

Chapter 4

The Role of Public Investment in a Growth Model with Asymmetric Information

I. Introduction

How to explain a country's economic growth has always been atop of economists' research agenda. The resurrection of the growth paradigm under the rubric "endogenous growth theory" not only restores the possibility of policy influence on growth, but further stimulates people's interest to search for the factors which can nurture growth. Among these factors, the role of financial intermediary to overcome market friction and allocate funds to their best use has attracted much attention. Thanks to progress in microeconomic theory (e.g. Rothchild & Stiglitz (1976)), we now better comprehend that asymmetric information, a well-known feature of credit markets, is hazardous to a country's welfare because the problem of credit rationing thus generated hinders the optional flow of financial resources to borrowers. Spurred by this understanding, a whole strand of literature discusses the pros and cons of credit rationing as regards growth in the 1990's. Bencivenga & Smith (1993) presume a negative impact of credit rationing on economic growth. In Bose & Cothren (1996), screening, credit rationing and a combination of these two characterize alternative forms of contracts to be chosen by the lender. They conclude that the equilibrium contract does not maximize economic growth. Shi (1996) challenges their point of view by suggesting that credit rationing may stimulate agents to undertake high quality but high risk projects whose success may create positive spillovers of new knowledge. In all these papers, the probability of success of a project is what distinguishes the type of its owner. My paper differs from theirs by introducing an additional factor to further subdivide project owners into smaller cliques, namely the hardness of a project owner's budget. The infamous "soft budget constraint" phenomenon implies the least control on an agent's profit and loss - and fosters serious problems of inefficiency. In my model, the kind of borrowers with such constraint is labeled public. On the contrary, private borrowers are those with less financial flexibility because of their hard budget constraint. All borrowers who plan to pursue projects that need funding will approach a bank (financial intermediary) for funds.

But the nature of the projects differs between the two kinds of borrowers. To be concrete, we will call the project owned by a public borrower a public project and a project owned by a private borrower a private project. A (successful) public project, for example construction of infrastructure, can exert positive externality on the productivity of private projects. In fact that is all it does. Associating these structural differences to a firm's ownership status in a growth model, I am going to examine the macroeconomic implications of credit rationing. Surprisingly, even under the shadow of soft budget constraint, the existence of public borrowers can be growth enhancing.

It is common for the government to design discriminatory policies in favor of public institutions. For instance, public borrowers can borrow money from banks with a minimum interest rate that can drive banks to financial distress. In my paper, these observations are captured by the public borrower's "first mover" advantage to borrow money from a bank with a zero interest payment. This advantage corresponds to a real life phenomenon : First, a low interest rate can help to mitigate state-owned enterprises' impulse to undertake high risk projects under the influence of soft-budget constraints. That is because if the interest rate is high, the expected profit from investment will be smaller unless the manager is willing to take a high risk and high return project. See Zou & Sun (1996). Second, banks may find it more beneficial to lend to state-owned enterprises rather than private enterprises, since by doing that they have a higher chance to collect the repayment of old debts. See Perotti (1993).

In order to maintain a balanced budget, banks will redeem the losses by asking for a high interest rate from private borrowers⁸. Consequently, some of these private borrowers may be "crowded out" by the high interest rate that the financial intermediary may charge. Stiglitz & Weiss (1981) develop a credit rationing model with two groups of people, safe group and risky group, borrowing money from banks. The safe group is willing to pay an interest rate below r_1 ; the risky group below r_2 ; and $r_1 < r_2$. They show that, as the interest rate slightly increases in the market, the mix of loan applicants changes. More importantly, all low risk borrowers may withdraw from the market. In my model, such an

⁸ The loss may also be financed by taxes. To keep things focused, we will exclude this alternative in this paper.

exogenous change of interest rate is provoked by the presence of public projects. Further, the separating equilibrium will show that low risk borrowers have a lower equilibrium probability to get funds than when there is no public project in the economy. In contrast to low risk borrowers, high risk borrowers can avoid this adverse effect since their probability to get funding is always unity.⁹ Though most of the "good" private projects in the sense of Stiglitz and Weiss do not materialize due to the high interest rate, the externality effect exerted by public projects can improve the productivity of private projects and to some extent alleviate the adverse effect of credit rationing on economic growth. How economic growth of a country will be affected by the interplay of these two opposing forces is the main theme of this paper.

We conclude that a country's growth hinges upon the positive externality of public projects. The magnitude of this effect will be measured by the probability of success of a public project. *Ceteris paribus*, an increase of this probability pushes the economy to a trajectory with a higher growth rate. We also compare the growth rate in situations with and without public projects. Interestingly, the growth rate with public project can be higher even if public projects are not that good.

The results of this paper lend support to the use of public investment in a country with a financial market where credit rationing contract induces self-selection between good and bad private investments. The government can remedy this situation by injecting public investment at the expense of availability of funds for some private "good" investments. Such a sacrifice will pay off if the quality of public investment is good enough.

In the next section, the basic model will be developed. Separating equilibrium contracts will be derived in section III. In section IV, we will discuss the growth of the economy. Section V will give two numerical examples to illustrate the main results of this paper. The last section will conclude and discuss some possible extensions.

⁹ Therefore, my model is of a stochastic credit rationing type : a portion of loan applicants get the credit they want, whereas the others do not get credit.

II. The Model

I will adopt the overlapping generations framework developed in Bose & Cothren (1996). For a graphical illustration of the difference between our models, see figure 1.¹⁰

This economy consists of an infinite sequence of two-period lived overlapping generations. All generations are identical in size and composition. The population size of each generation is normalized to one.

Young agents in each period are divided into two groups which have the same size. They are lenders and borrowers. A lender endowed a limited amount of funds is the only financial source in the economy. All borrowers can be subdivided into 2 different categories : public borrowers and private borrowers. Each of these borrowers has a potential project which needs funds to be implemented. To model public borrowers' "first mover" advantage, we assume that they are guaranteed enough money to invest and they are not required to pay any interest. From the private borrowers' viewpoint, the size of the cake (the amount of funds) shrinks because of the presence of public borrowers.

Each private project is either of high risk (H) or low risk (L) type. A high risk project has a lower possibility of success. Besides that, the two kinds of projects are alike. In this economy, I assume that a portion λ of private borrowers has a H-type project. By utilizing one unit of labor, each investment project can convert consumption goods into capital goods.

The i -type, $i \in \{H, L\}$ investment project can with probability P_i convert x units of time t outputs and one unit of labor into $Q \cdot x$ units of time $t+1$ capital. The project may fail with probability $1 - P_i$. We further assume that $1 \geq P_l > P_h \geq 0$. Young borrowers can provide one unit of labor to the market. Or, they can make use of labor to run the project. But they need to borrow external funds to develop their projects. If they cannot borrow funds, H type project borrowers will consume nothing for the rest of their life. L type

¹⁰ Since their model is an alteration of Bencivenga & Smith (1993), the comparison hereafter is done between mine and Bencivenga, Smith, Bose and Cothren's works (in short, it is denoted as B-S-B-C).

project borrowers can sustain their life by supplying labor in exchange for wage. Their output can be stored for consumption in next period and get a return $\beta_l \leq 1$ per unit of output. Borrowers will consume only in the second period. All public projects have the same probability P_u with $1 \geq P_u \geq 0$ of success.

If a private project is implemented successfully, the private borrower will upgrade to be a firm owner in the coming period. This owner is able to rent capital and hire labor at the rental rate, ρ_t and wage rate, w_t determined by the market competitively. Each firm assumes the following production function : $y_t = \gamma \psi_t^\alpha k_t^\theta L_t^{1-\theta}$.

y_t : units of output at time t .

ψ_t : average capital stock per firm at time t .

k_t : units of capital at time t .

L_t : units of labor at time t .

I assume that $\alpha = 1 - \theta$ and $\gamma = 1 + P_u$. As can be seen from the firm's production function, the higher is the chance of the public project to become successful (a high P_u), the higher the firm's output will be.

Each young lender is endowed with one unit of labor force which is supplied inelastically to the market to earn wage. The lender can convert his time t wage w_t into $Q \cdot \varepsilon \cdot w_t$ units of capital in time $t + 1$ and can rent this amount to firms. Or, the lender can lend the wage to a borrower in exchange for capital in $t + 1$. ε is assumed to be sufficiently smaller than P_h to guarantee that loans between borrowers and lenders occur. Borrowers and lenders are both risk neutral.

Lenders cannot observe the risk level of private investment projects so that the credit rationing problem occurs. But such asymmetry is absent in the public domain where all public borrowers face the same level of investment risk. This restrictive assumption yields us a simple model to depict our main idea.

At the beginning of each period, the lender will be required by the government to allocate an amount of money, q_u , to support a public project. It is assumed that q_u is a time invariant parameter which can be observed by all parties in the economy. Thereafter, the lender will announce a set of contracts. If these contracts are not dominated by others, one borrower will approach him and select a contract. By following Bose & Cothren (1996), we assume that each lender will be approached by one borrower only and the competition in the credit market will drive the lender's economic profit to the reservation level, which is normalized to zero.

The type- H private borrowers have an objective function of the form : $P_h \Pi_{ht} (Q\rho_{t+1} - R_{ht})q_{ht}$. Π_{ht} is the probability to borrow funds from banks. If the borrower borrows q_{ht} amount of funds, he will produce Qq_{ht} units of capital at time $t+1$ with probability P_h . $Qq_{ht}\rho_{t+1}$ is the rent he can earn. R_{ht} is the interest rate paid by this borrower. But if he cannot borrow anything, he will consume nothing next period. A type- L borrower's objective function assumes the following functional form : $P_l \Pi_{lt} (Q\rho_{t+1} - R_{lt})q_{lt} + (1 - \Pi_{lt})\beta_l w_t$. Recall that, if the type- L borrower cannot borrow anything, he can provide his effort to earn wage. We assume the condition $P_i(Q\rho_t - R_t) > \beta w_t > 0$ in what follows to ensure that private borrowers' expected payoff is increasing with Π_{it} .

III. The Separating Equilibrium Contract

The loan contract at time t is in the form $(R_{it}, q_{it}, \Pi_{it})$ where $i \in \{H, L\}$. R_{it} is the gross real rate of interest. q_{it} is the amount of loan offered to the borrower. Apparently, the maximum amount of funds at period t is w_t . Therefore, we have $w_t \geq q_{it} + q_u$. Π_{it} is the probability that the private borrower can borrow funds from the lender.

In equilibrium, the lender will design two different contracts C_L and C_H to separate high risk and low risk borrowers. These contracts will be designed in such a way that each type of agents will only accept the contract designed specifically for him (type L agent

accepts C_L and type H agent accepts C_H). In other words, these contracts must be incentive compatible.

For high risk borrowers, the equilibrium loan contract can be derived by solving the following program (HR) :

$$\begin{aligned} & \underset{\{q_{ht}, R_{ht}, \Pi_{ht}\}}{\text{Max}} P_h \Pi_{ht} (Q\rho_{t+1} - R_{ht}) q_{ht} \quad s.t. \\ & \Pi_{ht} (P_h R_{ht} q_{ht} - Q\varepsilon\rho_{t+1} q_{ht}) - Q\varepsilon\rho_{t+1} q_u = 0 \\ & w_t - q_u \geq q_{ht}, 1 \geq \Pi_{ht} \geq 0 \end{aligned}$$

The lender will maximize the type- H private borrowers' objective subject to three constraints. The first constraint is the lender's zero economic profit constraint. The second constraint specifies the maximum amount of available funds. The last constraint tells the feasible range for the probability Π_{ht} . The solution for this program details the maximum loan amount for high risk borrower $q_{ht} = w_t - q_u$, the gross rate of interest

$$R_{ht} = \frac{1}{P_h} [Q\varepsilon\rho_{t+1} (1 + \frac{q_u}{q_{ht}})] \quad \text{and the probability to get the loan } \Pi_{ht} = 1.$$

Two points warrant further comment about this contract. First, the amount of fund received by the type- H borrower, $w_t - q_u$ is smaller than when there is no public investment in the economy. Second, they need to pay a higher interest rate to get the funds if the government arbitrarily increases the amount of funds to public projects. This is most easy

$$\text{to see that by looking at } \frac{\partial R_{ht}}{\partial q_u} = \frac{Q\varepsilon\rho_{t+1} w_t}{P_h (w_t - q_u)^2} > 0.$$

The optimal contract for low risk type private borrowers is the result of the following program (LR):

$$\begin{aligned} & \underset{\{q_{lt}, R_{lt}, \Pi_{lt}\}}{\text{Max}} P_l \Pi_{lt} (Q\rho_{t+1} - R_{lt}) q_{lt} + (1 - \Pi_{lt}) \beta_l w_t \\ & \Pi_{lt} (P_l R_{lt} - Q\varepsilon\rho_{t+1}) q_{lt} - Q\varepsilon\rho_{t+1} q_u = 0 \\ s.t. \quad & P_h (Q\rho_{t+1} - R_{ht}) q_{ht} \geq \Pi_{lt} P_h (Q\rho_{t+1} - R_{lt}) q_{lt} \\ & w_t - q_u \geq q_{lt}, 1 \geq \Pi_{lt} \geq 0 \end{aligned}$$

with $q_{ht} = w_t - q_u$, $R_{ht} = \frac{1}{P_h} [Q\varepsilon\rho_{t+1}(1 + \frac{q_u}{q_{ht}})]$

Besides the zero economic profit constraint, the resource constraint and the feasible range constraint for Π_{lt} , an incentive compatibility constraint is required to induce type H private borrower to tell the truth regarding his investment risk. This is the second constraint in the above program.

From the first constraint, we can find $R_{lt} = \frac{1}{P_l} [Q\varepsilon\rho_{t+1}(1 + \frac{q_u}{\Pi_{lt}q_{lt}})]$. The second and the third constraints must be binding since q_{lt} only appears at their right hand sides. From the second constraint, we know that $q_{lt} = \frac{(Q\rho_{t+1} - R_{ht})q_{ht}}{(Q\rho_{t+1} - R_{lt})\Pi_{lt}} \Rightarrow \Pi_{lt}q_{lt} = \frac{(Q\rho_{t+1} - R_{ht})q_{ht}}{(Q\rho_{t+1} - R_{lt})}$.

Then, we can find another expression for $R_{lt} = \frac{Q\varepsilon\rho_{t+1}(Q\rho_{t+1}w_t - R_{ht}q_{ht})}{Q\varepsilon\rho_{t+1}q_u + P_lq_{ht}(Q\rho_{t+1} - R_{ht})}$.

By using this equation, we can derive

$\frac{\partial R_{lt}}{\partial q_u} = \frac{Q^2\varepsilon\rho_{t+1}^2(P_l - \varepsilon)}{[Q\varepsilon\rho_{t+1}q_u + P_lq_{ht}(Q\rho_{t+1} - R_{ht})]^2} [q_uq_{ht}\frac{\partial R_{ht}}{\partial q_u} + (Q\rho_{t+1} - R_{ht})w_t] > 0$. Similar to high risk borrowers, low risk borrowers will pay a higher interest rate if the government increases the amount of q_u . Note that R_{ht} and R_{lt} are related because of the binding incentive compatibility constraint. From the third constraint, we know $q_{lt} = w_t - q_u$.

In order to derive the equilibrium value of Π_{lt} as a function of q_{lt} , we put the equilibrium values of R_{ht} and R_{lt} into $q_{lt} = \frac{(Q\rho_{t+1} - R_{ht})q_{ht}}{(Q\rho_{t+1} - R_{lt})\Pi_{lt}}$. By using $q_{lt} = w_t - q_u$, we

find

$$\Pi_{lt} = \frac{P_lq_{ht} - \frac{P_l}{P_h}\varepsilon q_{ht} - \varepsilon q_u(\frac{P_l}{P_h} - 1)}{P_l(1 - \frac{\varepsilon}{P_l})q_{ht}}. \text{ Or, we can further simplify it into}$$

$$\Pi_{lt} = \frac{w_t P_l (1 - \frac{\varepsilon}{P_h}) - q_u (P_l - \varepsilon)}{w_t P_l (1 - \frac{\varepsilon}{P_l}) - q_u (P_l - \varepsilon)}$$

by using $q_{ht} = w_t - q_u$ in equilibrium. Obviously, the

optimal Π_{lt} is a time dependent parameter between zero and one. It is less than one since $P_l > P_h$. It must be greater than zero because the zero economic profit constraint will be violated otherwise. The lender cannot completely exclude all low risk borrowers for that will create negative revenues.

The following proposition will summarize what I have found up to here :

Proposition 1. *The lender will offer the following contracts C_H and C_L in the economy.*

$$R_{lt} = \frac{1}{P_l} [Q\varepsilon\rho_{t+1} (1 + \frac{q_u}{\Pi_{lt} q_{lt}})],$$

Contract C_L : $q_{lt} = w_t - q_u$,

$$\Pi_{lt} = \frac{w_t P_l (1 - \frac{\varepsilon}{P_h}) - q_u (P_l - \varepsilon)}{w_t P_l (1 - \frac{\varepsilon}{P_l}) - q_u (P_l - \varepsilon)}.$$

$$R_{ht} = \frac{1}{P_h} [Q\varepsilon\rho_{t+1} (1 + \frac{q_u}{q_{ht}})],$$

Contract C_H : $q_{ht} = w_t - q_u$,

$$\Pi_{ht} = 1.$$

Corollary 1. *If the government increases the amount of q_u , low risk borrowers will have a smaller probability to get funds.*

Proof :

$$\text{Note that } \frac{\partial \Pi_{lt}}{\partial q_u} = \frac{\varepsilon(P_l - \varepsilon)w_t P_l}{[w_t P_l (1 - \frac{\varepsilon}{P_l}) - q_u (P_l - \varepsilon)]^2} (\frac{1}{P_l} - \frac{1}{P_h}) < 0 \text{ since } P_l > P_h > \varepsilon > 0 . \blacksquare$$

It means that the problem of credit rationing is escalating along with the amount of q_u . This is the "crowding out" effect in my model. However, this effect does not apply to

high risk borrowers whose probability of being funded is equal to 1. Of course, the amount of fund they get decreases after the public investment is introduced.

Proposition 1 shows that the equilibrium contracts depend on the rate of return of capital. In the following section, I will show that the equilibrium growth rate not only depends on this rate of return of capital but also the externality effect exerted by public investment.

IV. The Growth Rate

First, we will compute the quantity of labor employed by each firm. Those private borrowers (low and high risk) who can successfully borrow funds from banks will become firms in the second period of their lives. Therefore, we have $0.25[\lambda P_h + (1-\lambda)P_l\Pi_t]$ firms operating at period t . $0.25(1-\lambda)(1-\Pi_t)$ of low risk private firms are credit rationed. So, the total labor supply in the economy is $0.5 + 0.25(1-\lambda)(1-\Pi_t)$. In other words, the quantity of labor employed by each firm is :

$$L_t = \frac{0.5 + 0.25(1-\lambda)(1-\Pi_t)}{0.25[\lambda P_h + (1-\lambda)P_l\Pi_t]} = \frac{2 + (1-\lambda)(1-\Pi_t)}{\lambda P_h + (1-\lambda)P_l\Pi_t}.$$

From the production function defined above, we can derive the wage rate and the marginal product of capital. $w_t = \frac{\partial y_t}{\partial L_t} = (1-\theta)(1+P_u)\psi_t^\alpha k_t^\theta L_t^{1-\theta} = (1-\theta)(1+P_u)k_t L_t^{-\theta}$

and $\rho_t = \frac{\partial y_t}{\partial k_t} = \theta(1+P_u)\psi_t^\alpha k_t^{\theta-1} L_t^{1-\theta} = \theta(1+P_u)L_t^{1-\theta}$. Since $L_t = \frac{2 + (1-\lambda)(1-\Pi_t)}{\lambda P_h + (1-\lambda)P_l\Pi_t}$,

$$\rho_t = \theta(1+P_u)L_t^{1-\theta} = \theta(1+P_u) \left[\frac{2 + (1-\lambda)(1-\Pi_t)}{\lambda P_h + (1-\lambda)P_l\Pi_t} \right]^{1-\theta} \quad \text{and}$$

$$w_t = (1-\theta)(1+P_u) \left[\frac{2 + (1-\lambda)(1-\Pi_t)}{\lambda P_h + (1-\lambda)P_l\Pi_t} \right]^{-\theta} k_t.$$

From these equations, one can see how the positive externality effect exerted by positive projects is influencing the economy's growth. A high P_u will increase the marginal product of labor, w_t , which will in turn help to accumulate capital stock for the economy.

Because of that, all private projects will have a higher chance to get funds in the future given a fixed q_u in each period.

The capital stock at $t + 1$ comes from those funded borrowers who can run their projects successfully, the successfully implemented public projects and the lenders who converted their wages into capital stock after they rationed the low risk type private borrowers. Algebraically, their contributions are equal to $0.25[\lambda P_h + (1 - \lambda)P_l \Pi_t]Q(w_t - q_u)$, $0.25P_u Qq_u$ and $0.25(1 - \lambda)(1 - \Pi_t)Q\epsilon w_t$ respectively.

The aggregate capital stock at time $t + 1$ is :

$$K_{t+1} = 0.25[\lambda P_h + (1 - \lambda)P_l \Pi_t]Qw_t + 0.25(1 - \lambda)(1 - \Pi_t)Q\epsilon w_t + 0.25\{P_u - [\lambda P_h + (1 - \lambda)P_l \Pi_t]\}Qq_u \dots\dots\dots(*)$$

Now we are ready to see how the "crowding out" effect is affecting the aggregate capital accumulation of the economy. Given P_u , an increase of investment in public projects (q_u) will reduce Π_t and w_t so that the first term of equation (*) will decrease. But the effect on the second term which is a multiple of $(1 - \Pi_t)$, w_t and other parameters is mixed. It means that less private borrowers can get funds to produce capital but more lenders who cannot lend their money out may help to do the job. But with a smaller amount of w_t , each of these lenders will produce a smaller amount of capital. In addition, by assuming a small ϵ , the effect from the reduction of the first term will dominate easily. To explain why w_t goes down when q_u goes up, recall that only private projects will become firms next period. As q_u goes up and then Π_t goes down, the number of firms in the economy will be getting smaller and L_t becomes larger. The supply of labor increases and labor demand decreases. As a consequence, w_t will go down.

Depending on the quality of public projects (or equivalently, the value of P_u), the last term of equation (*) can be bigger than, equal to or less than zero. Then we have the following three different cases to discuss.

Case (i) $P_u > \lambda P_h + (1 - \lambda)P_l\Pi_t$ (the probability of success of public projects is higher than the *expected* probability of success of private projects). Then, the last term of (*) is positive. The aggregate capital accumulation of this economy will improve.

Case (ii) $P_u = \lambda P_h + (1 - \lambda)P_l\Pi_t$. The last term of (*) vanishes. The aggregate capital accumulation deteriorates.

Case (iii) $P_u < \lambda P_h + (1 - \lambda)P_l\Pi_t$. The last term of (*) is negative. In this case, the capital accumulation is getting worse, too.

But these three cases are not mutually exclusive. Actually, they may happen cyclically in the course of the country's economic development. Recall that Π_t is an endogenous variable which is a function of w_t , q_u and other parameters. Consider $P_u > \lambda P_h + (1 - \lambda)P_l\Pi_t$ so that aggregate capital is always accumulating. Then, Π_t will go up since more low risk private borrowers get money from banks to produce capital. The left hand side of the inequality will become bigger and eventually, the inequality sign will be reversed. Given $\frac{\partial \Pi_t}{\partial q_u} < 0$, it is not optimal for the government to set $q_u = 0$ if it wants to maximize aggregate capital accumulation. Instead, the government can increase q_u to reduce Π_t so that the inequality sign reverses back to the original mode again. But by doing that, more low risk private projects will be crowded out from the economy. The government will continue to do so until, at some point, these two ways of accumulating capital - the private and the public - yield the same marginal return of investment. Implicitly, this argument indeed indicates that an optimal level of q_u exists in my model.

We can summarize the above discussion in the following intuitive way - the "quality" of public projects which substitute their private counter-parts are important to the capital accumulation of the economy. If their probability of success is higher, the chance to improve the capital accumulation will also be higher. But even with a high P_u , the government cannot completely exclude all private projects from the economy by replacing them with public projects for two reasons. First, only successful private projects

will become productive next period. Second, the high externality effect induced by a high P_u will gradually improve the chance of low risk borrowers to get funds. That will make the "crowding out" effect so costly that some private investment must remain.

The capital stock owned by each firm at time $t + 1$ is the following :

$$\begin{aligned}
k_{t+1} &= \frac{0.25[\lambda P_h + (1-\lambda)P_l\Pi_t]Q(w_t - q_u) + 0.25P_u Qq_u + 0.25(1-\lambda)(1-\Pi_t)Q\varepsilon w_t}{0.25[\lambda P_h + (1-\lambda)P_l\Pi_t]} \\
&= \frac{[\lambda P_h + (1-\lambda)P_l\Pi_t]Qw_t + (1-\lambda)(1-\Pi_t)Q\varepsilon w_t}{\lambda P_h + (1-\lambda)P_l\Pi_t} + \left\{ \frac{P_u - [\lambda P_h + (1-\lambda)P_l\Pi_t]}{\lambda P_h + (1-\lambda)P_l\Pi_t} \right\} Qq_u \\
&= \left\{ 1 + \frac{(1-\lambda)(1-\Pi_t)\varepsilon}{\lambda P_h + (1-\lambda)P_l\Pi_t} \right\} Qw_t + \left\{ \frac{P_u}{\lambda P_h + (1-\lambda)P_l\Pi_t} - 1 \right\} Qq_u
\end{aligned}$$

Note that when $q_u = 0$,

$$k_{t+1} = \left\{ 1 + \frac{(1-\lambda)(1-\Pi_t)\varepsilon}{\lambda P_h + (1-\lambda)P_l\Pi_t} \right\} Qw_t \text{ which is the result in Bose and Cothren (1996, p. 370).}^{11} \text{ This result will be used as a benchmark in the following discussion.}$$

V. Simulations

Since analytical results are difficult to obtain, I will use two examples to illustrate how the growth rate of an economy will be affected by the presence of public projects. In the first example, P_u assumes a value which is higher than P_h but smaller than P_l . The following table shows the simulation results.

Example 1 : $\lambda = 0.75$, $P_h = 0.4$, $P_l = 0.7$, $P_u = 0.45$, $Q = 2$, $\varepsilon = 0.1$, $q_u = 0.45$, $\theta = 0.3$

¹¹ Though with the same notations, w_t and Π_t in (*) are different to those in the benchmark. So the comparison can only provide intuitions about how the story goes. It is not a rigorous proof.

| t | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| g_t | - | 1.528 | 1.344 | 1.319 | 1.310 | 1.306 | 1.304 | 1.303 | 1.303 |
| K_t | 0.392 | 0.599 | 0.806 | 1.063 | 1.393 | 1.819 | 2.372 | 3.091 | 4.028 |
| w_t | 0.612 | 0.884 | 1.173 | 1.537 | 2.006 | 2.613 | 3.402 | 4.429 | 5.767 |
| Π_{it} | 0.528 | 0.745 | 0.797 | 0.823 | 0.839 | 0.849 | 0.856 | 0.861 | 0.864 |
| k_t | 1.000 | 1.393 | 1.834 | 2.394 | 3.117 | 4.054 | 5.273 | 6.859 | 8.926 |
| L_t | 5.398 | 4.795 | 4.666 | 4.603 | 4.566 | 4.543 | 4.527 | 4.515 | 4.507 |
| y_t | 4.719 | 6.052 | 7.817 | 10.11 | 13.08 | 16.96 | 22.00 | 28.57 | 37.13 |
| Y_t | 1.852 | 2.605 | 3.436 | 4.489 | 5.869 | 7.608 | 9.896 | 12.88 | 16.76 |
| C_t | 1.459 | 2.005 | 2.629 | 3.426 | 4.477 | 5.789 | 7.524 | 9.784 | 12.73 |

$g_t (= \frac{K_{t+1}}{K_t})$: growth rate at period t .

$C_t (= Y_t - K_t)$: aggregate consumption goods produced by all firms at period t .

The capital being accumulated per firm (k_t) and the amount of funds available per worker (w_t) are increasing starting from the first period. As a result, the chance for those low risk projects to get funds is gradually improving. Aggregate capital (K_t) is growing because k_t and the total number of firms are both increasing over time. The number of workers per firm (L_t) is decreasing since the number of firms is increasing. Thus, each firm will hire less labor. Total output per firm is increasing due to the reason that the increase of capital accumulation can completely offset the drain of labor force. It is remarkable to note that the trend of Y_t and C_t is rising all the time.

To appreciate the role of public projects in this economy, one can compare our results with the case when no public projects exist in the economy, that is when $q_u = 0$. (our benchmark). In that case, $\Pi_t = 0.875$, $w = 0.547$ and the growth rate, $g = \frac{k_{t+1}}{k_t} = \frac{K_{t+1}}{K_t}$ is a constant which equals 1.10. Low risk private borrowers have a better chance to be funded. Since there is no public projects, the amount of financial resource that is required to support the economy is smaller. Nonetheless, the growth rate is smaller than the one in example 1. Therefore, this example exhibits the positive role the public projects can play for the growth of an economy.

Example 2 : $\lambda = 0.75$, $P_h = 0.4$, $P_l = 0.7$, $P_u = 0.35$, $Q = 2$, $\varepsilon = 0.1$, $q_u = 0.45$, $\theta = 0.3$

In this situation, we have

| t | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| g_t | - | 1.324 | 1.099 | 1.090 | 1.089 | 1.092 | 1.097 | 1.103 | 1.111 |
| K_t | 0.352 | 0.465 | 0.512 | 0.557 | 0.607 | 0.662 | 0.726 | 0.801 | 0.890 |
| w_t | 0.547 | 0.661 | 0.716 | 0.773 | 0.836 | 0.907 | 0.990 | 1.088 | 1.205 |
| Π_{lt} | 0.295 | 0.609 | 0.664 | 0.701 | 0.729 | 0.752 | 0.771 | 0.787 | 0.800 |
| k_t | 1.000 | 1.145 | 1.229 | 1.319 | 1.419 | 1.534 | 1.670 | 1.831 | 2.023 |
| L_t | 6.189 | 5.159 | 5.007 | 4.909 | 4.836 | 4.778 | 4.730 | 4.691 | 4.659 |
| y_t | 4.836 | 4.875 | 5.124 | 5.423 | 5.774 | 6.189 | 6.690 | 7.293 | 8.019 |
| Y_t | 1.702 | 1.979 | 2.135 | 2.290 | 2.469 | 2.691 | 2.908 | 3.190 | 3.528 |
| C_t | 1.350 | 1.515 | 1.623 | 1.733 | 1.863 | 2.029 | 2.182 | 2.389 | 2.638 |

Note that the value of P_u is smaller than P_h in this case. Despite that, the economic growth rate can still be higher than the one in the benchmark case. That means, to sustain a growth rate higher than the benchmark case, the quality of public projects which will substitute private projects is not required to be strictly better than the latter.

If P_u is smaller than 0.33, the private sector will vanish in the sense that Π_{lt} is equal to 0 since P_u is too small to generate enough capital to support the development of both private and public projects. When most of the capital is being extracted by the public sector, nothing will be left over for private investors and all low risk borrowers will be crowded out as a result.

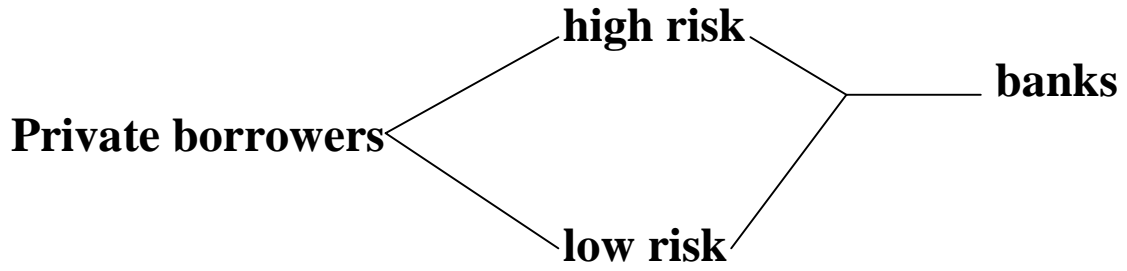
VI. Conclusion

The potentially beneficial role of public investment is confirmed by the above analysis in the context of an endogenous growth model. As the economy evolves with private projects of good or bad quality and the credit rationing contract is the only means available to the financial intermediary to induce self selection from borrowers, the government can consider public investment as a strategic instrument to crowd out some

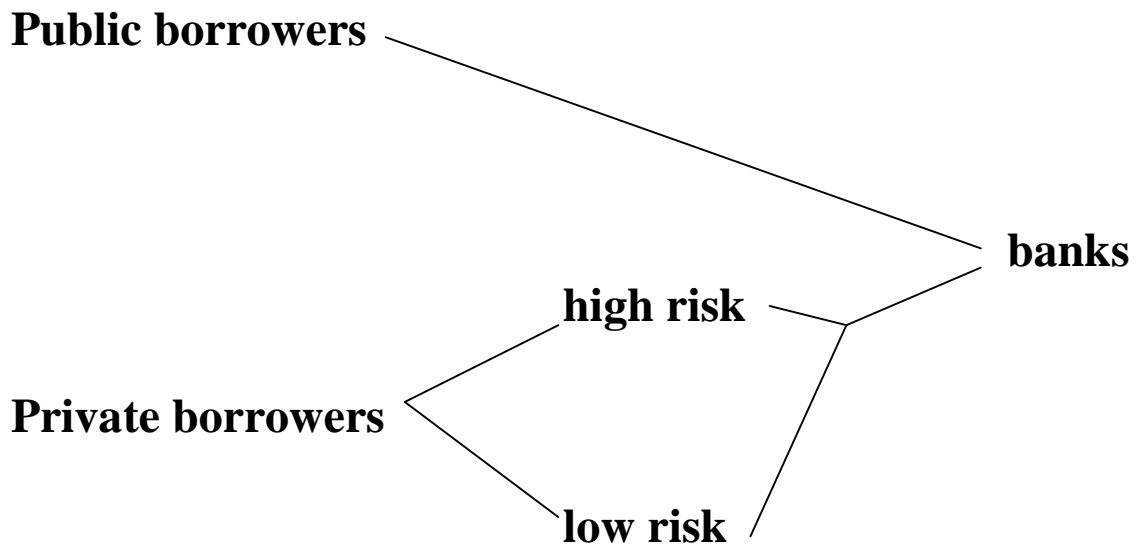
of these private projects in order to promote growth. The prerequisite is that all public projects are of good and equal type. One somewhat striking result we find is that, even if the probability of success of the public projects is smaller than those high risk private projects, the economic growth rate can still be higher than the benchmark case in which no public projects exist. This conclusion can be considered as a supplement to the view that positive economic growth is still possible when there is asymmetric information and its induced credit rationing problem in the economy, e.g. Shi (1996). In this paper, the possibility for borrowers to switch their project types is the main mechanics which makes higher capital accumulation thus higher growth possible. But we suggest that the government can perform in an active manner to alleviate the credit rationing problem so that a higher economic growth rate can be reached.

The "crowding out" effect in my model is describing the situation that more low risk borrowers will be rationed in the credit market with asymmetric information if there is public investment. But note that the meaning of this term does not exactly overlap with the one in the standard macroeconomic literature (see Barro, 1997). Even if there is no credit rationing, the government expenditure still can crowd out some private economic activities. However, one thing is common between our and the standard usage of the term. The crowding out effect works through the interest rate mechanism.

One conceivable extension of my paper is to endogenize the value of q_u , it will be interesting to study if there exists an (optimal) amount of funds for public projects which maximize the economic growth rate. If it exists, what properties does it have ? Second, if we relax the assumption about the absence of information asymmetry in the public sector, how will the model be complicated ?



B-S-B-C Model



My Model

Figure 1. A comparison between B-S-B-C model and my model.

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