CHAPTER 5 PROBLEM THEORY AND DEVELOPMENT

5.1 GENERAL OVERVIEW

There are two general approaches towards the analysis of moored structures used as breakwaters. They are frequency domain analysis and time domain analysis. Frequency domain analysis is usually used when the primary concern of the examiner is the breakwater's overall effectiveness. However, both methods are frequently used when examining forces that are developed in the mooring lines (Tsinker 1995). In this thesis, frequency domain analysis is used exclusively.

The linear analysis procedure used to analyze the breakwater's motions in water is similar to the free vibration theory discussed in Chapter 4. However, here three new elements are introduced. The motions are forced due to the waves passing over the structure, damping due to the fluid structure interaction is included, and an added mass term is included to account for the decreased response of the structure due to the presence of external water.

The equation of motion, in matrix form, that describes the motion of the breakwater is found as

$$[M + M_A]\{\ddot{\xi}\} + [C]\{\dot{\xi}\} + [K]\{\xi\} = \{F(t)\} \tag{5.1}$$

where [] and {} represent square matrices and column vectors, respectively. Given the 6 degrees of freedom of the structure, the square matrices are [6x6] (6 rows by 6 columns) and the column vectors are {6x1}. The left-hand side of (5.1) represents the forces acting on the body in still water. When the breakwater accelerates in still water, a hydrodynamic force is developed which is represented by $[M_A]\{\ddot{\xi}\}$. Similarly, a hydrodynamic force proportional to the breakwater's velocity is written as $[C]\{\dot{\xi}\}$. For the cylindrical breakwater, $[M_A]$ and [C] are functions of the frequency of motion. They are also dependent on the structure's shape and depth in water. These properties are constant for the breakwater of this thesis. The stiffness matrix [K] is due to the stiffness of the mooring lines, which anchor the breakwater to the ocean floor. The forces {F(t)} making up the right hand side of (5.1) are computed for a series of sinusoidal waves passing over the breakwater which is fixed in space (in water). These forces are a function of the wave amplitude, wave frequency, and incidence angle. In the context of linear theory, the wave forces as well as the body motions are proportional to the wave amplitude. In this study, a unit amplitude wave is used for all conditions. Although several important and rare topics such as three-dimensional analysis, oblique waves, and end effects are included in this thesis, there are limitations to its applicability. The primary assumption is that linear theory (small motion) prevails, as evident in the form of (5.1), and that linear wave theory is adequate. However, for regular waves and small motions of the structure, this assumption is very adequate. There is a general exception for moored structures. At very low frequencies, the mooring lines will act much more like catenaries, and the provided stiffness is highly nonlinear (Tsinker 1995). On the other hand, the breakwater of this thesis is an air-filled structure with a very large net bouyant force acting on it and Chapter 3 ensures the mooring lines are taut which causes the lines to respond linearly even at low frequencies.

5.2 LINEAR WAVE THEORY AND FLUID BEHAVIOR

Before proceeding to the question of how to solve for the new elements of (5.1) (M and K are the same as in Chapter 3), the assumptions and wave theory should be considered. It is assumed that the fluid is inviscid and incompressible. It is also assumed that the fluid particle motion is irrotational (i.e., the curl of the velocity vector is zero). Therefore, the velocity field can be written as the gradient of the velocity potential Φ.

For the examination of the moored cylinder as a breakwater, a new coordinate system will be chosen which is in accord with those more common to ocean engineering systems. The global XYZ system is fixed to the ocean floor directly below the center of gravity of the cylinder in its equilibrium configuration. The Z axis is defined as positive upwards, the Y axis is parallel to the cylinder's axis, and the X axis is positive in the direction of wave propagation for β=0. This new coordinate system is shown in Figure 5.1 with the incidence angle β. In addition to coordinate changes, nondimensional quantities will be dimensionalized and metric units will be used for all figures.

In response to the sinusoidal waves of unit amplitude and frequency ω (rad/s) passing over the structure, the cylinder will be assumed to undergo small steady-state vibrations about its equilibrium position at the driving frequency ω of the waves. The displacement of the structure will involve rigid body motions in each of the six previously established degrees of freedom and defined by

$$\xi_j(t) = \hat{\xi}_j e^{-i\omega t} \tag{5.2}$$

where j = 1 to 6 for each of the six degrees of freedom, surge, heave, sway, pitch, yaw, and roll, respectively, and the hat indicates a complex amplitude. Following Newman (1994) and Dewi (1997), the velocity potential can be written in the form

$$\Phi(x,y,z,t) = \mathrm{Re}\left[\left(\phi_I + \phi_D + \sum_{j=1}^{6} \phi_j(x,y,z)\hat{\xi}_j\right)e^{-i\omega t}\right] \tag{5.3}$$

where

$\phi_D e^{-i\omega t}$ is the potential of the diffracted waves,

$\phi_j e^{-i\omega t}$ is the radiation potential for mode (degree of freedom) j, and

$\phi_I e^{-i\omega t}$ is the incident wave potential.

The incident wave potential at some position z above the ocean floor is defined by Newman (1994) as

$$\phi_I = \frac{igA_I}{\omega}\frac{\cosh(kz)}{\cosh(kd)}e^{ik(x\cos\beta + y\sin\beta)} \tag{5.4}$$

where

A_I is the incident wave amplitude,

d is the water depth,

β is the incidence angle, and

k is the wave number. The wave number can be found from the following dispersion relation (Dean and Dalrymple 1991):

$$\omega^2 = kg\tanh(kd) \tag{5.5}$$

where g is the acceleration of gravity.

5.3 EXTERNAL FLUID CHARACTERISTICS

The motion of the fluid surrounding the air-filled breakwater is defined such that each of the ϕ_j potentials must satisfy Laplace's equation, $\nabla^2\phi_j=0$ (Dean and Dalrymple 1991). The following boundary conditions must also be satisfied:

For the free surface:

$$\frac{\partial\phi_j}{\partial z} - \frac{\omega^2}{g}\phi_j = 0 \tag{5.6}$$

For the ocean floor:

$$\frac{\partial \phi_D}{\partial z} = 0$$
$$\frac{\partial \phi_j}{\partial z} = 0 \qquad (5.7)$$

For the wet body surface, S_b:

$$\frac{\partial \phi_D}{\partial n} = -\frac{\partial \phi_I}{\partial n}$$
$$\frac{\partial \phi_j}{\partial n} = -i\omega n_j \qquad (5.8)$$

where

$$n_j = \vec{s}_j \cdot \vec{n} = u_j n_X + v_j n_Y + w_j n_Z \qquad (5.9)$$

where $\mathbf{s_j}$ denotes the displacement of panel j in the X, Y, and Z directions, respectively, and **n** points from the body surface to the external fluid and is normal to the breakwater surface.

A boundary integral method is used to analyze the fluid flow around the structure. Wehausen and Laitone (1960) provide the necessary Green's function for this method, namely

$$G(P,Q) = \frac{1}{\sqrt{R^2 + (z-\zeta)^2}} + \frac{1}{\sqrt{R^2 + (2d+z+\zeta)^2}} + 2\int_0^\infty \frac{\left(k + \frac{\omega^2}{g}\right)\cosh[k(z+d)]\cosh[k(\xi+d)]}{k\sinh(kd) - \frac{\omega^2}{g}\cosh(kd)} e^{-kd} J_0(kR)\,dk \qquad (5.10)$$

where

P=(x,y,z) is the coordinate of the field point,

Q=(ξ,η,ζ) is the coordinate of the source point, and

R=$\sqrt{(x-\xi)^2 + (y-\eta)^2}$

The application of Green's second identity gives a Fredholm equation of the second kind, which in turn results in values of the potential on the wet body surface. The Fredholm equation can be written as

$$-2\pi\phi_j(P)+\iint_{S_b}\phi_j(Q)\frac{\partial G(P,Q)}{\partial n_Q}dS_Q=\iint_{S_b}\frac{\partial\phi_j}{\partial n_Q}(Q)G(P,Q)dS_Q \tag{5.11}$$

The above integral is solved by approximating the cylindrical breakwater surface by a set of flat, quadrilateral panels with a known centroid for every panel. It is assumed that the potential across a given panel is constant and the value is assigned to the centroid. In order to evaluate Green's function (5.10), the FORTRAN subroutine FINGREEN developed by Newman (1985) is used.

5.4 DETERMINATION OF FORCE, ADDED MASS, AND DAMPING ELEMENTS

Recalling (5.1), $[M_A]$, [C], and {F} must be determined for each of the six degrees of freedom the structure undergoes. Following Newman (1994), the added mass terms are found in matrix form from

$$[M_A]_{ij}=\frac{1}{\omega^2}\mathrm{Re}\left(-i\omega\rho_w\iint_{S_b}\phi_j n_i dS\right) \tag{5.12}$$

where$[]_{ij}$ denotes a given matrix element of the square [6x6] matrix. Also, ρ_w is the density of salt (ocean) water. Likewise, the damping matrix [C] is found by

$$[C]_{ij}=\frac{1}{\omega}\mathrm{Im}\left(-i\omega\rho_w\iint_{S_b}\phi_j n_i dS\right) \tag{5.13}$$

and the total hydrodynamic force, which is composed of the incident wave force and the diffracted wave force, is found by

$$F_j^I+F_j^D=-i\omega\rho_w\iint_{S_b}(\phi_I+\phi_D)n_j dS \tag{5.14}$$

for each of the j =1 to 6 degrees of freedom. With the previously mentioned assumption dealing with a constant potential across each panel, and verifying convergence and the required number of panels to insure validity of results, each of the integrals above is given by a summation over the surface of the breakwater.

5.5 SOLVING FOR RESPONSE AMPLITUDES OF THE BREAKWATER IN WATER

Now (5.1) can now be solved in the frequency domain by inserting the adequate added mass, damping, and forcing terms for a given wave frequency. The response amplitudes ξ_j as defined earlier can be found by solving six simultaneous equations. The velocity potentials on the free surface are obtained from Green's Theorem (Dewi, 1997) as

$$\phi_j(P) = \frac{1}{4\pi}\iint_{S_b}(\phi_j(Q)\frac{\partial G(P,Q)}{\partial n_Q} - G(P,Q)\frac{\partial \phi_j(Q)}{\partial n_Q})dS_Q \tag{5.15}$$

and are part of the total velocity potential Φ summation of (5.3).

Finally, it is of prime importance to find the free surface elevation ζ in the proximity of (and especially behind) the breakwater. It can be found from

$$\zeta = -\frac{1}{g}\frac{\partial \Phi}{\partial t} = \frac{i\omega\Phi}{g} \tag{5.16}$$

5.6 PROCEDURE VALIDATION

The FORTRAN program used to solve for the added mass, damping, hydrodynamic forces, and velocity potentials for this breakwater was developed by Dewi (1997). As a result, the program has been previously verified with existing results in the literature for two-dimensional and three-dimensional arches, a rigid hemicylinder attached to the ocean floor, a vertical cylindrical column, and circular plates in contact with fluid. However, given the particular structure of this thesis, a moored, horizontal, rigid, cylindrical breakwater, the FORTRAN program should still be validated. As discussed in detail in Chapter 2, Evans et al. (1979) analyzed a long cylinder in waves using two-dimensional analysis and found that at the "tuning frequency" the transmission coefficient is zero. Creating a 304.8 m (1000 ft) long cylinder, these results have been verified. Although a three-dimensional analysis is used, the structure moves only in sway and heave for normal waves. Since end effects are negligible, the structure's motion can be described accurately in two dimensions. Figure 5.2 shows the magnitude of the waves over the proximity of the structure. The structure is located at X equal to 0 and extends from –152 m to 152 m in the Y direction. The waves advance from –X to +X. Figure 5.2 shows clearly that for a long structure, the magnitude of the waves after the structure is approximately zero. The plot is for a wave frequency ω of 1.25 rad/s which is the wet natural frequency ω_n of the cylinder. Here the term wet natural frequency is analogous to "tuning frequency" and represents the first natural frequency of the structure in water.

Figures 5.3 (a)-(f) show the free surface elevation near the breakwater at fractions of the wave period. The figures are not to scale and the structure appears much larger than it actually is. This is done in order to more specifically show the wave elevations near the structure. The location of the horizontal plane below the structure is not defined by Z=0 (ocean floor), but is approximately Z=3.0 m (the bottom of the structure). Figure 5.3 (e) seems strange, for the upstream waves are apparently reduced by the breakwater. This is not really the case. It should be recalled that the upstream waves represent the superposition of incident and deflected waves. At this time, the two waves are "out of phase" and somewhat cancel each other out. On closer inspection, it is clear that at some time t, the upstream waves are larger than at other times. Hence, depending on the phase of the incident and deflected waves, amplification or cancellation can take place. Figure 5.3 portrays the structure's effectiveness. Nevertheless, as will be shown in Chapter 6, for a more reasonable "short" breakwater under normal waves, end effects are considerable and the structure is not as effective. In addition, for oblique waves, the structural response is not two-dimensional and its effectiveness varies.