

Chapter 2: Privatization, Diffusion of Share Ownership, and Politics

1. Introduction

Since the 1980s, many national governments have pursued privatization programs. Although the word "privatization" encompasses a variety of government policies, from deregulation policies to the sale of state-owned enterprises to the private sector, a central feature of privatization policies is the change from public to private ownership. The scale of these programs varies across countries, but the trend toward privatization is worldwide.

The biggest privatization program in a capitalist country to date was implemented by the conservative Thatcher government in Britain, primarily in the 1980s¹. In 1979, at the beginning of the privatization process, state-owned enterprises accounted for about 10.5 percent of the country's GDP. Today almost everything has been privatized, from quasi-competitive activities, like transportation, automobile manufacturing, steel and port operations, to public utilities like telecommunication, gas, electricity, and water. With the exception of postal services, which are still in public hands, all British public utilities are now private enterprises.

The British Conservative government adopted its privatization plans with at least three intended (and declared) goals:

- improve the economic efficiency of the enterprise

¹ For a detailed study and evaluation of the British privatization program, see Vickers and Yarrow (1988).

- reduce the Public Sector Borrowing Requirement (PSBR) or, more generally, raise revenues for the government
- widen share ownership among small investors, employees, and the general public, encouraging them to buy and hold shares of privatized companies through various special arrangements (installment payments, loyalty bonuses, special discounts, and low share prices).

Whether the change in ownership structure is in itself sufficient to improve efficiency is the subject of continual debate. What is not controversial is that the sale of public enterprises can raise significant revenue. Privatization programs in the United Kingdom have raised tens of billions of pounds for the Treasury.

Nevertheless, these revenues could have been much higher. In fact, empirical evidence shows that the British government frequently underpriced shares of the newly privatized firms when it sold them to the public. For example, Jenkinson and Mayer (1988) and Vickers and Yarrow (1988) show that the average discount on offers for sale of state enterprises in Britain has been much higher, as a percentage of gross proceeds, than the discount usually observed in Initial Public Offerings of private issues².

Table 1 lists the major sales carried out by the British government from 1977 to 1991. For each sale, the table reports the percentage of the stock sold, the degree of underpricing (as measured by the percentage difference between the offer price per share and its market value at the end of the first day of trading), and the demand multiple. Most companies were sold using a fixed-price offer (when the government sets the price of the shares), while only few sales used a tender offer (when applicants bid a price above a standard minimum set by the government).

Typically premiums were high, and the issues were oversubscribed.

²Jenkinson and Mayer also report similar evidence for privatizations in France.

Table 1. Privatizations in The United Kingdom: 1977-1991.

Company	Date of sale	Stake sold (percentage)	Discount (percentage)	Demand multiple*
British Petroleum	June 1977	17	22.7	4.7
	November 1979	51	1	1.5
	September 1983	7	1 ^a	2.7
	October 1987	36.8	13 ^b	--- ^c
British Aerospace	February 1981	50	14	3.5
	May 1985	--- ^c	12	5.4
Cable and Wireless	November 1981	49	17.3	5.6
	December 1983	31	-1 ^a	0.7
	December 1985	22	0.0 ^a	2
Amersham	February 1982	100	32.4	25.6
Associated British Ports	February 1983	51.5	23.2	35
	April 1984	48.5	0.1 ^a	1.6
Jaguar	August 1984	100	8.5	8.3
British Telecom	December 1984	50.2	33	5
	December 1991	23.9	14.1	2.5
	July 1993	22	--- ^c	--- ^c
Enterprise Oil	July 1984	100	0 ^a	0.7
Britoil	November 1982	51	-8.8 ^a	0.3
	August 1985	49	12	10
Trustee Savings Bank	October 1986	100	35.5	8
British Gas	December 1986	100	9.2	4
British Airways	February 1987	100	35.2	32
Rolls Royce	May 1987	100	36.5	9.4
BAA	July 1987	100	18.8	8
British Steel	December 1988	100	3.3	3.3
Anglian Water	December 1989	100	48.5	2.2
N.W. Water Group	December 1989	100	35	1.6
Northumbria Water	December 1989	100	57	9.0
Severn Trent	December 1989	100	31	1.8
S.W. Water	December 1989	100	47	1.8
Southern Water	December 1989	100	41	3.4
Thames Water	December 1989	100	36	4.3
Welsh Water	December 1989	100	41	2.1
Wessex Water	December 1989	100	54	4.0
Yorkshire Water	December 1989	100	49	2.6
East Midlands Electric	December 1990	100	50.5	9.5
Eastern Electric	December 1990	100	48	9.2
London Electric	December 1990	100	42	8.1
Manweb	December 1990	100	66	15.4
Midlands Electric	December 1990	100	50	9.5
Norweb	December 1990	100	52	11.7
Northern Electric	December 1990	100	4.5	13.7
Seaboard	December 1990	100	2	13.2
South Wales Electric	December 1990	100	64	15.8
South Western Electric	December 1990	100	50	12.2
Southern Electric	December 1990	100	50	11.6
Yorkshire Electric	December 1990	100	59.5	7.7
National Power	March 1991	60	37.5	5.4
PowerGen	March 1991	60	37	5.4
Scottish Hydro-Electric	June 1991	100	22	3.0
Scottish Power	June 1991	100	15.5	3.0

Source: Enrico C. Perotti, (1995).

* Ratio of demand to supply of stock.

^a Tender sale.

^b Issued during the October 1987 stock crash. The price fall of the issue was less than the general price decline.

^c Information not available.

Even in the long term, premiums on new issues were high, confirming that shares of newly privatized firms were a good long-term investment. This suggests a poor pricing decision by the government. The very few cases in which the value of shares of privatized enterprises fell (or did not increase) were those that were in the midst of an industry wide crisis (Britoil, Enterprise Oil) and the partial sale of Cable and Wireless in 1983.

On the one hand, the underpricing phenomenon apparently contradicts the declared objective of raising revenues. On the other hand, if the government wants small investors to buy shares, then low prices might be justified. But why would the government care about diffusion of share ownership in the first place?

This paper provides one explanation of the government's desire for wide share ownership, and it shows that high diffusion of asset ownership and low share price can be chosen by a government that wants to maximize revenues.

Consider a dynamic game of complete information, in which a government first privatizes a company and then competes for votes against an opposition party. The government's objective is to choose the price of shares and the level of promotional effort to maximize its expected net revenues. Risk neutral voters decide how many shares to buy, if any. They have identical exogenous income but are differentiated by their transaction costs. Alternatively, the government can decide to sell the company to a non voting "foreign" investor, who has an unlimited budget and would buy the entire company for any price that does not exceed its true value. After the sale of the company to the public, the two parties announce what expropriation rate they would implement if they win the upcoming election. High promotional effort generates diffusion of asset ownership, which in turn lowers the expropriation rate that the two parties will simultaneously announce in equilibrium in the political game. I show that it is optimal for the incumbent party to underprice the stock with respect

to its true value. This is done to increase the size of the shareholders' interest group, which will favor the party announcing a lower expropriation rate in the second period; investors' anticipation of a low expropriation rate allows the government to sell the company at a higher price. The government is willing to sacrifice future revenues from expropriation because it may not be in power when those revenues are collected. Typically, the optimal policy is characterized by underpricing of shares, a positive level of promotional effort, and wide share ownership.

The existing literature on privatization appears not to have analyzed privatization and underpricing in such a political framework.

Sale methods and share prices in privatization have been analyzed by Branco and Mello (1991) and Perotti (1995), in the context of a signaling game between the government and investors. They offer the following explanation of underpricing and gradual or complete sales observed in privatization programs³. A government that decides to sell a firm to private investors can be of two types: a type committed to avoiding interference after the sale, or an uncommitted type, which cannot resist adopting ex-post policies that will reduce the value of the private stock. The government's type is private information. Both types strive to maximize sale proceeds, and the type that intervenes also gains from interference. Since investors are willing to pay more if there is no interference, the committed type wants to signal its commitment, while the intervening type has an incentive to imitate the other type's strategy. Using a two-period signaling model, the authors show that, for certain ranges of values of the parameters, a separating equilibrium exists in which the committed government sells only a fraction of the firm

³There is a major difference between the two analyses. While Branco and Mello assume that profits are exogenously given (equal to zero under public ownership, and growing at a constant rate when the firm is privatized), Perotti endogenously derives profits from specific assumptions concerning managers' incentives under the different ownership structures. Perotti makes also the rather restrictive assumption that managers of the firm are also the future private owners.

in period one and completes the sale in period two, while the uncommitted type sells the entire stock in the first period. There is a crucial value of the fraction sold in period one at (and below) which the uncommitted government's payoff from interference would be larger than the gains from avoiding interference in the first period and obtaining a higher price in the seasoned offer. For some parameter values, a separating equilibrium through partial sale only does not exist. In this case, the authors show that the committed government can combine underpricing with a partial sale offer to induce separation⁴. Underpricing is a dissipative signal and it affects the two types' payoffs in the same way. Therefore underpricing alone cannot generate separation. Also, an offer for sale must be chosen by the government to strategically manipulate the price of shares. If the government is a committed type, the model predicts that we should observe either tender offers with partial sales and no underpricing, or a fixed-price offer with partial sales and underpricing. The optimal equilibrium strategy for an uncommitted type is always to sell all at once through tender.

The present model differs in three fundamental ways from the signaling models of privatization. First, this model does not require that political parties have hidden characteristics: all actors in the model have complete information about the two parties' objectives. In fact, the two parties are identical with respect to their preferences and the decisions that they make in this model, although one party has the advantage of incumbency. Second, the rate of expropriation is determined endogenously, in the context of the electoral competition, when each party chooses to announce the rate of future expropriation that maximizes its chances of winning the election

⁴These results are similar to those obtained in the finance literature to explain underpricing in IPOs of private issues (see for example Allen and Faulhaber 1989, Grinblatt and Hwang, 1989, and Welch 1989). In that case, the asymmetry of information between investors and issuer regards the true value of the firm. As mentioned in the introduction, the average discount observed for the sales of state enterprises has been much higher than the discount observed for private issues.

and its future expected financial gains from expropriation. Third, the signaling models of privatization predict underpricing only when there is a partial sale of the state-owned company, but this result cannot explain the historical evidence (as presented in Table 1) of complete sales at prices lower than the companies' true values. The present model generates exactly this prediction, and it provides a rationale for underpricing and diffusion of share ownership as outcomes of a privatization policy aimed at maximizing financial and political gains.

Section (2) describes the timing of the model, the parties' objectives, and the voters' behavior, both as investors in the first period and as voters in the second period. Section (3) solves the election subgame, and section (4) studies the SPNE of the whole game. Section (5) contains final comments.

2. Timing and players' objectives

Consider a two-party democracy. The parties are indexed by $k \in \{G, O\}$, where G and O denote the party in power and the opposition party respectively. The voters are indexed by $i \in I \equiv [0, 1]$. Each voter i is characterized by $(c_i, b_i) \in [0, \bar{c}] \times [-r, r]$, where \bar{c} and r are exogenous, c_i measures voter i 's cost of engaging in transactions, and b_i measures voter i 's personal bias toward one party or the other, which exists for reasons outside the model.

The voters are distributed uniformly over $[0, \bar{c}] \times [-r, r]$; therefore, c_i and b_i are statistically independent of each other and across voters. The transaction cost c_i determines voter i 's investment behavior, while b_i determines voter i 's tendency to vote for one party or the other. Each (infinitesimal) voter is initially endowed with the same amount of exogenous

income, or wealth, W . The total mass of the population is normalized to one; therefore, W is the individual income as well as the total aggregate income in the economy.

A risk neutral “foreigner”, endowed with an unlimited budget, seeks to maximize his expected wealth. Alternatively, one can think of a rich domestic investor, who has only an infinitesimal, and therefore irrelevant, impact on the electoral process.

There are two periods. In the first period, the government G sells to the public a single state-owned enterprise. The government sells the entire firm, offering to private investors a certain total number of shares, here normalized to one for simplicity. The government announces a share price P , and it exerts promotional effort $e \in [0,1)$, with total cost $A(e)$. Since the total number of shares is normalized to one, P represents the sale value of the entire offering, as well as the price of a single share. Assume that $P < W$.

Each voter decides how many shares of the firm to buy, if any, and no voter is willing to pay more than the stock’s fair value. I also assume that, for a price that does not exceed the stock’s fair value, the government can sell any number of shares to the foreign investor. Since no potential investor is willing to pay for the shares more than their fair value and since, for a price not exceeding the fair value, the foreigner is always willing to purchase all unsold shares, in equilibrium the government never sets a price exceeding the stock’s fair value and, at the chosen price, it sells the entire offering.

In the second period, the government G and the opposition party O compete for votes in an election. Let S denote the fraction of the population that bought shares in the first period. The voters are partitioned into two interest groups: the shareholder group S , and the non-shareholder fraction $N \equiv 1-S$. The two parties simultaneously announce their platforms. In particular, they announce the expropriation rate τ_k that they would impose on private profits if

they win the election. The expropriation rate can be interpreted broadly as any confiscatory policy or profit tax that reduces the value of the private company. The extreme case of government interference is the re-nationalization of the private company, i.e. $\tau_k=1$.

After the platform announcements, the election is held and each voter i votes for one of the two parties. The newly privatized company makes profits equal to π , with $\pi>0$. Finally, the winning party k expropriates the private profits at the promised rate τ_k . I assume that the newly elected government fulfills its commitment. Of course, this assumption ignores the realistic issue of whether the elected party would actually keep its promises. The implicit assumption is that voters remember election promises, and each party faces prohibitively large political losses if it deviates from its announced platform when in power.

The time at which the government announces its choice of τ_G is a modeling assumption that requires some explanation. Although in period one the government is able to anticipate its optimal strategy in period two, I assume that it cannot precommit itself in the first period to a particular degree of future interference. This choice of the timing of commitment reflects a specific assumption about voters' behavior: voters in this model remember (and punish deviations from) election promises only, and they ignore promises made before the election campaign⁵. Although exogenously given in the model, this choice of the timing of commitment captures an important aspect of reality: in reality promises do count, but they tend to count less when they are made much in advance. It often happens that unexpected events and new

⁵ Alternatively, one could assume that private profits are random, and that in the first period some proxy of the profitability of the company is private information of the government. In such a model, the timing of the commitment of the government to implement a certain rate of expropriation would be derived endogenously.

circumstances cause old promises, and old commitments to those promises, to lose credibility over time.

2.1. The parties' objectives

The government and the opposition are self-interested, maximizing agents. They care only about their own financial gains and about their chances of winning the election. When both parties play their equilibrium strategies in the second period election game, μ_G denotes the government's probability of winning the election, and τ_G denotes the rate of expropriation that the government announces in equilibrium just before the election, and which it would implement if reelected.

The objective of the government is to choose the share price P and the level of promotional effort e that maximize

$$\mathbf{a}[P - A(e)] + \mathbf{m}_G(\mathbf{b} + \mathbf{a}t_G\mathbf{p}) \quad [1]$$

The exogenous parameter $\mathbf{a} \in (0,1)$ represents the fraction of net revenues from the sale $(P - A(e))$ and later expropriation $(t_G\mathbf{p})$ that directly benefits the government. One interpretation of α is that it is the percentage of financial earnings from the privatization policy that the party in power is able to use for its own personal benefit: money that the party holding office is able to invest on activities and/or goods that benefit only the party or its supporters. Alternatively, α can be interpreted as a measure of the degree of corruption in the political system.

The remaining fraction $(1-\alpha)$ of the financial revenues is spent by the party in office to provide a quantity q of public good. How q is defined will be discussed later, when the individual investment decision is analyzed. The exogenous parameter $b \geq 0$ represents the intrinsic (i.e. nonfinancial) benefit that the party receives from holding office. The future in this model is not discounted, therefore the discount rate is assumed to be zero.

$A(e)$ measures total costs of the promotional effort $e \in [0,1)$ undertaken by the government when it sells the company to the public. The promotion of the sale reduces the transaction costs of the potential investors, therefore increasing the fraction of people who buy shares. I assume that $\lim_{e \rightarrow 1} A(e) = +\infty$, with $A(0) = A'(0) = 0$ and $A', A'' > 0$ for all $e \in (0,1)$ ⁶.

In the first period, the government chooses a privatization “package” (P, e) that maximizes its objective in [1] subject to one constraint: to sell the company to any domestic or foreign investors, the government must choose a share price that does not exceed the future net expected value of the firm. Therefore the government must choose:

$$P \leq (1-\tau^*)\pi \quad [2]$$

where $\tau^* = m_G t_G + (1 - m_G) t_O$ is the expected rate of expropriation. The platforms t_G and t_O depend on both P and e . The government is otherwise unconstrained: in particular, the government will always be able to sell the entire company, because it can always choose to sell all the shares to the foreign investor at their true value.

The opposition party observes the government’s choices of P and e in the first period, and in the second period, before the election, it competes for votes against the party in power. The

⁶ Hereafter, single and double primes denote first and second derivatives of functions of one variable.

parties announce their platforms simultaneously. Specifically, the opposition announces the value of τ_o that maximizes:

$$m_o(\mathbf{b} + \alpha \tau_o \mathbf{p}) \quad [3]$$

where μ_o is the opposition party's probability of winning the election. For simplicity, assume that the values of the parameters α and β are the same for both parties.

The equilibrium of the political game, as shown in Section (3) of the paper, will be a symmetric equilibrium, in which both parties announce the same expropriation rate and are equally likely to win the election.

2.2. The voters' preferences

In the first period, each voter i , $i \in [0,1]$, decides how many shares of the state-owned company to buy (if any) at the announced stock price P . In the second period, voters vote, given the parties' platform announcements (t_g, t_o) .

Of the Sections that follow, Section (2.2.1.) studies the individual investment decision and Section (2.2.2.) studies individual voting behavior. Finally, Section (2.2.3.) derives the equations that describe each party's expected margin of victory, as well as its probability of winning the election.

2.2.1. The investment decision

At the beginning of the first period, each voter is endowed with the same amount of income W . Each voter i also has an idiosyncratic transaction cost parameter c_i . These

transaction characteristics are uniformly distributed over the real interval $[0, \bar{c}]$; therefore, for any $c \in [0, \bar{c}]$, the fraction of the population having a transaction parameter not exceeding c is $\frac{c}{\bar{c}}$.

Each voter must decide how many shares of the firm to buy. Let d_i denote the number of shares demanded by voter i . In making the investment decision, each voter considers his private payoff. Specifically, voters derive utility from the consumption of a public good and from personal income that they spend on private goods. They consume the quantity q of the public good, which is provided by the party in power using the fraction $(1-\alpha)$ of the revenues from the sale and from the expropriation. The unit price of the public good is normalized to one. Voter i also has a certain amount of disposable income y_i , which is the sum of his initial endowment W and any net financial gains that result from the individual investment decision. The utility that voter i derives from consumption is $V(q, y_i)$. Assume⁷ a quasilinear utility function, of the form $V(q, y_i) = f(q) + y_i$. Quasilinearity implies that:

$$\begin{aligned} V_{y_i} &= 1 \\ V_{y_i y_i} &= 0, \quad V_{q y_i} = 0 \end{aligned} \quad [4]$$

In addition, assume that $V(0,0)=0$, $V_q > 0$, and $V_{qq} < 0$.

Finally, if voter i buys any strictly positive number of shares, he will incur a utility loss from participating in the transaction. These transaction costs are higher for people having a higher value of the personal transaction characteristic c_i , and they are lower for higher levels of promotional effort e exerted by the government. This assumption reflects the idea that the party in power wants the information concerning the sale to be easily and cheaply available to all

⁷ From now on, subscripts denote partial derivatives of functions of several variables.

voters, and that, to make the sale attractive to potential buyers, it adopts special promotional campaigns⁸. Voter i 's transaction costs T_i equal:

$$T_i = \begin{cases} c_i - e & \text{if } c_i \geq e \\ 0 & \text{if } c_i < e \end{cases}$$

Voter i 's total utility is:

$$u_i = \begin{cases} V(q, y_i) - T_i & \text{if } d_i > 0 \\ V(q, W) & \text{if } d_i = 0 \end{cases} \quad \forall i \in [0,1]$$

where

[5]

$$q = (1 - \mathbf{a})[P - A(e) + \mathbf{t}p]$$

$$y_i = W + [(1 - \mathbf{t})p - P]\tilde{d}(d_i, S)$$

$$\tilde{d}(d_i, S) = \begin{cases} d_i & \text{if } Sd_i \leq 1 \\ \frac{1}{S} & \text{if } Sd_i > 1 \end{cases}$$

Here q denotes the quantity of the public good purchased by the government, y_i denotes voter i 's individual disposable income, τ represents the actual, realized expropriation rate, and $\tilde{d}(d_i, S)$ denotes the number of shares actually received by voter i . For a chosen price P and a given level of promotional effort e , the issue can be oversubscribed, in which case the government uses a rationing scheme that reduces each individual's demand by the same proportion. If S is the total fraction of the population that buys the stock, then $Sd_i > 1$ implies oversubscription, and each investor will receive $1/S$ shares. Voters anticipate this possibility when taking their investment decision.

⁸ The British Conservative Government, for example, distributed vouchers that could be used to reduce bills (such as phone bills or electricity bills) by all those who bought shares of the privatized public utilities.

Each voter i chooses d_i to maximize his utility in [5], subject to the constraint imposed by his limited income: $0 \leq d_i \leq \frac{W}{P}$.

Assume that changes in the individual (infinitesimal) demand d_i do not influence the choice of τ in the second period. Hence $\frac{dt}{dd_i} = 0$, implying $\frac{dq}{dd_i} = 0$. This assumption is reasonable in this framework. Since voters are distributed uniformly over $[0, \bar{c}] \times [-r, r]$, each voter is too small for his behavior to affect aggregate outcomes.

Inspection of the maximization problem reveals that only y_i is affected by the choice of d_i . Furthermore, since $(1-t)p \geq P$, y_i is non-decreasing in d_i . Therefore if voter i demands any shares, he would demand as many shares as possible, i.e. $d_i^* = W/P$.

We can conclude that those voters who demand shares demand the same number, W/P , while voters with sufficiently high transaction costs do not demand any shares: therefore the individual demand for shares, d_i^* , is either zero or W/P . The total demand for shares is $S \frac{W}{P}$.

The rationing rule becomes:

$$\tilde{d} = \begin{cases} \frac{W}{P} & \text{if } S \frac{W}{P} \leq 1 \\ \frac{1}{S} & \text{if } S \frac{W}{P} \geq 1 \end{cases} \quad [6]$$

with $\frac{W}{P} = \frac{1}{S}$ when the issue is exactly subscribed. If the issue is undersubscribed, then the government sells the remaining shares to the foreign buyer at price P .

The analysis of the investment decision concludes by deriving, for each price P and level of promotional effort e , the threshold value of the transaction parameter c_i that characterizes the marginal investor.

Adopting an arbitrary tie-breaking rule, when $e \leq c_i$ voter i buys shares of the company if and only if:

$$V(q, y_i) - (c_i - e) \geq V(q, W) \quad [7]$$

Therefore, given the price P and the level of promotional effort e , and given the anticipated rate of expropriation τ , the threshold value of c_i that characterizes the marginal investor is

$$\tilde{c} = V(q, y_i) - V(q, W) + e$$

From condition [7] and using the definitions of y_i and V , it follows that, for any $e > 0$, there is always a positive fraction of voters that has zero transaction costs and consequently buys shares, even when the government chooses to price the stock at $P = (1 - \tau)\mathbf{p}$. On the other hand, if there is no promotional effort ($e=0$), then only for a price P lower than the stock's value $(1 - \tau)\mathbf{p}$ will voters buy shares of the state-owned company.

The proportion of the total population with a value of the transaction parameter below the threshold is given by:

$$prob(c_i \leq \tilde{c}) = prob(c_i \leq V(q, y_i) - V(q, W) + e) = \frac{1}{\bar{c}} (V(q, y_i) - V(q, W) + e)$$

Therefore, recalling that $0 \leq S \leq 1$, the fraction of shareholders is defined as follows:

$$S = \begin{cases} \frac{1}{\bar{c}} (V(q, y_i) - V(q, W) + e) & \text{for } 0 \leq e \leq \bar{c} - [V(q, y_i) - V(q, W)] \\ 1 & \text{for } \bar{c} - [V(q, y_i) - V(q, W)] < e < 1 \end{cases} \quad [8]$$

The expression above defines the fraction of shareholders S as a function of the level of promotional effort the government decides to undertake, $e \in [0,1]$ ⁹, and it holds for any price P smaller than (or equal to) the fair value of the company $(1 - \tau)\pi$.

2.2.2. Individual voting behavior

After the Initial Public Offering, the population of voters is partitioned into the fraction of shareholders S , described by equation [8], and the fraction of non-shareholders N , with $N \equiv 1 - S$.

In the second period, after the government and the opposition announce their platforms t_g and t_o , each voter votes for the party he prefers. Voters favor the party that gives them the highest total utility. This depends on the announced expropriation rate, because the rate of expropriation will affect the quantity supplied of the public good, and, if the voter is a shareholder, the voter's disposable income.

I assume that individual utility, and so voting behavior, is also affected by a bias term ϵ_i , which is the sum of a personal, idiosyncratic bias b_i and a systematic bias δ :

$$\mathbf{e}_i = b_i + \mathbf{d} \quad i \in [0,1] \quad [9]$$

The idiosyncratic bias b_i is uniformly distributed over $[-r, r] \subset R$, and δ is uniformly distributed over $[-I, +I] \subset R$. Therefore, the total bias ϵ_i is distributed over the interval $[-I - r, I + r]$. The bias term captures the idea that the voting decision is influenced by non-policy related matters and ideology, as well as by policy issues other than the rate of

⁹ Since $e \in [0,1]$, it has to be $\left[\bar{c} - \left(V(q, y_i(\tilde{d})) - V(q, W) \right) \right] \geq 0$, which implies $V(q, y_i(\tilde{d})) - \bar{c} \leq V(q, W)$.

expropriation. The total bias ε_i can be in favor of (when positive) or against (when negative) the government.

For clarity of exposition, the analysis henceforth uses the double subscript ij to indicate the identity ($i \in [0,1]$) and the type ($j \in \{S, N\}$) of each voter respectively, and $u_{ij}(\mathbf{t}_k)$, with $k \in \{G, O\}$, to indicate voter ij 's utility as function of the rate of expropriation implemented by the party that wins the election. Finally, $U_{ij}(\mathbf{t}_k)$ takes voter ij 's bias term into account, and therefore it represents the total utility voter ij derives from the victory of party k .

Voter ij 's utility if party k wins is therefore:

$$U_{ij}(\mathbf{t}_k) = u_{ij}(\mathbf{t}_k) + w e_i \quad \forall i \in [0,1], k \in \{G, O\}, j \in \{S, N\}$$

with

$$u_{ij}(\mathbf{t}_k) = \begin{cases} V(q(\mathbf{t}_k), y(\mathbf{t}_k)) - T_i & \text{for } j = S \\ V(q(\mathbf{t}_k), W) & \text{for } j = N \end{cases} \quad [10]$$

$$e_i = b_i + \mathbf{d}$$

$$w = \begin{cases} 1 & \text{if } G \text{ wins the election} \\ 0 & \text{if } O \text{ wins the election} \end{cases}$$

where

$$q(\mathbf{t}_k) = (1 - \mathbf{a})[P - A(e) + \mathbf{t}_k \mathbf{p}]$$

$$y(\mathbf{t}_k) = \begin{cases} W + [(1 - \mathbf{t}_k) \mathbf{p} - P] \frac{1}{S} & \text{if oversubscription} \\ (1 - \mathbf{t}_k) \mathbf{p} \frac{W}{P} & \text{if no oversubscription} \end{cases}$$

Within each interest group, all voters have identical preferences over τ_k .

It is immediate that the non-shareholders' utility is strictly increasing in τ_k . Therefore, the non-shareholders always prefer full expropriation of private profits: the higher is the

expropriation rate, the bigger is the amount of public good, and the higher is the utility of a voter who does not own shares.

For a shareholder, a higher expropriation rate results in more public good, but it also reduces the shareholder's disposable income. This tradeoff implies that the optimal rate of expropriation for a shareholder can be smaller than one (Appendix 1 contains the analysis concerning the optimal expropriation rate for the representative shareholder).

How do voters cast their vote? Each will vote for the party whose announced platform \mathbf{t}_k , if implemented, would give him the higher utility, given his voting bias. Specifically, voter ij votes for the opposition O if and only if:

$$U_{ij}(\mathbf{t}_O) \geq U_{ij}(\mathbf{t}_G)$$

Therefore, adopting an arbitrary tie-breaking rule, the probability \mathbf{s}_{ij} that voter i in group j votes for the opposition is given by¹⁰:

$$\mathbf{s}_{ij} = \begin{cases} 1 & \text{if } \mathbf{e}_i \leq u_{ij}(\mathbf{t}_O) - u_{ij}(\mathbf{t}_G) \\ 0 & \text{if } \mathbf{e}_i > u_{ij}(\mathbf{t}_O) - u_{ij}(\mathbf{t}_G) \end{cases} \quad [11]$$

In the same way, voter ij votes for the government G if and only if

$$U_{ij}(\mathbf{t}_G) > U_{ij}(\mathbf{t}_O)$$

and the probability g_{ij} that voter i in group j votes for the government is $g_{ij} = 1 - \mathbf{s}_{ij}$.

¹⁰ Voting is assumed to be costless.

2.2.3. The plurality and the probability of winning the election.

Recalling that $e_i = b_i + \mathbf{d}$, it is easy to calculate the probability that a given voter supports the opposition. Let t be the value of the idiosyncratic bias b_i that makes voter ij indifferent between voting for the government or for the opposition:

$$t = u_{ij}(\mathbf{t}_O) - u_{ij}(\mathbf{t}_G) - \mathbf{d}$$

The distribution function of the idiosyncratic bias term b_i is uniform over the interval $[-r, r]$. Therefore:

$$\text{prob}(b_i \leq t) = \frac{1}{2r}(t+r) \quad \text{for } -r < t < +r$$

where $\frac{1}{2r}$ is b_i 's uniform density.

Then the probability that voter ij favors the opposition is:

$$\mathbf{s}_{ij} = \text{prob}(b_i \leq t) = \frac{1}{2r} [u_{ij}(\mathbf{t}_O) - u_{ij}(\mathbf{t}_G) - \mathbf{d} + r] \quad [12]$$

and the probability that voter ij votes for the government is $g_{ij} = 1 - \mathbf{s}_{ij}$.

The total fraction of votes cast for the opposition is the sum of the proportions of votes pro-opposition in the two interest groups:

$$S \left[\frac{1}{2r} (u_{iS}(\mathbf{t}_O) - u_{iS}(\mathbf{t}_G) - \mathbf{d} + r) \right] + N \left[\frac{1}{2r} (u_{iN}(\mathbf{t}_O) - u_{iN}(\mathbf{t}_G) - \mathbf{d} + r) \right]$$

Similarly

$$S \left[1 - \frac{1}{2r} (u_{iS}(\mathbf{t}_O) - u_{iS}(\mathbf{t}_G) - \mathbf{d} + r) \right] + N \left[1 - \frac{1}{2r} (u_{iN}(\mathbf{t}_O) - u_{iN}(\mathbf{t}_G) - \mathbf{d} + r) \right]$$

is the fraction of the population that favors the government.

The plurality for either party, as a function of the announced platforms τ_G and τ_O , is simply the difference between the fraction of the population that votes in favor of the party and the remaining fraction (that favors the political opponent). Specifically, the pluralities for the opposition party and the government are, respectively:

$$\begin{aligned} L_O(\mathbf{t}_G, \mathbf{t}_O) &= \frac{1}{r}[z - \mathbf{d}] \\ L_G(\mathbf{t}_G, \mathbf{t}_O) &= -L_O(\mathbf{t}_G, \mathbf{t}_O) \end{aligned} \quad [13]$$

where $z \equiv S(u_{iS}(\mathbf{t}_O) - u_{iS}(\mathbf{t}_G)) + N(u_{iN}(\mathbf{t}_O) - u_{iN}(\mathbf{t}_G))$ is the value of the systematic bias δ at which the pluralities are zero, and the election results in a tie.

Since there are only two parties, and everyone in the population votes, a party wins the election whenever its plurality is positive. Therefore, the parties' probability of winning the political competition is a discontinuous function of their pluralities.

Let $\mathbf{m}_k(\mathbf{t}_G, \mathbf{t}_O)$ indicate the probability that party k wins. Then:

$$\mathbf{m}_k(\mathbf{t}_G, \mathbf{t}_O) = \begin{cases} 1 & \text{if } L_k(\mathbf{t}_G, \mathbf{t}_O) > 0 \\ \frac{1}{2} & \text{if } L_k(\mathbf{t}_G, \mathbf{t}_O) = 0 \\ 0 & \text{if } L_k(\mathbf{t}_G, \mathbf{t}_O) < 0 \end{cases} \quad k \in \{G, O\}$$

The pluralities depend on the realization of the systematic bias δ , which is uniformly distributed over $[-I, +I]$, with density equal to $\frac{1}{2I}$.

The next step is to derive the probability that party k wins the election. From the distribution of δ :

$$\text{prob}(\mathbf{d} < z) = \frac{1}{2I}(z + I) \quad \text{for } -I < z < +I$$

and therefore:

$$\begin{aligned} \mathbf{m}_o(\mathbf{t}_G, \mathbf{t}_o) &= \text{prob}(L_o(\mathbf{t}_G, \mathbf{t}_o) > 0) = \text{prob}(\mathbf{d} < z) = \frac{1}{2\mathbf{I}}(z + \mathbf{I}) \\ \mathbf{m}_G(\mathbf{t}_G, \mathbf{t}_o) &= \text{prob}(L_G(\mathbf{t}_G, \mathbf{t}_o) > 0) = \text{prob}(\mathbf{d} > z) = 1 - \frac{1}{2\mathbf{I}}(z + \mathbf{I}) \end{aligned} \quad [14]$$

Each party's probability of winning the election, as expressed in [14], takes values between zero and one if:

$$-\mathbf{I} < u_{ij}(\mathbf{t}_o) - u_{ij}(\mathbf{t}_G) < +\mathbf{I} \quad \forall \mathbf{t}_o, \mathbf{t}_G \in [0,1], \forall i \in [0,1], \forall S, N \quad [15]$$

I assume that the value of λ is big enough to ensure that condition [15] is satisfied.

What this assumption means is the following. When the utility differentials for both shareholders and non-shareholders take values in the support of the systematic bias, then not only is there a positive probability that voter ij votes for either party, but also there is always a positive probability that, within each interest group, the plurality of either party is positive. Therefore, neither the government nor the opposition can guarantee themselves an overall majority, regardless of the other party's promise. This implies that each voter's probability of voting for one party is a differentiable function of that party's promise \mathbf{t}_K , implying that each party's probability of winning is a differentiable function of its campaign promise.

The derivation of the parties' probability of winning the election concludes the description of the model. In the Sections that follow, the game is solved by backward induction. Since, at each stage of the game, every player has exactly the same information, the game is a dynamic game of complete information, and the appropriate equilibrium concept is subgame perfection. Section (3) analyzes the subgame. Section (4) looks at the reduced game in the first period and discusses the equilibrium of the model.

3. The subgame

In period two, after the sale of the state-owned company to the private sector, the party in power and the opposition compete for votes, simultaneously announcing their choices of \mathbf{t}_G and \mathbf{t}_O . In doing so, each party commits itself to implement the announced rate of expropriation if it wins the election. Given the other party's choice, party k chooses the rate of expropriation τ_k that solves the following:

$$\begin{aligned} \max_{\mathbf{t}_k} & \quad [\mathbf{m}_k(\mathbf{t}_G, \mathbf{t}_O)(\mathbf{b} + \mathbf{a}\mathbf{t}_k\mathbf{p})] \\ \text{subject to} & \quad 0 \leq \mathbf{t}_k \leq 1 \end{aligned} \quad [16]$$

where $\mu_k(\tau_G, \tau_O)$, is defined by equation [14].

The unconstrained version of this maximization problem requires the following first-order condition at any critical point:

$$\frac{\mathcal{J}\mathbf{m}_k(\mathbf{t}_G, \mathbf{t}_O)}{\mathcal{J}\mathbf{t}_k} (\mathbf{b} + \mathbf{a}\mathbf{t}_k\mathbf{p}) + \mathbf{m}_k(\mathbf{t}_G, \mathbf{t}_O)\mathbf{a}\mathbf{p} = 0 \quad [17]$$

To simplify notation, from now on $\frac{\mathcal{J}V(q(\mathbf{t}_k), y(\mathbf{t}_k))}{\mathcal{J}q(\mathbf{t}_k)} \equiv V_q(\mathbf{t}_k)$ and $\frac{\mathcal{J}^2V(q(\mathbf{t}_k), y(\mathbf{t}_k))}{\mathcal{J}q^2(\mathbf{t}_k)} \equiv V_{qq}(\mathbf{t}_k)$

From the definitions in [13] and [14], and recalling from [4] that $V_y = 1$:

$$\frac{\mathcal{J}\mathbf{m}_k(\mathbf{t}_G, \mathbf{t}_O)}{\mathcal{J}\mathbf{t}_k} = \frac{1}{2I} \left(S \frac{du_{iS}(\mathbf{t}_k)}{d\mathbf{t}_k} + N \frac{du_{iN}(\mathbf{t}_k)}{d\mathbf{t}_k} \right) \quad [18]$$

with, using [4], [5] and [10]:

$$\begin{aligned} \frac{du_{iS}(\mathbf{t}_k)}{d\mathbf{t}_k} &= V_q(\mathbf{t}_k) \frac{dq(\mathbf{t}_k)}{d\mathbf{t}_k} + V_y \frac{dy(\mathbf{t}_k)}{d\mathbf{t}_k} = V_q(\mathbf{t}_k)[(1-\mathbf{a})\mathbf{p}] - \mathbf{p}\tilde{d} \\ \frac{du_{iN}(\mathbf{t}_k)}{d\mathbf{t}_k} &= V_q(\mathbf{t}_k) \frac{dq(\mathbf{t}_k)}{d\mathbf{t}_k} + V_w \frac{dW}{d\mathbf{t}_k} = V_q(\mathbf{t}_k)[(1-\mathbf{a})\mathbf{p}] \end{aligned}$$

where S , and therefore \tilde{d} , are determined in the first period, with either $\tilde{d} = W/P$ (if the issue is undersubscribed), or $\tilde{d} = 1/S$ (if there is oversubscription).

Because, by assumption, $\mathbf{a} \in (0,1)$, $\mathbf{b} \geq 0$, $\mathbf{p} > 0$, and $0 < \mathbf{m}_k(\mathbf{t}_G, \mathbf{t}_O) < 1$, conditions [17] and [18] require that, at any critical point:

$$\frac{\mathbf{m}_k(\mathbf{t}_G, \mathbf{t}_O)}{\mathbf{t}_k} = \frac{1}{2I} \left(S \frac{d u_{iS}(\mathbf{t}_k)}{d \mathbf{t}_k} + N \frac{d u_{iN}(\mathbf{t}_k)}{d \mathbf{t}_k} \right) < 0$$

Recalling that $N \equiv 1-S$, and directly from substitution, the necessary condition above simplifies to:

$$(1 - \mathbf{a})V_q(\mathbf{t}_K) < S\tilde{d} \quad [19]$$

which must hold at any critical point in the unconstrained problem.

Condition [19] tells us something about the marginal impact of expropriation, evaluated at the point implied by the policy \mathbf{t}_k that satisfies the first-order condition [17]. Recall that $V_y = 1$: then $(1 - \mathbf{a})V_q(\mathbf{t}_K) < S\tilde{d}$ implies that the aggregate marginal utility losses for domestic shareholders from lost income do exceed the aggregate marginal utility gains from increased consumption of the public good.

If condition [19] holds, then the optimal platform announcement for party k , if it exists, exceeds the representative shareholder's preferred rate of expropriation. Formally, any τ_k that satisfies condition [19] also satisfies $V_q(\mathbf{t}_K)(1 - \mathbf{a}) < \tilde{d}$: this implies that, for the representative shareholder, $\frac{d u_{iS}(\mathbf{t}_k)}{d \mathbf{t}_k} < 0$ at the equilibrium \mathbf{t}_k (see Appendix 1, equation A1). If $S=0$, then

condition [19] cannot hold for $\mathbf{t}_k \in [0,1]$, implying $\frac{\mathcal{J}\mathbf{m}_k(\mathbf{t}_G, \mathbf{t}_O)}{\mathcal{J}\mathbf{t}_k} \geq 0$, and therefore the best

choice for party k is to announce $\mathbf{t}_k = 1$.

From [17], and using [18], the sufficient second-order condition for an unconstrained global maximum for party k is:

$$\begin{aligned} & \frac{\mathcal{J}^2 \mathbf{m}_k(\mathbf{t}_G, \mathbf{t}_O)}{\mathcal{J}\mathbf{t}_k^2} (\mathbf{b} + \mathbf{a}\mathbf{t}_k \mathbf{p}) + 2\mathbf{a}\mathbf{p} \frac{\mathcal{J}\mathbf{m}_k(\mathbf{t}_G, \mathbf{t}_O)}{\mathcal{J}\mathbf{t}_k} = \quad [20] \\ & = \frac{1}{2\mathbf{l}} (\mathbf{b} + \mathbf{a}\mathbf{t}_k \mathbf{p}) \left(S \frac{d^2 u_{iS}(\mathbf{t}_k)}{d\mathbf{t}_k^2} + N \frac{d^2 u_{iN}(\mathbf{t}_k)}{d\mathbf{t}_k^2} \right) + \frac{\mathbf{a}\mathbf{p}}{\mathbf{l}} \left(S \frac{d u_{iS}(\mathbf{t}_k)}{d\mathbf{t}_k} + N \frac{d u_{iN}(\mathbf{t}_k)}{d\mathbf{t}_k} \right) < 0 \end{aligned}$$

If [20] is satisfied at every critical point, then the objective function is strictly concave and therefore, if a critical point exists, it is the unique maximum. Rewriting the first-order condition as

$$S \frac{d u_{iS}(\mathbf{t}_k)}{d\mathbf{t}_k} + N \frac{d u_{iN}(\mathbf{t}_k)}{d\mathbf{t}_k} = - \frac{2\mathbf{l}\mathbf{a}\mathbf{p}\mathbf{m}_k(\mathbf{t}_G, \mathbf{t}_O)}{\mathbf{b} + \mathbf{a}\mathbf{t}_k \mathbf{p}}$$

and then substituting it, together with

$$\frac{d^2 u_{ij}(\mathbf{t}_k)}{d\mathbf{t}_k^2} = V_{qq}(\mathbf{t}_K)(1-\mathbf{a})^2 \mathbf{p}^2 \quad \mathbf{j} \in \{S, N\}$$

into [20]¹¹, the second-order condition evaluated at the critical point becomes:

$$\begin{aligned} & \frac{1}{2\mathbf{l}} (\mathbf{b} + \mathbf{a}\mathbf{t}_k \mathbf{p}) \left[(1-N)V_{qq}(\mathbf{t}_K)(1-\mathbf{a})^2 \mathbf{p}^2 + NV_{qq}(\mathbf{t}_K)(1-\mathbf{a})^2 \mathbf{p}^2 \right] + \frac{\mathbf{a}\mathbf{p}}{\mathbf{l}} \left(- \frac{2\mathbf{l}\mathbf{a}\mathbf{p}\mathbf{m}_k(\mathbf{t}_G, \mathbf{t}_O)}{\mathbf{b} + \mathbf{a}\mathbf{t}_k \mathbf{p}} \right) = \\ & = \frac{1}{2\mathbf{l}} (\mathbf{b} + \mathbf{a}\mathbf{t}_k \mathbf{p}) V_{qq}(\mathbf{t}_K)(1-\mathbf{a})^2 \mathbf{p}^2 + \frac{\mathbf{a}\mathbf{p}}{\mathbf{l}} \left(- \frac{2\mathbf{l}\mathbf{a}\mathbf{p}\mathbf{m}_k(\mathbf{t}_G, \mathbf{t}_O)}{\mathbf{b} + \mathbf{a}\mathbf{t}_k \mathbf{p}} \right) < 0 \end{aligned} \quad [21]$$

¹¹ Recall that $V_{yy}=0$, $V_{qy}=0$, and that $S \equiv 1-N$.

Since $V_{qq}(\mathbf{t}_k) < 0$, [21] holds for any value of \mathbf{t}_k . Therefore, there exists a unique optimal expropriation rate that solves party k's second period (unconstrained) maximization problem.

If the FOC is not satisfied for a value of \mathbf{t}_k between zero and one, then the (constrained) optimum is either zero or one, depending on the sign of the first-order condition at those values. It might be that condition [19] never holds for a value of the expropriation rate between zero and one, in which case the left-hand side of equation [17] is always positive and therefore the boundary solution is $\mathbf{t}_k = 1$. If the fraction of shareholders S is very small, it might be optimal for party k to choose to announce full expropriation of private profits (this is always true for $S=0$). If this is the case, then everyone would anticipate the choice of $\tau_G=1$, and therefore, for any positive price P , nobody would buy shares. For $P=0$, a degenerate equilibrium of the game may exist: in the first period the government earns zero revenues "selling" the entire firm to the foreigner investor (without any promotion of the sale), and then it fully expropriates the private company in the second period. This is equivalent to not selling the firm at all.

On the other hand, if at $\mathbf{t} = 0$ a marginal increase in the value of the announced future expropriation greatly reduces the chances that party k wins the election, then the optimal choice for party k might be to promise a rate of expropriation equal to zero. All else equal, this is more likely to happen when the group of shareholders is large.

To conclude, equation [17] defines the government's optimal choice of \mathbf{t}_G as a function of \mathbf{t}_o , S , P , and e , as well as the parameters of the model. To derive the equilibria of the subgame, it is convenient to express the government's reaction function simply as $\mathbf{t}_G = f(\mathbf{t}_o)$. Equation [17] defines a symmetric solution to the opposition's problem, and

therefore $\mathbf{t}_o = f(\mathbf{t}_G)$ is the opposition party's reaction function. The next questions to be answered are whether there exists a (pure strategy) Nash equilibrium for this subgame, and whether that equilibrium is unique.

3.1. Existence

It is straightforward to show that there exists a Nash equilibrium in pure strategies. Because condition [21] holds at the unique optimum, the implicit function theorem guarantees that there exist well defined, continuous reaction functions $\mathbf{t}_G = f(\mathbf{t}_o)$ and $\mathbf{t}_o = f(\mathbf{t}_G)$. It follows that the composite function $f(f(\mathbf{t}_G))$ is continuous for $\mathbf{t}_G \in [0,1]$, and it maps a compact, convex set into itself. Therefore, according to Brouwer's fixed-point theorem, there exists a \mathbf{t}_G^* such that $f(f(\mathbf{t}_G^*)) = \mathbf{t}_G^*$.

3.2. Uniqueness

The subgame has a unique Nash equilibrium if:

$$0 < \frac{df(\mathbf{t}_o)}{d\mathbf{t}_o} < 1 \quad \forall \mathbf{t}_o \in \mathfrak{S} \equiv \{\mathbf{t}_o \in [0,1]: (1-\mathbf{a})V_q(\mathbf{t}_o) < S\tilde{d}\} \quad [22]$$

If [22] holds, then the two reaction functions cross only once in the interval $[0,1]$, and the equilibrium is unique and symmetric. It is shown next that condition [22] is always satisfied.

First of all, to derive $\frac{df(\mathbf{t}_o)}{d\mathbf{t}_o}$, apply the implicit function theorem to equation [17], which

defines the reaction function $\mathbf{t}_G = f(\mathbf{t}_o)$, and then differentiate it with respect to \mathbf{t}_o , obtaining (see the Appendix 1 for the algebra):

$$\frac{df(\mathbf{t}_o)}{d\mathbf{t}_o} = \frac{\mathbf{a}[(1-\mathbf{a})V_q(\mathbf{t}_o) - S\tilde{d}]}{2\mathbf{a}[(1-\mathbf{a})V_q(\mathbf{t}_G) - S\tilde{d}] + (1-\mathbf{a})^2(\mathbf{b} + \mathbf{a}\mathbf{t}_G\mathbf{p})V_{qq}(\mathbf{t}_G)}, \quad \mathbf{t}_o \in [0,1], \mathbf{t}_G = f(\mathbf{t}_o) \quad [23]$$

Since $\mathbf{t}_G = f(\mathbf{t}_o)$ solves [17], it must satisfy condition [19], namely $(1-\mathbf{a})V_q(\mathbf{t}_G) < S\tilde{d}$, and therefore, with $\mathbf{a} \in (0,1)$, $\mathbf{b} \geq 0$, $\mathbf{p} > 0$, and $V_{qq} < 0$ by assumption, the denominator of [23] is negative for all $\mathbf{t}_o \in [0,1]$. The sign of the numerator depends on the value of $V_q(\mathbf{t}_o)$. Since $V_{qq} < 0$, $V_q(\mathbf{t}_o)$ is decreasing in q , i.e. decreasing in \mathbf{t}_o . This admits the possibility that, at low values of \mathbf{t}_o , $(1-\mathbf{a})V_q(\mathbf{t}_o) > S\tilde{d}$, which implies that, at those values of \mathbf{t}_o , the numerator of [23] is positive, and the whole expression in [23] has a negative sign.

But recall condition [19] for the existence of an optimal (interior) value of \mathbf{t}_o : it requires $(1-\mathbf{a})V_q(\mathbf{t}_o) < S\tilde{d}$ at the optimum. Therefore, any Nash equilibrium of the subgame must be in the upward sloping part of the reaction functions, where, for both \mathbf{t}_G and \mathbf{t}_o , condition [19] is satisfied: any optimal response is in \mathfrak{S} .

Since \mathfrak{S} is a connected set, if the slope of the reaction function is smaller than one for $\mathbf{t}_o \in \mathfrak{S}$, then the two reaction functions cross only once in \mathfrak{S} , and therefore the equilibrium, which has to be in \mathfrak{S} , is unique.

From the symmetry of the two reaction functions it follows that the subgame perfect equilibrium is symmetric, i.e. $\mathbf{t}_G = \mathbf{t}_o$ at equilibrium. Hence, to conclude that condition [22]

holds for all $t_o \in \mathfrak{S}$, it suffices to show that it is verified at $t_G = t_o$. Therefore, evaluating [23]

at $t \equiv t_G = t_o$, condition [22] is verified when:

$$\frac{\mathbf{a}[(1-\mathbf{a})V_q(t) - S\tilde{d}]}{2\mathbf{a}[(1-\mathbf{a})V_q(t) - S\tilde{d}] + (1-\mathbf{a})^2(\mathbf{b} + \mathbf{a}t\mathbf{p})V_{qq}(t)} < 1$$

which becomes:

$$[(1-\mathbf{a})V_q(t) - S\tilde{d}] > 2[(1-\mathbf{a})V_q(t) - S\tilde{d}] + \frac{(1-\mathbf{a})^2}{\mathbf{a}}(\mathbf{b} + \mathbf{a}t\mathbf{p})V_{qq}(t)$$

The above disequality always holds, therefore the subgame has a unique Nash equilibrium, with $t_G \equiv t_o$: the two parties will announce the same expropriation rate in equilibrium, and they will both have equal chances of winning the election ($m_G \equiv m_o \equiv \frac{1}{2}$, from equation [14] and the definition of z).

In the next Section of the paper, the Subgame Perfect Nash equilibrium (SPNE) of the game is derived. It is shown that, through underpricing and promotional effort, the government increases share ownership, and that wide share ownership is instrumental in maximizing the government's total payoff.

4. The first period: privatization, promotional effort and underpricing.

In the first period, the government sells the state-owned company to the private sector. It chooses a share-price P and a level of promotional effort e to maximize its own objective, as given by equation [1]. From the analysis of the subgame, recall that the rate of expropriation t_G

announced by the government in the second period is a function of P , e , S , and the opposition's announcement \mathbf{t}_o : $\mathbf{t}_G = f(P, e, S(P, e, \mathbf{t}^*), \mathbf{t}_o)$. Symmetrically, the opposition's announcement is $\mathbf{t}_o = f(P, e, S(P, e, \mathbf{t}^*), \mathbf{t}_G)$. The fraction of shareowners S , as defined by equations [8], [5], and [6], is a function $S = g(P, e, \mathbf{t}^*)$ when the issue is undersubscribed, and it is a function $S = g(P, e, \mathbf{t}^*, S)$ when there is oversubscription. Therefore, before the government chooses P and e , S is itself a function of P , e , and the expected rate of expropriation $\mathbf{t}^* = \mathbf{m}_G \mathbf{t}_G + (1 - \mathbf{m}_G) \mathbf{t}_o$.

Since everyone anticipates that $\mathbf{t}_G = \mathbf{t}_o$, and therefore that the parties are equally likely to win the election ($\mathbf{m}_G = \mathbf{m}_o = \frac{1}{2}$), the government's optimization problem in [1] becomes:

$$\begin{aligned} & \max_{(P, e)} \left\{ \mathbf{a}[P - A(e)] + \frac{1}{2}(\mathbf{b} + \mathbf{a} \mathbf{p} \mathbf{t}_G(P, e, S, \mathbf{t}_o)) \right\} \\ & \text{subject to} \\ & P \leq (1 - \mathbf{t}_G(P, e, S, \mathbf{t}_o)) \mathbf{p} \quad [24] \\ & e \leq 1 \\ & P \geq 0, e \geq 0 \end{aligned}$$

Besides imposing the standard non-negativity constraints on the choice variables P and e , the government must choose a share price P that does not exceed the implied future value of the company, if it wants to sell the company at all.

One potential (degenerate) equilibrium of the game can be derived from simple inspection of the maximization problem in [24]: it is the possible SPNE characterized by $P^* = 0, e^* = 0, \mathbf{t}_G^* = \mathbf{t}_o^* = 1, S^* = 0$. From the analysis of the subgame, recall that the equilibrium announcement could be $\mathbf{t}_G^* = 1$, regardless of the size S of the shareholders' interest group. In this case, the best the government can do is to price the company at its true value, $P^* = 0$, and to

choose $e^*=0$. This is an equilibrium at which there is no underpricing, no promotion of the sale and full expropriation of private profits, and the entire company is sold to the foreign investor ($S^*=0$ from [8]). In this extreme case, the privatization policy does not serve the purpose of increasing the government's financial payoff, because the same revenues can be obtained if the company remains a state-owned company.

Another possible extreme outcome is an equilibrium in which the announcement in the second period is always $t_G^* = 0$. This is more likely for high values of S , of course. But $t_G^* = 0$ could also be the outcome of the subgame at low values of S , if citizens derive little utility from additional consumption of the public good, even when very few units of the public good are provided: in other words, if the marginal utility V_q derived from consumption of the first unit of the public good is very low, then the best campaigning strategy may be to announce zero expropriation of future private profits (see condition [19], recalling that it is not a sufficient condition for the existence of an interior solution). To conclude, there is the possibility of an equilibrium such that the government's optimal choices of P^* and e^* imply a value of $S^* > 0$ that supports either $t_G^* = 1$ or $t_G^* = 0$.

Since the most interesting case is an equilibrium with $0 < t_G^* < 1$, the following analysis assumes that the optimal platform announcements $t_G^* = t_o^*$ lie between zero and one. Consider the maximization problem in [24]. It is straightforward to show that some of the constraints are not binding. First, because of the assumption $\lim_{e \rightarrow 1} A(e) = +\infty$, the government will always choose $e^* < 1$. Second, it is easy to rule out any equilibrium with $P^* \leq 0$. In fact, if

the shares of the company are given away, then the government can do at least as well by keeping the firm: an equilibrium with $P^* \leq 0$, $e^* \geq 0$, and $0 < \mathbf{t}_G^* < 1$ is never optimal¹².

Therefore, the maximization problem in [24] becomes:

$$\begin{aligned} & \max_{(P,e)} \left\{ \mathbf{a}[P - A(e)] + \frac{1}{2}(\mathbf{b} + \mathbf{a}\mathbf{p}\mathbf{t}_G(P, e, S, \mathbf{t}_o)) \right\} \\ & \text{subject to} \\ & P \leq (1 - \mathbf{t}_G(P, e, S, \mathbf{t}_o))\mathbf{p} \\ & e \geq 0 \end{aligned} \quad [24']$$

Using the Kuhn-Tucker formulation, the Lagrangian of the problem in [24'] is:

$$L(P, e, \mathbf{g}) = \mathbf{a}[P - A(e)] + \frac{1}{2}(\mathbf{b} + \mathbf{a}\mathbf{p}\mathbf{t}_G^*(P, e, S, \mathbf{t}_o)) + \mathbf{g}[(1 - \mathbf{t}_G^*(P, e, S, \mathbf{t}_o))\mathbf{p} - P] \quad [25]$$

where \mathbf{g} is the multiplier. Assume that (P^*, e^*) is a solution of [24']. Then, there exists a multiplier $\mathbf{g}^* \geq 0$ such that (P^*, e^*, \mathbf{g}^*) satisfies:

$$\begin{aligned} \frac{\partial L(P, e, \mathbf{g})}{\partial P} &= 0 \\ \frac{\partial L(P, e, \mathbf{g})}{\partial e} &\leq 0, \quad e \frac{\partial L(P, e, \mathbf{g})}{\partial e} = 0 \\ \frac{\partial L(P, e, \mathbf{g})}{\partial \mathbf{g}} &\geq 0, \quad \mathbf{g} \frac{\partial L(P, e, \mathbf{g})}{\partial \mathbf{g}} = 0 \end{aligned}$$

Applied to [25], the conditions specified above become:

¹² Notice also that any equilibrium with zero promotional effort and full pricing of the sale cannot be sustained by an expropriation rate smaller than one: it cannot be that $P = (1 - \mathbf{t}_G^*)\mathbf{p}$, $e^* = 0$, and $0 < \mathbf{t}_G^* < 1$, because this would imply $S^* = 0$ from [8], and so in the second period it is optimal to announce $\mathbf{t}_G^* = 1$.

$$\begin{aligned} \frac{\partial L(P, e, g)}{\partial P} &= a + \frac{1}{2}ap \frac{dt_G(P, e, S, t_o)}{dP} + g(-p \frac{dt_G(P, e, S, t_o)}{dP} - 1) = 0 \\ \frac{\partial L(P, e, g)}{\partial e} &= -aA' + \frac{1}{2}ap \frac{dt_G(P, e, S, t_o)}{de} + g(-p \frac{dt_G(P, e, S, t_o)}{de}) \leq 0 \\ &\text{if } < 0, \text{ then } e^* = 0 \end{aligned} \quad [26]$$

$$\frac{\partial L(P, e, g)}{\partial g} = (1 - t_G(P, e, S, t_o))p - P \geq 0, \quad \text{if } > 0, \text{ then } g^* = 0$$

From [26], an equilibrium (P^*, e^*) with underpricing and positive promotional effort has to satisfy:

$$\begin{aligned} \left. \frac{\partial L(P, e)}{\partial P} \right|_{\substack{e=e^* \\ P=P^*}} &= a + \frac{1}{2}ap \frac{dt_G(P, e, S, t_o)}{dP} \Big|_{\substack{e=e^* \\ P=P^*}} = 0 \\ \left. \frac{\partial L(P, e)}{\partial e} \right|_{\substack{e=e^* \\ P=P^*}} &= -aA' + \frac{1}{2}ap \frac{dt_G(P, e, S, t_o)}{de} \Big|_{\substack{e=e^* \\ P=P^*}} = 0 \end{aligned} \quad [27]$$

with $g^* = 0$.

Assume that [27] characterizes a global maximum in (P^*, e^*) . Then from [27] it follows:

Proposition 1

Assume that there exists a global maximum that solves the maximization problem in [24']. Then,

a Subgame Perfect Nash Equilibrium with $0 < t_G^* < 1$, $P < (1 - t_G^*)p$ and $e^* > 0$ exists if

$$\begin{aligned} \left. \frac{dt_G(P, e, S, t_o)}{dP} \right|_{\substack{e=e^* \\ P=P^*}} &= -\frac{2}{p} \\ \left. \frac{dt_G(P, e, S, t_o)}{de} \right|_{\substack{e=e^* \\ P=P^*}} &= \frac{2A'}{p} \end{aligned} \quad [28]$$

Such an equilibrium will have $S^* > 0$.

The negative and positive signs of the total derivatives in [28] are intuitive. Ceteris paribus, the government always prefers a high price, low promotional effort, and a high

expropriation rate (see the objective function in [24']). For any given choice of e , if it was possible for the government to expropriate more at higher sale prices, namely if

$$\left. \frac{dt_G(P, e, S, t_o)}{dP} \right|_{P=P^*} > 0, \text{ then it would pay to do so, and } P^* < (1 - t_G)p \text{ could not be optimal.}$$

The same reasoning holds if the effect on the platform announcement of a decrease in

$$\text{promotional effort is positive, i.e. if } \left. \frac{dt_G(P, e, S, t_o)}{de} \right|_{e=e^*} < 0. \text{ For any given value of } P, \text{ it would}$$

pay to decrease the effort exerted in period one, and therefore to announce a higher expropriation rate in the second period. Both decisions would increase the government's payoff and consequently $e^* > 0$ could not be optimal.

To analyze the equilibrium described by Proposition 1, we need to focus on the two endogenous variables in the model, the fraction of share ownership S and the expropriation rate t_G . In the remaining part of this analysis, all the values presented are equilibrium values, although stars are dropped for simplicity.

Recall that the government's best reply function is a function $t_G = f(P, e, S, t_o)$, implicitly defined by the FOC in [17]. How does t_G change as S changes, ceteris paribus?

In general, it is:

$$\frac{dt_G}{dS} = \frac{\partial f}{\partial S} + \frac{\partial f}{\partial t_o} \frac{dt_o}{dS}$$

Because the equilibrium is symmetric in t , $\frac{dt_G}{dS} = \frac{dt_o}{dS}$, and the above equation becomes:

$$\frac{dt_G}{dS} = \left(\frac{1}{1 - \frac{\partial f}{\partial t_o}} \right) \frac{\partial f}{\partial S}$$

Applying the Implicit Function Theorem to [17], using [23] and the definitions in [13], [14], and [18], we obtain the following results:

Proposition 2

As share ownership S changes, the second period equilibrium announcement t_G changes as follows:

$$\frac{dt_G}{dS} = \frac{\frac{W}{P}(\mathbf{b} + \mathbf{a}pt_G)}{\mathbf{p} \left\{ \mathbf{a} \left[V_q(1 - \mathbf{a}) - S \frac{W}{P} \right] + (1 - \mathbf{a})^2 V_{qq}(\mathbf{b} + \mathbf{a}pt_G) \right\}} < 0 \quad \text{when } \tilde{\mathbf{d}} = \frac{W}{P} \text{ (undersubscription)}$$

and [29]

$$\frac{dt_G}{dS} = 0 \quad \text{when } \tilde{\mathbf{d}} = \frac{1}{S} \text{ (oversubscription)}$$

(See Appendix 2 for the proof)

For a given P and e , when the issue is not oversubscribed, an increase in share ownership causes a decrease of the rate of expropriation announced in the second period. This analytical result confirms the intuition that diffusion of share ownership can be instrumental in increasing revenues: higher share ownership implies lower future expropriation, and lower expropriation implies higher value of the company that goes on sale. This in turn allows the government to ask a higher price in the first period.

Once the issue is oversubscribed, any additional increase in S does not cause t_G to decrease. The intuition behind this result is simple. Recall from the analysis of the subgame, that the marginal impact of a change in t_G on the probability of winning the election, is given by:

$$\frac{\partial \mathbf{m}_k(t_G, t_o)}{\partial t_k} = \frac{1}{2I} \left(S \frac{du_{iS}(t_k)}{dt_k} + N \frac{du_{iN}(t_k)}{dt_k} \right) = \frac{1}{2I} [V_q(1-a)p - Sp\tilde{d}]$$

with $V_q(1-a)p$ measuring the aggregate marginal utility gains for the entire population from increased consumption of the public good due to an increase in t_G , and $SV_y p\tilde{d} \equiv Sp\tilde{d}$ measuring the aggregate marginal utility losses for domestic shareholders from lost income due to any increase in t_G . When there is oversubscription, $\tilde{d} = \frac{1}{S}$ and therefore the aggregate impact on the shareholder group S of any change in t_G is constant, and equal to p . In short, once the issue is oversubscribed, any further increase in share ownership does not constitute an incentive for the government to announce a lower expropriation rate in the second period, because such an announcement would not increase its chances of winning the election. Hence the result in Proposition 2.

Share ownership S , as defined by equations [8], [5] and [6], is a function $S = g(P, e, t^*)$ in case of undersubscription, and a function $S = g(P, e, t^*, S)$ in case of oversubscription, where $t^* = m_G t_G + (1 - m_G) t_o$ is the expected rate of expropriation. Since the second period equilibrium outcome is symmetric, with $t_G = t_o$ and $m_G = m_o = \frac{1}{2}$, we can interpret S as a function of t_G . Therefore, using [8], [5], and [6], S is explicitly defined as:

$$S = \frac{1}{\bar{c}} [V(q) + y - V(q) - W + e]$$

with

$$q = (1 - \mathbf{a})[P - A(e) + \mathbf{t}_G \mathbf{p}] \quad [30]$$

$$y = \begin{cases} (1 - \mathbf{t}_G) \mathbf{p} \frac{W}{P} & \text{if undersubscription} \\ W + [(1 - \mathbf{t}_G) \mathbf{p} - P] \frac{1}{S} & \text{if oversubscription} \end{cases}$$

How does S change as the anticipated equilibrium announcement \mathbf{t}_G changes, holding everything else constant? In general, it is:

$$\frac{dS}{d\mathbf{t}_G} = \frac{\partial g}{\partial \mathbf{t}_G} \quad \text{if undersubscription}$$

and

$$\frac{dS}{d\mathbf{t}_G} = \frac{\partial g}{\partial \mathbf{t}_G} + \frac{\partial g}{\partial S} \frac{dS}{d\mathbf{t}_G}, \quad \text{which is } \frac{dS}{d\mathbf{t}_G} = \left(\frac{1}{1 - \frac{\partial g}{\partial S}} \right) \frac{\partial g}{\partial \mathbf{t}_G}, \quad \text{if oversubscription.}$$

Using [30], the results are presented in the proposition below.

Proposition 3 *As the anticipated equilibrium value of \mathbf{t}_G changes, the change in share ownership is given by:* [31]

$$\frac{dS}{d\mathbf{t}_G} = \frac{1}{\bar{c}} \left(-\mathbf{p} \frac{W}{P} \right) < 0 \quad \text{when } \tilde{d} = \frac{W}{P} \text{ (undersubscription)}$$

$$\frac{dS}{d\mathbf{t}_G} = \frac{-\frac{1}{\bar{c}} \mathbf{p}}{1 + \frac{1}{\bar{c}} \frac{1}{S^2} [(1 - \mathbf{t}_G) \mathbf{p} - P]} < 0 \quad \text{when } \tilde{d} = \frac{1}{S} \text{ (oversubscription)}$$

(See Appendix 2 for the proof)

As the anticipated value of future expropriation rate decreases, the anticipated value of the firm, $VF=(1-\tau)\pi$, increases. For a given P and $e>0$, more investors will demand shares, and therefore S increases. Comparing the two equations in [31], the results indicate that, for any given reduction in expropriation rate, the increase in share ownership is proportionally less when the issue is oversubscribed than in case of undersubscription. This is simply explained by considering that, if the issue is oversubscribed, each voter would receive proportionally fewer shares. Therefore the expected returns to individual investment are lower than in the case of undersubscription, and, *ceteris paribus*, fewer additional voters would demand shares.

The results presented in Propositions 2 and 3 help to provide a graphical interpretation of the game and its possible equilibria, as presented in Figure (1). For a given price of the stock P and a given level of promotional effort e , the figure combines the results of the election game in the second period (the thin line) with a representation of the first period investment behavior (the thick line).

The second period outcome is summarized by representing the value of the firm, which is $VF=(1-\tau)\pi$, as a function of share ownership S . From Proposition 2, we know that, holding P and e constant, as S increases τ decreases. Therefore VF increases as S increases, as indicated by the positive slope of the thin line in Figure (1). The value of the firm increases with S only when the issue is undersubscribed: for any given $P>0$, there is always a positive value of S ($S=P/W$) beyond which the issue is oversubscribed (i.e. $S\frac{W}{P} > 1$), $\frac{dt_G}{dS} = 0$ according to Proposition 2, and therefore VF does not change with S . Notice also that $S=1$ always implies oversubscription.

The first period results concerning the investment decision are summarized by equation [30], which defines the fraction of shareholders S . Now consider representing S as a function of $(VF-P)$, the difference between the value of the company, $VF=(1-\tau)\pi$, and its price, P . S is therefore represented as a function of the degree of underpricing, as shown by the thick line in Figure (1). Given P , for any anticipated value of τ such that $VF < P$, the demand for shares is zero, i.e. $S=0$. As the anticipated value of τ decreases, VF increases. For a value of τ such that $VF=P$, given $e > 0$, there will be a fraction of voters with zero transaction costs who, according to condition [7], are interested in buying shares. Therefore S will be positive, i.e. $S > 0$ as indicated by the flat portion of the thick line in Figure (1). As the anticipated value of the rate of expropriation gets smaller, VF increases, $(VF-P) > 0$ increases, and, according to the results presented in Proposition 3, S increases. At first, there is undersubscription ($S < P/W$), and all voters who want to buy shares are able to obtain as many shares as they want. Eventually, VF increases to where $S=P/W$. At this point, further increases in VF lead to oversubscription, and this changes the impact of an increase of VF on S , creating the kink shown in the diagram. As previously explained, once the issue is oversubscribed, for the same price P and the same expropriation rate, therefore for the same value $VF=(1-\tau)\pi$ of the stock, each subscriber receives fewer shares. This implies that the expected returns to the investment are lower than they would otherwise be. It follows that, given the promotional effort e , therefore given each individual's transaction costs, fewer additional voters demand shares: an increase in VF has now a smaller effect on S than it would have if the issue was not oversubscribed.

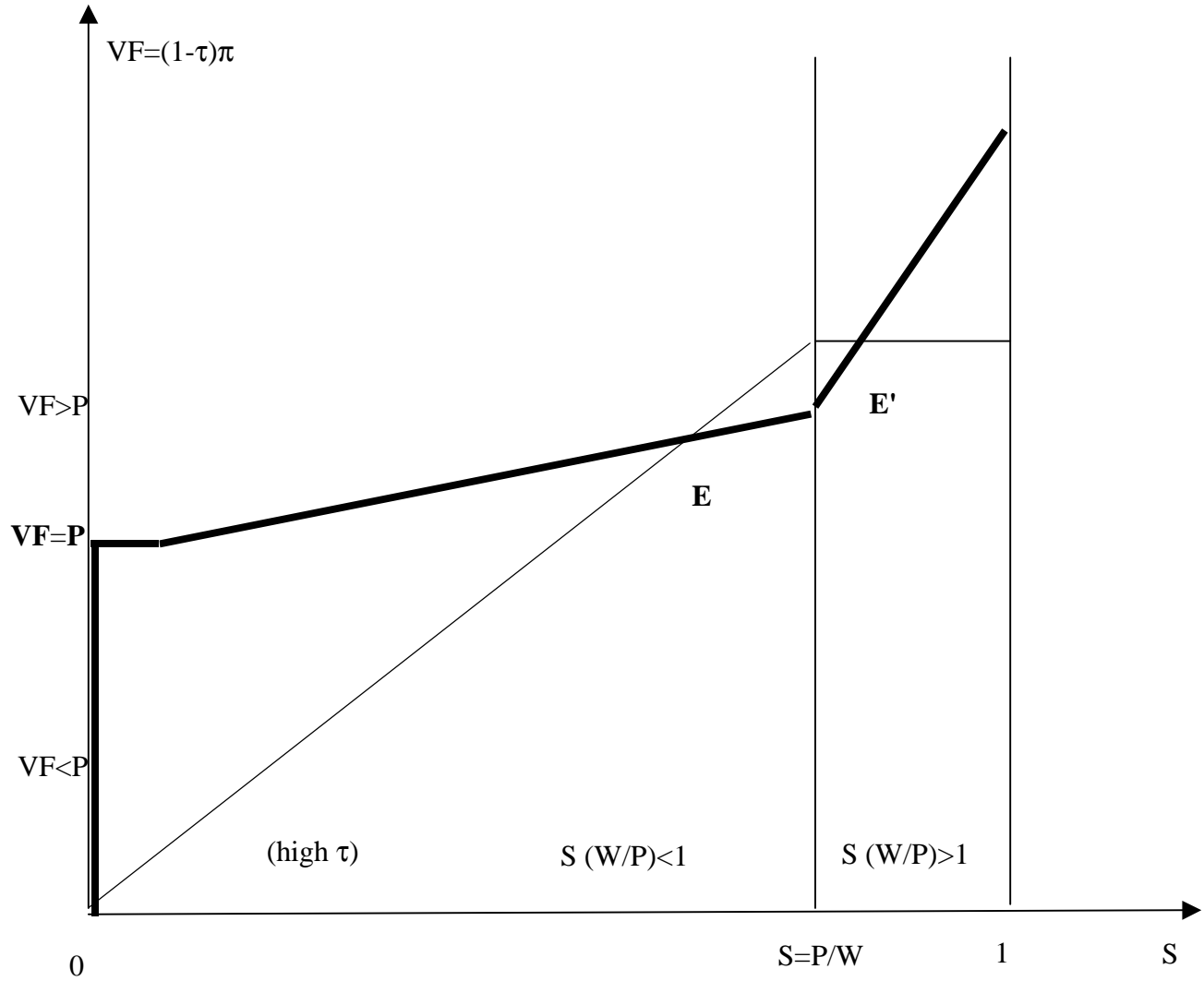


Figure (1): The Equilibria of the Game.

Figure (1) indicates the possibility of two equilibria¹³ characterized by underpricing and active promotion of the sale. One equilibrium (point E in the diagram) has relatively low S , no oversubscription, and high expropriation: the government prefers this equilibrium, because the higher rate of expropriation increases its revenues. The other equilibrium (point E' in Figure (1)) is characterized by relatively high S , oversubscription and rationing, and by relatively low expropriation, therefore low revenues for the government, *ceteris paribus*. The actual realization of one of the two possible equilibria depends on the endogenous determination of S , and S depends on which (second period) platform announcement the investors anticipate in the first period. The government may therefore be able to select its preferred (low S -high τ) equilibrium, by making a non-binding but credible announcement of its future platform.

Now consider the effect on $t_G = f(P, e, S, t_o)$, and therefore on the value of the firm $VF = (1 - \tau_G)\pi$, of a change in e , holding P and S constant. Intuitively, for a given price, higher e implies less public good, therefore the utility voters derive from the consumption of any additional unit of the public good increases, and this in turn means more incentive for the government to expropriate, given S . Higher expropriation reduces the firm's value VF .

Mathematically, we have to consider:

$$\frac{dt_G}{de} = \frac{\partial f}{\partial e} + \frac{\partial f}{\partial t_o} \frac{dt_o}{de} = \left(\frac{1}{1 - \frac{\partial f}{\partial t_o}} \right) \frac{\partial f}{\partial e}$$

¹³ For simplicity, the picture represents a linear relationship between VF and S in the case of undersubscription. From Proposition 2, it is obvious that such relationship will very likely be non-linear. Therefore additional equilibria, not shown in Figure (1), are possible.

The results are presented in the following Proposition.

Proposition 4 *Given the price P and the fraction of share ownership S , as advertising effort e changes, the change in t_G equals:* [32]

$$\frac{dt_G}{de} = \frac{(1-a)^2 A'V_{qq}(b+apt_G)}{p\{a[V_q(1-a)-S\tilde{d}]+(1-a)^2V_{qq}(b+apt_G)\}} > 0, \quad \tilde{d} = \begin{cases} \frac{W}{P} & \text{if undersubscription} \\ \frac{1}{S} & \text{if oversubscription} \end{cases}$$

(See Appendix 2 for the proof)

The result in [32] confirms that there exists a direct relationship between level of advertising effort chosen in the first period and rate of expropriation announced in the second period, given P and S . Ceteris paribus, as e increases (decreases), t_G increases (decreases), therefore the value of the firm VF decreases (increases). For an increase in e , Figure (2) illustrates this result graphically. The thin line, representing VF as a function of S , moves in the direction indicated by the dashed line in the figure.

What is the effect of a change in e on S , holding P and t_G constant? For a given price of shares and for a given anticipated value of future expropriation (therefore for a given degree of underpricing, $VF-P$), more (less) advertising implies that more (fewer) investors will buy the company's stock. Again, a dashed line in Figure (2) illustrates this effect graphically, for an increase in e . This result is presented in Proposition 5.

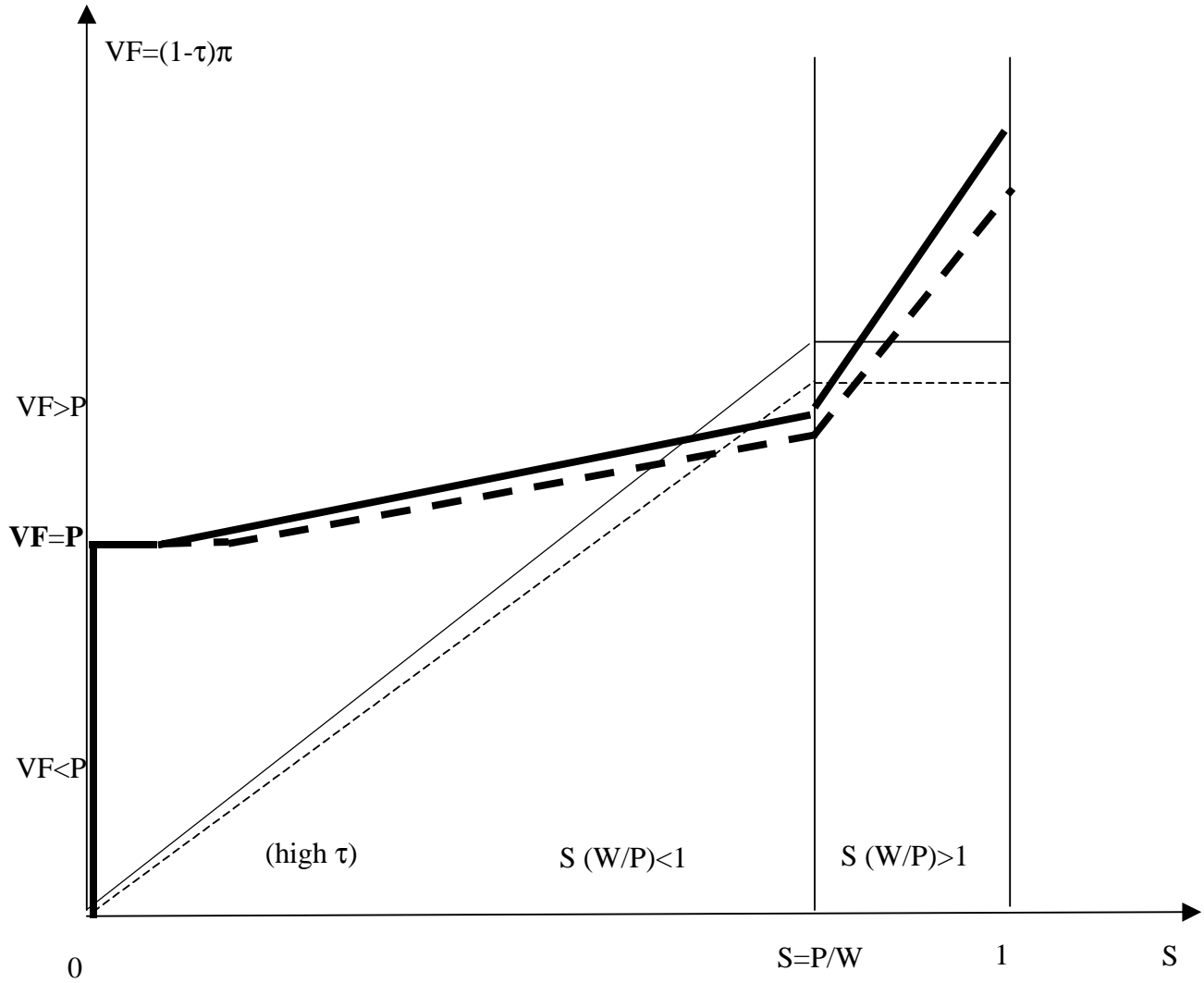


Figure (2): Effect of an increase in advertising effort.

Proposition 5 Holding P and t_G constant, a change in advertising effort e causes share ownership S to change as follows: [33]

$$\frac{dS}{de} \equiv \frac{\partial g}{\partial e} = \frac{1}{\bar{c}} > 0 \text{ if undersubscription}$$

$$\frac{dS}{de} = \left(\frac{1}{1 - \frac{\partial g}{\partial S}} \right) \frac{\partial g}{\partial e} = \frac{\frac{1}{\bar{c}}}{1 + \frac{1}{\bar{c}} \frac{1}{S^2} [(1 - t_G)P - P]} > 0 \text{ if oversubscription}$$

(See Appendix 2 for the proof)

Similarly to what obtained in Proposition 3, when the issue is oversubscribed the returns to individual investment are reduced by share rationing, and therefore the increase in S due to a given increase in e is proportionally less than in the case of undersubscription.

Figure (2) suggests that, for both equilibria, the overall effect on the equilibrium value of S of a change in e is uncertain. What the figure indicates is confirmed by the comparative statics results, which are presented in Proposition 6.

Proposition 6

A) If in equilibrium the issue is undersubscribed, and therefore $S = g(P, e, t_G)$, the effect on S of

a change in e is measured by $\frac{dS}{de} = \frac{\partial g}{\partial e} + \frac{\partial g}{\partial t_G} \frac{dt_G}{de}$ and it equals: [34]

$$\frac{dS}{de} = \frac{1}{\bar{c}} \left(1 - 2A' \frac{W}{P} \right)$$

With $A' > 0$, $P < W$, and $\bar{c} > 0$, the sign is indeterminate.

B) If in equilibrium the issue is oversubscribed, and therefore $S = g(P, e, t_G, S)$, the effect on S of a change in e is measured by

$$\frac{dS}{de} = \frac{\partial g}{\partial e} + \frac{\partial g}{\partial t_G} \frac{dt_G}{de} + \frac{\partial g}{\partial S} \frac{dS}{de} \text{ which is } \frac{dS}{de} = \left(\frac{1}{1 - \frac{\partial g}{\partial S}} \right) \left(\frac{\partial g}{\partial e} + \frac{\partial g}{\partial t_G} \frac{dt_G}{de} \right), \text{ and it equals:}$$

$$\frac{dS}{de} = \frac{\frac{1}{c} \left(1 - 2A' \frac{1}{S} \right)}{1 + \frac{1}{c} \frac{1}{S^2} [(1 - t_G)P - P]}$$

[35]

Again, the sign is indeterminate.

(See Appendix 2 for the proof)

The uncertainty concerning the effect of a change in e on the equilibrium value of S is explained considering the results presented in Proposition 1 and Proposition 5. In equilibrium, as e increases, on the one hand transaction costs are lowered and more voters will buy shares (Proposition 5), on the other hand the government will expropriate more in the second period, if elected (Proposition 1). Higher anticipated expropriation of profits will in turn reduce the investors' income from share ownership, and therefore fewer voters will demand shares. This trade-off determines the ambiguity of the results presented in Proposition 6, and it could be eliminated only introducing more restrictive assumptions on some parameters in the model.

Now consider the effect on $t_G = f(P, e, S, t_o)$, and therefore on the value of the firm $VF = (1 - \tau_G)\pi$, of a change in P , holding e and S constant. Mathematically:

$$\frac{dt_G}{dP} = \frac{\partial f}{\partial P} + \frac{\partial f}{\partial t_o} \frac{dt_o}{dP} = \left(\frac{1}{1 - \frac{\partial f}{\partial t_o}} \right) \frac{\partial f}{\partial P}$$

The results are presented in Proposition 7.

Proposition 7 For a given level of advertising effort e and fraction of share ownership S , the change in t_G due to a change in share price P is determined as follows:

A) Assume $\left| (1-a)^2 V_{qq} \right| > S \frac{W}{P^2}$

Then, if the issue is undersubscribed:

[36]

$$\frac{dt_G}{dP} = \frac{-(b + apt_G) \left[(1-a)^2 V_{qq} + S \frac{W}{P^2} \right]}{p \left\{ a \left[V_q (1-a) - S \frac{W}{P} \right] + (1-a)^2 V_{qq} (b + apt_G) \right\}} < 0$$

B) If the issue is oversubscribed:

[37]

$$\frac{dt_G}{dP} = \frac{-(b + apt_G)(1-a)^2 V_{qq}}{p \left\{ a \left[V_q (1-a) - 1 \right] + (1-a)^2 V_{qq} (b + apt_G) \right\}} < 0$$

(See Appendix 2 for the proof)

To understand the results presented in Proposition 7, consider again the marginal impact of a change in t_G on the probability of winning the election, which is given by:

$$\frac{\mathcal{J}m_k(t_G, t_O)}{\mathcal{J}t_k} = \frac{1}{2I} \left(S \frac{du_{iS}(t_k)}{dt_k} + N \frac{du_{iN}(t_k)}{dt_k} \right) = \frac{1}{2I} \left[V_q (1-a)p - Sp\tilde{d} \right]$$

Recall that $V_q (1-a)p$ measures the aggregate marginal utility gains for the entire population from increased consumption of the public good due to an increase in t_G , and $SV_p \tilde{d} \equiv Sp\tilde{d}$ measures the aggregate marginal utility losses for domestic shareholders from lost income due to an increase in t_G . Now consider the impact that a change in the price P has on

each of these components, holding e and S constant. As P increases, the amount of public good increases, the utility derived from the consumption of any additional unit of the public good decreases (at a rate measured by $\left| (1-\mathbf{a})^2 V_{qq} \right|$), and therefore the government has an incentive to announce less expropriation. This is the only effect of a change in price when the issue is oversubscribed, because, in case of oversubscription, the marginal impact of a change in expropriation rate on the shareholders' utility, as measured by $S\mathbf{p} \frac{1}{S} = \mathbf{p}$, is constant. Hence the result in part B of Proposition 7. On the other hand, when the issue is undersubscribed the aggregate marginal disutility of the shareholders due to a change in expropriation, as measured by $S\mathbf{p} \frac{W}{P}$, is a function of P . As P increases, the shareholders' disutility increases at a rate equal to $S\mathbf{p} \frac{W}{P^2}$. I assume that the aggregate impact of a change in price on the marginal utility that the entire population derives from consumption of the public good is greater than the impact P has on the shareholders' marginal utility. Hence the result in part A of Proposition 7.

These results are illustrated in Figure (3): for an increase in share price P , t_G decreases according to Proposition 7, and therefore the value of the firm $VF=(1-\tau_G)\pi$ increases, ceteris paribus (dashed thin line in the figure).

Figure (3) also shows the effect of a similar increase in price P on the fraction of share ownership S , holding e and τ_G (and therefore VF) constant: as P increases S decreases, as indicated by the dashed thick line in the figure. Using the definition of S in [30], Proposition 8 presents this result mathematically.

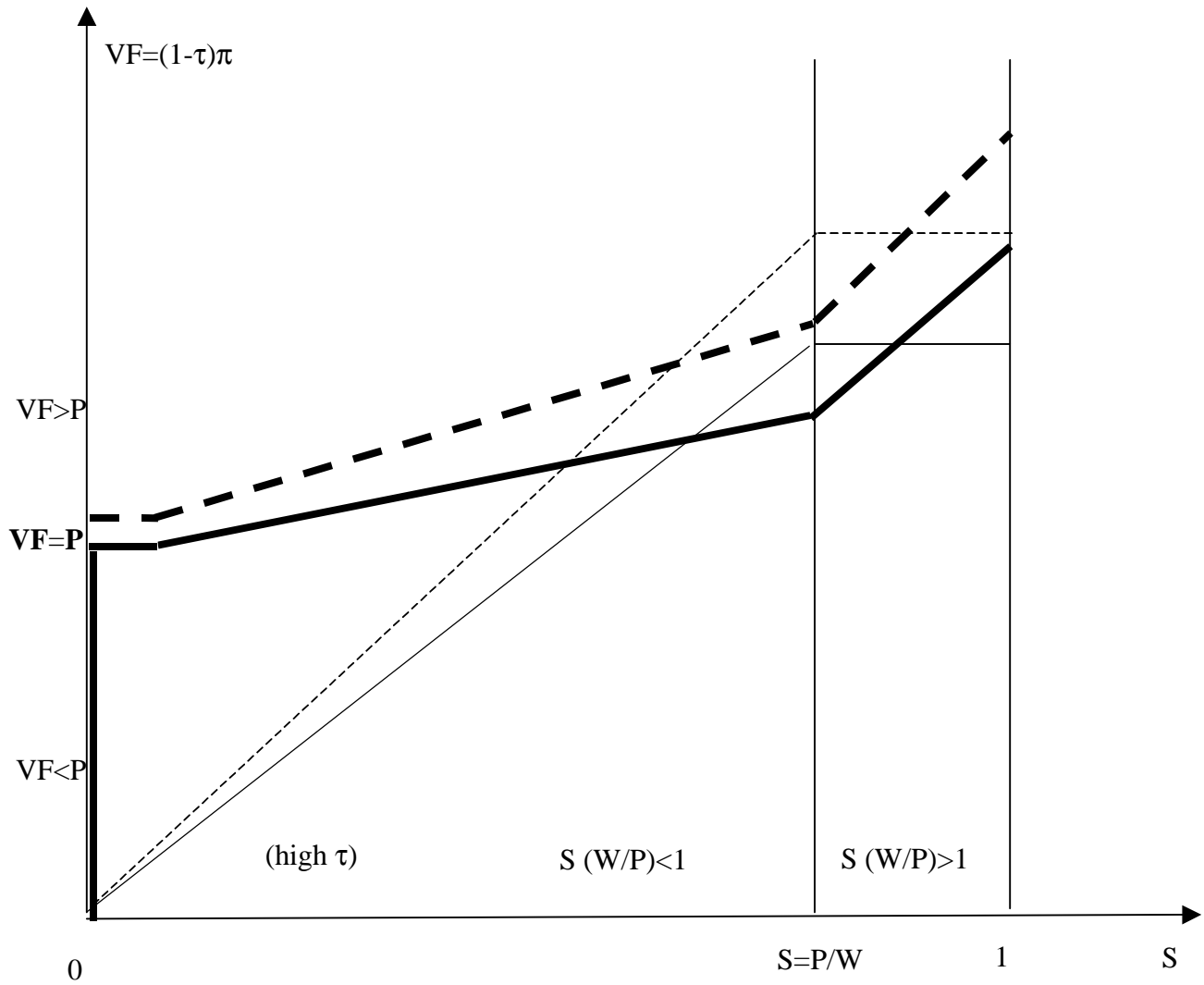


Figure (3): Effect of an increase in share price.

Proposition 8 Holding e and \mathbf{t}_G constant, a change in share price P causes share ownership S

to change as follows:

[38]

$$\frac{dS}{dP} \equiv \frac{\partial g}{\partial P} = \frac{1}{\bar{c}} \left[-(1 - \mathbf{t}_G) \mathbf{p} \frac{W}{P^2} \right] < 0 \text{ if undersubscription}$$

$$\frac{dS}{dP} \equiv \left(\frac{1}{1 - \frac{\partial g}{\partial S}} \right) \frac{\partial g}{\partial P} = \frac{-\frac{1}{\bar{c}} \frac{1}{S}}{1 + \frac{1}{\bar{c}} \frac{1}{S^2} [(1 - \mathbf{t}_G) \mathbf{p} - P]} < 0 \text{ if oversubscription}$$

(See Appendix 2 for the proof)

The next proposition describes the overall effect on the equilibrium value of S of a change in price P , when the equilibrium value of \mathbf{t}_G changes as well, according to the results presented in Proposition 1.

Proposition 9

A) If in equilibrium the issue is undersubscribed, and therefore $S = g(P, e, \mathbf{t}_G)$, the effect on S of

a change in P is measured by $\frac{dS}{dP} = \frac{\partial g}{\partial P} + \frac{\partial g}{\partial \mathbf{t}_G} \frac{d\mathbf{t}_G}{dP}$ and it equals: [39]

$$\frac{dS}{dP} = \frac{1}{\bar{c}} \frac{W}{P} \left(2 - \frac{\mathbf{p}}{P} (1 - \mathbf{t}_G) \right)$$

The sign is indeterminate.

B) If in equilibrium the issue is oversubscribed, and therefore $S = g(P, e, \mathbf{t}_G, S)$, the effect on S

of a change in P is measured by

$$\frac{dS}{dP} = \frac{\partial g}{\partial P} + \frac{\partial g}{\partial \mathbf{t}_G} \frac{d\mathbf{t}_G}{dP} + \frac{\partial g}{\partial S} \frac{dS}{dP} \text{ which is } \frac{dS}{dP} = \left(\frac{1}{1 - \frac{\partial g}{\partial S}} \right) \left(\frac{\partial g}{\partial P} + \frac{\partial g}{\partial \mathbf{t}_G} \frac{d\mathbf{t}_G}{dP} \right), \text{ and it equals:}$$

[40]

$$\frac{dS}{dP} = \frac{\frac{1}{\bar{c}} \frac{1}{S}}{1 + \frac{1}{\bar{c}} \frac{1}{S^2} [(1 - t_G)P - P]} > 0$$

(See Appendix 2 for the proof)

To understand these results, consider Proposition 8 together with Proposition 1. According to Proposition 8, the effect of higher share price on share ownership is negative. This result is derived holding t_G constant. But from Proposition 1, we know that in equilibrium higher share price in the first period determines lower expropriation rate in the second period. And anticipated lower expropriation translates in higher share ownership in the first period. The combination of these two opposite effects on S of a change in share price P determines the results presented in Proposition 9. In case of undersubscription, which of the two effects is stronger depends on the parameter values in the model, and without stricter assumptions on such values the sign is uncertain. In case of oversubscription, the equilibrium value of S increases as P increases, which indicates that the positive effect on S of a decrease in the equilibrium expropriation rate prevails.

5. Conclusions.

Empirical evidence shows that governments around the world have often sold companies to the private sector at big discounts, therefore forgoing revenues, and that they have engaged in heavy promotion and advertising of the sale, therefore incurring high financial costs. They also have often declared their interest in wide share ownership.

This paper provides an explanation for these observations. It shows that wide diffusion of share ownership may maximize government revenues from privatization, even though this diffusion must be achieved through expensive promotional activities and the underpricing of shares.

This result is obtained in a simple two-period model of complete information, with a government that first privatizes a company and then competes for votes against a political opponent. The government announces a low sale price and engages in promotion of the sale in the first period, and this generates diffusion of share ownership among the population of voters. In the second period, the two parties compete for votes announcing simultaneously their choice of a future rate of expropriation (taxation) of private profits. The lower is the price of the stock and the higher is the promotion expenditure in the first period, the bigger is the size of the shareholder interest group in the second period, and therefore the lower is the announced future rate of expropriation. The commitment to a low expropriation rate thus allows the government to set a higher price in the first period.

The results show the possibility of two equilibria, for a given price and level of promotional effort: one equilibrium occurs when the issue is oversubscribed, while the other characterizes a sale with no oversubscription. The government always prefers the latter equilibrium, because it produces a higher rate of expropriation and therefore higher revenues.

The model can be extended to include the possibility that private profits, which in the current analysis are exogenous, depend on diffusion of share ownership. In fact, any positive effect on profitability of increased corporate control is diluted when shares are dispersed among many small investors. If this is the case, any increase of the price of shares which could be

obtained by increasing diffusion of asset ownership and therefore committing to lower future expropriation, might be (partially) offset by the negative effect that higher diffusion would have on private profits.

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APPENDIX 1

The shareholders' preferred expropriation rate.

In the second period, the optimal platform announcement for the representative shareholder solves the problem below:

$$\begin{aligned} \max_{\mathbf{t}_k} V(q(\mathbf{t}_k), y(\mathbf{t}_k)) - (c_i - e) & \quad \mathbf{k} \in \{G, O\}, \forall c_i \in [0, \bar{c}], \forall e \in [0, 1) \\ \text{subject to} & \\ 0 \leq \mathbf{t}_k \leq 1 & \end{aligned}$$

with

$$q(\mathbf{t}_k) = (1 - \mathbf{a})[P - A(e) + \mathbf{t}_k \mathbf{p}]$$

and

$$y(\mathbf{t}_k) = \begin{cases} \mathbf{W} + [(1 - \mathbf{t}_k) \mathbf{p} - P] \frac{1}{S} & \text{if oversubscription} \\ (1 - \mathbf{t}_k) \mathbf{p} \frac{\mathbf{W}}{P} & \text{if no oversubscription} \end{cases}$$

The first order condition of the (unconstrained) maximization problem is

$$V_q(\mathbf{t}_k)(1 - \mathbf{a})\mathbf{p} + V_y(-\mathbf{p}\tilde{d}) = 0$$

where for simplicity $\frac{\mathcal{J}V(q(\mathbf{t}_k), y(\mathbf{t}_k))}{\mathcal{J}q(\mathbf{t}_k)} \equiv V_q(\mathbf{t}_k)$. By assumption $V_y = 1$, therefore the above

condition simplifies to:

$$(1 - a)V_q(\mathbf{t}_k) - \tilde{d} = 0 \quad [\text{A1}]$$

$$\text{where } \tilde{d} = \begin{cases} 1/S & \text{if oversubscription} \\ \mathbf{W}/P & \text{if no oversubscription} \end{cases}$$

Because of the assumption of quasilinearity, $V_{qy} = V_{yq} = 0$ and $V_{yy} = 0$. Therefore, the second order condition for the existence of a unique global maximum is always verified. This

proves the existence of a unique solution of the unconstrained maximization problem, but nothing excludes the possibility of a corner solution of the constrained maximization program. Only imposing more restrictive assumptions on the values of the partial derivatives of the utility function would guarantee an interior value for the shareholders' preferred rate of expropriation.

The subgame: uniqueness of the NE.

The solution of the government's maximization problem is $\mathbf{t}_G = f(\mathbf{t}_o)$ and it satisfies the first order condition:

$$2\mathbf{l} \mathbf{a} \mathbf{p} \mathbf{m}_G(f(\mathbf{t}_o), \mathbf{t}_o) + (\mathbf{b} + \mathbf{a} f(\mathbf{t}_o) \mathbf{p}) \left(S \frac{d u_{iS}(f(\mathbf{t}_o))}{d f(\mathbf{t}_o)} + N \frac{d u_{iN}(f(\mathbf{t}_o))}{d f(\mathbf{t}_o)} \right) \equiv 0$$

Differentiating with respect to \mathbf{t}_o , we have:

$$2\mathbf{l} \mathbf{a} \mathbf{p} \frac{d \mathbf{m}_G(f(\mathbf{t}_o), \mathbf{t}_o)}{d \mathbf{t}_o} + (\mathbf{b} + \mathbf{a} f(\mathbf{t}_o) \mathbf{p}) \left(S \frac{d \frac{d u_{iS}(f(\mathbf{t}_o))}{d f(\mathbf{t}_o)}}{d \mathbf{t}_o} + N \frac{d \frac{d u_{iN}(f(\mathbf{t}_o))}{d f(\mathbf{t}_o)}}{d \mathbf{t}_o} \right) + \mathbf{a} \mathbf{p} \frac{d f(\mathbf{t}_o)}{d \mathbf{t}_o} \left(S \frac{d u_{iS}(f(\mathbf{t}_o))}{d f(\mathbf{t}_o)} + N \frac{d u_{iN}(f(\mathbf{t}_o))}{d f(\mathbf{t}_o)} \right) \equiv 0$$

[A2]

Remember that:

$$\mathbf{m}_G(\mathbf{t}_G, \mathbf{t}_o) = 1 - \frac{1}{2\mathbf{l}} \left[S(u_{iS}(\mathbf{t}_o) - u_{iS}(\mathbf{t}_G)) + N(u_{iN}(\mathbf{t}_o) - u_{iN}(\mathbf{t}_G)) + \mathbf{l} \right]$$

and that

$$\frac{d u_{iS}(\mathbf{t}_k)}{d \mathbf{t}_k} = V_q(\mathbf{t}_K) [(1 - \mathbf{a}) \mathbf{p}] - \mathbf{p} \tilde{d}$$

$$\frac{d u_{iN}(\mathbf{t}_k)}{d \mathbf{t}_k} = V_q(\mathbf{t}_K) [(1 - \mathbf{a}) \mathbf{p}]$$

with $V_y = 1$.

To simplify the notation, $\frac{\mathbb{1}V(q(\mathbf{t}_k), y(\mathbf{t}_k))}{\mathbb{1}q(\mathbf{t}_k)} \equiv V_q(\mathbf{t}_k)$ and $\frac{\mathbb{1}^2V(q(\mathbf{t}_k), y(\mathbf{t}_k))}{\mathbb{1}q^2(\mathbf{t}_k)} \equiv V_{qq}(\mathbf{t}_k)$.

Computing all the derivatives and substituting them back in [A2]:

$$\begin{aligned} & (-S\mathbf{a}\mathbf{p}^2)(V_q(\mathbf{t}_o)(1-\mathbf{a})-\tilde{d}) + (S\mathbf{a}\mathbf{p}^2)(V_q(\mathbf{t}_G)(1-\mathbf{a})-\tilde{d})\frac{df(\mathbf{t}_o)}{d\mathbf{t}_o} + \\ & + (-N\mathbf{a}\mathbf{p}^2)V_q(\mathbf{t}_o)(1-\mathbf{a}) + (N\mathbf{a}\mathbf{p}^2)V_q(\mathbf{t}_G)(1-\mathbf{a})\frac{df(\mathbf{t}_o)}{d\mathbf{t}_o} + \\ & + (\mathbf{b} + \mathbf{a}f(\mathbf{t}_o)\mathbf{p})\left[S(V_{qq}(\mathbf{t}_G)(1-\mathbf{a})^2\mathbf{p}^2) + N(V_{qq}(\mathbf{t}_G)(1-\mathbf{a})^2\mathbf{p}^2)\right]\frac{df(\mathbf{t}_o)}{d\mathbf{t}_o} + \\ & + (S\mathbf{a}\mathbf{p}^2)(V_q(\mathbf{t}_G)(1-\mathbf{a})-\tilde{d})\frac{df(\mathbf{t}_o)}{d\mathbf{t}_o} + (N\mathbf{a}\mathbf{p}^2)V_q(\mathbf{t}_G)(1-\mathbf{a})\frac{df(\mathbf{t}_o)}{d\mathbf{t}_o} \equiv 0 \end{aligned}$$

which, recalling that $N \equiv 1-S$, simplifies to:

$$\begin{aligned} & S\mathbf{a}\mathbf{p}^2\tilde{d} - S\mathbf{a}\mathbf{p}^2\tilde{d}\frac{df(\mathbf{t}_o)}{d\mathbf{t}_o} - \mathbf{a}\mathbf{p}^2(1-\mathbf{a})V_q(\mathbf{t}_o) + \mathbf{a}\mathbf{p}^2(1-\mathbf{a})V_q(\mathbf{t}_G)\frac{df(\mathbf{t}_o)}{d\mathbf{t}_o} + \\ & + (\mathbf{b} + \mathbf{a}f(\mathbf{t}_o)\mathbf{p})(1-\mathbf{a})^2\mathbf{p}^2V_{qq}(\mathbf{t}_G)\frac{df(\mathbf{t}_o)}{d\mathbf{t}_o} - S\mathbf{a}\mathbf{p}^2\tilde{d}\frac{df(\mathbf{t}_o)}{d\mathbf{t}_o} + \mathbf{a}\mathbf{p}^2(1-\mathbf{a})V_q(\mathbf{t}_G)\frac{df(\mathbf{t}_o)}{d\mathbf{t}_o} \equiv 0 \end{aligned}$$

Finally:

$$\frac{df(\mathbf{t}_o)}{d\mathbf{t}_o} = \frac{\mathbf{a}[(1-\mathbf{a})V_q(\mathbf{t}_o) - S\tilde{d}]}{2\mathbf{a}[(1-\mathbf{a})V_q(\mathbf{t}_G) - S\tilde{d}] + (1-\mathbf{a})^2(\mathbf{b} + \mathbf{a}f(\mathbf{t}_o)\mathbf{p})V_{qq}(\mathbf{t}_G)} \quad \mathbf{t}_o \in [0,1]$$

which is equation [23] in Section (3.2.) of the paper.

APPENDIX 2**Proofs of Propositions 3, 5, 6, 8, and 9.**

From equation [30] in the paper, in equilibrium share ownership S is explicitly defined as:

$$S = \frac{1}{c} [V(q) + y - V(q) - W + e]$$

with

$$q = (1 - \mathbf{a})[P - A(e) + \mathbf{t}_G \mathbf{p}] \quad [\text{A1}]$$

$$y = \begin{cases} (1 - \mathbf{t}_G) \mathbf{p} \frac{W}{P} & \text{if undersubscription} \\ W + [(1 - \mathbf{t}_G) \mathbf{p} - P] \frac{1}{S} & \text{if oversubscription} \end{cases}$$

$$V_q > 0, V_{qq} < 0, A' > 0$$

Therefore, $S = g(P, e, \mathbf{t}_G)$ in case of undersubscription, and $S = g(P, e, \mathbf{t}_G, S)$ if the issue is oversubscribed.

In equilibrium, as advertising effort e changes, we have:

$$\frac{dS}{de} = \frac{\partial g}{\partial e} + \frac{\partial g}{\partial \mathbf{t}_G} \frac{d\mathbf{t}_G}{de} \quad \text{if undersubscription}$$

$$\frac{dS}{de} = \frac{\partial g}{\partial e} + \frac{\partial g}{\partial \mathbf{t}_G} \frac{d\mathbf{t}_G}{de} + \frac{\partial g}{\partial S} \frac{dS}{de} \quad [\text{A2}]$$

$$= \left(\frac{1}{1 - \frac{\partial g}{\partial S}} \right) \left(\frac{\partial g}{\partial e} + \frac{\partial g}{\partial \mathbf{t}_G} \frac{d\mathbf{t}_G}{de} \right) \quad \text{if oversubscription}$$

From Proposition 1 in the paper:

$$\frac{d\mathbf{t}_G}{de} = \frac{2A'}{\mathbf{p}} > 0 \quad [\text{A3}]$$

From [A1], we obtain:

$$1) \frac{\partial g}{\partial e} = \frac{1}{\bar{c}} \left[V_q \frac{\partial q}{\partial e} + \frac{\partial y}{\partial e} - V_q \frac{\partial q}{\partial e} + 1 \right] = \frac{1}{\bar{c}} > 0 \quad [\text{A4}]$$

$$2) \frac{\partial g}{\partial t_G} = \frac{1}{\bar{c}} \left[V_q \frac{\partial q}{\partial t_G} + \frac{\partial y}{\partial t_G} - V_q \frac{\partial q}{\partial t_G} \right] = \frac{1}{\bar{c}} \frac{\partial y}{\partial t_G} = \frac{1}{\bar{c}} (-\mathbf{p}\tilde{d}) < 0 \quad \text{with } \tilde{d} = \begin{cases} \frac{W}{P} & \text{if undersubscription} \\ \frac{1}{S} & \text{if oversubscription} \end{cases}$$

$$3) \frac{\partial g}{\partial S} = \frac{1}{\bar{c}} \left[V_q \frac{\partial q}{\partial S} + \frac{\partial y}{\partial S} - V_q \frac{\partial q}{\partial S} \right] = \frac{1}{\bar{c}} \frac{\partial y}{\partial S} = -\frac{1}{\bar{c}} [(1-t_G)\mathbf{p} - P] \frac{1}{S^2} < 0$$

The results presented in Proposition 3 follow from [A4.2] and [A4.3], the results in Proposition 5 follow from [A4.1] and [A4.3], and Proposition 6 follows from [A3] and [A4].

Consider now a change in the equilibrium share price P. We have:

$$\frac{dS}{dP} = \frac{\partial g}{\partial P} + \frac{\partial g}{\partial t_G} \frac{dt_G}{dP} \quad \text{if undersubscription} \quad [\text{A5}]$$

$$\begin{aligned} \frac{dS}{dP} &= \frac{\partial g}{\partial P} + \frac{\partial g}{\partial t_G} \frac{dt_G}{dP} + \frac{\partial g}{\partial S} \frac{dS}{dP} \\ &= \left(\frac{1}{1 - \frac{\partial g}{\partial S}} \right) \left(\frac{\partial g}{\partial P} + \frac{\partial g}{\partial t_G} \frac{dt_G}{dP} \right) \quad \text{if oversubscription} \end{aligned}$$

From Proposition 1 in the paper:

$$\frac{dt_G}{dP} = -\frac{2}{\mathbf{p}} < 0 \quad [\text{A6}]$$

Again using [A1], we obtain:

[A7]

$$1) \frac{\partial g}{\partial P} = \frac{1}{\bar{c}} \left[V_q \frac{\partial q}{\partial P} + \frac{\partial y}{\partial P} - V_q \frac{\partial q}{\partial P} \right] = \frac{1}{\bar{c}} \frac{\partial y}{\partial P} = \begin{cases} \frac{1}{\bar{c}} \left[-(1-t_G) \mathbf{p} \frac{W}{P^2} \right] < 0 & \text{if undersubscription} \\ \frac{1}{\bar{c}} \left(-\frac{1}{S} \right) < 0 & \text{if oversubscription} \end{cases}$$

$$2) \frac{\partial g}{\partial t_G} = \frac{1}{\bar{c}} \left[V_q \frac{\partial q}{\partial t_G} + \frac{\partial y}{\partial t_G} - V_q \frac{\partial q}{\partial t_G} \right] = \frac{1}{\bar{c}} \frac{\partial y}{\partial t_G} = \frac{1}{\bar{c}} (-\mathbf{p} \tilde{d}) < 0, \quad \tilde{d} = \begin{cases} \frac{W}{P} & \text{if undersubscription} \\ \frac{1}{S} & \text{if oversubscription} \end{cases}$$

$$3) \frac{\partial g}{\partial S} = \frac{1}{\bar{c}} \left[V_q \frac{\partial q}{\partial S} + \frac{\partial y}{\partial S} - V_q \frac{\partial q}{\partial S} \right] = \frac{1}{\bar{c}} \frac{\partial y}{\partial S} = -\frac{1}{\bar{c}} [(1-t_G) \mathbf{p} - P] \frac{1}{S^2} < 0$$

The results presented in Proposition 8 follow from [A7.1] and [A7.3], and the results presented in Proposition 9 follow from [A6] and [A7].

Proofs of Propositions 2, 4, and 7.

In order to derive the results presented in Propositions 2, 4, and 7, the Implicit Function Theorem has to be applied to the first order condition of the second period election problem, which implicitly defines $t_G = f(P, e, S, t_o)$:

$$\frac{\partial \mathbf{m}_G(\mathbf{t}_G, \mathbf{t}_O)}{\partial \mathbf{t}_G} (\mathbf{b} + \mathbf{a} \mathbf{t}_G \mathbf{p}) + \mathbf{m}_G(\mathbf{t}_G, \mathbf{t}_O) \mathbf{a} \mathbf{p} = 0 \quad [\text{A8}]$$

with

$$\begin{aligned} \frac{\partial \mathbf{m}_G(\mathbf{t}_G, \mathbf{t}_O)}{\partial \mathbf{t}_G} &= \frac{1}{2\mathbf{I}} \left[S \frac{du_{is}(\mathbf{t}_G)}{d\mathbf{t}_G} + (1-S) \frac{du_{in}(\mathbf{t}_G)}{d\mathbf{t}_G} \right] \\ &= \frac{1}{2\mathbf{I}} \left[S(V_q(1-\mathbf{a})\mathbf{p} - \mathbf{p}\tilde{d}) + (1-S)(V_q(1-\mathbf{a})\mathbf{p}) \right] = \\ &= \frac{1}{2\mathbf{I}} [V_q(1-\mathbf{a})\mathbf{p} - S\mathbf{p}\tilde{d}] \end{aligned}$$

and

$$\begin{aligned} \mathbf{m}_G(\mathbf{t}_G, \mathbf{t}_O) &= 1 - \frac{1}{2\mathbf{I}} [S(u_{is}(\mathbf{t}_O) - u_{is}(\mathbf{t}_G)) + (1-S)(u_{in}(\mathbf{t}_O) - u_{in}(\mathbf{t}_G)) + \mathbf{I}] \\ &= \frac{1}{2} - \frac{1}{2\mathbf{I}} [Su_{is}(\mathbf{t}_O) - Su_{is}(\mathbf{t}_G) + u_{in}(\mathbf{t}_O) - u_{in}(\mathbf{t}_G) - Su_{in}(\mathbf{t}_O) + Su_{in}(\mathbf{t}_G)] \end{aligned}$$

where

$$u_{is}(\mathbf{t}) = V(q) + y - T_i \quad T_i = \begin{cases} (c_i - e) & \text{if } c_i \geq e \\ 0 & \text{otherwise} \end{cases}$$

$$u_{in}(\mathbf{t}) = V(q) + W$$

and

$$q = (1-\mathbf{a})[P - A(e) + \mathbf{t}\mathbf{p}]$$

$$y = W + [(1-\mathbf{t})\mathbf{p} - P]\tilde{d} \quad , \quad \tilde{d} = \begin{cases} \frac{W}{P} & \text{if undersubscription} \\ \frac{1}{S} & \text{if oversubscription} \end{cases}$$

$$V_q > 0, V_{qq} < 0, A' > 0$$

Also recall that, according to condition [19] in the paper, it is:

$$V_q(1-\mathbf{a}) < S\tilde{d}$$

Proof of Proposition 2

Proposition 2 determines the effect of a change in S on the equilibrium value of \mathbf{t}_G ,

holding P and e constant. Recalling that the equilibrium is symmetric, and therefore $\frac{d\mathbf{t}_G}{dS} = \frac{d\mathbf{t}_O}{dS}$,

we have:

$$\begin{aligned}\frac{d\mathbf{t}_G}{dS} &= \frac{\partial f}{\partial S} + \frac{\partial f}{\partial \mathbf{t}_o} \frac{d\mathbf{t}_o}{dS} \\ &= \left(\frac{1}{1 - \frac{\partial f}{\partial \mathbf{t}_o}} \right) \frac{\partial f}{\partial S}\end{aligned}$$

Equation [23] in the paper defines $0 < \frac{\partial f}{\partial \mathbf{t}_o} < 1$:

$$\frac{\partial f}{\partial \mathbf{t}_o} = \frac{\mathbf{a}[(1-\mathbf{a})V_q - S\tilde{d}]}{2\mathbf{a}[(1-\mathbf{a})V_q - S\tilde{d}] + (1-\mathbf{a})^2 V_{qq}(\mathbf{b} + \mathbf{a}\mathbf{p}\mathbf{t}_G)} \quad [\text{A9}]$$

In differentiating the FOC in [A8] with respect to S to calculate $\frac{\partial f}{\partial S}$, one has to consider

the two possibilities of undersubscription and oversubscription separately.

Undersubscription

The following pieces of information are derived from the definitions in [A8]:

$$\frac{du_{iN}(\mathbf{t}_o)}{dS} = 0; \quad \frac{du_{iS}(\mathbf{t}_o)}{dS} = 0 \quad [\text{A10}]$$

$$\frac{du_{iN}(\mathbf{t}_G)}{dS} = \frac{\partial u_{iN}(\mathbf{t}_G)}{\partial \mathbf{t}_G} \frac{\partial f}{\partial S} = V_q \frac{\partial q}{\partial \mathbf{t}_G} \frac{\partial f}{\partial S} = V_q (1-\mathbf{a})\mathbf{p} \frac{\partial f}{\partial S}$$

$$\frac{du_{iS}(\mathbf{t}_G)}{dS} = \frac{\partial u_{iS}(\mathbf{t}_G)}{\partial \mathbf{t}_G} \frac{\partial f}{\partial S} = \left(V_q \frac{\partial q}{\partial \mathbf{t}_G} + \frac{\partial y}{\partial \mathbf{t}_G} \right) \frac{\partial f}{\partial S} = \left(V_q (1-\mathbf{a})\mathbf{p} - \mathbf{p} \frac{W}{P} \right) \frac{\partial f}{\partial S}$$

Therefore :

$$\begin{aligned}\frac{d\mathbf{m}_G(\mathbf{t}_G, \mathbf{t}_o)}{dS} &= -\frac{1}{2I} [u_{iS}(\mathbf{t}_o) + S \frac{du_{iS}(\mathbf{t}_o)}{dS} - u_{iS}(\mathbf{t}_G) - S \frac{du_{iS}(\mathbf{t}_G)}{dS} + \frac{du_{iN}(\mathbf{t}_o)}{dS} - \frac{du_{iN}(\mathbf{t}_G)}{dS} + \\ &\quad - u_{iN}(\mathbf{t}_o) - S \frac{du_{iN}(\mathbf{t}_o)}{dS} + u_{iN}(\mathbf{t}_G) + S \frac{du_{iN}(\mathbf{t}_G)}{dS}] \\ &= -\frac{1}{2I} \left[S\mathbf{p} \frac{W}{P} \frac{\partial f}{\partial S} - V_q (1-\mathbf{a})\mathbf{p} \frac{\partial f}{\partial S} \right]\end{aligned}$$

And:

$$\frac{d \frac{\partial \mathbf{m}_G(\mathbf{t}_G, \mathbf{t}_O)}{\partial \mathbf{t}_G}}{dS} = \frac{\partial^2 \mathbf{m}_G(\mathbf{t}_G, \mathbf{t}_O)}{\partial \mathbf{t}_G^2} \frac{\partial f}{\partial S} + \frac{\partial \mathbf{m}_G(\mathbf{t}_G, \mathbf{t}_O)}{\partial \mathbf{t}_G \partial S} = \frac{1}{2I} \left[V_{qq} (1-\mathbf{a})^2 \mathbf{p}^2 \frac{\partial f}{\partial S} - \mathbf{p} \frac{W}{P} \right]$$

Using the information in [A8] and [A10], the FOC is differentiated with respect to S to obtain:

$$\begin{aligned} & \frac{1}{2I} \left[V_{qq} (1-\mathbf{a})^2 \mathbf{p}^2 \frac{\partial f}{\partial S} - \mathbf{p} \frac{W}{P} \right] (\mathbf{b} + \mathbf{a} \mathbf{p} \mathbf{t}_G) + \frac{1}{2I} \left[V_q (1-\mathbf{a}) \mathbf{p} - S \mathbf{p} \frac{W}{P} \right] \mathbf{a} \mathbf{p} \frac{\partial f}{\partial S} + \\ & + \mathbf{a} \mathbf{p} \left(-\frac{1}{2I} \left[S \mathbf{p} \frac{W}{P} \frac{\partial f}{\partial S} - V_q (1-\mathbf{a}) \mathbf{p} \frac{\partial f}{\partial S} \right] \right) \equiv 0 \end{aligned}$$

Multiplying every term by (2) and isolating the terms with $\frac{\partial f}{\partial S}$:

$$\begin{aligned} & V_{qq} (1-\mathbf{a})^2 \mathbf{p}^2 (\mathbf{b} + \mathbf{a} \mathbf{p} \mathbf{t}_G) \frac{\partial f}{\partial S} + 2 \mathbf{a} \mathbf{p}^2 V_q (1-\mathbf{a}) \frac{\partial f}{\partial S} - 2 \mathbf{a} \mathbf{p}^2 S \frac{W}{P} \frac{\partial f}{\partial S} = \frac{W}{P} \mathbf{p} (\mathbf{b} + \mathbf{a} \mathbf{p} \mathbf{t}_G) \\ & \frac{\partial f}{\partial S} = \frac{\frac{W}{P} (\mathbf{b} + \mathbf{a} \mathbf{p} \mathbf{t}_G)}{2 \mathbf{a} \mathbf{p} \left[V_q (1-\mathbf{a}) - S \frac{W}{P} \right] + V_{qq} (1-\mathbf{a})^2 \mathbf{p} (\mathbf{b} + \mathbf{a} \mathbf{p} \mathbf{t}_G)} < 0 \end{aligned} \quad [\text{A11}]$$

Using [A11] and [A9], we finally obtain:

$$\frac{d \mathbf{t}_G}{dS} = \frac{\frac{W}{P} (\mathbf{b} + \mathbf{a} \mathbf{p} \mathbf{t}_G)}{\mathbf{p} \left\{ \mathbf{a} \left[V_q (1-\mathbf{a}) - S \frac{W}{P} \right] + V_{qq} (1-\mathbf{a})^2 (\mathbf{b} + \mathbf{a} \mathbf{p} \mathbf{t}_G) \right\}} < 0$$

which is the result presented in Proposition 2.

Oversubscription

Again, from [A8] we can derive the following pieces of information:

$$\frac{du_{iN}(\mathbf{t}_o)}{dS} = 0; \quad \frac{du_{iS}(\mathbf{t}_o)}{dS} = V_q \frac{\partial q}{\partial S} + \frac{\partial y}{\partial S} = -[(1-\mathbf{t}_o)\mathbf{p} - P] \frac{1}{S^2} \quad [\text{A12}]$$

$$\frac{du_{iN}(\mathbf{t}_G)}{dS} = \frac{\partial u_{iN}(\mathbf{t}_G)}{\partial \mathbf{t}_G} \frac{\partial f}{\partial S} = V_q \frac{\partial q}{\partial \mathbf{t}_G} \frac{\partial f}{\partial S} = V_q (1-\mathbf{a})\mathbf{p} \frac{\partial f}{\partial S}$$

$$\begin{aligned} \frac{du_{iS}(\mathbf{t}_G)}{dS} &= \frac{\partial u_{iS}(\mathbf{t}_G)}{\partial \mathbf{t}_G} \frac{\partial f}{\partial S} = \left(V_q \frac{\partial q}{\partial \mathbf{t}_G} + \frac{\partial y}{\partial \mathbf{t}_G} \right) \frac{\partial f}{\partial S} + V_q \frac{\partial q}{\partial S} + \frac{\partial y}{\partial S} = \left(V_q (1-\mathbf{a})\mathbf{p} - \frac{\mathbf{p}}{S} \right) \frac{\partial f}{\partial S} + \\ &\quad - [(1-\mathbf{t}_G)\mathbf{p} - P] \frac{1}{S^2} \end{aligned}$$

Therefore :

$$\begin{aligned} \frac{d\mathbf{m}_G(\mathbf{t}_G, \mathbf{t}_o)}{dS} &= -\frac{1}{2I} [u_{iS}(\mathbf{t}_o) + S \frac{du_{iS}(\mathbf{t}_o)}{dS} - u_{iS}(\mathbf{t}_G) - S \frac{du_{iS}(\mathbf{t}_G)}{dS} + \frac{du_{iN}(\mathbf{t}_o)}{dS} - \frac{du_{iN}(\mathbf{t}_G)}{dS} + \\ &\quad - u_{iN}(\mathbf{t}_o) - S \frac{du_{iN}(\mathbf{t}_o)}{dS} + u_{iN}(\mathbf{t}_G) + S \frac{du_{iS}(\mathbf{t}_G)}{dS}] \\ &= -\frac{1}{2I} \left[-S[(1-\mathbf{t}_o)\mathbf{p} - P] \frac{1}{S^2} - S \left[\left(V_q (1-\mathbf{a})\mathbf{p} - \frac{\mathbf{p}}{S} \right) \frac{\partial f}{\partial S} - [(1-\mathbf{t}_G)\mathbf{p} - P] \frac{1}{S^2} \right] + \right. \\ &\quad \left. - (1-S)V_q (1-\mathbf{a})\mathbf{p} \frac{\partial f}{\partial S} \right] \end{aligned}$$

And:

$$\frac{d \frac{\partial \mathbf{m}_G(\mathbf{t}_G, \mathbf{t}_o)}{\partial \mathbf{t}_G}}{dS} = \frac{\partial^2 \mathbf{m}_G(\mathbf{t}_G, \mathbf{t}_o)}{\partial \mathbf{t}_G^2} \frac{\partial f}{\partial S} + \frac{\partial \mathbf{m}_G(\mathbf{t}_G, \mathbf{t}_o)}{\partial \mathbf{t}_G \partial S} = \frac{\mathbf{p}}{2I} \left[V_{qq} (1-\mathbf{a})^2 \mathbf{p} \frac{\partial f}{\partial S} \right]$$

Using the information in [A8] and [A12], the FOC is differentiated with respect to S to

obtain:

$$\begin{aligned} &\frac{\mathbf{p}}{2I} \left[V_{qq} (1-\mathbf{a})^2 \mathbf{p} \frac{\partial f}{\partial S} \right] (\mathbf{b} + \mathbf{a}\mathbf{p}\mathbf{t}_G) + \frac{\mathbf{a}\mathbf{p}^2}{2I} [V_q (1-\mathbf{a}) - 1] \frac{\partial f}{\partial S} + \\ &- \frac{\mathbf{a}\mathbf{p}}{2I} \left[-[(1-\mathbf{t}_o)\mathbf{p} - P] \frac{1}{S} - S \left(V_q (1-\mathbf{a})\mathbf{p} - \frac{\mathbf{p}}{S} \right) \frac{\partial f}{\partial S} + [(1-\mathbf{t}_G)\mathbf{p} - P] \frac{1}{S} - (1-S)V_q (1-\mathbf{a})\mathbf{p} \frac{\partial f}{\partial S} \right] \equiv 0 \end{aligned}$$

which simplifies to:

$$\frac{\partial f}{\partial S} \left[\frac{\mathbf{p}}{2\mathbf{l}} V_{qq} (1-\mathbf{a})^2 \mathbf{p} (\mathbf{b} + \mathbf{a} \mathbf{p} \mathbf{t}_G) + \frac{\mathbf{a} \mathbf{p}^2}{2\mathbf{l}} [V_q (1-\mathbf{a}) - 1] - \frac{\mathbf{a} \mathbf{p}^2}{2\mathbf{l}} + \frac{\mathbf{a} \mathbf{p}^2}{2\mathbf{l}} V_q (1-\mathbf{a}) \right] \equiv 0$$

Since the term in parenthesis is negative¹⁴, it follows:

$$\frac{\partial f}{\partial S} = 0 \text{ and therefore } \frac{d\mathbf{t}_G}{dS} = \left(\frac{1}{1 - \frac{\partial f}{\partial \mathbf{t}_o}} \right) \frac{\partial f}{\partial S} = 0$$

which is the result presented in Proposition 2 in case of oversubscription.

Proof of Proposition 4

Proposition 4 establishes the effect of a change in advertising effort e on the equilibrium value of \mathbf{t}_G , holding P and S constant. Recalling again that the equilibrium is symmetric, and

therefore $\frac{d\mathbf{t}_G}{de} = \frac{d\mathbf{t}_o}{de}$, we have:

$$\begin{aligned} \frac{d\mathbf{t}_G}{de} &= \frac{\partial f}{\partial e} + \frac{\partial f}{\partial \mathbf{t}_o} \frac{d\mathbf{t}_o}{de} \\ &= \left(\frac{1}{1 - \frac{\partial f}{\partial \mathbf{t}_o}} \right) \frac{\partial f}{\partial e} \end{aligned}$$

From [A8], the following pieces of information are derived

$$\begin{aligned} \frac{du_{iN}(\mathbf{t}_o)}{de} &= V_q \frac{\partial q}{\partial e} = -(1-\mathbf{a})A'V_q; \quad \frac{du_{iS}(\mathbf{t}_o)}{de} = V_q \frac{\partial q}{\partial e} + \frac{\partial y}{\partial e} + \frac{\partial T_i}{\partial e} = -(1-\mathbf{a})A'V_q + 1 \\ \frac{du_{iN}(\mathbf{t}_G)}{de} &= V_q \frac{\partial q}{\partial \mathbf{t}_G} \frac{\partial f}{\partial e} + V_q \frac{\partial q}{\partial e} = V_q (1-\mathbf{a})\mathbf{p} \frac{\partial f}{\partial e} - (1-\mathbf{a})A'V_q \quad [A13] \\ \frac{du_{iS}(\mathbf{t}_G)}{de} &= V_q \frac{\partial q}{\partial \mathbf{t}_G} \frac{\partial f}{\partial e} + V_q \frac{\partial q}{\partial e} + \frac{\partial y}{\partial \mathbf{t}_G} \frac{\partial f}{\partial e} + \frac{\partial y}{\partial e} + \frac{\partial T_i}{\partial e} = V_q (1-\mathbf{a})\mathbf{p} \frac{\partial f}{\partial e} - (1-\mathbf{a})A'V_q - \mathbf{p}\tilde{d} \frac{\partial f}{\partial e} + 1 \\ \tilde{d} &= \begin{cases} \frac{W}{P} & \text{if undersubscription} \\ \frac{1}{S} & \text{if oversubscription} \end{cases} \end{aligned}$$

¹⁴ This can be easily verified, recalling that $V_{qq} < 0$ and referring to condition [19] in the paper.

Therefore :

$$\begin{aligned} \frac{d\mathbf{m}_G(\mathbf{t}_G, \mathbf{t}_O)}{de} &= -\frac{1}{2I} \left[u_{iS}(\mathbf{t}_O) + S \frac{du_{iS}(\mathbf{t}_O)}{de} - u_{iS}(\mathbf{t}_G) - S \frac{du_{iS}(\mathbf{t}_G)}{de} + \frac{du_{iN}(\mathbf{t}_O)}{de} - \frac{du_{iN}(\mathbf{t}_G)}{de} + \right. \\ &\quad \left. - u_{iN}(\mathbf{t}_O) - S \frac{du_{iN}(\mathbf{t}_O)}{de} + u_{iN}(\mathbf{t}_G) + S \frac{du_{iS}(\mathbf{t}_G)}{de} \right] \\ &= -\frac{1}{2I} \left[S[-(1-\mathbf{a})A'V_q + 1] - S \left[V_q(1-\mathbf{a})\mathbf{p} \frac{\partial f}{\partial e} - (1-\mathbf{a})A'V_q - \mathbf{p}\tilde{d} \frac{\partial f}{\partial e} + 1 \right] + \right. \\ &\quad \left. - (1-\mathbf{a})A'V_q - \left[V_q(1-\mathbf{a})\mathbf{p} \frac{\partial f}{\partial e} - (1-\mathbf{a})A'V_q \right] - S[-(1-\mathbf{a})A'V_q] + \right. \\ &\quad \left. + S \left[V_q(1-\mathbf{a})\mathbf{p} \frac{\partial f}{\partial e} - (1-\mathbf{a})A'V_q \right] \right] \end{aligned}$$

And:

$$\frac{d \frac{\partial \mathbf{m}_G(\mathbf{t}_G, \mathbf{t}_O)}{\partial \mathbf{t}_G}}{de} = \frac{\mathbf{p}}{2I} \left[V_{qq}(1-\mathbf{a})^2 \mathbf{p} \frac{\partial f}{\partial e} - V_{qq}(1-\mathbf{a})^2 A' \right]$$

Using the information in [A8] and [A13], the FOC is differentiated with respect to e to obtain:

$$\begin{aligned} \frac{\mathbf{p}}{2I} \left[V_{qq}(1-\mathbf{a})^2 \mathbf{p} \frac{\partial f}{\partial e} - V_{qq}(1-\mathbf{a})^2 A' \right] (\mathbf{b} + \mathbf{a}\mathbf{p}\mathbf{t}_G) + \frac{\mathbf{a}\mathbf{p}^2}{2I} \left[V_q(1-\mathbf{a}) - S\tilde{d} \right] \frac{\partial f}{\partial e} + \\ - \frac{\mathbf{a}\mathbf{p}}{2I} \left[\begin{array}{l} -S(1-\mathbf{a})A'V_q + S - S(1-\mathbf{a})V_q\mathbf{p} \frac{\partial f}{\partial e} + S(1-\mathbf{a})A'V_q + S\mathbf{p}\tilde{d} \frac{\partial f}{\partial e} - S + \\ -(1-\mathbf{a})A'V_q - (1-\mathbf{a})V_q\mathbf{p} \frac{\partial f}{\partial e} + (1-\mathbf{a})A'V_q + S(1-\mathbf{a})A'V_q + S(1-\mathbf{a})V_q\mathbf{p} \frac{\partial f}{\partial e} + \\ -S(1-\mathbf{a})A'V_q \end{array} \right] \equiv 0 \end{aligned}$$

Multiplying every term by $\frac{2I}{\mathbf{p}}$, and isolating the terms with $\frac{\partial f}{\partial e}$:

$$V_{qq}(1-\mathbf{a})^2 \mathbf{p}^2 (\mathbf{b} + \mathbf{a}\mathbf{p}\mathbf{t}_G) \frac{\partial f}{\partial e} + 2\mathbf{a}\mathbf{p} \left[V_q(1-\mathbf{a})\mathbf{p} - S\mathbf{p}\tilde{d} \right] \frac{\partial f}{\partial e} = V_{qq}(1-\mathbf{a})^2 \mathbf{p}A' (\mathbf{b} + \mathbf{a}\mathbf{p}\mathbf{t}_G)$$

which is:

$$\frac{\partial f}{\partial e} = \frac{(1-\mathbf{a})^2 A' V_{qq} (\mathbf{b} + \mathbf{a} \mathbf{p} \mathbf{t}_G)}{2\mathbf{a} \mathbf{p} [V_q (1-\mathbf{a}) - S\tilde{d}] + (1-\mathbf{a})^2 V_{qq} \mathbf{p} (\mathbf{b} + \mathbf{a} \mathbf{p} \mathbf{t}_G)} > 0 \quad [\text{A14}]$$

Therefore, using [A9] and [A14]:

$$\frac{d\mathbf{t}_G}{de} = \frac{(1-\mathbf{a})^2 A' V_{qq} (\mathbf{b} + \mathbf{a} \mathbf{p} \mathbf{t}_G)}{\mathbf{p} \{ \mathbf{a} [V_q (1-\mathbf{a}) - S\tilde{d}] + (1-\mathbf{a})^2 V_{qq} (\mathbf{b} + \mathbf{a} \mathbf{p} \mathbf{t}_G) \}} > 0$$

which is the result presented in Proposition 4 in the paper.

Proof of Proposition 7

Proposition 7 specifies what is the effect of a change in share price P on the equilibrium value of \mathbf{t}_G , holding e and S constant. Here we have to derive:

$$\begin{aligned} \frac{d\mathbf{t}_G}{dP} &= \frac{\partial f}{\partial P} + \frac{\partial f}{\partial \mathbf{t}_O} \frac{d\mathbf{t}_O}{dP} \\ &= \left(\frac{1}{1 - \frac{\partial f}{\partial \mathbf{t}_O}} \right) \frac{\partial f}{\partial P} \end{aligned}$$

As for the proof of Proposition 2, we need to analyze the two cases of undersubscription and oversubscription separately.

Undersubscription

The following pieces of information are derived from the definitions in [A8]:

$$\begin{aligned} \frac{du_{iN}(\mathbf{t}_O)}{dP} &= V_q \frac{\partial q}{\partial P} = (1-\mathbf{a})V_q; \quad \frac{du_{iS}(\mathbf{t}_O)}{dP} = V_q \frac{\partial q}{\partial P} + \frac{\partial y}{\partial P} = (1-\mathbf{a})V_q - (1-\mathbf{t}_O)\mathbf{p} \frac{W}{P^2} \\ \frac{du_{iN}(\mathbf{t}_G)}{dP} &= V_q \frac{\partial q}{\partial \mathbf{t}_G} \frac{\partial f}{\partial P} + V_q \frac{\partial q}{\partial P} = V_q (1-\mathbf{a})\mathbf{p} \frac{\partial f}{\partial P} + (1-\mathbf{a})V_q \quad [\text{A15}] \\ \frac{du_{iS}(\mathbf{t}_G)}{dP} &= V_q \frac{\partial q}{\partial \mathbf{t}_G} \frac{\partial f}{\partial P} + V_q \frac{\partial q}{\partial P} + \frac{\partial y}{\partial \mathbf{t}_G} \frac{\partial f}{\partial P} + \frac{\partial y}{\partial P} = V_q (1-\mathbf{a})\mathbf{p} \frac{\partial f}{\partial P} + (1-\mathbf{a})V_q - \mathbf{p} \frac{W}{P} \frac{\partial f}{\partial P} + \\ &\quad - (1-\mathbf{t}_G)\mathbf{p} \frac{W}{P^2} \end{aligned}$$

Therefore:

$$\frac{d\mathbf{m}_G(\mathbf{t}_G, \mathbf{t}_o)}{dP} = -\frac{1}{2I} \left[\begin{array}{l} S[(1-\mathbf{a})V_q - (1-\mathbf{t}_o)\mathbf{p} \frac{W}{P^2}] - S[V_q(1-\mathbf{a})\mathbf{p} \frac{\partial f}{\partial P} + (1-\mathbf{a})V_q - \mathbf{p} \frac{W}{P} \frac{\partial f}{\partial P} + \\ - (1-\mathbf{t}_G)\mathbf{p} \frac{W}{P^2}] + (1-\mathbf{a})V_q - [V_q(1-\mathbf{a})\mathbf{p} \frac{\partial f}{\partial P} + (1-\mathbf{a})V_q] - S[(1-\mathbf{a})V_q] + \\ + S[V_q(1-\mathbf{a})\mathbf{p} \frac{\partial f}{\partial P} + (1-\mathbf{a})V_q] \end{array} \right]$$

And:

$$\frac{d \frac{\partial \mathbf{m}_G(\mathbf{t}_G, \mathbf{t}_o)}{\partial \mathbf{t}_G}}{dP} = \frac{\mathbf{p}}{2I} \left[V_{qq}(1-\mathbf{a})^2 \mathbf{p} \frac{\partial f}{\partial P} + V_{qq}(1-\mathbf{a})^2 + S \frac{W}{P^2} \right]$$

Using the information in [A8] and [A15], the FOC is differentiated with respect to P.

After some algebra, we obtain:

$$\begin{aligned} (1-\mathbf{a})^2 V_{qq} \mathbf{p}^2 (\mathbf{b} + \mathbf{a} \mathbf{p} \mathbf{t}_G) \frac{\partial f}{\partial P} + 2\mathbf{a} \mathbf{p}^2 \left[(1-\mathbf{a})V_q - S \frac{W}{P} \right] \frac{\partial f}{\partial P} = \\ = -\mathbf{p} (\mathbf{b} + \mathbf{a} \mathbf{p} \mathbf{t}_G) \left[(1-\mathbf{a})^2 V_{qq} + S \frac{W}{P^2} \right] \end{aligned}$$

which is:

$$\frac{\partial f}{\partial P} = \frac{- (\mathbf{b} + \mathbf{a} \mathbf{p} \mathbf{t}_G) \left[(1-\mathbf{a})^2 V_{qq} + S \frac{W}{P^2} \right]}{2\mathbf{a} \mathbf{p} \left[(1-\mathbf{a})V_q - S \frac{W}{P} \right] + (1-\mathbf{a})^2 V_{qq} \mathbf{p} (\mathbf{b} + \mathbf{a} \mathbf{p} \mathbf{t}_G)} \quad [\text{A16}]$$

And therefore, using [A9] and [A16]:

$$\frac{d\mathbf{t}_G}{dP} = \frac{- (\mathbf{b} + \mathbf{a} \mathbf{p} \mathbf{t}_G) \left[(1-\mathbf{a})^2 V_{qq} + S \frac{W}{P^2} \right]}{\mathbf{p} \left\{ \mathbf{a} \left[(1-\mathbf{a})V_q - S \frac{W}{P} \right] + (1-\mathbf{a})^2 V_{qq} (\mathbf{b} + \mathbf{a} \mathbf{p} \mathbf{t}_G) \right\}}$$

which is the result presented in Proposition 7.

Oversubscription

The following pieces of information are derived from the definitions in [A8]:

$$\begin{aligned}\frac{du_{iN}(\mathbf{t}_O)}{dP} &= V_q \frac{\partial q}{\partial P} = (1-\mathbf{a})V_q; & \frac{du_{iS}(\mathbf{t}_O)}{dP} &= V_q \frac{\partial q}{\partial P} + \frac{\partial y}{\partial P} = (1-\mathbf{a})V_q - \frac{1}{S} \\ \frac{du_{iN}(\mathbf{t}_G)}{dP} &= V_q \frac{\partial q}{\partial \mathbf{t}_G} \frac{\partial f}{\partial P} + V_q \frac{\partial q}{\partial P} = V_q (1-\mathbf{a})\mathbf{p} \frac{\partial f}{\partial P} + (1-\mathbf{a})V_q & [A17] \\ \frac{du_{iS}(\mathbf{t}_G)}{dP} &= V_q \frac{\partial q}{\partial \mathbf{t}_G} \frac{\partial f}{\partial P} + V_q \frac{\partial q}{\partial P} + \frac{\partial y}{\partial \mathbf{t}_G} \frac{\partial f}{\partial P} + \frac{\partial y}{\partial P} = V_q (1-\mathbf{a})\mathbf{p} \frac{\partial f}{\partial P} + (1-\mathbf{a})V_q - \frac{\mathbf{p}}{S} \frac{\partial f}{\partial P} - \frac{1}{S}\end{aligned}$$

Therefore:

$$\frac{d\mathbf{m}_G(\mathbf{t}_G, \mathbf{t}_O)}{dP} = -\frac{1}{2\mathbf{l}} \left[\begin{aligned} & S[(1-\mathbf{a})V_q - \frac{1}{S}] - S[V_q(1-\mathbf{a})\mathbf{p} \frac{\partial f}{\partial P} + (1-\mathbf{a})V_q - \frac{\mathbf{p}}{S} \frac{\partial f}{\partial P} - \frac{1}{S}] \\ & + (1-\mathbf{a})V_q - [V_q(1-\mathbf{a})\mathbf{p} \frac{\partial f}{\partial P} + (1-\mathbf{a})V_q] - S(1-\mathbf{a})V_q + \\ & + S[V_q(1-\mathbf{a})\mathbf{p} \frac{\partial f}{\partial P} + (1-\mathbf{a})V_q] \end{aligned} \right]$$

And:

$$\frac{d \frac{\partial \mathbf{m}_G(\mathbf{t}_G, \mathbf{t}_O)}{\partial \mathbf{t}_G}}{dP} = \frac{\mathbf{p}}{2\mathbf{l}} \left[V_{qq} (1-\mathbf{a})^2 \mathbf{p} \frac{\partial f}{\partial P} + V_{qq} (1-\mathbf{a})^2 \right]$$

Using the information in [A8] and [A17], the FOC is differentiated with respect to P.

After some algebra, we obtain:

$$\begin{aligned}(1-\mathbf{a})^2 V_{qq} \mathbf{p} (\mathbf{b} + \mathbf{a} \mathbf{p} \mathbf{t}_G) \frac{\partial f}{\partial P} + 2\mathbf{a} \mathbf{p} [(1-\mathbf{a})V_q - 1] \frac{\partial f}{\partial P} &= -(\mathbf{b} + \mathbf{a} \mathbf{p} \mathbf{t}_G) (1-\mathbf{a})^2 V_{qq} \\ \frac{\partial f}{\partial P} &= \frac{-(\mathbf{b} + \mathbf{a} \mathbf{p} \mathbf{t}_G) (1-\mathbf{a})^2 V_{qq}}{2\mathbf{a} \mathbf{p} [(1-\mathbf{a})V_q - 1] + (1-\mathbf{a})^2 V_{qq} \mathbf{p} (\mathbf{b} + \mathbf{a} \mathbf{p} \mathbf{t}_G)} < 0 & [A18]\end{aligned}$$

And therefore, using [A9] and [A18]:

$$\frac{d\mathbf{t}_G}{dP} = \frac{-(\mathbf{b} + \mathbf{a} \mathbf{p} \mathbf{t}_G) (1-\mathbf{a})^2 V_{qq}}{\mathbf{p} \{ \mathbf{a} [(1-\mathbf{a})V_q - 1] + (1-\mathbf{a})^2 V_{qq} (\mathbf{b} + \mathbf{a} \mathbf{p} \mathbf{t}_G) \}} < 0$$

which is the result presented in Proposition 7.