SYSTEM DEPENDABILITY ANALYSIS AND EVALUATION

by

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(ABSTRACT)

Undependable systems are unable to accomplish the mission for which they were designed and cause tremendous loss [Airwar-Vietnam, 1978]. In most instances, timely maintenance is a vital necessity for a system to be kept in or returned to its operating condition, and to be able to successfully complete a specific mission. In a scarcer-resource but higher-competition environment, it is essential that system *dependability* be considered as a major system parameter and be evaluated during the early phases of the system design process.

Given a specific system mission profile, whether or not the mission can be successfully completed is based not only on such measures as performance, reliability, maintainability, and/or availability. There is a need for system designers and engineers to measure system dependability, as well. System dependability is a measure of effectiveness which allows for the consideration of maintenance in the life cycle as long as it does not inhibit the system from fulfilling its mission. It is the aim of this project to develop a methodology for the analysis and evaluation of system effectiveness through the utilization of the system dependability measure.

The concept and the mathematical model of system dependability is discussed. Effectiveness factors and relationships are described, a measure of system dependability has been defined, and a computer-based tool was developed to enable the accomplishment of trade-off analyses and the evaluation of various system configurations in terms of dependability. Maintenance requirements are addressed through the introduction of various combinations of failures, and failure distributions include the consideration of the exponential, Weibull, and log-normal cases.

Application of the dependability model is illustrated through a case study involving an aircraft radar subsystem as an example.

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CHAPTER I

INTRODUCTION TO SYSTEM DEPENDABILITY

Today, in a highly competitive environment, producers are searching for ways to gain a sustainable advantage in the marketplace. Competitiveness and profitability are two vital prerequisites for any enterprise to survive in this environment. Moreover, with the incorporation of new technologies in design, systems and products have become more complex. However, many of the systems in use are hardly meeting the needs for which they were developed, nor they are very cost-effective in terms of consumer utilization [Blanchard, 1992].

1.1 Background

System design and development are accomplished through the systems engineering process. *System Engineering* has been recognized as a systematic approach to simultaneously design products and their related processes, including manufacture and support, as illustrated in Figure 1-1 [Fabrycky and Blanchard, 1991]. More specifically, system engineering is a process employed in the evolution of man-made systems from the point when a need is identified through design, production and/or construction, ultimate deployment for consumer use, support, and retirement and phase-out of that system. The objectives of systems engineering include (1) improving the quality and effectiveness of systems or products through a better integration of requirements, and (2) reducing the system/product development cycle time through a better integration of activities and processes.

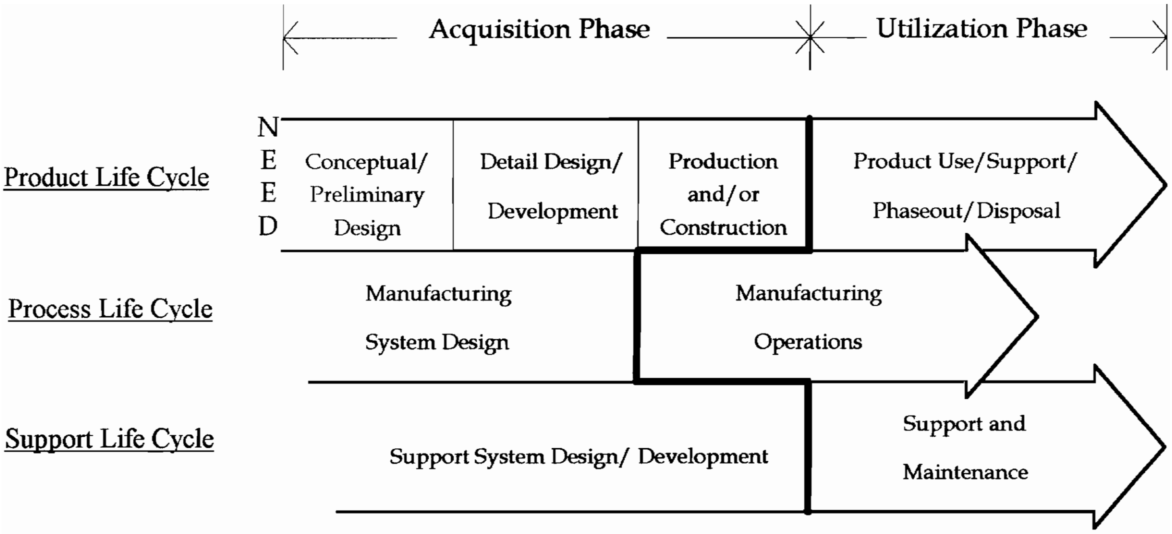


Figure 1-1. Product, process, and support life cycles [Adapted from Fabrycky and Blanchard, 1991].

1.2 Problem Statement

The system engineering process requires an appropriate application of scientific and engineering efforts to ensure that the ultimate product is operationally feasible. One of the more important design parameters of a system is *dependability*. Dependability is a measure of system status providing features and functions that contribute to the rapidity, accuracy, and successfulness with which a system can be kept in or returned to its specified condition, and accomplish the mission for which it was intended. Dependability is a function of reliability, maintainability, and/or supportability; it must be considered along with availability, performance, survivability, human factors, life-cycle cost, and other factors in systems design process.

Many systems in use today are highly sophisticated and do not fulfill their expectations when operating. Experience indicates that many systems are inoperative much of time, requiring extensive maintenance and the expenditure of scarce support resources. In most instances, timely maintenance is a vital necessity for a system to be kept in or returned to its operating condition, and to be able to successfully complete a specific mission. Undependable systems are unable to accomplish the mission for which they were designed and cause tremendous loss [Airwar-Vietnam, 1978]. In a scarcer-resource but higher-competition environment, it is essential that system dependability to be considered as a major system parameter and be evaluated during the early phases of system design process.

From the previous discussion, given a system mission profile, whether the mission can be successfully completed is based not only on the measures of reliability, maintainability, or availability. There is a need for system designers and engineers to measure the system dependability, or that the system mission can be successfully completed. It is the aim of this project to develop a methodology for the evaluation of system effectiveness through the utilization of the system dependability measure.

1.3 System Effectiveness and System Dependability

In this section, the system effectiveness concept and its relationship with system dependability are introduced. A combat aircraft is selected as the system for discussion of the concepts of system effectiveness, system dependability, and other system design characteristics (e.g., reliability, maintainability, availability, performance).

Figure 1-2 depicts a combat aircraft scenario and Figure 1-3 illustrates a typical operational mission profile. In this scenario, the aircraft attempts to get to the target area, to locate the intended target, to deliver one or more weapons that kill the target, and to return home base. A study of this scenario reveals that the ability of the aircraft system to accomplish these tasks is dependent upon:

(1) The availability of the aircraft system at the start of its mission.

(2) The dependability of the aircraft system during the mission.

(3) The aircraft performance capabilities and the target acquisition capabilities.

In this example, the aircraft system effectiveness can be expressed as a function of one or more figures of merit representing the extent to which the aircraft system is able to perform the intended function. A mission-oriented system effectiveness model is chosen to evaluate the system effectiveness (*SE*). This model considers the following measures [WSEIAC, 1965]:

(1) Measure of *availability* (*A*). Is the system or product working at the start of the mission?

A system cannot perform its mission unless it is operating. The probability that the system is in the operable and committable state *at the start* of a mission when the

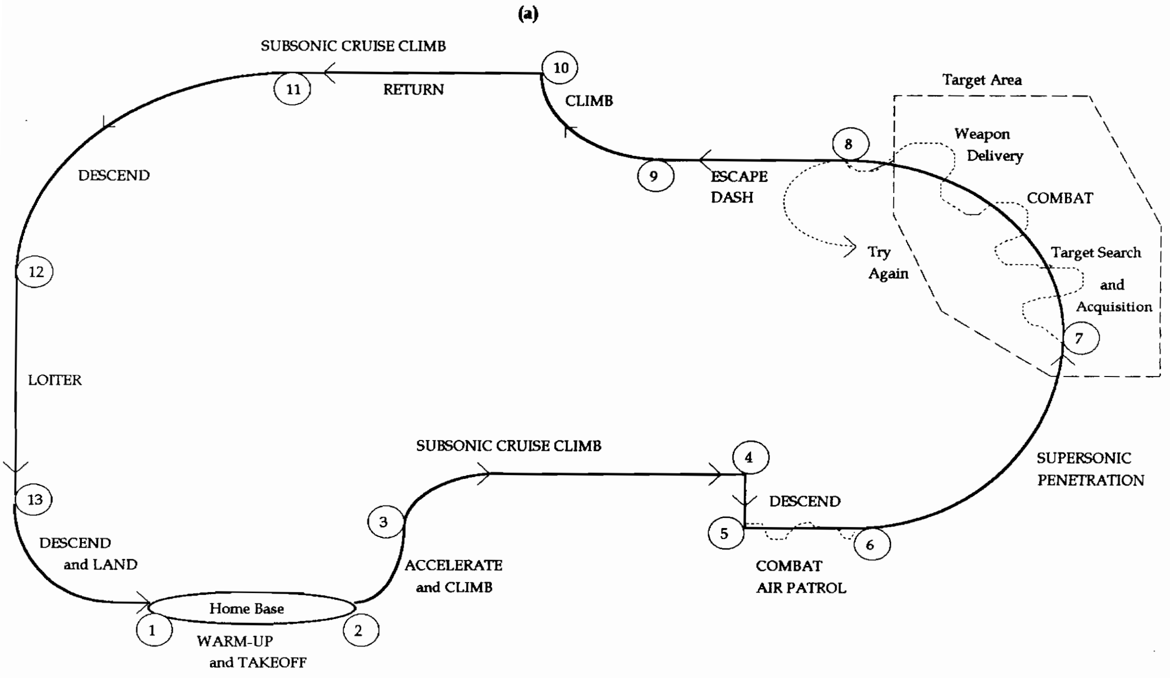


Figure 1-2. Typical system operational mission scenario.

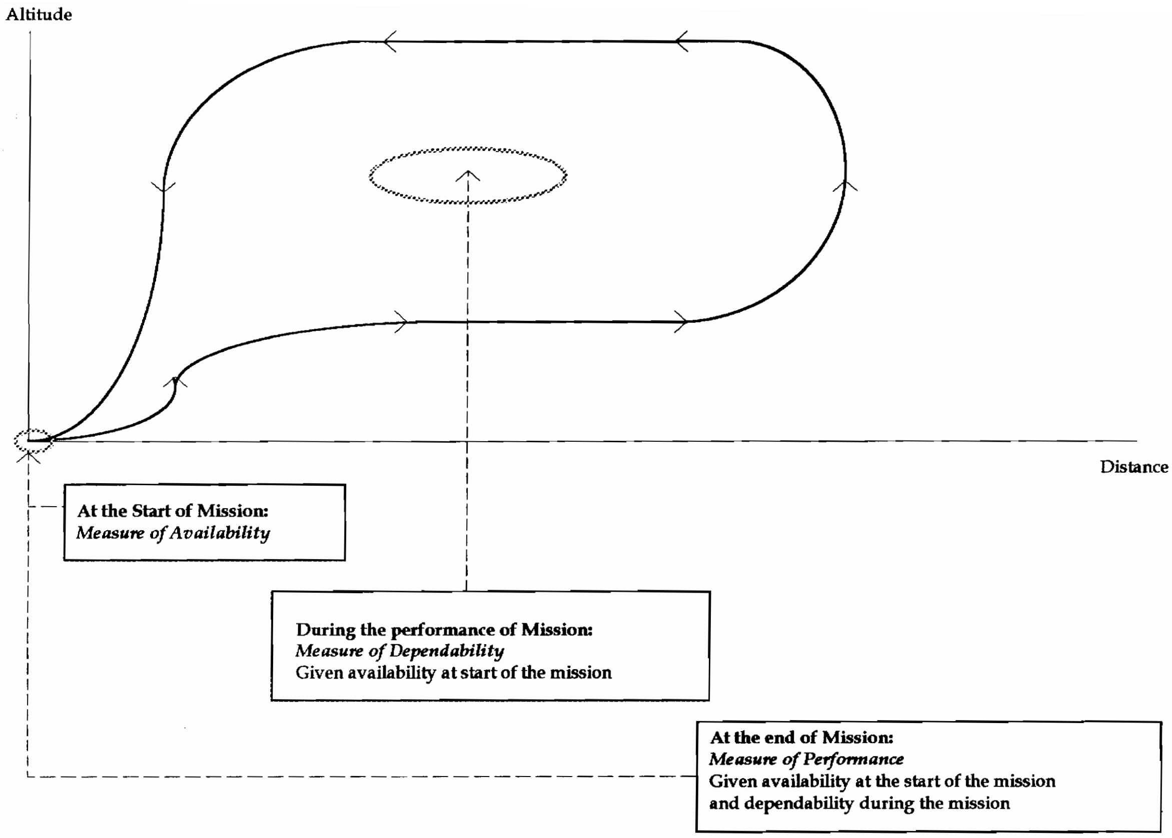


Figure 1-3. Typical system operational mission profile.

mission is called for at an unknown random point in time is called *availability* [Blanchard, 1992].

(2) Measure of *dependability* (*D*). If the system is working at the start of the mission, will it continue to work throughout the mission?

A system, however, still cannot perform its mission even though it is operating unless it accomplishes the mission and brings it to completion. The probability that the system will successfully complete its mission within the mission time providing a downtime per failure not exceeding a given time will not adversely affect the overall mission, given the system condition at the start of the mission (i.e., availability).

(3) Measure of *system performance parameters* (*P*). If the system worked throughout the specified mission scenario, will it achieve its objective?

These system performance parameters are measures of system *capability* such as the capability of aircraft performance, the range or weight of an airplane, destructive capability of a weapon system, and accuracy of a radar capability. In this instance, *performance* is the probability that a system will function given that it is available at the start of its mission and dependable throughout the mission scenario.

These three characteristics of system effectiveness are summarized in Figure 1-4 [Blanchard, 1969].

On the basis of this concept, the following function of general system effectiveness is stated:

$$SE = f(A,D,P) \cong A \cdot D \cdot P. \tag{1.1}$$

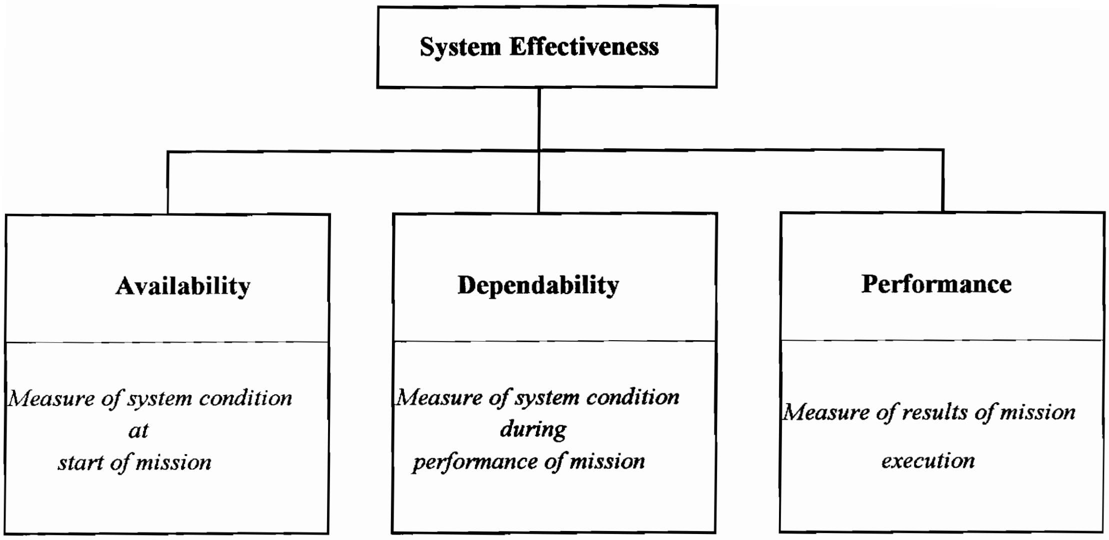


Figure 1-4. Dependability and system effectiveness[Adapted from Blanchard, 1969].

Trade-off studies are accomplished throughout system design and development in order to acquire the best balance among these parameters and to meet overall effectiveness requirements while reducing life-cycle cost. System dependability is an important measure of the system because of the mission success characteristic. System dependability can be expressed as a function of operating time (reliability) and downtime (maintainability and/or supportability). By considering system dependability, system performance can be assumed as being satisfactory even if a failure occurs, providing that the time required to repair the system after it has failed is sufficiently short and does not negatively impact on the completion of the mission. Consequently, system maintainability must be considered along with system reliability in determining the probability of the mission completion. The mission time is defined as the continuous time interval over which satisfactory system operation is required in support of the mission or the function undertaken by the whole system. More specifically, for the purposes of this project, *system dependability* is defined as:

> "the probability that given an operational system at time zero the system will successfully complete its mission within the mission time providing that any system failures occur during the mission can be corrected within a certain pre-determined interval of time so as to not adversely impact the overall mission" [Yang and Verma, 1994].

From this definition, a statistical model of system dependability has been developed to evaluate the system effectiveness, on a mission-oriented basis, during system design process. This project is focused on ***how to improve system effectiveness through the measurement of system dependability.***

1.4 Project Objective

The objective of this project is to develop a system dependability model for the purpose of evaluating the overall effectiveness of a system during its design process. This model is applicable to different instances by using different probability density functions to describe the system failure and/or repair-time characteristics. The exponential, Weibull, and log-normal distributions are discussed in this project report. A case study of designing an aircraft radar subsystem is presented to illustrate the application of the dependability model in selecting a better alternative when considering overall system effectiveness.

CHAPTER II

LITERATURE REVIEW

Chapter II provides a historical overview of system effectiveness, dependability, availability, reliability, maintainability, and capability. The aim of the chapter is to provide a background for understanding the improvement of system effectiveness through the measurement of system dependability.

2.1 System Effectiveness

While the term "system effectiveness" and its basic concepts can be traced back to at least the early 1940s, the organized impetus to apply these concepts in a practical and orderly fashion did not appear until the formation of the Weapon System Effectiveness Industry Advisory Committee (WSEIAC) in the 1960s [WSEIAC, 1965]. The general objective of WSEIAC was to provide technical guidance and support to Air Force Systems Command (AFSC) in the development of a technique to apprise management of current and predicted weapon system effectiveness at all phases of weapon system life.

System effectiveness, as viewed by WSEIAC, is a characteristic of the end product in service use. It is a measure of the ability of a system to successfully perform and accomplish its ultimate mission. Although such a concept could logically be expanded to include all of the elements necessary to the effective management of system programs, this was beyond the scope of the WSEIAC studies. Such innovations as the Program Definition Phase (DoD Directive 3200.9), Integrated Logistics Support (DoD Directive

4100.35), PERT, configuration management, to name a few, certainly deserve our careful study and cautious implementation. But the question remains - "are the systems that we produce really effective in accomplishing their missions?" It is this focus that the WSEIAC studies emphasized.

2.1.1 Why System Effectiveness?

The need for system effectiveness analysis arises from the increased demands for more performance; lower cost (life-cycle cost); longer operating time; more productivity (efficiency); and limited resources and test data. Some of the problems underlying the need for an integrated approach to system effectiveness are briefly summarized herein:

(1) Design for the Life Cycle.

In general, there are three coordinated life cycles progressing in parallel, as illustrated in Figure 1-1. Design engineers have focused primarily on the acquisition phase of the product life cycle, being involved in early design and analysis activities alone, and performance has been a main objective. However, experience indicates that a properly functioning product, which is competitive in the marketplace, cannot be achieved through efforts applied largely after the product comes into being. It must go beyond consideration of the life cycle of the product itself to concurrently embrace the life cycle of the manufacturing process as well as the life cycle of the product support capability. The objective is to ensure that the entire life of a system is considered from inception. An engineering design should take into account life-cycle outcomes as measured by performance, effectiveness, producibility, reliability, maintainability, supportability, availability, dependability, quality, disposability, and cost. It is best accomplished through a concurrent life-cycle approach, as illustrated in Figure 1-1, to ensure economic competitiveness [Fabrycky and Blanchard, 1991].

(2) Cost-Effectiveness Trade-off

Few deny the necessity of maintaining a good defense capability, yet there is increased emphasis to reduce defense costs. At the same time, there is emphasis to increase system effectiveness, with the possibility of new threats in mind. On the face of it, these two objectives are in conflict. The challenge is to obtain as efficient a defense posture as possible within the constraints of cost and effectiveness [WSEIAC, 1965]. Stated another way, we want to minimize cost when the effectiveness is constrained by some lower bound. Figure 2-1 illustrates the relationships between effectiveness and total life-cycle cost, where the objective is to design a system to meet a specified set of values (a given effectiveness level and an overall cost limitation) and yet be cost-effective. This is generally referred to as cost-effectiveness trade-off. Trade-off means allocating the national resources in a way that withstands the critical vision of hindsight.

2.1.2 System Effectiveness Models

Through the WSEIAC activity, a modeling concept was developed and used by Task Group II of the WSEIAC. The WSEIAC defined system effectiveness as "a measure of the extent to which a system may be expected to achieve a set of specific mission requirements and is a function of availability, dependability, and capability." The concept takes into account the entire system. It was realized that the primary concern of management was to derive a way to predict and evaluate the overall effectiveness of a system, setting each contributing system characteristic in its proper perspective.

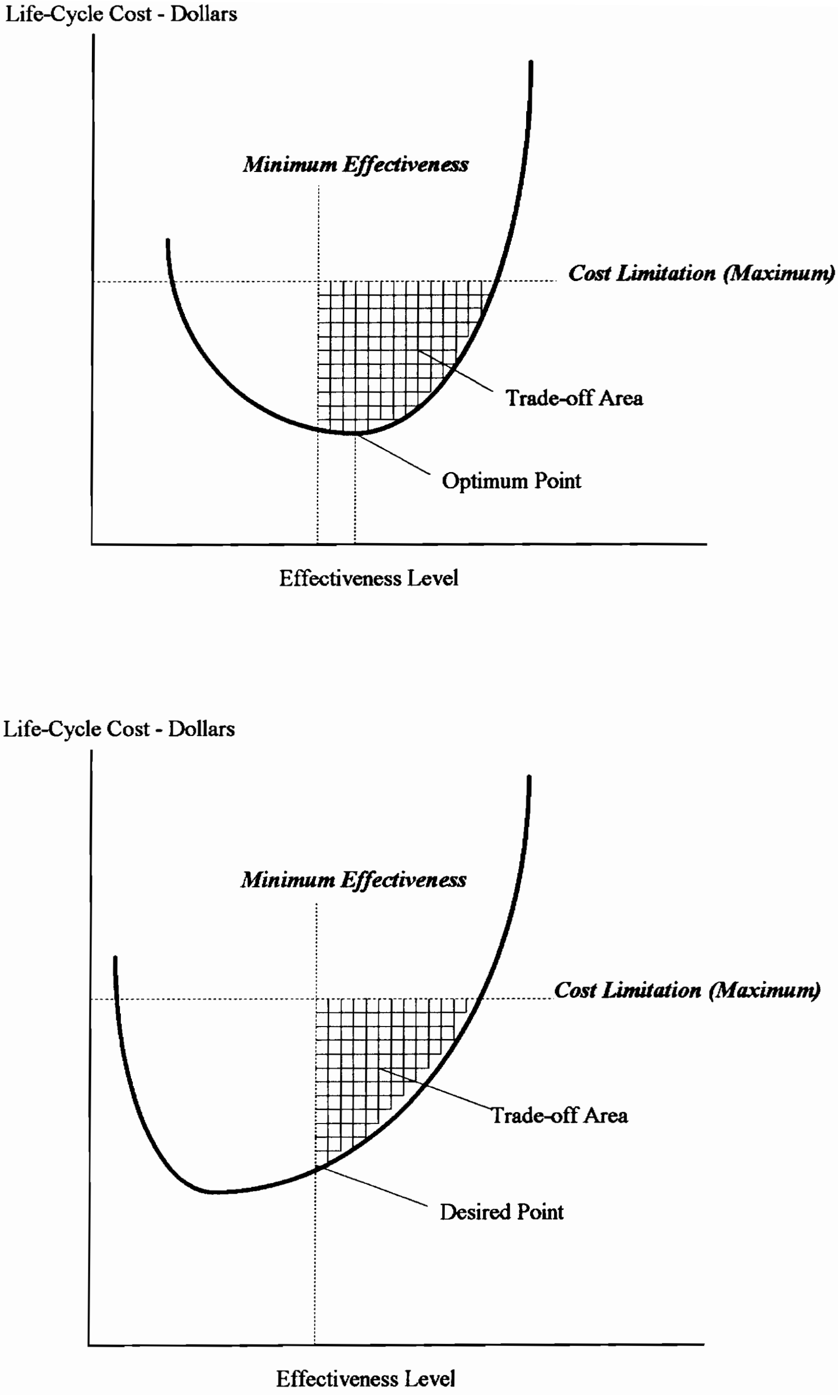


Figure 2-1. Effectiveness versus life-cycle cost.

Due to economic and technical constraints, it is no longer possible to fully simulate system performance through actual hardware and/or software prototypes. It is desirable to look at alternatives early through CAD (computer-aided design) methods and eliminate possible areas of risk prior to building prototypes. From the preceding considerations, the general role of analytic modeling is clear. The appropriate application of analytic models provides insight; they assist in making an empirical approach to system design economically feasible.

Figure 2-2 describes this modeling process in a general way. It is comprised of the following very broad categories of effort:

(1) Acquisition of relevant information.

(2) Definition of the system.

(3) Performance of the cost effectiveness analysis and optimization.

(4) Feedback of the results in terms of adaptations to elements of the system.

2.2 System Dependability, Availability, Capability, Reliability, and Maintainability

In general, system effectiveness is a measure of the ability of a system to meet a service demand. It is also a function of the capability of the system to operate in accordance with the engineering design concept. However, in practice, system effectiveness depends on the type of system and its mission requirements, and should consider system performance, dependability, availability, and supportability. A combination of these measures represents the technical characteristics of a system. By inspection, one can see that logistics affects the various elements of system effectiveness to a significant degree, particularly with regard to dependability and availability [Cunningham and Cox, 1972; Goldman and Slattery, 1967].

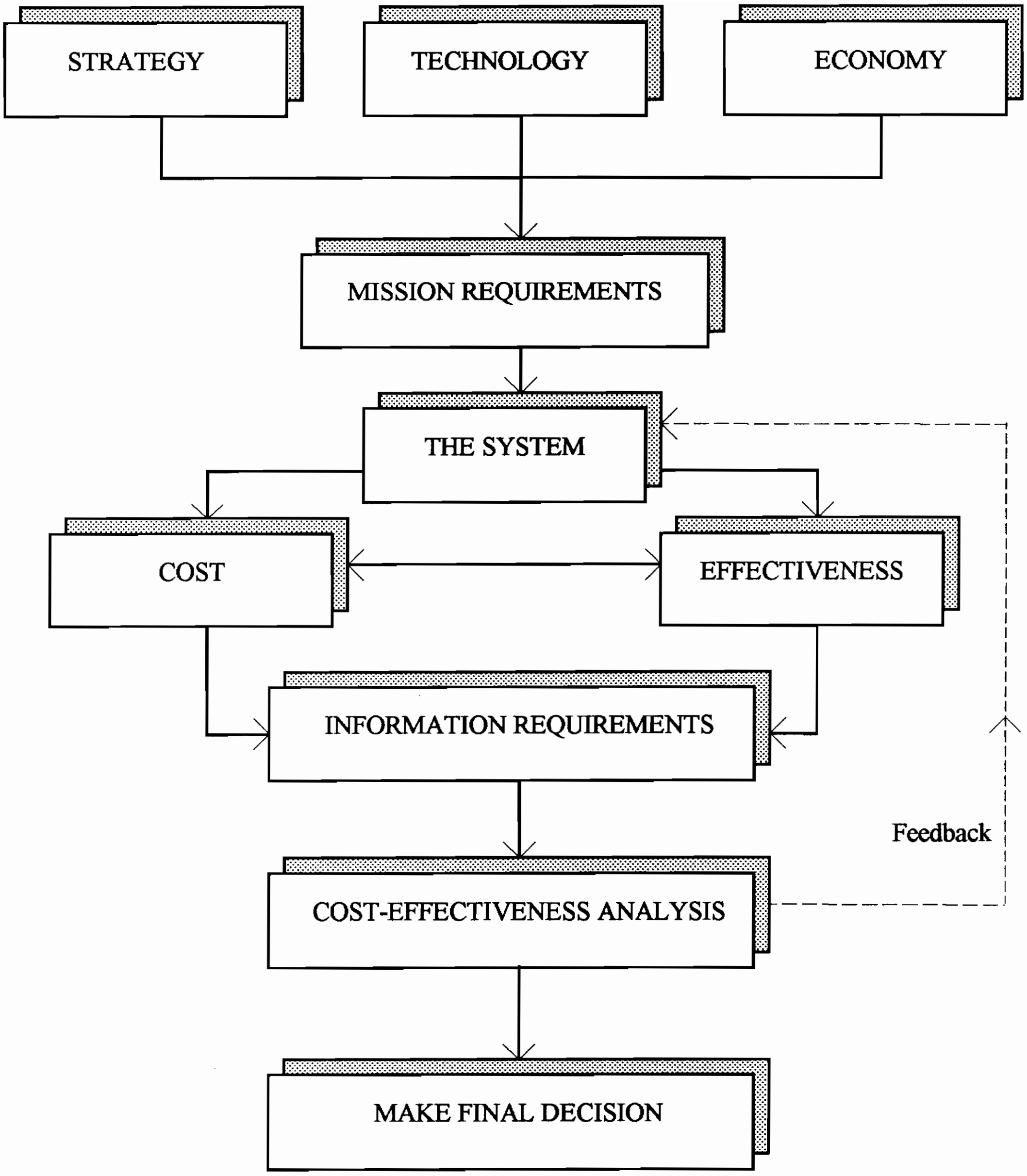


Figure 2-2. Concept of cost-effectiveness analysis.

Dependability can be used, together with availability and performance, to provide a more complete evaluation of a system's characteristics. *Availability* is a measure of the probability that a system will be ready for use at any time. The average operational and repair times give an indication of the net worth of the system to the user. *Dependability*, on the other hand, measures the probability of a successful performance of the system throughout its mission. Availability and dependability highly depend on engineering design as well as the elements of logistic support to ensure the desired system effectiveness is achieved. *Performance* measures the results of a mission that depends on the system design to perform certain functions during the specified mission. The relationship of these three characteristics of system effectiveness discussed above were illustrated in Figure 1-3, [Blanchard, 1969].

CHAPTER III

MEASURES OF SYSTEM DEPENDABILITY

This chapter is concerned with the development of the fundamental definitions and concepts for dependability. These concepts provide the basis for quantifying the dependability of a system, and only precise quantification allows comparison between systems or provides a logical basis for dependability improvement in a system.

3.1 The System Dependability Function

From the definition of dependability, system performance can be considered satisfactory even if a failure occurs, provided the time required to repair the system after it has failed is sufficiently short with respect to its mission time. Consequently, system maintainability must be considered with system reliability in determining the probability of the mission completion. The probability that a system will be capable of satisfactory operation for the duration of a given mission time is called the system dependability. The mission time is defined as the continuous time interval over which satisfactory system operation is required in support of one mission or function undertaken by the whole system.

Consider the probability, $P_A(t)$, that a failure will not occur during a given time interval, t, and the probability, $P_B(\textit{repair time} \leq t')$, that a failure will occur during this time interval (t), but will be repaired in a time not exceeding a given time interval, t'. Assume that either of these two situations would be acceptable. The probability that either one or the other event will occur is the sum of the two individual probabilities, and this new probability will be referred to as the system dependability, $D(t)$,

$$D(t) = P_A(t) + P_B(\text{repair time} \leq t'). \tag{3.1}$$

Assume that $P_f(t)$ is the probability that the system fails during time interval t, and $P_r(\textit{repair time} \leq t')$ is the probability that each repair will be completed in a time not exceeding t', then P_B in Equation (3.1) can be rewritten as

$$P_B = P_f(t)P_r(\text{repair time } \leq t'). \tag{3.2}$$

If t is made equal to the system's mission time, then $P_A(t)$ is the reliability, $R(t)$, of the system for the mission time (t) and because the probability that the system will either fail or not fail is equal to 1, therefore,

$$P_f(t) = 1 - R(t). \tag{3.3}$$

If t' is made equal to the average repair time per failure for each component, then, $P_f(\textit{repair time} \leq t')$ is called the operational maintainability of the system for the allowable repair time, t', and will be denoted by M. If $R(t)$ and M are substituted in Equations (3.1) and (3.2), Equation (3.3) is substituted in Equation (3.2), and Equation (3.2) is, in turn, substituted in Equation (3.1), the following mathematical definition of system dependability, $D(t)$, is obtained [NAVSHIPS, 1964]:

$$D(t) = R(t) + M(1 - R(t)) \tag{3.4}$$

where

$D(t)$ = the system dependability, or the probability that a system's mission will be successfully completed within the given mission time (t) providing a downtime per failure not exceeding a given time (t') will not adversely affect the overall mission.

$R(t)$ = the system reliability, or the probability that an system will operate without failure for the mission time (t).

M = the operational maintainability of the system, or the probability that, when a failure occurs, it will be repaired in a time not exceeding the allowable downtime (t').

In the following sections, exponential, Weibull, and log-normal distributed failure rates and dependability models are discussed.

3.2 The Failure Rate and Its Distributions

Let us now examine the concept of failure. A failure can be defined as "the termination of the ability of a system to perform a required function" [Villemeur, v.1, 1992]. A system will be said to have failed when it is no longer able to fulfill its function(s). The rate at which failures occur in a specified time interval is called the *failure rate* for that interval. The failure rate is defined as the proportion, by unit time, of a system which, having survived at any time t_i, has failed within the period of time $[t_i, t_i + \Delta t]$; when $\Delta t \to 0$, an instantaneous failure rate is obtained which is a function of t. The instantaneous failure rate can be expressed as

$$\lambda(t) = \lim_{\Delta t \to 0} \frac{R(t) - R(t+\Delta t)}{\Delta t \cdot R(t)} = -\frac{d}{dt} \ln R(t). \qquad (3.5)$$

Frequently, systems, especially, electronic equipment having a time-dependent failure rate are characterized by a "bathtub curve," as illustrated in Figure 3-1. Three periods are thus defined:

(1) *Early failure period.* That possible early period, beginning at a stated time and during which the failure rate decreases rapidly in comparison with that of the subsequent period.

(2) *Constant failure rate period.* That possible period during which the failure rate is relatively constant. This period is also sometimes called useful life.

(3) *Wear-out failure period.* That possible period during which the failure rate increases rapidly in comparison with the preceding period.

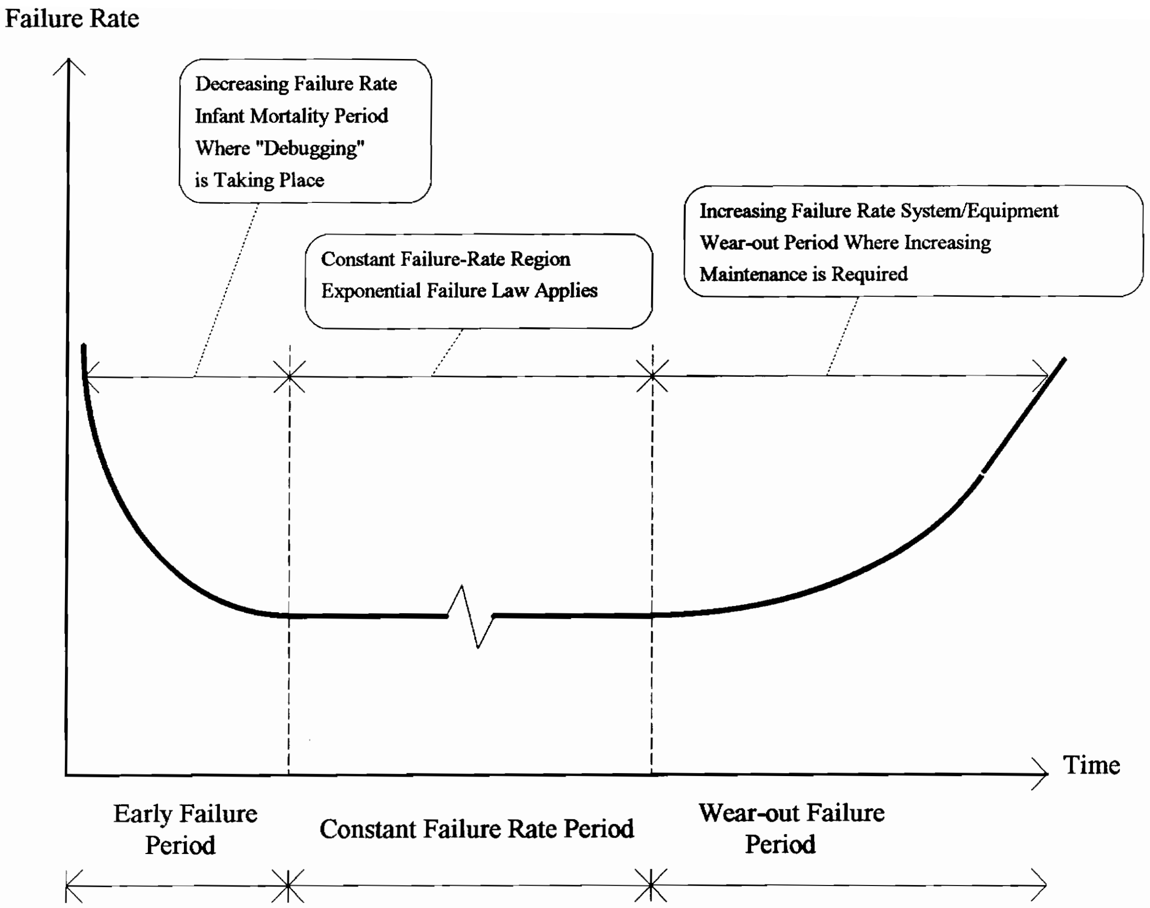
The failures taking place during these periods are respectively called *early failures*, *constant failures-rate region*, and *wear-out failures.*

3.2.1 The Exponential Distribution

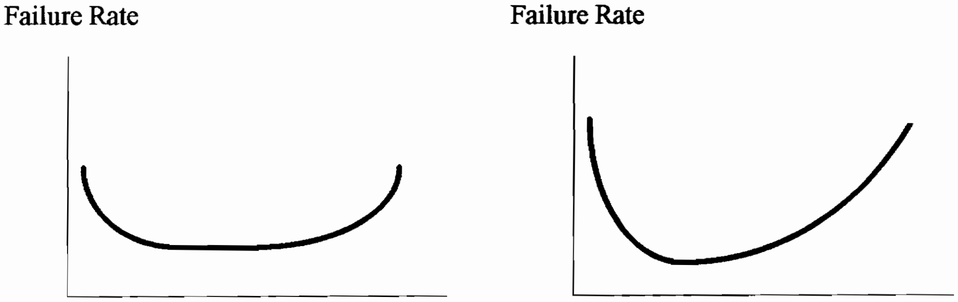
The exponential distribution is a very commonly used distribution in reliability engineering because it is assumed to be representative, both phenomenologically and empirically, of the time-to-failure distribution of components, equipment, and systems of complex nature with components of different and/or mixed life distributions, exhibiting a constant failure rate characteristic with operating time. Failures which result in a constant failure rate characteristic are called chance failures [Kececioglu, v.1, 1991]. Consequently, the time-to-failure distribution of chance failure is the exponential. In this project only the single-parameter exponential distribution will be discussed.

The single-parameter exponential probability distribution function is

$$f(t) = \lambda e^{-\lambda t} = \frac{1}{\theta} e^{-t/\theta}, t \geq 0, \lambda > 0, \theta > 0. \qquad (3.6)$$



Bathtub Curve Based on Time-Dependent Failure Rate



Electronic Equipment

Mechanical Equipment

Figure 3-1. Typical time dependent failure rate curve relationships [Blanchard, 1992].

where

λ = constant failure rate, in failures per unit of measurement period; e.g., failures per hour, per million hours, per million cycles, per million miles, per million rounds, etc.,

$\lambda = \frac{1}{\theta}$,

θ = mean time between failure (MTBF), or mean life,

e = the natural logarithm base $\cong$ 2.718281828,

and

t = operating time, life, or age, in hours, cycles, miles, rounds, etc.

Figure 3-2 illustrates Equation (3.6).

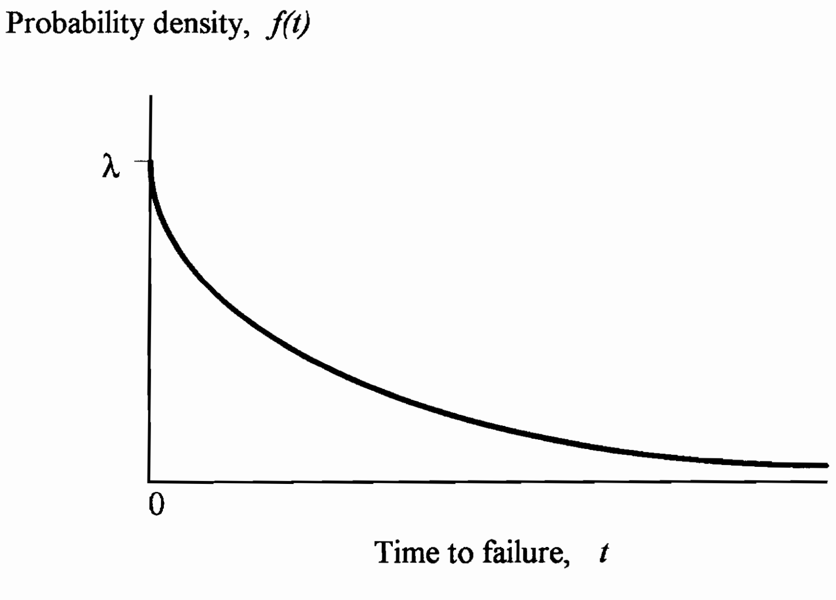


Figure 3-2. Plot of the single-parameter exponential time-to-failure distribution.

The reliability at time t is

$$R(t) = \int_t^\infty \tfrac{1}{\theta} e^{-t/\theta} \mathrm{d}t = e^{-t/\theta} \tag{3.7}$$

Mean life (θ) is the arithmetic average of the lifetimes of all items considered. The mean life (θ) for the exponential function is equivalent to mean time between failure (*MTBF*). Thus,

$$R(t) = e^{-t/MTBF} = e^{-\lambda t} \tag{3.8}$$

where λ is the instantaneous failure rate.

For many equipment, the life distribution after they have been "burned in", "broken in", and/or "debugged", follows the exponential curve up to the time wearout just starts to set in [Hahn and Shapiro, 1967]. When failures occur randomly in time in complex systems with many components and subsystems which undergo many deterioration mechanisms, each with its own failure rate, the time-to-failure data follow the exponential distribution [Kececioglu, v.1, 1991].

The exponential distribution is chosen in the development of system dependability model for the purpose of the simplicity of the problem.

3.2.2 Other Distributions

The exponential, or strictly the negative exponential, distribution is probably the most widely used distribution to represent time to failure. It is being used in this study because of mathematical convenience and simplicity. Other distributions represent time to failure and time to repair more accurately. The assumption of constant failure rates and the application of the exponential distribution considerably simplifies the problem. The constant failure rate period (or useful life) is the only region in which the exponential distribution is valid. Therefore, other distributions are essential for different types of system in operation.

It can be argued that the negative exponential distribution is a special case of the Weibull distribution with a shape parameter of $\beta = 1$. The Weibull distribution, has one very important property; the distribution has no specific characteristic shape. In fact, depending upon the values of the parameters in its reliability functions, it can be shaped to represent many distributions as well as shaped to fit sets of experimental data that cannot be characterized as a particular distribution other than as a Weibull distribution with certain shaping parameters. Weibull distribution has been shown to well represent the life characteristics of mechanical equipment [Billinton and Allen, 1992]. The discussion of the Weibull distribution is presented in Appendix B.

Experience has shown that in almost all cases the distribution of maintenance repair times for complex equipment, particularly for electronic equipment and systems, is log-normal [Blanchard and Fabrycky, 1990]. The log-normal distribution is related to the normal distribution and, likewise, is a two-parameter distribution. It dose not seem to be particularly suited for the representation of component lifetimes and because of this has not been considered in the past as an important distribution in reliability evaluation [Billinton and Allen, 1992]. This is still probably true with regard to mission-oriented systems. However, it now appears that the log-normal distribution can be a good fit to the distribution of component repair times and consequently is becoming an important distribution in the assessment of repairable systems. The log-normal distribution applies to most maintenance tasks and repair actions comprised of several subsidiary tasks of unequal frequency and time duration [SAE AE-9, 1987]. The discussion of log-normal distribution is presented in Appendix B.

3.3 Corrective Maintenance Cycle

Maintenance constitutes the act of diagnosing and repairing, or preventing, system failures. Maintenance time is made up of the individual task times associated with the required corrective and preventive maintenance actions for a given system or product. Maintainability is a measure of the ease and rapidity with which a system can be maintained, and is measured in terms of the time required to perform maintenance tasks.

Each time that a system fails, a series of steps is required to repair or restore the system to its full operational status. These steps include failure detection, fault isolation, disassembly to gain access to the faulty item, repair, and so on, as illustrated in Figure 3-3 [Blanchard, 1992]. Completion of these steps for a given failure constitutes a corrective maintenance cycle. Throughout the system use phase, there will be a number of individual maintenance actions involving the series of steps illustrated in Figure 3-3. The mean corrective maintenance time ($\overline{M}ct$), or the mean time to repair (*MTTR*) which is equivalent, is a composite value representing the arithmetic average of these individual maintenance cycle times.

3.4 Dependability Model

With regard to the distribution of repair times, the literature cites several cases. In some instances, the distribution of repair times often follows the normal or log-normal curve, particularly for electronic equipment [Blanchard, 1992; NAVSHIPS, 1964; Goldman and Slattery, 1967]. In other cases, a negative exponential distribution is assumed for the purposes of simplicity [Hahn and Shapiro, 1967; Kececioglu, v.1, 1991]. In this report, assume that the maintenance repair times follow a negative exponential distribution. Given this, one can use the repair rate or maintenance rate (Poisson distributed), μ which is the reciprocal of the *MTBM*, as the operational maintainability. The probability of no repair during time t is given as $e^{-\mu t}$. The probability of repair during the time t is $1 - e^{-\mu t}$. However, this approach discounts any probability that there will be multiple failures. Where multiple failures are a significant part of Equation (3.4), an alternate approach for assessing the value of M is presented as follows.

Define M as the ratio of failure rates restored within an available time to the failure rates of the total system under consideration.

$$M = \frac{\lambda^*}{\lambda_s} \tag{3.9}$$

where

M = the ratio of failure rates which can be restored within an available time to the failure rates of the total system under consideration,

λ^* = the failure rate, in all combinations, of those items which can be restored within the available time, and

λ_s = the total failure rate (the system failure rate) in the same combinations.

First, let us take Equation (3.4) and rewrite it, such that we consider the various combinations of failures possible:

$$D(t) = R(t)\left(1 + \frac{\lambda t}{1!}M_1 + \frac{(\lambda t)^2}{2!}M_2 + \frac{(\lambda t)^3}{3!}M_3 + \cdots + \frac{(\lambda t)^n}{n!}M_n\right) \tag{3.10}$$

where

M_n = the probability of correcting n malfunctions, (in n combinations), in the allowable time.

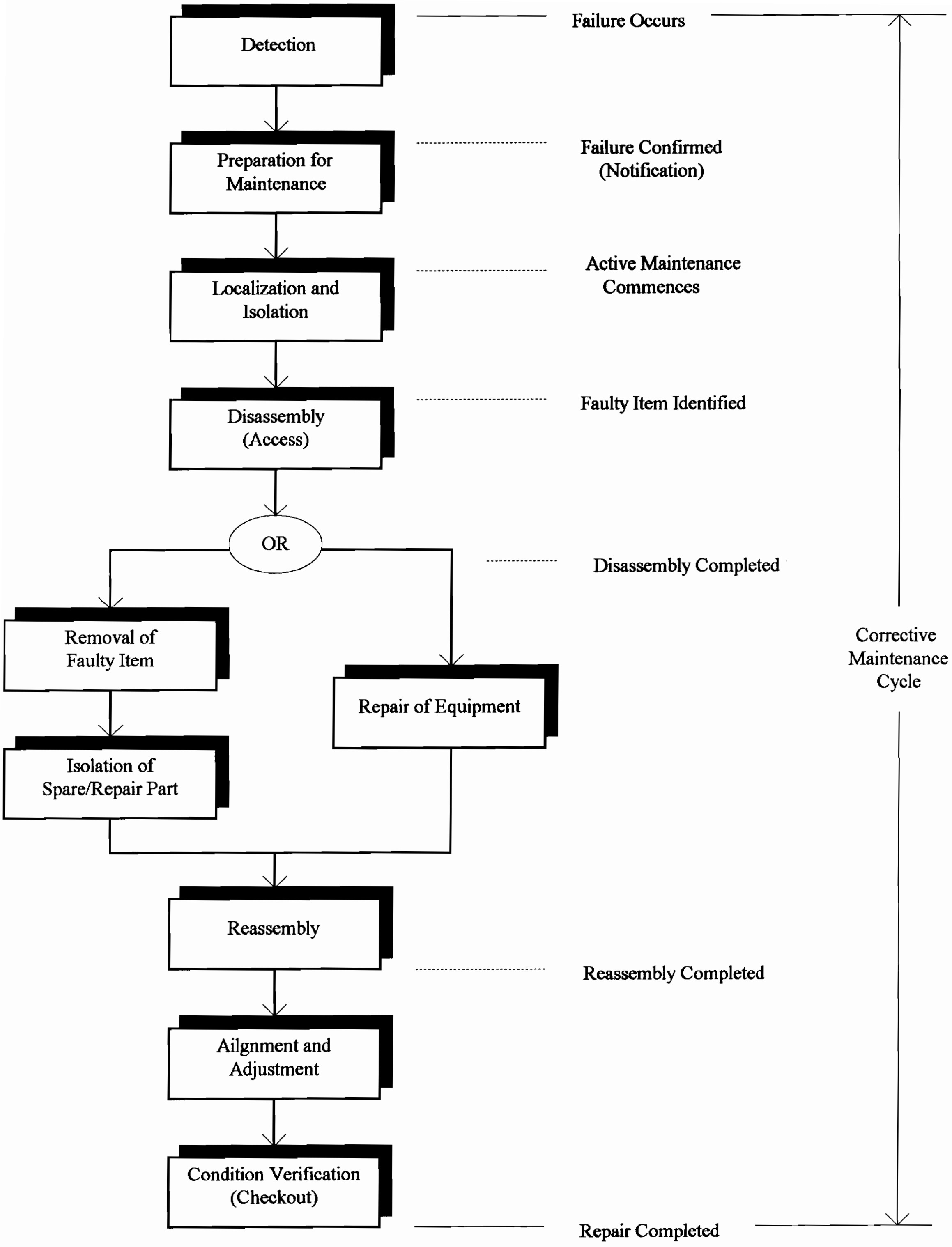


Figure 3-3. Corrective maintenance cycle [Blanchard, 1992].

Example 3.1

Consider an aircraft that has five critical electronic components of its avionics subsystem with failure rates (exponentially distributed) and average maintenance repair times as shown in Table 3-1.

Table 3-1. System components, failure rates and average maintenance repair times.

Compoment	Failure Rate	Ave. Repair Time (min.)
No. 1	$\lambda 1 = 0.001$	t1 = 15
No. 2	$\lambda 2 = 0.001$	t2 = 15
No. 3	$\lambda 3 = 0.002$	t3 = 20
No. 4	$\lambda 4 = 0.003$	t4 = 30
No. 5	$\lambda 5 = 0.003$	t5 = 50
	$\lambda s = 0.010$	

Further consider that each component is the lowest replaceable item. Additionally, the failure of any one component causes the entire avionics subsystem to be "down". We shall also take the time available for maintenance, when the aircraft is operating during its mission, to be within one hour. Then, for the mission time, $t = 5$ hours, the probability of no malfunctions in 5 hours is the mission reliability, $R(t) = e^{-\lambda t} = e^{-(0.01)(5)} = 0.9512$.

Referring back to Equation (3.10), we now solve for the value of the various M_n. By Equation (3.10), M_1 represents the percentage that a single malfunction (combination of one) can be restored in a time equal to or less than 1 hour. In this example, M_1 can be expressed as

$$M_1 = \frac{\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5}{\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5} = 1.0, \qquad (3.11)$$

because every single malfunction that occurs can be restore within 1 hour.

Now, consider what happens if one or more of the five components cannot be restored in the time available. This may occur if there are not sufficient resources available such as spares, personnel, or time. Therefore, if the time available for repair of the avionics subsystem during the mission were taken to be 30 minutes, then

$$M_1 = \frac{\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4}{\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5} = \frac{0.007}{0.01} = 0.7 \tag{3.12}$$

Since M_n is also a function of the frequency with which maintenance must be performed, it is appropriate to take the ratio of failure rates as a better estimate of the restoration probability.

As an interim step in demonstrating the ease with which the technique can be used, note that from the definition of M_1 in Equation (3.11), and from previous knowledge that the sum of the λ_i values is the system failure rate, λ_S, the denominator of Equation (3.11) will cancel the λ term in Equation (3.10). Therefore, considering only single malfunctions, Equation (3.10) can be rewritten as follows:

$$D(t) = e^{-\lambda t}(1 + \lambda^* t) \tag{3.13}$$

where

λ^* = the sum of the failure rates for those components which can be restored.

Example 3.2.

According to Example 1, let us now explore combinations of two malfunctions. The average maintenance repair times for restoring all combinations of two malfunctions are listed in Table 3-2.

TABLE 3-2. Task times for combinations of two malfunctions.

Combination	t1+t2	t1+t3	t1+t4	t1+t5	t2+t3	t2+t4	t2+t5	t3+t4	t3+t5	t4+t5	t1+t1	t2+t2	t3+t3	t4+t4	t5+t5
Task Times	15+15	15+20	15+30	15+50	15+20	15+30	15+50	20+30	20+50	30+50	15+15	15+15	20+20	30+30	50+50
Sum of Times	30	35	45	65	35	45	65	50	70	80	30	30	40	60	100

If we continue with the same allowable time (60 minutes) as previously used, we see that only those combinations involving t_5 cannot be restored within the available time. Note that this situation is the same as only having one spare for component 5, whereas there are 2 spares for each of the others. Then, by the previous definition of the M_n, we can derive an expression for M_2.

$$\mathrm{M}_2 = \frac{\left[\frac{1}{2}\sum_{i=1}^{4}\lambda_i{}^2 + \sum_{i,j=1,2;i\neq j}^{3,4}\lambda_i\lambda_j\right]}{\left[\frac{1}{2}\sum_{i=1}^{5}\lambda_i{}^2 + \sum_{i,j=1,2;i\neq j}^{4,5}\lambda_i\lambda_j\right]} \tag{3.14}$$

Therefore, considering only single malfunctions and combinations of two malfunctions, the dependability equation becomes:

$$D(t) = e^{-\lambda t} + \left(\sum_{i=1}^{5}\lambda_i\right)te^{-\lambda t} + \left[\frac{1}{2}\sum_{i=1}^{4}\lambda_i{}^2 + \sum_{i,j=1,2;i\neq j}^{3,4}\lambda_i\lambda_j\right]t^2e^{-\lambda t} \tag{3.15}$$

The format of Equation (3.22) with regard to combinations of two can be tested. First, assume that the probability that component 1 will fail twice during the time period of interest, and the other components do not fail, this can be given by:

$$\frac{(\lambda_1 t)^2}{2}(e^{-\lambda_1 t})(e^{-(\lambda-\lambda_1)t}) = \frac{(\lambda_1 t)^2}{2}(e^{-\lambda t}) \tag{3.16}$$

and so on for components 2, 3, 4, and 5. For the other combinations, we can examine the probability that component 1 and 2 each fail once during the time period of interest and the others do not fail. This is given by:

$$\left[(\lambda_1 t)e^{-\lambda_1 t}\right]\left[(\lambda_2 t)e^{-\lambda_2 t}\right]e^{-(\lambda-\lambda_1-\lambda_2)t} = (\lambda_1\lambda_2)t^2 e^{-\lambda t} \tag{3.17}$$

and so on for all remaining combinations. Now, let us sum the terms according to whether the single or double combinations are equal to or less than the available time:

$$\begin{aligned} D(t) = e^{-\lambda t} + \left(\sum_{i=1}^{5}\lambda_i\right)te^{-\lambda t} + \frac{1}{2}t^2\left(\lambda_1{}^2+\lambda_2{}^2+\lambda_3{}^2+\lambda_4{}^2\right)e^{-\lambda t} \\ + t^2(\lambda_1\lambda_2+\lambda_1\lambda_3+\lambda_1\lambda_4+\lambda_2\lambda_3+\lambda_2\lambda_4+\lambda_3\lambda_4)e^{-\lambda t} \end{aligned} \tag{3.18}$$

Note that Equation (3.15) and Equation (3.18) are identical. Similar techniques can be used to show that, for combinations of three, four, or more malfunctions the resulting equation will have the form of:

$$\begin{aligned} D(t) = e^{-\lambda t}\Bigg\{ & 1 + t\left(\sum_{i=1}^{n}\lambda_i^*\right) + t^2\left[\left(\frac{1}{2}\sum_{i=1}^{n}\lambda_i^{*\,2}\right) + \left(\sum_{i,j=1,2;i\neq j}^{n-1,n}\lambda_i\lambda_j^*\right)\right] \\ & + t^3\left[\left(\frac{1}{6}\sum_{i=1}^{n}\lambda_i^{*\,3}\right) + \left(\frac{1}{2}\sum_{i,j=1,2;i\neq j}^{n-1,n}\lambda_i^{*\,2}\lambda_j^*\right) + \left(\sum_{i,j,k=1,2,3;i\neq j\neq k}^{n-2,n-1,n}\lambda_i\lambda_j\lambda_k^*\right)\right] \\ & + t^4\Bigg[\left(\frac{1}{24}\sum_{i=1}^{n}\lambda_i^{*\,4}\right) + \left(\frac{1}{6}\sum_{i,j=1,2;i\neq j}^{n-1,n}\lambda_i^{*\,3}\lambda_j^*\right) + \left(\frac{1}{4}\sum_{i=1;i\neq j}^{n-1}\lambda_i^{*\,2}\sum_{j=2}^{n}\lambda_j^{*\,2}\right) \\ & \quad + \left(\frac{1}{2}\sum_{i=1;i\neq j\neq k}^{n}\lambda_i^* \sum_{j,k=2,3}^{n-1,n}\lambda_j\lambda_k^*\right) + \left(\sum_{i,j,k,l=1,2,3,4;i\neq j\neq k\neq l}^{n-3,n-2,n-1,n}\lambda_i\lambda_j\lambda_k\lambda_l^*\right)\Bigg]\Bigg\} \end{aligned} \tag{3.19}$$

Equation (3.19) is the general case, applicable through combinations of four. Combinations of five or more can be deduced through the same reasoning. The star (∗) is intended to connote that only those terms are summed that have repair times in the appropriate combinations which are equal to or less than the allowable time. Then, continue on for as many combinations as will reasonably contribute to the dependability. The results of measuring as many as combinations of five malfunctions are shown in Table 3-3. We do not need to look at any higher combinations, since there is not a significant improvement in dependability.

Table 3-3. Results of dependability evaluation for Examples 3.1 and 3.2.

Number of malfunction combination	Dependability
0	0.951229
1	0.998791
2	0.999195
3	0.999196
4	0.999196
5	0.999196

3.5 Computerized Dependability Model

From the discussion in Section 3.4, a computerized model has been developed to measure the system dependability with exponentially distributed time to failure. The logic flow chart of the computer model is illustrated in Figure 3-4. The program listing and outputs for Examples 3.1 and 3.2 are presented in Appendix A. This program is capable of measuring as many as combinations of *five* malfunctions.

3.6 Dependability Analysis and Evaluation

Incorporating quality features into the design early in the life cycle is a multidisciplinary effort. It is critical to select the right individuals who understand the design process with a representative from each functional area. The key is not a continuation of the sequential approach for design, but a concurrent approach. It is a challenge to ensure that all elements of the system are developed and/or procured concurrently and integrated in a timely manner as one progresses through the life cycle, and the selection of tools, methods, and models must support this objective. From a broad perspective, any engineering design and development activity includes the utilization of analysis and evaluation tools as depicted in Figure 3-5 [Blanchard, Fabrycky, and Verma, 1994; Blanchard and Fabrycky, 1990].

Analysis also includes the process of synthesizing alternatives or candidate design configurations. In the context of this discussion, synthesis may be defined as the process of assembling a set of conceptual solutions or design alternatives or candidates which seem to satisfy the more significant system specifications, such as availability, dependability, performance, reliability, maintainability, and/or supportability. The outcome from the analysis phase is a reduced but feasible set of alternatives or candidate designs. Evaluation

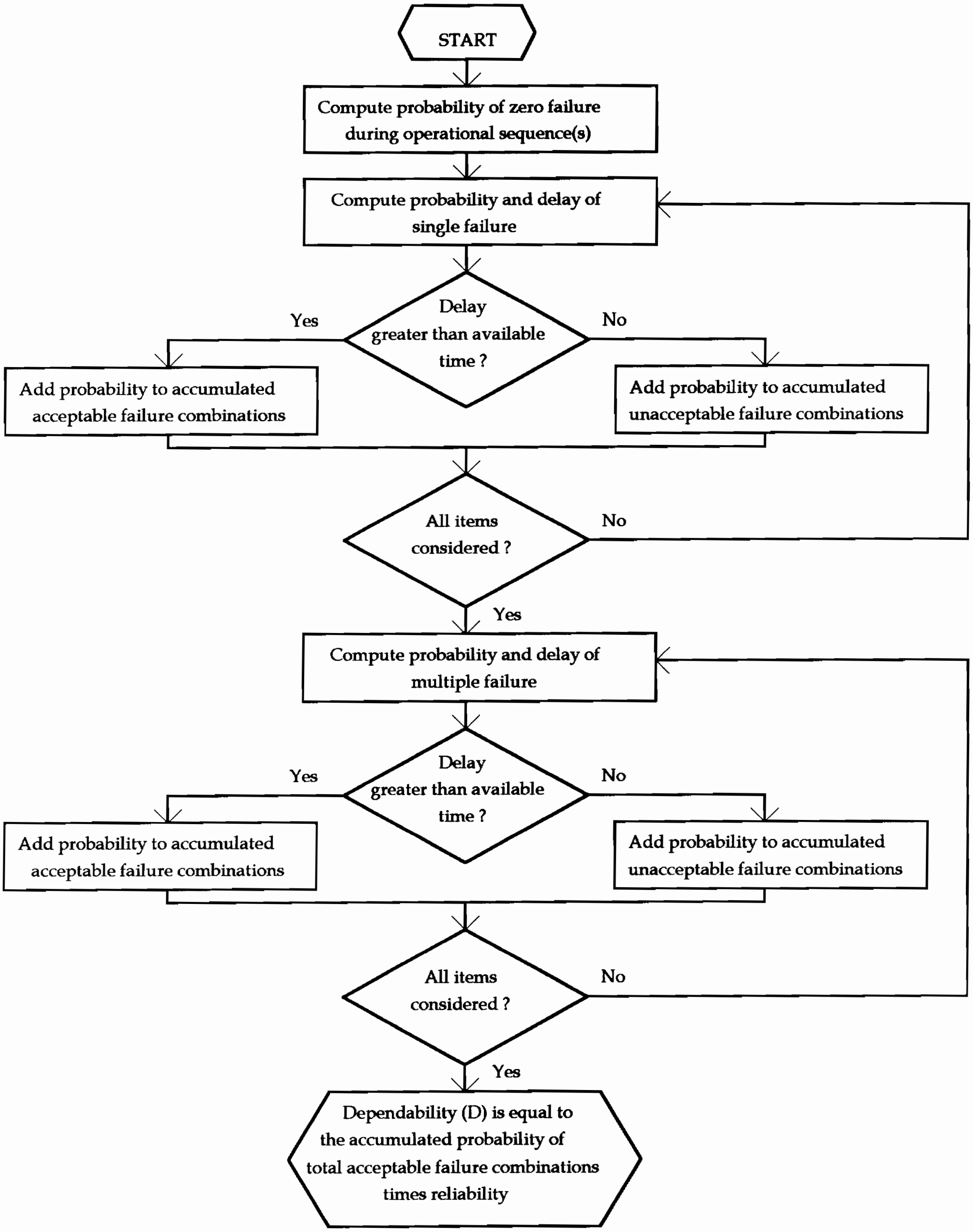


Figure 3-4. System dependability measurement flow chart.

involves the anticipation of system behavior. It involves the derivation, estimation, and prediction of values for all relevant design parameters and variables in order to determine acceptability in terms of the initially stated requirements, objectives, and specifications. Ultimately, the iterative process of analysis and evaluation includes the task of selecting the "optimal" or preferred approach [Blanchard and Fabrycky, 1990].

It is important to understand that the analysis, synthesis, and evaluation activities in system design are not only iterative in nature, but also continuously changing in terms of scope and resolution throughout the progression of the system design and development process.

In system design, dependability analysis is a continuous process to synthesize (Block 2 and 3 in Figure 3-5), analyze (Block 4), and evaluate (Block 5) the effectiveness of a system configuration. Dependability analysis starts from specifying the requirements for dependability at system level; allocating these requirements to lower-level elements; evaluating alternative design concepts to meet the allocated requirements; selecting a preferred approach; assessing the design configuration in terms of meeting the requirements; and making recommendation for design improvement as required. This iterative process of continuous refinement is illustrated in Figure 3-6.

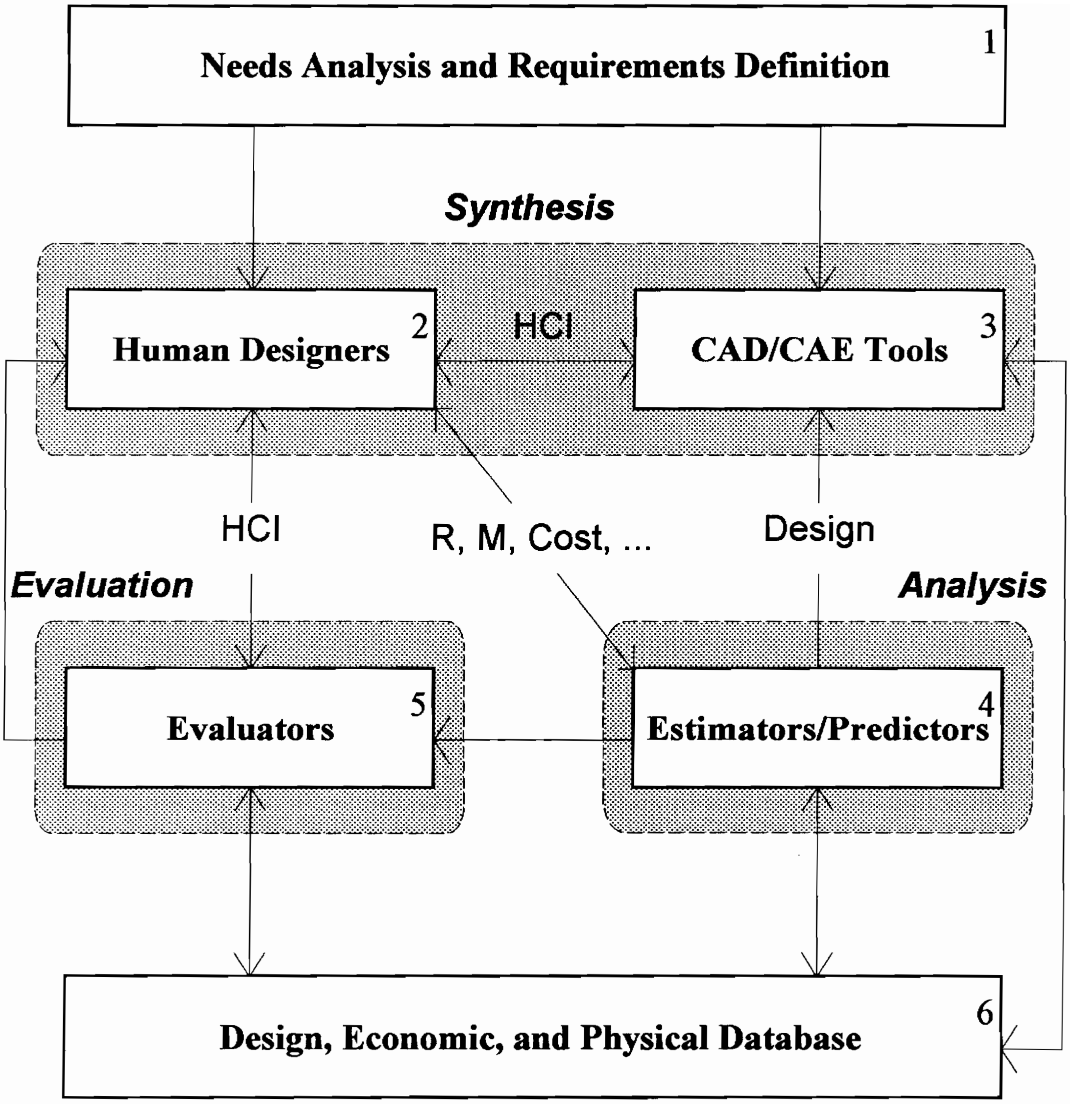


Figure 3-5. A morphology for concurrent engineering design [Adapted Blanchard and Fabrycky, 1990].

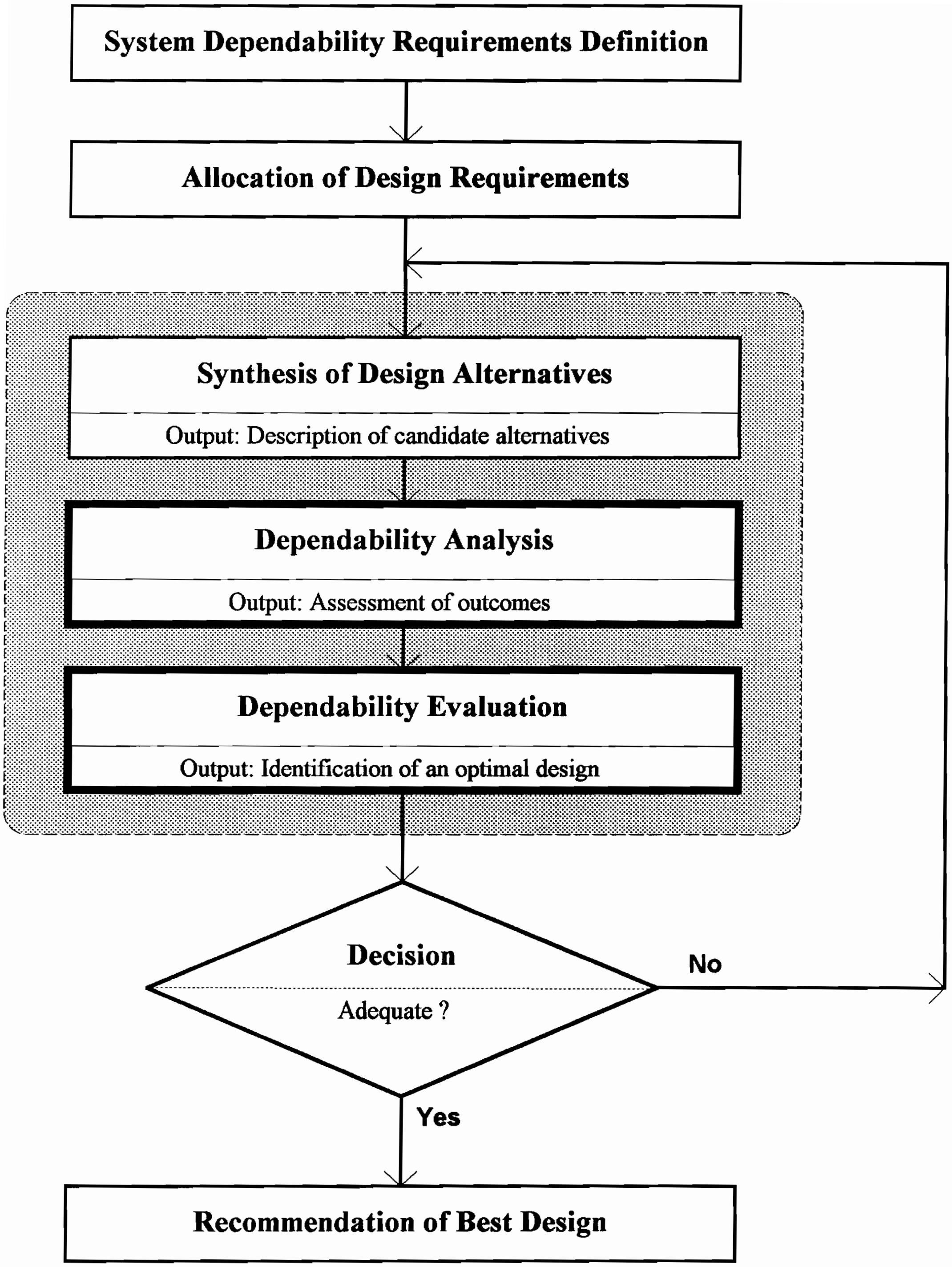


Figure 3-6. Dependability analysis and evaluation in system design.

CHAPTER IV

CASE STUDY

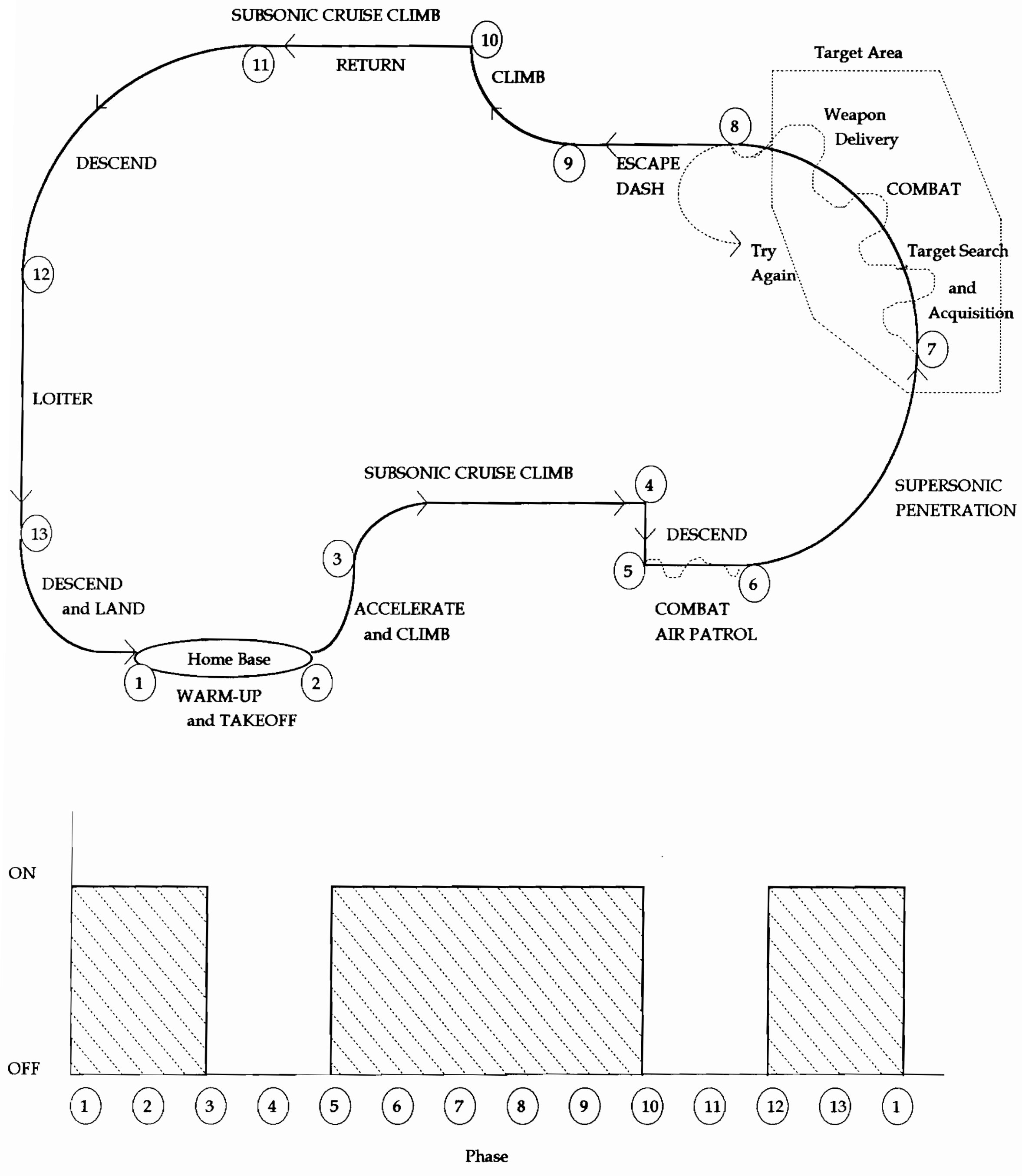
The objective of this chapter is to show the applicability of the dependability model developed in Chapter III. An aircraft radar subsystem is presented to illustrate an application of the dependability model. The example originates from the radar system design given in Chapter 6 (Problem 32) of Blanchard, Logistics Engineering and Management [Blanchard, 1992, pp. 197 - 200].

4.1 An Example of Aircraft Radar Subsystem Design

A need has been identified which will require the addition of a new performance capability to an existing aircraft radar subsystem. This new radar subsystem, referred to as System "XYZ", is to be installed in a combat aircraft that will be deployed at five operational bases. Each base will have a maximum force level of 20 aircraft, with the bases being activated in series (e.g., base 1 at the end of year 2, base 2 at the end of year 3, etc.).

The mission of the aircraft radar subsystem is defined through the aircraft operational profiles in several phases as shown in Figure 4-1.It is anticipated that each radar system will be utilized on the average of 4 hours per days, 365 days per year. It is assumed that all functions of the System "XYZ" are fully operational throughout this time period. The aircraft crew members will be assigned to operate several different systems throughout flight, and it is assumed that 1% of his time is allocated to System XYZ.

To achieve the stated operational objectives, there is a need to acquire system that will meet certain performance and effectiveness requirements. The required system must



Radar System "XYZ" Mission Profile

Figure 4-1. Mission scenario by phases.

exihibit a dependability of at least 0.990, a MTBM of at least 300 hours, a 500-hour MTBF, and a $\overline{M}$ of 30 minutes or less. Further, budget limitation require that the life-cycle cost not exceed $5,000,000 in inflated dollars. Advanced program planning indicates that a full complement of units must be in operation five years after the start of the program, and that this capability must be maintained through the fifteenth year. The total number of systems in operational use for each year of the projected life cycle is shown in Table 4-1. The significant program milestones and projected number of units in operational use are presented in Figure 4-2. This forms the basis for defining the unit life cycle, the major life-cycle functions, and the life-cycle cost.

Based on a review of the available sources of supply, there is no known existing system that will completely fulfill the need; but there are two new candidate design configurations that should suffice, assuming that all design goals are met.

The objective is to evaluate each configuration in terms of (1) the dependability of the System "XYZ", (2) the life-cycle cost of the System "XYZ", and (3) the cost-effectiveness figure of merit (FOM); and to recommend the preferred configuration.

TABLE 4-1. The total number of System XYZ in operation.

Program Year																
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
0	20	40	70	105	105	105	105	105	105	104	103	102	101	100	50	10

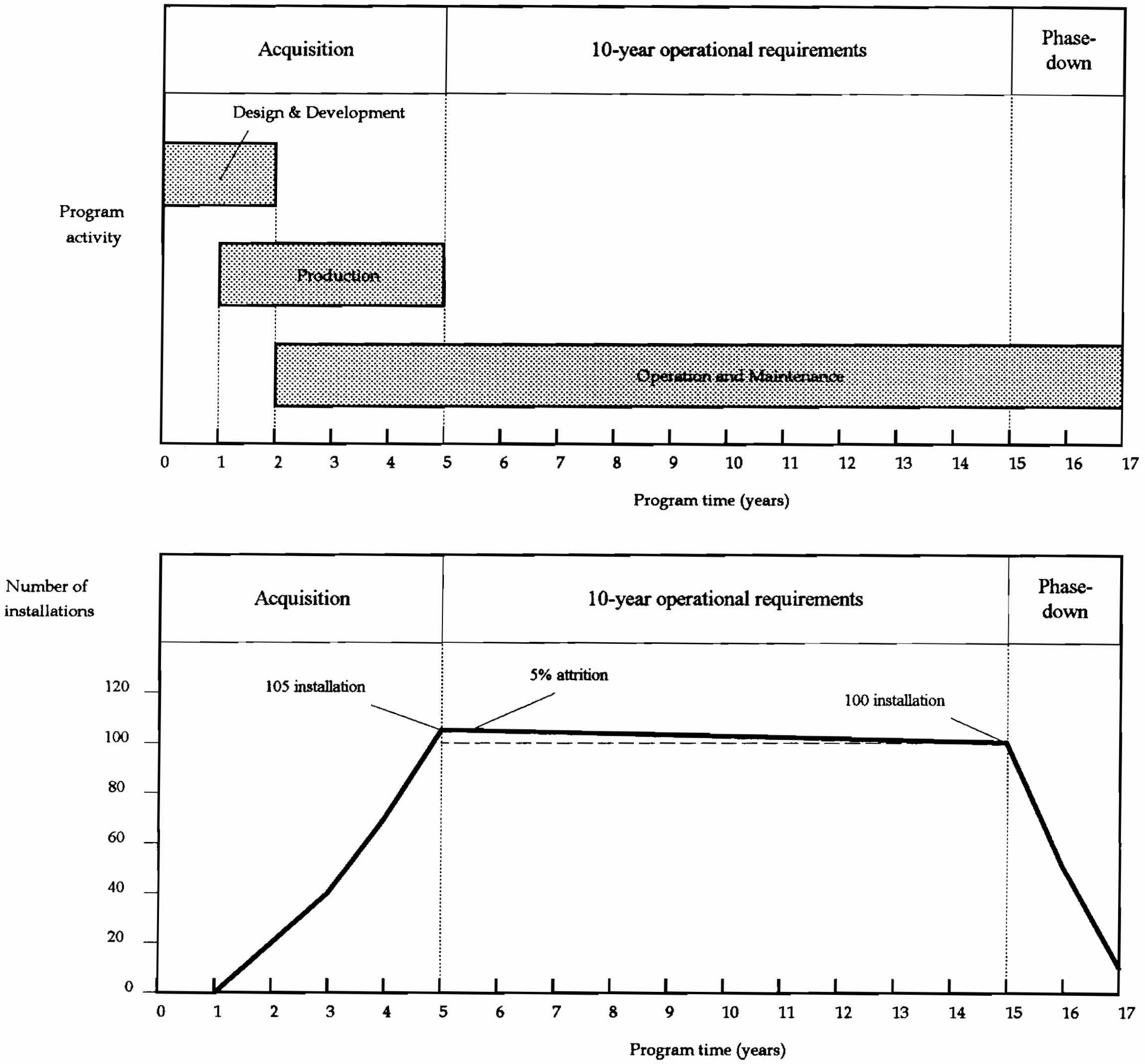


Figure 4-2. Radar System "XYZ" program profile.

Based on a review of the available sources of supply, there is no known existing system that will completely fulfill the need; but there are two new candidate design configurations that should suffice, assuming that all design goals are met. The objective is to evaluate each configuration in terms of the system dependability (top level) and life-cycle cost and to recommend the preferred configuration..

System "XYZ" is a newly designed entity, and each of the two candidate configurations is packaged in three units: Unit A, Unit B, and Unit C, as illustrated in Figure 4-3. The predicted reliability and maintainability factors associated with each of the two candidate configurations are noted in Table 4-2.

Relative to the maintenance concept, the two configurations of System "XYZ" incorporate a built-in self-test capability that enables rapid system checkout and fault isolation to the unit level. No external support equipment is required at the aircraft. In the event of a no-go condition, fault isolation is accomplished to the unit and the applicable unit is removed, replaced with a spare, and sent to the intermediate-level maintenance shop (located at the operational base) for corrective maintenance. Unit repair is accomplished through module replacement, with the modules being discarded-at-failure (i.e., modules are assumed as being nonrepairable). Scheduled (preventive) maintenance is accomplished for Configuration A (Unit B) and Configuration B (Unit A), as noted in Table 2, in the intermediate shop every six (6) months. No depot maintenance is required; however, the depot does provide backup supply and support functions as required.

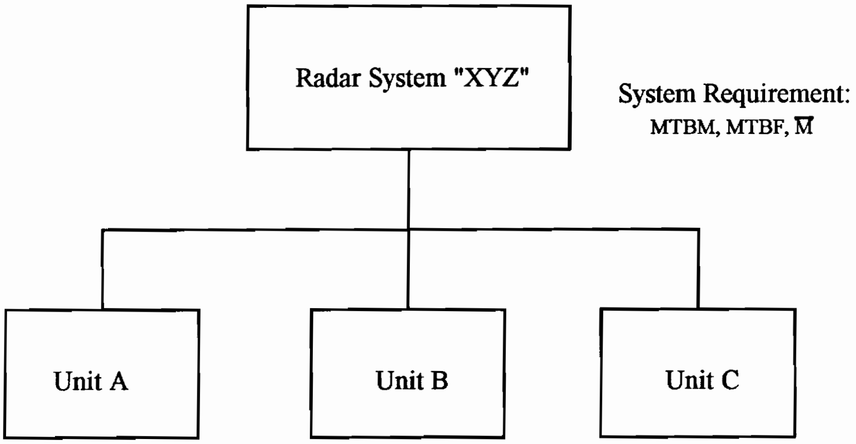


Figure 4-3. Hierarchy of system "XYZ".

Table 4-2. Reliability and maintainability factors

Parameter[1]	Configuration A	Configuration B
System Level (Organizational Maintenance)		
MTBM	600 hr	450 hr
MTBF	831 hr	567 hr
$\overline{M}$	30 min	30 min
Unit Level (Intermediate Maintenance)		
Unit A		
MTBM	2000 hr	1250 hr
MTBF	2000 hr	2915 hr
$MTBM_S$	~	2190 hr
MTTR	6 hr	6 hr
$\overline{Mpt}$	~	25 hr
Unit B		
MTBM	1162 hr	855 hr
MTBF	2475 hr	855 hr
$MTBM_S$	2190 hr	~
MTTR	6 hr	5 hr
$\overline{Mpt}$	20 hr	~
Unit C		
MTBM	3333 hr	4000 hr
MTBF	3333 hr	4000 hr
MTTR	4 hr	3 hr

4.1.1 Dependability Analysis and Evaluation

Considering the system dependability, for instance, System "XYZ" on the combat aircraft is turned on and off during the flight (as shown in Figure 4-1). The System "XYZ"

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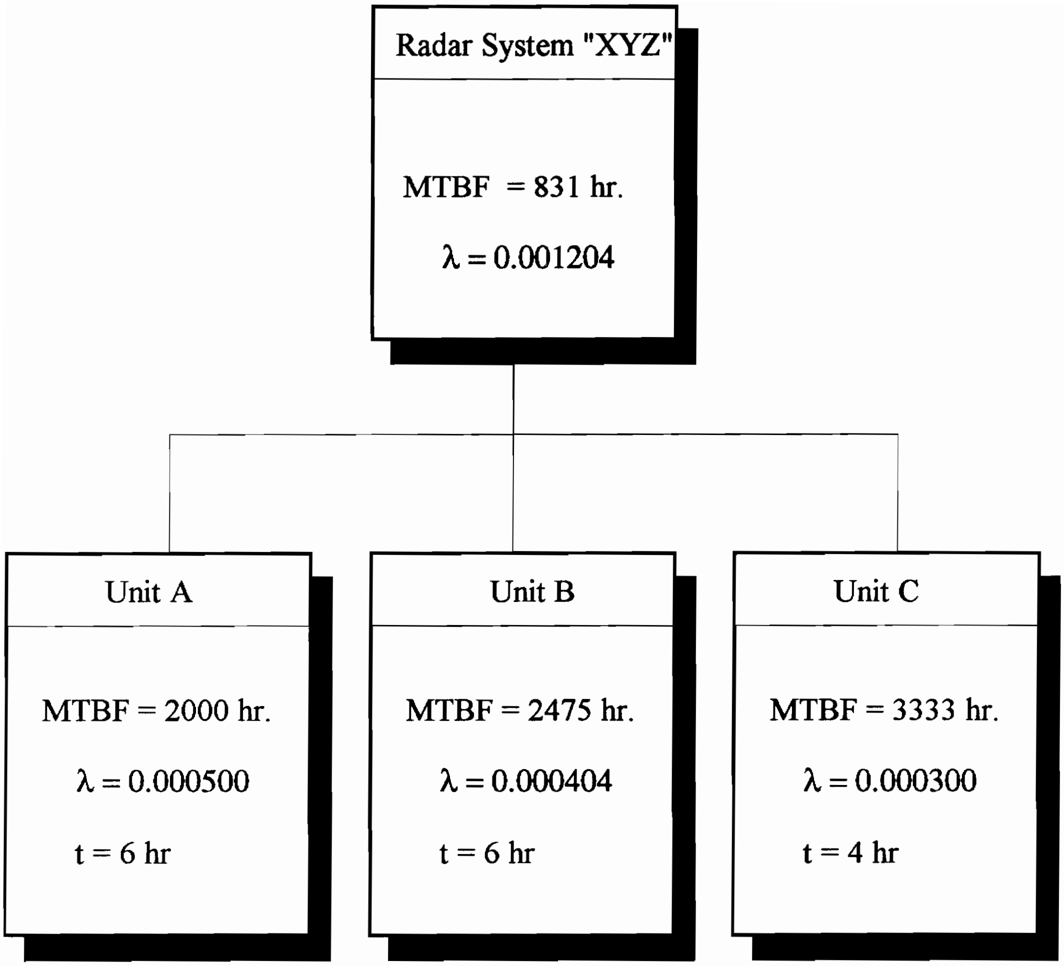
[1] Assume that MTBF = $MTBM_u$. When there is no scheduled maintenance, MTBM = $MTBM_u$.

operational mission profile includes the following phases with total mission duration = 3 hours:

Phase 1 → 2: Warm-up and takeoff (10 minutes), System "XYZ" on.
Phase 2 → 3: Accelerate and climb (20 minutes), System "XYZ" on.
Phase 3 → 4: Subsonic cruise climb, System "XYZ" off.
Phase 4 → 5: Descend, System "XYZ" off.
Phase 5 → 6: Combat air patrol (30 minutes), System "XYZ" on.
Phase 6 → 7: Supersonic penetration (30 minutes), System "XYZ" on.
Phase 7 → 8: Combat (30 minutes), System "XYZ" on.
Phase 8 → 9: Escape dash (10 minutes), System "XYZ" on.
Phase 9 → 10: Climb (20 minutes), System "XYZ" on.
Phase 10 → 11: Subsonic cruise climb, System "XYZ" off.
Phase 11 → 12: Descend, System "XYZ" off.
Phase 12 → 13: Loiter (20 minutes), System "XYZ" on.
Phase 13 → 1: Descend and land (10 minutes), System "XYZ" on.

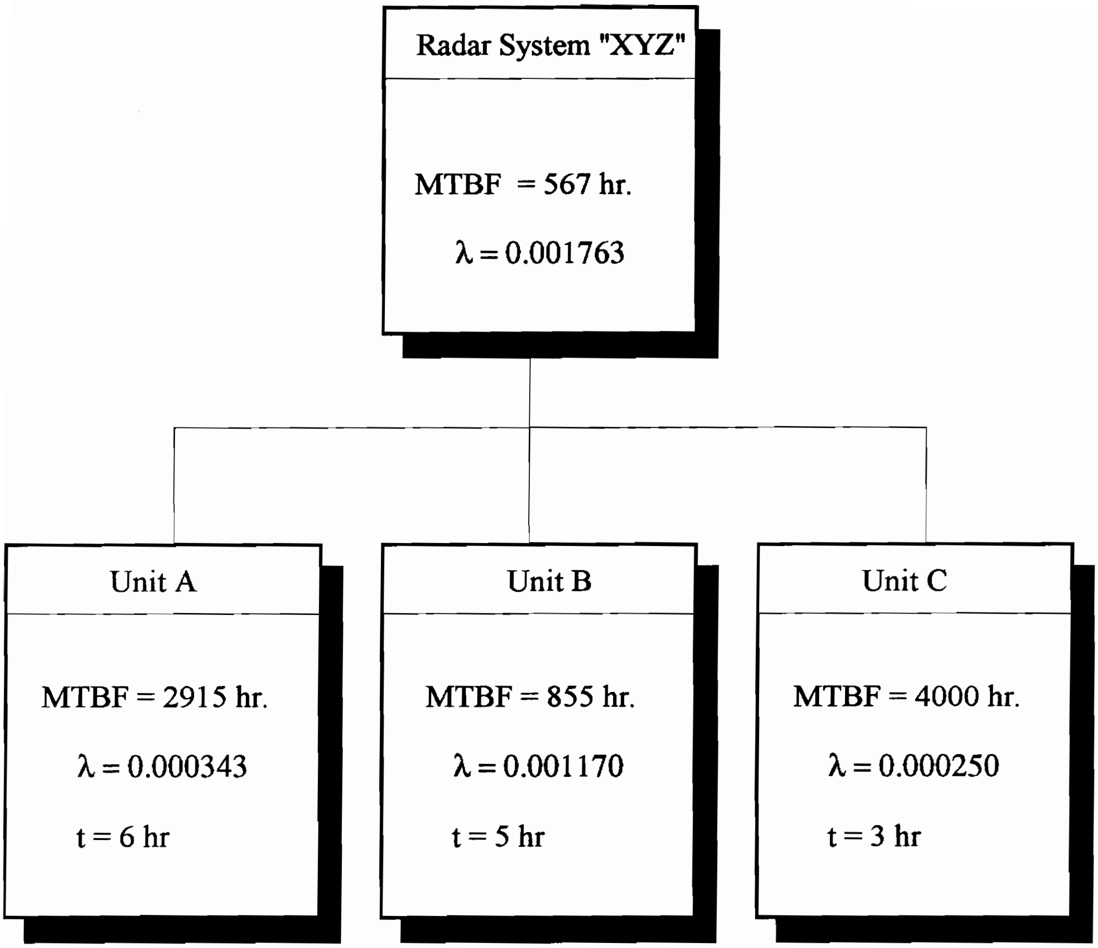
System "XYZ" is designed to function continuously throughout the phases whenever required (turned on). However, sometimes the radar system might fail during the flight because of the failures or malfunctions of its units. In such an event, the built-in self-test equipment enables rapid system checkout and fault isolation to the unit level. The fail unit will be replaced immediately with a fully functional standby redundancy. In case both the main unit and the redundancy are down, the radar is unavailable. Otherwise, if the spare is available and functions continuously throughout the entire mission, the mission can be completed successfully. Figures 4-4 and 4-5 indicates the top-level system of the System "XYZ" dependability requirements (Configurations A and B) supported by factors established at the subsystem level and on down (also indicated in Table 4-2).

Now let us calculate the value of system dependability. Assume that the maximum time available for repair during the mission is equal to ten (10) hours. The mission time duration for System "XYZ" is 3 hours. Assume that the time to failure of Units A, B, and



Note: t indicates the average repair time for each unit.

Figure 4-4. System dependability factors (Configuration A).



Note: t indicates the average repair time for each unit.

Figure 4-5. System dependability factors (Configuration B).

C for both Configurations A and B are exponentially distributed. Additionally, the failure of any one unit causes the entire radar System "XYZ" to be "down".

Let us calculate dependability value for Configuration A first. The probability of no malfunctions in 3 hours is the mission reliability,

$$D(t) = R(t) = e^{-\lambda t} = e^{-(0.001204)(3)} = 0.996395.$$

Considering single malfunctions, by Equation (3.20),

$$\begin{aligned} D(t) &= e^{-\lambda t}(1 + \lambda^{*} t) \\ &= e^{-(0.001204)(3)}[1 + (0.000500 + 0.000404 + 0.000300) \times 3] \\ &= 0.999994 \end{aligned}$$

Considering single malfunctions and combinations two malfunctions,

$$\begin{aligned} D(t) &= e^{-\lambda t}(1 + \lambda^{*}_{i} t) + \left[\left(\tfrac{1}{2}\lambda^{*2}_{i} + \lambda_{i}\lambda^{*}_{j}\right)\right] t^{2} e^{-\lambda t} \\ &= 0.999993 + [(0.5)(0.000300^{2}) + (0.000500 \times 0.000300) \\ &\quad + (0.000404 \times 0.000300)] \times 3^{2} \times e^{-(0.0001204)(3)} \\ &= 0.999996 \end{aligned}$$

The results, calculated by developed dependability model (program listed in Appendix A), are shown in Table 4-3 with different combinations of malfunctions for the two configurations. The dependability values after combinations of 2 malfunction are the same without any improvement of dependability. The results indicate the probability that System "XYZ" can successfully accomplish its mission. On the basis of the *system dependability*, Configuration A is preferred as summarized in Table 4-3.

Table 4-3 Combat aircraft System "XYZ" dependability with combinations of different malfunctions

Configuration A	
Number of malfunction combinations	Dependability
0	0.996395
1	0.999994
2	0.999996
3	0.999996
4	0.999996
5	0.999996
Configuration B	
Number of malfunction combinations	Dependability
0	0.994725
1	0.999986
2	0.999989
3	0.999989
4	0.999989
5	0.999989

4.1.2 Life-Cycle Cost Analysis

Assume that System "XYZ" life-cycle costs are broken down into the three categories represented by the blocks in the program profile as shown in Figure 4-3 (i.e., Design and Development, Production, and Operations and Maintenance).

In an attempt to simplify the problem, the following additional factors are assumed:

(1) Design and development costs for System "XYZ" (to include labor and material) are:

Configuration A: $ 750,000 ($ 400,000 in year 1 and $ 350,000 in year 2).

Configuration B: $ 730,000 ($ 380,000 in year 1 and $ 350,000 in year 2).

(2) Design and development costs for special support equipment at the intermediate level of maintenance are:

Configuration A: $ 600,000 ($ 400,000 in year 1 and $ 200,000 in year 2).

Configuration B: $ 580,000 ($ 380,000 in year 1 and $ 200,000 in year 2).

(3) System XYZ operational models are produced and delivered in the year prior to the operational deployment need (i.e., 20 models are produced and delivered in year 2, etc.). Production costs for each System "XYZ" are:

Configuration A: $ 24,000 / system.

Configuration B: $ 22,000 / system.

(4) Special support equipment is required at each intermediate maintenance shop (for corrective maintenance of units) at the start of the year when System "XYZ" operational models are deployed (i.e., base 1 at the beginning of year 3). In addition, a backup special support equipment set is required at the depot when the first operational base is activated. Special support equipment is produced and delivered at a cost of (recurring and amortized nonrecurring costs are included):

Configuration A support equipment: $ 20,000.

Configuration B support equipment: $ 18,000.

(5) Spare units are required at each intermediate maintenance shop at the time of base activation. Assume that one (1) Unit A, one (1) Unit B, and one (1) Unit C constitute a set of spares, and that the cost of a set is equivalent to the cost of a production system (i.e., $ 24,000 for Configuration A and $ 22,000 for Configuration B). Also, assume that one set of spares is stocked at the depot when the first operational base is activated.

Additional spares constitute components (i.e., module, subassemblies, parts, etc.). Assume that the material costs are $ 300 per corrective maintenance action, and $100 per preventive maintenance action. These cost factors include amortized inventory maintenance costs. In the interest of simplicity, the effects of the total logistics pipeline and maintenance shop turnaround time on spares are ignored in this problem.

(6) Maintenance facilities (as defined here) includes resources required for System "XYZ" maintenance and support, above and beyond spares, personnel and data. This includes the use of intermediate-level maintenance facilities and the sustaining maintenance support of special support equipment. A burden rate of $ 1 per direct maintenance man-hour associated with the prime equipment is assumed.

(7) Maintenance data include the preparation and distribution of maintenance reports, failure reports, and related data associated with each maintenance action. Maintenance data costs are assumed to be based on a rate of $ 30 per maintenance action.

(8) For each maintenance action at the system level, one low-skilled technician at $ 15 per direct maintenance man-hour is required on a full-time basis. For the purpose of simplicity, it is assumed that this rate is an average value, applied throughout the life cycle, and includes direct, indirect, and inflationary factors. $\overline{M}$ is 30 minutes for each of the two configurations.

(9) For each corrective maintenance action involving Unit A, Unit B, or Unit C, two technicians are required on a full-time basis (i.e., duration of the MTTR value). One (1) low-skilled technician at \$ 15 per hour and one (1) high-skilled technician at \$ 35 per hour are required. Direct, indirect, and inflationary factors are considered in these average values.

(10) For each preventive maintenance action involving units (Configurations A and B), one high-skilled technician at \$ 30 per hour is required on a full-time basis (i.e., duration of the $\overline{M}$pt value).

(11) For operation of System "XYZ", the allocated crew operator cost is \$ 40 per hour.

There are two different configurations being proposed for System "XYZ", and each of the two configurations will meet the required mission needs in terms of performance. Throughout the program time span of 17 years there are events associated with the design and development, test and evaluation, production, operation and maintenance of System "XYZ". These events represent individual costs which are identified on a year-to-year basis, total for each year, and discounted to the present value. Discounting is accomplished based on the assumption that other alternatives exist, and that the various configurations of System "XYZ" are being evaluated on a relative basis. If the intent is to evaluated each of these configurations in terms of future budgetary requirements (i.e., the dollars that are required each year for future system operation and support), then the discounting factor does not apply.

The total system cost is made up of three parts based on the plan of System "XYZ" (illustrated in Figure 4-3):

(1) Design and Development;

(2) Production; and

(3) Operations and Maintenance.

The System XYZ cost breakdown structure (CBS) [Blanchard and Fabrycky, 1990; Fabrycky and Blanchard, 1991], as presented in Figure 4-6, is developed to provide a mechanism for initial cost allocation, cost categorization, and cost monitoring and control. The CBS is used as a basis for assessing the life-cycle cost of each alternative being considered. The CBS links objectives and activities with resources, and constitutes a logical subdivision of cost by functional activity area and major element of System "XYZ". The System "XYZ" life cycle cost is summarized in Table 4-4.

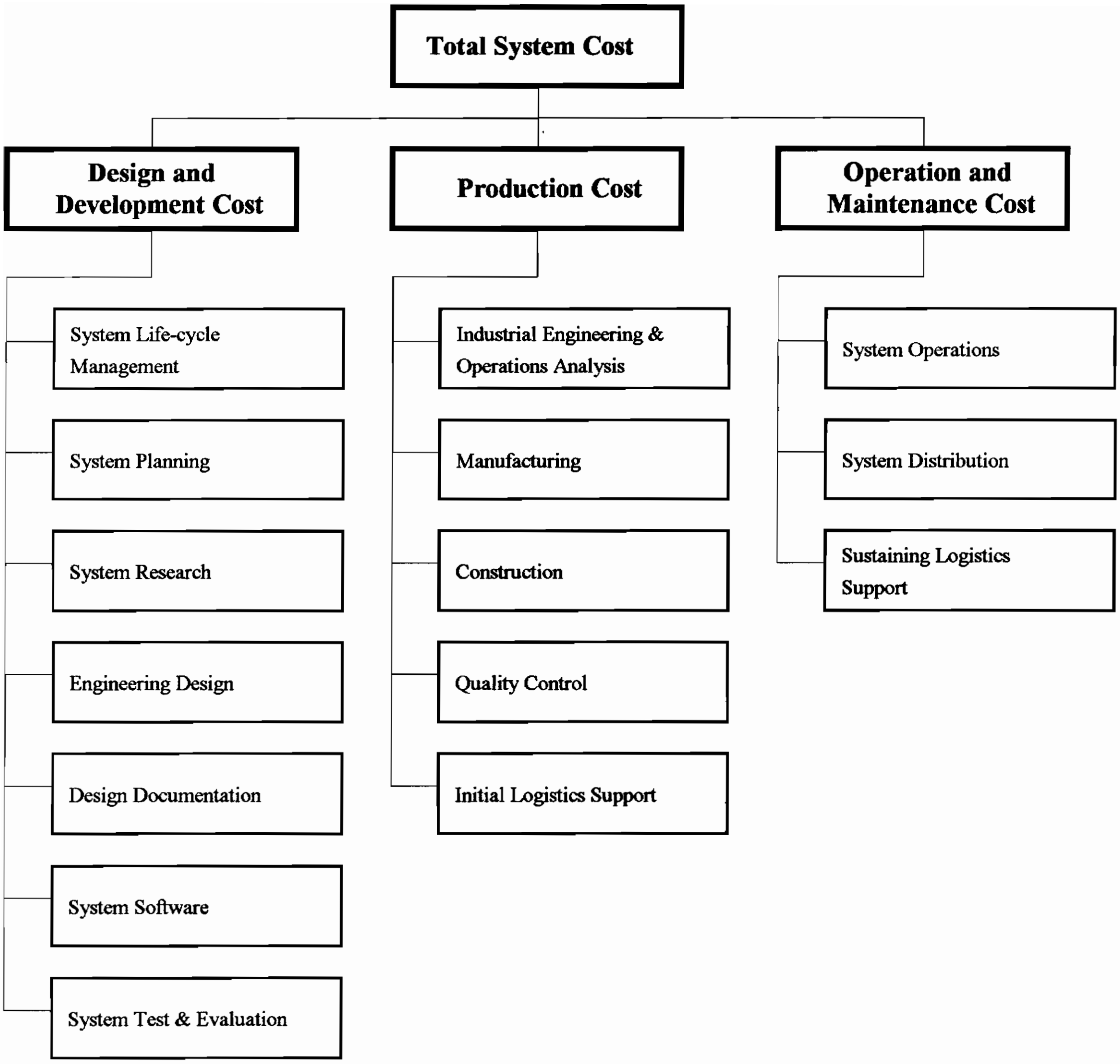


Figure 4-6. System "XYZ" cost breakdown structure (CBS).

Table 4-4. System XYZ Life Cycle Cost Summary ($) (based on an interest rate of 10%).

Program Activity					Life	Cycle	(year)			
	1	2	3	4	5	6	7	8	9	10
Configuration A										
1. System Design and Development										
a. System "XYZ"	400,000	350,000	-	-	-	-	-	-	-	-
b. Support Equip.	400,000	200,000	-	-	-	-	-	-	-	-
2. Production										
a. System "XYZ'	-	480,000	480,000	720,000	840,000	-	-	-	-	-
b. Support Equip.	-	40,000	20,000	40,000	20,000	-	-	-	-	-
3. Operation and Maintenance										
	0	48,000	76,417	152,942	207,666	275,191	275,191	275,191	275,191	272,567
Total Actual Cost	800,000	1,118,000	576,417	912,942	1,067,666	275,191	275,191	275,191	275,191	272,567
Total Present Cost	727,280	924,027	433,062	623,539	662,914	155,345	141,228	128,377	116,709	105,102
Cumulative PC	727,280	1,651,307	2,084,369	2,707,908	3,370,822	3,526,167	3,667,395	3,795,772	3,912,481	4,017,583
Configuration B										
1. System Design and Development										
a. System "XYZ"	380,000	350,000	-	-	-	-	-	-	-	-
b. Support Equip.	380,000	200,000	-	-	-	-	-	-	-	-
2. Production										
a. System "XYZ'	-	440,000	440,000	660,000	770,000	-	-	-	-	-
b. Support Equip.	-	36,000	18,000	36,000	18,000	-	-	-	-	-
3. Operation and Maintenance										
	0	44,000	84,258	170,564	243,565	330,789	330,789	330,789	330,789	328,062
Total Actual Cost	760,000	1,070,000	542,258	866,564	1,031,565	330,789	330,789	330,789	330,789	382,062
Total Present Cost	690,916	884,355	407,398	591,863	640,499	186,730	169,761	154,313	140,288	126,501
Cumulative PC	690,916	1,575,271	1,982,669	2,574,532	3,215,031	3,401,761	3,571,522	3,725,835	3,866,123	3,992,624

Table 4-4. System "XYZ" life-cycle cost summary ($) (based on an interest rate of 10%) (continued).

Program Activity					Life	Cycle	(year)	Total
	11	12	13	14	15	16	17	Cost ($)
Configuration A								
1. System Design and Development								
a. System "XYZ"	0	0	0	0	0	0	0	750,000
b. Support Equip.	0	0	0	0	0	0	0	600,000
2. Production								
a. System "XYZ"	0	0	0	0	0	0	0	2,520,000
b. Support Equip.	0	0	0	0	0	0	0	120,000
3. Operation and Maintenance								
	272,527	269,473	267,466	264,304	262,405	131,141	26,307	3,351,979
Total Actual Cost	272,527	269,473	267,466	264,304	262,405	131,141	26,307	$7,341,979
Total Present Cost	95,521	85,854	77,485	69,591	62,820	28,536	5,206	$4,442,596
Cumulative PC	4,113,104	4,198,958	4,276,443	4,346,034	4,408,854	4,437,390	4,442,596	$4,442,596
Configuration B								
1. System Design and Development								
a. System 'XYZ"	0	0	0	0	0	0	0	730,000
b. Support Equip.	0	0	0	0	0	0	0	580,000
2. Production								
a. System "XYZ"	0	0	0	0	0	0	0	2,310,000
b. Support Equip.	0	0	0	0	0	0	0	108,000
3. Operation and Maintenance								
	328,022	324,983	321,125	318,398	315,961	156,654	32,153	3,990,901
Total Actual Cost	328,022	324,983	321,125	318,398	315,961	156,654	32,153	$7,718,901
Total Present Cost	114,972	103,540	93,030	83,834	75,641	34,088	6,363	$4,504,092
Cumulative PC	4,107,596	4,211,136	4,304,166	4,388,000	4,463,641	4,497,729	4,504,092	$4,504,092

(1) Design and Development Costs

Non-recurring costs associated with the design and development of System XYZ and for required special support equipment are entered in Table 4-4.

(2) Production Costs (Investment)

The costs of operational systems and special support equipment are noted before. These figures include both recurring production costs and amortized non-recurring costs covering initial setup. A summary of these costs is presented in Table 4-5.

(3) Operations and Maintenance Costs

Operations and maintenance costs are primarily based on the frequency of maintenance (or the quantity of maintenance actions) and the logistic support resources required for such maintenance. The quantity of maintenance actions (particularly corrective maintenance) is a function of system utilization (total hours of system operation) and the MTBM factor. Total system operating hours by year (assuming a 365-day year) are noted in Table 4-6.

The assumed average quantity of maintenance actions for System XYZ is based on operating hours divided by MTBM factors, and is noted in Table 4-7.

In determining maintenance factors (in this instance), a good approach is to (1) determine the maintenance actions for each unit of each configuration applicable to intermediate level maintenance, and (2) summarize these actions to provide the total quantity of maintenance actions are based on a function of the operating time and the $MTBM/MTBM_S$ for each unit as shown in Tables 4-8 and 4-9.

Table 4-5. Production Costs.

Configuration	Cost by Program Year					Total
	1	2	3	4	5	Cost ($)
Configuration A	0	20 systems	20 systems	30 systems	35 systems	105 systems
1. System Design and Development						
a. System "XYZ"	-	480,000	480,000	720,000	840,000	2,520,000
b. Support Equip.	-	40,000	20,000	40,000	20,000	120,000
Total	-	520,000	500,000	760,000	860,000	$ 1,640,000
Configuration B	0	20 systems	20 systems	30 systems	35 systems	105 systems
1. System Design and Development						
a. System "XYZ"	-	230,000	230,000	460,000	460,000	2,310,000
b. Support Equip.	-	36,000	18,000	36,000	18,000	108,000
Total	-	266,000	248,000	496,000	478,000	$ 1,418,000

Table 4-6. System "XYZ" operating hours.

Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9	Year 10
0	0	29,200	58,400	102,200	153,300	153,300	153,300	153,300	151,840

Table 4-6. System "XYZ" operating hours (continued).

Year 11	Year 12	Year 13	Year 14	Year 15	Year 16	Year 17
151,840	150,380	148,920	147,460	146,000	73,000	14,600

Table 4-7. Corrective maintenance actions.

Configuration	Program Year									
	1	2	3	4	5	6	7	8	9	10
Configuration A										
Unit A	0	0	15	29	51	77	77	77	77	76
Unit B	0	0	25	50	88	132	132	132	132	131
Unit C	0	0	9	18	31	46	46	46	46	46
Total	0	0	49	97	170	255	255	255	255	253
Configuration B										
Unit A	0	0	23	47	82	123	123	123	123	122
Unit B	0	0	34	68	120	179	179	179	179	178
Unit C	0	0	7	15	26	38	38	38	38	38
Total	0	0	64	130	228	340	340	340	340	338

Table 4-7. Corrective maintenance actions (continued).

Configuration	Program Year							Total
	11	12	13	14	15	16	17	
Configuration A								
Unit A	76	75	75	74	73	37	7	896
Unit B	131	129	128	127	126	63	13	1,539
Unit C	46	45	45	44	44	22	4	538
Total	253	249	248	245	243	122	24	2,973
Configuration B								
Unit A	122	120	119	118	117	58	12	1,432
Unit B	178	176	174	173	171	85	17	2,090
Unit C	38	38	37	37	37	18	4	447
Total	338	334	330	328	325	161	33	3,969

Table 4-8. Preventive maintenance actions.

Configuration	Program Year									
	1	2	3	4	5	6	7	8	9	10
Configuration A										
Unit B	0	0	13	27	47	70	70	70	70	69
Configuration B										
Unit A	0	0	13	27	47	70	70	70	70	69

Table 4-8. Preventive maintenance actions (continued).

Configuration	Program Year							Total
	11	12	13	14	15	16	17	
Configuration A								
Unit B	69	69	68	67	67	33	7	816
Configuration B								
Unit A	69	69	68	67	67	33	7	816

Table 4-9. Total maintenance actions (system level).

Configuration	Program Year									
	1	2	3	4	5	6	7	8	9	10
Config. A	0	0	62	124	217	325	325	325	325	322
Config. B	0	0	77	157	275	410	410	410	410	407

Table 4-9. Total maintenance actions (system level) (continued).

Configuration	Program Year							Total
	11	12	13	14	15	16	17	
Config. A	322	318	316	312	310	155	31	3,789
Config. B	407	403	398	395	392	194	40	4,785

After determining the quantity of maintenance actions (estimated for the life cycle), the next step is to determine the average resources consumed per maintenance action. Resources include both human and material resources. Human resources (in this instance) are measured in terms of maintenance man-hours consumed per maintenance action. The maintenance man-hours of System XYZ are noted in Tables 4-10 and 4-11.

The figures in Table 4-11 include both the low-skilled and high-skilled technicians for corrective maintenance (split evenly). For instance, the corrective maintenance man-hours for Unit A of Configuration A for year 3 = (15 maintenance actions)(6 hrs.)(2 technicians) = 180 MMH.The maintenance man-hours = (maintenance actions)(MTTR or $\overline{M}pt$)(Qty. of personnel).

Table 4-10. Maintenance man-hours at system level (low skill technician).

Configuration	Program Year									
	1	2	3	4	5	6	7	8	9	10
Config. A	0	0	31	62	108	163	163	163	163	161
Config. B	0	0	39	79	138	205	205	205	205	203

Table 4-10. Maintenance man-hours at system level (low skill technician) (continued).

Configuration	Program Year							Total
	11	12	13	14	15	16	17	
Config. A	161	159	158	156	155	77	15	1,895
Config. B	203	202	199	197	196	97	20	2,393

In addition to maintenance man-hour consumption, the man-hours associated with the operation of System "XYZ" must be determined as shown in Table 4-12. The operator man-hours = (system operating hours)(1%)

Table 4-11. Maintenance man-hours at Intermediate level (corrective and preventive).

Configuration	Program Year									
	1	2	3	4	5	6	7	8	9	10
Configuration	A									
Corrective M.										
Unit A	0	0	180	348	612	924	924	924	924	912
Unit B	0	0	300	600	1,056	1,584	1,584	1,584	1,584	1,572
Unit C	0	0	72	144	248	368	368	368	368	368
Total	0	0	552	1,092	1,916	2,876	2,876	2,876	2,876	2,852
Preventive M.										
Unit A	0	0	260	540	940	1,400	1,400	1,400	1,400	1,380
Configuration	B									
Corrective M.										
Unit A	0	0	276	564	984	1,476	1,476	1,476	1,476	1,464
Unit B	0	0	340	680	1,200	1,790	1,790	1,790	1,790	1,780
Unit C	0	0	42	90	156	228	228	228	228	228
Total	0	0	658	1,334	2,340	3,494	3,494	3,494	3,494	3,472
Preventive M.										
Unit B	0	0	325	675	1,175	1,750	1,750	1,750	1,750	1,725

Table 4-11. Maintenance man-hours at Intermediate level (corrective and preventive) (continued).

Configuration	Program Year							Total
	11	12	13	14	15	16	17	
Configuration	A							
Corrective M.								
Unit A	912	900	900	888	876	444	84	10,752
Unit B	1,572	1,548	1,536	1,524	1,512	756	156	18,468
Unit C	368	360	360	352	352	176	32	4,304
Total	2,852	2,808	2,796	2,764	2,740	1,376	272	33,524
Preventive M.								
Unit B	1,380	1,380	1,360	1,340	1,340	660	140	16,320
Configuration	B							
Corrective M.								
Unit A	1,464	1,440	1,428	1,416	1,404	696	144	17,184
Unit B	1,780	1,760	1,740	1,730	1710	850	170	20,900
Unit C	228	228	222	222	222	108	24	2,682
Total	3,472	3,428	3,390	3,368	3,336	1,654	338	40,766
Preventive M.								
Unit A	1,725	1,725	1,700	1,675	1,675	825	175	20,400

Table 4-12. System "XYZ" operator man-hours.

Configuration	Program Year									
	1	2	3	4	5	6	7	8	9	10
System "XYZ"	0	0	292	584	1,022	1,533	1,533	1,533	1,533	1,519

Table 4-12. System "XYZ" operator man-hours (continued).

Configuration	Program Year							Total
	11	12	13	14	15	16	17	
System "XYZ"	1,518	1,504	1,489	1,475	1,460	730	146	17,871

The next step is to determine operator and maintenance personnel costs by applying the above MMH and maintenance action values, and the individual cost factors. The results are listed in Table 4-13.

For corrective maintenance at the intermediate level, one-half of the maintenance man-hours are at $ 15 per hour and one-half are at $ 35 per hour. Spare part costs are related to Unit spares (one set per intermediate level maintenance shop and one set at the depot), and Component spares which are a function of individual maintenance actions. Spares costs are summarized in Table 4-14.

System "XYZ" operations and maintenance costs are summarized in Table 4-15. Maintenance facility cost are based on the quantity of corrective and preventive maintenance actions multiplied by the dollar rate per maintenance action.

A comparison of Configurations A and B using the present equivalent total life-cycle cost is presented in Table 4-16. The costs are listed for those major categories of the cost breakdown structure that are relevant to this analysis.

Table 4-13. System "XYZ" personnel costs($)

Program Activity	Life Cycle (year)									
	1	2	3	4	5	6	7	8	9	10
Configuration A										
1. Operator Cost										
	0	0	11,680	23,360	40,880	61,320	61,320	61,320	61,320	60,760
2. Maintenance Cost										
Organization	0	0	465	930	1,620	2,445	2,445	2,445	2,445	2,415
Intermediate:										
a. Corrective M.	0	0	13,800	27,300	47,900	71,900	71,900	71,900	71,900	71,300
b. Preventive M.	0	0	7,800	16,200	28,200	42,000	42,000	42,000	42,000	41,400
Total Cost	0	0	33,745	67,790	118,600	177,665	177,665	117,665	117,665	175,875
Configuration B										
1. Operator Cost										
	0	0	11,680	23,360	40,880	61,320	61,320	61,320	61,320	60,760
2. Maintenance Cost										
Organization	0	0	585	1,185	2,070	3,075	3,075	3,075	3,075	3,045
Intermediate:										
a. Corrective M.	0	0	16,450	33,350	58,500	87,350	87,350	87,350	87,350	86,800
b. Preventive M.	0	0	9,750	20,250	35,250	52,500	52,500	52,500	52,500	51,750
Total Cost	0	0	38,465	78,145	136,700	204,245	204,245	204,245	204,245	202,355

Table 4-13. System "XYZ" personnel costs($) (continued).

Program Activity	Life Cycle (year)							Total
	11	12	13	14	15	16	17	Cost ($)
Configuration "A"								
1. Operator Cost								
	60,720	60,160	59,560	59,000	58,400	29,200	5,840	714,840
2. Maintenance Cost								
Organization	2,415	2,385	2,370	2,340	2,325	1,155	225	28,425
Intermediate:								
a. Corrective M.	71,300	70,200	69,900	69,100	68,500	34,400	6,800	838,100
b. Preventive M.	41,400	41,400	40,800	40,200	40,200	19,800	4,200	489,600
Total Cost	175,835	174,145	172,630	170,640	169,425	84,555	17,065	2,070,965
Configuration "B"								
1. Operator Cost								
	60,720	60,160	59,560	59,000	58,400	29,200	5,840	714,840
2. Maintenance Cost								
Organization	3,045	3,030	2,985	2,955	2,940	1,455	300	35,895
Intermediate:								
a. Corrective M.	86,800	85,700	84,750	84,200	83,400	41,350	8,450	1,019,150
b. Preventive M.	51,750	51,750	51,000	50,250	50,250	24,750	5,250	612,000
Total Cost	202,315	200,640	198,295	196,405	194,990	96,755	19,840	2,381,885

Table 4-14. Spares costs ($).

Program Activity	Program Year									
	1	2	3	4	5	6	7	8	9	10
Configuration A										
1. Spare Units Costs	0	48,000	24,000	48,000	24,000	0	0	0	0	0
2. Component Spares:										
a. Corrective M.	0	0	14,700	29,100	51,000	76,500	76,500	76,500	76,500	75,900
b. Preventive M.	0	0	1,300	2,700	4,700	7,000	7,000	7,000	7,000	6,900
Total Cost	0	48,000	40,000	79,800	79,700	83,500	83,500	83,500	83,500	82,800
Configuration B										
1. Spare Units Costs	0	44,000	22,000	44,000	22,000	0	0	0	0	0
2. Component Spares:										
a. Corrective M.	0	0	19,200	39,000	68,400	102,000	102,000	102,000	102,000	101,400
b. Preventive M.	0	0	1,300	2,700	4,700	7,000	7,000	7,000	7,000	6,900
Total Cost	0	44,000	42,500	95,100	109,000	109,000	109,000	109,000	109,000	108,300

Table 4-14. Spares costs ($) (continued).

Program Activity	Program Year							Total
	11	12	13	14	15	16	17	Cost ($)
Configuration A								
1. Spare Units Costs	0	0	0	0	0	0	0	144,000
2. Component Spares:								
a. Corrective M.	75,900	74,700	74,400	73,500	72,900	36,600	7,200	891,900
b. Preventive M.	6,900	6,900	6,800	6,700	6,700	3,300	700	81,600
Total Cost	82,800	81,600	81,200	80,200	79,600	39,900	7,900	1,117,500
Configuration B								
1. Spare Units Costs	0	0	0	0	0	0	0	132,000
2. Component Spares:								
a. Corrective M.	101,400	100,200	99,000	98,400	97,500	48,300	9,900	1,190,700
b. Preventive M.	6,900	6,900	6,800	6,700	6,700	3,300	700	81,600
Total Cost	108,300	107,100	105,800	105,100	104,200	51,600	10,600	1,404,300

Component spares = (Maintenance Actions)($ of Material).

Table 4-15. Summary of System XYZ operations and maintenance costs ($).

Program Activity	Program Year									
	1	2	3	4	5	6	7	8	9	10
Configuration A										
1. Personnel Cost	0	0	33,745	67,790	118,600	177,665	177,665	177,665	177,665	175,875
2. Material Cost										
a. Spare Units	0	48,000	24,000	48,000	24,000	0	0	0	0	0
b. Component Spares	0	0	16,000	31,800	55,700	83,500	83,500	83,500	83,500	82,800
3. M. Facilities	0	0	812	1,632	2,856	4,276	4,276	4,276	4,276	4,232
4. MaintenanceData	0	0	1,860	3,720	6,510	9,750	9,750	9,750	9,750	9,660
Total Cost	0	48,000	76,417	152,942	207,666	275,191	275,191	275,191	275,191	272,567
Configuration B										
1. Personnel Cost	0	0	38,465	78,145	136,700	204,245	204,045	204,245	204,245	202,355
2. Material Cost										
a. Spare Units	0	44,000	22,000	44,000	22,000	0	0	0	0	0
b. Component Spares	0	0	20,500	41,700	73,100	109,000	109,000	109,00	109,000	108,300
3. M. Facilities	0	0	983	2,009	3,515	5,244	5,244	5,244	5,244	5,197
4. Maintenance Data	0	0	2,310	4,710	8,250	12,300	12,300	12,300	12,300	12,210
Total Cost	0	44,000	84,258	170,564	243,565	330,789	330,789	330,789	330,789	328,062

Table 4-15. Summary of System XYZ operations and maintenance costs ($) (continued).

Program Activity	Program Year							Total
	11	12	13	14	15	16	17	Cost ($)
Configuration A								
1. Personnel Cost	175,835	174,145	172,630	170,640	169,425	84,555	17,065	2,070,965
2. Material Cost								
a. Spare Units	0	0	0	0	0	0	0	144,000
b. Component Spares	82,800	81,600	81,200	80,200	79,600	39,900	7,900	973,500
3. M. Facilities	4,232	4,188	4,156	4,104	4,080	2,036	412	49,844
4. MaintenanceData	9,660	9,450	9,480	9,360	9,300	4,650	930	113,670
Total Cost	272,527	269,473	267,466	264,304	262,405	131,141	26,307	$3,351,979
Configuration B								
1. Personnel Cost	202,315	200,640	198,295	196,405	194,990	96,755	19,840	2,381,885
2. Material Cost								
a. Spare Units	0	0	0	0	0	0	0	132,000
b. Component Spares	108,300	107,100	105,800	105,100	104,200	51,600	10,600	1,272,300
3. M. Facilities	5,197	5,153	5,090	5,043	5,011	2,479	513	61,166
4. Maintenance Data	12,210	12,090	11,940	11,850	11,760	5,820	1,200	143,550
Total Cost	328,022	324,983	312,125	318,398	315,961	156,654	32,153	$3,990,901

Table 4-16. Life-cycle cost breakdown.

Cost Category	Configuration A		Configuration B	
	P.V. Cost	Percent of Total (%)	P.V. Cost	Percent of Total (%)
Design and Development				
a. Prime System	652,915	14.70	634,733	14.09
b. Support Equipment	528,940	11.90	510,758	11.34
Sub-total	1,181,855	26.60	1,145,491	25.43
Production				
a. Prime System	1,770,660	39.86	1,623,105	36.04
b. Support Equipment	87,824	1.98	79,041	1.75
Sub-total	1,858,484	41.84	1,702,146	37.79
Operation and Maintenance				
a. Personnel Cost	837,199	18.84	962,847	21.38
b. Spare Units	105,389	2.37	96,607	2.14
c. Component Spares	393,565	8.86	514,255	11.42
d. Maintenance Facilities	20,150	0.45	24,725	0.55
e. Maintenance Data	45,954	1.04	58,021	1.29
Sub-total	1,402,256	31.56	1,656,452	36.78
Grand total	**$4,442,596**	100 %	**$4,504,092**	100 %

4.1.3 Evaluation of Alternatives

The problem is to select the best of the two alternatives on the basis of present equivalent total life-cycle cost. A comparison of Configurations A and B using this criterion is presented in Tables 4-4 and 4-16. The costs are listed for those major categories of the cost breakdown structure that are relevant to this analysis.

Referring to Figure 4-7, the initial concern is to determine whether the candidates meet the specified requirements (i.e., dependabilit of 0.990, the budget constraint of 5 million for the life-cycle cost, the MTBM of 300 hours, the MTBF of 500 hours, and the $\overline{M}$ of 30 minutes). In this instance, both alternatives meet the requirements and fall within the trade-off area identified in Figure 4-7.

When evaluating two or more alternatives on a relative basis, the future cost estimations for each alternative must be reduced to their present equivalent amounts. The present equivalent costs for Configurations A and B are presented in Table 4-4. On the basis of the *present value of the estimated life cycle cost,* Configuration A is preferred as summarized in Table 4-17. The cost profiles are illustrated in Figure 4-8 based on an interest rate of 10%.

The results of this analysis support Configuration A as the preferred alternative on the basis of present equivalent life-cycle cost. Note that the design and development cost is higher for Configuration A; however, the overall life-cycle cost is lower, owing to a significantly lower operation and maintenance cost. This would tend to indicate that the equipment design for reliability pertaining to Configuration A is somewhat better. Although this increased reliability results in higher design and development costs, the anticipated number of maintenance actions is lower, resulting in lower operation and maintenance costs.

Evaluation criteria	Configuration A	Configuration B
Effectiveness D	0.996395	0.994725
Budget constraint (P.V. Cost)	$ 4,442,596	$ 4,504,092
Performance	Requirement met	Requirement met

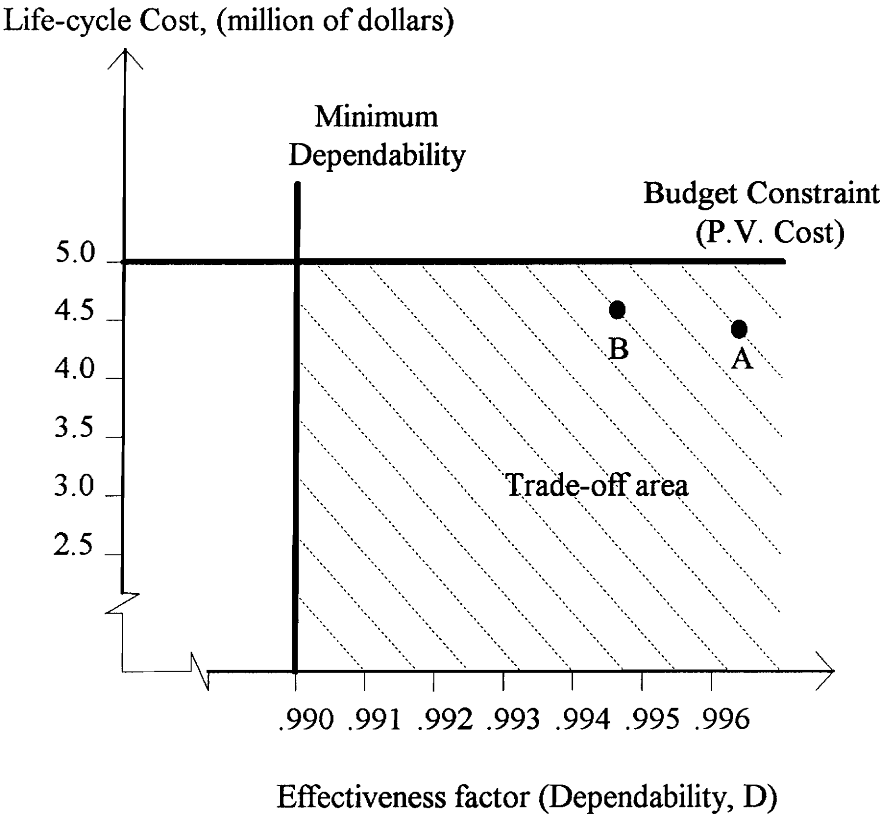


Figure 4-7. Effectiveness versus unit cost.

Table 4-17. Summary of results.

Year	Interest Factor (10%)	Configuration A Cost ($)		Configuration B Cost ($)	
		Undiscounted	Discounted	Undiscounted	Discounted
1	0.9091	800,000	727,280	760,000	690,916
2	0.8265	1,118,000	924,027	1,070,000	884,355
3	0.7513	576,417	433,062	542,258	407,398
4	0.6830	912,942	623,539	866,564	591,863
5	0.6209	1,067,666	662,914	1,031,565	640,499
6	0.5645	275,191	155,345	330,789	186,730
7	0.5132	275,191	141,228	330,789	169,761
8	0.4665	275,191	128,377	330,789	154,313
9	0.4241	275,191	116,709	330,789	140,288
10	0.3856	272,567	105,102	328,062	126,501
11	0.3505	272,527	95,521	328,022	114,972
12	0.3186	269,473	85,854	324,983	103,540
13	0.2897	267,466	77,485	321,125	93,030
14	0.2633	264,304	69,591	318,398	83,834
15	0.2394	262,405	62,820	315,961	75,641
16	0.2176	131,141	28,536	156,654	34,088
17	0.1979	26,307	5,206	32,153	6,363
Total		$7,341,979	$4,442,596	$7,718,901	$4,504,092

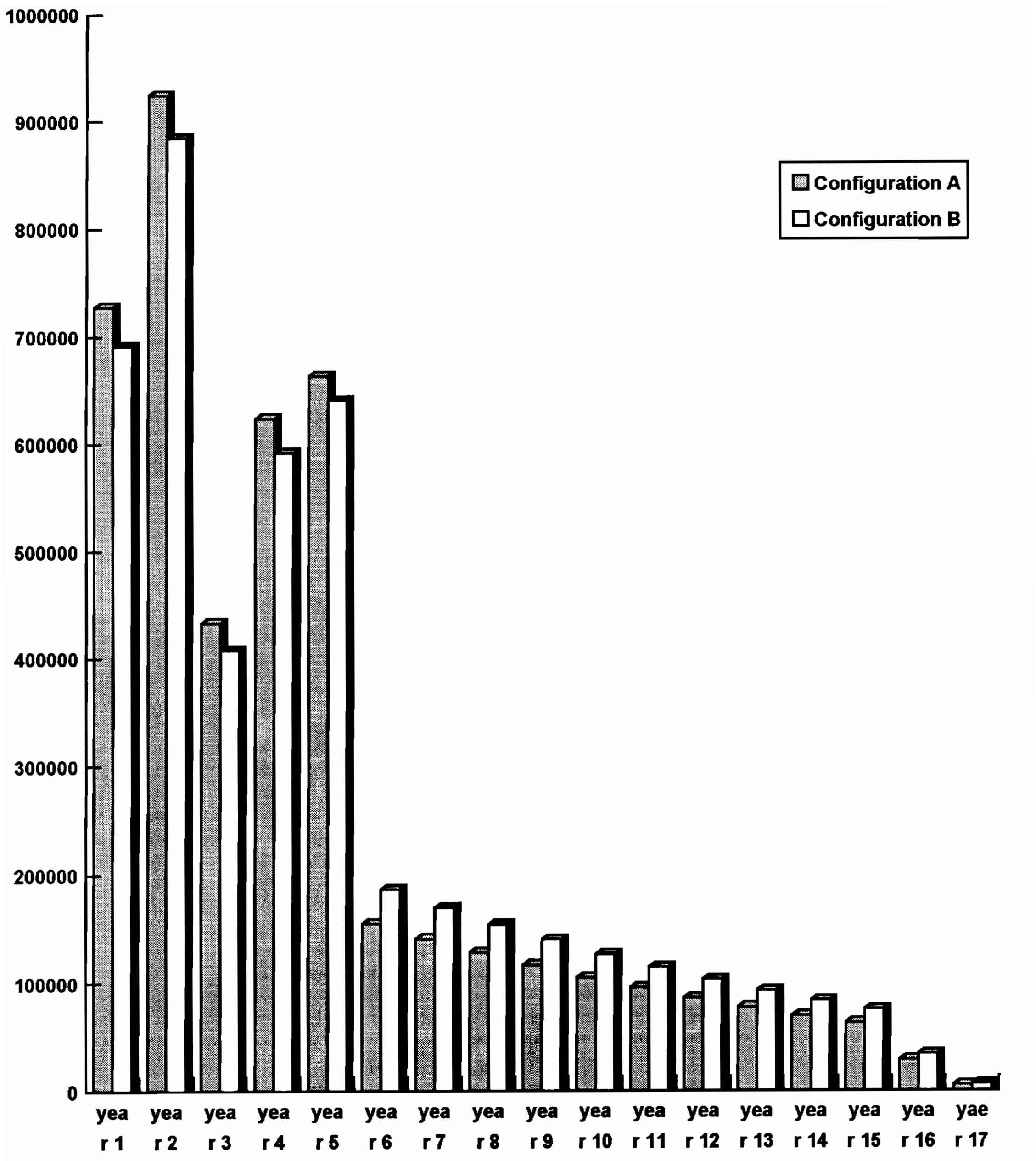


Figure 4-8. Cost profiles for radar subsystem alternatives (discounted).

Prior to a final decision on which alternative to select, the analyst should accomplish a break-even analysis to determine the point in time when Configuration A becomes more economical than Configuration B. Figure 4-9 indicates that the break-even point, or the point in time when Configuration B becomes less costly, is approximately 11 years and 4 months after the program start. This point is early enough in the life cycle to support the decision. If this corssover point were much further out in time, the decision might be questioned.

4.1.4 Cost-Effectiveness Analysis

The design and development of a system or product that is cost-effective, within the constraints specified by operational and maintenance requirements, is a prime objective. Cost-effectiveness relates to the measure of a system in terms of mission fulfillment (*system effectiveness*) and total *life-cycle cost.* Cost-effectiveness, which is similar to the standard cost-benefit analysis factor employed for decision-making purposes in many industrial and business applications, can be expressed in various terms (i.e., one or more figures of merit), depending on the specific mission or system parameters that one wishes to measure [Blanchard, 1992]. The prime ingredients of cost-effectiveness are illustrated in Figure 4-10.

In this example, the cost-effectiveness analysis involves the evaluation of two alternative radar subsystems using dependability and life-cycle cost as decision criteria. This specific cost-effectiveness figure of merit (FOM) can be expressed as

$$\text{FOM} = \frac{\text{dependability}}{\text{life-cycle cost}} \tag{4.1}$$

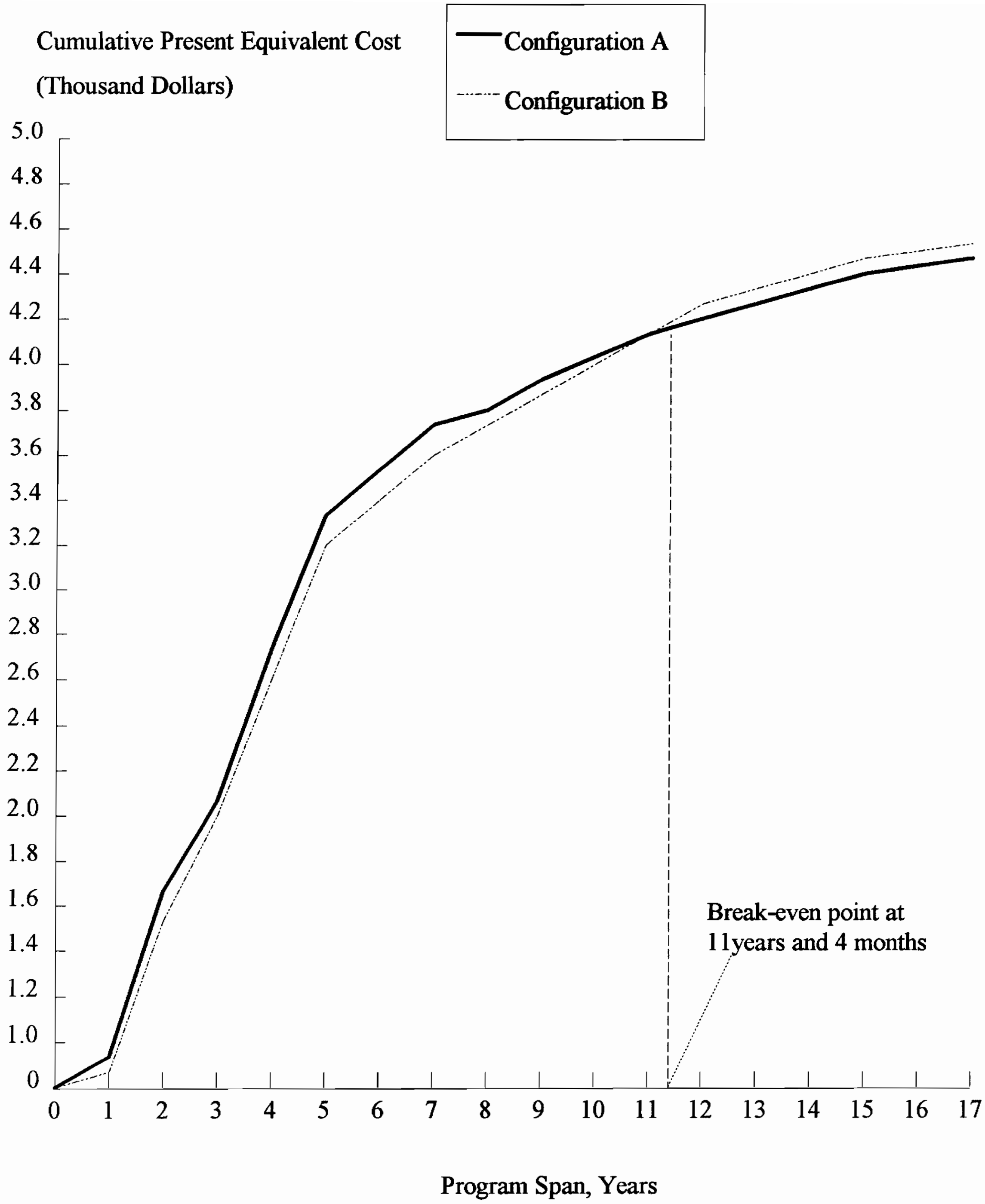


Figure 4-9. Investment payback (break-even analysis).

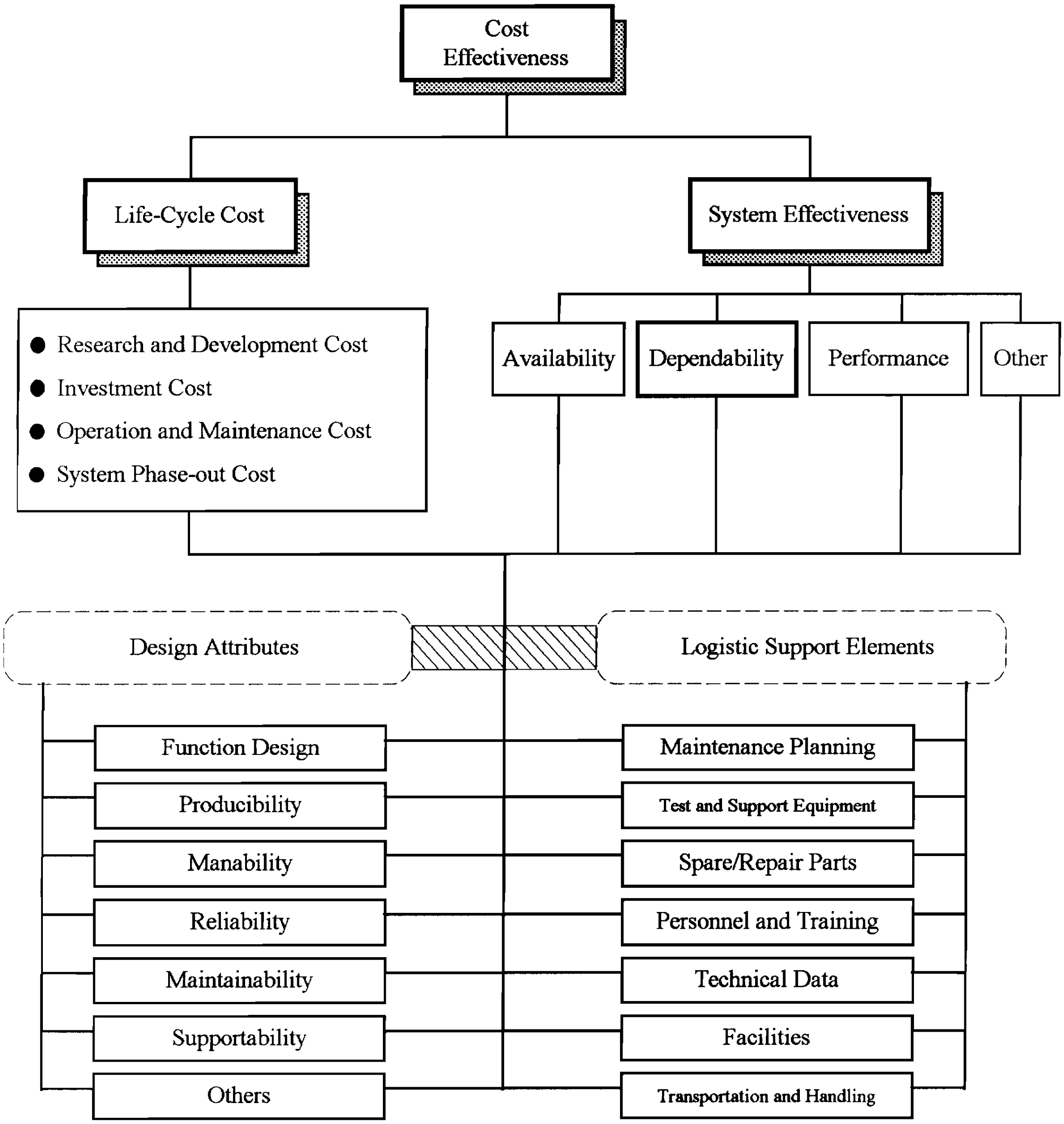


Figure 4-10. Basic ingredients of cost-effectiveness [Blanchard, 1992].

The FOM value for Configuration A is

$$\text{FOM} = \frac{\text{dependability}}{\text{life-cycle cost}} = \frac{0.996395}{\$4,442,596} = 2.2428 \times 10^{-7}$$

and Configuration B is

$$\text{FOM} = \frac{\text{dependability}}{\text{life-cycle cost}} = \frac{0.994725}{\$4,504,092} = 2.2085 \times 10^{-7}$$

Therefore, Configuration A is preferred.

CHAPTER V

CONCLUSIONS AND FUTURE RESEARCH

The aim of this chapter is to present a summary and conclusions from this project and report. Also, some suggestions of future research are given.

5.1 Summary and Conclusions

The concept of system dependability and the mathematical model of dependability have been discussed. Effectiveness factors and relationships have been described, a measure of system dependability has been defined, and a computer-based tool has been developed to enable the accomplishment of trade-off analyses and the evaluation of various system configurations in terms of dependability. Maintenance requirements have been addressed through the introduction of various combinations of failures, and failure distributions include the consideration of the exponential, Weibull, and log-normal cases. Application of the dependability model is illustrated through a case study involving an aircraft radar subsystem as an example to illustrate the cost-effectiveness approach, using life-cycle cost factors as a function of dependability. The computerized model was used to facilitate the system dependability analysis and evaluation in the case-study application.

System dependability is an important measure of system effectiveness. Throughout the evaluation and trade-off analysis of system dependability, the system effectiveness can be better predicted so as to select a better design alternative. A high level of effectiveness is a desired result, outcome, and consequence of system operation. System effectiveness is a measure of the ability of the system to accomplish its objective. The measures of system effectiveness must be "tailored" to the specific mission. It can also be thought of as the probability that the system will achieve its objective. The proper question for analysts, engineers, managers, politicians, and/or decision-makers is not "will it work," but rather "what is the probability that it is adequate and will it accomplish the objectives?" The objective of this project report was to provide insights into the quantification and the issues of system dependability.

The objective of developing a mathematical model for evaluating the system dependability has been met. The developed model enables the estimation of dependability figures of merit at the system, mission, function, and element level. The combinations of malfunction are also estimated. Appendix A presents the computerized model in order for most analyses and calculation.

5.2 Future Research

There are several ways to further improve the dependability model by extending the work present in this project and report. These include:

(1) Expand the model to consider additional system design parameters, such as availability, supportability, etc., in order to acquire an optimal balance among these parameters and to meet overall effectiveness requirements while reducing life-cycle cost.

(2) Consider multifunctional systems performing a series of missions.

(3) Consider Markov approach or Markov modeling in the evaluation of system dependability.

(4) Consider simulation techniques in the evaluation of system dependability.

(5) Consider other distributions of time to failure and time to repair as in Appendix B.

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Appendix A

Computer Program List and Output

/* depp.c Program to calculate system dependability for exponentially distributed time-to-failure with multiple failures. A logic flow chart for this program is illustrated in Figure A-1.

*/

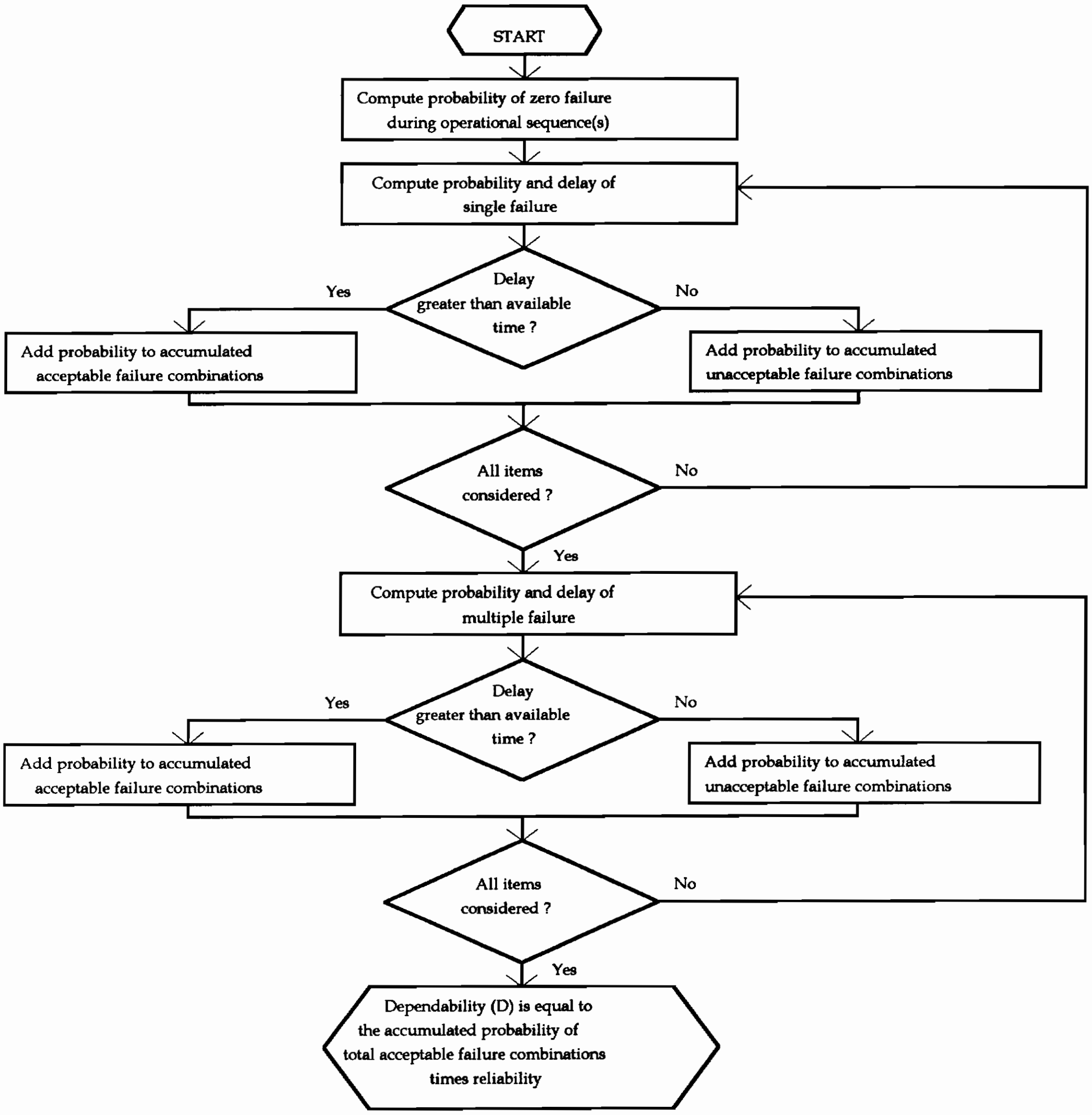


Figure A-1. System dependability measurement flow chart.

```
/* Definitions

    Arrays:
        Failure[][0]    Failure rates of components
        Failure[][1]    Average repair times for components

    Variables:
        f         Total system failure rate
        d         System dependability
        m         number of components
        n         The desired combination number of malfunctions (less than or equal to 5)
        t         Mission time
        ta        The time available for repair
        pi, pij   Probability of correcting malfunctions
        mm, mn, mo, mp, mq   integers
*/

#include <stdio.h>
#include <math.h>

main()
{

/* Define variable types */

float f, t, ta, d;
 float p0, p1, p2, p21, p22, p3, p31, p32, p33;
 float p4, p41, p42, p43, p44, p45;
 float p5, p51, p52, p53, p54, p55, p56, p57;
 int m, n, mm, mn, mo, mp, mq;
 float failure[50][2];

/* Get inputs */
printf("Input the number of components: ");
 scanf("%d", &m);
 printf("Input the desired combination number of malfunctions(<=5 ):");
 scanf("%d", &n);
 printf("Input the time available for repair (in minutes):");
 scanf("%f", &ta);
 printf("Input the mission time (in hours):");
 scanf("%f", &t);
 for (mm=0; mm<m; mm++) {
   printf("Input failure rate and average repair time for components %d:", mm+1);
   scanf("%f %f", &failure[mm][0], &failure[mm][1]);
   }
```

```
/* Calculate Probability of correcting different combinations of malfunctions */

/* Combination of zero */
 if (n >= 0)
   p0 = 1;

/* Combination of one */
 if (n >= 1) {
    p1=0;
    for (mm=0; mm<m; mm++) {
          if (failure[mm][1] <= ta)
          p1=p1+failure[mm][0];
    }
 }

/* Combinations of two */
 if (n >= 2) {
    p21=0;
    p22=0;
    for (mm=0; mm<m; mm++) {
          if ((2*failure[mm][1])<=ta)
          p21=p21+((1/2)*failure[mm][0]*failure[mm][0]);
          for (mn=mm+1; mn<m; mn++) {
            if ((failure[mm][1]+failure[mn][1])<=ta)
            p22=p22+(failure[mm][0]*failure[mn][0]);
          }
    }
 }
 p2=p21+p22;

/* Combinations of Three */
 if (n >= 3) {
    p31=0;
    p32=0;
    p33=0;
    for (mm=0; mm<m; mm++) {
          if ((3*failure[mm][1])<=ta)
          p31=p31+((1/6)*failure[mm][0]*failure[mm][0]);
          for (mn=0; (mn!=mm) && (mn<m); mn++) {
            if ((2*failure[mm][1]+failure[mn][1])<=ta)
            p32=p32+((1/2)*failure[mm][0]*failure[mm][0]*failure[mn][0]);
          }
          for (mn=mm+1; mn<m; mn++) {
            for (mo=mn+1; mo<m; mo++) {
              if ((failure[mm][1]+failure[mn][1]+failure[mo][1])<=ta)
```

```
                p33=p33+(failure[mm][0]*failure[mn][0]*failure[mo][0]);
             }
          }
    }
}
p3=p31+p32+p33;

/* Combinations of four */
if (n >= 4) {
    p41=0;
    p42=0;
    p43=0;
    p44=0;
    for (mm=0; mm<m; mm++) {
          if ((4*failure[mm][1])<=ta)
          p41=p41+((1/24)*failure[mm][0]*failure[mm][0]*failure[mm][0]
                            *failure[mm][0]);
          for (mn=0; (mn!=mm) && (mn<m); mn++) {
             if ((3*failure[mm][1]+failure[mn][1])<=ta)
             p42=p42+((1/6)*failure[mm][0]*failure[mm][0]*failure[mm][0]
                                 *failure[mn][0]);
          }
          for (mn=m+1; mn<m; mn++) {
             if (((2*failure[mm][1])+(2*failure[mn][1]))<=ta)
             p43=p43+((1/4)*failure[mm][0]*failure[mm][0]
                                 *failure[mn][0]*failure[mn][0]);
          }
          for (mn=0; (mn!=mm) && (mn<m); mn++) {
             for (mo=0; (mo!=mn) && (mo<m); mo++) {
                if ((2*failure[mm][1]+failure[mn][1]+failure[mo][1])<=ta)
                p44=p44+((1/2)*failure[mm][0]*failure[mm][0]*failure[mn][0]
                                    *failure[mo][0]);
             }
          }
          for (mn=mm+1; mn<m; mn++) {
             for (mo=mn+1; mo<m; mo++) {
                for (mp=mo+1; mp<m; mp++) {
                      if ((failure[mm][1]+failure[mn][1]+failure[mo][1]
                                               +failure[mp][1])<=ta)
                      p45=p45+(failure[mm][0]*failure[mn][0]*failure[mo][0]
                                                      *failure[mp][0]);
                }
             }
          }
    }
```

```
 }
 p4=p41+p42+p43+p44+p45;

/* Combinations of five */
 if (n>=5) {
  p51=0;
  p52=0;
  p53=0;
  p54=0;
  p55=0;
  p56=0;
  p57=0;
  for (mm=0; mm<m; mm++) {
   if ((5*failure[mm][1])<=ta)
   p51=p51+((1/120)*failure[mm][0]*failure[mm][0]*failure[mm][0]
                  *failure[mm][0]*failure[mm][0]);
   for (mn=0; (mn!=mm) && (mn<m); mn++) {
    if ((4*failure[mm][1]+failure[mn][1])<=ta)
    p52=p52+((1/24)*failure[mm][0]*failure[mm][0]*failure[mm][0]
                  *failure[mm][0]*failure[mn][0]);
   }
   for (mn=0; (mn!=mm) && (mn<m); mn++) {
    if ((3*failure[mm][1]+2*failure[mn][1])<=ta)
    p53=p53+((1/12)*failure[mm][0]*failure[mm][0]*failure[mm][0]
                  *failure[mn][0]*failure[mn][0]);
   }
   for (mn=0; (mn!=mm) && (mn<m); mn++) {
    for (mo=0; (mo!=mn) && (mo<m); mo++) {
       if ((3*failure[mm][1]+failure[mn][1]+failure[mo][1])<=ta)
       p54=p54+((1/6)*failure[mm][0]*failure[mm][0]*failure[mm][0]
                  *failure[mn][0]*failure[mo][0]);
    }
   }
   for (mn=0; (mn!=mm) && (mn<m); mn++) {
    for (mo=0; (mo!=mn) && (mo<m); mo++) {
       if ((2*failure[mm][1]+2*failure[mn][1]+failure[mo][1])<=ta)
       p55=p55+((1/4)*failure[mm][0]*failure[mm][0]
                  *failure[mn][0]*failure[mn][0]*failure[mo][0]);
    }
   }
   for (mn=0; (mn!=mm) && (mn<m); mn++) {
    for (mo=0; (mo!=mn) && (mo<m); mo++) {
       for (mp=0; (mp!=mo) && (mp<m); mp++) {
        if ((2*failure[mm][1]+failure[mn][1]+failure[mo][1]
                              +failure[mp][1])<=ta)
```

```
          p56=p56+((1/2)*failure[mm][0]*failure[mm][0]*failure[mn][0]
                          *failure[mo][0]*failure[mp][0]);
         }
      }
    }
    for (mn=mm+1; mn<m; mn++) {
     for (mo=mn+1; mo<m; mo++) {
        for (mp=mo+1; mp<m; mp++) {
         for (mq=mp+1; mq<m; mq++) {
          if ((failure[mm][1]+failure[mn][1]+failure[mo][1]
                               +failure[mp][1]+failure[mq][1])<=ta)
          p57=p57+(failure[mm][0]*failure[mn][0]*failure[mo][0]
                                 *failure[mp][0]*failure[mq][0]);
         }
        }
      }
    }
  }
 }
 p5=p51+p52+p53+p54+p55+p56+p57;

/* Calculate System Dependability */
 f=0;
 for (mm=0; mm<m; mm++)
   f=f+failure[mm][0];
 d=exp(-f*t)*(p0+(p1*t)+(p2*t*t)+(p3*t*t*t)+(p4*t*t*t*t)+(p5*t*t*t*t*t));

/* Print out System Dependability */
 printf("System Dependability, D= %f ", d);
}
```

Program Output

- Consider of zero combination of malfunction

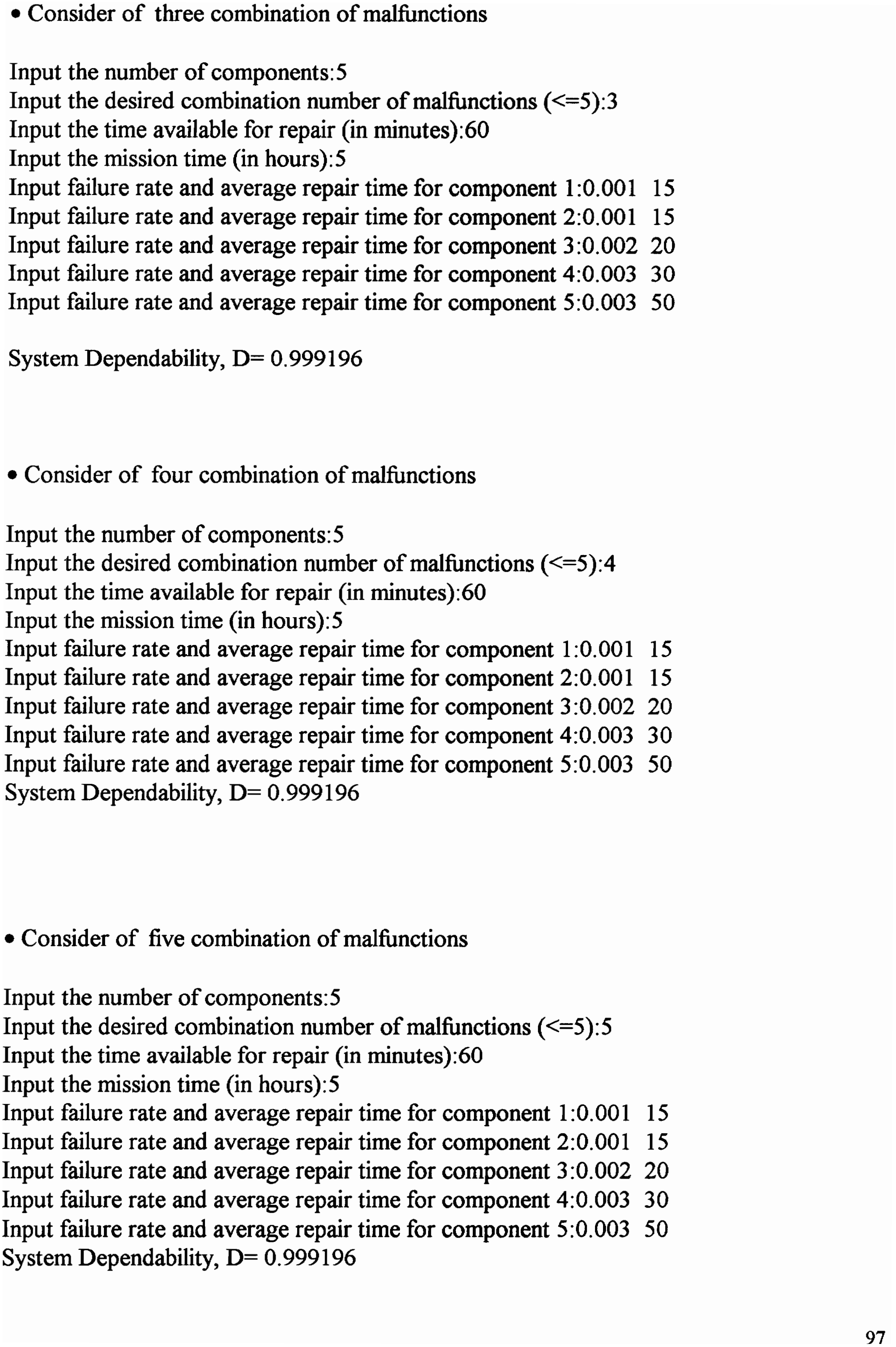
```
Input the number of components: 5
Input the desired combination number of malfunctions(<=5): 0
Input the time available for repair( in minutes): 60
Input the mission time (in hours): 5
Input failure rate and average repair time for component 1:0.001  15
Input failure rate and average repair time for component 2:0.001  15
Input failure rate and average repair time for component 3:0.002  20
Input failure rate and average repair time for component 4:0.003  30
Input failure rate and average repair time for component 5:0.003  50
System Dependability, D= 0.951229
```

- Consider of single combination of malfunctions

```
Input the number of components:5
Input the desired combination number of malfunctions (<=5):1
Input the time available for repair (in minutes):60
Input the mission time (in hours): 5
Input failure rate and average repair time for component 1:0.001  15
Input failure rate and average repair time for component 2:0.001  15
Input failure rate and average repair time for component 3:0.002  20
Input failure rate and average repair time for component 4:0.003  30
Input failure rate and average repair time for component 5:0.003  50
System Dependability, D= 0.998791
```

- Consider of two combination of malfunctions

```
Input the number of components:5
Input the desired combination number of malfunctions (<=5):2
Input the time available for repair (in minutes):60
Input the mission time (in hours):5
Input failure rate and average repair time for component 1:0.001  15
Input failure rate and average repair time for component 2:0.001  15
Input failure rate and average repair time for component 3:0.002  20
Input failure rate and average repair time for component 4:0.003  30
Input failure rate and average repair time for component 5:0.003  50
System Dependability, D= 0.999195
```



- Consider of three combination of malfunctions

```
Input the number of components:5
Input the desired combination number of malfunctions (<=5):3
Input the time available for repair (in minutes):60
Input the mission time (in hours):5
Input failure rate and average repair time for component 1:0.001  15
Input failure rate and average repair time for component 2:0.001  15
Input failure rate and average repair time for component 3:0.002  20
Input failure rate and average repair time for component 4:0.003  30
Input failure rate and average repair time for component 5:0.003  50

System Dependability, D= 0.999196
```

- Consider of four combination of malfunctions

```
Input the number of components:5
Input the desired combination number of malfunctions (<=5):4
Input the time available for repair (in minutes):60
Input the mission time (in hours):5
Input failure rate and average repair time for component 1:0.001  15
Input failure rate and average repair time for component 2:0.001  15
Input failure rate and average repair time for component 3:0.002  20
Input failure rate and average repair time for component 4:0.003  30
Input failure rate and average repair time for component 5:0.003  50
System Dependability, D= 0.999196
```

- Consider of five combination of malfunctions

```
Input the number of components:5
Input the desired combination number of malfunctions (<=5):5
Input the time available for repair (in minutes):60
Input the mission time (in hours):5
Input failure rate and average repair time for component 1:0.001  15
Input failure rate and average repair time for component 2:0.001  15
Input failure rate and average repair time for component 3:0.002  20
Input failure rate and average repair time for component 4:0.003  30
Input failure rate and average repair time for component 5:0.003  50
System Dependability, D= 0.999196
```

Appendix B

Other Distributions

In this Appendix, the Weibull and log-normal distributions are discussed. The mathematical models of system dependability with Weibull and log-normal distributions are presented.

B.1 The Weibull Distribution

The Weibull distribution is probably the most widely used distribution in reliability engineering because of the many shapes it attains for various values of β [Kececioglu, v.1, 1991]. The Weibull probability density function is given by

$$f(t)=\frac{\beta(t-\gamma)^{\beta-1}}{(\theta-\gamma)^{\beta}}e^{-\left(\frac{t-\gamma}{\theta-\gamma}\right)^{\beta}} \tag{B.1}$$

where

$f(t)\geq 0,\quad t\geq\gamma,\quad \beta>0,\quad \theta>\gamma,\quad -\infty<\gamma<\infty,$

t = operating time,

β = shape parameter or Weibull slope,

θ = scale parameter or characteristic life, and

γ = location parameter or minimum life.

The shape parameter β is a pure number. The parameters θ and γ have the same units of *t*, such as hours, miles, cycles, actuations, etc., to failures. The parameter γ may assume all values and provides an estimate of the earliest time a failure may be observed. A negative γ may indicate failure prior to the beginning of the test, during production, in storage, in transit, during checkout prior to the start of a mission, or prior to actual operation. Various forms of the failure density function are illustrated in Figure B-1 [Kapur, 1977], where γ is taken as zero.

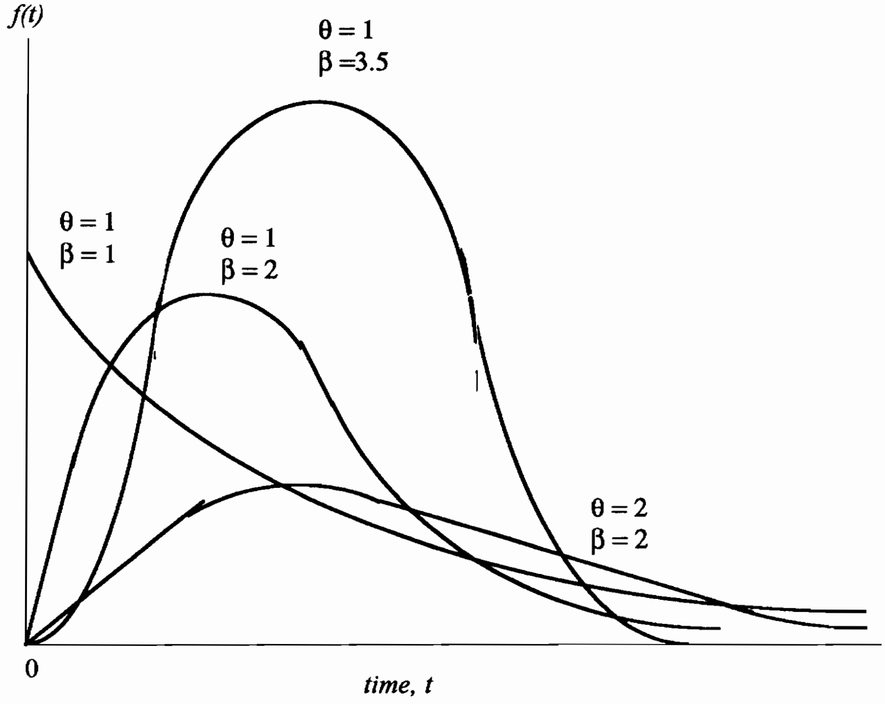


Figure B-1. Plot of the density function for the Weibull distribution [Kapur, 1977].

It can be shown that, for $t \geq \gamma$,

$$R(t) = 1 - F(t) = e^{-\left(\frac{t-\gamma}{\theta-\gamma}\right)^{\beta}} \tag{B.2}$$

The time-to-failure of electron tubes, relays, capacitors, and ball bearings have been shown to be well represented by the Weibull distribution [Kececioglu, v.1, 1991]. While the exponential distribution represents the life characteristics of systems quite well, the Weibull distribution well describes the life characteristics of parts and components [Lipson and Sheth, 1973]. In particular, the negative exponential distribution is a Weibull distribution with shape parameter β = 1; the Rayleigh distribution is a special Weibull distribution with β = 2; and when β ≅ 3.5, the Weibull distribution is a good approximation to the normal distribution [Kececioglu, v.1, 1991].

Referring to Equations (3.4) and (B.1), if the time-to-failure is Weibull distributed and the time-to-repair is negative exponential distributed, then Equation (3.4) can be rewritten and simplified into a convolution of a Weibull distribution and a negative exponential distribution. The convolution is at least approximately a Weibull distribution [Sols, 1992]. In this case, for all system components the cumulative distribution functions of time to failure and time to repair are given by:

a) Distribution of time to failure: $F(t) = 1 - e^{-\left(\frac{t-\gamma}{\theta-\gamma}\right)^{\beta}}$ (B.3)

b) Distribution of time to repair: $\Re(t) = 1 - e^{-\mu t}$ (B.4)

Let $\gamma = 0$, Equation (B.2) can be written as

$$F(t) = 1 - e^{-\left(\frac{t}{\theta}\right)^{\beta}} = 1 - e^{-(\lambda t)^{\beta}} \tag{B.5}$$

Therefore, the system reliability with Weibull time-to-failure distribution can be expressed as

$$R(t) = 1 - F(t) = e^{-(\lambda t)^{\beta}} \tag{B.6}$$

The system dependability with Weibull time-to-failure distribution can then be expressed as

$$D(t) = R(t)\left(1 + \frac{\lambda t}{1!}M_1 + \frac{(\lambda t)^2}{2!}M_2 + \frac{(\lambda t)^3}{3!}M_3 + \cdots + \frac{(\lambda t)^n}{n!}M_n\right) \tag{B.7}$$

where $R(t) = e^{-(\lambda t)^{\beta}}$.

M_n = the probability of correcting n malfunctions, (in n combinations), in the allowable time.

Consider of four combinations of malfunctions, the system dependability can be expressed as

$$\begin{aligned}D(t) = e^{-(\lambda t)^{\beta}} \Biggl\{ & 1 + t\left(\sum_{i=1}^{n} \lambda_i^*\right) + t^2\left[\left(\frac{1}{2}\sum_{i=1}^{n} \lambda_i^{*\,2}\right) + \left(\sum_{i,j=1,2;i\neq j}^{n-1,n} \lambda_i \lambda_j^*\right)\right] \\ & + t^3\left[\left(\frac{1}{6}\sum_{i=1}^{n} \lambda_i^{*\,3}\right) + \left(\frac{1}{2}\sum_{i,j=1,2;i\neq j}^{n-1,n} \lambda_i^{*\,2} \lambda_j^*\right) + \left(\sum_{i,j,k=1,2,3;i\neq j\neq k}^{n-2,n-1,n} \lambda_i \lambda_j \lambda_k^*\right)\right] \\ & + t^4\left[\left(\frac{1}{24}\sum_{i=1}^{n} \lambda_i^{*\,4}\right) + \left(\frac{1}{6}\sum_{i,j=1,2;i\neq j}^{n-1,n} \lambda_i^{*\,3} \lambda_j^*\right) + \left(\frac{1}{4}\sum_{i=1;i\neq j}^{n-1} \lambda_i^{*\,2} \sum_{j=2}^{n} \lambda_j^{*\,2}\right)\right. \\ & \left. + \left(\frac{1}{2}\sum_{i=1;i\neq j\neq k}^{n} \lambda_i^* \sum_{j,k=2,3}^{n-1,n} \lambda_j \lambda_k^*\right) + \left(\sum_{i,j,k,l=1,2,3,4;i\neq j\neq k\neq l}^{n-3,n-2,n-1,n} \lambda_i \lambda_j \lambda_k \lambda_l^*\right)\right]\Biggr\}\end{aligned}$$

(B.8)

B.2 The Log-normal Distribution

The log-normal failure density function (in practice, because of the discussion in Section 3.2.2, it is more likely to be a repair density function) is defined as

$$f(t) = \frac{1}{\sigma t\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\ln t - \tau}{\sigma}\right)^2\right] \qquad (B.9)$$

where $f(t) \geq 0, t \geq 0, -\infty < \tau < \infty, \sigma > 0,$

τ = mean of the natural logarithms of the times to failure, the mean wearout life,

t = operating time, and

σ = standard deviation of the natural logarithms of the times to failure, the shape

$$\text{parameter} = \sqrt{\frac{\left[\sum(\ln t - \tau)\right]^2}{n-1}} \tag{B.10}$$

Various forms of the log-normal density function are illustrated in Figure B-2.

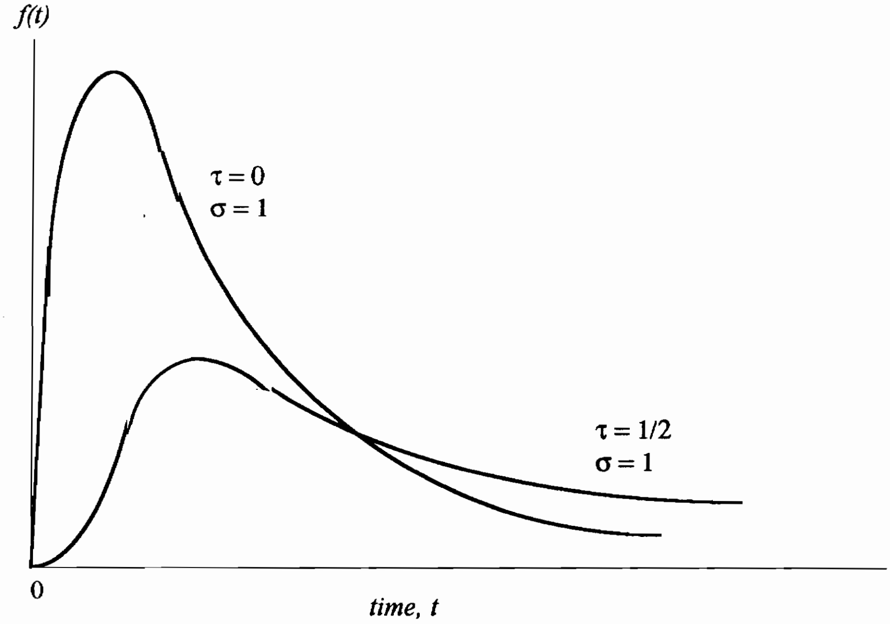


Figure B-2. The failure density function for the log-normal distribution.

The cumulative distribution function for the log-normal is

$$F(t) = \frac{1}{\sigma\sqrt{2\pi}} \int_0^t \frac{1}{x} \exp\left[-\frac{1}{2}\left(\frac{\ln x - \tau}{\sigma}\right)^2\right] dx \tag{B.11}$$

and this can be related to the standard normal deviate z by

$$F(t) = P[\mathrm{t} \le t] = P[z \le Z] \tag{B.12}$$

where

$\mathbf{t}$ = the random variable, the time to failure,

Z = the standardized random variate = $\dfrac{\ln t - \tau}{\sigma}$.

The system reliability can then be expressed as

$$R(t) = 1 - F(t) = P[\mathbf{t} > t] = P\left[z > \frac{\ln t - \tau}{\sigma}\right] \qquad \text{(B.13)}$$

The probability value of system reliability can be found in a normal distribution table for a $Z = \dfrac{\ln t - \tau}{\sigma}$.

If the time to failure is exponentially distributed and the time to repair is log-normally distributed, then system dependability can be expressed as Equation (B.14).

$$D(t) = R(t)\left(1 + \frac{\lambda t}{1!}M_1 + \frac{(\lambda t)^2}{2!}M_2 + \frac{(\lambda t)^3}{3!}M_3 + \cdots + \frac{(\lambda t)^n}{n!}M_n\right) \qquad \text{(B.14)}$$

where $R(t) = e^{-(\lambda t)}$.

M_n = the probability of correcting n malfunctions, (in n combinations), in the allowable time.

As was stated previously in Section the log-normal distribution is the most commonly used distribution in maintainability analysis. However, if the log-normal distribution function with $\sigma < 0.2$, it may be seen that the log-normal distribution may represent the times to failure when considering of wearout [Kececioglu, v.1, 1991]. Thus, the $R(t)$ term in Equation (B.14) can be expressed as

$$R(t) = 1 - F(t) = P[\mathbf{t} > t] = P\left[z > \frac{\ln t - \tau}{\sigma}\right] \qquad \text{(B.15)}$$

The values for the log-normal distribution are easily computed by using the standard normal table. Consider of four combinations of malfunctions, the system dependability can be expressed as

$$\begin{aligned} D(t) = R(t)\Bigg\{ & 1 + t\left(\sum_{i=1}^{n} \lambda_i^*\right) + t^2\left[\left(\frac{1}{2}\sum_{i=1}^{n} \lambda_i^{*\,2}\right) + \left(\sum_{i,j=1,2;i\neq j}^{n-1,n} \lambda_i \lambda_j^*\right)\right] \\ & + t^3\left[\left(\frac{1}{6}\sum_{i=1}^{n} \lambda_i^{*\,3}\right) + \left(\frac{1}{2}\sum_{i,j=1,2;i\neq j}^{n-1,n} \lambda_i^{*\,2} \lambda_j^*\right) + \left(\sum_{i,j,k=1,2,3;i\neq j\neq k}^{n-2,n-1,n} \lambda_i \lambda_j \lambda_k^*\right)\right] \\ & + t^4\Bigg[\left(\frac{1}{24}\sum_{i=1}^{n} \lambda_i^{*\,4}\right) + \left(\frac{1}{6}\sum_{i,j=1,2;i\neq j}^{n-1,n} \lambda_i^{*\,3} \lambda_j^*\right) + \left(\frac{1}{4}\sum_{i=1;i\neq j}^{n-1} \lambda_i^{*\,2} \sum_{j=2}^{n} \lambda_j^{*\,2}\right) \\ & + \left(\frac{1}{2}\sum_{i=1;i\neq j\neq k}^{n} \lambda_i^* \sum_{j,k=2,3}^{n-1,n} \lambda_j \lambda_k^*\right) + \left(\sum_{i,j,k,l=1,2,3,4;i\neq j\neq k\neq l}^{n-3,n-2,n-1,n} \lambda_i \lambda_j \lambda_k \lambda_l^*\right)\Bigg]\Bigg\} \end{aligned}$$

(B.16)