USE OF FRACTURE MECHANICS PARAMETERS TO CHARACTERIZE COMMINUTION

by

BIN HAO

Report submitted to the Graduate Faculty of the Virginia Polytechnic Institute and State University in partial fulfillment of the requirements for the degree of

MASTER OF ENGINEERING

in

Mining Engineering

APPROVED:

M. Karfakis, Chairman

M. Karmis, Department Head

G. Adel

E. Topuz

February, 1996

Blacksburg, Virginia

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Committee Chairman: Mario G. Karfakis

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(ABSTRACT)

This report is to investigate the use of fracture mechanics parameters (fracture toughness, specific work of fracture) to characterize comminution process. Comminution is a very important industrial process and is extremely low in efficiency. Establishment of a crushing index based on fracture mechanics principles is of great significance for improved machine design and enhanced efficiency. Single particle fracture study has been reviewed because it is considered the most elementary process in and provides the basis for comminution.

Rock fragmentation can be best described by fracture mechanics principles and concepts. The most fundamental concept in fracture mechanics is fracture toughness. Extensive review has been done on the fracture toughness application to rock fragmentation problems, and has found it has not been successfully used in comminution process. Further study is necessary to investigate the link between comminution and fracture toughens. Interrelation of fracture toughness with other rock properties has been studied. Loading rate effects on fracture toughness has also been reviewed.

Fracture toughness testing for rock materials has also been studied. The SCB (Semi-Circular Bend specimen) method has been selected for its sound analytical background and ease of operation. A experimental proposal is made based upon the survey results. Single particle fracture is proposed to be conducted on the Allis-Chalmers High Energy Crush Test System, which, compared with other test apparatus, more closely simulates the actual crushers. Detailed procedures on how to use the test system has been given in the report.

ACKNOWLEDGMENTS

I wish to express my sincere appreciation to Dr. Karfakis, committee chairman and my academic advisor, for his guidance and help throughout this study.

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Three years in the mining department at Virginia Tech has been a wonderful and unforgettable experience. I wish to thank all the people in the department who helped me with my study and life.

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INTRODUCTION

Comminution is an important and widely used industrial process. It is the most energy-intensive and extremely low in efficiency (Flavel et al., 1988). US consumption for the size reduction process was reported as 29 billion KWH of electrical energy per annum in 1981. It is also known that only 1% of the input energy is actually used for size reduction of the newly created surfaces, with the rest of energy wasted in the process (NMAB, 1981). Study of comminution fundamentals for improvement of efficiency in comminution devices is of utmost importance. Establishment of a universal crushing index will invariably be of great significance in improved machine design, selection and comparison of comminution machinery. This index should have a sound theoretical basis.

Single particle fracture is the most elementary process in comminution, and provides the basis for comminution. This paper first gives a summary of the previous results in single particle fracture study in the areas of testing methods, breakage mechanism, energy requirement, size distribution, fracture pattern, and energy input versus product size relationship.

Fracture mechanics best describes rock fragmentation process. LEFM principles and fracture toughness have found wide application in solving rock fragmentation problems. Application of fracture toughness in rock fragmentation has been reviewed. Testing methods for fracture toughness and influencing factors are then discussed.

The purpose of this study is to try to link the rock fragmentation process to fracture mechanics principles and concepts. Although some attempts have been made in the areas such as hydraulic and explosive fracturing, and tunnel boring, fracture toughness has not been successfully applied to rock crushing process. Further study in this area is necessary.

An experimental program has been proposed at the end of the paper which will try to establish a link between fracture mechanics parameters to actual comminution process. The fracture mechanics parameters (fracture toughness, fracture energy, and energy index) are determined by the SCB (semi-circular bending specimen) method which has a sound analytical background and the instrument to be utilized has been used successfully in the rock lab for fracture toughness determinations (Akram, 1991). The single particle crushing test will be conducted on a Allis-Chalmers High Energy Crush Test System to determine the energy requirement and size distribution. Compared with the previous single particle test instrument, the HECT system can more closely simulate the actual crushing process (Allis-Chalmers, 1985). Actual crushing test on a laboratory crusher will also be proposed to be conducted to verify the results.

1. SINGLE PARTICLE FRACTURE STUDY

Comminution study started early this century. Numerous papers have been published over the past century. Many of the efforts in comminution study have been focused on single particle crushing tests. Most of the single particle fracture study occurred during the past three decades. A wide range of test methods have been developed, with the aim to gain a better understanding of the various aspects which contribute to the particle breakage mechanism, and finally apply the developed theories to predict and compare comminution machine performance, and in helping the design of new devices. Single particle fracture is the most elementary process in comminution, and provides the basis for comminution (Bergstrom, 1962; Gilvarry & Bergstrom, 1963; Yashima, 1979; Krough, 1980; Bourgeois et al., 1992).

1.1 PREVIEW OF SINGLE PARTICLE FRACTURE STUDY

Numerous studies have been conducted on single particle fracture over the past three decades. Various testing methods and set-up have been devised. The testing methods usually include slow compression and impact. Specimen geometry range from spheres, discs, rock beams to irregular shaped specimen. Test set-up utilizes the pendulum apparatus, drop weight, single impact machine, free fall, MTS machine and Ultrafast Load Cell.

Fracture of single spheres is by far the most frequently used method because of its symmetry in force application and fracture pattern for easy analysis, also because point contact loading is typical of the mechanics of conventional crushers. The theories of stress state, deformation and the strain energy accumulation are all well-established (Timoshenko, 1951; Schonert, 1979, 1987; Bergstrom & Solleberger, 1962). Fracture of glass, limestone, sand-cement and various other rock spheres have been reviewed, and they all show a much similar breakage pattern and size distribution (Bergstrom, 1962; Arbiter et al., 1969; Bergstrom & Solleberger, 1962). The fracture pattern, stress distribution within the particle, spatial distribution of the fragments, energy transfer and kinetic aspects have been studied with each spherical specimen. Cylindrical specimen and rock or cement beams are also often used, especially in the impact fracture test and in the study of energy transfer (Charles et al., 1955; Lundberg, 1976; Santurbano et al., 1991; Bascoul, 1987). Three modes of load application are frequently used in single particle fracture study, (1) slow compression; (2) single impact; and (3) double impact. In single impact the particle is broken either by a falling media or by free fall of the particle onto a hard surface. From the efficiency point of view, slow compression is better than high velocity impact (Arbiter et al., 1969), but some materials are easily broken by impact than by slow compression because of their rate-hardening property.

1.2 BASIC THEORY GOVERNING PARTICLE FRACTURE

Griffith's theory provides the basis for single particle fracture study (Griffith, 1921). The theory states that failure of brittle solids is caused by the extension of the inherent flaws called Griffith flaws or cracks which are distributed within the body of the solid. When stress is applied, these cracks act as stress raisers which generate stress concentration at the crack tip. The theory has been verified by the fact that the actual strength of a material is much lower than its theoretical value. The theoretical tensile strength of materials should be close to about 1/10 of their elastic moduli, due to the presence of cracks and flaws, the measured strengths lie between $E/10^3$ and $E/10^2$. By eliminating the pre-existing cracks the tensile strength can approach the theoretical estimates (Lange, 1973).

Based on the crack theory, Griffith also established the crack extension criterion in terms of energy and stress requirements. This has laid the foundation for the development of fracture mechanics which deals solely with the material failure caused by the extension of internal or artificial cracks. When developing the energy balance equation, Griffith only considered the free surface energy as the only energy consumption. In actuality, the energy consumed in the process of plastic deformation at the crack tip is considerably greater than the free surface energy. Irwin (1947) and Orowan (1949) later modified Griffith theory by incorporating the crack-tip plastic deformation.

The original Griffith theory and criterion were developed for crack problems in tension, yet the predominant mechanism of rock breakage is compression induced tensile and shear failure of the rock body. The Griffith theory has been found not directly applicable to compressive problems. Many people accordingly have studied crack propagation in compression, based on Griffith theory which should be more useful and can be directly applied to solve rock problems.

Brace and Byerlee (1966) believed that the mechanics of propagation of a macroscopic fracture in compression may have little to do with the mechanics of the propagation of a single crack, and that the fundamental mechanism for compressive failure is the crack interaction, i.e., macroscopic fracture is formed from an array of suitably oriented cracks. Bienawski (1967) has postulated and experimentally confirmed the several stages of crack extension under compression. Kenmeny et al., (1987) have developed two models of crack propagation under compression, the axial splitting and shear faulting models.

Recent studies in single particle fracture are based on the application of LEFM (linear elastic fracture mechanics) principles and concepts. Fracture mechanics tries to give a quantitative description of the transformation of a material body from intact to a broken one by crack propagation. Rock fragmentation can be best described by LEFM. The most basic concept in LEFM, fracture toughness, has been found to be an intrinsic material property for rock materials. It characterizes the resistance which the material offers to crack propagation, it's also a measure of energy in the fragmentation process. Fracture toughness has successfully used to predict fragmentation process such rock blasting and hydraulic fracturing (Margolin et al., 1982; Roegiers et al., 1982).

Despite all the attempts to gain a better understanding of the rock fracture process, the basic mechanism governing rock fragmentation and comminution is still not very well understood. But the application of LEFM to comminution process will certainly shed some light to a better understanding.

1.3 PRODUCT SIZE DISTRIBUTION AND ENERGY-SIZE RELATIONS

The physical constitution of particulate substance resulting from comminution operations is described by means of their particle size distribution, which is often expressed as the percentage of weight finer than x (a specified particle size), and the curve of this percentage vs size x is called the particle size distribution curve, the function representing this curve is the distribution function (breakage function). The frequency curve is the derivative of the distribution curve.

Rock breakage in comminution process is closely related to the breakage function which describes its product size distribution. Particle size distribution results from the crack development within the particle, and is also closely related to the fragmentation energy and energy intensity (loading rate). Different rock fragmentation process can be represented by their final breakage function (Bergstrom, Sollenberger and Mitchell, 1962; Gilvarry and Bergstrom, 1963; Bergstrom, 1963; Arbiter, et al., 1968; Yashima and Saito, 1979; Krough, 1980; Grady, et al., 1981; Clark, 1987; Malcolm, et al.; 1988; Bourgeois, et al., 1992).

There have been numerous efforts to represent the particle size distribution in a mathematical expression. Beke (1964) has given a critical review of most of these expressions, and he argued because of the complexity of comminution process, there should exist no laws of absolute accuracy and reliability to define the mechanism of the process, and so a statistical approach is assumed for this purpose.

1.3.1 Gilvarry Crack Theory

Gilvarry theory (Gilvarry, 1961) relates the Grifftih flaw to the size distribution. According to Gilvarry, the Griffith cracks are evenly distributed within the material body. Gilvarry & Bergstrom (1963) has stated that the Griffith flaws associated with fracture of a specimen are flaws that occur within the volume of the body, on the new surfaces created within its interior and on new edges within the body. Fracture proceeds by activation of flaws in the volume of the specimen, in the fracture surfaces throughout its interior, and in the edges produced by interaction of fracture surfaces. Accordingly Gilvarry has proposed that the distribution of fragment size has the form of Poisson distribution. The fraction y, by weight or volume, passing a seive of mesh size x is given by:

$$y = 1 - e^{[\{\frac{x}{k}\} + \{\frac{x}{j}\}^2 + \{\frac{x}{i}\}^3]} \qquad (1)$$

where k= an average spacing of activated edge flaws, j^2 = an average amount of surface containing one activated surface flaw, and i^3 = an average volume corresponding to an activated volume flaw. Gilvarry theory has been found to correctly describe the size distribution of single fracture.

Parameters usually used to characterize product size distribution include the size modulus which has been found to be closely related to the flaw distribution of the material, the distribution modulus which describes the uniformity of the product, and median or mean size.

1.3.2 The Breakage Function Resulting From Single Fracture

Figure 1 shows the size distribution of glass spheres obtained by Bergstrom (1963). The Gilvarry equation indicates two peaks in the differential probability dy/dx vs. particle size curve as is obtained from the experiment. According to Gilvarry, the several humps that appear in the size distribution curve result from the fragments produced by the activation of new surface and volume flaws within the specimen. The third peak was explained to be the result of the use of experimental specimen of finite size, this peak arises from fragments produced by activation of surficial flaws on the exterior surface of the specimen. This has been experimentally confirmed using large number of rock specimens.

From the y vs. x curves it can also be seen that in the fine size range the slope of the curve is unity which, in accord with the Gilvarry equation, means that in this range the activation of edge flaws is the predominant mode of fracture.

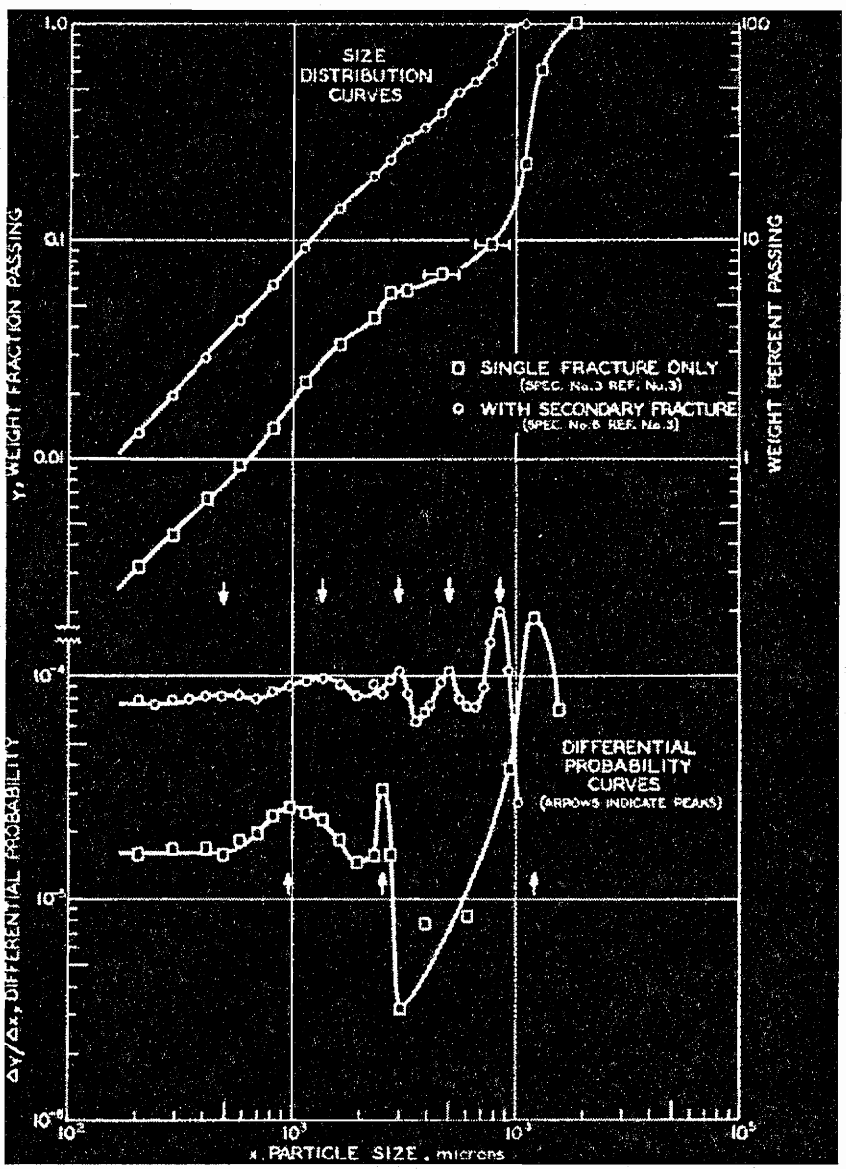


Figure 1. Size distribution and differential probability curves for two glass spheres by single fracture (in geltin) and with secondary fracture (in steel retaining ring) respectively . (after Bergstrom, 1963)

Size distribution of single spherical fracture is characterized by a residue (coarse product) with a distribution modulus of 2.5 and 5.0 as determined by Arbiter et al., (1969) for sand-cement spheres and a complement (fine product) with a fairly constant distribution modulus ranging between 0.2 - 0.3 (Arbiter, 1969). Arbiter et al., also found that the distribution modulus of the complement seems to be insensitive to both variations in energy and the method of breakage, though its size modulus is strongly dependent on energy input. Bergstrom (1963) arrives at identical results.

When log-log coordinates are used the distribution curves tend to be straight lines (Krough, 1978). It can be seen from Figure 2 that the fineness increases with energy input, but the increase only starts above a certain energy level, and this level corresponds to a crushing frequency of about 50%.

1.3.3 Energy-Size Relationship

The three laws of comminution are the basic energy-size relations which produced much controversy in the filed of comminution, yet they all have their application in a certain range. Of them Bond's theory (the third theory of comminution) is the more often used. In this theory, the work input is proportional to the new crack tip length produced in particle breakage, and equals the work represented by the product minus that represented by the feed. The crack tip length is equivalent to the square root of one-half of the surface area, and crack length is proportional to $(1/\sqrt{P} - 1/\sqrt{F})$, for practical applications, the particle diameter can be expressed in terms of 80% passing size (Yap et al., 1987),

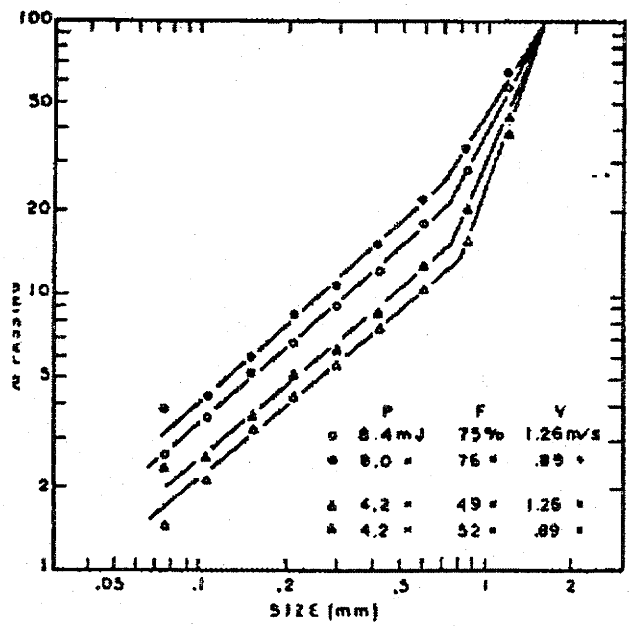


Figure 2. Size distribution of calcite at two energy levels and two impact velocities (P is the input energy level, F is the breakage probability and V is the impact velocity) (after Krough, 1978)

$$W = W_i[\frac{10}{\sqrt{P}} - \frac{10}{\sqrt{F}}] \quad (2)$$

where W= energy input to the mill, W_i = work index, P = square seive opening which 80% of the product pass, F = square seive opening which 80% of the feed pass. Bond's work index has proven to be a valuable tool in selecting and comparing grinding machines.

Application of the laws of comminution in single particle crushing has been studied by Yashima et al., (1979), who used the Lewis' general equation,

$$d[\frac{E}{M}] = -C_3 \frac{1}{x^n} dx \quad (3)$$

where E= fracture energy, M = mass of the specimen, x = particle size, C_3 = constant, n= index. In the equation when n takes the appropriate numbers the three laws of comminution are obtained. On integration Equation (3) becomes:

$$\frac{E}{M} = \frac{C_3}{n-1} \frac{1}{x^{n-1}} \quad (4)$$

Yashima's experiment gives the results in the form of

$$\frac{E}{M} = (\frac{1}{xa})^{1/a} (\frac{1}{x^{-1/a}}) \quad (5)$$

which shows that n = 1-1/a. He thus concludes that in single particle crushing under slow constant loading rate, in a coarse size range of borosilcate glass, the Rittinger's law is applicable, it also holds good for quartz, feldspar and quartz glass.

Relationships between strain energy, sphere compressive strength and particle size in single particle crushing has also been determined experimentally by Yashima,

$$\frac{E}{V} = C_6 S_t^{5/3} \tag{6}$$

and

$$\frac{1}{k_{100}} = C_7 S_t^{5/3} \tag{7}$$

where E is the stored strain energy in the particle volume V, S_t is the compressive strength, k_{100} is the product modulus, C_6 and C_7 are constants.

Bergstrom (1963) has established the following relationship between the specific fracture energy and the product size modulus,

$$\frac{E}{M_{gel}} = \frac{C_1}{k} \text{ and } \frac{E}{M_{steel}} = \frac{C_2}{k} \tag{8}$$

where C_1 and C_2 are constants, k is the product size modulus, M_{gel} and M_{steel} are the particle mass broken in gel and steel respectively. Many experimental results indicate that this relationship is more characteristic of single particle fracture.

1.3.4 Factors Which Influence Size Distribution

These are essentially the same factors influencing single particle fracture which will be discussed in detail later in this paper. They include material inhomogenity, material deformation response, mode of force application, loading rate, particle size, the level of energy input and energy intensity.

1.4 FRACTURE OF SPHERICAL PARTICLES

Particles of spherical shape are frequently used to study the stress distribution, crack development, breakage pattern, size distribution, fracture energy and kinetic energy aspects. Of them glass sphere (Bergstrom, 1962, Bergstrom and Sollenberger, 1962) is used because it is considered a fairly homogeneous and brittle material which makes it ideal for fracture testing.

1.4.1 Fracture Pattern

Many people studied the fracture pattern of spherical fractures and they all reached essentially identical results (Arbiter et al., 1969; Bergstrom and Sollenberger, 1962; Bergstrom, 1963). Bergstrom and Sollenberger (1962) conducted experiment on glass spheres and have found that no network of visible internal cracks developed prior to fracture. The cone which has been formed with the contact area as its base is pressed into the sphere, and tensile stress arises in the sphere shell causing cracks in meridional planes. The sphere was fractured into several large fragments (Figure 3), and all the larger fragments which are in the shape of lungs have substantially the same outward velocity, secondary fracture occurs with these primary fragments crushing against the retaining ring, after that not much further action occurs. The ratio of kinetic energy to the strain energy is approximately 59%. The cone from the contact area is a compacted mass of fine fragments in which numerous smaller particles are formed along with those originated

Figure 3. Fragmentation from a typical sphere. No secondary fracture was permitted. Lunes arranged according to their relative position in the sphere (after Bergstrom, 1963)

from the core of the sphere. Slow compression at room temperature tends to break the sphere into very few pieces, while impacting at high and low temperature causes meridional cracks.

Results from limestone spheres test by Santurbano and Fairhurst (1991) have shown that the fracture pattern is essentially the same as that of glass spheres (Figure 4). A cone of finely broken materials also develops below the contact zone, at the free end opposite to this cone, a larger intact cone is formed. The remaining material breaks into orange-like slices whose largest dimensions are parallel to the direction of impact.

Arbiter et al, (1969) tested the fracture of sand-cement spheres and have obtained similar results. He also finds that slow compression (1). requires half as much energy to initiate fracture as free-fall impact; (2). produces a significantly smaller amount of fine product than free-fall and double impact.

Steel and gelatin retaining rings were used in Bergstrom's (1962A) test to study the kinetic energy (and hence the secondary crushing) effects, and the resultant fragment size distribution was plotted as log percentage passing vs log particle size, the fragment size modulus was determined from this plot (Figure 1).

Results have shown that the kinetic energy of the produced fragments had a significant effect on particle fracture, as was indicated from the energy-product size relationship curve which showed that the kinetic energy possessed by the fractured fragments is large enough to cause secondary comminution. 45% of the stored strain

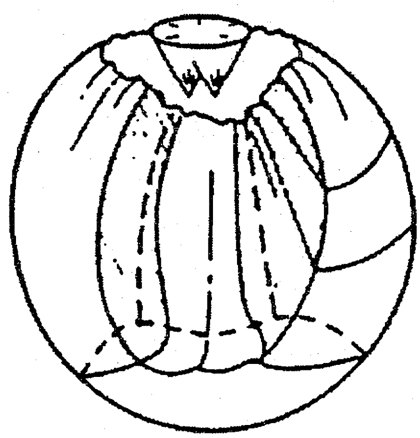


Figure 4. Cone and slice fracture pattern in an impacted sphere
(after Gildemeister and Schonert, 1972).

energy in the particle is converted into kinetic energy of the fragmented parts, and if used properly, this kinetic energy can be utilized for secondary crushing.

1.5 FACTORS INFLUENCING PARTICLE FRACTURE

The initiation and propagation of cracks within a particle are determined by the stress field, which depends upon the number and direction of the applied forces, the deformation rate, the particle deformation behavior, and the particle size and shape. Particle fracture is also influenced by the structural failure and the flaw pattern that causes stress concentrations, the crack pattern determines the size and shape of the created fragments and new surfaces. But the factors have an interacting relationship, e.g. the stress field is dependent upon the deformation rate, the material deformation behavior and material properties, meanwhile, the deformation behavior also depends on the material properties, the temperature, and the deformation rate. In interdependence scheme the state of stress plays a key role in the breakage process (Scheonert, 1979).

1.5.1 Material Properties

Material strength is an important parameter affecting fracture process. First the internal flaws in the material cause stress concentration which reduces the material strength. Secondly, Material strength increases with strain rate. Rinehart (1965) has found that there is a ten fold increase in strength under dynamic loading. Failure loads have been found by Everell (1973) to affect the selection function in particle breakage.

Brittleness which has been defined as the ratio of hardness to toughness (Marshall, 1974) can also be thought of as a material property influencing material breakage. It can be regarded as the measure of the materials' ability to absorb energy. It is obvious that materials with the high brittleness index will be easy to comminute, while those with low index values will be difficult to comminute. Brittleness offers a valuable approach to material classification for ease of comminution. Brace and Byerlee (1966) have found that materials with high brittleness index relax the strain energy by crack propagation rather than by plastic deformation. Particle britlleness also affects the spatial arrangements of the fractured products (Schonert, 1991) , because the very brittle materials such as quartz will explode at the breakage point due to the high stress energy density in relation to material strength. The fragment velocity from primary fracture is considerably higher than that of softer particles which break smoothly and the fragments move away only little and form a narrow cluster.

Material mechanical function (deformation response) is another material property affecting particle breakage, although it is heavily dependent on the loading condition. The difference between elastic and plastic deformations is essential primarily because inelastic deformation reduces the stress level and dissipates the stress energy as heat, then much more energy is needed for releasing cracks.

1.5.2 Loading Rate Effect in Single Particle Fracture

Many people have studied the effect of loading rate on the fracture pattern, energy utilization, product size distribution, and the material strength (Arbiter et al., 1969; Marshall, 1974; Yashima, 1979; Schonert, 1979; Schonert, 1987).

According to Schonert (1991), strain rate sensitivity of the material is an important aspect of loading rate influence. Some materials embrittle with increasing rate because viscous flow is suppressed. They will break more easily when stressed at a high velocity. The spatial distribution of the fractured fragments also can be influenced by the loading rate, especially for softer materials like limestone. Since the fly-off velocity for such materials is low, increasing the stressing velocity can narrow the spatial distribution and thus cause more energy loss due to internal friction.

Schonert (1972) has shown through experiment that at constant reduction ratio slow compression is much more efficient than impact in terms of energy utilization, but the differences tend to diminish at higher energy input. Arbiter et al., (1969) explained the reasons why slow compression is more efficient than impact as: (1). High velocity impact wastes energy by getting more energy into the particle than is necessary to disrupt it; (2). Not enough time is allowed for the crack to fully propagate; (3). Certain materials which exhibit plastic flow under slow compression tend to be brittle at high loading rates.

Bergstrom (1962), through experiment, found that loading rate had little effect on the energy-product size relationship for the glass spheres, but an increase in energy requirement for fracture was observed with the increase in loading rate.

Yashima et al., (1979)conducted single fracture of spheres of eight different rocks and found that at slow loading rate, not much effect is observed on the strain energy and the sphere compressive strength, but they both show a tendency to increase with increase of loading rate.

Arbiter et al., (1969), compared slow compression and low-velocity impact with spheres of sand-cement and glass, they found geometrically similar stress fields are produced by slow compression and impact, therefore similar fracture pattern and fragment distribution are resulted, but the energy requirement is drastically different, because compared with slow compression, low velocity impact required about 80% more energy to initiate fracture and about double the energy is required to produce substantially similar size distribution. Arbiter attributed the higher requirement to the following reasons. (a). With impact a great number of internal cracking takes place yet without contributing to the actual fragmentation; (b). possible higher residual kinetic energy in the fragments as compared with slow compression; and (c). energy losses into the hammer-sphere-anvil system due to vibration and to possible deformation localized at the points of contact.

Santurbano et al., (1991) have found that the peak force and degree of breakage increase with the impact velocity.

Dynamic transient loading has been studied by Lundberg (1976) using the Split Hopkinson bar method. He found that the stress pulse with the highest amplitude and the longest duration causes the highest degree of comminution. Fracture pattern has revealed that the main crack orientation is axial, parallel to the direction of stress wave propagation and to the compressive stress. This is identical to the results of static tests, but the degree of comminution is much higher in dynamic tests. Lundberg argued that the reason is that when loading quasi-statically, the most critical crack extends through the specimen a time which is very short compared to the time during which the load is applied. Whereas when loading by a stress wave, initiated cracks extend through the specimen more slowly than load propagates, thus extensive crack initiation may occur before general failure of the specimen prevents further increase of the load.

The other important finding of the test is the critical load corresponding to the rapid increase in relative energy absorption, which is defined by the author as WL/WI, WL is the energy absorbed by the specimen, and WI is the energy of the incident stress pulse. WL is obtained by subtracting the transmitted and reflected energy from the energy of the incident stress pulse. Lundberg has found that when the load acting on the specimen approaches 1.8 times the static compressive strength for Bohus granite, and 1.3 times the static compressive strength for Solenhofen limestone, there occurs a marked increase in the relative energy absorption.

In summary, the energy input, peak fracture force and degree of comminution all increase with loading rate, while the product size modulus remain unchanged. Slow compression is more efficient than impact.

1.5.3 Particle Size

There are several aspects of particle size influence on the deformation behavior of the particle (Schonert, 1972).

(a). Flaw size consideration. The flaw size decrease with decreasing particle size, therefore the stress level has to be increased to cause cracks. Hence the particle breakage strength is actually increased. Yishima et al., (1979) has experimentally determined the tensile strength to be related to particle volume as,

$$S_t \propto V^{\frac{1}{m}} \tag{9}$$

(b). Stored elastic energy consideration. The elastic energy stored inside a particle is proportional to its volume, while the energy needed for crack release is proportional to the cross section. Thus with decreasing particle size there is a greater decrease in stored elastic energy than the energy required for crack propagation. This is also equivalent to an increase in particle strength. The strain energy-volume relation has been determined from the Timoshenko's theory and the above tensile strength vs particle volume relation as (Yashima et al., 1979),

$$E = C_2 V^{-1/m} \tag{10}$$

in which C_2 is a constant characterizing the material properties.

(c). Particle size also effects the breakage function or fragment size distribution, because the crack pattern is determined by the stress field which depends on the homogeneity and the deformation behavior of the particle, yet both of these two factors change with the particle size.

1.6 ENERGY ASPECTS IN COMMINUTION

Comminution is an energy balance process. According to Griffith (1921) and Irwin (1957), the basic energy balance equation is in the following form:

$$\frac{dU}{da} \geq \gamma_e = \gamma + \gamma_{pl} \tag{11}$$

The item on the left of the equation is the energy expenditure per unit crack extension, it should be greater than the sum of the free surface energy and the crack-tip plastic energy, this sum is called the effective surface energy. The specific fracture energy is defined as the energy consumption per unit crack area, theoretically it is equal to twice the effective surface energy,

$$\overline{R} = 2\gamma_{eff} = \frac{W_f}{A} \tag{12}$$

Fracture energy is a fundamental parameter in particle fracture. Study of material's ability to absorb strain energy before fracture will provide insight into the finding of an intrinsic material property in governing the fracture process. Karfakis (1993) has proposed the concept of energy index to describe this material characteristics. The energy index (Figure 5) is defined as the ratio of stored elastic energy to the irreversible plastic deformation energy. In the fracture process, it is this irreversible plastic energy which actually helps to propagate the crack, leading to final fracture. The stored elastic energy dissipates as heat and kinetic energy of the fragmented product. This ratio will be unique for different materials, but it is affected by the loading rate because material deformation behavior is a loading rate dependent process.

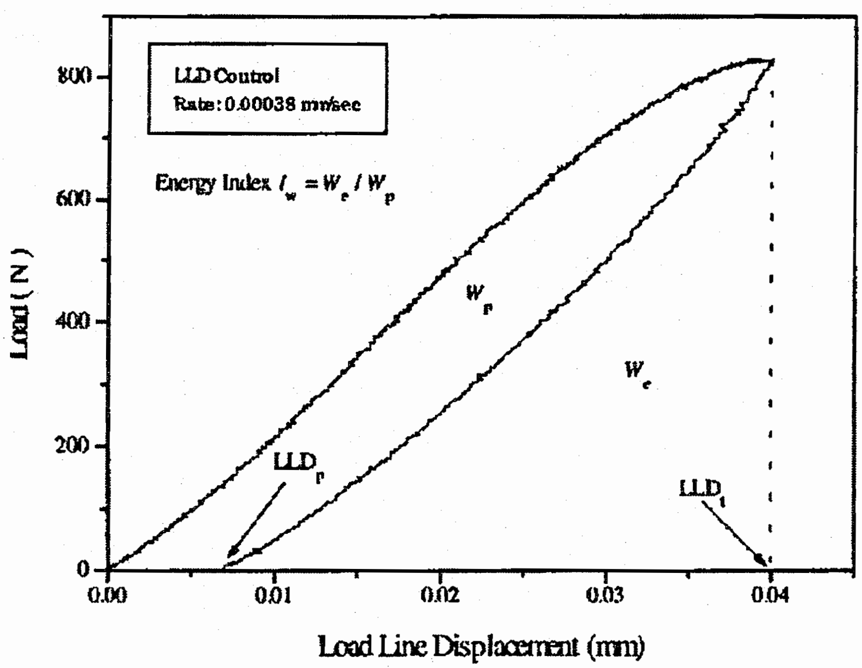


Figure 5. Energy index determination (after Karfakis, 1991)

The energy transfer process has also been studied by several workers. It has been found that this process is mainly determined by the material property and the loading rate.

Many people have studied the single particle fracture energy. In comminution specific energy of fracture is more commonly used which is the fracture energy per unit area produced. It is determined by the integration of the load-displacement curve. With the recording of force-time records, it is easier and more accurate to determine its value (Bourgeois, et al., 1992; Santurbano and Fairhurst, 1991).

Rock crushing is basically a dynamic process. The energy transfer in dynamic loading has been observed by some researchers. Dynamic fracture through Hopkinson bar by Santurbano and Fairhurst has revealed that the formation of the crushed zone in the immediate vicinity of the point of impact is the major source of energy dissipation. Banthia, et al, (1987) used drop weight impact machine to break rock particle and found that energy transfer to the particle is a time-dependent process, and energy lost is higher for stronger and stiffer materials.

In rock fragmentation, the materials' ability to absorb energy before failure is one important aspect which attracted the attention of many people. Various indexes have been proposed to characterize this ability. Charles and Rruyn (1956) tried to describe the dynamic energy absorption index in an impact breakage system, and have found that the ratio of squared strain to the original total kinetic energy measures the strain energy absorption.

To summarize, the influencing factors on energy utilization include the material properties especially the stress-strain behavior, the level of energy input, the loading rate and particle size.

1.7 SOME RECENT STUDIES OF SINGLE PARTICLE FRACTURE

Recent studies by King's group at the University of Utah have been conducted on the Ultrafast Load Cell Device (ULCD) (Figure 6). This test set-up is based on a well-established theory of mechanics. One of its outstanding feature is that a low impact energy can be applied, and the energy input can be adjusted by varying the height of the falling ball. An accurate record of force-time history during the impact fracture process can be obtained, from which the minimum force to fracture is determined. The fracture energy, defined as the that absorbed by the particle up to the point of first fracture, is then accurately calculated. Breakage function is determined by breaking a large number of rock samples under identical test conditions. Their experimental results are mainly concerned with the mass specific fracture energy (fracture energy per unit mass of rock sample) distribution.

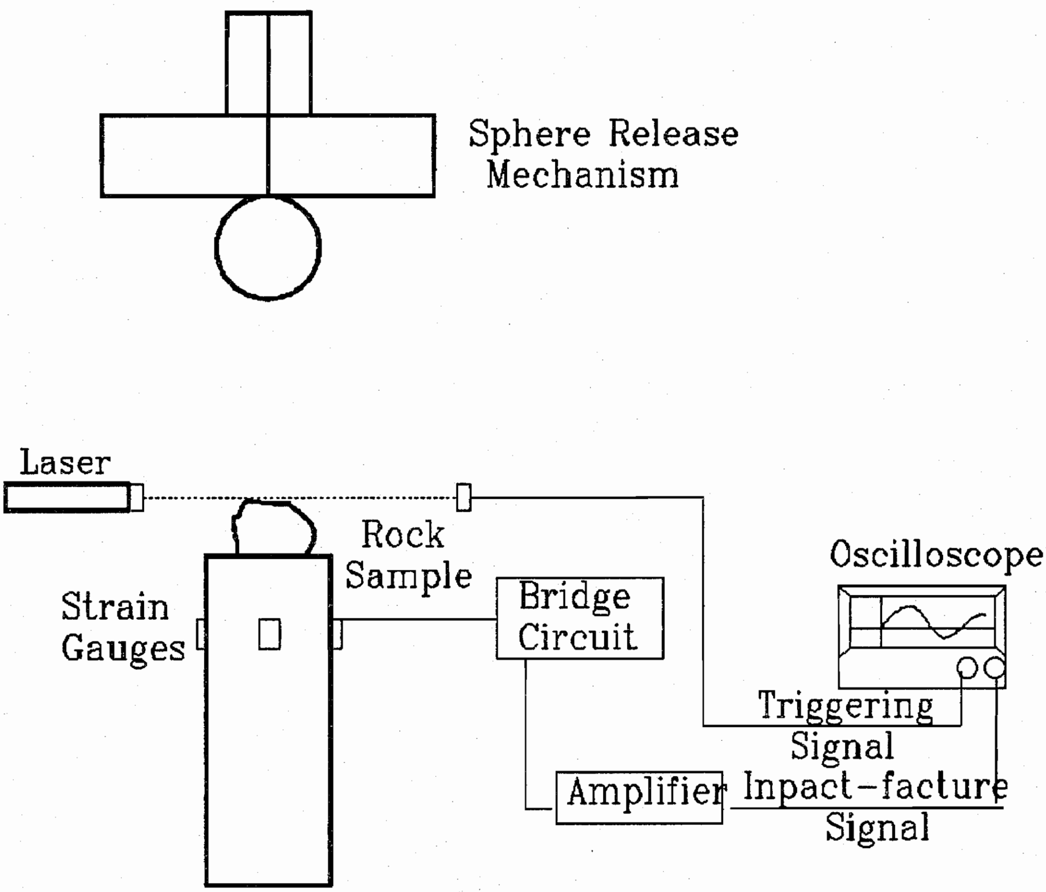


Figure 6. Schematic representation of ULCD test set-up
(after King et al., 1991)

2. SINGLE FRACTURE CHARACTERIZATION AND COMMINUTION CRITERION

Fracture criterion is one very important aspect in comminution. Although many theoretical criteria have been established in fracture mechanics, their specific application to comminution has not been very successful. Comminution criterion should be simple to determine, have sound analytical basis, and relate the fracture mechanism to the fracture mechanics principles. Of the fracture criterion study reviewed in this paper, only the grindability study relates the feed and product size to the energy requirement (Gutsche et al., 1992). Oka and Majima (1971) have found that the square of the tensile strength can be used as an index of comminution.

Gutsche et al., (1991) has arrived at the energy-reduction ratio relationship,

$$\frac{X_f}{X_{50}} = jE + c \tag{13}$$

in which X_f is the median size of the feed, X_{50} is the median size of the product, E is the energy input, c is a constant, and j is defined as the grindability which is used as an index for grinding. The grindability for various rock materials have been determined by Gutsche, and it has been found that harder materials have low grindability values and break coarse into narrow size distribution with high levels of energy input, while softer materials have high grindability values and break fine into wide size distributions with low levels of energy input.

Gutsche's grindability index can be a suitable criterion in comminution, but it is not based on a thorough understanding of the fracture mechanism, it is rather a result of empirical work. If the final comminution criterion, arrives at based on the fracture mechanism should come up with the same results, the it is purely coincidence. Oka and Majima's conclusion of using the square of tensile strength as an index of comminution also has the same problem that it is not based on the analysis of the fracture mechanism which occurs inside the particle.

Several other people have tried to characterize fracture process. Schonert (1991) has proposed that the fracture properties of particulate material should be characterized in terms of three fundamental properties, (1). the particle strength σ_p; (2). the mass specific breakage energy $E_{M,Br}$; and (3). the breakage fragment size distribution $B^{*}(x, x', E_{M,B})$.

Krough (1980) believed that the fracture characterization values must be independent of the type of comminution process, the testing to obtain these values must be easy to accomplish, and the measuring points must be precisely defined. He proposed to use (1). the distribution of strengths of particles of a given size; (2). the strength of particles as a function of its size; and (3). the size distribution of the product material. Based on these stipulations he proposed the three basic functions (the energy function, the breakage probability function, and the breakage function) to characterize comminution ability. The probability of breakage is a function of the energy input. By 100% regularity, no particles will break below a certain energy level, while all the particles will break at an energy input above the same level. In the breakage probability vs energy input cure, the energy function is defined as the energy necessary to break 50% of the particles, and it is a straight line when the energy function is plotted against particle size.

It is obvious that when studying the basic elements of crushing, the crack distribution in a brittle material should be a major element in a theory of comminution (Krough, 1980), Rose tried to make a universal comminution model by combining Griffith's law with expressions giving the probability density of the occurrence of a crack of a certain area and the relation between stress and impact energy.

Bergstrom has stated (Flavel et al, 1988) that the breakage function should be linked to the breakage energy to establish a free threshold breakage energy or comparing various comminution devices. Schonert's proposal to use specific energy is close to a solution by fracture mechanics principles. The relations between specific energy and breakage function have been determined although for limited rock types. The test methods and procedures to determine the selection and breakage function and specific breakage energy are all well-established and have good theoretical background (Hanisch, 1986). They have also been successfully used in many applications. But little is known about the elementary processes leading to selection and breakage functions which are average quantities resulting from a great number of elementary processes (Herbst and Lo, 1992). The fracture mechanics associated with the comminution of brittle particulate material is still not well understood (Bourgeois and Herbst, 1992).

The basic problems can be summarized in the three fundamental questions (Schmidt and Rossmanith, 1983), (1). How can one predict the failure load of the flawed structures? (2). What combination of load and flaw geometry parameters lead to failure? (3). What material parameters are the fracture processes governed by? Over the past few decades LEFM has been used to investigate the crack propagation with great success, the LEFM theory is based on linear elasticity and is directly related to the Griffith theory, and plastic deformation can be accounted for if it is of small scale at the crack tip.

Stress intensity factor provides the answer to the above-stated three questions. K is from a straightforward analysis of stresses at crack tip. Stress intensity factor quantifies the intensity of stress singularity at a crack tip, and from fracture mechanics, the crack will advance when the stress intensity reaches a critical value, K_C, assuming the crack tip is in a state of plane strain, this value of K_C is the plane strain fracture toughness, and this fracture toughness is a measure of material constant. Numerous publications on rapid fracture, fatigue fracture propagation, stress corrosion, and crack branching have shown the stress intensity factor to be the controlling fracture parameter. This concept has also been shown to be applicable to anisotropic materials.

Some people have recently come to realize the significance of fracture mechanics concepts in the comminution process, and research has been directed toward using fracture toughness to characterize comminution process. Bourgeois et al., (992) have stated that a useful measure of the ability of a material to withstand load without fracture is the fracture toughness. Bearman et al., (1989) have shown a very strong correlation between fracture toughness and the power consumption in a laboratory crusher and this is a good indication that this parameter could have useful application in the analysis of industrial operations. But there is no reported research on fracture toughness application to rock crushing problem. A full evaluation of this parameter as applied to crushing needs to be done. Systematic theoretical and experimental work needs to be conducted.

In summary, comminution criterion should, first of all be based on the mechanism of fracture (Clark, 1992), i.e. the presence and distribution of cracks within the particle and the criterion for crack extension. The geometry and mode of force application should be taken into account. Secondly, the criterion should relate to breakage function and relates this function to the energy requirement. It should establish a procedure to determine the free threshold energy which is used to compare comminution machines (Flavel, 1988). And finally, this criterion should be easy to determine. Based on the above discussion, intuitively, fracture toughness, is an ideal candidate for comminution criterion. Although more theoretical and experimental work should be conducted to confirm this hypothesis, which is the purpose of this paper.

3. FRACTURE TOUGHNESS APPLICATION TO ROCK FRAGMENTATION PROBLEMS

In light of the above discussion, it is necessary to review the fracture toughness application to rock fragmentation problems. This is a rather recent event. This review will hopefully provide some clue as to the comminution characterization using fracture toughness.

Fragmentation of the rock material can be best described using fracture mechanics concepts. Fracture mechanics aims to give a quantitative description of the transformation by crack growth of an intact structural component to a broken one. In its most basic form, it relates the maximum permissible stress to the size and location of a crack. Fracture toughness characterizes the resistance which a material offers to propagation of cracks, a concept commonly used to predict the onset of unstable crack growth. It is also a measure of the energy needed for crack propagation.

Fracture toughness application in rock mechanics started only recently from the later seventies. Most of the published papers appear in the US Symposium on Rock Mechanics starting from the eighties. The ISRM publication (1988) summarizes some of the applications, which are (1) A parameter for classification of rock materials; (2) An index of fragmentation process such as tunnel boring and model scale blasting; (3) A material property in the modeling rock fragmentation like rock cutting, hydraulic fracturing, gas driven fracture, explosive stimulation of gas well, radial explosive fracturing, and crater blasting as well as stability analysis. Some of the above applications are summarized below.

Fracture toughness, as a measure of energy in comminution, has been used by Ingraffea et al., (1982) to predict the performance of TBM machines. SR specimen is used to evaluate the fracture toughness of a substantial number of different rocks from three ongoing TBM projects. The results are compared with the obtained toughness values from other test methods. The fracture toughness values are also compared with other strength measures such as uniaxial tensile and compressive strength, which shows that the K_{IC} variability is smaller than that of other strength measures. A data store for fracture toughness and TBM performance values will be set up to establish the link and attempt to use fracture toughness as an index characterizing machine performance.

In hydraulic fracturing, Roegiers et al., (1982) performed testing using Short Rod and Notched Burst methods on cores from downhole to obtain fracture toughness, which, along with the measured tensile strength, will enable to determine the maximum horizontal fracture stress. In modeling crack propagation for explosive stimulation of gas well, Travis and Davis (1980) have used fracture toughness as a crack extension criterion in the model. In radial explosive fracturing using propellant, Warpinski et al., (1979) calculated the resultant stress intensity factor from the gas pressure and crack closing, then used the critical value (fracture toughness) to determine the final crack length penetrating radially into the wellbore.

Blasting is another filed in which K_{IC} has begun to find wide application (Margolin and Adams, 1982; Rustan, 1983).

Guo (1990) used fracture toughness to predict drilling performance. The analysis of rock breakage mechanism in drilling suggested that rock cutting performance should be linked to fracture properties, i.e., rock fracture mechanics. He then measured the values of six rocks, these values are then correlated with the penetration rates measured on the same rock. A general trend has been observed between the penetration rates and rock fracture toughness. The penetration rates decreases with increasing toughness. Similar relations have also been observed with other rock properties, i.e., uniaxial compressive strength and Brazilian tensile strength.

It can be concluded that there will be many areas in rock fragmentation in which fracture toughness will find wide application. This is an area of great potential and worth further study.

4. INTERRELATION BETWEEN ROCK FRACTURE TOUGHNESS AND OTHER PROPERTIES

The fracture toughness concept offers a new perspective of viewing the strength behavior of brittle materials. Strength alone can no longer describe the failure mechanism. The nature of fracture toughness as an intrinsic material property has led some people to study the interrelations between its value and other strength measures. In characterizing rock fragmentation process, the commonly used rock properties are the compressive and tensile strength, the rock hardness and specific energy of breakage. Fracture toughness has also become widely used in recent years. It is of interest to study the interrelations between the fracture toughness and other rock properties, if any.

Fracture toughness, as a critical value of stress intensity factor, is derived from the crack-tip stress state. Thus toughness values should be intimately related to the characteristic stress value at the crack tip. When the actual stress resulted from stress concentration due to the crack reaches a critical value, the materials tensile strength, crack propagation is initiated. This critical stress is also called fracture stress, and is a material property. The tensile strength can be predicted by the critical stress intensity factor using a penny-shaped crack.

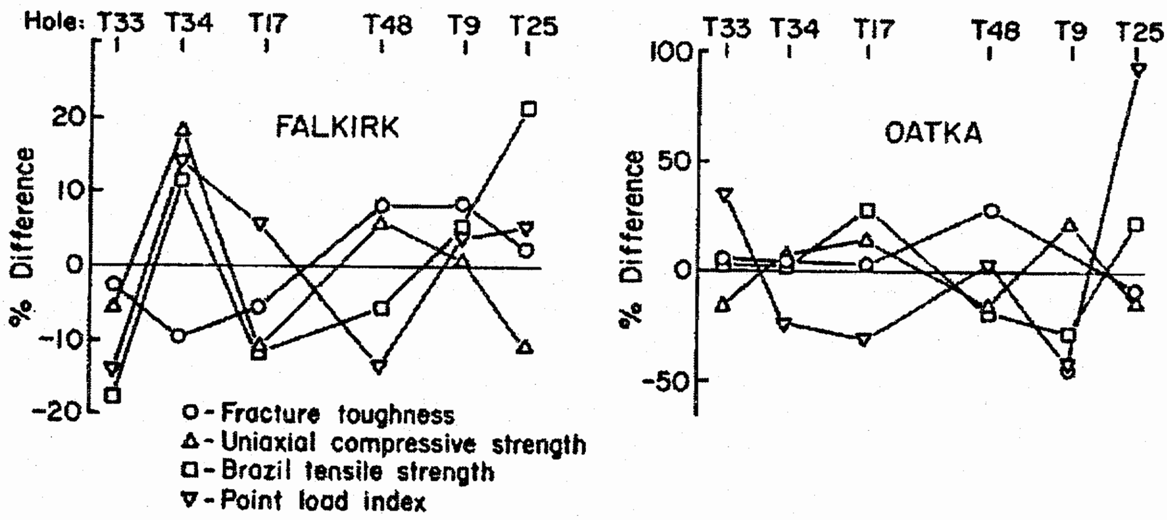
$$\sigma_c = \frac{\pi}{2}\frac{K_c}{\sqrt{c_0}} \tag{14}$$

Likewise if the size of the crack and the fracture stress are known, fracture toughness can be calculated from this equation.

The relations for metallic materials seem to have some general trend. Dowling (1993) has quoted the opposite trend of fracture toughness versus yield strength for AISO 4340 steel The yield strength increases while the fracture toughness actually decreases. Actually such an opposite trend between yield strength and fracture toughness is fairly common in metallic materials. Low strength in a tension test is usually accompanied by high ductility and also by high fracture toughness, while high strength is often related to low ductility and low fracture toughness (Landgraf, 1992).

Not much study has been done on fracture toughness as related to other properties for rock materials. Ingraffea, et al., (1982) have compared the fracture toughness values of Indiana limestone from the Short Rod specimens with the point load strength, uniaxial compressive strength and the indirect tensile strength from the Brazilian test. The variability of these parameters along the tunnel axis is plotted (Figure 7), which shows that while a general trend exists between the toughness and other measures, these trends are not consistent. No obvious conclusions can be drawn as to their relationship. In general, the variation of fracture toughness is less than that of other measures.

Contrast to the findings of this study, Whittaker et al, (1992) have studied the interrelations between the fracture toughness and other rock properties, such as hardness, strength and flexture rigidity. Statistical techniques have been utilized on the available fracture toughness data collected from the results of more than fifty previous people. The



Fogure 7. Comparision of variability of strength
(after Ingraffea et al., 1982)

rock samples include basalt, coal, dolostone, granite, limestone and sandstone. They have arrived at a linear relationship between the fracture toughness and hardness, uniaxial tensile and compressive strength,

$$K_{IC} = a + b \times Y \tag{15}$$

where a & b are the regression coefficients for the different parameters of Y which is the material properties (hardness, strength). Figure 8 shows one of the interrelation plot of fracture toughness versus compressive strength.

In the study of fracture toughness as applied to drilling process, Guo (1990) has established some of the interrelations between fracture toughness and uniaxial compressive strength and Brazilian tensile strength, and found that the general trends are that the higher compressive and tensile strengths are accompanied by a higher fracture toughness, and vise versa. The result indicates some linear relationship between toughness and the strength measures. Guo's results agree well with that of Whittaker. Another result in Guo's experiment is that the a closer correlation exists between the fracture toughness and Brazilian tensile strength than between the toughness and the uniaxial compressive strength. Guo explained the reason may be that the uniaxial compressive strength is an index-like property; the measurement involves some complexities such as multiple failure mechanism and end effects. Fracture toughness, on the other hand, is an intrinsic property determined by controlled single crack propagation and based on firm failure analysis, the Brazilian test is based on relatively thorough failure analysis, and is close to the fracture

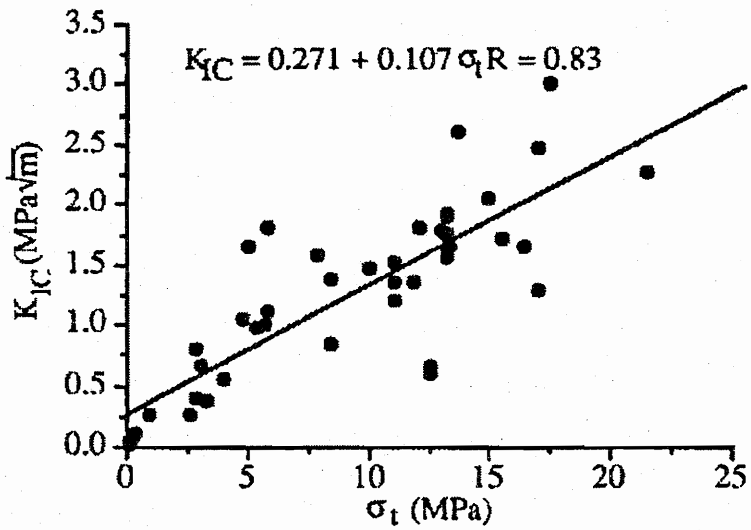


Figure 8. Correlation of mode I fracture toughness with uniaxial tensile strength (after Whittaker et al., 1992)

toughness measurement in nature. Therefore, the Brazilian tensile strength is more comparable with fracture toughness. This is essentially the same explanation that Ingraffea (1982) has given to account for the more scattered data of compressive and tensile strength than those of fracture toughness.

5. LOADING RATE EFFECT ON FRACTURE TOUGHNESS

Strain rate has been found to be an important factor influencing rock strength. Green and Perkins (1968) have found rock strength increases with increasing strain rate. Chong et al., (1980) has found that for oil shale,

$$\sigma_u = A_0 + B_0 \log \varepsilon \tag{16}$$

where σ_u is the ultimate compressive strength, A_0, B_0 are regression coefficients, and ε is the strain rate. Strain-rate dependency failure mechanism has also been proposed by Chong to explain the observed behavior.

The strain rate effects on fracture toughness is not well understood. Some studies have shown that loading rate does not have a significant influence on fracture toughness (Rummel and Winter, 1980; Costin, 1981; Winter, 1983, Ouchterlony, 1983). Ouchterlony has argued that the loading rate is not very critical in the fracture toughness testing of rock. Others have found that loading rate is an important parameter influencing the measured value of fracture toughness. Kemeny and Cook (1987) believed that the strain energy release rate G depends on the loading rate. From the standpoint of the formation of FPZ, an increase in loading rate should result in a decrease in fracture toughness, especially at high loading rate such as in dynamic fracturing, because the fracture process takes place so fast that it does not allow enough time for the FPZ to fully develop. As a consequence smaller value of fracture toughness should be observed (Whittaker, 1992).

The effect of loading rate on toughness for metals has been observed by Dowling (1993). A higher rate of loading usually lowers the fracture toughness of ASTM A572 steel (Figure 9). But from the plot the effect between slow ($\varepsilon \approx 10^{-5}$ 1/s) and dynamic loading ($\varepsilon \approx 10$ 1/s) is not pronounced. It has generally accepted that fracture toughness for metals decreases with increasing loading rate, whereas. yield strength shows the opposite effect (Figure 10). Shah, (1987) has observed that for concrete if fracture toughness is expressed using the definition and test methods developed for metals then K_{IC} increases with compressive strength (Figure 11). This is contrary to the generally accepted observation that high strength concretes are more brittle, that is, less tough, and he believed the problem lies in the test methods, the test methods for metals are not completely applicable to concrete and rock materials. The way to define K_{IC} which is more appropriate to concrete and rocks should also be set apart from that for metals.

Costin (1981) has done some dynamic testing on oil shale using bend specimen. He has found that the dynamic test results from the small 3-point bending specimens (width = 25 mm) are in agreement with the results from static tests on the largest specimen (width = 100 mm), but are somewhat larger than the results of static tests on the same size specimens. The apparent fracture toughness increased by approximately 30% from the static to the dynamic values from the tests on small specimen (width = 25 mm). If the Charpy test (impact test) results are included, the increase would be slightly greater. Similar rate effects have been observed in other types of tests on oil shales (Lipkin et al.,

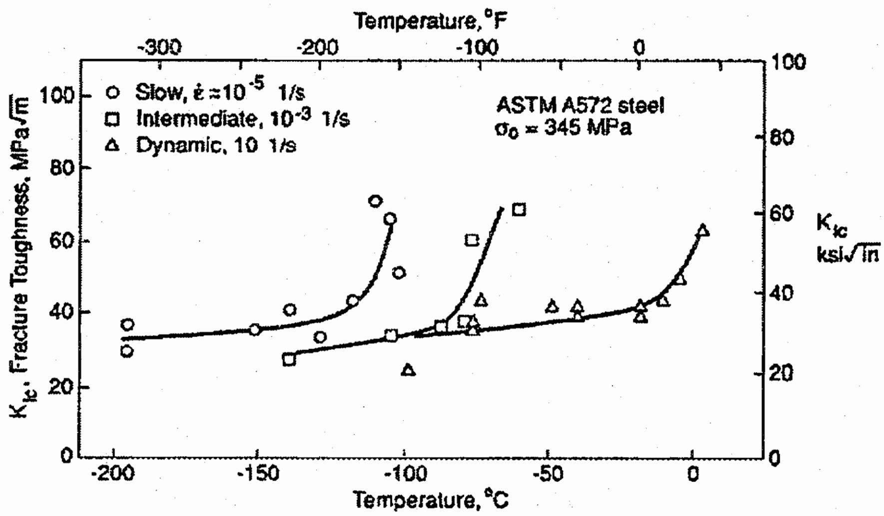


Figure 9. Effect of loading rate on the fracture toughness of ASTM A572 steel (after Dowling, 1993)

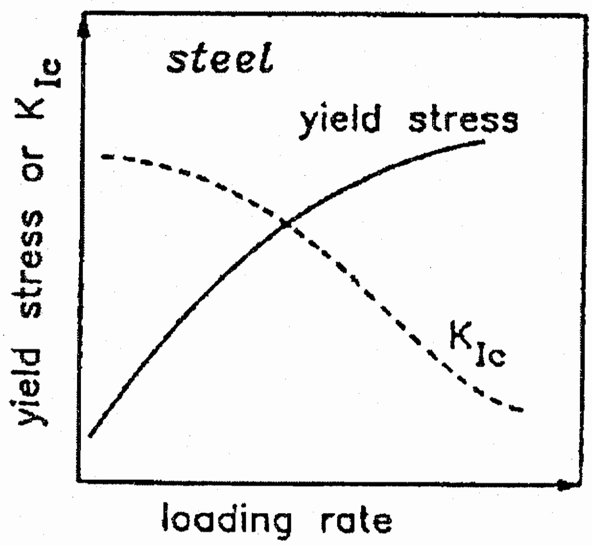


Figure 10. Effect of loading rate on fracture toughness and yield stress of metals at constant temperarure (after Shah, 1987)

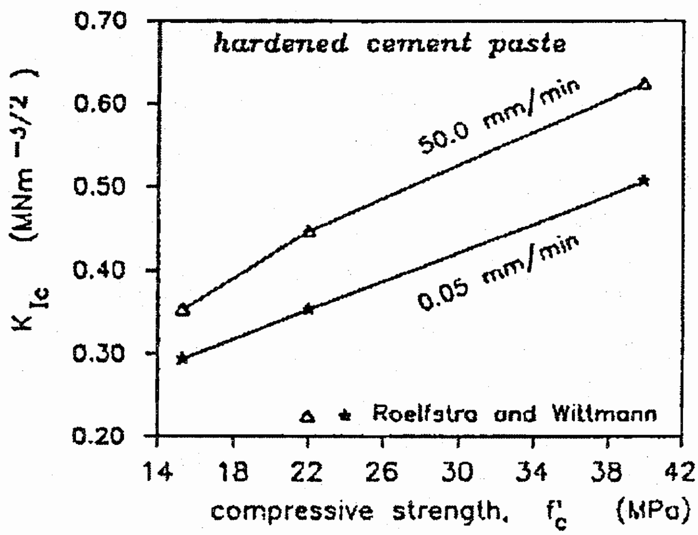


Figure 11.Effect of compressive strength and loading rate on fractrue toughness (LEFM)of hardened cement paste (after Shah, 1987)

1979; Olsson,) as well as with other types of rocks (Donath & Fruth, 1971). Thus the dynamic loading significantly reduces the size of the crack tip process zone or perhaps alter the actual fracture mechanism, leading to an increase in the apparent fracture toughness in finite sized specimens.

Lim et al., (1994) has found through tests on SCB specimen that within the range of 0.01-0.05 mm/min. fracture toughness of Johnstone (a synthetic material much resembling mudstone) increases with loading rate, but the effect depends on material properties. The increase in fracture toughness is much more marked for stronger materials than for weaker materials. In the test with increasing saturated water content, the apparent fracture toughness exhibits less dependence on the loading rate. This is believed by Haberfield and Johnston to be largely due to the change in pore-water pressures in the crack tip yield zone. The high tensile stress produced around the crack tip causes negative pore-water pressures to be created. In the stronger specimens, the negative pore-water pressures are unable to dissipate quickly enough at high loading rates due to its lower porosity. This leads to an apparent increase in fracture toughness. On other hand, weaker specimens have a relatively greater porosity which allows the negative pressure to dissipate quicker. Thus less dependence on the loading rate would be observed for the weaker specimens.

In conclusion, the mechanism of loading rate influence on fracture toughness is not clear. Contradicting results have been obtained from limited experiments. Formation of process zone seems to govern the effect of loading rate, but results from experiments contradict the explanation. More complete tests need to be conducted to arrive at a sound conclusion..

6. BASIC CONSIDERATION FOR FRACTURE TOUGHNESS TESTING OF FOR ROCK MATERIALS

6.1 DEVELOPMENT OF ROCK FRACTURE TOUGHNESS TESTING

With the development of fracture mechanics principles application to rock problems, test methods for rock fracture toughness has begun to mature. Fracture toughness of a wide variety of rocks have been obtained.

The fundamental goal of fracture mechanics testing is to obtain a reproducible and representative value of the fracture toughness of a material. Since this value is an intrinsic material property it should not depend on the testing methods, specimen size, crack length and loading configuration. Yet this can only be achieved by the proper designing of the testing procedures with the properly prepared specimen with the proper size.

The early attempted methods of using ASTM-E399 standards (Schmidt, 1976, 1977; Whittaker, 1993)) have now largely abandoned due to its inadequacy to rock materials (Ouchterlony, 1988). Specimens of core shape have received the dominant attention fro the past two decades (Ouchterlony, 1983; Barker, 1977; Senseny, 1984; Chong & Kuruppu, 1984). Testing methods for rock materials have begun to mature. The basic requirements for rock material testing have been studied by many people (Schmidt, 1979; Costin, 1981; Ouchterlony, 1982; Barton, 1987). In 1988 the standard methods have been proposed by ISRM (ISRM, 1988). Other test methods have also been put forward (Karfakis, 1990; Guo, 1992; Atkinson, 1980).

6.2 BASIC REQUIREMENTS IN ROCK FRACTURE TOUGHNESS TESTING

6.2.1 Specimen Size Effect

Of all the factors affecting measured values of fracture toughness, specimen size is probably the most important and the most studied. Size dependency of fracture toughness for rocks have been observed by many people. The general trend is that the apparent fracture toughness increases with crack length until a platen value is reached indicating the true fracture toughness value, independent of specimen size (Schmidt, 1976; Schmidt and Lutz, 1979; Costin, 1981). It is generally agreed that the size-dependency of apparent fracture toughness is mainly due to the crack tip non-linear region. The size requirement is to assure the measurement of fracture toughness in the plane strain condition, and to assure that the crack tip non-linear region is sufficiently small in comparison with the smallest characteristic dimension of the specimen, thus small scale yielding is satisfied. Since plane strain or stress condition does not affect the fracture process zone (Schmidt 1980), hence the size and shape of FPZ will not change whether the FPZ is located on the specimen free surface or deep in the specimen. Thus directly from the ASTM-E399 standards for metals, the size requirement for rock materials becomes:

$$\begin{matrix} a \\ W-a \end{matrix} \geq 2.5 \cdot [\frac{K_{IC}}{\sigma_t}]^2 \qquad (17)$$

In the equation the yield strength σ_y for metals has been replaced by the ultimate tensile strength σ_t of rocks. a is crack length, W is the width of specimen. Schmidt and

Lutz (1979) have found this size requirement applies to Westerly granite (Figure 12). They also have experimentally confirmed that specimen thickness has no influence on the measured fracture toughness (Figure 13). The minimum specimen thickness should be greater than the microcracking size.

$$B \geq r_{mc} = \frac{27}{32\pi} \cdot \left(\frac{K_{IC}}{\sigma_t}\right)^2 \approx 0.269 \cdot \left(\frac{K_{IC}}{\sigma_t}\right)^2 \tag{18}$$

where r_{mc} is the size of the microcracking zone, B is specimen thickness. A number of investigators have found that the factor of 2,5 is somewhat conservative for rocks. Chong and Kuruppu (1985) has determined the factor to be 2.0 for oil shale using SCB specimen.

$$\left.\begin{matrix} a \\ W-a \end{matrix}\right\} \geq 2.0 \cdot \left[\frac{K_{IC}}{\sigma_t}\right]^2 \tag{19}$$

Lim et al., (1994) has found no discernible difference in apparent fracture toughness with SCB specimen of 55 to 144 mm in diameter. Whereas results from other methods show a size dependency (Schmidt and Lutz, 1979 with Short Rod, Haberfiled and Johnston, 1990 with Single Edge Cracked Beam under 3-point bending). Sun and Ouchterlony (1986) found that the apparent fracture toughness is more sensitive to the ligament length than to the crack length. Lim's test on soft rock revealed that the remaining ligament does not significantly affect the apparent fracture toughness. The smallest ligament length of 14.7 mm is much smaller than observed from other methods (30 mm from Haberfield and Johnston with SECB). Since the minimum ligament length required for determining valid fracture toughness provides an indication of the process zone size, Lim thus concluded that the SCB technique appears to induce a smaller process

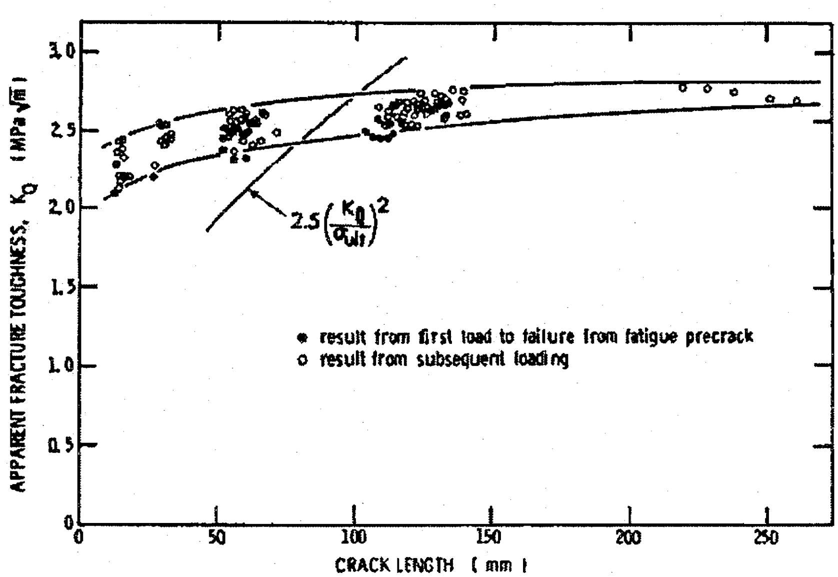


Figure 12. Apparent fracture toughness of Westerly granite versus crack length for all compact specimens (after Schmidt and Lutz, 1979).

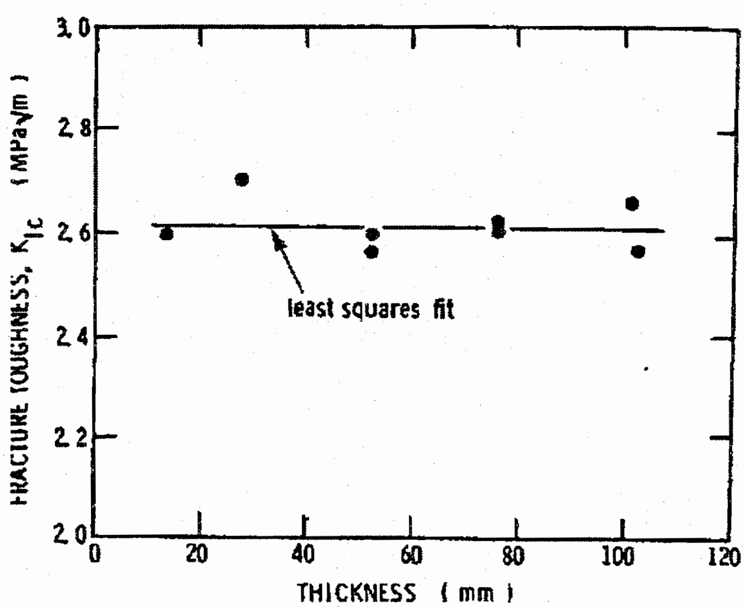


Figure 13. Effect of specimen thickness on fractrue toughness of Westerly granite (after Schmidt and Lutz, 1979).

zone size compared with the SECB technique. Therefore considerably smaller specimens can be employed to obtain valid fracture toughness values.

For Short Rod specimen with chevron notch, Yi et al., (1991) have found that (1) The Level I fracture toughness is independent of specimen size when the specimen size satisfies the following relationship,

$$D \geq 1.25 \cdot (\frac{K_{SR}^{c}}{\sigma_t}) \tag{20}$$

where D is the specimen diameter, K_{SR}^{c} is fracture toughness of SR specimen with a chevron notch, and σ_t is the uniaxial tensile strength of the material. (2). The Level II fracture toughness is independent of specimen size.

The size of the specimen is also influenced by the grain size of rock materials. Nolen-Hoeksema and Gordon (1987) have found that the minimum specimen size to grain size ratio to be in the range of (20-40):1 for a valid fracture toughness value.

Generally, size requirement for test methods using ASTM methods for metals are applicable to rocks using same methods, although smaller size requirements are frequently obtained. The size requirement for method using core-based specimen varies with test methods and specimen configuration. Since no theoretical basis exists for size limitation, experiments should be done to determine the size requirement for each method.

6.2.2 Fracture Toughness Evaluation

The apparent fracture toughness K_Q is calculated using the critical applied load P_Q and critical crack length a_c.

$$K_Q = f(P_Q, a_c) \tag{21}$$

The calculated results of K_Q cannot be considered as a fracture property of the material since its measurement does not take into account the effects of crack tip non-linearity inherently existing at the crack tip upon unloading.

6.2.2.1 Plasticity Factor

Because sometimes it is impractical to employ sufficiently large specimen to meet the size requirement, sub-sized specimens have to be used. Thus the small scale yielding is not satisfied. The measured fracture toughness is adjusted by using the non-linear characteristic of the P vs LPD records resulting from cyclical loading and unloading the notched specimen (Barker, 1977, 1979; Ouchterlony, 1986). The chevron-notched Short Rod specimen is the method that uses the plasticity correction factor. The plane strain fracture toughness K_{IC} is obtained by the following relation:

$$K_{IC} = \sqrt{\frac{1+p}{1-p}} K_Q \tag{22}$$

6.2.2.2 Crack Resistance Curve (R-curve) Method in Evaluating K_{IC}

The specific work of fracture $\overline{R}$ is defined as,

$$\overline{R} = 2\gamma_{eff} = W_f / A = \int F \cdot d\delta_F / A \tag{23}$$

The fracture energy consumption rate R is obtained through,

$$R = \frac{d}{da_c}(W - E) \tag{24}$$

Relation between R and $\overline{R}$ is expressed as,

$$\overline{R} = \frac{1}{A}\int_0^A R \cdot da_c \tag{25}$$

Figure 14 and Figure 15 show the specific work of fracture and fracture energy consumption rate curve against dimensionless crack length. The plateau level in the $\overline{R}$-curve is considered a material property. Plateau level in the R-curve is also judged the most characteristic of R-value for rock (Ouchterlony, 1983). The ideal linear-elastic and isotropic materials will have a bilinear R-curve with a flat plateau. Since the R-curve is a material property independent of starting crack length a_0 the crack-extension resistance in terms of stress intensity factor is expressed as $K_R(a - a_0)$. The crack resistance is then compared with the crack extension force $K_I(a,P)$ curves, the unique crack extension force curve that develops tangency to the crack resistance curve defines the critical load, which is used to calculate the fracture toughness (Figure 16).

6.2.2.3 J-integral Method

From the NLFM (non-linear fracture mechanics), J-integral is an extension of LEFM analysis to account for inelastic behavior and is a governing fracture parameter in those

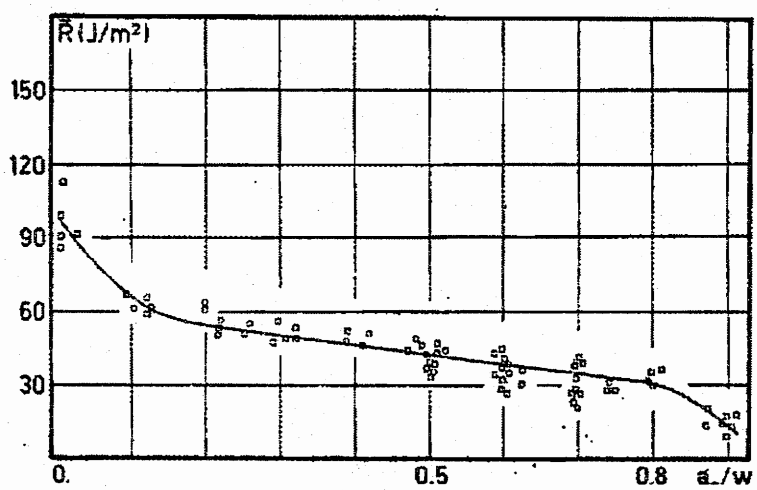


Figure 14. Specific work of fracture $\overline{R}$ for SENB of Chelmsford granite (after Ouvhterlony, 1983)

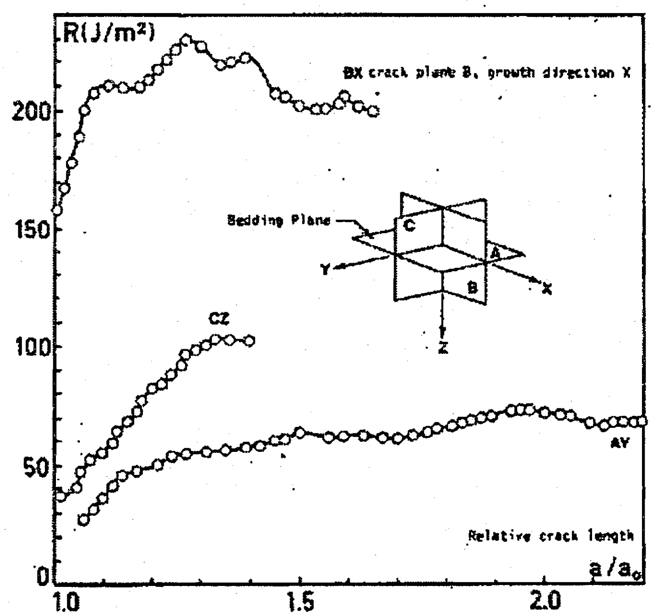


Figure 15. Fracture energy consumption rate R for Indiana limestone (after Ouchterlony, 1983)

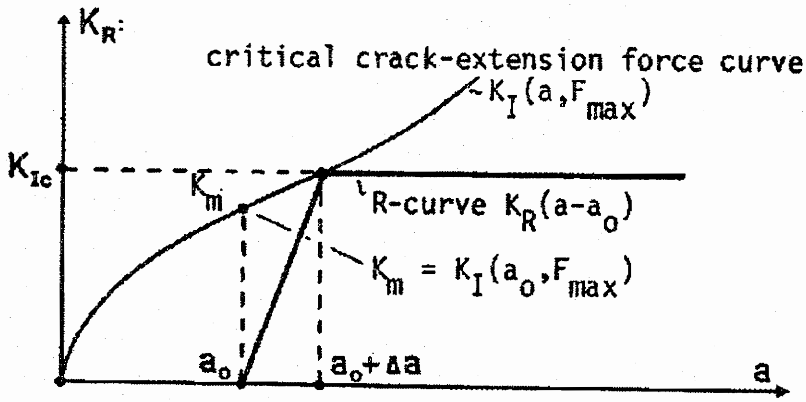


Figure 16. Simple bilinear R-curve and critical stress intensity factor curve
(after Ouchterlony, 1983)

materials exhibiting substantial non-linear characteristics. Ouchterlony (1983) has given a very detailed description of the use of this method.

6.2.2.4 Direct Use of Apparent Fracture Toughness

With subsized specimen, Ingraffea (1978) proposed to use the apparent fracture toughness K_Q directly in application rather than resort to the more complicated methods such J-integral. The basis of this direct use of K_Q is that K_{IC} is often used even when plastic process zone exists at crack tip. The presumption of utilization of apparent fracture toughness is the crack length is the dominant factor affecting the decrease in apparent fracture toughness in the region where the crack length can no longer be considered large compared to the size of the inelastic zone. Or it can be equivalently assumed that the apparent fracture toughness versus crack length relation is a material property curve, where K_{IC} is the limiting value of K_Q. His experimental results on Indiana Limestone using five different specimen and crack configurations confirmed his assumption.

7. COMMONLY USED METHODS FOR ROCK FRACTURE TOUGHNESS DETERMINATION

7.1 ISRM STANDARD TEST METHODS

Fracture toughness testing for rocks has utilized a wide variety of specimen types and methods. The resulting values are generally not comparable (Ouchterlony, 1982; Barton, 1982; Karfakis et al., 1986), implying that the fracture toughness values thus obtained do not represent a material property. In respect of the features of rock materials and the availability of rock samples, it has been found that core-based specimens with chevron notches have numerous advantages over other forms for determining fracture toughness (Ouchterlony, 1986). As a result the ISRM standard testing methods have been established based on core-based specimen of round bar and short rod.

The ISRM suggested methods, together with the SCB specimen, can also be used to determine the fracture toughness along different orientations on a single core (Chong et al., 1987) (Figure 17).

The required formulas to calculate fracture toughness are based on LEFM as well as NLFM principles, assuming isotropic material behavior.

From this literature review, the Short Rod specimen with chevron notch is probably the most widely used (Ingraffea, 1982; Bubsey et al., 1982).

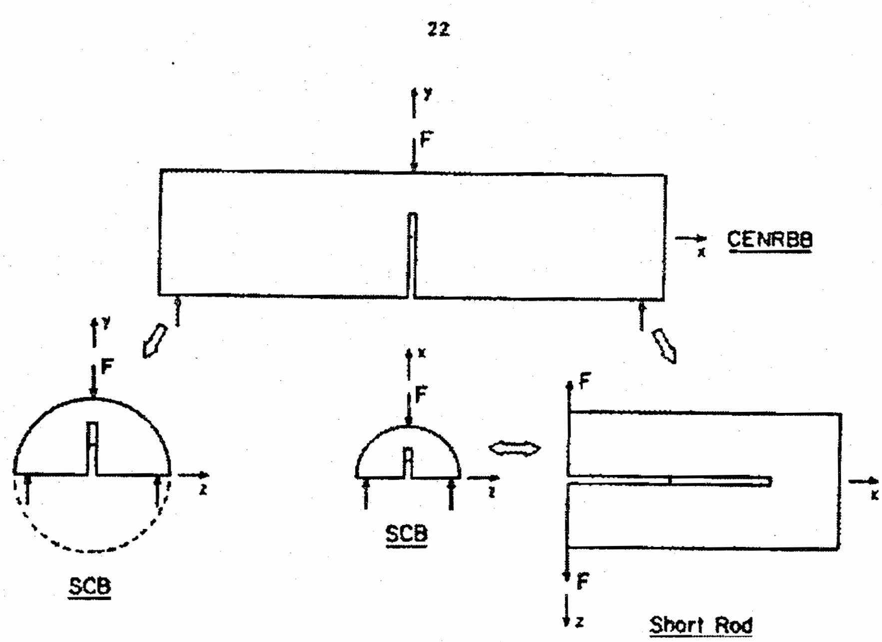


Figure 17. Complete fracture toughness characterization from a single core (after Chong et al., 1987)

7.1.1 Chevron Notch

In fracture toughness testing, specimen configurations which exhibit some degree of crack growth stability offer certain advantages over the more conventional specimens. Chevron notch possesses such characteristic (Figure 18). The V shape generates a relatively long period of stable crack growth under increasing load before the point at which the fracture toughness is evaluated. In such a way a sharp natural crack is automatically formed in the specimen and the crack resistance of the material should become fully developed with the initial crack growth. Barker (1977) has given a theoretical analysis of chevron notch to demonstrate the stable crack growth. The critical crack length a_c has been found to be independent of specimen material properties, and only dependent on the specimen geometry and loading configuration.

With the use of chevron-notched specimen, no precracking is needed, because the V-shaped chevron notch results in the crack front to be gradually increased as the crack propagates. The applied load has to increase for any further crack propagation (Figure 16).

The chevron shape in a specimen also allows the stress intensity factor to assume a minimum (under constant load) after a prescribed crack growth which is indicated Y_{kmin} in the fracture toughness evaluation formula. The crack growth process before the minimum is reached is therefore stable and serves as a means to create a sharp crack front (Yi, *et. al.*, 1991).

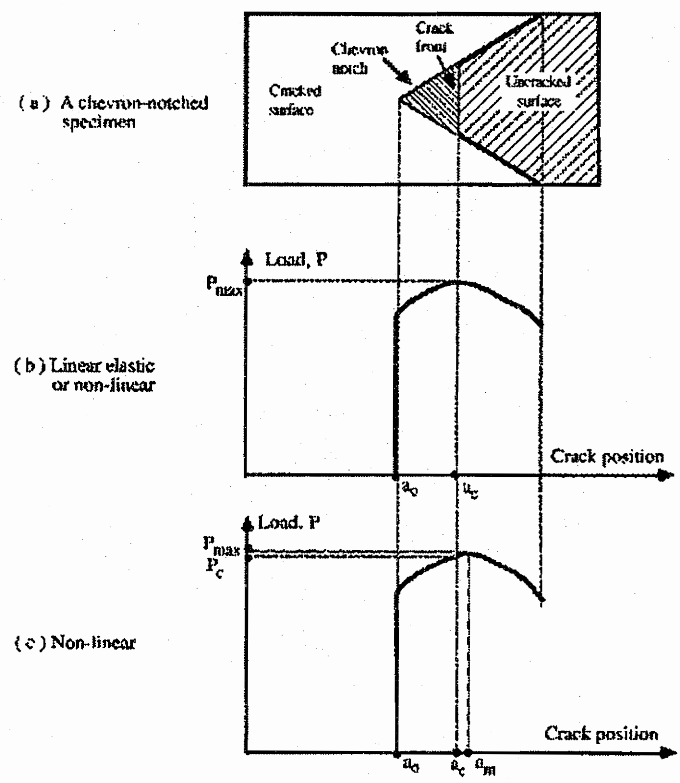


Figure 18. Chevron notch and load-crack front relationship
(after Whittaker, 1992)

7.1.2 The Chevron Notched Short Rod Method (SR)

The SR method was introduced by Barker (1977). The specimen configuration and the complete set of formulas have been given by ISRM (1988) and Ouchterlony (1986, 1988).

The crack extension force curves for SR specimen are concave upwards with a minimum at a specific crack length, $a = a_c$. (Figure 19). Before this critical length under constant load, K_I decreases with increasing crack length. Beyond this critical length K_I increases with crack length, thus the crack extension tends to become unstable. When the material is assumed to be brittle, then a flat upper R-curve is exhibited. The tangential point of the R-curve to the K_I curves the critical condition at which the applied load , P, reaches a maximum corresponding to a_c and the dimensionless stress intensity factor Y_λ^C reaches a minimum. Therefore the fracture toughness for a chevron-notched short rod specimen, K_{SR}, is calculated from the maximum load, P_{max}, and the minimum dimensionless stress intensity factor $Y_{K\,min}^C$ as follows:

$$K_{SR} = \frac{P_{max}}{D^{3/2}} Y_{K\,min}^C \tag{26}$$

There are several approaches to evaluate the dimensionless stress intensity factor $Y_{K\,min}^C$, which is a constant independent of material type for a given geometry. They all generally yield accurate results for $Y_{K\,min}^C$ of around 24.0 (Barker, 1977; Bubsey *et al.*, 1982; Beech and Ingraffea, 1982; Wang *et al.*, 1984; Newman, 1984: Hsiao and Rabaa,

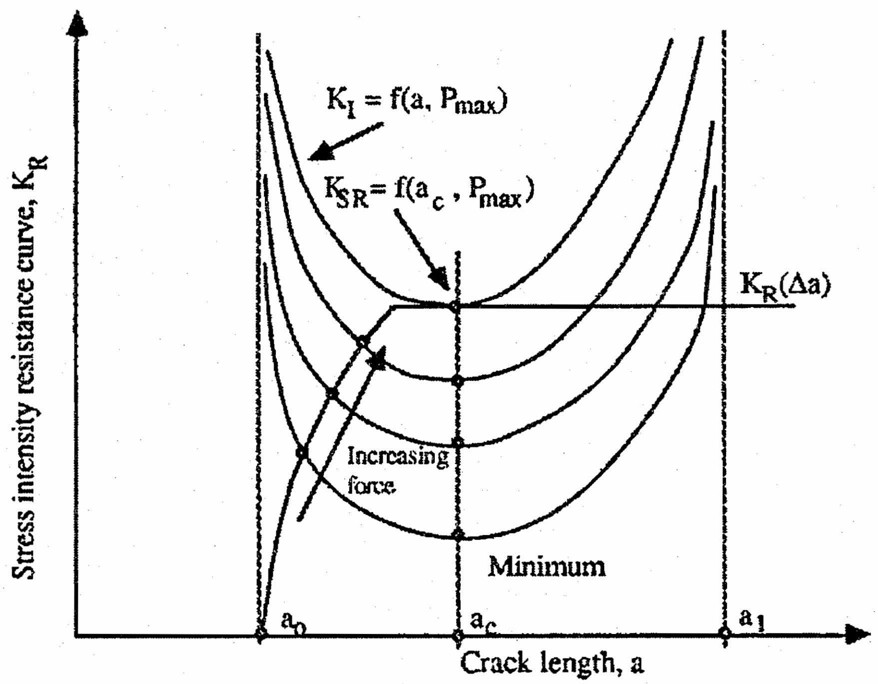


Figure 19. Fracture resistance curves of SR specimen and an assumed R-curve with a flat upper part for determining the point for evaluating fracture toughness (after Ouchterlony, 1982)

1987; Munz *et al.*, 1980; Wu, 1984). The $Y^{C}_{K\,min}$ = 24.0 is adopted by ISRM for W/D = 1.45 and a_0/D = 0.48. The level I fracture toughness is calculated from Equation (26). The level II fracture toughness accounts for the crack tip non-linearity, and is determined from the loading-unloading lines on the P vs CMOD curve. The plasticity correction factor, *p*, decreases with increasing specimen size (Barker, 1979).

Chevron-notched SR specimen provides a way of evaluating fracture toughness with small size specimen without going through the complicated process of J-integral or COD (crack opening displacement) determination. Plane strain can still be achieved on sub-sized SR to assure proper K_{IC} determination for materials exhibiting elastic-plastic behavior (Barker, 1979).

For generalized application, the short rod specimen must be calibrated over a range of specimen proportions and chevron-notch configurations.

The advantages of short rod chevron-notch specimen are (1). crack development at the chevron tip during the early stage of testing loading; (2). convenient calculation of K_{IC} from the maximum test load and a calibration factor which depends on the specimen geometry and manner of loading (Bubsey et al., 1982); and (3). No measurement of crack length or fatigue precracking is needed (Barker, 1979).

7.1.3 The Chevron Edge Notched Round Bar in Bending (CB)

The specimen configuration and ISRM suggested dimensions are given in the ISRM publication (1988). This method was developed by Ouchterlony and co-worker (Ouchterlony, 1980; Ouchterlony and Sun, 1983).

In evaluating the fracture toughness, the materials are assumed to be very brittle, and exhibit flat upper R-curve (Figure 19), then the maximum load P_{max} occurs at the critical crack length, a_c, corresponding to the minimum value of the dimensionless stress intensity factor Y_K^C. Hence the fracture toughness K_{CB} is determined as follows:

$$K_{CB} = \frac{P_{max}}{D^{3/2}} \cdot Y^C_{K\,min} \qquad (27)$$

The minimum value of the dimensionless stress intensity factor Y_K^C is independent of material property and is only the function of a_0 only.

$$Y^C_{K\,min} = \frac{1}{D}[1.835 + 7.15(\frac{a_0}{D}) + 9.85(\frac{a_0}{D})^2] \qquad (28)$$

7.2 SEMI-CIRCULAR BEND (SCB) METHOD

The SCB method was proposed by Chong & Kuruppu in 1984 (Chong & Kuruppu, 1984). Its evaluation and application have then matured through the work of several people. Karfakis (1986) has given a critical review of the SCB method compared with other test methods. Chong et al., (1987) has used several methods for the evaluation of fracture toughness determined by this method. Akram (1993) has used SCB to study the chemical environment influence. Lim et al., (1994a, 1994b) extended the application to soft rocks. Size effects and mixed mode fracture have also been studied by Lim et al., using the SCB method.

SCB specimen have a sound analytical basis and allow the fracture toughness to be determined without measurement of the fracture length. Most testing methods available for fracture toughness determination require a measurement of an area under the load-displacement curve, necessitating a precise load line displacement measurement. Displacements due to the loading frame and load cell have to be adjusted. However, for SCB specimen, a crack mouth opening displacement using an appropriate device such as a crack opening displacement gage is more accurate and convenient than a load line measurement of displacement (Chong and Kuruppu, 1984).

SCB specimen has been developed based on the circular core specimen with a center crack subjected to diametrical compression which was used by Chong et al., (1982) to determine the fracture toughness of oil shale. The SCB is just one half of the specimen that Chong et al has used (Chong and Smith, 1984).

Three different methods have been used by Chong et al., (1987) to calculate the fracture toughness based on the experimental observations, (1) elastic stress intensity; (2) the J-integral; and (3) the compliance method. For oil shale all methods yield agreeable results.

Size effects have also been studied using oil shale by Chong et al. They have found that the smallest size specimen (R = 22.4 mm, a = 11.43 mm) yield results that agree well with those of larger sizes. Test results have shown that the minimum size requirement for plane strain fracture toughness is$2.0(K_{IC}/\sigma_t)^2$. Lim et al., (1994) has evaluated the size effect with soft rock. Specimen diameter, thickness and notch length have been studied. The results agree well with Chong's.

In the numerical modeling of SCB specimen, one quarter of the disk is used because of geometry. The precracked disk is modeled under displacement loading. The effect of crack length on the non-dimensional stress intensity factor is investigated by (1) the strain energy method; (2) the ellipse method; and (3) the stress method (using Westergaard's solution for stress at the crack tip element). Close agreement has been observed for the results from the three methods (Figures 20 and 21).

The kinetics of the proposed fracture model is shown schematically in Figure 22. The load line displacement (q) and crack mouth opening (Δv) are obtained by Chong and Kuruppu (1984).

$$q = R-(1-\bar{r})(R-a)-[\bar{r}(R-a)+a+z]\frac{\cos(\alpha+\theta)}{\cos\alpha}+(s-1)\sin\theta+z\cos\theta \tag{29}$$

in which,

$$\theta = \sin^{-1}\{\frac{(\Delta v+2)\cos\alpha}{2[\bar{r}(R-a)+a+z]}\}-\alpha \tag{30}$$

The non-dimensionalized stress intensity factor was investigated by (1). strain energy method (Dixon and Strannigan, 1972); and (2). ellipse method (Woo and Kuruppu, 1982). The results are shown in Figure 20, from which,

$$\overline{K}_I = \frac{K_I}{\sigma_0\sqrt{\pi a}} \tag{31}$$

where

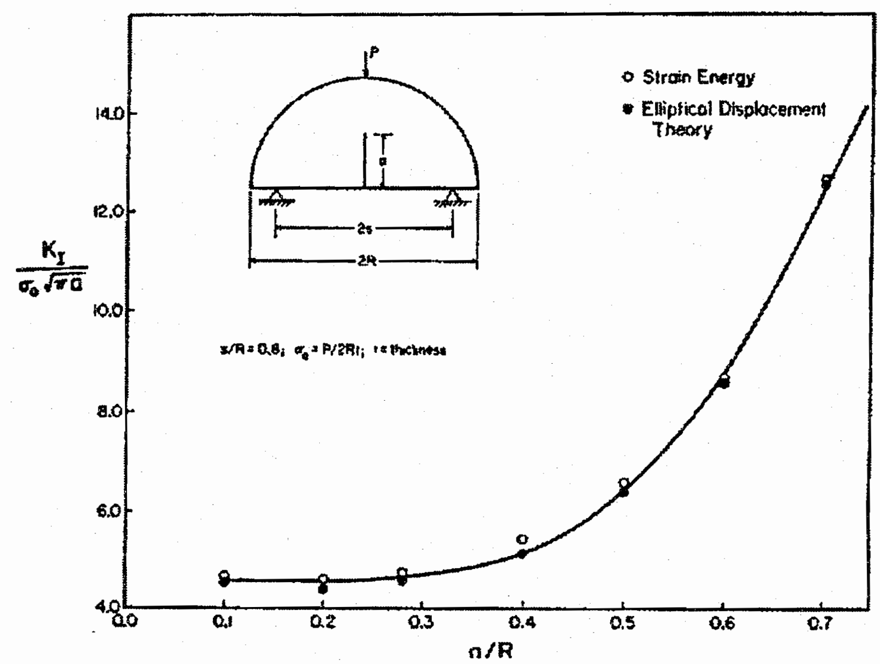


Figure 20. Mode I stress intensity factor for SCB specimen (after Chong et al., 1987)

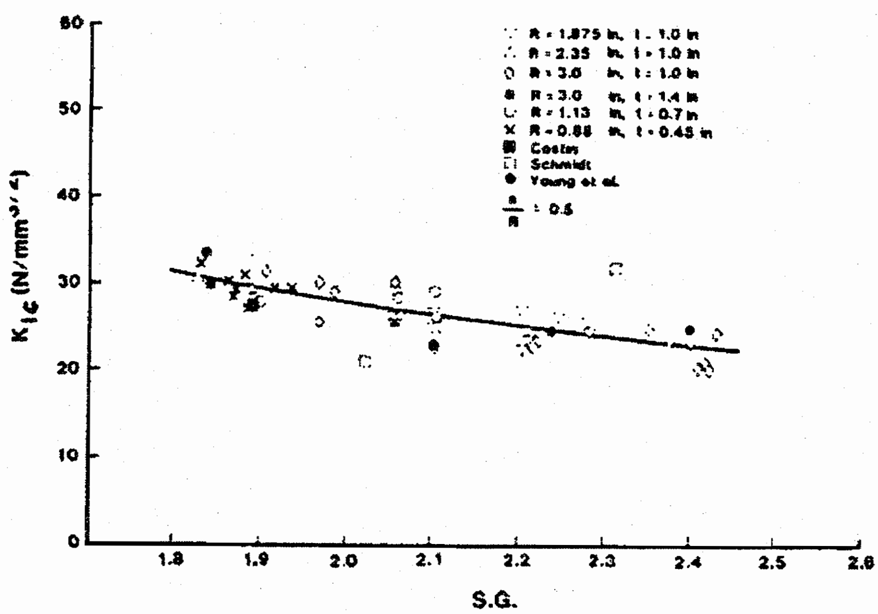


Fig 21. Fracture toughness versus specific gravity using average of stress intensity factor, compliance and J-integral methods (after Chong et al., 1987)

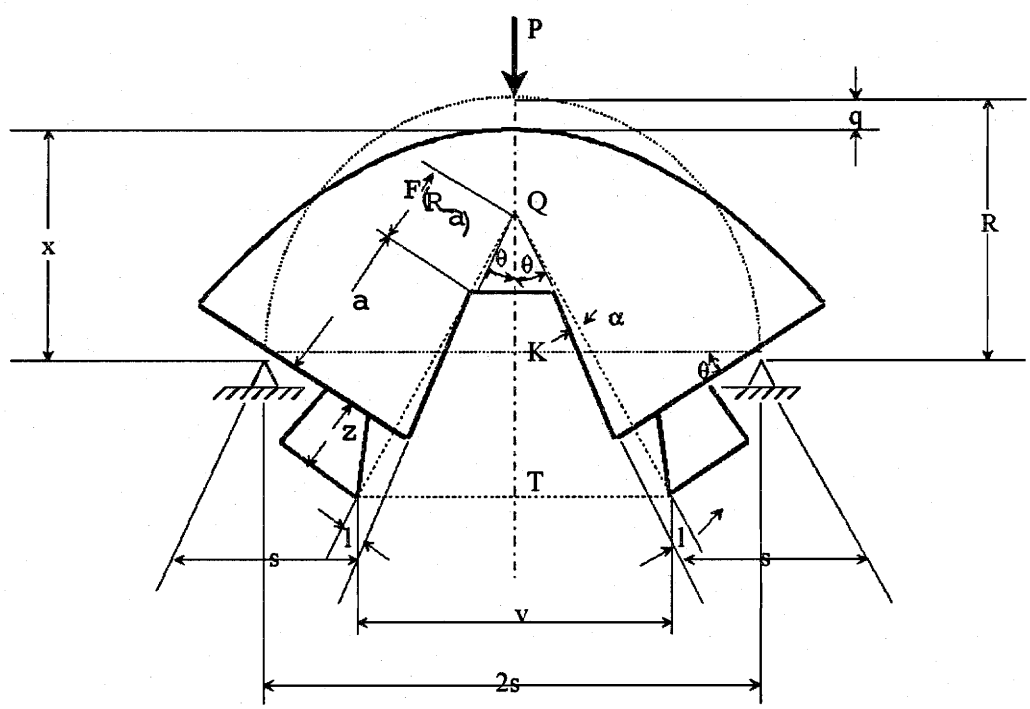


Figure 22. Kinetics of semi-circular fracture model
(after Karfakis, et al., 1986)

$$\sigma_0 = \frac{P}{2Rt} \tag{32}$$

Thus

$$K_I = \overline{K}_I \sigma_0 \sqrt{\pi a} \tag{33}$$

at fracture

$$K_{IC} = \overline{K}_I \sigma_u \sqrt{\pi a} \tag{34}$$

where

$$\sigma_u = \frac{P_Q}{2Rt} \tag{35}$$

P_Q is the critical load.

The SCB specimen currently used requires fatigue precracking. If chevron notch is used precracking can be avoided (Karfakis et al., 1986). To have a full evaluation of SCB specimen, more work needs to be done on size effects.

The major advantages of SCB method are summarized below.

1. Sound analytical basis
2. Easy preparation of test sample
3. Loading configuration well suited for LLD (load-line displacement) measurement
4. Rock anisotropy easily studied
5. Especially adaptable to small, compact specimen
6. Readily used for mixed-mode fracture study

7.3 BRAZILIAN TEST IN DETERMINING ROCK FRACTURE TOUGHNESS

Guo and Aziz (1992). have developed a method of using Brazilian test. The main feature of this approach lies in that there is no need to introduce any crack or notch in the disk specimen, while a diametrical crack is induced during loading, which finally leads to the failure of the specimen.

The Brazilian method of testing fracture toughness is based on the observation that most rocks in biaxial stress fields fail in tension along the loading diameter of the disc specimen and diametrical crack propagation in disc specimens due to tension is often observed. The stress distribution for a disc specimen under diametrical compression has been well-established (Hondros, 1959; Chong et al., 1979, 1980, 1984; Fairhurst, 1964; Jaeger and Cook, 1979). The theoretical basis for calculating fracture toughness is based on the expression of the opening-mode stress intensity factor for an edge crack in a semi-infinite medium subject to tension. For an internal crack of length 2c in an infinite plate, where the crack is subjected to a normal tensile stress $\sigma(x)$ which varies along its length, the analytical expression for the stress intensity factor of mode I, K_I is,

$$K_I = BP\Phi(c/R) \qquad (36)$$

where P is the applied load, B and $\Phi(c/R)$ are defined as:

$$B = \frac{2}{\pi^{3/2} R^{1/2} t\alpha} \qquad (37)$$

and,

$$\Phi(\frac{c}{R}) = (\frac{c}{R})^{1/2} \int_0^{\frac{c}{R}} [\Phi(r/R)/\{(\frac{c}{R})^2 - (\frac{r}{R})^2\}^{1/2}] d(\frac{r}{R}) \qquad (38)$$

In equation (36), when the stress intensity factor is replaced by the fracture toughness K_{IC}, the critical load is predicted as follows,

$$P_c = \frac{K_{IC}}{B\Phi(c/R)} \tag{39}$$

The dimensionless stress intensity factor $\Phi(c/R)$ versus c/R (crack length/specimen radius) is studied by Guo using numerical method. In the $\Phi(c/R)$ versus dimensionless crack length c/R curve, two typical regions clearly indicate the unstable crack propagation and crack arrest. According to Guo this phenomenon has been observed in many tests, and the diametrical crack propagation terminates at about R/5 from the outer boundary without further load increase. This critical position thus determines the point to evaluate the fracture toughness. This can also be shown from the load versus c/R curve, the minimum load corresponds to the critical point. The toughness values obtained by this method by Guo have shown good agreement with those from ISRM standard methods.

8. EXPERIMENTAL PROPOSAL

8.1 BACKGROUND

(1). Single particle fracture is fundamental in understanding the comminution process. Many test methods have been devised using various shapes of rock samples over the past few decades. Though some theories are well-established and some results are well-accepted, the basic mechanism in rock breakage is still not fully understood (Bergstrom, Sollenberger and Mitchell, 1962; Gilvarry and Bergstrom, 1963; Bergstrom, 1963; Brace & Byerlee, 1966; Arbiter, et al., 1968; Lundberg, 1976; Jaeger, 1966, 1979; Yashima and Saito, 1979; Krough, 1980; Banthia, 1987; Malcolm, et al., 1988; Schonert, 1974, 1979, 1987, 1991; Bourgeois, et al., 1992).

(2). Fracture mechanics has been successfully applied to rock problems. Fracture toughness testing for rocks has become mature through the efforts of many people. Fracture mechanics parameters are increasingly used in a wide variety of areas in rock engineering (Ouchterlony, 1988, 1989; Karfakis, et al., 1986, 1988; Schmidt, 1976; Whittaker, 1992; Hassanzadeh and Hillerborg, 1987; ISRM, 1988; Chong, 1987, 1988).

(3). Some attempts have been made to establish a suitable criterion in comminution and provide a universal index (Bergstrom et al., 1961; Suzuki and Tanaka, 1968; Jomoto and Majima, 1971; Everell, 1972; Yigit, 1976). Yet it is obvious from the extensive review, that despite the fact that single fracture study and fracture mechanics application have both achieved considerable results, a link between these two is still missing in comminution. Applying suitable fracture mechanics parameter to predict fracture in comminution and to compare comminution machines possesses great potential and is worth further study.

8.2 PURPOSE OF THE EXPERIMENT

The purpose of the proposed experiment is to establish a link between fracture mechanics parameters and rock crushing process.

Fragmentation of rock material can be best described using fracture mechanics concepts, which aims to give a quantitative description of the transformation by crack growth of an intact structural component to a broken one. The most important parameter in fracture mechanics is fracture toughness which characterizes the resistance to crack growth, It is also a measure of the energy needed for crack propagation. Fracture toughness has been used as an index of fragmentation process such as tunnel boring and blasting (Lindqvist, 1982; Rustin, et al., 1983; Nelson and Fong, 1986). It hasn't been introduced in the field of comminution where it may yet be the one parameter that can best describe the fragmentation characteristics of rocks.

The ability of rock materials to store strain energy before fracture can be characterized by the concept of energy index, which is defined as the ratio between the energy elastically accumulated to the energy used to generate a fracture (Ouchterlony, 1988; Haramy and Mcdonnell, 1988; Karfakis, 1989). This energy index is an important parameter characterizing rock fracture process. Fracture toughness, specific fracture energy and energy index will be experimentally determined using well-established techniques.

Rock breakage in comminution process is closely related to the breakage function which describes its product size distribution. Particle size distribution results from the crack development within the particle, and is also closely related to the fragmentation energy and energy intensity (loading rate). Different rock fragmentation process can be represented by their final breakage function (Bergstrom, Sollenberger and Mitchell, 1962; Gilvarry and Bergstrom, 1963; Bergstrom, 1963; Arbiter, et al., 1968; Yashima and Saito, 1979; Krough, 1980; Grady, et al., 1981; Clark, 1987; Malcolm, et al.; 1988; Bourgeois, et al., 1992).

Correlation will be made to link energy to the breakage characteristics to establish a procedure to determine the free crushing threshold energy for comparing various mechanical comminution devices, and finally to arrive at a universal standardized crushability index of comminution criterion.

8.3 EXPERIMENTAL PROGRAM

8.3.1 Rock Types and Sample Preparation

Six types of rocks have been selected for the tests. (1) dolomite limestone; (2) Westerly granite, (3) Sioux quartzite, (4) Hokie stone, (5) sandstone, (6) Siltstone. They cover a wide range of rock properties occurring massively in nature. They all have well-established mineral composition and mechanical properties and are relatively free from fractures. Mineralogically the fist three represent three distinct varieties. Sioux quartzite is a high silica (almost 100%) monomineralic rock, Westerly granite is a polymineralic rock, and dolomitic limestone is a sedimentary rock. Fracture toughness values for some of the rock types have already been determined.

Rock samples for the fracture mechanics tests utilizes 4" diameter, 1" thick half disc. Akram (1993) has a very detailed description of the procedures to prepare the sample. The single particle crushing test will use cylindrical rock samples approximately two inches in diameter and one inch in thickness. They are prepared from rocks blocks obtained from a dimension stone quarry near Blacksburg. The 2 inch cores are first drilled by a diamond core bit, then the cores are cut into one inch discs which are polished and measured to the specification. At least 8 identical specimens are prepared for each test, and their respective dimensions are recorded to arrive at the accurate volume to be used for determining product size distribution.

8.3.2 Single Particle Fracture Test

8.3.2.1 Theoretical Background

Cylindrical rock samples will be used as single particles. Theoretical considerations for a system composed of an elastic cylinder gripped by a pair of parallel platens is well established (Jaeger and Cook, 1979) and experimental load-deformation curves can be readily verified by established theoretical procedures (Figure 23). Cylindrical specimens are easy to prepare (unlike spherical samples) and have simple geometry which is more likely to produce consistent and repeatable tests (unlike irregular specimens) (Guo and Aziz, 1992; Clark, 1992).

The stress distribution, failure initiation (Figure 24), test machine stiffness, loading effects and the successive stages of failure have all been studied and have well-established conclusions (Hondros, 1959; Fairhurst, 1964; Hudson, 1972; Chong et al., 1980; Clark, 1992).

8.3.2.2 Test Program

For each rock type six samples will be tested for crushing energy, rock strength, and breakage function. The exact measurement will be made for each rock sample. Two crushing speeds and two crushing throws will be chosen to test their effects on the results. Therefore a total of 24 samples will be tested for each rock type as seen in the table below.

Table I. Crushing Test Design

Number of Samples	Throw (Displacement) 1"	Throw (Displacement) 0.5"
Crushing Speed 50 in./sec	6	6
Crushing Speed 10 in./sec	6	6

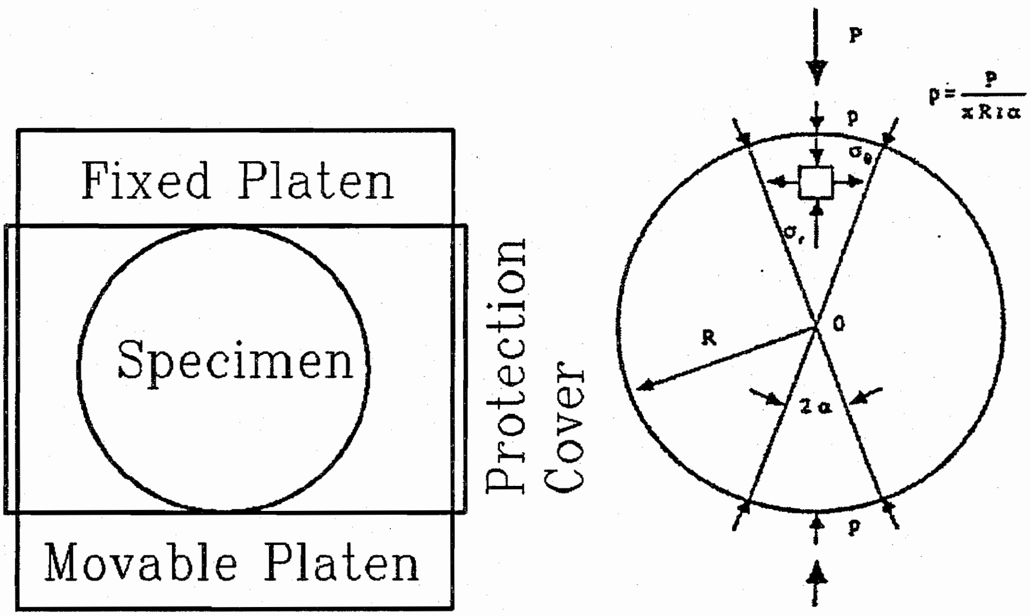


Figure 23. Cylindrical specimen under diametrical compression
and its theoretical background

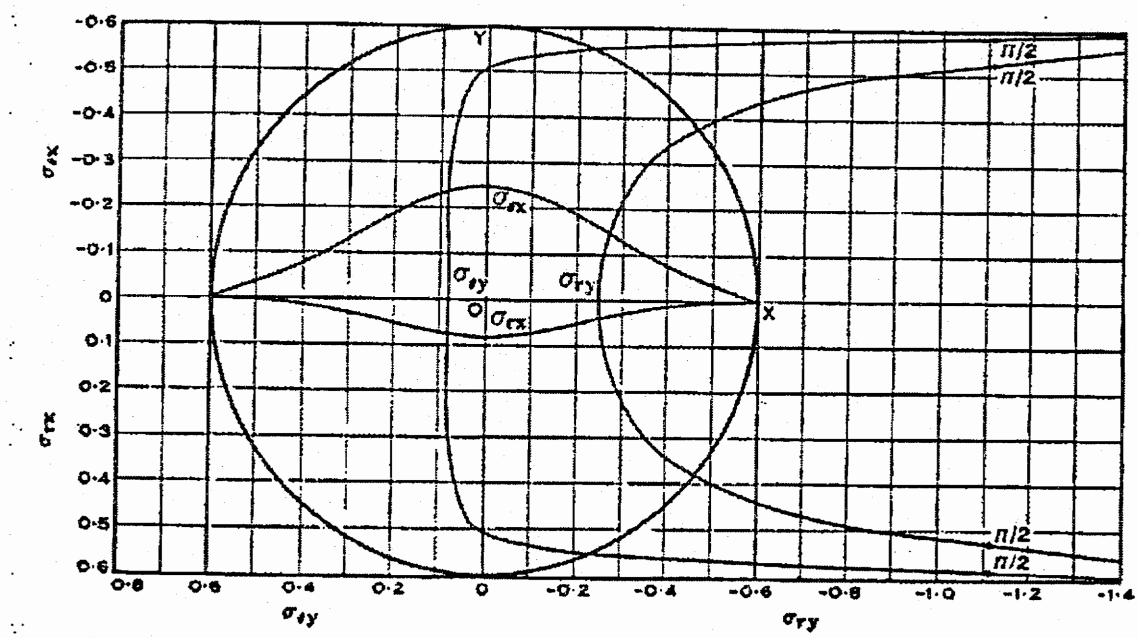


Figure 24. Theoretical stress distribution along the vertical (OY) and horizontal (OX) diameters (after Hondros, 1959)

The final test results foe each specified crushing speed and throw will be tabulated as:

Table II. Crushing Test Results

Rock Type	Sample Dimensions	Crushing Speed	Crushing Throw

Sample No	Total Fracture Energy	Specific Fracture Energy	Ultimate Tensile Strength	Product Size Distribution
1				
2				
3				
4				
5				
6				

8.3.3 Fracture Mechanics Test

Fracture mechanics test will be conducted to determine the fracture toughness, specific fracture energy and energy index. The semi-circular bend (SCB) will be used which yields accurate results of test parameters without considering sample size and crack length measurement (Karfakis, et al., 1986,). MTS 810 closed loop servohydraulic testing machine will be used for the fracture mechanics test. Since the elasto-plastic behavior of rocks is highly dependent on loading rates (Arbiter, et al., 1969; Marshall, 1974; Schonert, 1979, 1987), tests will be conducted at different LLD (load line displacement) rates to assess the effect of loading rate on the energy index.

8.3.3.1 Fracture Toughness Test

SCB specimen have a sound analytical basis and allow the fracture toughness to be determined without measurement of the fracture length. Most testing methods available for fracture toughness determination require a measurement of an area under the load-displacement curve, necessitating a precise load line displacement measurement. Displacements due to the loading frame and load cell have to be adjusted. However, for SCB specimen, a crack mouth opening displacement using an appropriate device such as a crack opening displacement gage is more accurate and convenient than a load line measurement of displacement (Chong and Kuruppu, 1984).

SCB specimen has been developed based on the circular core specimen with a center crack subjected to diametrical compression which was used by Chong et al., (1982) to determine the fracture toughness of oil shale. The SCB is just one half of the specimen that Chong et al has used (Chong and Smith, 1984).

Three different methods have been used by Chong et al., (1987) to calculate the fracture toughness based on the experimental observations, (1) elastic stress intensity; (2) the J-integral; and (3) the compliance method. For oil shale all methods yield agreeable results.

Size effects have also been studied using oil shale by Chong et al. They have found that the smallest size specimen (R = 22.4 mm, a = 11.43 mm) yield results that agree well with those of larger sizes. Test results have shown that the minimum size requirement for plane strain fracture toughness is $2.0 \cdot (K_{IC} / \sigma_t)^2$. Lim et al., (1994) has evaluated the size effect with soft rock. Specimen diameter, thickness and notch length have been studied. The results agree well with Chong's.

The formulas and for fracture toughness determination have been in Chapter 7. Akram (1993) has a detailed description of the procedures.

8.3.3.2 Specific Work of Fracture and Energy Index

During crack extension, energy is consumed to create new surface area. The less energy a material absorbs, the lower its fracture resistance. Specific fracture energy will also be determined in the test in which the load versus load line displacement is recorded until the specimen has virtually no residual strength. From this data the total work of fracture W_f is calculated as:

$$W^f_{SCB} = \int_0^\infty P d(LLD) \tag{40}$$

The specific work of fracture is determined as:

$$R_{SCB} = W^f_{SCB} / A \tag{41}$$

Energy index is calculated by:

$$R = W_E / W^f_{SCB} \tag{42}$$

where W_E is the elastic energy.

Different loading rate for the fracture mechanics test will be used. Six samples for each rock type will be tested. Test results will be tabulated.

Table III. Fracture Mechanics Test Design

Sample Number	Sample t and R	Ligament Length	Ligament Area	Fracture Toughness	Total Fracture Energy	Specific Fracture Energy	Elastic Energy	Energy Index
1								
2								
3								
4								
5								
6								

8.4 TEST EQUIPMENT

For the fracture mechanics test, the MTS 810 closed loop servohydraulic 110 kips testing system will be used. Akram (1993) in his mater's thesis has a detailed description of the equipment function, test set-up, data-acquisition and reduction, experimental procedures, and fracture toughness and energy index determination. These will not be discussed in this paper. The distinguishing feature for the fracture toughness test of this research is to use different loading rate, preferably the high loading rate compatible with those used for the crushing test.

Single particle fracture test will be done on the Allis-Chalmers high energy crushing test (HECT) system to obtain the breakage energy, compressive strength, breakage function and other rock breakage characteristics. Compared with the previous single particle test apparatus, the high energy crushing machine is able to supply excess energy, which is more analogous to the processing environment of commercial crushing devices (Flavel, 1988). Crushing force and displacement can be measured during the test which accurately yields the crushing energy. An outstanding feature of the test system is the flexibility available to control the force or displacement during the test to closely follow a prespecified profile. The crushing test will be run under displacement control with an inverted haversine profile of displacement versus time which closely simulates the crushing cycle on an actual crusher, insofar as the relative displacement between the mantle and chamber is concerned. Also high speed test can be achieved to obtain the actual crushing situation and to take into account the effect of crushing speed on energy consumption. With the test system the range of throws (displacement) can also be varied whose effects can also be considered. Breakage function is obtained by sieving and weighing the fractured product after each test (Allis-Chalmers, 1985).

The crushing test is carried out under displacement control. The crushing force is measured by a load cell on the stationary platen and the displacement of the hydraulic ram is measured by a linear variable differential transducer (LVDT). These two signals are transmitted to the Data 6000 waveform analyzer (Figure 25) for further processing and to the disk-drive for data storage. Other parameters of the test results such as peak force, crushing energy and actual displacement can be directly processed displayed by the Data 6000 system.

The main features of the HECT system are listed below.

1. 55 kips MTS stiff, closed loop servohydraulic dynamic system
2. 2m/sec., 35 kips hydraulic ram for fragmentation /breakage study
3. Providing excessive energy -- more analogous to commercial crushers
4. Able to simulate actual crushing cycle of the displacement profile
5. Generating high and adjustable crushing speed
6. High speed data-acquisition and processing system
7. On-line plotter

8.5 TEST PROCEDURES FOR SINGLE PARTICLE FRACTURE TEST

The step-by-step procedure for conducting the crushing test are outlined in the operators manual. The following only lists the important steps unique to the tests of this paper and the procedures which can not find in the manual.

A) Test set-up and feed preparation

The test set-up is identical to a Brazilian test except that it is of high loading rate and the resulted fragments are collected for size distribution determination. The cylindrical

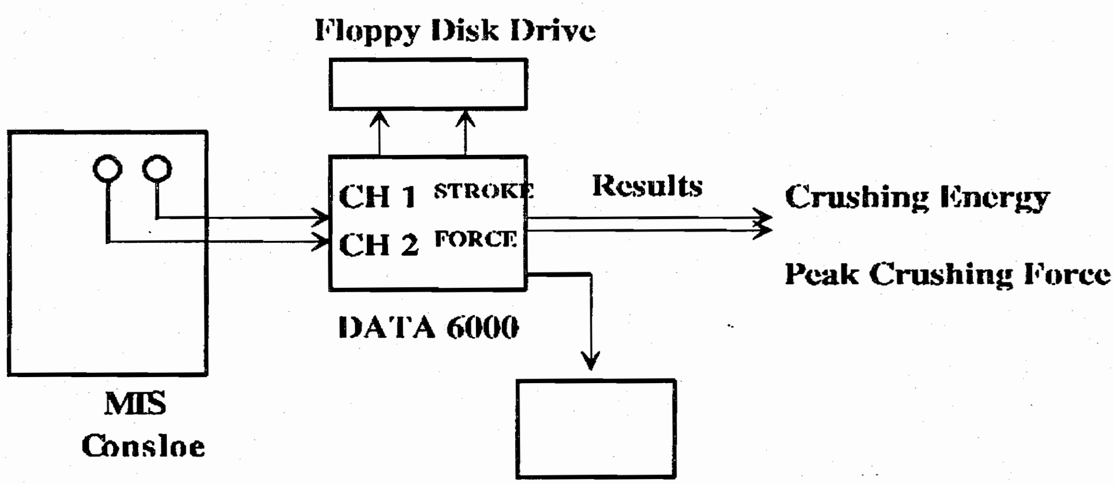


Figure 25. Schematics of HECT and data acquisition system

specimen is placed at the center of the bottom moving platen and the plastic door is closed and latched for safety and preventing flying away of the fragments.

B) Data 6000 initial preparation

The detailed procedures for Data 6000 set-up and preparation are outlined in the Allis-Chalmers Operators Manual (Allis-Chalmers, 1985). The most difficult step is to set the triggering point which can only be accomplished through several trial runs of the test. The right triggering voltage should ensure that data is recorded at the onset of the crushing and that the complete crushing cycle is stored. The DELAY function has sometimes to be utilized along with the TRIGGER for this purpose.

C) Set-up preparation for specific test condition and set-up calibration

The test calls for fracture energy and size distribution under varying displacement and loading rate. The set-up for varying displacement is relatively simple. Under a certain percentage of the stroke position, the SPAN is set according to the proportion to get the desired throw (displacement). This displacement setting can also be verified through the displayed results on the Data 6000 screen and on the plotted result curves. Care should be taken to use the minimum possible stroke range to get an accurate displacement setting. Calibration can be done by several trail runs of the tests. Once one setting has been calibrated, tests can be carried out with no further calibration since each test comes with two displacement readings, their correlation can be used to verify the correctness of the setting.

The loading rate set-up has been found to be extremely intricate. Since this test equipment has never been used under varying loading rate conditions, and all the available instructions manuals only gives test setting for one loading rate at about 20 inches per second, other loading rate set-ups are difficult to obtain for a stable test condition. Primary tests with the machine have found that the loading rate setting depends not only on the RATE knob on the control panel but also on the displacement setting. Yet through careful selection of these control knobs and most importantly through dozens of trial runs, it is possible to achieve loading rate between 10 and 70 inches per second.

D) Execution of the test, data acquisition and storage, data processing

After the completion of the above set-ups, the START button is pushed to execute the test. Data storage and processing can be done easily with the very powerful Data 6000 system, and the final results can be recorded on the plotter.

8.6 LABORATORY CRUSHING TEST

Limited crushing tests using different laboratory size crushers will also be conducted to verify the conclusions drawn from fracture mechanics and single particle crushing tests.

8.7 Summary

1. Fracture toughness and energy index can be accurately measured from the SCB specimen and using existing lab equipment.

2. Loading rate effects on fracture toughness and energy index can be studied.

3. Compared with other test equipment, the High Energy Crush Test System can more closely simulate the actual crushing process.

4. Size distribution, fracture energy and the loading rate effects can be obtained from the HECT system.

9. PRIMARY OBSERVATION, DISCUSSIONS AND CONCLUSIONS

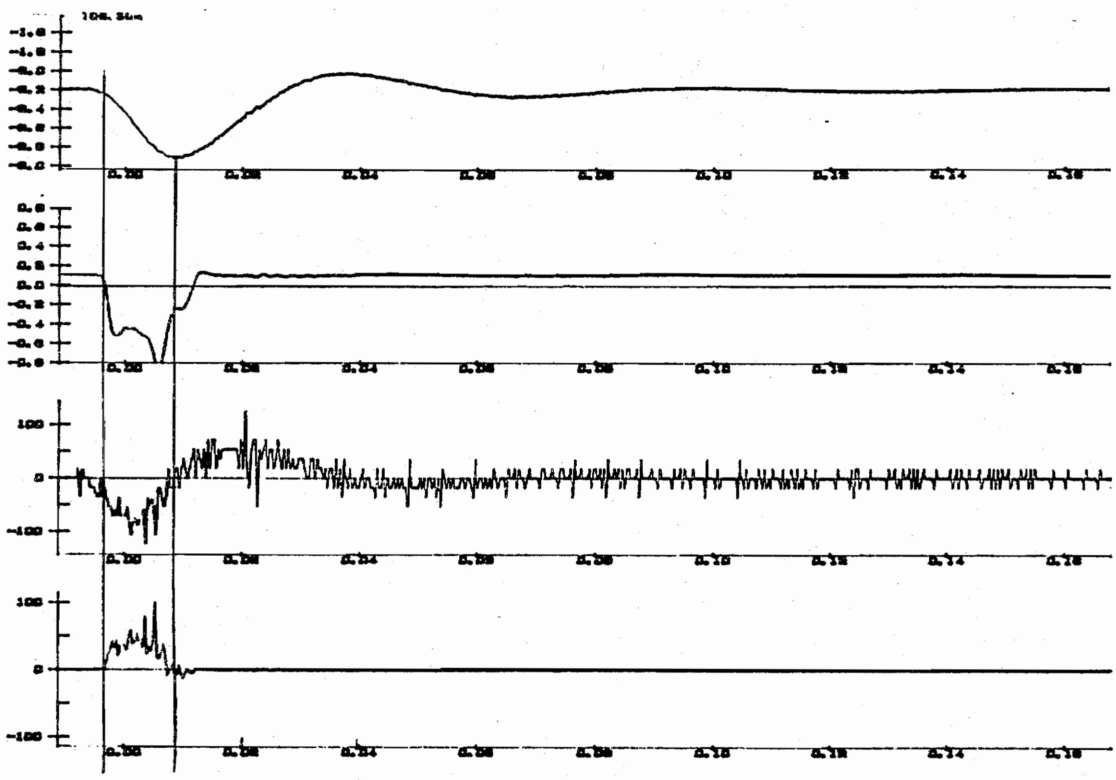
9.1 PRIMARY OBSERVATIONS

9.1.1 Single Particle Crushing Test

Some primary crushing tests have been conducted using several types of rock samples. Figures 26 and 27 are the plotted results showing the displacement, crushing velocity, force and energy.

For the purpose of obtaining the accurate breakage function, secondary crushing is strictly avoided. The plastic protection cover is able to ensure, to a great extent, that the fragments from the primary crushing will not be further crushed. Yet with very large displacement and very small final crushing gap between the moving and the fixed platens, secondary crushing will invariably occur. One way to deal with this is to observe the force-time record on the plotted results. When secondary crushing occurs a rebounding in fracture force will be evidenced on the plot. Thus the appropriate displacement and crushing gap can be achieved to avoid secondary crushing.

Figure 28 is the fracture pattern. It can be seen from this diagram that the cylindrical disc is basically broken into two half discs, which agrees with the results of Brazilian test. The very fine products result from the two contact areas, in which excessive compressive stresses are generated. Along the diameter cutting the disc in two, some smaller fragments are also produced. When fracture occurs, the impact velocity is so high



Rock sample: Sandstone, Equipment settings: Stroke = 1”, Rate = 99 Hz

Calculated results

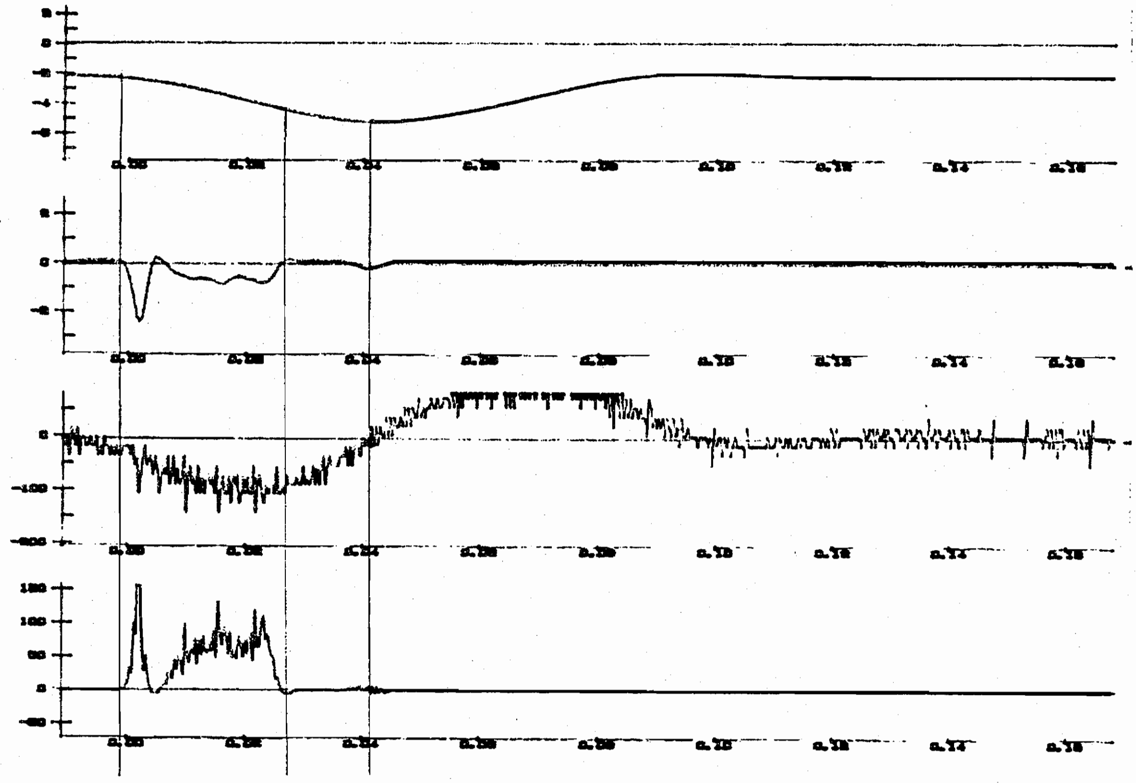
Mean force = 106.28 mv, Area (energy consumption) = 81.49 Pts

Peak force = 3413 lbs, Peak stroke = 447.07 mft

Figure 26. Single particle fracture test results

(graph shown from top down: displacement, crushing force,

crushing speed and crushing energy



Rock sample: Westerly Granite, Equipment settings: Stroke = 1", Rate = 10 Hz

Calculated results

Mean force = 112.50 mv, Area (energy consumption) = 304 Pts

Peak force = 8794 lbs, Peak stroke = 1.625 in

Figure 27. Single particle fracture test results

(graph shown from top down: displacement, crushing force,

crushing speed and crushing energy

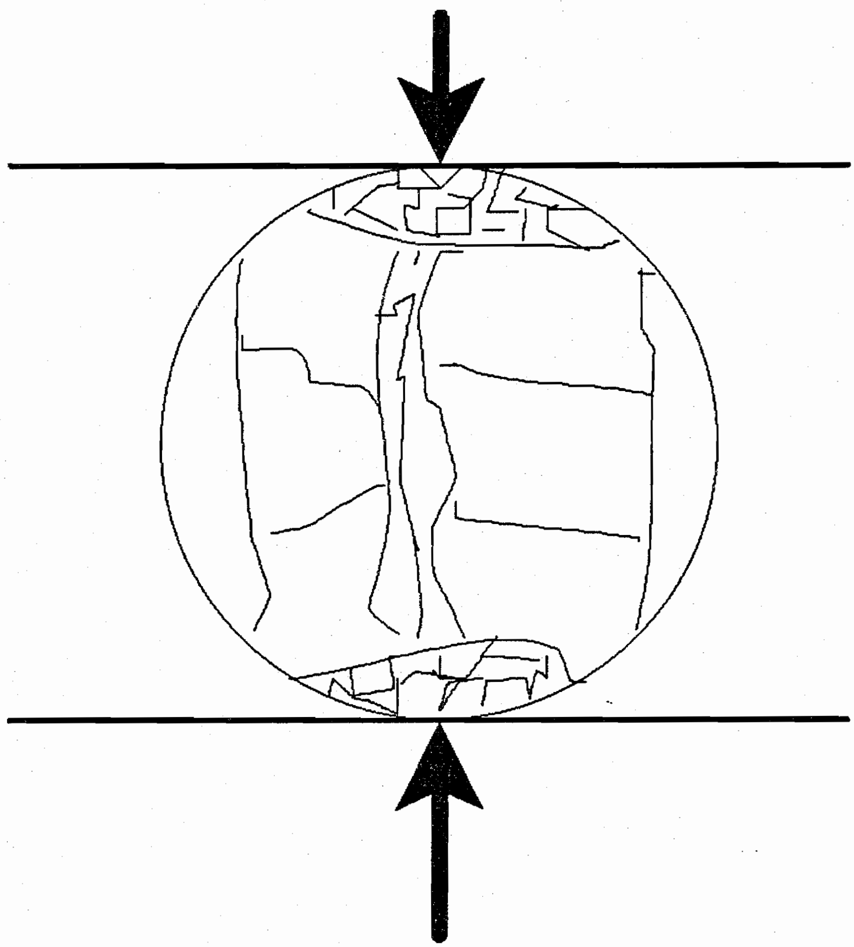


Figure 28. Breakage pattern for cylindrical specimen under impact

that the rock sample breaks in a splash of a second. This makes it impossible to observe the fracture pattern as it is occurring. By analysis of the resulted fragments, it is postulated that most of the fragments come from the crushing of the two half disc. All the test have shown that the two parts which are relatively unbroken are the parts farthest from the loaded diameter.

Primary results for loading rate and displacement effects are also observed in the test. For same type of rock under constant displacement, the fracture force and energy increases with loading rate. High loading also results in finer product. But due to the high energy consumption, it is less efficient than slow compression. Under constant loading rate, greater displacement causes increase in product fineness.

Material property also plays a role in the fracture process. Harder materials tend to break more violently into finer fragments with higher energy consumption and higher fracture force, while softer materials break gently with more very fine products and fewer fragments larger than the fragments from harder materials.

9.1.2 Loading Rate Effects on Fracture Mechanics Parameters With Regard to Comminution Device Selection

From the discussions on loading rate effects on fracture toughness, the proposed fracture mechanics tests will also study its influence on fracture toughness and energy index. Since the elasto-plastic behavior of rock materials is loading rate dependent, test results will be helpful, from the viewpoint of energy input, in the selection of comminution devices for different rock types. The loading rate study will also establish the most efficient loading rate with the existing machines.

9.2 DISCUSSIONS ON DATA ANALYSIS AND FRACTURE TOUGHNESS APPLICATION TO COMMINUTION

It can be concluded through this literature review that rock crushing is probably one of the few last areas of rock fragmentation to which fracture mechanics concepts have not been successfully applied. It is also felt that the link between rock crushing and fracture mechanics principles will provide the key to the solution in establishing a suitable crushing criterion. Applying suitable fracture mechanics parameter to predict fracture in comminution and to compare comminution machines possesses great potential and is worth further study.

In comminution, the population balance model considers size reduction of two basic components: the fracture event (represented by the breakage distribution function), and the fracture process (represented by the selection function) (Kelly and Sporriswood, 1990). The work in this report concentrated on the fracture event of single particle fracture, but its potential application to the whole comminution process cannot be underestimated because the concept of fracture toughness as an index of comminution is based on the strict analysis of the fracture mechanism.

The analysis of test results will be the defining step into finishing the study and arriving at a sound conclusion. Since there are no available data at this point, the future work discussed here will be focused on the principles used in the test result analysis. The potential application of fracture toughness as a comminution index will also be discussed.

9.2.1 Data Analysis -- Principles and Methods

From the perspective of fracture mechanics toward the understanding of comminution, the principle of data analysis for single particle study is to correlate fracture toughness to the force and energy input as well as the resulting fragment size distribution. Although various previous studies have been conducted, it is the first time to use fracture toughness as an index of comminution.

First, the ease of comminution from the single particle fracture test can be characterized by (1) the fracture force (maximum force, mean force and the force to first fracture), (2) fracture energy, and (3) the product size distribution. Then the fracture mechanics parameters (fracture toughness and energy index) are to be related to the ease of comminution as described above. With the help of some of the statistical tools available, the first step is to seek the possible relations between them. When the resulted values of fracture toughness are plotted against the fracture force and the mass specific fracture energy, some primary conclusions may be drawn. Also from the theoretical point of view, fracture toughness can be related to specific fracture energy assuming brittle fracture. Since rock materials are not perfectly elastic, the experimental relationship between fracture toughness and energy can be compared with the theoretical relations.

From the point of view of comminution, the established fracture toughness as a comminution index should be incorporated into a comminution model to take the place the existing index such as Bond's work index or grindability index, which are not based on the strict analysis of the fracture mechanism but a results of empirical work. The results from single particle study should also be able to be incorporated into the model describing multiparticle comminution system. Basically, fracture toughness, much like the tensile strength of a material in tension, is a material property in brittle fracture. In the selection function (specific rate of breakage), not only must the applied load exceed the particle strength for a breakage event to occur, fracture toughness should also play a major role. It might just well be the dominant parameter. Everell (1973) has studied the relation between selection function and failure load and have arrived at the following expression:

$$S(x) = K_x \cdot (Psc_x)^{\alpha} \tag{43}$$

where S(x) is the element of the selection function or the specific rate of breakage for the size x, K_x is the value of a coefficient for size x, Psc_x is the average failure load for particle s of x, and α is a constant. Due to the nature and role of fracture toughness in brittle fracture, the correlation between selection function and fracture toughness should shed more light to the comminution process. The influence of fracture toughness as an intrinsic material property governing fracture should be studied for this purpose.

9.2.2 Fracture Toughness Related to the Breakage Function

The breakage function is closely related to the fracture mechanism of the single particle. As discussed above, in the proposed experiment the size distribution consists of the very fine fragments resulting from the compressive failure in the immediate vicinity of the contact area, and a coarse distribution resulting from the main fracture due to the induced tensile stress in the center. In the data analysis of the test results, the fracture toughness will be closely correlated to the size distribution. This correlation will also be examined against loading rate, displacement, crushing gap and possibly particle size for future study. Since the dominant fracture mode is the tensile fracture from within the particle, and fracture toughness is the governing parameter controlling the creation and propagation of cracks, the correlation between fracture toughness and size distribution may help to predict breakage function based on theoretical analysis, i.e. given the rock type and comminution device, the size distribution may be accurately predicted.

9.2.3 Fracture Toughness and Grindability

Grindability is a measure of a mineral's resistance to reduction in size, it is also a measure of the energy needed for its comminution. It is widely used for sizing mills and designing milling circuits (Gutsche et al., 1992). According to Gutsche, grindability, besides depending on Mohs hardness, is a complex function of the chemical structure of the material, flaw distribution in the particles, lattice defects and cleavage planes and also of the mill and its operating conditions. It is determined by measuring the energy required to a certain reduction ratio. Its determining equation is given by Equation (13). Harder materials have low grindability values and break coarse into narrow size distribution with high levels of energy input, while softer materials have high grindability values and break fine into wide size distributions with low levels of energy input.

Fracture toughness also characterizes the material resistance to comminution. Yet unlike grindability it has a very strict analytical basis. It also is a function of the material inhomengenity and loading configuration and it is a measure of the energy consumption in comminution. The results with regard to comminution are the same in that harder and stronger materials have lower values of fracture toughness. The fundamental difference lies in the fact that grindability is an empirical value but fracture toughness can be accurately developed through sound theoretical analysis. Intuitively, fracture toughness should be more useful in comminution device designing. This, of course, needs to be experimentally confirmed. This is one part that will be conducted in future work.

9.2.4 Selecting Comminution Device and Predicting its Performance Based on Study of Crushing Parameters

The more direct application of fracture toughness as a comminution index lies in the selection of existing machinery. Since fracture toughness is correlated to the crushing parameters such as crusher jaw gap and crushing speed. A direct comparison of the crushing force and energy consumption will determine the type of rocks that can be best crushed using minimal force and energy requirement. Loading rate is another very important parameter in comminution device selection. The proposed experiments will also determine, with the existing machines, the optimum crushing speed for different rock types.

In light of the above discussion, the determined fracture toughness values can be used to select the suitable comminution device and predict its performance for different types of rocks.

9.3 CONCLUSIONS

Fracture toughness is the most basic intrinsic material property in governing the fracture process of rock materials. It can be used as an index of comminution for selection of crushers or grinding mills. Unlike grindability, it has a rigid theoretical background which enables the comminution process to be predicted on a scientific basis. The study in this report will stimulate further research work toward the better understanding of comminution process using fracture mechanics parameters and concepts.

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