ANALYSIS OF FINK TRUSSED BENTS

BY

MOMENT AND THRUST DISTRIBUTION

by

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I. INTRODUCTION

It is not uncommon for engineers, when analyzing certain types of structures, to make simplifying assumptions and approximations which will lead to fairly accurate results that may be used in the design of these structures. The time saved and the results obtained for the most part justify the use of these approximate methods rather than the undertaking of a more rigorous analysis.

In recent years however, new methods of analysis have been developed which have shortened the time and labor involved in the analysis of many statically indeterminate structures. The "modern" methods, so called, which are basically systems of successive approximations and convergence rely to a great extent on one's understanding of the physical picture of what is taking place in the structure. In most instances, if the processes are carried out far enough, "exact" solutions will result without having to resort to the time consuming "classical" methods involving the principles of work and energy.

In the analyses of trussed bents, assumptions are very often made which simplify the analyses a great deal. There is reason to believe however that these assumptions are not always valid for certain types of structures. It was with this view in mind that the investigations on the Application of Moment Distribution to Trussed Bents[3] and on the Analysis of Trussed Bents by Moment Distribution[4] were undertaken by Joseph E. Spagnuolo and Grover Lee Rogers respectively.

The results obtained by Spagnuolo and Rogers showed that the effect of thrust in trussed bents can not always be neglected and that there are cases when thrust should definitely be taken into account. These

investigations were limited to trussed bents made up with parallel chord trusses.

When trussed bents made up with non-parallel chord trusses are considered however, it is felt that the thrusts must be taken into account. The reason for this feeling is that these bents no longer approach the girder type bent but rather approach the arch, where thrust is an important factor. This fact concerning arches was brought out by Professor Hardy Cross in his book, Continuous Frames in Reinforced Concrete,[2] and it is treated accordingly by his method of moment and thrust distribution.

It is the purpose of this thesis to carry on the investigations of trussed bents started by Spagnuolo and Rogers to see whether or not the method of moment and thrust distribution can be applied to Fink trussed bents, and also to determine if such a method would be practical as compared to the "classical" methods.

II. REVIEW OF LITERATURE

Although the method of moment distribution was known and had
been put to use by a limited few in the late 1920's, it was not un-
til the early 1930's that the method was formally published by Pro-
fessor Cross. Its first public appearance was in the 1932 Transactions[1]
of the American Society of Civil Engineers. Since then many papers and
textbooks have been published on the subject which have discussed the
method and which have introduced certain refinements and short-cuts to
the method.

Moment distribution has been applied practically to many and
varied types of structures with remarkable ease and efficiency. It
has been found however, that in some types of problems it is not suffi-
cient to balance only moments but that the balancing of thrusts developed
by the lengthening and shortening of the members must also be taken
into account. This is particularly true in arches. The method of
moment and thrust distribution is discussed in detail by Professor Cross
in his book, Continuous Frames in Reinforced Concrete.[2]

One type of problem that has been overlooked by most writers is
the application of moment distribution to trussed bents. Professor
L. C. Maugh of the University of Michigan is the one writer who appears
to have done the most on this particular subject. In his book,
Statically Indeterminate Structures,[5] he has applied moment distri-
bution to some types of trussed bents satisfactorily by introducing the
use of an equivalent column in the bent and then determining the
appropriate fixed end moments, stiffness and carry over factors.

Throughout his work however, he has neglected to take into account the effect of thrusts and as a consequence a certain discrepancy arises in the results obtained by Maugh's method and in those obtained by the "classical" methods.

In 1940 and in 1948, theses on the application of moment and thrust distribution to trussed bents with parallel chord trusses were written by Spagnuolo[4] and Rogers[5] respectively at Virginia Polytechnic Institute. It was found that the effect of thrust in some cases was considerable and therefore should be taken into account. The method of moment and thrust distribution was applied to these bents and it was found that it could be used practically.

Both the methods of Maugh and Spagnuolo are discussed herein, and they are applied to the cases analyzed.

III. INVESTIGATION

A. General

The purpose of this investigation as stated previously is primarily to determine a manner by which the method of moment and thrust distribution may be applied to trussed bents made up with non-parallel chords, and secondly to see if such a method would be practical to use.

The investigation has therefore been carried out for a bent made up with a Fink truss with a ratio of height of truss to span of one-quarter. The ratio of area of web members to area of chord members was taken as one-half, in all cases considered. In most instances, whenever possible, the necessary constants involved were worked out for a general case. However, in order to arrive at a solution, it was found necessary to apply the method to a specific case, and for simplicity, a purely academic problem was chosen. The case considered is that of a single aisle, hinged-end bent, loaded with a unit panel load. Then, to show the practicality of the method, a triple aisle, hinged-end bent loaded with a unit panel load was analyzed.

For both of these cases, solutions have been worked out using the method of least work, the Maugh method of the equivalent column with no thrust effects considered, and the method of moment and thrust distribution by using Maugh's equivalent column and by using the Spagnuolo method.

Before a problem may be solved by moment and thrust distribution,

certain constants which apply to the structure being analyzed must be known. These constants which are later determined and defined are:

1. Fixed-end moment

2. Fixed-end thrust

3. Moment stiffness factor

4. Thrust stiffness factor

5. Carry-over factor for moment

6. Carry-over factor for thrust

The method of least work has been used to determine the truss constants, whereas the methods of moment-area and conjugate beam have been used to determine the column constants.

In the figure on Plate I is a line diagram of the truss and bent under consideration. Bow's notation is used for the designation of the particular members, and the notation as shown on the diagram will be used throughout the investigation. Below the figure are listed the length, area and $L/_A$ ratio of each member in terms of L, the span length of the truss, and a, the area of the chord members, respectively, where $L/_A$ is the ratio of the length to area of each member.

B. Moment and Thrust Distribution Constants

1. For Trusses.

a) Fixed-end moments and fixed-end thrusts.

The fixed-end moments and fixed-end thrusts are the moments and thrusts that act at the ends of a truss which is fixed at both ends. These moments and thrusts are developed in order to keep the ends of the truss from rotating or translating under the action of the external loading.

PLATE I

LENGTHS, AREAS AND $\frac{L'}{A}$ RATIOS OF TRUSS MEMBERS

MEMBER	L'	A	$\frac{L'}{A}$
AC	0.2795 L	a	0.2795 L/a
BD	0.2795 L	a	0.2795 L/a
bd	0.2795 L	a	0.2795 L/a
ac	0.2795 L	a	0.2795 L/a
CG	0.3125 L	a	0.3125 L/a
cg	0.3125 L	a	0.3125 L/a
EF	0.3750 L	a	0.3750 L/a
CD	0.1398 L	2a	0.0699 L/a
DE	0.3125 L	2a	0.1563 L/a
dE	0.3125 L	2a	0.1563 L/a
cd	0.1398 L	2a	0.0699 L/a
GF	0.3366 L	a	0.3366 L/a
gF	0.3366 L	a	0.3366 L/a

For a fixed-end Fink truss, however, it is not possible to solve for these values of moment and thrust directly since the point at which this thrust acts is not known. Therefore, it was found necessary to solve for fixed-end reactions, X, Y and Z, (Figure 1a) which are then combined into a moment and a single force acting at a particular point which is determined later. A unit panel load is taken for the external loading.

Figure 1a.

One manner by which these fixed-end reactions may be obtained is by the use of Castigliano's Principle of Least Work. The procedure is as follows:

i. Make the truss statically determinate by removing as many redundant reactions as necessary, as shown in Figure 1b. Then determine the stresses, S_o, which are present due to the original loading.

Figure 1b.

ii.   Apply unit forces, one at a time, at all points where the re-
dundant reactions acted, and determine the stresses, S_x, S_y, S_z, due to
these forces.   See Figures lc, d, e.

Figure lc.

Figure ld.

Figure le.

Then by the law of superposition the resultant stress in any member
will be

$$S = S_o + XS_x + YS_y + ZS_z \quad \dots\dots\dots \text{ (1)}$$

Using Least Work, we know that the total strain energy in the
structure is equal to the summation of the strain energy in all the
members,

$$U = \sum \frac{S^2 L}{2AE} \quad \dots\dots\dots\dots\dots\dots \text{ (2)}$$

where S is as given in (1), L is the length of a member, A is the cross-
sectional area of a member, and E is the modulus of elasticity of the material.

Castigliano's Principle states that the partial derivative of the total work, U, with respect to a redundant force is equal to the distortion of a body in the direction of the force. Since, in our case, there can be no translation or rotations of the ends, there can be no distortion. Therefore making use of this relationship, together with the equations (1) and (2) we arrive at the following expressions:

$$\frac{\partial U}{\partial X} = \sum \frac{S_o S_x L}{AE} + \sum X \frac{S_x^2 L}{AE} + \sum Y \frac{S_x S_y L}{AE} + \sum Z \frac{S_x S_z L}{AE} = 0 \quad (3)$$

$$\frac{\partial U}{\partial Y} = \sum \frac{S_o S_y L}{AE} + \sum X \frac{S_x S_y L}{AE} + \sum Y \frac{S_y^2 L}{AE} + \sum Z \frac{S_y S_z L}{AE} = 0 \quad (4)$$

$$\frac{\partial U}{\partial Z} = \sum \frac{S_o S_z L}{AE} + \sum X \frac{S_x S_z L}{AE} + \sum Y \frac{S_y S_z L}{AE} + \sum Z \frac{S_z^2 L}{AE} = 0 \quad (5)$$

All the necessary coefficients involved in the above equations may be found in Plate II. Substituting them in the equations (3), (4), and (5), and solving for X, Y and Z we obtain the values shown in Figure 1f.

Figure 1f.

TABLE A
PLATE II

FIXED END REACTIONS FOR UNIT PANEL LOAD

MEMBER	$\frac{L'}{A}\cdot\frac{a}{L}$	S_o	S_x	S_y	S_z	$S_o S_x \frac{L'}{A}\cdot\frac{a}{L}$	$S_o S_y \frac{L'}{A}\cdot\frac{a}{L}$	$S_o S_z \frac{L'}{A}\cdot\frac{a}{L}$	$S_x^2 \frac{L'}{A}\cdot\frac{a}{L}$	$S_x S_y \frac{L'}{A}\cdot\frac{a}{L}$	$S_x S_z \frac{L'}{A}\cdot\frac{a}{L}$	$S_y^2 \frac{L'}{A}\cdot\frac{a}{L}$	$S_y S_z \frac{L'}{A}\cdot\frac{a}{L}$	$S_z^2 \frac{L'}{A}\cdot\frac{a}{L}$
AC	0.2795	−3.3541	0	−0.5709	−0.2595	0	0.5352	0.2433	0	0	0	0.0911	0.0414	0.0188
BD	0.2795	−2.9069	0	−0.5709	−0.2595	0	0.4638	0.2108	0	0	0	0.0911	0.0414	0.0188
bd	0.2795	−2.9069	0	−0.2595	−0.5709	0	0.2108	0.4638	0	0	0	0.0188	0.0414	0.0911
ac	0.2795	−3.3541	0	−0.2595	−0.5709	0	0.2433	0.5352	0	0	0	0.0188	0.0414	0.0911
CG	0.5125	+3.0000	+1.0000	+0.5107	+0.2321	0.9375	0.4788	0.2176	0.3125	0.1596	0.0725	0.0815	0.0370	0.0168
cg	0.3125	+3.0000	+1.0000	+1.1606	−0.4178	0.9375	1.0881	−0.3917	0.3125	0.3627	−0.1306	0.4209	−0.1515	0.0546
EF	0.3750	+2.0000	+1.0000	+1.1606	+0.2321	0.7500	0.8705	0.1741	0.3750	0.4352	0.0870	0.5051	0.1010	0.0202
CD	0.0699	−0.8944	0	0	0	0	0	0	0	0	0	0	0	0
DE	0.1563	+1.0000	0	+0.4642	0	0	0.0726	0	0	0	0	0.0337	0	0
de	0.1563	+1.0000	0	0	+0.4642	0	0	0.0726	0	0	0	0	0	0.0337
cd	0.0699	−0.8944	0	0	0	0	0	0	0	0	0	0	0	0
GF	0.3566	0	0	+1.0000	0	0	0	0	0	0	0	0.3566	0	0
gf	0.3566	0	0	0	+1.0000	0	0	0	0	0	0	0	0	0.3366
Σ						2.6250	3.9631	1.5257	1.0000	0.9575	0.0289	1.5976	0.1521	0.6817

b) <u>Thrust Stiffness</u>

Thrust stiffness is defined as the thrust which is necessary to produce a unit translation with no rotation of one end, while the other end remains fixed. Again, as in the case of the fixed-end moments and thrusts, the point at which this thrust must act is not known. Therefore the reactions that will produce a unit translation with no rotation of one end will be solved for. (See Figure 2a.) These reactions, X and Y, may then be combined into a single force which will be the thrust stiffness.

Figure 2a.

Using the same energy approach as before the total stress in any member is

$$S = XS_x + YS_y + ZS_z \dots\dots\dots\dots\dots\dots (6)$$

where S_x, S_y, S_z are the stresses due to unit forces in the respective directions and locations of the X, Y and Z reactions. The total strain energy is then

$$U = \sum \frac{S^2 L}{2AE} \dots\dots\dots\dots\dots\dots\dots\dots (7)$$

Taking partial derivatives with respect to X, Y, and Z, the following relationships result:

$$\frac{\partial U}{\partial X} = 1 = \sum X \frac{S_x^2 L}{AE} + \sum Y \frac{S_x S_y L}{AE} + \sum Z \frac{S_x S_z L}{AE} \qquad (8)$$

$$\frac{\partial U}{\partial Y} = 1 = \sum X \frac{S_x S_y L}{AE} + \sum Y \frac{S_y^2 L}{AE} + \sum Z \frac{S_y S_z L}{AE} \qquad (9)$$

$$\frac{\partial U}{\partial Z} = 0 = \sum X \frac{S_x S_z L}{AE} + \sum Y \frac{S_y S_z L}{AE} + \sum Z \frac{S_z^2 L}{AE} \qquad (10)$$

Substituting the values from Plate III in the above equations, we obtain the results shown in Figure 2b. By summing up the X and Y forces, the thrust stiffness is then found as $1.0020 \frac{Ea}{L}$.

$\Delta_L = 1$ $\Delta_R = 0$

$0.0133 \frac{Ea}{L}$ $0.0133 \frac{Ea}{L}$

$1.0352 \frac{Ea}{L}$ $1.0352 \frac{Ea}{L}$

$0.0332 \frac{Ea}{L}$ $0.0332 \frac{Ea}{L}$

$0.0133 \frac{Ea}{L}$ $0.0133 \frac{Ea}{L}$

Figure 2b.

c) Thrust Carry-Over Factor

The thrust carry-over factor is the ratio of the thrust induced at the fixed end of a truss to the thrust stiffness when one end is translated through a unit displacement with no rotation. From Figure 2b it is seen that the magnitude of the thrust carry-over factor is equal to unity.

TABLE 13

TRUSS THRUST STIFFNESS

MEMBER	$\dfrac{L'}{A}\cdot\dfrac{a}{L}$	S_x	S_y	S_z	$S_x^2\,\dfrac{2L'}{A}\cdot\dfrac{a}{L}$	$S_xS_y\,\dfrac{L'}{A}\cdot\dfrac{a}{L}$	$S_xS_z\,\dfrac{L'}{A}\cdot\dfrac{a}{L}$	$S_y^2\,\dfrac{2L'}{A}\cdot\dfrac{a}{L}$	$S_yS_z\,\dfrac{L'}{A}\cdot\dfrac{a}{L}$	$S_z^2\,\dfrac{2L'}{A}\cdot\dfrac{a}{L}$
AC	0.2795	0	-0.6149	-.2595	0	0	0	0.1057	0.0446	0.0188
BD	0.2795	0	-.6149	-.2595	0	0	0	0.1057	0.0446	0.0188
bd	0.2795	0	-0.2795	-0.5709	0	0	0	0.0218	0.0446	0.0911
ac	0.2795	0	-0.2795	-0.5709	0	0	0	0.0218	0.0446	0.0911
CG	0.3125	1.0000	+0.5500	+0.2321	0.3125	0.1719	0.0725	0.0945	0.0399	0.0168
cg	0.3125	1.0000	+1.2500	-0.4178	0.3125	0.3906	-0.1306	0.4883	-0.1632	0.0546
EF	0.3750	1.0000	+1.2500	+0.2321	0.3750	0.4688	0.0870	0.5859	0.1088	0.0202
CD	0.0699	0	0	0	0	0	0	0	0	0
DE	0.1563	0	+0.5000	0	0	0	0	0.0391	0	0
dE	0.1563	0	0	+0.4642	0	0	0	0	0	-0.0337
cd	0.0699	0	0	0	0	0	0	0	0	0
GF	0.3366	0	+1.0770	0	0	0	0	0.3904	0	0
gf	0.3366	0		+1.0000	0	0	0	0	0	0.3366
Σ					1.0000	1.0313	0.0289	1.8533	0.1639	0.6817

d) Moment Stiffness

Moment stiffness is defined as the moment required to rotate one end of the truss through a unit angle with no translation, while the other end remains fixed. It is here that a difficulty arises, since the point at the end of the truss which must be kept from translating is not known.

In the case of parallel chord trusses, this point is readily obtained, and it is dependent only on the areas of the top and bottom chords. This is not true for non-parallel chord trusses.

Therefore in order to determine the moment stiffness of the truss being dealt with, it was found necessary to assume a point or equivalent joint about which the end of the truss would rotate. Two such assumptions are made. The first assumption is based on the Spagnuolo method. This makes the top portion of the column, which serves as the end vertical of the truss, infinitely stiff. Then the point midway/between the knee brace and the lower chord of the truss is taken as the equivalent joint. The second assumption is to assume the equivalent joint at the top of an equivalent column as determined by the Maugh method.

The first assumption is the more simplifying of the two, and based on it the appropriate moment stiffness may be obtained for the general case. The second assumption however, is dependent upon the height of the column, and it therefore limits the value of the moment stiffness to a special case.

Figure 3a shows the free-body diagram of the truss for determining the value of the moment stiffness. The force, T, must be located so

as to prevent translation, while the moment, M, produces a unit rotation
of the end.

Figure 3a.

By the Castigliano's Principle the total strain energy again is

$$U = \sum \frac{S^2 L}{2AE} \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \quad (11)$$

where

$$S = MS_m + TS_t + XS_x \quad \dots\dots\dots\dots\dots\dots\dots\dots \quad (12)$$

S_m is the stress in any member due to a unit moment applied in the
same manner as M, and S_t and S_x are the stresses due to unit forces
applied in the directions of T and X respectively. By Least Work we
know that the partial derivative of the total strain energy with respect
to a moment is equal to the rotation of a body, and as before the partial
derivative of the total strain energy with respect to a force represents
the translation of a body. Using these relationships together with
(11) and (12) we arrive at the following equations:

$$\frac{\partial U}{\partial M} = 1 = \sum M\frac{S_m^2 L}{AE} + \sum T\frac{S_m S_t L}{AE} + \sum X\frac{S_m S_x L}{AE} \quad (13)$$

$$\frac{\partial U}{\partial T} = 0 = \sum M\frac{S_m S_t L}{AE} + \sum T\frac{S_t^2 L}{AE} + \sum X\frac{S_t S_x L}{AE} \quad (14)$$

$$\frac{\partial U}{\partial X} = 0 = \sum M\frac{S_m S_x L}{AE} + \sum T\frac{S_t S_x L}{AR} + \sum X\frac{S_x^2 L}{AE} \quad (15)$$

In Plate IV are found the values of the constants involved in the above equations based on the Spagnuolo method, which assumes that the force, T, must act midway between the lower chord and the knee brace. The values of M, T, and X obtained are shown in Figure 3b.

$$M_L = 6.2027 \frac{Ea}{L}$$
$$T_L = 1.0759 \frac{Ea}{L}$$

$$M_R = 1.1503 \frac{Ea}{L}$$
$$T_R = 1.0759 \frac{Ea}{L}$$

$$1.1117 \frac{Ea}{L}$$

$$-0.0386 \frac{Ea}{L}$$

Figure 3b.

Before the moment stiffness may be determined by the use of the Maugh method of the equivalent column, certain finite dimensions must be known. Therefore let us take H = 12 and L = 16 for the bent shown in Plate I.

This equivalent column method, which was introduced by L. C. Maugh[5] makes it possible to substitute a column which will change the bending moment diagram as shown in Figure 4. The substitution is made so that the strain energy in the column, due to the lateral thrust forces, will be the same in both cases. Considering a column with a uniform cross section, we have

$$\int_0^{h_1} \frac{(H_y)^2}{2EI}dy \;+\; \int_0^{h_2} \frac{(H \circ \frac{h_1}{h_2} \cdot y)^2}{2EI}dy \;=\; \int_0^{h^1} \frac{(H_y)^2}{2EI}dy \qquad (16)$$

from which we obtain

$$h^{1^3} = h_1^3 \left(1 + \frac{h_2}{h_1}\right) \qquad (17)$$

PLATE IV

TRUSS MOMENT STIFFNESS (SPAGNUOLO METHOD)

MEMBER	$\frac{L'}{A}\cdot\frac{a}{L}$	S_m	S_t	S_x	$S_m^2\frac{L'}{A}\cdot\frac{a}{L}$	$S_m S_t\frac{L'}{A}\cdot\frac{a}{L}$	$S_m S_x\frac{L'}{A}\cdot\frac{a}{L}$	$S_t^2\frac{L'}{A}$	$S_t S_x\frac{L'}{A}$	$S_x^2\frac{L'}{A}\cdot\frac{a}{L}$
AC	0.2795	+0.3074	-0.3074	-0.2595	0.0264	-0.0264	-0.0223	0.0264	+0.0223	0.0188
BD	0.2795	+0.3074	-0.3074	-0.2595	0.0264	-0.0264	-0.0223	0.0264	+0.0223	0.0188
bd	0.2795	+0.1397	-0.1397	-0.5709	0.0055	-0.0055	-0.0223	0.0055	+0.0223	0.0911
ac	0.2795	+0.1397	-0.1397	-0.5709	0.0055	-0.0055	-0.0223	0.0055	+0.0223	0.0911
CG	0.3125	+0.2250	+0.7750	+0.2321	0.0158	+0.0545	0.0163	0.1877	+0.0562	0.0168
cg	0.3125	-0.1250	+1.1250	-0.4178	0.0049	-0.0439	0.0162	0.3995	-0.1469	0.0546
EF	0.3750	-0.1250	+1.1250	+0.2321	0.0059	-0.0527	-0.0109	0.4746	+0.0979	0.0202
CD	0.0699	0	0	0	0	0	0	0	0	0
DE	0.1563	-0.2500	+0.2500	0	0.0098	-0.0098	0	0.0098	0	0
dE	0.1563	0	0	+0.4642	0	0	0	0	0	0.0337
cd	0.0699	0	0	0	0	0	0	0	0	0
GF	0.3366	-0.5385	+0.5385	0	0.0976	-0.0976	0	0.0976	0	0
gF	0.3366	0	0	+1.0000	0	0	0	0	0	0.3366
Σ					0.1978	-0.2133	-0.0675	1.2330	+0.0964	0.6817

For our particular problem we have $h_1 = 12$ and $h_2 = 2$. Substituting these values in (17), we find $h' = 12.64$.

Original
M Diagram

Equivalent Column
and Moment Diagram

Figure 4.

If we then consider the top of the equivalent column, h', as the equivalent joint of the truss, and apply a corresponding moment and thrust at this point to produce a unit rotation with no translation of one end, in a manner similar to that shown in Figure 3a, we can obtain the moment stiffness of the truss based on this second assumption. The constants involved in equations (13), (14) and (15), which also apply for this case, are found in Plate V, and the values of M, T, and X solved for are shown in Figure 5.

$\theta_L = 1$
$\Delta_L = 0$

$\theta_R = 0$
$\Delta_R = 0$

$3.9178 \frac{Ea}{L}$

$M_L = 13.6545 \frac{Ea}{L}$
$T_L = 4.0441 \frac{Ea}{L}$

Figure 5.

$M_R = 5.4274 \frac{Ea}{L}$
$T_R = 4.0441 \frac{Ea}{L}$

$0.1360 \frac{Ea}{L}$

PLATE V

TRUSS MOMENT STIFFNESS (MAUGH METHOD)

MEMBER	$\frac{L'}{A}\cdot\frac{a}{L}$	S_m	S_t	S_x	$S_m^2\frac{2L'}{A}\cdot\frac{a}{L}$	$S_m S_t\frac{L'}{A}\cdot\frac{a}{L}$	$S_m S_x\frac{L'}{A}\cdot\frac{a}{L}$	$S_t^2\frac{2L'}{A}\cdot\frac{a}{L}$	$S_t S_x\frac{L'}{A}\cdot\frac{a}{L}$	$S_x^2\frac{2L'}{A}\cdot\frac{a}{L}$
AC	0.2795	0.3074	-0.4181	-0.2595	0.0264	-0.0359	-0.0223	0.0489	0.0303	0.0188
BD	0.2795	0.3074	-0.4181	-0.2595	0.0264	-0.0359	-0.0223	0.0489	0.0303	0.0188
bd	0.2795	0.1397	-0.1901	-0.5709	0.0055	-0.0743	-0.0223	0.0101	0.0303	0.0911
ac	0.2795	0.1397	-0.1901	-0.5709	0.0055	-0.0743	-0.0223	0.0101	0.0303	0.0911
CG	0.3125	+0.2250	+0.6940	+0.2321	0.0158	0.0488	0.0163	0.1505	0.0503	0.0168
cg	0.3125	-0.1250	+1.1700	-0.4178	0.0049	-0.0457	0.0163	0.4278	-0.1528	0.0546
EF	0.3750	-0.1250	+1.1700	+0.2321	0.0059	-0.0549	-0.0109	0.5133	0.1019	0.0202
CD	0.0699	0	0	0	0	0	0	0	0	0
DE	0.1563	-0.2500	+0.3400	0	0.0098	-0.0133	0	0.0181	0	0
dE	0.1563	0	0	+0.4642	0	0	0	0	0	0.0337
cd	0.0699	0	0	0	0	0	0	0	0	0
GF	0.3366	-0.5385	+0.7324	0	0.0976	-0.1328	0	0.1806	0	0
gF	0.3366	0	0	+1.0000	0	0	0	0	0	0.3366
Σ					0.1978	-0.4183	-0.0675	1.4083	0.1206	0.6817

.00743

e) Carry-Over Factor for Moment

In determining the moment stiffness it was found necessary to apply a moment at one end while the other end remained fixed. At the fixed end of the truss then, a moment was induced which prevented that end from rotating. The ratio of the moment induced at the fixed end to the moment applied at the free end is called the carry-over factor.

From Figure 3b we find that the magnitude of the carry-over factor based on the Spagnuolo method is 1.0692, and from Figure 5 the value based on the equivalent column method is 0.3843.

2. For Columns

a) Moment Stiffness

Moment stiffness is now defined as the moment required to rotate one end of the column through a unit angle with no translation. For the cases being investigated, hinged end columns are involved. Since no moment can act at the hinged end, we determine the moment stiffness keeping one end hinged and thereby eliminate the process of carrying-over moments to the bottom of the column.

In order to solve for the moment stiffness the principle of the conjugate beam is used. This method involves the use of a fictitious column loaded with the $\frac{M}{EI}$ diagram of the given column. Then, the slope of the given column at any point is equal to the shear in the conjugate column at the same point. We are interested in the case when the slope is unity at the top of the column. Therefore in the conjugate column the shear must be equal to one at that point.

i. Spagnuolo Column:

Figure 6 illustrates the procedure involved for determining the

moment stiffness for the Spagnuolo column.

Free Body   Moment   M/EI   Conjugate
Diagram   Diagram   Diagram   Column

Figure 6.

$$R_1 = \text{Slope} = \theta = 1 = 1 \times \frac{12h \times 12}{2EI \times 13} \times 8 = \frac{576h}{13EI} \qquad (18)$$

$$\text{or} \qquad h = \frac{13EI}{576}$$

The moment stiffness, then, equals h x 13 = 0.2935EI. If we now assume the moment of inertia of the column to be equal to ten times the area of the chord of the truss, (I = 10a), we then have the column moment stiffness as

$$M = 2.935Ea$$

ii. Maugh Equivalent Column

The same procedure is used for determining the moment stiffness for the equivalent column. (Figure 7)

Free Body   Conjugate
Diagram   Column

Figure 7.

$$R_1 = \frac{1}{2} \left(\frac{12.64h}{EI} \quad x \quad 12.64 \right) x \frac{2}{3} = 1 \tag{19}$$

$$h = \frac{3EI}{12.64 \text{ x } 12.64}$$

then $M = 12.64h$

and for $I = 10a$, $M = 2.3734Ea$

b) **Thrust Stiffness**

As in the case of the truss, the thrust stiffness is the lateral
thrust required to produce a unit displacement of one end of the
column while the other end remains hinged, for the case being dealt
with. The method of moment area is used for determining this value.
This principle states that the deflection from the elastic curve at
any point, measured normal to the original position of the curve from
a tangent at any other point, is equal in magnitude to the moment of
the M/EI area between the two points, about an ordinate through the
first point.

i. Spagnuolo Column

Referring to Figure 8 we may determine the thrust stiffness, h, for the Spagnuolo column.

| Free Body Diagram | Moment Diagram | M/EI Diagram |

Figure 8.

$$\triangle = 1 = \frac{1}{2} \times \frac{12h \times 8 \times 12}{EI} \qquad (20)$$

or $h = \frac{EI}{48 \times 12}$

for I = 10a

h = 0.01738 Ea

ii. Maugh Equivalent Column

In a manner similar to that shown above, we find the thrust stiffness for the equivalent column. (Figure 9)

Free Body Diagram	Moment Diagram	M/EI Diagram

Figure 9

$$\triangle = 1 = \frac{1}{2} \times \frac{12.64h \times 12.64}{EI} \times \frac{2}{3} \times 12.64$$

or $h = \frac{3EI}{(12.64)^3} = 0.001488 \text{ EJ}$

for $I = 10a$

$h = 0.01488 \text{ Ea}$

c) Thrust Carry-Over Factor

The thrust carry-over factor for the column is the ratio of the thrust produced at the top of the column to that produced at the bottom of the column. As may be seen in Figures 8 and 9, the magnitude of this value is one.

3. Sign Convention

The sign convention that is used in working out solutions to the cases discussed is shown in Figure 10.

Figure 10.

Moments producing clockwise rotation of the joint and thrusts that act
to the right on the joint are considered positive.

By inspection of some of the preceding figures, it is seen that
all the carry-over factors are negative for both moment and thrust.
This characteristic, it might be noted, is also true for most arches.

4. Distribution of Moments and Thrusts

When an unbalanced moment or thrust exists at a joint, it must
be distributed among the members framing into that joint in proportion
to their relative stiffnesses. For unbalanced moments, the amount that
is distributed to a member is the moment stiffness of that member divided
by the sum of the stiffness factors of all the members at the joint.
The unbalanced thrusts are distributed in a similar manner, i.e. they
are distributed in proportion to the relative thrust stiffnesses.

5. Effects Produced by Balancing Moments and Thrusts

At this point it might be well to note the effects produced by
the balancing of moments and thrusts.

a) **For the truss**

i. It was seen when determining the moment stiffness, that in order to keep the truss from translating, an accompanying thrust was produced. Therefore whenever moments are balanced, thrusts accompanying these moments must also be accounted for. A similar operation must be carried out when balancing thrusts, since, in order to keep the truss from rotating while translating, an accompanying moment must be developed.

ii. The effect of carrying-over moments and thrusts already has been discussed. However, as in the case of the balancing operation, thrusts and moments accompanying the carried-over moments and thrusts respectively, must also be accounted for.

b) For the column

Figure 11 illustrates the effect of changes in moment and thrust on the column. Whenever a moment is produced on the column, a thrust is also produced to keep the column in equilibrium. Similarly, when a thrust is produced on the column, an accompanying moment results.

Figure 11

C. Applications.

Case I - Single Aisle, Hinged-end Bent Loaded with a Unit Panel Load (Referring to Plate I, H = 12 and L = 16)

1. Solution by Least Work

Using the method of Least Work, the structure must first be made statically determinate. Then an appropriate force must be applied to restore the bent to its original position. This is shown in Figure 12.

Figure 12.

The actual stress in any member will then be equal to the stress, S_o, in the member due to the original load, plus the stress, S_h, due to a unit thrust times some constant, H.

$$S = S_o + HS_h \quad \dots\dots\dots\dots\dots\dots\dots\dots \quad (22)$$

In a similar fashion, the moment in the columns of the trussed bent is equal to the moment, M_o, due to the loading on the statically determinate structure plus the moment, M_h, produced by the unit thrust times the same constant H.

$$M = M_o + Hm_h \quad \dots\dots\dots\dots\dots\dots\dots\dots \quad (23)$$

For Case I, $M_o = 0$.

Using the same energy approach as previously, the total strain energy is

$$U = \sum \frac{S^2 L}{2AE} + \int \frac{M^2 ds}{2EI} \quad \dots\dots\dots\dots\dots \quad (24)$$

from which we obtain

$$\frac{\partial U}{\partial H} = \sum \frac{S_o S_h L}{AE} + \sum H\frac{S_h^2 L}{AE} + \int H\frac{m_h^2}{EI}ds \qquad (25)$$

From Plate VI and Figure 13, we obtain the constants involved in

this equation, and substituting these values in it, we find H = 0.2513.

$$\int_0^{14} \frac{m_h^2}{EI}ds = 2 \int_0^{12} \frac{y^2}{EI}dy + 2 \int_0^2 \frac{(6y)^2}{EI}dy = 1344/EI$$

for I = 10a

$$\int_0^{14} \frac{m_h^2}{EI}ds = 134.4/EI$$

Figure 13.

2. Solution by Moment and Thrust Distribution with the use of the

Spagnuolo Column

The solution of Case I by moment and thrust distribution is worked

by the standard method developed by Professor Cross. The moments in this

solution are balanced about an equivalent joint which is considered to be

midway between the knee brace and lower chord of the truss. As mentioned

previously, care must be exercised to be certain of including all the

effects produced by the balancing of moments and thrusts. The work

involved in evaluating the necessary constants required for the solution

PLATE VI

CASE I - SOLUTION BY LEAST WORK

MEMBER	$\frac{L'}{A} \cdot \frac{a}{L}$	S_o	S_h	$S_o S_h \frac{L'}{A} \cdot \frac{a}{L}$	$S_h^2 \frac{L'}{A} \cdot \frac{a}{L}$
AC	0.2795	-3.3541	6.2606	-5.8691	10.9550
BD	0.2795	-2.9069	6.2606	-5.0866	10.9550
CG	0.3125	3.0000	0.3999	0.3749	0.0500
½EF	0.1875	2.0000	-4.4999	-1.6875	3.7967
CD	0.0699	-0.8944	0	0	0
DE	0.1563	1.0000	-3.4997	-0.5470	1.9143
GF	0.3366	0	-7.5392	0	19.1322
			½ Truss	-12.8153	46.8032
			Total Truss	-25.6306	93.6064

has already been performed, and these values follow. The actual distribution of moments and thrusts is shown on Plate VII.

Fixed-end moment = 0.8302

Fixed-end thrust = 2.5176

Moment stiffness (truss) = 0.3877 Ea

Moment stiffness (column) = 2.9350 Ea

Carry-over factor of moment = -1.0692

Thrust stiffness (truss) = 0.0626 Ea

Thrust stiffness (column) = 0.0174 Ea

Carry-over factor for thrust = -1.0

Moment accompanying balancing thrust, $\Delta M_h = \pm \dfrac{0.0668}{0.0626} \times \Delta H$

$\qquad\qquad = \pm 1.0671 \, \Delta H$

Moment accompanying carried-over thrust $= -\Delta M_h$

Thrust accompanying balancing moment, $\Delta H_m = + \dfrac{0.0672}{0.3877} \times \Delta M$

$\qquad\qquad = + 0.1733 \times \Delta M$

Thrust accompanying carried-over moment $= -\Delta H_m$

Since both the bent and loading are symmetrical about the center line of the bent, it was only necessary to work with half the bent. Because of symmetry also, it was possible to combine the moments accompanying the balancing thrusts with the moments accompanying the carried-over thrusts. The same was also true for the thrusts accompanying moments.

In the distribution process it is seen that moments converge rapidly, whereas the thrusts converge rather slowly. Therefore rather than follow a methodical pattern of going through a cycle of moment distribution for each cycle of thrust distribution, it was found convenient to

PLATE VII

CASE I - SOLUTION BY MOMENT AND THRUST DISTRIBUTION

(SPAGNUOLO COLUMN)

Thrust acc. Moment, $\Delta H_m = \mp 2 \times 0.17335 \, \Delta M = 0.3467 \, \Delta M$

Moment acc. Thrust, $\Delta M_h = \mp 2 \times 1.0671 \, \Delta H = 2.1342 \, \Delta H$

MOMENT

$S = 2.9350$ $S = 0.3877$ $S = 2.9350$
$C.O.F. = -1.0692$

THRUST

$S = 0.0174$ $S = 0.0626$ $S = 0.0174$
$C.O.F. = -1.0$

OPERATION	0.8833	0.1167	0.1167	0.8833	0.2175	0.7825	0.7825	0.2175	OPERATION
FEM		+0.8302	-0.8302			-2.5176	+2.5176		FET
ΔM acc ΔH	-7.1188	+4.2044	-4.2044	-7.1188	+0.5476	+1.9700	-1.9700	-0.5476	Bal.
Bal	+1.8410	+0.2432	-0.2432	-1.8410		+1.9700	-1.9700		C.O.
C.O.		+0.2600	-0.2600		-0.1416	+0.0843	etc.		ΔH acc ΔM
ΔM accΣBal₁	+2.8743	-1.6975	etc.		-0.4160	-1.4967			Bal.
Bal	-1.2691	-0.1677				-1.4967			C.O.
C.O.		-0.1793			+0.3255	+1.1712			Bal.
ΔM accΣBal₂	+0.2366	-0.1398				+1.1712			C.O.
Bal	+0.0729	+0.0096			-0.2547	-0.9165			etc.
						-0.9165			
					+0.1993	+0.7172			
						+0.7172			
					-0.1560	-0.5612			
						-0.5612			
					+0.1221	+0.4391			
						+0.4391			
					-0.0955	-0.3436			
						-0.3436			
					+0.0747	+0.2689			
						+0.2689			
					-0.0585	-0.2104			
						-0.2104			
					+0.0458	+0.1646			
						+0.1646			
					-0.0358	-0.1288			
						-0.1288			
	ΣBal.₁ = -0.2211				+0.0280	+0.1008	ΣBal.₁ = -0.7954		
						+0.1008			
					+0.0976	-0.0581			
					-0.0305	-0.1098			
						-0.1098			
					+0.0239	+0.0859			
						+0.0859			
					-0.0187	-0.0672			
						-0.0672			
					+0.0146	+0.0526			
						+0.0526			
					-0.0114	-0.0412			
						-0.0412			
					+0.0090	+0.0322			
						+0.0322			
					-0.0071	-0.0252			
						-0.0252			
					+0.0055	+0.0197			
						+0.0197			
					-0.0043	-0.0154			
						-0.0154			
					+0.0033	+0.0121			
						+0.0121			
	ΣBal.₂ = -0.0182				-0.0026	-0.0095	ΣBal.₂ = -0.0655		
	-3.3631	+3.3631			+0.2586	-0.2586			
	-3.3843	+3.3843			+0.2602	-0.2602			

} 1 cycle

} 20 cycles for complete convergence

$\dfrac{M}{h^1} = 0.2603$

balance only thrusts for a number of cycles and then take into account the combined thrust effect on the moments, in one operation.

3. Solution by Moment and Thrust Distribution with the use of the Maugh Equivalent Column.

This solution of Case I by moment and thrust distribution assumes the equivalent joint to be at the top of the Maugh equivalent column. The procedure is identical with that used in the solution based on the Spagnuolo column. The constants involved in the solution are listed below and the actual solution is shown on Plate VIII.

Fixed-end moment = 0.0761

Fixed-end thrust = 2.5176

Moment stiffness (truss) = 0.8534 Ea

Moment stiffness (column) = 2.3734 Ea

Carry-over factor for moment = -0.3843

Thrust stiffness (truss) = 0.0626 Ea

Thrust stiffness (column) = 0.0149 Ea

Carry-over factor for thrust = - 1.0

Moment accompanying balancing thrust, $\Delta M_h = + \dfrac{0.0893}{0.0626} \times \Delta H$

$$= + 1.4265 \, \Delta H$$

Moment accompanying carried-over thrust = $- \Delta M_h$

Thrust accompanying balancing moment, $\Delta H_m = + \dfrac{0.2523}{0.8534} \times \Delta M$

$$= + 0.2962 \Delta M$$

Thrust accompanying balancing moment, $= -\Delta H_m$

4. Solution by the Maugh Method

The Maugh method makes use of the equivalent column, and the solution of the problem is merely one in moment distribution. The

CASE I - SOLUTION BY MOMENT AND THRUST DISTRIBUTION

(MAUGH COLUMN)

Thrust acc. Moment, $\Delta H_m = + \Delta M : \dfrac{0.2528}{0.8534} \times 2 = +0.5925\,\Delta M$

Moment acc Thrust, $\Delta M_h = + \Delta H = \dfrac{0.0893}{0.0626} \times 2 = +2.8530\,\Delta H$

MOMENT

S = 2.3734   S = 0.8534   S = 2.3734
C.O.F. = - 0.3843

THRUST

S = 0.0149   S = 0.0626   S = 0.0149
C.O.F. = -1.0

OPERATION	0.7355	0.2645	0.2645	0.7355	0.1923	0.8077	0.8077	0.1923	OPERATION
FEM		-0.0761	+0.0761			-2.5176	+2.5176		FEM
ΔM acc. ΔH	-6.1190	+5.8016	-5.8016	+6.1190	+0.4841	+2.0335	-2.0335	-0.4841	Bal
Bal	+0.2894	+0.1041	-0.1041	-0.2894		+2.0335	-2.0335		C.O.
C.O.		+0.0400	-0.0400		-0.0229	+0.0617	-0.0617	+0.0229	ΔH acc ΔM
ΔM accΣBal._1	+2.7441	-2.6051	etc.		-0.3985	-1.6738	+1.6738	+0.3985	Bal
Bal	-0.1317	-0.0473				-1.6738	etc.		C.O.
					+0.3219	+1.3519			Bal
						+1.3519			C.O.
					-0.2600	-1.0919			Bal
						-1.0919			
					+0.2100	+0.8819			
						+0.8819			
					-0.1696	-0.7123			
						-0.7123			
					+0.1370	+0.5753			
						+0.5753			
					-0.1106	-0.4647			
						-0.4647			
					+0.0894	+0.3753			
						+0.3753			
					-0.0722	-0.3031			
						-0.3031			
					+0.0583	+0.2448			
						+0.2448			
					-0.0471	-0.1977			
						-0.1977			
					+0.0380	+0.1597			
						+0.1597			
					-0.0307	-0.1290			
						-0.1290			
					+0.0248	+0.1042			
						+0.1042			
					-0.0200	-0.0842			
						-0.0842			
					+0.0162	+0.0680			
						+0.0680			
					-0.0131	-0.0549			
						-0.0549			
					+0.0106	+0.0443			
						+0.0443			
					-0.0085	-0.0358			
						-0.0358			
					+0.0069	+0.0289			
						+0.0289			
					+0.0104	-0.0280			
					-0.0022	-0.0091			

$\Sigma\,\text{Bal}_{+1} = -0.2172$ $\Sigma\,\text{Bal}_{.1} = -0.9131$

-3.2172	+3.2172			+0.2523	-0.2523		

} 2 cycles

} 22 cycles for complete convergence

-3.1884	+3.1884			+0.2521	-0.2521		

$$\frac{M}{h^1} = \frac{3.1884}{12.64} = 0.2522$$

effect of thrusts has been neglected. The values of the necessary

constants for this solution follow.

Fixed-end moment = 0.0761

Moment stiffness (truss) = 0.3534 Ea

Moment stiffness (column) = 2.3734 Ea

Moment carry-over factor = - 0.3843

The solution of the problem is shown on Plate IX.

5. Comparison of Results

Plate X shows the results obtained for Case I by the various

solutions just discussed.

CASE II - Triple-Aisle, Hinged-end Bent Loaded with a Unit Panel Load.

(Truss in each aisle, and H and L dimensions are the same as for

Case I.)

1. Solution by Least Work

The method of solving Case I by Least Work is similar to that

shown for Case II. In this case however, there are five redundant re-

actions. A free-body diagram of the structure is shown in Figure 14.

Figure 14.

PLATE IX

CASE I - SOLUTION BY THE MAUGH METHOD

(THRUST EFFECTS NEGLECTED)

$S = 2.3734$ $S = 0.8534$ $S = 2.3734$

$S/\Sigma S$	COL.	TRUSS		TRUSS	COL.
	0.7355	0.2645	C.O.F. = -0.3843	0.2645	0.7355
		-0.0761		+0.0761	
	+0.0560	+0.0201		-0.0201	-0.0560
		+0.0077		-0.0077	
	-0.0057	-0.0020		+0.0020	+0.0057
		-0.0010		+0.0010	
	+0.0007	+0.0003		-0.0003	-0.0007
	+0.0510	-0.0510		+0.0510	-0.0510

Thrust at the bottom of column,

$$H = \frac{M}{h}1 = \frac{0.510}{12.64} = 0.0403$$

PLATE X

CASE I - COMPARISON OF RESULTS

Least Work	$H = 0.2513$
Moment and Thrust Distribution (Spagnuolo Column)	$H = 0.2602$
	$\frac{M}{h} = 0.2603$
Moment and Thrust Distribution (Maugh Column)	$H = 0.2521$
	$\frac{M}{h} = 0.2522$
Maugh Method (Thrust neglected)	$\frac{M}{h} = 0.0403$

Again the structure is made statically determinant (Figure 15) and unit forces are individually placed at the points where the redundant reactions acted. The resultant stress in any member is then

$$S = S_o + H_a u_a + H_b u_b + H_c u_c + V_b u_b + V_c u_c \qquad (26)$$

and the moment in the bent is

$$M = M_o + H_a m_a + H_b m_b + H_c m_c + V_b n_b + V_c n_c \qquad (27)$$

where u_a, u_b, u_c, v_b, and v_c are the stresses in any members due to the respective unit forces, and m_a, m_b, and m_c are the moments due the respective horizontal unit forces, and n_b and n_c are the moments due to the respective vertical unit forces.

Figure 15.

For this case

$$M_o = n_b = n_c = 0$$

The total strain energy in the bent is then

$$U = \sum \frac{S^2 L}{2AE} + \int \frac{M^2 d_s}{2EI} \qquad (28)$$

The redundant forces may then be solved for by taking into account the fact that there can be no displacements at the reactions. From this relationship the following equations may be obtained:

$$\frac{\partial U}{\partial H_a} = \sum \frac{S_o u_a L}{AE} + \sum H_a \frac{u_a^2 L}{AE} + \sum H_b \frac{u_a u_b L}{AE} + \sum H_c \frac{u_a u_c L}{AE} + \sum V_b \frac{u_a v_b L}{AE}$$

$$+ \sum V_c \frac{u_a v_c L}{AE} + \int H_a \frac{m_a^2}{EI} ds + \int H_b \frac{m_a m_b}{EI} ds + \int H_c \frac{m_a m_c}{EI} ds = 0 \qquad (29)$$

$$\frac{\partial U}{\partial H_b} = \sum \frac{S_o u_b L}{AE} + \sum H_a \frac{u_a u_b L}{AE} + \sum H_b \frac{u_b^2 L}{AE} + \sum H_c \frac{u_b u_c L}{AE} + \sum V_b \frac{u_b v_b L}{AE}$$

$$+ \sum V_c \frac{u_b v_c L}{AE} + \int H_a \frac{m_a m_b}{2EI} ds + \int H_b \frac{m_b^2}{2EI} ds + \int H_c \frac{m_b m_c}{2EI} ds = 0 \qquad (30)$$

$$\frac{\partial U}{\partial H_c} = \sum \frac{S_o u_c L}{AE} + \sum H_a \frac{u_a u_c L}{AE} + \sum H_b \frac{u_b u_c L}{AE} + \sum H_c \frac{u_c^2 L}{AE} + \sum V_b \frac{u_c v_b L}{AE}$$

$$+ \sum V_c \frac{u_c v_c L}{AE} + \int H_a \frac{m_a m_c}{EI} ds + \int H_b \frac{m_b m_c}{EI} ds + \int H_c \frac{m_c^2}{EI} ds = 0 \qquad (31)$$

$$\frac{\partial U}{\partial V_b} = \sum \frac{S_o v_b L}{AE} + \sum H_a \frac{u_a v_b L}{AE} + \sum H_b \frac{u_b v_b L}{AE} + \sum H_c \frac{u_c v_b L}{AE} + \sum V_b \frac{v_b^2 L}{AE}$$

$$+ \sum V_c \frac{v_b v_c L}{AE} = 0 \qquad (32)$$

$$\frac{\partial U}{\partial V_c} = \sum \frac{S_o v_c L}{AE} + \sum H_a \frac{u_a v_c L}{AE} + \sum H_b \frac{u_b v_c L}{AE} + \sum H_c \frac{u_c v_c L}{AE} + \sum V_b \frac{v_b v_c L}{AE}$$

$$+ \sum V_c \frac{v_c L}{AE} = 0 \qquad (33)$$

Because of the similarities in Case I and Case II it is readily seen that the coefficients of the terms containing the moments produced by the unit forces, are all identical. That is

$$\int \frac{m_a^2}{EI} ds = \int \frac{m_a m_b}{EI} ds = \int \frac{m_a m_c}{EI} ds = \int \frac{m_b^2}{EI} ds = \int \frac{m_b m_c}{EI} ds = \int \frac{m_c^2}{EI} ds = \int_0^{14} \frac{m_h^2}{EI} ds \ (fig.13) = \frac{134.4}{EI}$$

$$(34)$$

The evaluation of the remaining constants involved in equations (29) – (33) is shown in Plate XI, and the results obtained from the above equations are as follows:

$$H_a = 0.3281$$

$$\left. \begin{array}{l} H_b = 0.0414 \\ H_c = -0.0792 \end{array} \right\} \quad \text{mean value} = 0.0603$$

$$\left. \begin{array}{l} V_b = 3.8836 \\ V_c = 3.9456 \end{array} \right\} \quad \text{mean value} = 3.9145$$

From symmetry it is known that $H_b = H_c$ and $V_b = V_c$. Therefore the Least Work solution is somewhat in error. From the small magnitudes of these reactions when compared with the constants in the equations, it would seem that a small discrepancy in the tabulated quantities might have a relatively large effect on the reactions. These discrepancies in the tabulated quantities may then have been caused by not having carried out the solution to enough decimal places. The results, comparatively speaking, were not enough in error however to warrant the reworking of the problem, particularly when the time and labor involved for the working of it are considered.

2. Solution by Moment and Thrust Distribution with the Use of the Spagnuolo Column.

This solution is carried out in a manner similar to that shown in case one. Again, because of symmetry, only one half the structure is involved in the solution. The necessary constants required are the same as those listed in part 2 of Case I, and the solution itself is shown on Plate XII.

PLATE XI

CASE II - SOLUTION BY THE METHOD OF LEAST WORK

AISLE	MEMBER	$\frac{L'\cdot a}{A\cdot L}$	S_o	u_a	u_b	u_c	v_b	v_c	$S_o u_a \frac{L'\cdot a}{A\cdot L}$	$S_o u_b \frac{L'\cdot a}{A\cdot L}$	$S_o u_c \frac{L'\cdot a}{A\cdot L}$	$S_o v_b \frac{L'\cdot a}{A\cdot L}$	$S_o v_c \frac{L'\cdot a}{A\cdot L}$	$u_a^2 \frac{L'\cdot a}{A\cdot L}$	$u_a u_b \frac{L'\cdot a}{A\cdot L}$	$u_a u_c \frac{L'\cdot a}{A\cdot L}$	$u_a v_b \frac{L'\cdot a}{A\cdot L}$	$u_a v_c \frac{L'\cdot a}{A\cdot L}$	$u_b^2 \frac{L'\cdot a}{A\cdot L}$	$u_b u_c \frac{L'\cdot a}{A\cdot L}$	$u_b v_b \frac{L'\cdot a}{A\cdot L}$	$u_b v_c \frac{L'\cdot a}{A\cdot L}$	$u_c^2 \frac{L'\cdot a}{A\cdot L}$	$u_c v_b \frac{L'\cdot a}{A\cdot L}$	$u_c v_c \frac{L'\cdot a}{A\cdot L}$	$v_b^2 \frac{L'\cdot a}{A\cdot L}$	$v_b v_c \frac{L'\cdot a}{A\cdot L}$	$v_c^2 \frac{L'\cdot a}{A\cdot L}$
I	AC	0.2795	-12.2984	6.2610	0	0	1.4907	0.7453	-21.5216	0	0	-5.1241	-2.5619	10.9564	0	0	2.6087	1.3042	0	0	0	0	0	0	0	0.6210	0.3105	0.1553
	BD	0.2795	-11.8512	6.2610	0	0	1.4907	0.7453	-20.7390	0	0	-4.9378	-2.4687	10.9564	0	0	2.6087	1.3042	0	0	0	0	0	0	0	0.6210	0.3105	0.1553
	bd	0.2795	-22.5843	6.2610	0	0	3.2796	1.6398	-39.5214	0	0	-20.7019	-10.3509	10.9564	0	0	5.7391	2.8696	0	0	0	0	0	0	0	3.0062	1.5031	0.7515
	ac	0.2795	-23.0315	6.2610	0	0	3.2796	1.6398	-40.3040	0	0	-21.1118	-10.5559	10.9564	0	0	5.7391	2.8692	0	0	0	0	0	0	0	3.0062	1.5031	0.7515
	CG	0.3125	11.0000	0.4000	0	0	-1.3334	-0.6667	1.3750	0	0	-4.5836	-2.2918	0.0500	0	0	-0.1667	-0.0833	0	0	0	0	0	0	0	0.5556	0.2778	0.1389
	cg	0.3125	-11.4000	0.4000	0	0	2.4000	1.2000	-1.4250	0	0	-8.5500	-4.2750	0.0500	0	0	0.3000	0.1500	0	0	0	0	0	0	0	1.8000	0.9000	0.4500
	EF	0.3750	10.0000	-4.5000	0	0	-1.3334	-0.6667	-16.8750	0	0	-5.0003	-2.5001	7.5938	0	0	2.2501	1.1250	0	0	0	0	0	0	0	0.6668	0.3334	0.1667
	CD	0.0699	-0.8944	0	0	0	-1.3334	-0.6667	-16.8750	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	DE	0.1563	1.0000	-3.5000	0	0	0	0	0	0	0	0	0	1.9147	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	dE	0.1563	17.0000	-3.5000	0	0	-2.6667	-1.3334	0	0	0	-7.0857	-3.5430	1.9147	0	0	1.4588	0.7294	0	0	0	0	0	0	0	1.1115	0.5558	0.2779
	cd	0.0699	-0.8944	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	GF	0.3366	0	-7.5392	0	0	0	0	0	0	0	-66.6380	-33.3190	19.1322	0	0	14.5770	7.2885	0	0	0	0	0	0	0	0	0	0
	gF	0.3366	34.4650	-7.5392	0	0	-5.7442	-2.8721	-81.4616	0	0	-66.6380	-33.3190	19.1322	0	0	14.5770	7.2885	0	0	0	0	0	0	0	11.1064	5.5532	2.7766
II	AC	0.2795	-31.9762	6.2610	6.2610	0	4.0249	3.1305	-55.9567	-55.9567	0	-35.9719	-27.9784	10.9564	10.9564	0	7.0434	5.4782	10.9564	0	7.0434	5.4782	0	0	0	4.5278	3.5217	2.7391
	BD	0.2795	-31.5256	6.2610	6.2610	0	4.0249	3.1305	-55.1742	-55.1742	0	-35.4689	-27.5871	10.9564	10.9564	0	7.0434	5.4782	10.9564	0	7.0434	5.4782	0	0	0	4.5278	3.5217	2.7391
	bd	0.2795	-31.5290	6.2610	6.2610	0	3.1305	4.0249	-55.1742	-55.1742	0	-27.5871	-35.4689	10.9564	10.9564	0	5.4782	7.0434	10.9564	0	5.4782	7.0434	0	0	0	2.7391	3.5217	4.5278
	ac	0.2795	-31.9762	6.2610	6.2610	0	3.1305	4.0249	-55.9567	-55.9567	0	-27.9784	-35.9719	10.9564	10.9564	0	5.4782	7.0434	10.9564	0	5.4782	7.0434	0	0	0	2.7391	3.5217	4.5278
	CG	0.3125	-3.3995	0.4000	0.4000	0	1.7334	-0.1333	-0.4249	-0.4249	0	-1.8415	0.1416	0.0500	0.0500	0	0.2167	-0.0167	0.0500	0	0.2167	-0.0167	0	0	0	0.9390	-0.0722	0.0056
	cg	0.3125	-3.3995	0.4000	0.4000	0	-0.1333	1.7334	-0.4249	-0.4249	0	0.1416	-1.8415	0.0500	0.0500	0	-0.0167	0.2167	0.0500	0	-0.0167	0.2167	0	0	0	0.0056	-0.0722	0.9390
	EF	0.3750	18.0000	-4.5000	-4.5000	0	-2.0000	-2.0000	-30.3750	-30.3750	0	-13.5000	-13.5000	7.5938	7.5938	0	3.3750	3.3750	7.5938	0	3.3750	3.3750	0	0	0	1.5000	1.5000	1.5000
	CD	0.0699	-0.8944	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	DE	0.1563	17.0000	-3.5000	-3.5000	0	-2.6667	-1.3334	-9.2999	-9.2999	0	-7.0857	-3.5430	1.9147	1.9147	0	1.4588	0.7294	1.9147	0	1.4588	0.7294	0	0	0	1.1115	0.5558	0.2779
	dE	0.1563	17.0000	-3.5000	-3.5000	0	-1.3334	-2.6667	-9.2999	-9.2999	0	-3.5430	-7.0857	1.9147	1.9147	0	0.7294	1.4588	1.9147	0	0.7294	1.4588	0	0	0	0.2779	0.5558	1.1115
	cd	0.0699	-0.8944	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	GF	0.3366	34.4650	-7.5392	-7.5392	0	-5.7442	-2.8721	-87.4616	-87.4616	0	-66.6380	-33.3190	19.1322	19.1322	0	14.5770	7.2885	19.1322	0	14.5770	7.2885	0	0	0	11.1064	5.5532	2.7766
	gF	0.3366	34.4650	-7.5392	-7.5392	0	-2.8721	-5.7442	-87.4616	-87.4616	0	-33.3190	-66.6380	19.1322	19.1322	0	7.2885	14.5770	19.1322	0	7.2885	14.5770	0	0	0	2.7766	5.5532	11.1064
III	AC	0.2795	-23.0315	6.2610	6.2610	6.2610	1.6398	3.2796	-40.3040	-40.3040	-40.3040	-10.5559	-21.1118	10.9564	10.9564	10.9564	2.8696	5.7391	10.9564	10.9564	2.8696	5.7391	10.9564	2.8696	5.7391	0.7515	1.5031	3.0062
	BD	0.2795	-22.5843	6.2610	6.2610	6.2610	1.6398	3.2796	-39.5214	-39.5214	-39.5214	-10.3509	-20.7019	10.9564	10.9564	10.9564	2.8696	5.7391	10.9564	10.9564	2.8696	5.7391	10.9564	2.8696	5.7391	0.7515	1.5031	3.0062
	bd	0.2795	-11.8512	6.2610	6.2610	6.2610	0.7453	1.4907	-20.7390	-20.7390	-20.7390	-2.4687	-4.9378	10.9564	10.9564	10.9564	1.3042	2.6087	10.9564	10.9564	1.3042	2.6087	10.9564	1.3042	2.6087	0.1553	0.3105	0.6210
	ac	0.2795	-12.2984	6.2610	6.2610	6.2610	0.7453	1.4907	-21.5216	-21.5216	-21.5216	-2.5619	-5.1241	10.9564	10.9564	10.9564	1.3042	2.6087	10.9564	10.9564	1.3042	2.6087	10.9564	1.3042	2.6087	0.1553	0.3105	0.6210
	CG	0.3125	-11.4000	0.4000	0.4000	0.4000	1.2000	-0.6667	-1.4250	-1.4250	-1.4250	-4.2750	-2.2918	0.0500	0.0500	0.0500	0.1500	0.3000	0.0500	0.0500	0.1500	0.3000	0.0500	0.1500	0.3000	0.4500	0.9000	1.8000
	cg	0.3125	11.0000	0.4000	0.4000	0.4000	-0.6667	-1.3334	1.3750	1.3750	1.3750	-2.2918	-4.5836	0.0500	0.0500	0.0500	-0.0833	-0.1667	0.0500	0.0500	-0.0833	-0.1667	0.0500	-0.0833	-0.1667	0.1389	0.2778	0.5556
	EF	0.3750	10.0000	-4.5000	-4.5000	-4.5000	-0.6667	-1.3334	-16.8750	-16.8750	-16.8750	-2.5000	-5.0003	7.5938	7.5938	7.5938	1.1250	2.2501	7.5938	7.5938	1.1250	2.2501	7.5938	1.1250	2.2501	0.1667	0.3334	0.6668
	CD	0.0699	-0.8944	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	DE	0.1563	17.0000	-3.5000	-3.5000	-3.5000	-1.1334	-2.6667	-9.2999	-9.2999	-9.2999	-3.5430	-7.0857	1.9147	1.9147	1.9147	0.7294	1.4588	1.9147	1.9147	0.7294	1.4588	1.9147	0.7294	1.4588	0.2779	0.5558	1.1115
	dE	0.1563	1.0000	-3.5000	-3.5000	-3.5000	0	0	-0.5471	-0.5471	-0.5471	0	0	1.9147	1.9147	1.9147	0	0	1.9147	1.9147	0	0	1.9147	0	0	0	0	0
	cd	0.0699	-0.8944	0	0	0	-2.8721	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	GF	0.3366	34.4650	-7.5392	-7.5392	-7.5392	-5.7442	-2.8721	-87.4616	-87.4616	-87.4616	-33.3190	-66.6380	19.1322	19.1322	19.1322	7.2885	14.5770	19.1322	19.1322	7.2885	14.5770	19.1322	7.2885	14.5770	2.7766	5.5532	11.1064
	gF	0.3366	0	-7.5392	-7.5392	-7.5392	-2.8721	-5.7442	0	0	0	0	0	19.1322	19.1322	19.1322	0	0	19.1322	19.1322	0	0	19.1322	0	0	0	0	0
									-919.6488	-683.3292	-236.3196	-468.3914	-468.3914	280.8396	187.2264	93.6132	105.3439	105.3439	187.2264	93.6132	70.2291	87.7867	93.6132	17.5572	35.1148	60.3692	50.1552	60.3692

PLATE XII

CASE II – SOLUTION BY MOMENT AND THRUST DISTRIBUTION (SPAGNUOLO COLUMN)

MOMENT

	COL.	TRUSS		COL.	TRUSS	℄ of Truss
S = 0.8833	S = 2.935	S = 0.1167	S = -1.0692	S = 0.3877	S = 0.1045	
OPERATION	0.8833	0.1167		0.3877	0.1045	OPERATION
FEM		+0.8302		-0.8302	+0.8302	FEM
ΔM acc. ΔH	-7.1188	+2.1022		+0.9229	+0.9229	ΔM acc. ΔH
ΔM acc. C.O.		-0.9229		-2.1022	+0.9229	ΔM acc. C.O.
Bal	+4.5130	+0.5963		-0.6376		C.O.
C.O.		-0.3457		+0.3233	+0.3233	Bal
ΔMacc. ΔH	-3.2929	+0.9727		+0.0797	+0.0797	ΔM acc. ΔH
ΔM acc. C.O.		-0.0797		-0.9727	+0.0797	ΔM acc. C.O.
Bal	+2.4252	+0.3204		-0.3426	+0.3457	C.O.
C.O.		-0.1116		+0.1044	+0.1044	Bal
ΔM acc. ΔH	-0.6318	+0.1867		+0.0760	+0.0760	ΔM acc. ΔH
ΔM acc. C.O.		-0.0760		-0.1867	+0.0760	ΔM acc. C.O.
Bal	+0.5589	+0.0738		-0.0789	+0.1116	C.O.
C.O.		-0.0205		+0.0192	+0.0192	Bal
ΔM acc. ΔM	-0.2964	+0.0874		+0.0116	+0.0116	ΔM acc. ΔH
ΔM acc. C.O.		-0.0116		-0.0874	+0.0116	ΔM acc. C.O.
Bal	+0.2130	+0.0281		-0.0300	+0.0205	C.O.
				+0.0105	+0.0105	Bal
FINAL M	-3.6298	+3.6298		-3.7207	+3.9458	FINAL M

THRUST

	COL.	TRUSS		TRUSS	COL.	TRUSS
S = 0.0174	S = 0.7825	S = 0.0626	S = -1.0	S = 0.0174	S = 0.1220	S = 0.4390
OPERATION	0.2175	0.7825	-1.0	0.4390	0.1220	0.4390	OPERATION
FET		-2.5176	+2.5176	-2.5176		+2.5176	FET
Bal	+0.5476	+1.9700	-1.9700				C.O.
C.O.		-0.8649		+0.8649	+0.2402	+0.8649	Bal
ΔH acc. ΔM	-0.3472	+0.1033	+0.0560	+0.0560	-0.1882	+0.0560	ΔH acc. ΔM
ΔH acc. CO		-0.0560		+0.0560		+0.0560	ΔH acc. C.O.
Bal	+0.2533	+0.9115	-0.9115	+0.8649		+0.8649	C.O.
C.O.		-0.0747		+0.0747	+0.0207	+0.0747	Bal
ΔH acc. ΔM	-0.1863	+0.0555	+0.0181	+0.0181	-0.0608	+0.0181	ΔH acc. ΔM
ΔH acc. C.O.		-0.0181		+0.0181		+0.0181	ΔH acc. C.O.
Bal.	+0.0486	+0.1750	-0.1750	+0.0747		+0.0747	C.O.
C.O.		-0.0712		+0.0712	+0.0198	+0.0712	Bal
ΔH acc. ΔM	-0.0430	+0.0128	+0.0053	+0.0053	-0.0112	+0.0053	ΔH acc. ΔM
ΔH acc. C.O.		-0.0035		+0.0188		+0.0188	ΔH acc. C.O.
Bal	+0.0223	+0.0819	-0.0819	+0.0712	+0.0030	+0.0712	C.O.
C.O.		-0.0109		+0.0109		+0.0109	Bal
ΔH acc. ΔM	-0.0164	+0.0049	+0.0018	+0.0018	-0.0062	+0.0018	ΔH acc. ΔM
ΔH acc. C.O.		-0.0018		+0.0049		+0.0049	ΔH acc. C.O.
Bal	+0.0055	+0.0189		-0.0189	+0.0017	+0.0189	C.O.
				+0.0060		+0.0060	Bal
FINAL T	+0.2847	-0.2847		+0.2907	+0.0190	-0.3098	FINAL T

$$\Delta H_m = 0.1733 \, \Delta M$$

$$\Delta H_h = 1.0671 \, \Delta H$$

3. Solution by Moment and Thrust Distribution with the Use of the Maugh Equivalent Column.

The procedure for this solution is identical with the previous one. The constants that are needed are the same as those listed in part 3 of Case I, and the solution is shown on Plate XIII.

4. Solution by the Maugh Method.

The constants required for this solution are the same as those shown in part 4 of Case I. The solution is shown on Plate XIV.

5. Comparison of Results.

Plate XV shows the results of the solution obtained for Case I by the various methods used.

PLATE XIII

CASE II - SOLUTION BY MOMENT AND THRUST DISTRIBUTION (MAUCH COLUMN)

MOMENT

	S = 2.3734 COL.	S = 0.8534 TRUSS	S = 2.3734 COL.	S = 0.8534 TRUSS	OPERATION
	0.7355	0.2645	-0.3843	0.20915	0.5817	0.20915
		-0.0761		-0.0761	-2.7340	-0.0761	FEM
	-6.1190	+2.9008		+1.2961		+1.2961	ΔM acc ΔH
		-1.2961		-2.9008		+1.2961	ΔM acc C.O.
	+3.3762	+1.2142		-0.4666	+1.2873		C.O.
		-0.1779		+0.4629		+0.4629	Bal
	-2.3169	+1.0983		-0.0565		-0.0565	ΔM acc. ΔH
		+0.0565		-1.0983	+0.1188	-0.0565	ΔM acc.C.O.
	+0.9856	+0.3544		-0.1362		+0.1779	C.O.
		-0.0890		+0.2316	+0.6441	+0.2316	Bal
	-0.0051	+0.0023		-0.0054	+0.0126	-0.0054	ΔM acc. ΔH
		+0.0054		-0.0023		-0.0054	ΔM acc.C.O.
	+0.0635	+0.0229		-0.0088		+0.0890	C.b.
		+0.0060		-0.0155	-0.0433	-0.0155	Bal
	+0.0253	-0.0116		+0.0116	-0.0177	+0.0131	ΔM acc. ΔH
		-0.0131		+0.0116		+0.0131	ΔM acc.C.O.
		-0.0017		+0.0007		-0.0060	C.O.
	-0.0049			-0.0008	-0.0165	-0.0058	Bal.
	-3.9953	+3.9953		-2.6041	-0.7485	+3.3526	FINAL M

THRUST

OPERATION	S = 0.0149 COL.	S = 0.0149 TRUSS	S = 0.0626	S = 0.0626 TRUSS	S = 0.0149 COL.	S = 0.0149 TRUSS	OPERATION
	0.1985	0.8077	-1.0	0.4468	0.1064	0.4468	
FET		-2.5176		+2.5176		-2.5176	FET
Bal	+0.4841	+2.0355		-2.0355			C.O.
C.O.		-0.9086		+0.9086	+0.2163	+0.9086	Bal
ΔH acc. ΔM	-0.2671	+0.3596		+0.1571	-0.1018	+0.1371 ΔH acc.ΔM
ΔH acc. C.O.		-0.1371		-0.3596		+0.1371 ΔH acc.C.O.
Bal	+0.1833	+0.7699		-0.7699		+0.9086	C.O.
C.O.		+0.0396		-0.0396	-0.0094	-0.0396	Bal
ΔH acc. ΔM	-0.0780	+0.1050		+0.0686	-0.0510	+0.0686 ΔH acc.ΔM
ΔH acc. C.O.		-0.0686		-0.1050		+0.0686 ΔH acc.C.O.
Bal	+0.0004	+0.0016		-0.0016		-0.0396	C.O.
C.O.		-0.0038		-0.0038	-0.0010	-0.0038	Bal
ΔH acc. ΔM	-0.0050	+0.0068		+0.0046	+0.0034	+0.0046 ΔH acc.ΔM
ΔH acc. C.O.		+0.0046		-0.0068		-0.0046 ΔH acc.C.O.
Bal	-0.0020	-0.0082		+0.0082		-0.0038	C.O.
C.O.		-0.0057		+0.0057	+0.0014	+0.0057	Bal.
ΔH acc. ΔM	+0.0004	-0.0006		-0.0017	+0.0013	-0.0017 ΔH acc.ΔM
ΔH acc. C.O.		+0.0017		+0.0006		-0.0017 ΔH acc.C.O.
Bal	+0.0008	+0.0034		-0.0034		+0.0057	C.O.
				+0.0004	+0.0001	+0.0004	Bal
FINAL T	+0.3169	-0.3169		+0.3175	+0.0595	-0.3765	FINAL T

PLATE XIV

CASE II – SOLUTION BY THE MAUGH METHOD (THRUST EFFECTS NEGLECTED)

COL.	TRUSS		TRUSS	COL.	TRUSS	℄ of Truss
0.7355	0.2645	−0.3843	0.20915	0.5817	0.20915	
−0.07611			+0.0761		−0.0761	
+0.0201			−0.0077			
			+0.0016	+0.0045	+0.0016	
+0.0560	−0.0560		+0.0700	+0.0045	−0.0745	

$$\frac{M}{h} = \frac{0.0560}{12.64} = 0.0044 \qquad \frac{M}{h} = \frac{0.0045}{12.64} = 0.0004$$

+0.0.560

PLATE XV

CASE II – COMPARISON OF RESULTS

	H_a	$H_b = H_c$ (mean)	$V_b = V_c$ (mean)
Least Work	0.3281	0.0603	3.9145
M & T Dist. (Spagnuolo Col.)	0.2846	0.0190	4.0057
M & T Dist. (Maugh Col.)	0.3169	0.0593	3.8930
Maugh Method	0.0044	0.0004	4.0079

−44−

IV. CONCLUSIONS

From the preceding investigations, the following conclusions may be drawn:

1. That moment and thrust distribution can be applied to Fink type trussed bents.

2. That the method is practical and can be carried out with relative ease as compared to the classical methods. This conclusion is brought out strongly in cases where a high degree of indeterminacy exists.

3. That the results obtained check very closely with those obtained by the method of least work. This is particularly true for the solutions based on Maugh's equivalent column. The results obtained from the solutions based on the Spagnuolo column, although not as accurate as those based on the equivalent column, still fall within the accuracy desired by structural engineers.

4. That the necessary constants required by the moment and thrust distribution method may be compiled in chart form in terms of various parameters such as the span length, the ratio of the area of web members to the area of chord members, etc. This will be a definite advantage as far as the time element involved in the solution is concerned. Since the standardization of truss construction is more than likely in the not too distant future, the charting of the constants would be a perfectly feasible undertaking.

It is here that the method based on the Spagnuolo column would be a great advantage, since all the constants required are obtainable in general terms and are dependent upon fewer parameters, when compared to the same method based on the equivalent column. It is also true however,

as seen from the cases investigated, that a lesser degree of accuracy is obtainable with this method.

5. That the effect of thrust cannot be neglected when analyzing Fink type trussed bents.

V. SUMMARY

Before the method of moment and thrust distribution c an be applied to trussed bents, certain constants must be determined. These constants can be obtained without too much difficulty as has been shown.

Once these bent constants are known, the moment and thrust distribution may then be performed about an equivalent joint. For Fink type trussed bents the location of this equivalent joint is dependent upon either of two assumptions which make use of the Spagnuolo column and the Maugh equivalent column.

The actual process of the distribution of the thrusts and moments follows the pattern developed by Professor Cross. Care must be taken however in carrying out the procedure to be certain of including all the effects that accompany the balancing process.

VI. <u>ACKNOWLEDGEMENT</u>

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