysis and to stimulate his recall of the experience. As previously mentioned, I generated a list of codes from my reading of the data and information contained in NCTM Standards documents concerning mathematics classrooms, the role of mathematics teachers, and the role of students. Constructing possible codes from this information was appropriate since Mr. Lester's vision of implementing reform-oriented strategies was largely based on such information. After this, I re-read both sets of transcripts (Mr. Lester's and my copies) side by side and wrote a page-by-page comparison. These were analysis notes that helped in refining the search for patterns by noting similarities as well as discrepancies in mine and Mr. Lester's perceptions of the data. For example, in Unit One I wrote [with Mr. Lester's codes in brackets]: "The teacher [listens] to ideas and [poses] questions but students mostly give single [responses]. The teacher displays a reluctance to have any structure that allows for student expression of ideas." These comparisons were used to determine the types of discussion that occurred frequently enough to assert a pattern.

Interview Data

Interview data was essentially collected in two parts; first, during observations of the classroom and second, after the semester had ended. Primary analysis began as interview tapes were transcribed and read. Preliminary ideas were written as marginal notes such as: teacher's attempt to connect Standards, change over the semester, teacher talking about success, intentions to use small-groups, overcoming student reluctance, and teacher use of professional language. As analysis continued, the concept of progressive focusing provided a guide for the gradual shift from describing events and processes to developing and testing explanations (Hammersly and Atkinson, 1983). To test explanations, concepts were noted and propositions were developed to further guide the analysis.

Concepts that suggested direction concerning the interview data were generalized from the data. For example, the following three concepts were developed: control of discussion, proof of professional knowledge, and student powerlessness. Corresponding propositions were developed as follows: (1) Mr. Lester controlled discussion as an expression of authority; (2) Mr. Lester used professional language in interviews to show that he was reform-minded; and (3) Students often simply responded to teacher-directed questions or prompts rather than acting as contributing members of discussion. With these emerging themes, I began to refine my understanding of the data.

By re-reading the interview data and continuing to read related literature, categories were being developed. The following codes that emerged from developed concepts and propositions and the reading of related literature helped to further the analysis: perceptions, plans, expectations, authority, Standards, and struggles. For example, in previous studies the concept of teacher authority provided a reference for my analysis. The interview data was re-read with these codes and I used a grid to organize the information.

Finally, after synthesizing the information, I color-coded the interview data by seven final categories as the story line emerged. These were: actual practice (what Mr. Lester did during the semester), vision of discussion (Mr. Lester's comments that led to defining his vision of reform), reflections on his own teaching (Mr. Lester's professional background and his perception of himself as a teacher), intentions (Mr. Lester's attempts to implement reform ideas), struggles (the challenges evident in Mr. Lester's practice), authority (comments and actions that reveal Mr. Lester's implied authority), and professional experience (Mr. Lester's professional development opportunities). The data were divided into color-coded groups so appropriate quotes and remarks could be applied as the case report was written.

Creating a Reconstruction

Wolcott (1994) gives three main guiding principles for the "doing of something" with your data: description, analysis, and interpretation. He plainly states, "Tell the story. Then tell how that happened to be the way you told it" (p. 16). In leading to a reconstruction of the data, the final stage was discounting the data to aid in developing trustworthiness of findings as the data were reflected on in light of the context in which they were collected. For instance, by re-reading personal log entries I could reflect on any possible researcher impact on the setting. Or, through observational notes I could reflect on Mr. Lester's spontaneous remarks or students' attitudes (unsolicited data) that contributed to my perception of the class environment.

Before writing the final case report, the report was read by Mr. Lester as a "member check" to support credibility of findings. In this study, Mr. Lester provided acknowledgment that the findings were credible although he provided little specific feedback. For instance, in conversations with him concerning the final report, he recognized my characterization of the struggles involved in reforming class discussions. When asked to provide written or verbal comments by providing his perceptions of specific incidents in the report though, he did not provide substantial information. Lincoln and Guba (1985) note that member checking also occurs informally during the course of the investigating (e.g. when Mr. Lester read class transcripts and participated in subsequent interviews). Essentially this helps to answer the question: Is the researcher's reconstruction recognizable to the participant?

In response to the research questions, the following assertions (Erickson, 1986) were developed in the context of data analysis as a basis for writing the final report: Assertion 1: The patterns of verbal interactions existing in Mr. Lester’s class discussions were largely teacher-directed and followed a traditional initiation-reply-evaluation pattern. Assertion 2: Mr. LesterÌs role in attempting to facilitate whole-class mathematical discussions in his Algebra class was that of a director who maintained discussion, posed and promoted questions, and provided clarification of ideas. Assertion 3: The struggles confronting Mr. Lester in attempting to implement mathematical discussions in his Algebra class were unrealized intentions, classroom challenges, student resistance, and teacher authority.

The case reporting mode was the basis for writing the results of the case study. Lincoln and Guba (1985) write that this is appropriate for providing thick description which is essential in making clear the complexities of the situation and the ways in which these interact to form a portrayal of the investigation. Writing the report was done in concurrence with continued reading of the literature to focus on how this study fit into the body of mathematics education research in the context of current reform.

Planning for Trustworthiness

As a quantitative study depends upon validity and reliability, a qualitative study strives for trustworthiness (Lincoln & Guba, 1985). It must be shown that the study "has represented those multiple constructions adequately, that is, that the reconstructions that have been arrived at via the inquiry are credible to the constructors of the original multiple realities" (p. 296). In establishing the traditional requirements of internal validity, external validity, and reliability, I drew on Lincoln and Guba's terms to describe these within the framework of a qualitative study: credibility, transferability, and dependability.

Credibility (Internal Validity)

Internal validity conventionally deals with the extent to which the findings of a study match reality. That is, are the interpretations of the researcher credible? In establishing credibility, prolonged engagement with persistent observation is needed. Through persistent observations in the classroom during the semester that included consistent observational notetaking and daily audiorecording during the two units of study, prolonged engagement was accomplished. Negative case analysis also aided in establishing credibility. Ely (1991) notes that this is "the search for evidence that does not fit into our emergent findings and that leads to a re-examination of our findings" (p. 98). Lincoln and Guba regard it as "a process of revising hypotheses with hindsight" (p. 309) in order to reduce the number of exceptional incidents to make the data more credible. Member checking also adds to credibility by ensuring that the report captures the essential meanings in the data.

Triangulation also aided in establishing credibility. Through multiple sources of data (students, teacher, researcher) and methods of data collection (observations, interviews, recordings) data were triangulated. This triangulation of findings is described as "watching for the convergence of at least two pieces of data" (Ely, 1991, p. 97). The combination of observational notes, incidental notes and transcriptions of interviews and whole-class discussions provided multiple accounts of what was happening in the classroom. Taylor and Bogdan (1984) note that "by drawing on other types and sources of data, observers also gain a deeper and clearer understanding of the setting and people being studied" (p. 68).

Transferability (External Validity)

Transferability is akin to external validity in traditional studies for which random sampling provides for generalization of findings to a larger population. Generalization is not the goal of interpretive study, rather the study strives to provide a comparative understanding of settings to classify a particular case. Erickson (1986) suggests this in saying:

> The task of the analyst is to uncover the different layers of universality and particularity that are confronted in the specific case at hand - what is broadly universal, what generalizes to other similar situations, what is unique to the given instance. This can only be done, interpretive researchers maintain, by attending to the details of the concrete case at hand. Thus the primary concern of interpretive research is particularizability. (p. 130)

To provide transferability, the researcher provides " a sufficient base to permit a person contemplating application in another receiving setting to make the needed comparisons of similarity" (Lincoln & Guba, p. 360). An adequately "thick" description that can be used in judging the possibility of transfer is provided through keeping thorough notes and memos. An in-depth description of data collection and data analysis procedures provides a knowledge base concerning the study that is available to others.

Dependability (Reliability)

Reliability refers to replication of the study in the traditional sense. A case study of one teacher, in a particular class, at a specific time, can never be completed replicated. In fact, because different settings at different times offer different circumstances, the attempt to replicate in this sense is not meaningful. Rather, dependability is attested to through examining the process and products of the inquiry. This was largely provided through the audit trail and personal log. By keeping a justifiable and organized record of the data, findings, and interpretations the product can be examined and we can attest " that it is supported by data and is internally coherent "(Lincoln & Guba, p. 318). A qualitative case study strives to describe and explain a setting as it occurs. In commenting on this measure, Merriam (1988) states: "Since there are many interpretations of what is happening, there is no benchmark by which one can take repeated measures and establish reliability in the traditional sense" (p. 170).

Limitations and Ethical Considerations

Every research endeavor is limited to some extent. In completing research in the classroom there are obviously a myriad of issues that may confront the researcher and participants. Thus, boundaries as to the scope of the research must be addressed. This project was limited by the research questions and the resources of the researcher. The research questions limited the scope of the research to analyzing the patterns of verbal interaction that occurred within one secondary classroom. Because there was only one researcher, resources were limited. Because the project did not intend to measure any change or evaluate the acquisition of skills or amount of learning occurring during the semester, the projected length of the study was limited. The recording of class discussions were limited to the units of study agreed upon with the teacherÌs consent. Strauss and Corbin (1990) also provide that the analysis process limits your final constructions to findings that exist in the actual data. They emphasize that the focus is not on what you think you might find, rather "what you can't find in our data becomes one of the limitations in your study" (p. 112).

I reviewed the Ethical Standards of the American Educational Research Association and obtained approval for the study through the Virginia Tech Internal Review Board for Research Involving Human Subjects. The approved IRB proposal and informed consent form provided statements of risks and benefits to participants so there was complete disclosure to participants and parents of participants. I also contacted parents individually by phone, through permission of the school, to answer any questions concerning the extent of the study.

Through my observations of Mr. Lester's Algebra class, conversations, and working relationship with him, I have had the opportunity to view mathematics classroom reform from a different perspective. That is, instead of being the instructor responsible for facilitating reform, I had the chance to observe and reflect on someone else's practice. During this case study, my analysis and reflection have provided an avenue to examine the complexities of mathematics teaching through the lens of current reform recommendations. My journey as a researcher through this case study was at times uncertain. Because there were changes in the focus of the case study as I collected data, I provide next a reflective account of the eventual direction of this study.

My Reflections on the Research Process

In the Fall of the 1995-96 school year, I spent approximately two weeks doing daily observations in an Algebra Two class taught by Mr. Lester. With intentions to collect data in the Spring of the year with Mr. Lester's consent, we had agreed that my observations in this class would serve as a trial for several data collection techniques and to develop a picture of Mr. Lester's pedagogical strategies and classroom environment (specifically examining his intent to utilize small-group and whole-class discussions as an integral part of the environment).

During this time of observation, there were several instances of students generating questions and discussions, and working in small groups. Although there were instances of more traditional discussion patterns and teacher-led discussion, Mr. Lester often attempted to introduce discussions through conceptually-based arguments. Based on Mr. Lester's prior teaching experience and study in mathematics education, there seemed to be ample ground for his continuing experimentation with reform-recommended mathematics teaching strategies. To experiment with collecting data of discussions in the class, I received consent to audiotape and videotape small-group discussions. I found that videotaping (with my limited resources) was awkward and difficult to implement effectively. I also took fieldnotes of whole-class discussions and observations. With this information, I was led to develop the basis of the resulting case study. That is, a study of a secondary mathematics teacher who was interested in and planning to develop and use student-centered, mathematical discussions in his mathematics class.

The Original Research Focus

The Spring semester began in January 1996. Because Perry High School was on a semester block schedule, Mr. Lester was just beginning to teach two Algebra classes and one Geometry class. Working with his knowledge of reform recommendations and his desire to use strategies to enact reform, Mr. Lester planned to create a more student-centered environment (compared to a traditional Algebra class) in which mathematics topics were freely discussed. My focus was centered on examining the patterns in whole-class and small-group discussions, how these were related (if at all), and Mr. Lester's role in the implementation of discussion. This original focus produced the following research questions that subsequently were revised:

1*) What patterns of verbal interactions concerning the construction of mathematical knowledge exist within small-group discussions?*

2) What patterns of verbal interactions concerning the construction of mathematical knowledge exist in whole-class discussions?

3) What relationships exist between small-group and whole-class discussions?

4) What is the role of the teacher in this class in establishing and maintaining discussions?

With the methodology outlined in my research proposal, I began to informally observe in Mr. Lester's Algebra classes as a preliminary tool to decide which class would participate in the study. Mr. Lester and I discussed which class would be most amenable to audiotaping. After initial observations and continued conversations with Mr. Lester, we decided that I would focus on the fourth block Algebra class (this class was more cooperative in providing consent and working with the teacher). I began observing the class, taking notes in my personal log, and audiotaping discussions when Mr. Lester attempted to use small-groups to help facilitate whole-class discussions. During my initial observations (conducted informally before the two main units of study) I wrote in my log about working with small-groups:

> Remember that I need to look at days when there are small and whole-class discussions on the same day (not taping on days when largely teacher-led traditional questioning). I really like this class in terms of its attitude - the students are cooperative and listen. I think that they could be involved more with notes or questioning if they had more opportunities to give explanations at certain times (March 11).

At this point in the study, I was sensing that the students had the potential to productively contribute to ensuing discussions, although opportunities to do so were limited.

On March 25, I began to collect classroom discussion data in whole-class and small-group settings. I was also completing short pre-post lesson interviews with Mr. Lester concerning his plans for the class and resulting reflections of what happened during each class session. I taped some small-group discussions to acclimate the students to my presence and purpose and determine the best arrangement for this procedure. As I listened to audiotapes of the small-group discussions, I realized that there was not enough mathematical discussion occurring to warrant a productive analysis. The structure of the groups was minimal which partially accounted for the lack of student-generated discussion.

With this realization, I talked with an advisor about how to more effectively collect data. We devised a schedule in which I would collect data over two particular units of study, use pre-post lesson conversations, and complete three interviews (one at the beginning of the first unit, one between units, and one at the end of the second unit). Mr. Lester and I chose the units according to his schedule and our agreement that the units be representative of typical Algebra class topics. The first unit started on April 9 with a whole-class exam review. At this point, I began daily audiotaping of whole-class and small-group discussions. The next day (April 10), I wrote:

> Students are working in groups, but Mr. Lester did not "officially" set up the groups, so I did not tape them today. I did observe and see that they were engaged in the work.

By the end of that first week, I confirmed to myself that collecting data in small-groups was going to be troublesome:

> Group two today has one person missing. Also, they are not arranged in their groups. The teacher did not have the students arrange their desks into groups. I think this is part of [the teacher] not setting up the accepted norms for groupwork at the beginning of the semester. The teacher expected them to work in groups and know what to do. It made me frustrated today that the teacher did not arrange them in groups so that they will be engaged in discussion. Also, students work individually because the task does not require them to work together. (April 12)

I wanted to remain detached from impacting the discussions in the class, so I did not offer any suggestions for grouping students.

On Monday, I taped two groups (although there was a fire drill), but I recognized that my data was probably not going to be substantial:

> The whole-class seems to be talking about other stuff [as opposed to Algebra] except for one pair and a group of males (who were not being taped). This is a concern because I wonder what to do about days when they don't discuss the mathematics. How do I use this? Or do I discount this information? When I listen to these tapes, I'll get a better idea of what they discussed. (April 15)

The discussions were minimal and provided little data for analysis of any patterns of discussion in small-groups. Through the thirteen days of the first unit, I only had three days in which there was the possibility of any group activity worthy of audiotaping. Near the end of the unit, I reflected on the status of the study:

> I've typed most of the observation notes, plus transcriptions. Group two will not give as much information because they were not adjourned regularly (i.e. did not function as a complete group each time I taped). As I finished transcriptions and typing notes [here are] some things I noticed: typical class discussion has only a few students involved who regularly ask or answer questions; pattern of discussion is teacher-led and student-response; students are still not completely adjusted to norms of a discussion-oriented environment; and students who take the initiative to regularly participate often recognize their mistakes, use effective strategies, and ask thought-provoking questions. (April 23)

Alarmed to some degree that Mr. Lester was not utilizing small-groups as an integral part of the class, I began to focus on the struggles that he seemed to encounter daily in the attempts to create an environment conducive to the discussion that he envisioned. I was slightly frustrated myself and wondered about his seeming lack of effective strategies (based on my personal knowledge of his extensive experience in examining reform recommendations). What struggles were evident in Mr. Lester's attempts to facilitate discussion? How did this relate to his beliefs about current mathematics education reforms? How can we address the difficulties of a teacher struggling to reflect in his practice, his philosophy of creating a student-centered environment?

Through these reflections, I determined that my emerging data provided information that could feasibly be woven together to illuminate the struggles in implementing Standards-like philosophies that seemed to plague many mathematics teachers.

The Emerging Perspective

In examining this situation, I concluded that it would be worthwhile to continue observing and audiotaping during the second unit of study originally chosen, but with a new perspective. I determined that I needed to narrow and revise the focus of my study. Specifically the following observations were made: (1) Mr. Lester's plans for the Algebra class, based on his beliefs about the teaching and learning of mathematics, were not being attained as expected; (2) Whole-class discussions occurred in the class but were largely teacher-directed; (3) Small-group discussions rarely occurred and were unstructured when utilized; and, (4) Through conversations with Mr. Lester, it became evident that he was experiencing frustrations in attempting to facilitate the types of discussions that he had envisioned. As a result of these observations, the role of the teacher in establishing and maintaining discussion in the classroom acquired a different focus. Part of my narrowing process was to limit my focus in the classroom data to whole-class discussions.

Some revision was needed in terms of my original research questions. Because small-group data was minimal, the questions aimed at examining patterns in small-group discussions and any relationship to whole-class discussions could not be substantially addressed. The other questions remained, but with changed emphasis as a result of adjusting in line with the substance of the data that emerged in the classroom. The original question, "What is the role of the teacher in this class in establishing and maintaining discussions?," led to two other pertinent questions: (1) What is the emerging role of the teacher in attempting to facilitate whole-class mathematical discussions in a secondary Algebra class?, and (2) What issues does the teacher confront in attempting to implement mathematical discussions in the classroom?

The rationale for this perspective was based in the growing need to examine current reform recommendations (i.e. Standards-like ideas) in mathematics classroom practice. In this case study, Mr. Lester was well-versed in reform recommendations for the mathematics classroom, and from all accounts considered to be a teacher who embraced Standards-based concepts. Although he was not attempting to implement any particular project curriculum, his plans and ideas were consistent with current recommendations. Indeed, the content of his instructional plans were non-traditional in focus. Even with this emphasis, Mr. Lester encountered struggles in developing a student-centered, discussion-oriented Algebra class. The environment of his classroom was not as he had envisioned. Near the end of the second unit of observation, Mr. Lester talked to me about the class environment and I wrote:

> Today, Mr. Lester mentioned that it's amazing that if he puts the students in rows and gives them a worksheet, it's amazing how they sit and do work. This seems to be "evidence" of the students willingness to accept the "normal" [traditional] classroom environment opposed to the struggle of accepting the "discussion" environment. The attempt to implement a Standards-like class environment seems to be dependent on teacher decision making as it applies to structuring the class. The class today is situated in rows. Some students "do" work, while others chat. One girl says, "This is the most boring class we've had." That's interesting to me because, for Mr. Lester it seems to be a relief from facilitating, since on the unboring days he often found it harder to manage. Some students are explaining to others how to complete the problems. (May 16)

In later conversations with him, Mr. Lester implied that students' "liking rows" was largely a sign of student resistance to his non-traditional stance. Although there is some merit to this explanation, the fact that he never actively established a "non-traditional norm" for classroom discussion provides clues for the lack of student participation in discussions.

On the last two days of my observations, the class was completing reviews for final exams. Interestingly, Mr. Lester set up this review in such a way that individuals, pairs, or groups (self-selected) were responsible for presenting solutions to the given problems. Students did this without hesitation. The last two days of classroom transcripts revealed an environment in which there is little teacher-talk, except to provide direction and questions (see Chapter Five). Although this does not provide conclusive data for the notion that Algebra students are indeed willing and able to become the generators of productive discussion, it does provide a picture of what can feasibly occur when the teacher creates a conducive environment.

Based on my analysis of data and my journey through this study, the following chapters illustrate Mr. Lester's experiences with attempting to reform the discussion in his Algebra class. The results provide information that can add to current research in the context of mathematics education reform concerning mathematics teachers' implementation of reform-oriented ideas.

CHAPTER FOUR

EXPERIENCES IN MATHEMATICS EDUCATION REFORM

This chapter provides a picture of Mr. Lester's vision of reform by outlining his professional background and experiences in working with other mathematics educators interested in current reform issues. Three main topics are addressed: Mr. Lester's mathematics teaching background, his prior experiences with reform efforts, and his current role in implementing reform practices in his Algebra class. These are outlined to provide an understanding of how Mr. Lester's experiences with and knowledge concerning reform issues shaped the translation of his vision into current teaching practice.

Developing a Vision of Reform

Neil Lester, a veteran mathematics teacher of 22 years, is interested in and actively involved in attempting to reform his mathematics classroom practice. Through his years as a mathematics teacher, he has consistently sought out opportunities for developing professional knowledge in his field. He has worked through the schools in which he taught and with university educators to learn about and share knowledge concerning reform in mathematics teaching and learning. He has participated in local, state, and national reform efforts at varying levels. At the time of this study (1995 -96) Mr. Lester was teaching secondary mathematics, at Perry High School, in Virginia.

Teaching Background

Mr. Lester's classroom teaching experience began in 1974, at a high school in Virginia. In his first teaching experience he taught Algebra and Geometry classes. At that time, these classes were distinguished from lower level mathematics classes by labelling them as academic. As Mr. Lester described, "Back then, the majority of math that students [took] was in fact the general math...the majority of the students didn't take the academic courses." After three years of developing as a new teacher, Mr. Lester took the opportunity to teach at a nearby middle school. There he taught seventh grade mathematics and an Algebra class consisting of eighth grade students. Again these were considered to be higher level courses intended for a select group of students who were successful in the school mathematics tradition.

With the gaining of teaching experience and exposure to classes at both the middle and high school levels, Mr. Lester began to take initiative in developing a vision for reforming instructional practice in mathematics classrooms. For example, one summer, he and other mathematics teachers at the middle school completed some writing with the intent of identifying skills that their students needed to master according to the Standards of Learning (SOLs) for Virginia. As a result, they developed a program in which each teacher would focus on several topics during the year. Each teacher would teach a specific topic for three weeks with a group of students. Students would then change classes by attending another class that would aid them in addressing mathematical skills for which they needed more instruction. Mr. Lester described this practice as an alternative to traditional teaching:

> So, for three weeks [my class] may have been concepts and fractions, and the next hree weeks [my class] may have been computation with decimals, then the next three weeks [my class] might have been solving problems with percent, the next three [we] may have been some other skill or concept. And, I thought it was kind of neat, kind of innovative.

This experience can be perceived as a contributing factor to Mr. Lester's increasing development of a reform vision based in the context of his changing classroom practice.

Mr. Lester's next professional move was to Perry County where he is currently employed as a mathematics teacher. During his twelve years at Perry High School, Mr. Lester has taught mostly Geometry and periodically Algebra, Algebra Two, and Trigonometry. Throughout this time he garnered a wealth of classroom teaching experience and continued to reflect on his practice. Specifically, he considered how his conceptions about teaching and learning could impact his role and the role of students in the classroom, identifying the NCTM Curriculum and Evaluation Standards as a guide for his vision:

> When the Standards came out ... I think, to a much greater degree they reinforced the things I had thought. But, of course, they went much further, gave me a little bit more of a direction, instead of just me thinking, what do I think I ought to do next or what

> direction should I go. The Standards pretty much fueled what I was trying to do.

As many mathematics teachers who were thinking about their classrooms in contrast to traditionally held conceptions of mathematics instruction, it was after more exposure to the Standards, that Mr. Lester discovered a professional niche for his views concerning reform strategies:

> Well ... I think by the time the late 80's rolled around I was, you know, trying to do a lot of different things in my classroom. But then I was sort of, I really was basically doing it on my own, sort of stumbling around in the dark, wasn't really sure exactly what I wanted to do. I really got into a lot of problem solving and student explanations and class discussions and things because I really got excited about the kinds of things that students could create without having to be told.

Here, Mr. Lester's beginning vision of reform evidences the uncertainty that accompanies changing practice. He talks about the excitement of student learning that is more "creation oriented," but explains his uncertainty about the direction he would take.

As cited in Ball's (1993) report of teaching elementary school mathematics, the struggles that are encountered by teachers in changing educational environments are characteristic to teaching. Practice in changing environments is often uncertain. In commenting on her encountered struggles in implementing change, she says:

> Still, aiming to create a practice that is, at once, honest to mathematics and honoring of children clearly heightens the uncertainties. The conception of content is more uncertain than a traditional view of mathematics as skills and rules, the view of children as thinkers more unpredictable. (p. 394)

Similarly, Mr. Lester professed aims to use instructional strategies that would build on student learning of concepts through their own development. In informal conversations with him, he frequently mentioned his support of constructivist principles underlying mathematics education reform. If these conceptions led to a pedagogy that did give students more power in creating mathematical meanings, then as Ball notes, Mr. Lester might expect more unpredictability in his practice.

Experiences with Reform Efforts

During the late 1980's and early 1990's, Mr. Lester also became more involved in working with other mathematics educators who were interested in current reform movements in mathematics education. As the result of a recommendation from another teacher, he was asked to be a reviewer for the analysis of the Fourth National Assessment for Educational Progress (NAEP) report. This experience evolved into the beginning of Mr. Lester's immersion into the National Council of Teachers of Mathematics (NCTM) Standards documents. After a summer of working with the NAEP report he explained, "The Fall of the year they started, they had a draft of the Standards [Curriculum and Evaluation], and they asked me if I'd be interested in participating in some discussions of that, so I did." Mr. Lester was genuinely interested in and willing to share in the developing discussions.

These experiences initially served as a catalyst for more involvement, as stated by Mr. Lester:

> So when I got involved with [reviewing the] NAEP, I started reading some of [the report], you know it sort of reinforced a lot of things. I mean obviously it was much more well-developed than I was at the time, but it sort of gave some credence to some things that I had thought about and students, what they were learning and not learning, and you know it pointed out weaknesses in the programs, I mean it really did.

This statement subtly implies that through discussion with other educators, Mr. Lester was able to not only discover what he perceived as weaknesses in mathematics programs with which he was involved, but also to recognize that some of his practices would need to change. Conversations in which educators have opportunities to openly express their uncertainties are important. As Ball (1993) writes, "... the incentives for honest and constructive conversation are lacking [in the professional community of teachers]" and:

> On the one hand, acknowledging pedagogical difficulty is too often tantamount to admitting processional incompetence. On the other hand, the tone of some articles and workshops seems to convey that there is a "right way" to motivate children, to teach place value, or to respond to certain kinds of questions from students. Between these two opposing approaches to problems of practice lies little territory for thoughtful teachers to discuss with others the uncertain challenges of their work. (p. 395)

Mr. Lester credits his involvement in discussion with other educators as a partial validation of his own vision of reform. Through these discussions he could thoughtfully reflect on the challenges offered by reform recommendations.

Involvement in professional activities seemed to become more common for Mr. Lester, especially as the NCTM Standards began to become noticed by mathematics teachers and other educators around the country. Mr. Lester's own professional activity heightened. In the Spring of 1989 he attended the NCTM Conference in Orlando where the Standards were first introduced and he came back with information to be used in introducing the Standards to his school and district. Along with a university professor, "We had a presentation for all the math teachers in Perry County where we showed one of NCTM's [promotional] videos." The presentation started conversations in the county and his school about the impact that the Standards would have on mathematics classrooms in the coming decade and several teachers expressed interest. Although a few teachers including Mr. Lester began to reflect more heavily on the guiding influence of the Standards, he admits, "In terms of a county wide effort that was pretty much the end of it until the Spring of 92."

Mr. Lester was also involved in attending mathematics seminars through the graduate program in which he was enrolled. These seminars provided more time for discussion with other mathematics educators and some direction in terms of his thinking about his own instruction and mathematics teaching in general:

> When we were looking at the draft of the Standards, and then after the Standards came out, we also just kept that seminar, you know that same [mathematics education] seminar. I mean they actually had that for several years and we would look at research on learning and research on problem solving and research on that and I guess there were four or five semesters of those. There again, a lot of stuff that I was reading as a result of that, sort of gave me answers to a lot of the questions about how students were or were not learning, and sort of reinforced a lot of things that I had seen or helped me to understand a lot of the things that I have seen.

His continued reading of professional literature combined with reflection on his own practice, kept Mr. Lester working on reshaping his ideas concerning reform. He comments that some of his questions about student learning had been answered to some extent. Also, the further discussions reinforced his understandings of ideas he encountered in the current reform literature. These experiences continued to build Mr. Lester's base of knowledge and strategies that potentially could impact his teaching practice.

At Perry High School several mathematics teachers along with Mr. Lester were talking about the Standards during the 1992 - 93 school year. This appeared to be evidence of the further development of ideas consistent with reform efforts:

> I think the teachers in Perry were sort of dialoguing and starting to share ideas and actually were becoming a little bit more active. Some of us had attended a workshop on the graphing calculator when it first came out. I think it was fifteen days on the graphing calculators, where the Standards weren't addressed specifically, but you were with fifteen other teachers who were really interested in changing how they taught and so you naturally had these dialogues which I'm sure carried over into the classrooms. Other teachers...started getting involved in reform type stuff. We [Perry High School mathematics department] started to be really aggressive in terms of attending conferences and things.

Through his continuing initiative in learning about current reforms, Mr. Lester became involved in revisions of curriculum for the county. The mathematics revision team for the county was designed to have the teachers examine the county mathematics curriculum in attempts "to try to line it up with the Standards and at the end of that year, as a result of us having redefined who we were, we would be better prepared to adopt textbooks. That was the plan." The redefinition came slowly and Mr. Lester felt somewhat at odds with several teachers. The team decided to compile a notebook of information for dissemination to those teaching Algebra One and talked about using the benchmarks in the Standards as a guide:

> We put research articles in there, put sample lesson plans in there, sample assessments, etc... It was a very, very primitive effort and a lot of the stuff that was put in there was very traditional in nature. Cause even though the people on the reform team [said] "Yeah, yeah, yeah we want to do this, we want to do this," very few of them had any real understanding of what that kind of reform meant. So typically what they tended to do was include things that they had used successfully in their classrooms.

Interestingly, Mr. Lester almost indicts the teachers who he perceives to be lacking any in-depth knowledge concerning reforms in mathematics education. His explicit stance is that their use of successful "traditionally used" activities is tantamount to ignoring recommended reforms.

His self-reflection begins to encompass a kind of comparison of his extensive examination of reform efforts with the behavior of other mathematics teachers who were not necessarily immersed in or endorsing current reform ideas. As Weiss (1995) reported, although secondary mathematics teachers have extensive preparation in mathematics, they are generally less supportive of the use of reform-oriented strategies and feel less confident in doing so. Even so, "more than two-thirds of those [teachers] in grades 7 - 12 consider themselves to be 'master' mathematics teachers" (p. 13). In contrast to this, Mr. Lester was outwardly supportive of changing instruction to include more reform-oriented strategies and seemed confident that his vision of reform could be achieved. Mr. Lester's comparative description of other teachers' attitudes reveals the struggles inherent in the context of reform. As the aforementioned dialogues moved towards reforming actual classroom practice, teachers were confronted with expectations of changing the environment of their mathematics classrooms. A commitment to embracing current reform ideas would involve more than the adoption of new texts. Instruction would also have to shift and the class environment would necessarily change (NCTM, 1991).

At this point, Mr. Lester was also on the verge of accepting a temporary position with a nearby university as a mathematics instructor. This position involved teaching some freshmen level mathematics courses and working with preservice teachers in the mathematics education program. Through previous relationships formed through the masters degree program and involvement in seminars, Mr. Lester had worked as a cooperating teacher for several years with student teachers. The opportunity arose for him to teach a secondary mathematics methods course for student teachers, so he took it. Mr. Lester was temporarily on leave from working daily in the mathematics classroom at Perry High School, but his ties to the county remained.

The math revision team, previously headed by him, was now steeped in the process of textbook adoption. The manner in which Mr. Lester talked about his encounters with teachers during this process changed slightly after he began working at the university. The change seems subtle in the words he used to describe his perception of what the county mathematics teachers were thinking:

> Yeah, we don't like this book. It doesn't have enough skill problems. In the book they wanted of course, at the end of each section there was an entire page that didn't contain any words. I mean it was solve the equations and then there were sixty equations. Or they said things like, "Well there aren't enough [of] those C type problems, not enough of those really, really hard equations." And the teachers who were more familiar with the Standards would say things like, "Well, why do students need to solve those equations?" Any student who would actually ever use an equation like that is going to solve it on the computer, or solve it on the calculator, and probably is never going to solve an equation like that.

His remarks implicitly point again to a division between teachers in terms of their attention to the Standards as it pertained to reform in their classrooms. His constructed perception draws a line between a Standards-intending teacher and a non-Standards, or traditional teacher. There was a seemingly unavoidable polarization developing between them. Mr. Lester used language similar to that in the Standards when he talked about this perceived division. As he described the mathematics teachers in Perry County who he considered to be working within a pre-Standards conception of teaching mathematics, he said, " They were really hung up with symbol manipulation, memorization, recall, those kinds of things and they were not focusing on the concepts or understanding at all." Those teachers who were conducive to the idea that Standards-type reform in the mathematics classroom was at least worthwhile to consider, were described differently: "Yeah, at least we thought that working towards that was important. But we were pretty much roadblocked by that one group of teachers that said, 'We don't think that those things are important'."

Research suggests that experienced mathematics teachers will continue to rely on their own models of effective teaching and tend to incorporate pieces of reform that are consistent with their prior views (Smith, 1996). Even with teachers who express a positive reception to reform, their classroom reform "is more a matter of assimilating surface features to their current teaching practice than giving thoughtful consideration to the basic principles" (p. 395).

"Doing" Reform in Current Practice

When Mr. Lester returned to full-time classroom teaching at Perry High School during the1995-96 school year, the reality of reconciling his continuing development of professional knowledge with current classroom practice was at the forefront of his planning. His two year appointment as a college instructor had been filled with immersing himself in the Standards documents and other mathematics education research as he worked with pre-service teachers and continued his own graduate education. As he began to change his instructional strategies towards ones that he perceived to be more Standards-like, uncertainty in practice was evident. For example, in his expressions about student feedback he talked about his struggle with students' perceptions of his class environment:

> I mean, I've had students come back, and say I wish I were in your math class again, but while they were in my math class they couldn't wait to get out. So I always felt that, I wished you would have told me that while you were in my math class. [I've had students recently who said] that they really enjoy the way I do math, because it is different. They

> feel like they can be more risk takers, they feel like their ideas are, well, I can have good ideas. You know, I was never allowed to have good ideas in a math class before.

Mr. Lester's reflections here suggest that translating his vision as a Standards-intending teacher into effective instructional strategies presented challenges for him and his students.

This study began in the second semester of the 1995-96 in one of Mr. Lester's Algebra One classes. The first semester served as a kind of re-acclimation to the high school environment. However, his ample previous secondary teaching experience made acclimation a minor aspect of the year's experience. During a pre-study conversation with Mr. Lester, he outlined his ideas for reforming his practice. He mentioned that he had used the Summary of Changes in Content and Emphases in 9 -12 Mathematics (NCTM, 1989, p. 126-127) as a guide for his planning. This guide calls for decreased attention to: word problems by type, the use of factoring to solve equations and simplify rational expressions, operations with rational expressions, and paper-and-pencil graphing of equations by plotting points. Attention is to be increased in: the use of real-world problems, the use of technology to develop conceptual understanding, and applications of mathematics concepts. In Mr. Lester's vision, he related his plans to create a classroom environment that benefited the content and social learning needs of students. He envisioned the teacher's role as a risk taker, supporter, listener, and questioner. He thought that the classroom should be a safe place for students to take risks; a non-threatening environment that is thought and question provoking. He also planned to use graphing calculators and manipulatives as an integral part of the class.
Concerning this goal he stated:

> At the beginning of the year I say, look, I'm not in this classroom to teach you a whole bunch of stuff. But we talk about how much knowledge there is versus how much we can possibly learn in a year and I told them I want you to know up front, one of my philosophies is, there are some skills I want you to have because you need some foundation skills to build other skills. But I'm really more about you becoming a better learner of mathematics than I am concerned about you remembering a bunch of skills. I can't teach you everything there is to know, but if I can give you the skills, then next year,

> the year after, the year after, you'll be able to learn a lot more.

This comment conveys the idea that Mr. Lester wanted students to acquire thinking and problem solving skills that will be a "foundation to build other skills." In this light, he appears to be embracing the constructivist principles that are the basis for mathematics education reform. However, there is a seeming contradiction to these ideas when he continues, "I can give you the skills."

In terms of the role of students, Mr. Lester originally planned to use small-groups (he called them "co-ops") to facilitate his belief that "each student should strive to be a generator of mathematical ideas". From this perspective, discussion would involve negotiation of meanings through mutual communication in which students are encouraged to develop ideas (Wood, Cobb & Yackel, 1993). He expressed the belief that students are capable of generating ideas that could add to the development of conceptual understanding:

> At least if they hear other students coming up with the ideas, it will be easier for them to accept, than it would be if I just had to tell them. That's been my experience in the past. Any time a student comes up with the idea, everybody believes it more than if the teacher tells them.

Again, Mr. Lester alludes to the constructivist views that build on the social interaction of student ideas. This belief necessarily implies that students become an integral part of "coming up with ideas" in an environment that allows them to express their ideas to other students in meaningful ways.

From my observations over the semester, there was much in the Algebra class that complimented Mr. Lester's plans for reforming his practice. Mr. Lester usually began the class (a ninety minute block) with an entry activity intended to get students started on time and to motivate the discussion for the day. Although the patterns of discussion were fairly traditional, the class was not representative of conventional lecture in which "silence, memorization, and imitation have long been the hallmarks of conventional mathematics instruction" (Silver, 1996, p. 128). In fact, from an observer's stance, it was evident that Mr. Lester spent a great deal of time and effort trying to generate non-traditional problems and tasks. His teaching plans and views reflected more of a process-oriented approach than a content-oriented approach (Thompson, 1984). That is, Mr. Lester represented his view of mathematics as discovery of relationships rather than the acquisition of skills and facts. However, similar to one of the teachers in Thompson's study, Mr. Lester's professed views about mathematics teaching were observable in his effort and planning, but, were often not apparent in his practice.

The tasks that Mr. Lester developed were indicative of his extensive preparation in reform-oriented recommendations for mathematics instruction. He incorporated problem sets that aimed at the development of general concepts rather than relying heavily on the textbook as the sole guide for skill-based problems. Students were not asked to complete traditional skill worksheets that were repetitive and stressed basic recall of facts.

Mr. Lester also used manipulatives when he thought that it would aid students in developing ideas for further discussion. For example, he spent a great deal of time using algebra tiles to develop the idea of multiplication as an area. At the beginning of the unit he started by showing a grid of 1 X 1 unit squares with a large rectangle that was divided into several smaller regions. He asked the students to determine the area of the large rectangle. He used this to illustrate multiplication (26 X 35) by dividing the large rectangle into smaller regions and added as follows: 600 + 180 + 100 + 30 = 910 + 26 X 35. The next day the class was asked to use algebra tiles to construct a rectangle that matched a model he had put on the overhead. Then, they tried to identify the dimensions of the rectangle. This led to asking students to construct their own rectangle given particular dimensions (e.g. $(x + 3)(2x + 3)$) and then (similar to multiplying 26 X 35) find the area of the rectangle. They spent four classes using the tiles to construct rectangles, leading to deconstructing (factoring) and also graphed quadratics with the calculators. The next week, these ideas led to finding solutions for quadratic equations by factoring and using the quadratic formula. To motivate the quadratic formula, Mr. Lester used the tiles to represent the quadratic x2 + 8x + 5 and asked if it could be made into a rectangle (which led to examining completing the square and then the quadratic formula).

Mr. Lester was incorporating into his class many of the recommended activities for reform to which he had been exposed. It was evident from the amount of material he gathered, that he spent a lot of time searching for "better" ways to present topics for discussion. He also worked to develop lessons that covered content outlined in the Virginia SOLs while also focusing on ideas that could potentially facilitate discussion. He often used the Standards as a guide, but also admitted that in the context of practice, they appeared to offer a somewhat idealistic view of the level of mathematics content for some secondary students. The Standards offer an explanation for the appearance of an excessive amount of content at the secondary level by saying:

> When this content is evaluated, however, it should be remembered that the proposed 5 -8 curriculum will enable students to enter high school with substantial gains in their conceptual and procedural understandings of algebra, in their knowledge of geometric concepts and relationships, and in their familiarity with informal, but conceptually based, methods for dealing with data and situations involving uncertainty. (p. 125)

But, Mr. Lester addresses his concern in attempting to work with secondary students in this context. The fact was that their background was not consistent with the Standards vision of a middle school curriculum. Mr. Lester's perception of his students' understandings or lack of understandings was focused on their lack of readiness rather than on their difficulties in making sense of concepts.

Like many other teachers being studied in the context of reform (Peterson, 1990), Mr. Lester presents a conservative view of the students' abilities to make sense of mathematics rather than acting on the idea that children possess much informal mathematical knowledge that could provide for fruitful discussion. His frustration with feeling unsuccessful at facilitating worthwhile whole-class discussions is seen here:

> To have a conversation that is rich in mathematical ideas, requires some level of maturity. That's why I'm trying to tone it down, I'm thinking in terms of let's try to have a conversation that maybe a fifth grade teacher would have in their math class, while trying

> to focus on ninth grade math. But I'm saying let's try not to get them too far, and I'm thinking that maybe in the algebra class there were a lot of students I lost, just because, even if they gave it their best shot, they hadn't developed the thinking and conversation and discipline and those things that would be necessary to participate in that kind of conversation.

Here, Mr. Lester presented his view that in his Algebra class he had provided an appropriate environment in which students were encouraged to participate in discussion. His reasoning for the lack of student involvement is based on the view that they were unprepared to participate. This seems contradictory to Mr. Lester's goal as expressed in his plans for the semester. His goal in terms of students' mathematical behavior was in "changing their way of thinking about mathematics to something that somebody has to give me, to something I can develop for myself; something that I can only use if someone shows me how to do it into, well, I now have the confidence to use my mathematics in new situations." Developing and utilizing strategies to effect this type of behavior presented a continuing challenge.

The descriptions of Mr. Lester's perceptions of his classroom practice point to the frustrations that arise when faced with the prospect of fulfilling professional beliefs. Mr. Lester was now in the position of constructing a feasible path towards the connection of his beliefs and practice. His background experience in professional development activities and study of professional literature in mathematics education certainly served as sound preparation. Involvement in introducing the NCTM Standards to county mathematics teachers and participation in conferences gave him a solid basis of information concerning the implementation of Standards-like strategies. Still, practicing his vision offered challenges. Ball (1992) places this challenge in perspective when reflecting on her own practice in the context of reform: "Still, placing the text of the Standards alongside a few minutes of classroom work makes clear that while the Standards provide guidance for my work, they do not - indeed, cannot - prescribe it" (p. 14).

When my study of Mr. Lester's Algebra class began, he had an extensive teaching background and ample exposure to pedagogical reform efforts in mathematics education. With one semester completed after his return to the secondary classroom, the struggles inherent in reform were now ones that Mr. Lester had to face first-hand. He was faced with the process of attempting to use instructional strategies consonant with his ideas concerning reform. The next two chapters provide views of Mr. Lester's actual class discussions and the challenges encountered in attempting to develop discussions in an environment based on his reform-oriented conceptions.

CHAPTER FIVE

DOCUMENTING CLASS DISCUSSIONS

This chapter presents a view of the class discussions that occurred in Mr. Lester's Algebra class. In particular, the patterns of verbal interactions were examined to document the role of the teacher in attempting to facilitate class mathematical discussions. The patterns that emerged in Mr. Lester's class discussions were largely teacher-directed and resembled a traditional initiation-reply-evaluation sequence. In attempting to facilitate class mathematical discussions, his role was that of a director who maintained discussion largely through posing and promoting questions, and providing clarification of ideas.

Class Discussion Patterns

Patterns of interaction developed in classrooms are the result of social interaction and are jointly constructed by the teacher and students. Wood, Cobb, and Yackel (1993), call these patterns "hidden regularities" that either encourage or limit opportunities for student engagement in meaningful learning in the classroom. Verbal interactions occurring during whole-class discussions act to encourage or inhibit the sharing of ideas, questions, and reasonings among members of the class. In examining class discussions in Mr. Lester's Algebra class, the dominant occurring pattern was one in which the teacher elicited and maintained the direction of discussion; there were limited opportunities for student-initiated discussion.

Mr. Lester's descriptions of his plans for his Algebra class included a desire for students to become more participatory in discussions. For example, as outlined in Chapter Four, he wanted an environment in which students participated in generating thoughts, questions, and ideas. As the semester progressed, it was evident through observations and his comments that the discussion was not as fertile as desired. Mr. Lester expressed this struggle when he talked about the lack of conversation skills of the students. Classroom discussion, instead of developing as a shared process in which mathematical discussions would lead to understandings of concepts, actually could be divided into distinguishable categories of teacher-talk and student-talk. Mr. Lester's role during class discussions was largely teacher-centered, even in cases where he was evidently attempting to promote discussion. Through his attempts to generate and maintain discussions, he often re-explained student questions or responses to provide clarification. Although this appeared to be an attempt to use and encourage student ideas, the result was that he either explicitly or implicitly controlled the direction of discussion. Students rarely participated to the extent that their contributions would generate a class discussion without teacher prompting. This whole-class pattern resembles the initiation-reply-evaluation pattern found in many classroom studies. In this instance, though, Mr. Lester often avoided the evaluation portion of the pattern as he consciously refrained from the typical use of short evaluative phrases such as "Good answer", or "That's correct". Rather, he used rephrasing or re-explaining of student responses as a form of reinforcement. Unfortunately, this tended to reduce further student questioning and to some extent accounted for the lack of student-driven discussion.

The verbal interactions that existed in class discussions were coded according to teacher-talk and student-talk (see Chapter Three for methodology). Each class session during the two units was characterized by several instances of many of these interactions. The interactions that occurred most frequently were: maintaining, clarification of ideas, and responding. Others that occurred frequently were: promoting, posing, and presenting. Sequences of these interactions formed patterns that actually acted to discourage rather than encourage participation in discussion. The nature of the exchanges during whole-class discussions illustrate that through Mr. Lester's attempts to maintain discussion by paraphrasing and clarifying students' ideas, the focus inadvertently became teacher-centered as opposed to student-centered. Next, I provide a picture of actual class discussions to illustrate the sequences of verbal interactions that formed patterns of whole-class discussions.

Sequences of verbal interactions were constructed by the participants each time a class discussion occurred. The sequences consist of teacher-talk and student-talk. As sequences were repeated, it was natural for patterns to develop. In Mr. Lester's class, several sequences of verbal interactions contributed to the teacher-directed nature of class discussions. Discussions resembled traditional patterns (Edwards & Westgate, 1994) in which Mr. Lester talked the most, asked many questions requiring only brief answers, and communication was limited among class members. Mr. Lester used a turn allocation procedure in which he often directed student responses towards one answer (Wood, 1995). These patterns will be outlined by the following:

the pattern of posing-responding-maintaining, the pattern of promoting-responding-clarification, and the pattern of extending-responding.

Posing-Responding-Maintaining

The pattern of posing-responding-maintaining was similar to the traditional initiation-reply-evaluation pattern of classrooms. Mr. Lester posed questions in an attempt to elicit student thinking, a student responded, then Mr. Lester maintained the discussion by taking the lead. In the following illustration the class is looking at solving systems of equations. Mr. Lester first asks the class to pick ordered pairs for the equation n + d = 13 as he graphs them at the overhead to form a line. Then, he tries to get the students to find ordered pairs for the second equation in the system [5n + 10d = 90] by posing a question. He most often maintains the discussion by repeating a student's response.

Teacher: Can you give me some ordered pairs on the second equation?

Ryan: 4 and 7.

Teacher: Five times four is 20 and 7 times 10 is 70, so 4 nickels, 7 dimes. Okay.

Ryan: Will that work? Yeah, it will. ((kind of talking to himself))

Teacher: Erika?

Erika: 10 dimes and 0 nickels..

Teacher: 10 dimes and 0 nickels

Erika: Nine dimes.

Teacher: How about nine dimes?

Erika: Nine dimes.

Teacher: Nine dimes and no nickels. Stefan, give me an ordered pair.

Stefan: Uhm, I don't have one. ((Another student looks over and tells him one)) 18 nickels and no dimes.

Teacher: Okay that'll work. And let's get one more in there. Yeah, Davis.((raised hand))

Davis: Five dimes and 8 nickels.

Teacher: Five dimes and 8 nickels. Obviously if we connect these ((draws on overhead)),what's our solution? ((students say things here, but inaudible)) 5 dimes, 8 nickels. Does that work? Is 5 dimes and 8 nickels the solution? Our solution is in fact, what was it again?

Several: 5 and 8.

Teacher: Five dimes and 8 nickels. So we can solve the system if we can write two equations and then if we can graph those equations we can find their intersection. ((They continue to another problem)) Gotta come up with an equation. We need some equations here. See if we can write equations for a and b. Someone give me an equation for the first one there. Tammy?

Tammy: 15 TF plus 9 MC equals 81.

Teacher: I'm going to just put one letter for each one okay? [15T + 9m = 81] How about the second one? Jamel?

Jamel: 13T plus 11m equals 83.

Teacher: Okay, okay. Two good equations, alright. Now suppose I said okay let's graph it. Give me some ordered pairs. What's going to happen because you got numbers like 11 and 13 and 15 and 9 and 83...What's coming up with ordered pairs going to be like?

Student: Messed up.

Teacher: In reality, more situations have fractions than not.

In this illustration, Mr. Lester posed questions and listened to student responses; however, the structure of the discussion allowed for little expansion of student thinking. In the first question posed, the student responded by giving an ordered pair, and then Mr. Lester provided the explanation of how that ordered pair fits into the equation. He maintained the discussion, but on his terms. He continued to solicit responses to generate other ordered pairs, but does not explore students' reasons for their responses.

Later in the same unit, the class is asked to get into groups and talk about the solutions to some exercises in which polynomials are divided by monomials. After reconvening as a class to share their solutions, Mr. Lester began by posing a question that seemed to be addressing a familiar mistake in dividing algebraic expressions:

Teacher: How about the third one Charles? (8x2y + 12xy2 + 20x2y2)/4xy

Charles: 2x plus 3y plus 5xy.

Teacher: 2x plus 3y plus 5xy that's what you said? Anybody like Charles to explain that or... Charles, we've got a request that you explain how you got that one.

Charles: Divided the four by the numbers.

Teacher: Wait a minute, you said something about four by the numbers so you're saying 8x2y divided by 4xy. Four divided the eight gives you two. And the x, you have x2 divided by x, that's just one of the x's divides out and you have an x. What happens to the y's?

Charles: They're completely gone.

Teacher: y divided by y is?

Charles: Zero.

Teacher: Well, no, y divided by y isn't zero. What's y divided by y?

Student: One ((says quietly)).

Teacher: ((still talking to Charles)) I bet you know ((prompts)). What's three divided by three? What's ten divided by ten, what's twenty divided by twenty? So what's y divided by y?

Charles: One.

Teacher: Now, what happens when I multiply by one?

Charles: Stays the same.

Teacher: So the first term would just be 2x. The second term would be?

Charles: 3y.

Here, Mr. Lester although attempting to provide guidance for a familiar misconception ended up leading the discussion. He explicitly asked Charles to provide an explanation for his response, but accepted Charles' minimal explanation.

Another similar pattern that acted to discourage student participation was the pattern of promoting-responding-clarification. The difference in this pattern is subtle. In contrast to posing, promoting is initiating discussion about a topic in general.

Promoting-Responding-Clarification

The pattern of promoting-responding-clarification illustrated a procedural pattern in which Mr. Lester promoted an idea, students responded, and then Mr. Lester clarified the student response. In the following illustration, the classroom arrangement was changed to rows of desks for a tutorial-like lesson on solving systems of equations. The previous day had been filled with confusion as the students worked in groups, so rearranging the desks was an attempt to provide some structure to the environment to facilitate the lesson. Mr. Lester attempted to promote student thinking about the strategy involved in using graphing as a technique to solve systems of equations:

> Teacher: To use the slope-intercept method of graphing a line we need to have an equation that says [y equals mx + b]. To get that out of this equation [2y = 3x - 3] divide both sides by 2, so we get y = 3/2 x - 3/2. What does that tell us?
>
> Joel: I got negative four over two.
>
> Teacher: What information does that give us?
>
> Erika: The slope and y intercept.
>
> Teacher: The slope and y intercept. Which one's which?
>
> Erika: The slope's with the x. The slope's the one with the number in front of the x and ..
>
> Teacher: Okay, for this problem what's my y intercept?
>
> Erika: Negative 3 over 2. ((the teacher plots this point on overhead))
>
> Teacher: How do I use slope to graph it there?
>
> Erika: Over 2 and up 3.
>
> Teacher: Okay, over 2 and up 3. Or you can go back 2 and down 3 and again hopefully you notice that if we do either one of those things we end up with points that are on a straight line. Yes Joel?
>
> Joel: What happened to the negative 4? ((Teacher reminds him that at the beginning of that problem he told them to change the negative 4 to a negative 3))
>
> Teacher: The issue here is not how we graph. The issue is how do we solve a system of equations by graphing? Can someone pick up from there? Once we've got the equation graphed? What do we know? How do we solve a system of equations by graphing?
>
> Nathan: We know, y intercept and slope.
>
> Teacher: Well, we might know the y intercept and the slope but really that's not the purpose of the problem is it?

Nathan: We know where points intersect, the lines. ((Nathan corrects himself))

Teacher: We know where the lines intersect, sure. And where do the lines intersect?

Nathan: At (3, 3).

Teacher: At (3,3), right here. There's our intersection. That's the point (3,3). Where your lines intersect is the solution to your system. So in this system the lines intersect at the point (3,3). Which means that's the one ordered pair that is a solution to both equations.

As Mr. Lester clarified Nathan's response, this tended to reinforce the view that the teacher's explanation is the valid one.

Another illustration of the promoting-responding-clarification pattern occurred near the end of the first unit. The class was working through examples of solving a system of linear equations. After solving a system by eliminating the x variable and then solving for y, Mr. Lester prompted the class to talk about how to solve the same system by eliminating y and solving for x. He took a suggestion from a student but then proceeded to provide a re-explanation for clarification:

Teacher: Let's look at what that gives us. ((the teacher writes on overhead as students provide answers)) [$9x + 6y = 45$; $8x - 6y = 24$] So, you accomplished exactly the same thing we accomplished over here. The first time we went through it we multiplied by three and two because that gave us x coefficients that were the same. Then you're saying let's do that same process again but with the y's in mind...What can I do to combine these two equations? Will I add or subtract?

Nolan: I have no idea.

Teacher: Well, what are you trying to have happen?

Nolan: Trying to get rid of x values.

Teacher: Not x values.

Nolan: The y.

Teacher: Yeah the y in this case. So are we adding or subtracting?

Nolan: Adding.

In this scenario, Mr. Lester attempted to promote a discussion of strategy use. Nolan refrains from giving any in-depth response.

The next day, the students were asked to work in pairs to develop a system of equations for a problem:

Teacher: Let's put m here, yeah, I personally think that would be a good choice. [7f + 8m = 67] That gives you one equation. Okay, second equation? Somebody else give me a second equation. Jill, can you give me a second equation?

Jill: 11b + 7d = 83.

Teacher: Okay, except I'm going to use f. You might have different letters. That's okay. Any questions about those two equations? [11f + 7m = 83] How about a third equation? What do I need for a third equation, Lucy?

Lucy: You don't need a third equation.

Teacher: Why not?

Lucy: There's only two variables.

Teacher: Ah, very good, Lucy. Yeah, we only need two equations because we only have two variables. Okay, solve it. Take a few minutes and solve that please. ((pairs look over problem; some do problem individually then check with each other; teacher walks around checking what pairs are doing; after approximately 10 minutes)) Anyone who attempted to do the problem by substitution? Can someone tell me why they didn't use substitution?

Student: Too many numbers.

Teacher: And so what would happen as a result?

Several: Fractions.

Teacher: You're gonna have a lot of fractions. If you do it with substitution your gonna have to solve one of these, you're either going to have to divide by 7 or by 11 or by 8. If you take any of those numbers and divide by that you're gonna end up with a lot of fractions. For that reason alone, substitution probably would not be your best bet.

In some ways, Mr. Lester seemed to be attempting to model here by using student ideas and then providing clarification using his own input. However, the amount of explanation provided for clarification purposes provided the perception that Mr. Lester was looking for a teacher-accepted strategy.

In the second unit, the class was using algebra tiles to look at factoring after a review of multiplying binomials. A student had just finished showing an arrangement of tiles for the factoring of x2 + 7x + 12:

> Teacher: So, what are the dimensions then? In other words, what's the length and width?
>
> Lucy: ((gives answer inaudible))
>
> Teacher: x plus 3 times x plus 4? x plus 3 and x plus 4, okay. How would we write the answer? How would we write that? What would the factors be?
>
> Student: In parentheses.
>
> Teacher: In parentheses, x plus 3 and x plus 4. [x2+ 7x + 12 = (x+3)(x+4)] There's something really important about factoring that we're going to use later to solve problems and solve equations. What's the last operation that's performed here?
>
> Student: Addition.
>
> Teacher: What's the last operation that's performed over here? ((right of the equal sign))
>
> Student: Addition, I mean multiplication.
>
> Teacher: Don't you do what's inside the parentheses first and then the operation between parentheses. Factoring is a way of changing an expression from addition to multiplication. Okay, this process is called factoring and that's what you're going to be asked to do. ((teacher groups students and gives them a polynomial to factor using tiles)) See if you can make a rectangle out of x2+5x+4. ((the teacher walks around to desks)) Okay. Here's an arrangement I've seen on several desks. So, Ned, what are the dimensions of that rectangle now?
>
> Ned: Uh, x plus 4 and x plus 1.
>
> Teacher: Okay, So we can write this as x plus 4 times x plus 1. That would be the factorization. Now, Andria said that she had a different arrangement. How was yours different? ((the teacher refocuses class))

Andria: The rectangle (inaudible).

Jill: Leave on the side. ((teacher moving pieces at overhead))

Andria: Can I just go up there and show how to do that?

Teacher: Sure, good idea. ((She puts pieces on overhead to show her group's arrangement))

Andria: There you go.

Teacher: Okay. So we have a different arrangement. You have to recognize that it might be possible to rearrange the parts of the rectangle in a different way but still, what are the dimensions of that particular rectangle? It's still x plus 1 times x plus 4.

This dialogue was approached as a discussion of ideas before the students were asked to work in groups to find solutions. Mr. Lester attempted to connect Andria's response by paraphrasing or providing information to clarify what she said instead of having her expand on the group's strategies. For example, Andria noticed that her arrangement of tiles was different and she showed how her group arranged the tiles to construct the correct dimensions. But, she was not encouraged to explicitly express the group's strategy use, nor were other students asked to reflect on their idea. The result is that the teacher directs much of the discussion and students provided one-phrase responses.

Extending-Responding

The pattern of extending-responding illustrated more closely Mr. Lester's envisioned role of getting students involved in generating discussion. In the following, he began the discussion by asking students to recall the procedure for graphing a line. The following dialogue illustrates student responses to questions by the teacher that required only short responses. Mr. Lester's rewording of student responses tended to advance these types of responses. The class was being asked to recall procedures for graphing a line. After the first student response, Mr. Lester attempted to ask Tim to extend on his response, but by providing a quick evaluative response, he failed to give Tim the time to provide an explanation:

Teacher: Given the slope of a line and a point, what we need to do is write the equation of the line.

Tim: Use $y = mx + b$ and point.

Teacher: Yeah, yeah, we're going to use y = mx + b. What are we going to do with that Tim?

Tim: That's the equation.

Teacher: That's the general equation

Tim: Uhm, multiply 5 and 3.

Teacher: Really what you're saying is ((teacher says to student that what he is saying is correct and the teacher is rewording it)) Let's put a five in for m because five is the slope and put three in for x because the ordered pair they gave us, three is the x value. And then we also have to put -2 in place of y.

At this point in the discussion, Mr. Lester attempted to get the students to extend their thinking about the procedure for graphing a line. He continued to extend the idea of finding the y-intercept by connecting to the graph of the equation:

Teacher: Now, what do we do to figure out what b is then? So what's my equation then?

Allison: y = -2x + b

Teacher: No, I think you're in a time warp ((teacher talks to a student about the problem they are on, she was not attentive to where they were in the discussion, she may have been working on other problems or just not paying attention. The teacher then shows how to find the equation using the graph, plots the point (3, -2)) What does it mean if my line has a slope of five?

Nolan: Go over one and up five.

Teacher: Go over one and up five. ((teacher draws on the overhead)) If I continue to move in that direction I'm never going to get y intercept, hit the y axis. So, what can I do instead?

Nolan: Go back five and over one.

Teacher: Close, the other way, back one down five. I still haven't gotten to my y axis, what can I do?

Nolan: Go back another one, down five more. ((Nolan and teacher continue to discuss this as teacher draws line until it crosses y axis))

Teacher: That point is going to be my y intercept. What's it going to be? Can you tell me? I started at - 2 didn't I, I went down five, that takes me to : : ((Here teacher leads students as he points on graph on overhead and several students respond to each move))

Several: Negative seven, negative twelve, looks like neg. 17.

Teacher: Isn't that already what we found the y intercept to be?

In these student responses, there were several opportunities for Mr. Lester to redirect the answers to other students or ask the responding student to expand on the response to facilitate more discussion. This did not occur. Instead, a typical question-response pattern resulted. Mr. Lester's language also contributed to this pattern. In his reaction to a student response he said, "Really what you're saying is", which reinforced the idea that the teacher's explanation was the valid one.

How did the resulting patterns of verbal interactions contribute to the characteristic of Mr. Lester's role in the classroom? He expressed the desire to develop his role as a facilitator who would encourage student participation. The next section examines his role in the context of my observations, transcript data, and Mr. Lester's comments about whole-class discussions.

Mr. Lester's Role in Class Discussions

The consistent use of verbal patterns that reinforced the perception of teacher control in classroom discussion may have inadvertently increased teacher authority contrary to Mr. Lester's envisioned role. As a result, his actual role was that of a director who maintained the structure and content of discussion. For example, the following illustrates Mr. Lester's role in directing discussion. He asked the class to recall a mathematical term. Because the students did not initially respond, he provided clarification by rephrasing student responses. He continued the discussion through a sequence of posing questions and maintaining the discussion.

Teacher: The first word start with O and the second starts with P. ((pause)) It's an ordered pair. So every solution to that equation is an ordered pair. How many are there?

Student: Infinite.

Teacher: How do we identify or find an ordered pair?

Roger: Put one number in and solve it like an equation.

Teacher: That's a good strategy. Give us an example.

Roger: Put 2 in for x, subtract by 6, divide by 3 [teacher writes $3(2) + 2y = 12$, $6 + 2y = 12$, $2y = 6$, $y=3$]

Teacher: If (we) put two in for x and 3 in for y, it works. The equation is true. Can someone come up with another one?

Emma: I can. x is, uh, x is one.

Teacher: Okay, let's do it. [teacher writes $3(1) + 2y = 12$, $3 + 2y = 12$, $2y = 9$, $y = 4\ 1/2$ $(1, 4\ 1/2)$]

Emma: Any y would be 6, no, no.... ((teacher continues writing as Emma talks))

Teacher: How many can we find?

Several: Infinite, a zillion

Teacher: Okay, everybody find me another pair.

After Roger recalls a procedure for finding an ordered pair, Mr. Lester commented on strategy use, but Roger is not given the chance to present his strategy. Although exploring multiple strategies can enhance class discussion, there is no follow-up on modeling what it means to have a good strategy. (i.e. the student continues to give a specific procedure rather than a general strategy) At the end of this dialogue there was an opportunity to expand on the student response concerning the number of ordered pairs that can be found, but it is not clear why Mr. Lester did not choose to expand on this. The methods employed by Mr. Lester were contradictory to his intentions to build shared meanings through class discussion. The students who do participate are given few chances to include their meanings in the final analysis.

Another similar illustration of Mr. Lester's role as a director of discussion shows him maintaining the discussion after an initial introduction in which he posed a question about methods for finding solutions to a system of equations. This represents a "typical session" in Mr. Lester's whole-class discussion in which the teacher's statements and questions generated a sequence of one-thought student responses:

Teacher: What do you think? Let me suggest one is better by substitution and one by linear combination.

Rachel: I think number eleven would be better by substitution.

Teacher: Why?

Rachel: Cause it's got the y = mx + b form.

Teacher: The y is already written in mx + b form. So why does that make it easier to substitute?

Rachel: Cause you already know what y is.

Teacher: So, what should I do to finish the rest of that problem?

Rachel: Substitute 5x - 8 for y.

Teacher: What would I get then? ((Rachel talks as teacher writes [x + 2(5x - 8) = -5]. The teacher goes on to say that the problem is nicely set up for substitution which is not unusual; that often you'll have equations where one of the variables is isolated , so substitution is a very common method.))

Teacher: So what do I do next?

Rachel: Solve, do it. Ten x minus 18, uh 16. Then add 16 to both sides, the divide by, no, you add x's together. ((teacher writes)) [x + 10x - 16 = - 5, 11x = 11, x = 1]

Teacher: Now what?

Rachel: Divide by 11.

Teacher: There's x, where's y? What's y Ned?

Ned: You have to do it again.

Teacher: I could do it again, you're right. But I don't want to do all that work.

Ned: Put one in place of x in the other equation.

Teacher: Yeah, put one in place, back up in there for x. ((the teacher continues to solve the equation by asking students to give steps))

When Ned suggested, "You have to do it again," an opportunity arose for Mr. Lester to have him explain his intention for using that strategy. Mr. Lester's response though, further reinforced the perception of his role as mathematical authority. At the beginning of this dialogue, Mr. Lester asked "Why?" in a way that suggested his genuine interest in exploring the student response. The student provides the beginning of a possibly productive discussion. Mr. Lester then turned to directing the strategy by asking basic recall questions to maintain the discussion. In this case, the student was inadvertently denied a chance to expand on the solution strategy. Again, the development of shared meanings is thwarted.

Through Mr. Lester's coding of class discussion transcripts, my observations, and interviews it became apparent that Mr. Lester's role emerged in part through his perceptions of what was occurring in the classroom. For example, when asked, "Would you have done anything differently?", Mr. Lester responded that "enough good conversations" occurred to present opportunities for students to develop ideas. This implies that Mr. Lester perceived his loosely structured approach to discussion as at least partially consistent with his envisioned role. Because of this, Mr. Lester did not necessarily feel that he was projecting a more authoritative role in discussion.

In the second unit, for example, the class was reviewing methods for dividing polynomials. Mr. Lester asked the students to work together to simplify polynomial expressions. The language he used in attempting to connect students' thinking, however, implied that the teacher's ideas were the basis for the validity of other ideas. The exercise (9x3 + 3x - 6) / (3x) is being discussed:

Teacher: Okay, I'm seeing this. ((stops discussion and asks students to look at overhead)) [3x2 + 1] I'm seeing 3x2 and I'm seeing plus one. Is that right? Everybody agree with that? That's okay?

Several: Yeah, yes.

Teacher: But I've got some concern about this last term. I saw a lot of plain old twos.

Several: Negative two, 2x, -2 over x. ((students are calling out answers))

Teacher: Rodney says it's negative 2 over x. I don't know if he's right or not.

Explain how you got -2 over x.

Rodney: Divide 3 by 6, I mean divide 6 by 3. And there's no x, so you can't divide x you just leave it down there.

Teacher: I heard someone over here say you can't do anything with the x so you just leave it alone. Something like that? That's what you said isn't it? But what did you do? did you leave the x? What do you think? We're talking in pretty generic terms… "leave the x".

Nolan: I think it'd be 3x2 + 1 minus 2x. Because when you divide 6 by 3 you get 2 and x is left over so you put it beside the 2. Because when you divided 3x by uh, 9x3, you just left the x beside.

Teacher: Okay. I can see what you're thinking. I think what I would have to help you recall is what the numerator and denominator of a ratio represents. The numerator is multiplication and the denominator is division. So in this original problem this denominator is saying we want to divide by 3 and we want to divide by x. Then you said, cause we couldn't actually do that we just made it 2x, 2 times x, so you're saying wherever you can't divide you just change it into multiplication?

Nolan: Uhm, that problem up there is 2 over x. You can't divide so you can multiply and it'

Teacher: ((gives a counterexample; asking student if he thinks this is correct))
[5/2 implies 5*2 = 10]

Nolan: No

Teacher: I really think you're thinking good ideas. I just want to make sure that what you're thinking is consistent with other things that you already know. Is this what you ordinarily do if that wouldn't divide nice and pretty? But you're saying here that since two won't divide nice and pretty, we're going to change it to 2 times x.
[2/x implies 2*x]

Nolan: It's not going to work.

Teacher: Nah. It's not that you didn't have a good idea. Sometimes they don't work out the way we want them to. In fact, what Randall had come up with, Randall's answer was the mathematically correct one. We can't perform the division by x, so we simply leave it as division.

In this excerpt, Mr. Lester coded his role as modelling (along with clarification). His perception appeared to be that if he saw this as modelling of thinking, then students would begin to see that their ideas were valid.

In interview comments, Mr. Lester said that students hadn't been asked in previous mathematics classes to "think" in the same way that he encouraged, implying that he was encouraging students to think in alternative ways. However, actual discussions reveal that students were not explicitly asked to explain their thinking or strategies. Mr. Lester perceived that he was challenging students to provide explanations, but students often resorted to providing only simple responses.

To further illustrate how Mr. Lester's actual role tended to construct a lack of student empowerment in generating and participating in discussion, the following provides a view of Mr. Lester's impending frustration with attempting to include student ideas. The class is entering equations into the calculator in order to look at solving systems of equations:

Jason: I don't think that's right, I'm just guessing.

Teacher: Well, why not?

Jason: Well it IS right, ((students laugh)) plus four, three.

Teacher: Three what?

Several: C.

Teacher: Well, there's going to be a little problem there.[47=2x + 3y]

Jason: y, x

Teacher: Unfortunately we're going to be limited to x and y's because the calculator doesn't have a c and d mode ....You really have to write everything in terms of x and y. That's going to be a limitation. That's going to be something that you have to provide. There's one equation. How many equations do we need?

Several: Two.

Teacher: Why two?

Nolan: Because you want to know where the intersection is.

Teacher: And if I have two variables, I need two lines. So what's my second equation gonna be?

Student: Just do that backwards

Teacher: Well, that'd be the same equation. What other piece of information is in the problem?

Nolan: y=4, y=4...

Teacher: Wait, wait...((teacher tells them that it is not helpful to just shout out anything that comes to mind without thinking.)) You write equations by looking at entire sentence ideas. Equations are not phrases, remember we discussed that? ((Jason raises his hand))

Jason: I think I've got an equation for the second one. Uh, one equals...

Teacher: One what?

Jason: Uh, one cassette.

Teacher: What do you want to use for cassettes?

Jason: y

Teacher: So, one y.

Jason: And four something.

Teacher: What did he buy?

Jason: No wait, 3.

Teacher: Read the entire sentence.

Jason: He bought 3.

Rachel: He bought 5.

Several: Three CD's.

Jason: He purchased 3 CD's.

Teacher: Which letter?

Jason: x.

At the beginning of this excerpt, Jason is not convinced of the validity of his ideas as Mr. Lester attempted to direct his responses. When Mr. Lester subsequently asked how many equations are needed, several students responded "Two". In an attempt to get students to provide an explanation, Mr. Lester proceeded to ask, "Why two?" However, the question appeared to convey the perception that a particular response had already been formed by the teacher. Nolan's response appears perfunctory and Mr. Lester quickly clarifies, then asked another question. He continued to ask questions that had students responding in the same fashion. The end of the discussion seems to be the beginning of an exchange in which student responses are contributing to the direction of the discussion, but these are not followed-up in any significant manner. Towards the end of the semester, Mr. Lester did begin to see more possibilities for worthwhile student discussion. I provide next a look at discussion in which students were becoming more involved in generating and sharing ideas.

A Glimpse of Reformed Discussion

It wasn't until near the end of the semester that the discussions in Mr. Lester's class began to develop some characteristics consistent with reform recommendations. Specifically, the use of manipulatives and explicit directions for student presentation of solutions contributed to the development of discussion that was less teacher-directed. Using algebra tiles to look at the concept of factoring, a couple of students were sharing solutions by writing on the overhead. Mr. Lester posed a question after a student initiation of the idea that the algebra tiles needed to form a rectangle when using them to represent the factors of a polynomial. The discussion developed into one that was shared among several students and Mr. Lester:

Teacher: Well, if it's a rectangle can you have something sticking outside? ((several students say "well it's not that much")) The problem is that even if you look real close here, that red sticks out on the side.

Andria: Something needs to be done.

Teacher: One of the problems we have in using algebra tiles is

Stefan: They're not even.

Teacher: Well, they're not supposed to be even.

Stefan: Well, they're not square.

Teacher: They're square, they're not even. You can't put together five little ones or six yellows to get a red. See this ways not going to work.

Stefan: Can I try another?

Teacher: You want to try a different arrangement?

Stefan: I'm going to try. ((he puts only yellow squares up on screen; some students laugh; some say you have to use all of them))

Several: You've got to use all of them.

Stefan: It's a little hard.

Teacher: Give him some ideas, what can he try there?

Student: Try red ones sideways.

Andria: Do little yellow things too?

Student: How many are there?

Stefan: You've got to use all these?

Teacher: Can you think of some of the arrangements we had yesterday? When you multiplied polynomials? How were the shapes arranged? Can you maybe use that same kind of arrangement idea? ((Stefan still at overhead))

Erika: Put red one right under square and then put others in. Make a box. ((she goes up to overhead; teacher asks where?)) It won't work anyway.

Andria: Stefan, it's not going to work. ((Stefan keeps rearranging pieces))

Teacher: Obviously not going to work Stefan. Somebody else want to give it a shot? Go ahead Jon. ((Jon goes up to overhead))

Teacher: Those yellows don't line up. That's what Stefan tried. ((Jon rearranges from there into a rectangle; students very attentive))

Andria: Is that considered a square or a rectangle?

Teacher: Well, a rectangle is just a special, excuse me a square is just a special kind of rectangle. If it's a square it is a rectangle. Alright, Jon's got it. That would certainly be a rectangle. Very nice. Excellent. So we could take this area and make a rectangle out of it and now we can figure out the dimensions of that rectangle. What are the dimensions of that rectangle then? How long is this rectangle?

Jill: x plus 3.

Teacher: x plus three, isn't that x plus 3.

Jill: When you go down, it's x plus 2.

Andria: That big blue thing is x.

There were more ideas exchanged here in contrast to most discussion that prompted simple student responses. Students who participated were interjecting ideas and questions that provided direction to the discussion.

Because the overall pattern of discussion was largely teacher-directed, the incidences of student-directed discussion were not as prolific as student's simply responding. Incidences of student presentation of ideas occurred more frequently when manipulatives were an integral part of the lesson or when Mr. Lester purposefully structured the environment for students to present solutions. In the following, students were working with algebra tiles to determine the area of a rectangle with dimensions of (x + 3) and (2x + 3). Mr. Lester walked around the class checking the progress of groups. He prompts students to "fill in" the tiles if they have constructed the correct dimensions:

Teacher: How do I get this rectangle constructed?((a student from one group gives an explanation of what they did as the teacher places tiles on the overhead))

Stefan: Okay, we took two big black x's, then we took three big long lines, right there.

Teacher: So that's 2x plus three?

Stefan: Yes sir. Then we put them little tiny squares, put three of them in a row. There you go. Then right beside that you put three more, beside that one we did three more.

Teacher: Okay, so right now you're saying that just what I have there right now, 2x plus three across the bottom and I have x plus 1,2,3 on the side?

Stefan: Yep.

Teacher: So we have the right dimensions and all I have to do is to fill in. It doesn't have to be arranged exactly the same. Okay, you can have different arrangements. We can move things around and get the same result.

Stefan: Alright, now I have them three long bars, rectangles. Yes sir, I have them there in between the red and the yellow squares, rectangles.

Teacher: Okay, this is one possible solution to the situation. The order really does not affect the area at all. I still have my 2x plus three and x plus three. What's the area? ((The teacher gives the groups time to ponder that question; walks around and talks to individual groups)) What do we have for the area of this rectangle then? 2x plus three time x plus three, the length times the width and what's our area?

Brittany: 2x.

Teacher: What do we have? We've got two large x-squares.

Brittany: Nine x.

Teacher: Plus nine x.

Brittany: Plus nine.

Teacher: Plus nine, okay.((Teacher writes)) [(2x+3)(x+3)=2x2+9x+9]

This discussion illustrates the presentation of student thinking even though Mr. Lester physically modelled the tiles. The discussion seemed to be dominated by teacher-talk, but this time the direction came from verbal explanations presented by the student. When Mr. Lester asked for the areas of the arrangement, a student provided a verbal response, but was not given the opportunity to further present her strategy.

The last two days of the unit was a test review in which Mr. Lester provided a structure that not only encouraged but demanded student presentation of solutions. Each group of students was given overhead transparencies or dry erase boards on which they recorded their solution strategies to test-review problems. Interestingly, it is not until this point in the semester (near the end) that the class took on characteristics representative of Mr. Lester's plans for the student generation of and presentation of solutions. The following is the presentation of two solutions to factoring exercises in which the students were asked to factor the polynomial
x3 + 6x2:

Lara: To factor polynomials which is what we want to do in this problem, you have to find a common factor, and the common factor in this problem is x to the second, cause there's x to the second in here and in x to the third there's two x's. Then you make parentheses and since this is an x3 and x2 here, you put x, cause x2 times x is x3. And then you want to add 6 cause x2 times 6 is 6x2. Then we made an example one. Andria will explain that.

Andria: Like she said before, we're trying to find common denominators, I mean factors. In this case it's x2 since both have x2. [4x2 + x3 = x2(4 + x)] This one you're just going to bring the four over here.

Although the students who presented the solution provided clear explanations, there was not much time left in the class to expand on their ideas. That is, Mr. Lester did not have time at this point to redirect questions to other students in the class. In another problem, the students presented a solution for multiplying two binomials.

> Brittany: I took x minus 7 times x plus 7. I multiplied this x times that x, then x times plus 7, 7x, then -7 times x, -7x, then -7 times 7 which is -49. Both of 7x's cross out each other so the answer is x2 - 49.
>
> Teacher: Anybody see anything there they're not sure about? Uh, can I ask you to think about rephrasing something you said. You said the positive 7x and the negative 7x sort of cross each other out. Can you think of a more mathematical way of saying that? Both cross each other out, is there some (other way) we can say that?
>
> Student: Cancel.
>
> Teacher: Well, cancel doesn't mean anything mathematically. What do those two add to give you?
>
> Brittany: Zero.
>
> Teacher: They add up to zero. Let's say that. They add up to zero. Let's say something that suggests or conveys what's actually happening. Cancelling out has no meaning whatsoever but, they add up to zero, makes sense. Perfect Brittany, thanks.
>
> Stefan: What if I said they equal zero?
>
> Teacher: Well, they add to equal zero. They don't each equal zero.
>
> Stefan: When you combine them they equal zero.
>
> Teacher: That would be good.

Here again, the couple of involved students seemed to be contributing to the discussion in a worthwhile way. Mr. Lester prompted some discussion, but the class as a whole did not appear to be involved.

Summary

The results presented in this chapter add to the confirmation of results in similar studies (Gregg, 1995; Haimes, 1996) that have suggested that traditional practices in mathematics classrooms prevail, even in cases where the teacher's intentions and professed beliefs are reform-oriented. As illustrated by sequences of verbal interactions that occurred in Mr. Lester's class discussions, a traditional initiation-reply-evaluation pattern prevailed. This was in contrast to the discussion pattern (Voight, 1995) that was more consistent with Mr. Lester's stated intentions for facilitating discussion in his Algebra class.

Mr. Lester's role during class discussions was largely teacher-centered, even in cases where he was evidently attempting to promote student-initiated discussion. As suggested in Barnes (1992), the decisions concerning communication and the establishment of roles in the classroom will characterize the setting. This was illustrated in Mr. Lester's class as his decisions during whole-class discussions tended to reinforce his teacher-centered role. The structure of class discussion also limited chances for student negotiation of ideas. Thus, the possibilities for creating shared meanings in the spirit of constructivist principles were miniscule.

Specifically, these results suggest that even well-informed teachers, with reform-oriented views and a wealth of reform-oriented knowledge, may not know how to take a more experimental approach in the implementation of new forms of discussion in mathematics classes. How did Mr. Lester deal with his experiences in attempting to implement his intended visions of a reform-oriented environment? The next chapter outlines his struggles with this experience.

CHAPTER SIX
STRUGGLES IN IMPLEMENTING REFORM

As the semester progressed, it became evident that the nature of Mr. Lester's classroom environment contributed to whole-class discussions that were inconsistent with his vision. Mr. Lester experienced struggles between his intentions to reform and the reality of implementing whole-class discussions of mathematics. That is, as he endeavored to implement instructional strategies consistent with his visions of reform, difficulties were encountered. This chapter documents and analyzes the struggles that Mr. Lester encountered through examining interviews during which he reflected on the semester. By examining his comments and actual practice, contradictions were revealed to evidence his struggles with classroom challenges, teacher authority, and student resistance. These struggles led to the recognition of unrealized intentions that made it difficult to see significant reform in Mr. Lester's practice.

Analyzing Mr. Lester's Struggles

The interviews completed with Mr. Lester focused on reflections on his current and previous practice, experiences with reform, and perceptions of implementing reform in his Algebra class. This section outlines the struggles evident in Mr. Lester's changing practice as he tried to incorporate mathematical discussions in a student-centered way. One of the main tenets of his vision was that students should be involved in the discussion of mathematical concepts. For instance, at the beginning of the semester, Mr. Lester intended to use small-groups to facilitate discussion of concepts. In reflecting on his practice over the semester, he recognized that it was difficult to implement different strategies:

> And I really think that I made the assumption that because there is so much group work being done at the middle school, that there was some understanding of how the students could operate in a group situation, and that's obviously not the case. That was just a bad strategy on my part, the way I did that. I haven't done a very good job with that.

This quote captures Mr. Lester's willingness to be self-evaluative in reflecting on his struggles in practicing reform-oriented strategies (in this case, using groups to facilitate discussion). His intentions to use small-groups to facilitate discussion was not productive which resulted in few opportunities for students to participate in this manner. Based on similar quotes and through examining Mr. Lester's classroom discussions, the struggles that he encountered are discussed next in three sections: Classroom Challenges, Perceptions of Student Resistance, and Teacher Authority.

Classroom Challenges

Mr. Lester's struggles in the classroom were evident throughout my observations and interviews with him. He often sounded frustrated with the challenges involved in trying to implement changes in the class structure, his role, and the role of students. For example, his original plan to use small groups as an integral part of the class became difficult to continue. He often perceived students to be unmotivated or disinterested in developing discussion. In talking about students' connecting ideas to develop an understanding of mathematics concepts, Mr. Lester said, "I don't think that the thought processes which would make the connection be made are taking place." Mr. Lester's perception here contradicts his previous comments concerning how his students acquired more thinking skills from being involved in non-traditional discussions in his class:

> They tended to lead group activities, they tended to have a much more academically mature or learning mature vision of what mathematics could be and what they could do with their math and those kinds of things.

He further struggled with the perception that students were not investing in the class:

> That would require a thinking and investment in time and concern, and I don't see that happening. I don't really see an investment there. And that investment is, like I said, they'll answer a question in class and then the next day they can't answer the same question. That to me is an indication that there is no investment between one class and the next.

The lack of student thinking and investment in class is seen as a direct result of their lack of commitment to preparation in out of class situations. Mr. Lester's challenge was to create incentives that would encourage student investment.

Examples of classroom discussion revealed that there was little incentive for students to invest to the extent he desired. For example, in the second unit when the class was discussing solutions to quadratic equations, Mr. Lester encouraged students to notice a connection between the solutions and the equation of a quadratic:

> Teacher: Now, yesterday we were graphing, got a graph something like ((draws on overhead)) [parabola opening up through -6 and -2 on x axis] We found it had two solutions. Do you remember what those two solutions were? x2 + 8x +12 was the equation ((writes that on overhead)). We got one solution was -2 and one was -6 and we tried to figure out relationships between solutions and actual equations. ((no student response)) Do you remember what the connection was between the solutions and equation? Randall, do you remember?
>
> Randall: If you add them together it would equal one of them?
>
> Teacher: No. Did you do the assignment last night? Solving quadratics and looking at factors. Did you factor any of those quadratics? Do you remember the connection between the equation and its factors and the solution? ((writes (x+6)(x+2) on overhead, under the original equation)) What was the connection? Here's my equation, here's its graph, here's its factors and here are solutions. How are all these things connected?
>
> Nathan: The 6 and 2 can go together to make 8?
>
> Teacher: Well, when I factor this, 6 plus 2 gives me 8. That's how we found the factors. We did use the fact that 6 plus 2 gave us 8 and 6 times 2 gave us 12, that's how we factored. But once we have it factored, how are all those things related? How can we get one from the other? How can we use, if I had the factors could you give me the solution? If I give you the graph can you tell me the solution? Suppose I have ((draws graph on overhead)) and it goes through that point. Can you give me the equation? All these things are connected. Do you see these numbers anywhere?
>
> Elizabeth: If they're negative, they are going to be positive in parentheses.
>
> Teacher: Okay. Elizabeth said, look this is negative but that's positive. What's going on

> here? There is a direct connection between solutions and the factors. What's the connections? Anybody remember what we said yesterday?

Mr. Lester's frustration is evident in this episode. He poses several questions in a rapid-fire manner, leaving little room for students to participate. The students rarely had opportunities to explore their answers to teacher-directed questions, ponder their own questions, or openly share their reasonings and strategies. His frustrations continued to offer challenges in the contradictions that arose between his vision and practice.

To cope with this discontinuity between his intentions and actuality, Mr. Lester questioned the students' motivations. In Gregg's (1995) study of a high school teacher coping with similar contradictions, he notes:

> It is important to note that Ms. Weston did not appear to consider students' conceptions or interpretations of mathematics when accounting for their difficulties... She described individual student difficulties in terms of an inability to remember ideas or simply an inability to understand, but never in terms of any specific mathematical constructions. Further, in all cases where students ran into difficulty, Ms. Weston assisted them by asking leading questions to guide them to the solution she had in mind. (p. 460)

Mr. Lester's reaction to classroom struggles was similar as he credited the students' inabilities as a continual source of difficulty.

The classroom environment that Mr. Lester envisioned was one in which students would be actively engaged in the discussion of mathematics topics. The teacher would facilitate this environment by providing worthwhile tasks and setting up situations in which students were compelled to share ideas and questions. Although he spent much time developing non-traditional tasks and lessons aimed at prompting discussion among students (e.g. graphing calculator exercises, using algebra tiles), he struggled with implementing strategies that would encourage students to use non-traditional activities in mathematically powerful ways. For example, Mr. Lester used calculators throughout the class and largely professed alignment with

NCTM recommendations for calculator use. However, when faced with his students' lack of basic algebraic skills, he struggled to find an appropriate way to encourage calculator use while also developing important conceptual underpinnings:

> If they know how to enter negatives into the calculator then they're not making any decisions in terms of negative numbers. So, if they were allowed to use calculators, they never developed the algebraic skills and if they're not allowed to use calculators, they're convinced they can't do the arithmetic. Right there, there's a (no win) situation. If I'm teaching it again next year, I've got to find out, figure out a way to help them to develop a lot higher level of self confidence before we introduce any algebraic ideas.

It appeared that Mr. Lester experienced difficulties in encouraging students to both use calculators while also making decisions in terms of the concept of negative numbers. His experience and background may have provided him with a broad base of information concerning reform issues, however, when faced with difficult challenges he seemed to struggle with direction.

Perceptions of Student Resistance

When several parties are involved in a change process, each party arrives at their own perception of what happened during the process. In classroom discussions, the two parties are usually the teacher and the students. When the teacher wants to encourage discussion that requires thoughtful reflection by both parties, students may initially resist if this is not consistent with their prior experience (Borasi, 1990;Pimm, 1996; 1990; Silver & Smith, 1996). In Mr. Lester's class, his perception of the struggles encountered in changing practice were partially based on his view that his students were resistant to change and therefore would continually resist participating in ways that would enhance discussion. From observations and conversations with students, this was not necessarily the case.

In reflecting on the semester, Mr. Lester described his view of student behavior in terms of his previous semester's class when he tried to involve them in discussion of concepts:

> Well, (to use the old buzzword) I think the Algebra Two students, their paradigm is more established. I probably had less success with my students last [semester], getting them, breaking them out of the mold of what mathematics is, that's a good way to think of it. It was harder for me, they looked at, some of, a lot of the things that I tried to do, they looked at it as, "Well, this is just silly", or they were just too old for it to be neat. But they sort of fought that kind of thinking skill more. They really wanted more gimme, gimme, gimme, skill, skill, skill ,show me how, show me how.

Translating this thought to his current Algebra class, he continued:

> The algebra students [the class in this study] at least, maybe because I got them in their first high school math class, didn't really have a set expectation. And so, [they] were not more receptive, but at least not as resistant to that kind of stuff. They didn't fight it, they weren't as recalcitrant.

Meeting students at their level continued to pose a challenge for Mr. Lester, although he handled the struggle by easing his description of students' resistance by saying that the students marginally accepted the "kind of stuff" that was different in his class.

Mr. Lester appeared to hold the perception that his students were willing to participate in the discussion of ideas as opposed to simply accepting teacher explanations. However, upon closer examination, his statements concerning student acceptance of what he perceived to be a non-traditional environment reveal a contradiction. For example, Mr. Lester described an instance in which the class was working in small groups and he was walking around to each group to examine their progress. A girl asked him a question about solving a system of equations and instead of providing one answer, Mr. Lester was evasive in an attempt to challenge the student's thinking. He tried to encourage her to discuss her strategy. He said:

> "Well, look at the system that you have. If you add what happens?" and she worked it out, "and if you subtract, what happens?", and she worked it out and she got an answer both ways. Of course one of them gave her x and one of them gave her y. And I said,

> "That's why I said yes". You said " Do you add or subtract?", [and] I said, "Yes, actually I was answering your question". She didn't like that. She was kind of giggling. It was a nice conversation we had, but it was obvious that she was uncomfortable that there could have been more than one right way to get it. But, that's hard work. I mean it really is harder. It's stressful because the students are very… they become agitated very quickly and become very demanding…and very intolerant.

When the student in this episode reacted to his evasiveness, Mr. Lester perceived it to be a sign of her resistance to participate in a non-traditional manner. His conception of her resistance fueled his struggle as he continued to view practice in this way as difficult and demanding above the traditional route. How would Mr. Lester reconcile his notions that students should be more involved in student-centered discussion and at the same time his perception that students were resistant to an environment that empowered students? Changing his conceptions about students abilities to become empowered would be difficult.

It is difficult to determine whether Mr. Lester perceived student resistance as a lack of motivation on the part of the students or as a lack of maturity. Silver and Smith (1996) address the issue of student resistance by commenting:

> Unless the classroom environment is safe for thinking and speaking, students will be reluctant to propose their tentative ideas and hypotheses, to question assertions that are puzzling to them, or to share their alternative interpretations. (p. 22)

Was Mr. Lester's classroom environment conducive to student speaking and proposing of ideas? From my observations, this was not apparent. In his vision he wanted to create such an environment, but became easily frustrated in class when he perceived students to be resisting his approach.

In the following example during a class in which they had been using algebra tiles, Mr. Lester asked for student strategies but then seemed frustrated when there was no immediate clear route:

Teacher: Anybody else have an idea? Yes Tyler?

Tyler: Take the 2x and times it by the three.

Teacher: So, where are you talking about on this diagram. Which one?

Tyler: It doesn't matter.

Teacher: Well, it does matter.

Tyler: It does?

Teacher: Yeah, when you multiply you have to multiply a length times a width and these are both lengths. I couldn't multiply,

Tyler: Okay, multiply that 2x by that 3, 6x.

Teacher: But where does that go? What up here is 6x? See anything up here that's 6x?

Tyler: No, you multiply to get 6x.

Teacher: Right. And where do you see 6x up here?

Tyler: Right there.

Teacher: Where, right there? ((He asks her to show where and she points to the diagram))

Mr. Lester's perceptions of student resistance continued through the semester to increase his difficulties in developing the types of discussions he imagined. He often times remarked that the students' reluctance to shift towards more student-centered roles was based on their lack of confidence:

> If you look at their math ability, that was just an extra obstacle for them, and me to have to overcome. And, again there's that paradigm, you know, "Gee I've always been an 'A' student …so how come all the sudden this Algebra, I can't get A's." But then you look at the skills, the actual skills that they have, and they don't have skills that you'd think a student would need to be successful in Algebra One. If the student's not having confidence in their multiplication of whole numbers, that sort of interferes with any confidence they can have in solving equations or factoring a polynomial.

Not only did Mr. Lester perceive that students' responses to his ideas of reform were based largely on grades, he also supposed that their confidence was lacking to the point that their skill development suffered.

Mr. Lester's perception of student resistance due to their lack of understanding and ability left him sounding much like a "traditional" mathematics teacher. He comments here on his rendition of what a student might be thinking:

> I can do a lot of things that are in the book, yeah. I can do a lot of things that are on the typical assignments that we have. But, if I have the problem solving skills but no tools to actually finish it, then I never get anything done successfully. I can't do, I can't do the situation or context problems. I can't do the simplify these expression problems. I can't do the solve the ... there's nothing I can do.

His surprising solution for teachers was:

> So, we've got to be careful that we don't focus too much on the concept and the understanding without developing the tools that the student needs.

This comment revealed Mr. Lester's continuing frustration with reconciling his desire to have students develop concepts and understandings in an environment in which student discussion was center.

Teacher Authority

In Mr. Lester's Algebra class, whole-class discussions were largely teacher-directed. Through his struggle to develop strategies consistent with an environment for open-ended discussion, he managed to imply his authority in making mathematical decisions. The term teacher authority as used to describe Mr. Lester's struggles refers to the traditional role of authority in a teacher-dominated classroom environment. In mathematics classrooms the traditional view of mathematical authority is one in which answers to all mathematical problems are known and teachers are the route towards mathematical truth (Smith, 1996). Mr. Lester did not begin the semester by designing an environment that forced students to comply with his mathematical thinking processes. In contrast, he aimed to offer students the freedom to think about and discuss concepts by providing little structure to the environment.

Mr. Lester's intentions to provide open discussions was consistent with reform-oriented ideas concerning the sharing of mathematical authority between teachers and students. As stated in Wilson & Lloyd (1996):

> Sharing mathematical authority with students challenges teachers to learn to encourage and listen to the varied interpretations of their students. Doing so demands that the teacher structure the classroom in ways that focus students on investigation and discussion of mathematics in more student-centered activities. (p. 7)

Mr. Lester's strategies in discussion, however, resulted in struggles with sharing authority. As the semester went on, his direction in leading discussion and asking questions left little room for student intervention. The desire for a student-centered environment gave way to one that was autocratically controlled in spite of Mr. Lester's visions for reform.

Contradictions in Mr. Lester's vision of less teacher authority and actual practice contributed to his struggles in developing class discussions. He talked about wanting to provide an environment in which students would acquire power:

> I'm thinking that in the long run, I can probably have taught them more mathematical skills but I won't be giving them much math power if that's all I do. I'd rather give them a minimum of calculation, algorithmic skills and, then fill in as much as I possibly can with trying to compensate for their lack of thinking, reasoning kind of experiences.

In practice though, he perceived that students developed ideas and concepts at a minimal level. The result was often Mr. Lester taking the authority for assigning meaning to explanations:

> And maybe in my eagerness, when somebody came up with the idea, I maybe thought okay, that was a signal that if we had one person come up with the idea maybe if we talk about it a little more we can sort of spread that idea around. I don't think that ever

> happened. Looking back on it. Even though they came up with the idea, they never had the confidence to believe that they had the idea.

Even in this case, where Mr. Lester stated a belief that students were capable of developing the basis for a concept, he discredits the students' confidence in building on their ideas.

When talking about his vision of student participation in class discussion, Mr. Lester noted that student generated ideas would be validated by other students. He said, "At least if they hear other students coming up with the ideas it will be easier for them to accept than it would be if I just had to tell them." In direct contradiction to this idea, Mr. Lester's comments concerning his experience over the semester illustrated his continued struggle with sharing authority:

> So this year [the semester after this study], I'm doing a little bit more of giving them the ideas, then we're talking about it to see how it fits. So, sort of a pseudo-constructivist thing. I'm doing a little bit more of the construction and then we're taking a look back and we're saying how does this fit? Where can we tie this in with, what can we do with this idea. At least if I give them the idea or the method first, you know, they latch onto it as being valid because it came from me as opposed to [a student's] method or whatever.

Though he had laudable intentions, he reached a point where he decided that to reconcile his vision he could use cut-and-paste strategies that would make the class run more smoothly from his perspective. The fact that he decided to "do the construction" himself lessened students' opportunities to participate in meaningful explanations. Mr. Lester took the authority to steer students, but still wanted to reassure that they would have opportunities to discuss concepts through discussion, after the initial steps.

The lack of student participation in discussion continued to be a struggle for Mr. Lester. In his reflections he concluded that maybe he did not provide enough guidance in encouraging students to share ideas. He believed that students' made discoveries about some concepts, but needed his authority in developing conceptual understanding:

> And then, when we tried to build off that, they, the rest of them still weren't sure. "Well, is that a good idea? Because that was [a student's] idea, that's not Mr. Lester and we know [this student], he's in here with us, so he's not very smart". And, that's their perception.

Mr. Lester's perception relied on the idea that students inherently view the teacher's explanations in discussion as the valid ones. Although most students' prior experience in the school mathematics tradition does indeed bolster this feeling, there is no reason to expect that they can not adjust accordingly, if the teacher relinquishes a traditional authoritative stance. In Mr. Lester's class, his authority derived more from the environment he created, the accepted norms that emerged, and patterns of discussion than from students' lack of desire to validate each other's ideas. Pirie (1996) comments on this in her writing about the role of listening in mathematics classrooms:

> Classroom communication has two inherent dangers that can be reduced by a teacher listening sensitively. The first of these dangers results from a perceived imbalance of power, which can inhibit students from questioning the language used by the teacher. [Students do not ask] when they realize that their understanding does not match that of the teacher. They watch and listen and try to guess at the meaning. The second danger is more universal; until the talk ceases to be compatible with the thinking of one of the participants, each participant will be assuming that all have a common understanding of the words being used (p. 114).

Students in Mr. Lester's Algebra class appeared to be inhibited to some extent; however, it is more likely that because he maintained authority in discussion, students perceived that Mr. Lester was not listening attentively and therefore resigned themselves to become passive recipients.

In dealing with struggles of authority, Mr. Lester reflected on his perception that he attempted to give students more "freedom" through his efforts at reforming classroom discussion. His frustrations through the struggle result in his thinking that the prior experience of students was a major factor in what occurred in his class:

> Let's face it, their first eight years of math they were never given the option of making a decision. The teacher said, "Do it this way, don't do it this way". And so maybe I just gave them something that they weren't prepared to handle. I mean, I would have kind of, maybe I was a little idealistic and I was thinking, "You know, seven years after the Standards were published, maybe they've had a few experiences but they really haven't."

In comparison to the description of traditional teachers that Mr. Lester's provided earlier, he did appear to be less-than-traditional in his approach. He rarely intimated that his methods for solving problems or reasoning about a concept were the only valid ones. Instead, the atmosphere was one in which his authority was implied, consciously or not. He explained his struggles by lamenting that the Algebra class could have been more democratic or student-centered if only the students' prior mathematics teachers would have established classrooms based on Standards-like ideas.

Teacher authority stems from a long-term tradition in school mathematics. Teachers invariably find it safer and more efficacious to revert to relied upon strategies when faced with difficulties in reforming their classroom practice. Although Mr. Lester obviously utilized ideas based on his experience with reform-oriented recommendations, he displayed some unwillingness to release authority in ways that could empower students in class discussions. He appeared to be unconvinced that students could develop and maintain thoughtful and mathematically powerful discussion. In struggling with this apparent contradiction in his professed vision and actual practice in his classroom, Mr. Lester reflects on the lack of student generation of ideas in his class:

> And that never really went over as well as I would hoped it would. Not like, "Wow, gosh this person came up with an idea before Mr. Lester had to tell him. I wonder if I can do that?" That's what I was hoping would happen. Maybe that same validity question was what was actually happening? And if Joe made it up or Betty made it up then is it

> really mathematics or is it just they were the first ones to find it? So we'll have to do it their way. And isn't that a lot of how we teach students? Like, going back to the idea of order of operations. "Well, we do it this way so everybody gets the same answer." Well, that sounds like it's sort of fabricated. And then we brainwash kids; "Well, 'cause the first person did it this way, that's the way we all do it." I mean I can hear teachers saying that. I mean, literally and figuratively I can hear teachers saying that in my mind.

It is evident that Mr. Lester was genuinely concerned that students were not getting the chance to participate in a meaningful exchange of ideas, but continued to struggle to understand why those chances did not materialize. He continued by reflecting on his belief that students can and often do possess powerful ideas:

> And then if I try to build off a student's wonderful, I mean mathematically pure, rich mathematically valid insight; if I'm building off of that, are the students really seeing it as really mathematically valid? Or is it just one of those things that ,"[A student] made that up and Mr. Lester likes [that student], so from now on we'll do it [that] way."? And I'm wondering how much of that happened last year.

Mr. Lester's struggles left him with perceived deficiencies in his classroom practice. Through these conversations he continued to struggle with his identity as a non-traditional, Standards-like mathematics teacher.

The struggles that Mr. Lester encountered over the semester resulted in aspects of his practice that were contradictory to his stated intentions. These results are outlined next.

Unrealized Intentions

From anecdotal teaching experience, we know that the intentions teachers have to attempt "new things" in their classrooms are sometimes met, exceeded, or are not realized to the extent imagined. The constraints of daily classroom routines can create a feeling of being well-intentioned, but not effective, if significant results are perceived as lacking. Mr. Lester expressed his dissatisfaction with results based on his intentions to include student ideas as an integral part of the Algebra class environment. In one instance, he had set up small groups using a jigsaw format to try to generate discussion of a topic. He explained what happened:

> There were times earlier in the year where I did some things [like jigsaw] and it worked out really well in terms of the operational aspects. They conformed to it, but the effect of it, you know, somebody from each [group] feeling confident on a particular topic and then using that confidence to develop other ideas or sort of mesh with the other ideas of the students in the [group], they take everything at surface value, they don't really see, they don't really get into, "Oh wow, I learned something." or "Oh, that worked out really well, I understand something." [They say], "Well, I've done my work." It's really on a very superficial level. I get the work done, I have an answer.

He had planned early in the year to utilize small-groups to aid in the facilitation of whole-class discussion. Because he did not structure the class environment in ways that compelled students to interact, Mr. Lester did not realize the intended results. That is, the extent to which students participated in whole-class discussions was minimal. Mr. Lester also expressed frustration with the apparent lack of student motivation to explore through discussion. For example, he expressed his perception that the students' "conformed" to the group setup, but saw little impact of student-to-student discussion in the development of ideas. He further expressed his frustration by recognizing that he tended to "give up" over time on the prospect of realizing his intentions for developing a student-centered environment:

> I know I haven't pursued it with the enthusiasm that I started with. And I know I should have, but quite frankly I've had to spend so much time doing a lot of administrative garbage for the class, and I don't blame them [the students], I really don't blame them. So many of those students have been put in that class without the proper background to be successful. Quite honestly, when a student can't multiply three times five in their head, it's going to be pretty hard for them to do things algebraically when they don't have any confidence in their arithmetic skills. And for me to say, "Well, that's okay, you can still do algebra", although I believe it's true, to them it flies in the face of logic and they don't buy into it.

Here, Mr. Lester acknowledges the feeling that he could have done more, but offers reasons for his lack of continued enthusiasm. Although he expressed his frustration in terms of students' backgrounds, he was careful to place no blame on students.

Mr. Lester's stated intentions for a more student-centered environment relied much on the Standards descriptions of reform-oriented classroom environments. He intended to develop a more student-centered environment in which students actively participated in discussion. In reflecting on these intentions he says:

> I think that, (pause) I think I'm having to force myself to come back down a little closer to reality than I would like. Recognizing that, although, I still think the Standards is where I want to take my classes or that's giving me the image or the focus or where I want to go, my student's experiences with that kind of learning are so limited, that they can't function at a high school level, or they can't function within the Standards at the high school level.

It is evident that Mr. Lester, equipped with knowledge of reform recommendations, continued to reflect on the strategies he used in the context of the Standards. He comments on his intentions to create an environment in which students would become more aware of learning mathematics as a constructive activity. That is, students would perceive the learning of mathematics as developing understandings of concepts as opposed to the traditional acceptance of ideas given by the teacher. He perceived that he had attempted to provide an environment in which thinking and questioning skills were developed:

> To implement the Standards at the high school level you are of course assuming that the students have experienced that kind of teaching and learning experience for the past eight years. Now I've known all along they haven't, and so I really tried to focus a little bit less on the math itself and more on giving them the thinking and reasoning skills, questioning skills and mathematical insights that the Standards would help, would tend to give the students aside from the mathematics itself.

Mr. Lester wonders about the feasibility of implementing reform-oriented strategies when students at the secondary level arrive with little experience in problem solving or discussion of their reasoning.

Although Mr. Lester's stated intentions were to dispel traditional notions of the teacher as the possessor of knowledge, his language suggests a different outlook. In his attempt to refrain from "giving" students the ideas (concerning Algebra topics), he takes authority in "giving" students thinking and reasoning skills to make them more powerful. His language here appears to contradict his vision. A similar study of a secondary Algebra class (Haimes, 1996) found that the teacher's actions in her implementation of a reform curriculum were not consistent with her original intentions. Haimes suggests that the teacher in his study, although well-versed in pedagogical knowledge, eventually adopted a curriculum prioritized by "content coverage, methods and procedures were emphasized, and teacher-centered pedagogical practices were adopted" (p. 599). Mr. Lester experienced similar struggles.

Mr. Lester's intentions to involve students in the discussion of ideas and change their thinking about learning mathematics met with little success when compared to his stated plans for the semester. As he continued to reflect on the semester he talked about these feelings of doubt:

> Well, I didn't feel really successful, I'll tell you that. But it's the kind of thing, I know it sounds, well ... it's not the kind of thing you are going to see the benefits of at the time. I have a couple of [the students] actually in geometry who were in that math class and they seem to be much more receptive to participating in that kind of conversation. They have a little bit more confidence than the other students.

Mr. Lester's feeling that he was less than successful in his attempts to include students in meaningful discussions, illustrated the unrealized intentions that resulted from struggles over the semester. To reconcile the inefficacy of this situation, Mr. Lester commented that there wasn't enough time to effectively develop discussion in beneficial ways. Although he said that he didn't feel successful, Mr. Lester perceived success on the part of students in developing confidence in their acceptance of an environment that is less traditional. Even with the evident struggles, Mr. Lester maintained that his Algebra class was different from a traditional mathematics class based on comments from a former student:

> [He liked my] class because there always seemed to be so much more going on than just getting mathematics. He was, I think he was encouraged to think of things he'd never thought of, and things that he thought of were validated as opposed to, "That's not what we're talking about now" or " That's not the way we want to do it".

This comment points to the efficacy for teachers in focusing on aspects of their classes that produce the feeling that some goals are realized (e.g. Mr. Lester's perception that students developed more confidence in participating in non-traditional conversations).

It is evident that through Mr. Lester's struggles that intentions to change are not sufficient; however, he continually contended that students were exposed to enough of a Standards-like environment to impact their development. As Smith (1996) writes:

> [When reacting to reform] experienced teachers use their own models of effective mathematics teaching to interpret the reform and incorporate only those elements consistent with their views. Their positive reception of the reform is more a matter of assimilating surface features to their current teaching practice than giving thoughtful consideration to the basic principles. (p. 395)

This applies to Mr. Lester's case in the sense that although his intentions were to have a less teacher-centered environment, the actuality was that he struggled to incorporate elements of reform that required deeper changes in practice.

Changing Classroom Practice

Examining changes in mathematics classroom practice seems to be an on-going endeavor of mathematics educators at all levels. In the current environment of reform in mathematics classrooms, changing practice as Mr. Lester was attempting raises issues concerning the struggles encountered by experienced mathematics teachers who are inclined towards classroom reform. While it is important that mathematics teachers develop diverse pedagogical strategies to implement reform-oriented recommendations, they must also realize that changing practice is often complex and on-going. As Pimm (1996) writes:

> Mathematics education seems particularly prone to the belief in the single new idea: do this (whether using calculators, teaching mathematics through problem solving, working collaboratively, stressing the basics, employing manipulatives, and so on), and all your mathematics teaching problems will be solved. (p. 11)

In practice, the use of reform-oriented strategies and techniques must be continually reflected upon and revised, often amidst other duties. That is, if students are involved in reform-oriented experiences (e.g. problem-solving with calculators, using manipulatives to develop the concept of multiplication) with little continued exposure, it is improbable that there will be lasting effects.

Mr. Lester alluded to this notion when he talked about how reforms in mathematics classrooms will require consistency over grade levels and will continue to pose difficulties for teachers:

> You might have one third grade teacher that is really big time into it [using Standards-like mathematics strategies] and so [those] students get a really good experience. But then when they go to fourth grade and fifth grade, teachers that don't, it's lost. And literally it would be lost. So until there's that consistency K to 12 ... (pause) ... it's going to constantly be that kind of painful experience for anybody that wants to break out of that mold.

He expressed the idea that if students are not exposed to reform-oriented environments at the elementary level, then secondary teachers will continue to experience struggles in changing practice along those lines. In relationship to this, he mentioned that the students did not have a solid content background which made it difficult to progress in Algebra:

> Our curriculum for Algebra at the high school level is based on the belief or the understanding that certain SOL's were covered in certain chapters in grades 5, 6, 7 and 8, that build up to Algebra. These students, most of them couldn't tell you what a variable was [and] that's in the SOL's for grade five. I think maybe I wasn't [as] alert to those lack of prerequisites as I should have been. Or I wasn't as sensitive to it.

Even as Mr. Lester admitted that attempting to change his practice was "painful", he wondered if there were something he could have done differently to accommodate the students' prior experiences with mathematics learning.

The goal of many mathematics educators is to share reform efforts with classroom teachers in ways that will practically impact their practice. The literature seems to be replete with conceptions of lasting reforms that include the use of teacher's voices, the expanding of collaborative research projects, and the idea that reform is systemic and a process of teacher learning (Fullan & Miles, 1992). Even with these efforts, teachers will find that the challenge of reform will involve struggles. Mr. Lester, who willingly participated in lending his voice to this project, expressed an overall feeling of dissatisfaction with not being able to reach students in the way he intended:

> Let's face it, their first eight years of math they were never given the option of making a decision. The teacher said, "Do it this way, don't do it this way." Maybe I just gave them something that they weren't prepared to handle. I mean I would have, kind of, maybe I was a little idealistic. And I was thinking, you know, seven years after the Standards were published maybe they've had a few experiences. But they really haven't.

Ball (1993) comments on the continuing difficulties in addressing the struggles inherent in changing classroom practice:

> On the one hand, acknowledging pedagogical difficulty is too often tantamount to admitting professional incompetence. On the other hand, the tone of some articles and workshops seems to convey that there is "a right way" to motivate children, to teach place

> value, or to respond to certain kinds of questions from students. Between these two opposing approaches to problems of practice lies little territory for thoughtful teachers to discuss with others the uncertain challenges of their work. (p. 395)

Ball makes it clear that although many teachers embrace reform-oriented efforts, it is often difficult to openly address the uncertainties experienced in attempting to change their practices.

In this study, the struggles that Mr. Lester experienced were sometimes difficult for him to express. The feeling of frustration at not finding the "right way" to involve students in the way he intended, was evident in his classroom behavior as well as his responses during interviews. His responses revealed the difficulty in openly addressing his uncertainties in practice. His frustrations also resulted in an overall environment of distrust. Mr. Lester's authority in class discussion and overall demeanor towards student participation resulted in low student participation. A summary of these results is given next.

Summary

This chapter focused on the struggles experienced by a veteran mathematics teacher who was well-versed in reform-oriented recommendations. Knowing that there are often struggles accompanying change processes, it would be expected that some difficulties would arise. In Mr. Lester's case, however, it was interesting to note that even with a strong background in reform knowledge, his struggles with challenges in the classroom, student resistance, and teacher authority pervaded. Other studies (Haimes, 1996; Lloyd, 1996; Prawat, 1992a; Wilson & Lloyd, 1995, 1996) are also finding that experienced teachers with willingness to use reform-oriented strategies continue to struggle with changing their practice in profound ways.

To address the struggles of reform, teachers and researchers hope to provide informative strategies to aid teachers in transition. Confrey (1990) proposes a model of non-traditional instruction based on constructivist principles. In this model, Confrey describes six components as strategies of instruction for teachers in transition: the promotion of student autonomy, the development of reflective processes, the construction of case histories, the identification and negotiation of tentative solution paths, the retracing and group discussion of the paths, and the adherence to the intent of the materials (p. 107). This model suggests that teachers embracing reform-oriented beliefs consider students as valid negotiators through group and class discussions. Confrey outlines three assumptions for teachers who embrace these goals. Teachers must build models of student's understandings of mathematics. This assumption implies that teachers will need to create varied ways of gathering evidence for judging the strength of students' constructions. A second assumption is that instruction is inherently interactive. Teachers and students must interact to form tentative solution paths to problems. In contrast to the traditional view, a teacher with reform-oriented goals accepts that negotiation is possible. Finally, the student must decide on the adequacy of his construction. Mr. Lester's envrionment and actions unfortunately did not induce negotiation among class members.

In continuing to provide guidelines for teachers like Mr. Lester who are involved in or amenable to change efforts in mathematics classrooms, the NCTM's (1996 - 1997) position statement on the professional development of teachers of mathematics states that:

> Responsibility for the continued transformation of school mathematics rests, ultimately, with classroom teachers. To shoulder the responsibility teachers must have access to ongoing, informed, carefully and creatively designed professional development opportunities. The Council supports the growth of mathematics teachers by providing a range of professional development opportunities; developing and updating standards for the quality of these opportunities; and furthering professional development efforts through dissemination of effective models. (p. 22)

However, even with these statements concerning professional development, many in-service activities aimed at examining reform issues still serve as a presentation of steps that will improve teaching in school classrooms, while failing to address the prior experiences of teachers. Silver and Smith (1996) address this when examining the challenge of developing new forms of instructional practice aimed at changing discussion in the classroom:

> The creation of discourse communities in mathematics classrooms is especially challenging at this time because most teachers lack personal experience with such environments. Most teacher education programs do not furnish prospective teachers with

> extensive experience with mathematical discourse, nor do most graduate-degree programs for teachers. In-service workshops are also an insufficient source of such experience. Moreover, since most teachers have learned the mathematics they know in traditional classrooms, they are being asked to create instructional environments with which they have had little direct experience as teachers or as learners. Helping teachers to move away from a pedagogy of isolation and recitation and toward a form of instruction rich in collaboration and communication is likely to require new forms of experience and support. (p. 26 - 27)

Recognizing the strength of veteran teachers' traditional conceptions of mathematics classroom environments (such as sharing of authority) supports the notion that teachers like Mr. Lester will require new forms of professional development activities to support their efforts.

The final chapter considers how this study in particular can add to the evidence that mathematics teachers in the current context of reform need time and support to meet these challenges.

CHAPTER SEVEN
SUMMARY AND IMPLICATIONS OF THE STUDY

In this chapter, I summarize the main results of this case study of Mr. Lester. Additionally, I propose some implications of this study for the professional development of mathematics teachers and areas for future research.

Summary

The purpose of this study was to broaden our understanding of the implementation of reform recommendations (e.g. NCTM Standards-like curriculum and instruction) in secondary mathematics classrooms. In particular, this case study examined the experience of one secondary mathematics teacher during his efforts to facilitate discussions in a secondary Algebra class. Mr. Lester, the teacher who participated in this study, was a veteran mathematics teacher with an extensive background in learning about and participating in reform efforts in mathematics education. The study was completed in Mr. Lester's Algebra classroom during the 1995-96 school year; and, subsequently in interviews with him in the Summer of 1996 and the Fall of 1997.

The study examined the patterns of verbal interactions that occurred in Mr. Lester's class, the role that Mr. Lester played in facilitating discussion, and the struggles that Mr. Lester encountered in attempting to facilitate discussion according to his vision of reform. These areas of focus were derived from the current emphasis in creating classroom environments that are rich in mathematical reasoning and discussion (NCTM, 1991). A secondary classroom was chosen because it is a fertile area for further research concerning current reform efforts of mathematics teachers. Mathematics teachers at the secondary level may be more confident in the subject matter and thus are able to make changes in content emphases (Lloyd, 1996; Wilson & Lloyd, 1997), but less amenable to utilizing reform-oriented instructional techniques (Weiss, 1995). Completing research at the secondary level can be informative for teachers, mathematics educators, and others in the mathematics education community.

Mr. Lester's classroom was observed for one semester. Classroom discussions in two mutually chosen units of study (approximately two weeks each in length) were audio-recorded daily and transcribed to form the data used to examine the patterns of verbal interactions in the Algebra class. Mr. Lester participated in interviews throughout the two units. I also audio-recorded and transcribed these interviews. Analysis, using methods adapted from Taylor and Bogdan (1984), Lincoln and Guba (1985), and Spradley (1989), began during my observations in the classroom and continued over the course of the study. As I interviewed Mr. Lester at the end of the semester, I continued to read related literature, look for emerging themes, and develop propositions as part of my analysis. Empirical assertions were generated and data was searched to establish evidence for the assertions (Erickson, 1986).

In Chapter Four, through analysis of conversations with Mr. Lester, I provide an examination of his vision of reform. Mr. Lester envisioned a classroom environment similar to that represented in the Standards documents. That is, he wanted to create an environment in which students were actively involved in discussion and problem-solving. Mr. Lester remarked that activities should involve students in the use of real-world problems, the use of technology to develop conceptual understanding, and in applications of mathematics concepts. He described the teacher's role as one of a risk-taker, supporter, listener, and questioner.

Mr. Lester's classroom practice evidenced his development of plans to involve students in non-traditional tasks. He incorporated problem sets aimed at developing general concepts rather than relying heavily on skill-based worksheets. He used manipulatives to aid in conceptual understanding. In practice though, the inconsistent use of small-groups and continued hindering of student discussion led to a largely teacher-directed environment as outlined in Chapter Five.

The verbal interactions concerning the construction of mathematics knowledge that existed in whole-class discussions in Mr. Lester's class consisted largely of the following: teacher posed questions, teacher promoted statements and ideas, teacher maintained discussions in which the teacher often provided clarification of student responses, and teacher extended explanations. Sequences of these interactions formed patterns that largely contributed to the central nature of Mr. Lester's role in the class. Students most often simply responded to prompts and questions.

Mr. Lester expressed intentions to create a classroom environment consistent with constructivist precepts. For example, he wanted students to take initiative in generating discussion; he sought to gather data using manipulatives and physical materials; he intended to ask open-ended and thought-provoking questions; and he attempted to make considerations for shifting instruction to accommodate student understanding. With exposure to these ideas and prior experiences with reform efforts, his resulting role in practice was surprising to some extent. That is, his role emerged as one that followed fairly traditional patterns in which he maintained most of the authority for developing and maintaining discussion. His intentions in using these strategies (e.g. attempting to pose thought-provoking questions) in discussion may have been to increase student involvement; however, often led to student ideas being shaped by the teacher.

From daily observations, it was evident that although he continued attempting to promote discussion, students were not actively participating. Mr. Lester structured the class in ways that rarely encouraged student discussion. There was little use of explicit encouragement for students to elaborate on their ideas. He experienced difficulty in accepting student autonomy. The difficulty in realizing his vision of increased student involvement in the presenting and discussing of ideas apparently could not be totally dependent on the "non-traditional" tasks that he developed and utilized. Rather, the structure of the class environment seemed to provide the larger challenge. How could he provide the opportunity for students to explore uncertain ideas on their own terms, while at the same time ensure that there was some direction and meaningfulness to their discussions? How could he direct discussion without appearing to provide an authoritative stance? As Ball (1992) writes:

> The quality of the tasks is crucial, but, equally important is the nature of the classroom discourse - the ways in which ideas are developed interactively in the class. And the environment of the class - the kinds of norms that are established, the ethos of collaboration and respect, the patterns and expectations for thinking and interacting - combines with this attention to discourse, considerably extending and complicating what counts as mathematics pedagogy. Emphasizing reasoning in the use and development of mathematical ideas means involving students in teaching and learning as they rarely have been before. (p. 7)

Reforming practice in terms of whole-class discussions, then, became much more than developing non-traditional problem sets and allowing students to manipulate materials.

What struggles did Mr. Lester encounter in attempting to implement reform in his classroom practice? He often displayed frustration and expressed difficulty in changing students' views concerning classroom discussion of mathematics. Through examining conversations with Mr. Lester, Chapter Six presents struggles categorized into the following: Classroom Challenges, Perceptions of Student Resistance, and Teacher Authority. In trying to change his behavior, Mr. Lester was in a situation that required him to revise his strategies in the midst of uncertainty. Although Mr. Lester desired to have his students interacting in mathematical discussions of concepts, he rarely employed any specific strategies (e.g. writing prompts, explicit direction for students to reveal their reasoning, structured cooperative groups) to develop the necessary connections between students' reasoning and discussion. As a result, his practice concealed many intentions to create a student-centered, Standards-like environment.

It was apparent that Mr. Lester held views consistent with the constructivist ideas inherent in the Standards documents, but encountered struggles in attempting to utilize strategies in line with these views. His professed intentions to encourage discussion did not easily translate into realized practice. In daily classroom struggles, an almost immediate observation was that the unstructured use of small-groups and whole-class discussions contributed to difficulty in gaining student participation. The norms of discussion established by Mr. Lester were traditional in nature. Thus, students participated in less mathematically powerful ways. For instance, students rarely publicly shared their reasonings of solutions. In class discussions, he posed questions aimed at eliciting students' strategies, but did not specifically ask students to provide their explanations. In spite of his intentions and activities, students appeared to feel uncomfortable in initiating discussion.

Mr. Lester also perceived that students persistently resisted his attempts to include them in discussions. His perception of student resistance was in some part a result of his own contradictory stances on students' abilities to participate in discussion. Mr. Lester would talk about students' abilities to develop ideas and share strategies, but then through frustration would conclude that they wanted him to give ideas and reasonings as the mathematical authority. His reactions appeared to result in his retreating to a less student-centered environment and he became more, as opposed to less, authoritative in his instruction. He tended to direct discussion in ways that implied to students that their questions or ideas were under-valued even as he contended that he wanted students to participate more fully.

Implications of this Study

The findings of this study suggest that classroom mathematics teachers at the secondary level, even those with extensive experience and grounded knowledge in current reform initiatives, will need to further develop strategies for facilitating a Standards-like environment in their classrooms. The gathering of Standards-based materials and activities has aided many teachers in developing more student-centered lessons. However, in utilizing reform-oriented recommendations to effectively change practice, teachers are involved in on-going adaptations of their practice. Until teachers develop strategies that can enact sustaining change, mathematics classrooms may remain largely teacher dominated.

This study, in particular, points to the need specifically for strategy use in facilitating discussion in mathematics classrooms. Mr. Lester widely participated in reform-oriented programs and activities. He was adept at acquiring and adapting information to develop non-traditional lessons. His struggles with translating this extensive knowledge into desired whole-class discussions contributes to the idea that there is a continuing need for teachers to utilize strategies that explicitly include student thinking and reasoning in discussion.

This idea offers a challenge for mathematics teachers to become less authoritative in their conceptions of teaching and learning mathematics. The conceptions of a teacher concerning teaching and learning impacts on their practice (Thompson, et al. 1994; Bauersfeld 1980; Prawat 1992). In adopting a constructivist approach to teaching and learning, teachers must become willing to realign their instructional strategies. As Prawat writes:

> Most of the problems associated with implementing a constructivist approach to teaching could be overcome if teachers were willing to rethink not only what it means to know

> subject matter, but also what it takes to foster this sort of understanding in students. This is a tall order. Such change is unlikely to occurwithout a good deal of discussion and reflection on the part of teachers. Identifying what is problematic about existing beliefs, however, is an important first step in the change process. (p. 361)

What was problematic in Mr. Lester's case? Overall, his approach in implementing strategies appeared to be indirect. Prawat terms this type of approach "naive constructivism"; the idea that the "key task [of the teacher] is to watch over the environment, ensuring that it affords enough opportunity for students to be involved in interesting and engaging educational activity" (p. 369). This approach provides a false sense of student autonomy on the part of the teacher. Teachers themselves must be willing to seriously reflect on their practices if reform in the current context is desired.

In concert with this, pre-service and in-service educators should examine the development of programs aimed at implementing specific tenets of the NCTM Standards. That is, the proliferation of informational sessions that have provided descriptions of pedagogical strategies can be changed to better reflect actual practice. Mathematics educators can help in reforming teaching practice by providing effective models that build on classroom teachers' prior experiences and expertise. Providing models that aid teachers in reform is a continuing challenge for mathematics educators as outlined by Lester et al. (1994):

> To recommend that teachers adopt a drastically different posture toward mathematics instruction in their classrooms is one thing; to help them learn how to change is quite another. It is no small task to alter ways of thinking about and doing mathematics that have been bred by a dozen or more years of transmission-mode instruction. Simply put, we believe that teachers are prone to teach the way they themselves were taught. (p. 153)

In my observations, Mr. Lester seemed to be ill-equipped to drastically alter his instruction according to his intentions. He seemed to be sure of his desire to change and obviously possessed the required abilities and skills to change. Even so, he continually struggled with implementing strategies to change. He indirectly faulted students' unwillingness to change and other teachers emphasis on rote learning, for the lack of results. Realizing that teachers will continue to rely on tried practices (Smith, 1996) as a basis for their continual growth, researchers can benefit by designing studies based in teachers' natural environments.

When Mr. Lester abandoned the tried-and-true ways of instruction, he found struggles in gaining student participation in discussion and he perceived that students were unwilling to cooperate. As a result, he relied on a more teacher-centered environment, while still trying to design lessons that were more open-ended. The contradictions that arose only seemed to present more confusion to students. Could Mr. Lester have been provided with strategies (e.g. listening more thoroughly to student responses, asking more specific question to students, redirecting solutions and responses to other students to expand participation) that would have alleviated some confusion? I think so. As Ball (1992) writes concerning teachers:

> [Teachers] may also not know how to take a more experimental approach to their work, for the pressure to appear competent, smooth, and sure of one's methods and results predominates. Thoughtfully constructed curriculum materials, articles describing teaching and efforts by others to try particular ways of working, would further comprise useful resources for constructing new ways of working with students at all level of mathematics education. (p. 17)

Teacher educators and practicing mathematics teachers can help in this endeavor by sharing time in classrooms and reflecting on their own teaching to aid in developing working strategies that will effectively encourage all in the mathematics community.

Finally, the mathematics education community should continue the examination of mathematics teachers confronting changes in classroom practice. Mathematics educators and researchers who continue to call for reform in mathematics instruction must realize the complexity of the task. As Mr. Lester discussed his difficulties, it was evident that he was genuinely interested in helping students to become more mathematically powerful in their learning. At the same time, he realized that he often felt a lack of control in this endeavor. We can expand our efforts to understand the difficulties inherent in reforming mathematics teaching practice within the constraints of public school classrooms. The continued examination of struggles in actual classrooms can form a basis of information for involving teachers in the development of relevant, effective strategies for reform in mathematics classrooms.

REFERENCES

Artzt, A.F. (1996). Developing problem-solving behaviors by assessing communication in cooperative learning groups. In P. Elliot (Ed.), Communication in mathematics, K - 12 and beyond (pp. 116 - 125). Reston, VA: NCTM.

Ball, D.L. (1990). The mathematical understandings that prospective teachers bring to teacher education. Elementary School Journal, 90, 449 - 466.

Ball, D.L. (1992). Implementing the NCTM standards: Hopes and hurdles (Issue paper 92-2). East Lansing, MI: National Center for Research on Teacher Learning. (ERIC Document Reproduction Services No. 352264).

Ball, D.L. (1993). With an eye on the mathematical horizon: Dilemmas of teaching elementary school mathematics. Elementary School Journal, 93, 373 - 397.

Barnes, D. (1992). From communication to curriculum. Portsmouth, NH: Boynton/Cook Publisher's Inc.

Bauersfeld, H. (1992). Classroom cultures from a social constructivist's perspective. Educational Studies in Mathematics, 23, 467 - 481.

Blanck, G. (1990). Vygotsky: The man and his cause. In L. Moll (Ed.), Vygotsky and education (pp. 31 - 58). Cambridge University Press.

Briggs, C. (1986). Learning how to ask. New York, NY: Cambridge University Press.

Briscoe, C. (1991). The dynamic interactions among beliefs, role metaphors, and teaching practices: A case study of teacher change. Science Education, 75(2), 185 - 199.

Brooks, J. G., & Brooks, M. G. (1993). In search of understanding: The case for constructivist classrooms. Alexandria, VA: ASCD.

Bruer, J. (1993). Schools for thought: A science of learning in the classroom. Cambridge, MA: The MIT Press.

Bruner, J. (1986). Actual minds, possible worlds. Cambridge, MA: Harvard University Press

Bruner, J. (1990). Acts of meaning. Cambridge, MA: Harvard University Press.

Chilcott, J. (1987). Where are you coming from and where are you going? The reporting of ethnographic research. American Educational Research Journal, 24, 199 - 218.

Cicourel, A. (1974). Cognitive sociology: Language and meaning in social interaction. New York, NY: The Free Press.

Cobb, P. (1990). Multiple perspectives. In L. Steffe & T. Wood (Eds.), Transforming children's mathematics education: International perspectives (pp. 200 - 215). Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

Cobb, P., Yackel, E. & Wood, T. (1993). Theoretical orientation. In T. Wood, P. Cobb, E. Yackel, & D. Dillon (Eds.), Rethinking elementary school mathematics: Insights and issues, Journal for Research in Mathematics Education Monograph No. 6 (pp. 21 - 32). Reston, VA: NCTM.

Cobb, P. et al. (1992). Interaction and learning in mathematics classroom situations, Educational Studies in Mathematics, 23, 99 - 122.

Cobb, P. & Bauersfeld, H. (1995). The emergence of mathematical meaning: Interaction in classroom cultures. Hillsdale, NJ: Lawrence Erlbaum Associates.

Cobb, P. (1995). An emerging practice of teaching. In Cobb, P. & Bauersfeld, H.(Eds.), The emergence of mathematical meaning: Interaction in classroom cultures. Hillsdale, NJ: Lawrence Erlbaum Associates.

Cohen, D. K. (1990). A revolution in one classroom: The case of Mrs. Oublier. Educational Evaluation and Policy Analysis, 12(3), 327 - 345.

Cohen, E. (1994). Restructuring the classroom: Conditions for productive small groups. Review of Educational Research, 64, 1 - 35.

Confrey, J. (1990). What constructivism implies for teaching. In R. Davis, C. Maher, & N. Noddings (Eds.), Constructivist views on the teaching and learning of mathematics, Journal for Research in Mathematics Education Monograph No. 4 (pp. 107 - 124). Reston, VA: NCTM

Cuban, L. (1992). Managing dilemmas while building professional communities. Educational Researcher, 4 - 11.

Damon, W. & Phelps, E. (1989). Critical distinctions among three approaches to peer interaction. In N. Webb (Ed.), Peer interaction, problem-solving, and cognition: Multidisciplinary perspectives, International Journal of Educational Research, 13, 9 - 19. Davidson, N. & Kroll, D.L. (1991). An overview of research on cooperative learning related to mathematics. Journal for Research in Mathematics Education, 22, 362 - 365.

Driver, R. & Scott, P. (1995). Mind in communication: A response to Erick Smith. Educational Researcher, 24, 7, 27 - 28.

Edwards, A.D. & Westgate, D.P.G. (1994). Investigating classroom talk. Philadephia, PA: The Falmer Press.

Eisenhart, M.A. (1988). The ethnographic research tradition and mathematics education research, Journal for Research in Mathematics Education, 19, 99 - 114.

Ely, M. (1991). Doing qualitative research: Circles within circles. Philadelphia, PA: The Falmer Press.

Erickson & Mohatt (1982). Cultural organization of participation structures in two classrooms of indian students. In G. Spindler (Ed.), Doing the ethnography of schooling: Educational anthropology in action (pp. 132 - 173). New York, NY: Holt, Rinehart and Winston.

Erickson, F. (1986). Qualitative methods in research on teaching. In M. C. Wittrock (Ed.), The handbook of research on teaching (3rd ed., pp. 119 - 161). New York: Macmillan.

Flavell, J., Miller, P. & Miller, S. (1985). Cognitive Development. Englewood Cliffs, NJ: Prentice Hall.

Friedrichs, J. & Ludthe, H. (1975). Participant observation. England: Saxon House/DC Heath

Gardener, H. (1991). The unschooled mind: How children think and how schools should teach. New York: Harper Collins Publishers, Inc.

Gibbs, W. & Orton, J. (1994). Language and mathematics. In A. Orton & G. Wain (Eds.), Issues in teaching mathematics. New York, NY: Cassell.

Good, T. & Brophy, J. (1987). Looking in classrooms. New York, NY: Harper and Row.

Graves, N. & Graves, T. (1989). What is cooperative learning: Tips for teacher and trainers. Santa Cruz, CA: Cooperative College of California.

Gregg, D. (1995). Gender-related differences in interaction patterns in elementary school inquiry mathematics classrooms. Proceedings of the Seventeenth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education (pp. 36 - 41).

Gregg, J. (1995). The tensions and contradcitions of the school mathematics tradition. Journal for Research in Mathematics Education, 26, 442 - 466.

Haimes, D. H. (1996). The implementation of a "function" approach to introductory algebra: A case study of teacher cognitions, teacher actions, and the intended curriculum. Journal for Research in Mathematics Education, 27, 582 - 602.

Hamm, M. & Adams, D. (1992). The collaborative dimensions of learning. Norwood, NJ: Ablex Publishing Corporation.

Hammersly, M. & Atkinson, P. (1983). Ethnography: Principles in practice. NewYork, NY: Routledge.

Hammersly, M. (1990). Classroom ethnography: Empirical and methodological essays. Philadelphia, PA: Open University Press.

Hargreaves, A. (1994). Changing teachers, changing times: Teachers' work and culture in the postmodern age. New York: Teachers College Press

Jacob, E. (1987). Qualitative research traditions: A review. Review of Educational Research, 57, 1, 1 - 50.

Johnson, D. & Johnson, R. (1987). Learning together and alone: Cooperation,competition, and individualization. Englewood Cliffs, NJ: Prentice-Hall, Inc.

Johnson, D.W. (1991). Learning together and alone: Cooperative, competitive, and individualistic learning. Englewood Cliffs, NJ: Prentice-Hall, Inc.

Kagan, S. (1985). Cooperative learning resources for teachers. Mission Viejo, CA: Resources for Teachers.

Kamii, C. (1985). Young children reinvent arithmetic: Implications of Piaget's theory. New York: Teachers College Press.

Lampert, M. (1991). Connecting mathematical teaching and learning. In E. Fennema, T. Carpenter & S. Lamon (Eds.), Integrating research on teaching and learning mathematics (pp. 121 - 152). Albany, NY: State University of New York Press.

Lester, F. K., Masingila, J., Mau, S., Lambdin, D., Pereira dos Santos, V. M., & Raymond, A. (1994). Learning how to teach via problem solving. In D. B. Aichele & A. F. Coxford (Eds.), Professional development for teachers of mathematics (pp. 152 - 166). Reston, VA: NCTM.

Little, J. (1993). Teachers' professional development in a climate of educational reform, Educational Evaluation and Policy Analysis, 15(2), 129 - 151.

Lincoln, Y. & Guba, E. (1985). Naturalistic inquiry. Newbury Park, CA: Sage.

Lloyd, G. (1996). Transforming instruction about functions: One veteran teacher's experience with an innovative secondary mathematics curriculum. Unpublished doctoral dissertation, The University of Michigan.

Lo, J., Wheatley, G. & Smith, A. (1994). The participation, beliefs, and development of arithmetic meaning of a third-grade student in mathematics class discussions. Journal for Research in Mathematics Education, 25, 30 - 49.

Lofland, J. & Lofland, L. (1984). Analyzing social settings. Belmont, CA: Wadsworth, Inc.

Mayer, R. (1992). Cognition and instruction: Their historic meeting within educational psychology. Journal of Edcuational Psychology, 84, 405 - 412.

Mehan, H. & Wood, H. (1983). The reality of ethnomethodology. Malabar, FL: R.E. Krieger.

Merriam, S. B. (1988). Case study research in education: A qualitative approach. San Francisco, CA: Jossey-Bass Publishers.

Minstrell, J. & Simpson, V. (1996). A classroom environment for learning: Guiding students' reconstructiomn of understanding and reasoning. In R. Glaser & L. Schauble (Eds.), (pp. 175 - 202). Innovations in learning: New envrionments for education. Mahwah, NJ: Lawrence Erlbaum Associated, Inc.

National Council of Teachers of Mathematics. (1989). Curriculum and evaluation standards for school mathematics. Reston, VA: NCTM.

National Council of Teachers of Mathematics. (1991). Professional standards for teaching mathematics. Reston, VA: NCTM.

National Coucil of Teachers of Mathematics. (1995). Assessment standards for school mathematics. Reston, VA: NCTM.

National Council of Teachers of Mathematics. (1996 - 97). Handbook: NCTM goals, leaders, and positions. Reston, VA: NCTM.

National Research Council. (1989). Everybody counts: A report to the nation on the future of mathematics education. Washington, DC: National Academy Press.

Nespor, J. (1987). The role of beliefs in the practice of teaching. Journal of Curriculum Studies, 19(4), 317 - 328.

Noddings, N. (1989). Theoretical and practical concerns about small groups in mathematics. The Elementary School Journal, 89, 607 - 623.

Noddings, N. (1990). Constructivism in mathematics education. In R. Davis, C. Maher, & N. Noddings (Eds.), Constructivist views on the teaching and learning of mathematics, Journal for Research in Mathematics Education Monograph No. 4 (pp. 7 - 18). Reston, VA: NCTM

Orton, A. (1994). Learning mathematics: Implications for teaching. In A. Orton & G. Wain (Eds.), Issues in teaching mathematics. New York, NY: Cassell.

Patton, M.J. (1991). Qualitative research on college students: Philosophical and methodological comparisons with the quantative approach. Journal of College Student Development, 32, 389 - 396.

Peterson, P.L. (1990). Doing more in the same amount of time: Cathy Swift, Educational Evaluation and Policy Analysis, 12(3), 261 - 280.

Peterson, P. (1992). Revising their thinking: Keisha Colemans and her third-grade mathematics class. In Marshall, H. (Ed.), Redefining student learning: Roots of educational change (pp. 151 - 176). Norwood, NJ: Ablex Publishing Corporation.

Phillips, D.C. (1995). The good, the bad, and the ugly: The many faces of constructivism. Educational Researcher, 24, 5 - 12.

Pimm, D. (1996). Diverse communications. In P. Elliot (Ed.), Communication in mathematics, K - 12 and beyond (pp. 10 - 19). Reston, VA: NCTM.

Pirie, S. & Kieren, T. (1992). Creating constructivist environments and constructing creative mathematics. Educational Studies in Mathematics, 23, 505 - 528.

Pirie, S. (1996). Is anybody listening? In P. Elliot (Ed.), Communication in mathematics, K - 12 and beyond (pp. 105 - 115). Reston, VA: NCTM.

Prawat, R. S. (1992a). Are changes in views about mathematics teaching sufficient? The case of a fifth-grade teacher. Elementary School Journal, 93(2), 195 - 211.

Prawat, R. S. (1992b). Teachers' beliefs about teaching and learning: A constructivist perspective. American Journal of Education, 354 - 395.

Richards, J. & Von Glasersfeld, E. (1980). Jean Piaget, psychologist of epistemology. Journal for Research in Mathematics Education, 11, 29 - 36.

Russell, T. & Munby, H. (1991). Reframing: The role of experience in developing teachers' professional knowledge. In D. Schon (Ed.), The reflective turn: Case studies in educational practice (pp. 164 - 187). Columbia University, NY: Teacher's College Press.

Sainsbury, M. (1992). Meaning, communication and understanding in the classroom. Brookfield, VT: Ashgate Publishing Co.

Schifter, D. & Fosnot, C.T. (1993). Reconstructing mathematics education: Stories of teachers meeting the challenge of reform. New York: Teacher's College Press.

Schoenfeld, A.H. (1989). Ideas in the air: Speculations on small group learning, environmental and cultural influence on cognition and epistemology (pp. 71 - 88). In N. Webb (Ed.), Peer interaction, problem-solving, and cognition: Multidisciplinary perspectives. International Journal of Educational Research, 13, 1 - 119.

Schratz, M. & Mehan, H. (1993). Gulliver travels into a math class. In search of alternative discourse in teaching and learning. In Saljo, R. (Ed.), Learning discourse: Qualitative research in education, International Journal of Educational Research, 19, 247 - 264.

Silver E.A. & Smith, M.S. (1996). Building discourse communities in mathematics classrooms: A worthwhile but challenging journey. In P. Elliot (Ed.), Communication in mathematics, K - 12 and beyond (pp. 20 - 28). Reston, VA: NCTM.

Silverman, D. (1993). Intrepreting qualitative data. Thousand Oaks, CA: Sage.

Simon, M. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. Journal for Research in Mathematics Education, 26, 114 - 145.

Sinclair, H. (1990) Learning: The interactive recreation of knowledge. In L. Steffe & T. Wood (Eds.), Transforming children's mathematics education: International perspectives (pp. 19 - 29). Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

Slavin, R. (1985). Learning to cooperate, cooperating to learn. New York, NY: Plenum Publishing.

Smagorinsky, P. (1995). The social construction of data: Methodological problems of investigating learning in the sone of proximal development. Review of Educational Research, 65, 191 - 212.

Smith, J.P. (1996). Efficacy and teaching mathematics by telling: A challenge for reform. Journal for Research in Mathematics Education, 27, 387 - 402.

Spradley, J. (1980). Participant observation. Orlando, FL: Harcourt Brace Jovanovich.

Stake, R. (1988). Case study methods in educaitonal research: Seeking sweet water. In R.M. Jaeger (Ed.), Complementary methods for research in education. Washington, DC: AERA.

Steffe, L. & Tzur, R. (1994). Constructing mathematical knowledge: Epistemology and mathematics education. Studies in mathematics education series. Bristol, PA: Palmer Press Taylor & Frances (ERIC Document Reproduction Service No. 378 043).

Strauss, A. & Corbin, J. (1990). Basics of qualitative research: Grounded theory procedures and techniques. Newbury Park, CA: Sage.

Taylor, S. & Bogdan, R. (1984). Introduction to qualitative research methods. New York, NY: John Wiley and Sons.

Tharp, M & Uprichard, A.E. (1992). A problem-solving inquiry-oriented approach to learning mathematics: Student/teacher interactions of rule-based learners. Paper presented at the Annual Meeting of the American Educational Research Association. (ERIC Document Reproduction Serivces No. 355090)

Tharp, R. & Gallimore, R. (1988). Rousing minds to life: Teaching, learning, and schooling in social context. New York: Cambridge University Press.

Thompson, A.G. (1984). The relationship of teachers' conceptions of mathematics and mathematics teaching to instructional practice. Educational Studies in Mathematics, 15, 105 - 127.

Thompson, A.G. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D. Grouws (Ed.), Handbook of research on mathematics teaching and learning (pp. 127 - 146). New York: Macmillan Publishing Company.

Thompson, A.G., Philipp, R., Thompson, P., & Boyd, B. (1994). In D. B. Aichele & A. F. Coxford (Eds.), Professional development for teachers of mathematics (pp. 79 - 91). Reston, VA: NCTM.

Vergnaud, G. (1990). Epistemology and psychology of mathematics education. In Nesher, P. & Kilpatrick, J. (Eds.), Mathematics and cognition (pp. 14 - 30). New York, NY: Cambridge University Press.

Voight, J. (1994). Negotiation of mathematical meaning and learning mathematics. Educational Studies in Mathematics, 26, 275 - 298.

Voight, J. (1995). Thematic patterns of interaction and sociomathematical norms. In Cobb, P. & Bauersfeld, H.(Eds.), The emergence of mathematical meaning: Interaction in classroom cultures (pp. 163 - 202). Hillsdale, NJ: Lawrence Erlbaum Associates.

vonGlasersfeld, E. (1981). The concepts of adaptation and viability in a radical constructivist theory of knowledge. In I. Sigel, D. Brodzinsky & R. Golinkoff (Eds.), New directions in Piagetian theory and practice (pp. 87 - 95). Hillsdale, NJ: Lawrence Erlbaum Associates.

von Glasersfeld, E. (1990). Environment and communication. In L. Steffe & T. Wood (Eds.), Transforming children's mathematics education: International perspectives (pp. 30 - 38). Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

vonGlasersfeld, E. (Ed.). (1991). Radical constructivism in mathematics education. Dordrecht: Kluwer.

Weiss, I. R. (1995). Mathematics teachers' responses to the reform agenda: Results of the1993 national survey of science and mathematics education. Paper presented at the Annual Meeting of the American Educational Research Association (San Francisco, CA). (ERIC Document Reproduction Service No. ED 387 346)

Whitt, E. (1991). Artful science: A primer on qualitative research methods. Journal of College Student Development, 32, 406 - 415.

Wilson, M.R. & Lloyd, G. (1995). High school teachers experiences in a student-centered mathematics curriculum. In D.T. Owens, M.K. Reed, & G.M. Millsaps (Eds.), Proceedings of the Seventeeth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education (Vol. 2, pp. 162 - 167). Columbus, OH: The ERIC Clearinghouse for Science, Mathematics, and Environmental Education.

Wilson, M.R. & Lloyd, G. M. (1996). Sharing mathematical authority: New roles for teachers. Paper presented at the research presession of the 74th annual meeting of the National Council of Teachers of Mathematics, April 23-24, 1996, San Diego: CA.

Wilson, M.R. & Lloyd, G.M. (1997). The challenge to share mathematical authority with students: High school teachers reforming classroom roles and activities through curriculum implementation. Manuscript under review.

Wilson, S.M. (1990). A conflict of interests: The case of Mark Black. Educational Evaluation and Policy Analysis, 12(3), 293 - 310.

Wolcott, H. (1994). Transforming qualitiative data: Description, analysis, and interpretation. Thousand Oaks, CA: Sage.

Wood, T. (1993). Creating an environment for learning mathematics: Social interaction perspective. In T. Wood, P. Cobb, E. Yackel, & D. Dillon (Eds.), Rethinking elementary school mathematics: Insights and issues, Journal for Research in Mathematics Education Monograph No. 6 (pp. 15 - 20). Reston, VA: NCTM.

Wood, T. (1995). Mathematical learning and small-group interaction: Four case studies. In Cobb, P. & Bauersfeld, H. (Eds.), The emergence of mathematical meaning: Interaction in classroom cultures (pp. 203 - 228). Hillsdale, NJ: Lawrence Erlbaum Associates.

Wood, T., Cobb, P. & Yackel, E. (1993). The nature of whole-class discussion. In T. Wood, P. Cobb, E. Yackel, & D. Dillon (Eds.), Rethinking elementary school mathematics: Insights and issues, Journal for Research in Mathematics Education Monograph No. 6 (pp. 55 - 68). Reston, VA: NCTM.

Woods, P. (1986). Inside schools: Ethnography in educational research. New York, NY: Routledge & Kegan Paul.

Yackel, E., Cobb, P. & Wood, T. (1993). Developing a basis for mathematical communication within small groups. In T. Wood, P. Cobb, E. Yackel, & D. Dillon (Eds.), Rethinking elementary school mathematics: Insights and issues, Journal for Research in Mathematics Education Monograph No. 6 (pp. 33 - 44). Reston, VA: NCTM.

Yin, R.K. (1989). Case study research: Design and methods. Newbury Park, CA: Sage.

APPENDICES

APPENDIX A
CODING SHEET FOR MR. LESTER

Directions for examining whole-class discussion transcripts

The transcript for whole-class discussions is to be read and coded by the participating teacher using the steps below:

1) Underline and color code teacher dialogue using codes below:
    a) promoting - you were trying to promote discussion of a topic
    b) posing - you were trying to elicit students' thinking with a question
    c) listening - you were trying to listen to a student idea
    d) maintaining - you were trying to keep discussion going
    e) clarifying - you were trying to ask a student to clarify their idea
    f) extending - you were trying to ask a student to extend their idea
    g) mathematical language - deciding to "attach" notation or "definitions" to generated ideas
    h) modeling - you were trying to model participation in discussion
    i) other - pick a label to describe what's happening
2) Circle and color code student dialogue using codes below:
    j) generalizing - student makes a generalization
    k) discovering - student expresses an idea discovered through discussion
    l) responding - student simply responds to a prompt
    m) initiating - student initiates a problem or question
    n) questions - student responds to an idea by asking a question
    o) presenting - student presents solution to a problem
    p) explores - student explores an example or counterexample
    q) other - pick a label to describe what's happening
3) Write any thoughts other than codes on a separate piece of paper and cite the page number and approximate location on page.

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RESEARCH

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PROFESSIONAL EXPERIENCE

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1997 - present — *Assistant Professor, Mathematics/Education*
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1994 - 1996 — *Instructor, Basic Applied Agricultural Mathematics*;
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Taught class and developed syllabus, instructional materials, and assessment materials

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1992	*Instructor, Pre-Calculus*; Virginia State University, Petersburg, Va. Taught graphing calculator intensive class with math department syllabus.

Supervision

1993 - 1997	*Supervisor of Secondary Mathematics Student Teachers*; Virginia Tech, Blacksburg, Va. Assisted in teaching methods classes and developing evaluation instruments; observed student aides and student teachers in classroom teaching situations; held conferences with student teachers and cooperating teachers.

Administration

1988 - 1990	*Graduate Assistant for Mathematics Department Head*; Virginia State University, Petersburg, Va. Performed clerical duties, library research, and assisted in mathematics classes

Public Schools

1996	*Instructor, Summer Honors Middle School Program*; Roanoke City Public Schools, Roanoke, Va. Developed lessons and materials for teaching informal geometry concepts and a credit geometry class for prospective Governor's School students
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1991 - 1992	*Instructor, VSU Summer Academy* (a summer program aimed at enriching the math and science opportunities for minority students); Virginia State University, Petersburg, Va. Developed and taught lessons in problem solving skills.
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Nelson, Rebecca S. (February 1997). Utilizing whole-class discussions to develop mathematical knowledge: One teacher's dilemmas in implementing reform. Paper presented at the Eastern Educational Research Association Conference, Hilton Head, SC.

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