Evaluation of Ranked Set Sampling for Estimating Shrub Phytomass in Appalachian Oak Forests

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by

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INTRODUCTION

Ranked set sampling was introduced by McIntyre (1952) in estimating pasture yields. As described by McIntyre, the ranked set sampling procedure is implemented by first selecting a set of n elements at random from a population, and then raking those elements from lowest to highest by inspection of the variable of interest. The lowest ranked element is then measured. A second set of n elements is selected and ranked, and the second lowest element is measured. This process is continued until n such sets have been selected and n elements have been measured. The entire procedure can be repeated as many times as deemed necessary.

McIntyre claimed that the ranked set estimator of the population mean was unbiased regardless of errors in ranking, and that with perfect ranking the variance of the mean from ranked set sampling would be less than that of random sampling when the number of measured elements is the same for both methods. Halls and Dell (1966) concluded that ranked set sampling was more efficient than simple random sampling in estimating pasture yields. A theorectical underpinning for ranked set sampling was provided by Dell and Clutter (1972), who also demonstrated that ranked set sampling is more efficient than random sampling even when errors in ranking are present.

Ranked set sampling is clearly advantageous when measurement of an element is time consuming or costly and sample elements can be reliably ranked. The effectiveness of this technique has already been demonstrated for estimating forage and pasture yields. The objective of this study is to evaluate the effectiveness of ranked set sampling for estimating shrub phytomass in forest stands.

METHODS

The study was conducted on the southeast slope of Potts Mountain in Craig County, Virginia. This area is typical of second growth Appalachian oak forests on upland sites underlain by acid sandstone and shales in the Ridge and Valley Province of Virginia. A complete description of the study area is available in McEvoy et al. (1980) and Martin (1979).

Four major vegetation types along a decreasing moisture gradient were identified in the study area. The types were mixed hardwood, mixed oak, mixed oak and pine, and mixed pine. In each of the four vegetation types a 20 m by 20 m area was subjectively located, within which 16 5 m by 5 m plots were nested.

Actual Ranked Set and Random Samples

The ranked set sample was collected by randomly selecting four sets of four plots from the 16 plots in each vegetation type. The plots in each set were ranked on the basis of ocularly estimated shrub phytomass (1 m to 5 m in height). The shrub phytomass on the lowest ranked plot in the first set was measured by harvesting and weighing all vegetation between 1 m and 5 m in height. The shrub phytomass on the second lowest ranked plot in the second set was measured, and the process continued until one plot in each set had been measured. At the conclusion of sampling four sets of four plots had been ranked in each vegetation type, and shrub phytomass had been measured on four plots, one in each of the four ranks.

A random sample of 16 plots was gathered to provide a comparison with the ranked set sample. Four plots of the 16 in each vegetation type were randomly selected without replacement and without regard to the ranked set plot selection. On each selected plot the shrub phytomass was harvested and weighed.

The shrub phytomass on all remaining plots was harvested and weighed after the ranked set and random samples had been collected.

Simulated Ranked Set and Random Samples

A simulation trial of ranked set sampling and random sampling was performed within each vegetation type. The simulation was conducted to compare the sampling methods using a larger sample size than was originally available, and to test the procedure suggested by McIntyre (1952) and

Dell and Clutter (1972) in which sample size is unequally allocated among the ranks on the basis of estimated variance within the ranks.

Three sampling methods were simulated: (1) ranked set sampling with equal allocation in each rank, (2) ranked set sampling with allocation in each rank proportional to the estimated variance of that rank and (3) random sampling.

The phytomass values of the 16 plots in each vegetation type were used as the simulation population. All ranking in the ranked set sample methods was perfect, i.e. without error.

Ranked set sampling with equal allocation in each rank was simulated by randomly generating eighty sets of four plots, without replacement in each set, from the 16 plots in each vegetation type. The plots in each set were ranked on the basis of previously determined phytomass. In twenty sets the lowest ranked plots were measured, i.e. previously determined phytomass was treated as having been measured. In another twenty sets the second ranked plots were measured with the procedure being repeated until 20 plots in each of the ranks had been measured.

Sample sets for the unequal allocation ranked set sample were generated in the same manner as those for the equal allocation ranked set sample. The number of samples to be measured in each rank was calculated as the ratio of variance in a rank from the equal allocation sample to the sum of the variances for all four ranks from the equal allocation sample. Therefore, ranks with larger variance were sampled more intensively than those with smaller variances. The sample sizes in each rank and vegetation type are shown in Table 1.

The random sample was generated by successive random selection of one plot from the 16 plots in each vegetation type until 80 plots in each type had been selected.

Table 1. Number of measured plots in each rank* and vegetation type for simulated ranked set samples with uneaqual allocation.

	VEGETATION TYPE			
Rank	Mixed Hardwood	Mixed Oak	Mixed Oak & Pine	Mixed Pine
1	50	42	62	24
2	18	30	6	20
3	10	6	5	27
4	2	2	7	9

* Highest rank is 1.

RESULTS

Actual Ranked Set and Random Samples

The ranked set estimate of the mean was much closer to the mean of all 64 plots than was the random sample estimate (table 2). Also, the variance of the mean and coefficient of variation from the ranked set sample were smaller than those for random sampling.

The relative precision of random sampling to ranked set sampling was 1.66. This value does not exceed the approximate limit given by McIntyre for the case of perfect ranking, and is in line with the relative precision values with ranking errors given by Dell and Clutter.

Simulated Ranked Set and Random Samples

Comparisons of the sample means with the population mean are inconclusive (table 3). Each sample mean was nearest the population mean in at least one vegetation type and farthest in at least one other type. However, the random and equal allocation ranked set means were closer to the population mean more often than was the unequal allocation ranked set mean.

Both types of ranked set samples had smaller variance and coefficient of variation than did the random sample in each vegetation type (table 3). The unusually high values of relative precision between random and ranked set sampling, however, probably reflect the effects of perfect ranking and many repeated samples drawn from a small sample base as much as they reflect the advantage of ranked set sampling.

The variance of unequal allocation ranked set sampling was less than that for equal allocation in only two vegetation types, and the coefficient of variation was less in only one type (table 3). While this result was unexpected, it appears to be real; both ranked set samples were drawn from the same sample base and both were perfectly ranked.

Table 2. Summary of random and ranked set sample results for 16 measured plots across all vegetation types.

Sampling Method	Mean Phytomass (kg/ha)	Variance of the Mean (x 10^6)	Coefficient of Variation of the Mean (%)
All 64 Plots	2536	0.15	15
Random	1976	4.54	108
Ranked Set	2356	2.73	70

Table 3. Summary of simulated random and ranked set (RS) sample results with 80 measured plots within each vegetation type.

Sampling Method	Mean Phytomass (kg/ha)	Variance of the Mean (x 10^6)	Coefficient of Variation of the Mean (%)
		Mixed Hardwood	
All 16 Plots	656	0.03	28
Random	684	0.50	103
RS-Equal Allocation	652	0.06	38
RS-Unequal Allocation	591	0.05	38
		Mixed Oak	
All 16 Plots	1096	0.09	28
Random	1014	1.15	106
RS-Equal Allocation	1021	0.11	33
RS-Unequal Allocation	1025	0.14	37
		Mixed Oak and Pine	
All 16 Plots	1927	0.17	21
Random	1939	2.70	85
RS-Equal Allocation	1727	0.19	25
RS-Unequal Allocation	1826	0.26	28
		Mixed Pine	
All 16 Plots	6467	0.87	14
Random	7012	6.91	37
RS-Equal Allocation	6436	1.78	21
RS-Unequal Allocation	5892	1.38	20

CONCLUSIONS

Ranked set sampling was more efficient than random sampling in both the actual and simulated sampling trials. Therefore, the number of plots measured for a ranked set sample can be considerably reduced from that planned for a random sample without sacrificing precision. Since ranking shrub phytomass is many times easier and quicker than clipping and weighing the contents of a plot, a considerable savings in sampling time and effort can be achieved.

The literature on ranked set sampling suggests that there is an additional gain in precision to be had by unequally allocating measured plots to the different ranks. In this study there was little or no precision gained by unequal allocation. Since unequal allocation requires slightly more presampling effort than equal allocation, the unequal allocation scheme does not appear to be worthwhile for estimating shrub phytomass in forest stands, and may not be worthwhile in other applications.

LITERATURE CITED

Dell, T. R. and J. L. Clutter. 1972. Ranked set sampling theory with order statistics background. Biometrics 28: 545-555.

Halls, L. K. and T. R. Dell. 1966. Trial of ranked-set sampling for forage yields. Forest Science 12: 22-26.

Martin, W. L. 1979. Above-ground phytomass and nutrients in the understory vegetation in an Appalachian Oak Forest in Virginia. Unpublished M.S. thesis, Department of Forestry, V.P.I. & S.U., Blacksburg, Virginia. 89 pp.

McEvoy, T. J., T. L. Sharik, and D. W. Smith. 1980. Vegetative structure of an Appalachian Oak Forest in Southwest Virginia. Am. Mid. Nat. 103: 96-105.

McIntyre, G. A. 1952. A method of unbiased selective sampling, using ranked sets. Aust. J. Agric. Res. 3: 385-390.